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A NEW METHOD OF MEASURING POWER SYSTEM CAPACITY

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16. ABSTRACT A new method of measuring the steady-state impedance and therefore the steady-state strength of an electric power system is developed. This method, called the "Two Load Method," can be used to measure the impedance of a system without the necessity of measuring the voltage-current phase angle or short-circuit current. Only a single high-accuracy voltmeter and two resistive loads are required.			
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**A NEW METHOD OF
MEASURING POWER
SYSTEM CAPACITY**

by
**M. L. Jacobs
H. Vu**

Division of Research and Laboratory Services
Engineering and Research Center
Denver, Colorado

December 1984



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INTRODUCTION

The procedure developed in this report is limited to steady-state conditions. Power system strength under transient or subtransient conditions is not considered. If the measuring point is far away from power sources, the steady-state, transient, and subtransient system strengths are approximately equal.

The strength of a power system is defined as the VA (volt-ampere) output when the system is short circuited. Ideally, the system strength should be infinite, but actually it is limited by the internal impedance of the system. A strong system has a very low internal impedance, while a weak system's impedance is relatively high.

The most accurate method of measuring the strength of a power system is to measure the open circuit voltage and the short-circuit current. The product of open circuit voltage and short-circuit current is the system strength in VA. Measuring the short-circuit current results in temporary power outages and is therefore not always feasible.

The most common method of measuring system strength is to load the system to between one-half and two PU (per unit) current and measure the voltage across the load. This voltage, along with the open circuit voltage and an assumption that the system impedance can be adequately modeled by a single element (usually an inductor), is sufficient to calculate the system strength.

Many systems are known to have sufficiently high X/R (reactance/resistance) ratios that the simple inductor model of system impedance is quite adequate. However, some systems exist with ratios low enough that modeling the system impedance as a simple inductor results in very large errors in the calculated system strength. See Appendix A for a discussion of this problem. For these systems, the system impedance must be modeled as a resistance in series with a reactance if an accurate system strength calculation is to be made. Since this model has two unknown elements, two independent measurements must be made to allow calculation of the system strength. An obvious choice for the second measurement is the phase angle between voltage and current when the system is loaded and the voltage drop measurement is made. Modeling the system as in figure 1, equations (1) and (2), can be written:

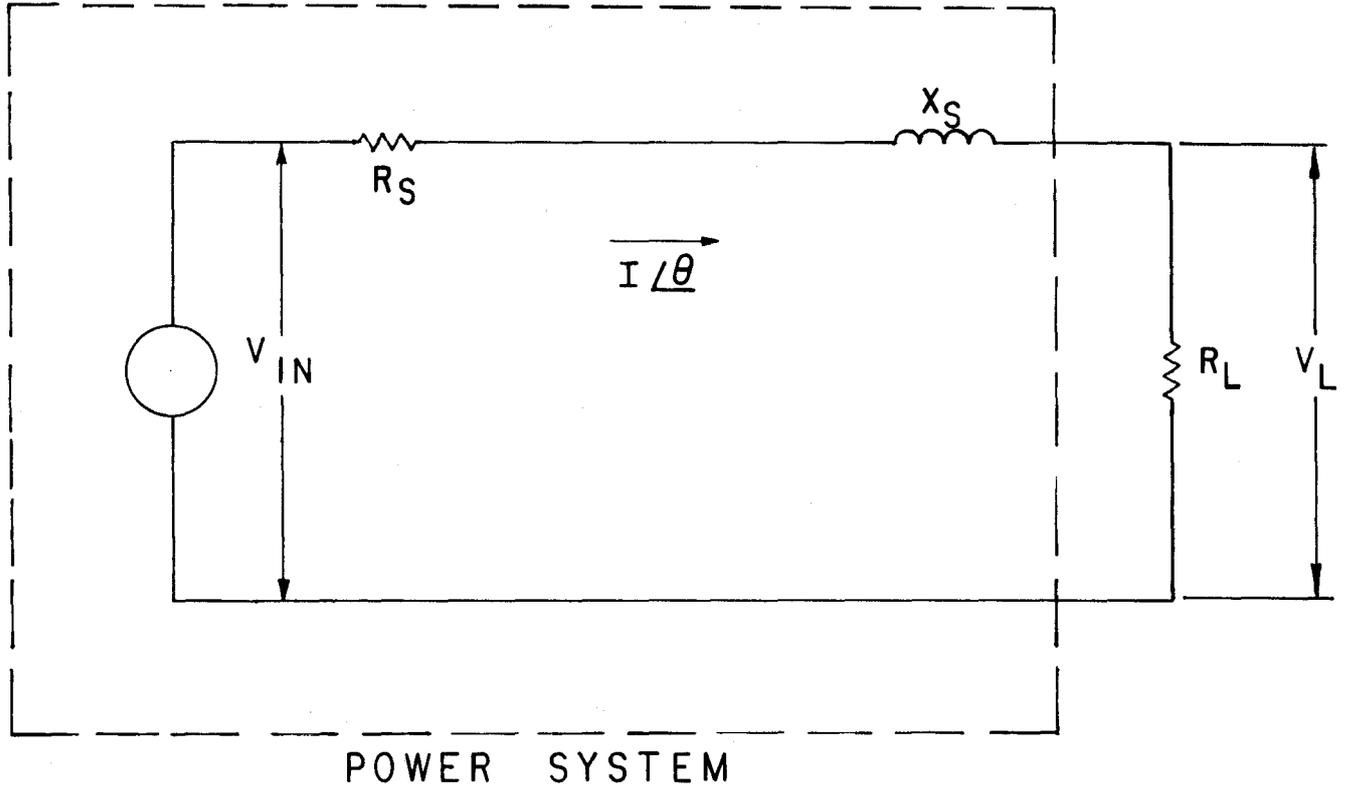


Figure 1. – Low X/R ratio model.

$$I = \frac{V_L}{R_L} = \frac{V_{IN}}{\sqrt{(R_S + R_L)^2 + X_S^2}} \quad (1)$$

$$\frac{X_S}{R_S + R_L} = \tan \theta \quad (2)$$

Equations (1) and (2) can be solved simultaneously to yield:

$$X_S = \left[\frac{V_{IN}}{V_L} \sin \theta \right] R_L \quad (3)$$

$$R_S = \left[\frac{V_{IN}}{V_L} \cos \theta - 1 \right] R_L \quad (4)$$

If the current is used as the phase angle reference, the sign of X_S will be positive for inductive reactance, and negative for capacitive reactance. It can be shown that the bracketed term in equation (4) is always ≥ 0 when $-90^\circ \leq \theta \leq +90^\circ$.

Accurate measurement of the phase angle requires an accurate, high resolution phase angle meter, or a high quality dual-channel oscilloscope, both of which are expensive and may not be readily available.

An alternate procedure is to make another voltage drop measurement using a different load resistance. This procedure has the disadvantage of requiring two loads and tests, but has the advantage of requiring only a single voltmeter. High accuracy digital voltmeters are readily available and are quite inexpensive, especially when compared to high quality phase angle meters and oscilloscopes. This procedure can be considered a generalization of the method now commonly used to measure the strength of power systems.

This report describes the theoretical development of the "TWO LOAD METHOD" and its application to several different systems.

CONCLUSIONS

1. The Two Load Method is an easy way to measure a power system's impedance and therefore strength without the necessity of measuring voltage-current phase angles or short-circuit currents. Only a high accuracy voltmeter, two resistive loads, and two switches are required.
2. When using the Two Load Method, the system's rated current should be known approximately. The two values of R_L should be chosen such that the current in figure 1 varies from a minimum of about one-half to a maximum of about twice the rated current. If the rated current is not known, then trial and error should be used to find the two values of R_L , the use of which should give voltage drops of from about 5 to 10 percent at the load.
3. The Two Load Method has a very high sensitivity to load voltage measurement error. Therefore, care must be taken when collecting data. Inaccurate measurements can result in the calculation of imaginary values of L_s , because of the square root function in equation (11). Five to ten data measurements should be taken and the average values used in the system impedance calculations to minimize or eliminate this potential source of error.
4. The Two Load Method provides a way of checking the validity of the assumption (often made in power system analysis) that the system impedance consists primarily of reactance. If this assumption is made, but is *not* valid and the system strength is calculated in the conventional manner, a large error can result in the calculated system strength. See Appendix A for an example of this.

5. If the power system under consideration includes variable autotransformers, then the effective system resistance is nonlinear because of the V-I (voltage-current) characteristics of the transformer brushes. These nonlinear characteristics can be eliminated from the calculations by modeling the brush characteristics as constant voltage sources with linear internal resistances. An additional error will be introduced by this modeling process. Refer to Appendix B for a detailed discussion of this problem.

THEORETICAL DEVELOPMENT

In reality, a power system is neither linear nor time invariant. Generator voltages are periodically adjusted, and loads are continuously changing. From a practical standpoint, however, the power system can be considered linear (fig. 1). All voltages and currents are RMS scalar quantities. If the system is linear and time invariant, the following equation may be written:

$$V_L = \frac{R_L}{\sqrt{(R_S + R_L)^2 + X_S^2}} V_{IN} \quad (5)$$

Note that $V_L \xrightarrow{\cong} V_{IN}$ when $R_L \rightarrow \infty$

By taking two measurements with different loads, R_{L1} and R_{L2} , equations (6) and (7) can be developed:

$$V_{L1} = \frac{R_{L1}}{\sqrt{(R_S + R_{L1})^2 + X_S^2}} V_{IN1} \quad (6)$$

$$V_{L2} = \frac{R_{L2}}{\sqrt{(R_S + R_{L2})^2 + X_S^2}} V_{IN2} \quad (7)$$

where the subscripts 1 and 2 designate the first and second measurements, respectively. If the system is truly time invariant, $V_{IN1} = V_{IN2}$. In practice, V_{IN1} and V_{IN2} will vary slightly between measurements as the generators are adjusted, taps are changed on transformers, compensation is added or removed from the system, or customers loads change.

By rearranging and squaring equations (6) and (7), equations (8) and (9), respectively, can be developed:

$$(R_S + R_{L1})^2 + X_S^2 = R_{L1}^2 \left(\frac{V_{IN1}}{V_{L1}} \right)^2 \quad (8)$$

$$(R_s + R_{L2})^2 + X_s^2 + R_{L2}^2 \left(\frac{V_{IN2}}{V_{L2}} \right)^2 \quad (9)$$

By subtracting equation (8) from (9):

$$(R_s + R_{L2})^2 - (R_s + R_{L1})^2 = R_{L2}^2 \left(\frac{V_{IN2}}{V_{L2}} \right)^2 - R_{L1}^2 \left(\frac{V_{IN1}}{V_{L1}} \right)^2$$

Solving for R_s :

$$R_s^2 + 2R_s R_{L2} + R_{L2}^2 - R_s^2 - 2R_s R_{L1} - R_{L1}^2 = R_{L2}^2 \left(\frac{V_{IN2}}{V_{L2}} \right)^2 - R_{L1}^2 \left(\frac{V_{IN1}}{V_{L1}} \right)^2$$

$$2R_s(R_{L2} - R_{L1}) = R_{L2}^2 \left(\frac{V_{IN2}}{V_{L2}} \right)^2 - R_{L2}^2 - R_{L1}^2 \left(\frac{V_{IN1}}{V_{L1}} \right)^2 + R_{L1}^2$$

$$R_s = \frac{R_{L2}^2 \left[\left(\frac{V_{IN2}}{V_{L2}} \right)^2 - 1 \right] - R_{L1}^2 \left[\left(\frac{V_{IN1}}{V_{L1}} \right)^2 - 1 \right]}{2(R_{L2} - R_{L1})} \quad (10)$$

From equation (8):

$$X_s = \sqrt{R_{L1}^2 \left(\frac{V_{IN1}}{V_{L1}} \right)^2 - (R_s + R_{L1})^2} \quad (11)$$

Equations (10) and (11) give the values for the components of system impedance in terms of the two load resistances, measured load voltages, and system no-load voltages.

Notice that this procedure cannot distinguish between inductive and capacitive reactance. Usually this is no problem. Power systems are rarely, if ever, capacitive. Even when compensating capacitors are used, only part of the inductance is compensated to avoid resonance problems. However, a check is available should it be necessary. Equation (11) can be solved for either inductance or capacitance as follows:

$X_s = \omega L_s$ for inductance, from which

$$L_s = \frac{X_s}{\omega} \quad (12)$$

$X_s = \frac{1}{\omega C_s}$ for capacitance, from which

$$C_s = \frac{1}{\omega X_s} \quad (13)$$

Normally, only one of the numbers L_s or C_s will be reasonable. If L_s is reasonable, C_s will be very large; while if C_s is reasonable, L_s will be very small. Generally, no difficulty should be encountered deciding whether the system is inductive or capacitive. In the rare event that both equations (12) and (13) produce reasonable numbers, an additional test would have to be performed; for instance, loading the line with an appropriately chosen inductor to see whether the load voltage rises or dips, and by how much.

The problem of deciding whether the reactance is inductive or capacitive normally occurs so rarely that no further consideration is given to it in this report.

SYSTEM DESCRIPTIONS

System strength values were obtained on the following systems using the Two Load Method of measurement:

System A (fig. 2)

A 120-V single-phase 50-A source supplied from the low-voltage side of a substation transformer located about ¼ mile away.

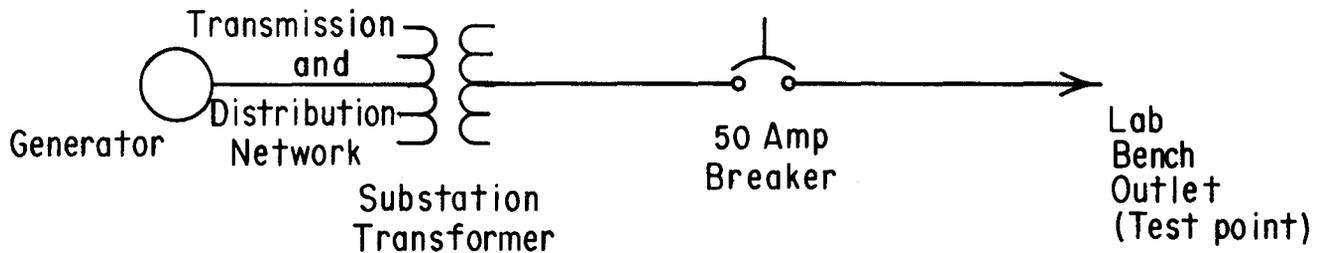


Figure 2. – System A schematic.

System B (fig. 3)

System A in series with a small capacity (0.5 kVA) dual primary-dual secondary transformer, type DU ½.

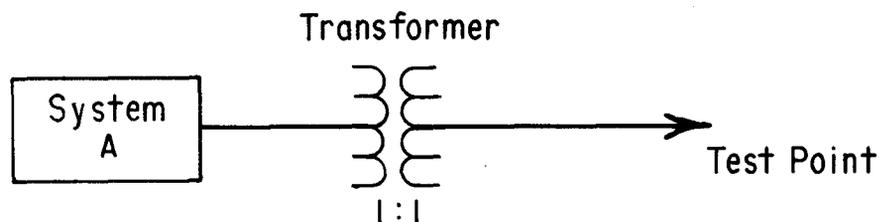


Figure 3. – System B schematic.

Subsystem B1: Only a single primary and single secondary were used.

Subsystem B2: Both primary windings were paralleled as well as both secondary windings.

System C (fig. 4)

System A in series with a powerstat variable autotransformer, type 3PN126 (120 V input, 0 - 140 V output, 1.8 KVA).

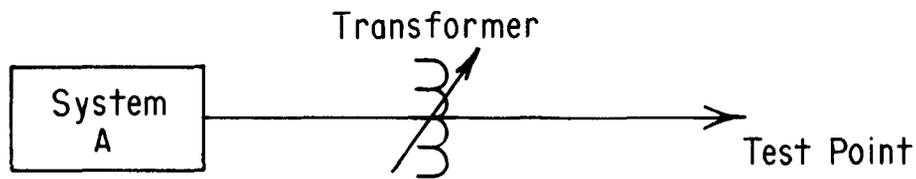


Figure 4. – System C schematic.

Subsystem C1: The autotransformer ratio equals 1:1.

Subsystem C2: The autotransformer ratio equals 1:1.184 (maximum output voltage).

System D (fig. 5)

System A in series with two variable autotransformers; the transformer of system C and a General Radio Model W20MT3 (120 V input, 0 - 140 V output, 2.5 kVA).

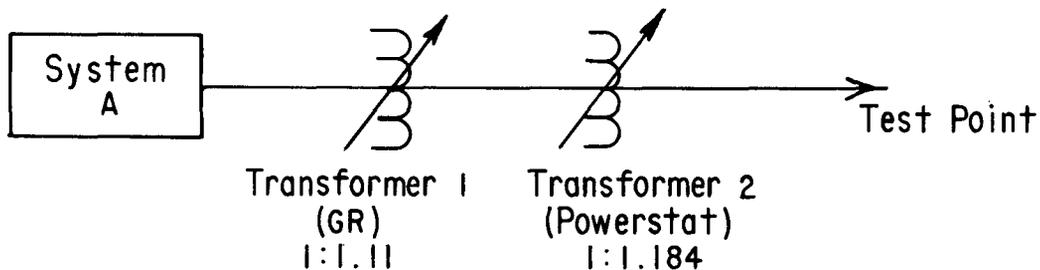


Figure 5. – System D schematic.

These transformer ratios were chosen to provide 155 volts at the test point. This voltage was required for an unrelated project for which a knowledge of the system strength was required.

RESULTS

The system strength for the circuit shown in figure 6 is the open circuit voltage, V_{oc} , times the short circuit current, I_{sc} .

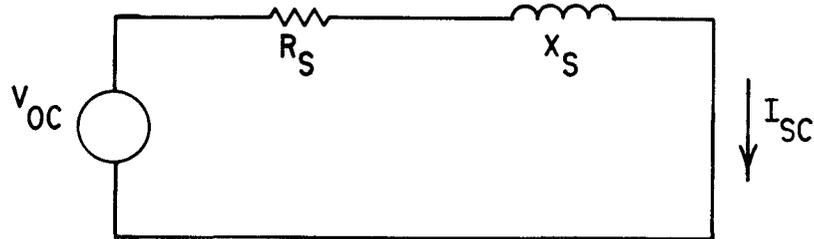


Figure 6. – Power system circuit for system strength calculations.

$$I_{sc} = \frac{V_{oc}}{\sqrt{(R_s^2 + X_s^2)}}$$

Therefore, the system strength, S , is:

$$S = \frac{(V_{oc})^2}{\sqrt{R_s^2 + X_s^2}} \quad (14)$$

where V_{oc} = measured V_{in} ,

R_s = Equivalent system resistance, and

X_s = Equivalent system reactance.

Table 1 provides the relevant measurements and calculations for the systems previously described.

Table 1. – Resistance, reactance, inductance, and strength for various systems

System	R_s (ohm)	X_s (ohm)	L_s (mH)	S (KVA)
A	0.13	0.14	0.37	75
B1	2.4	2.6	7.0	4.2
B2	1.3	1.9	5.1	6.4
C1	0.20	0.38	1.0	33
C2	0.37	0.35	0.93	37
D	0.55	0.26	0.70	39

EQUIPMENT ARRANGEMENT AND TEST PROCEDURE

The test equipment was arranged as shown schematically in figure 7.

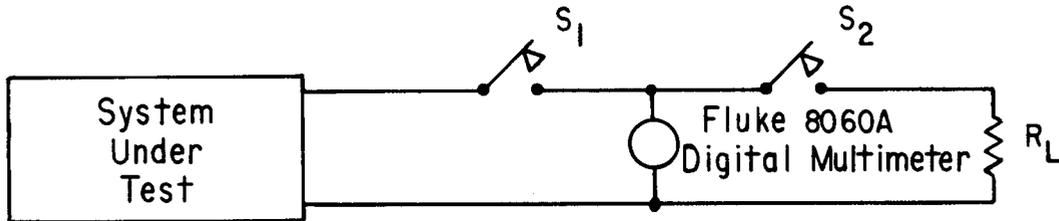


Figure 7. – Equipment arrangement.

The test procedure was then as follows:

Start with S_1 and S_2 open;

- 1) Place the DMM (digital multimeter) in "a-c volts" mode;
- 2) Close S_1 , and measure V_{IN} ;
- 3) Open S_1 ;
- 4) Place the DMM in "ohms" mode;
- 5) Close S_2 , and measure R_L ;
- 6) Place the DMM in "a-c volts" mode;
- 7) Close S_1 , and measure V_L ;
- 8) Open S_1 and S_2 ;
- 9) Repeat steps 1 through 8 five to ten times;
- 10) Change R_L ; and
- 11) Repeat steps 1 through 9 with new R_L .

ANALYSIS OF DATA

Procedures Common to all Systems

The load resistances were measured with a Fluke Model 8060A true rms DMM (Digital Multimeter). The multimeter leads had a measured resistance of 0.07 ohms, which was not in series with the source and load. The load resistance as seen from the voltage measurement point (between S_1 and S_2 on figure 3) was actually the measured resistance minus the DMM lead resistance. In the data tables that follow, the measured R_L is shown in the columns labeled R_{L1} (Raw) and R_{L2} (Raw), while the actual load resistance ($R_{L\text{measured}} - 0.07$ ohm) is shown in the columns labeled R_{L1} (Corrected) and R_{L2} (Corrected).

The input impedance of the DMM in the "a-c volts" mode is approximately 11 megohms. This value is so high compared to the load resistances used (approximately 50 ohms max) that it has no noticeable effect on the load resistance as seen from the system under test when S1 is closed. For this reason, no correction was made for system loading by the DMM.

Systems A and B

The corrected load resistances, along with measured load and input voltages, were averaged over ten runs for system A and five runs for system B. The average values were used in equations (10) and (11) to calculate the values for R_{ST} and X_S . L_S was calculated from equation (12). R_{ST} consisted of the system resistance plus the test setup wiring and S1 resistances. These were measured and found to total 0.04 ohm. Therefore, the system resistance was:

$$R_S = R_{ST} - 0.04 \quad (15)$$

This is the value recorded in the Results section.

The measurements and calculations are shown in the following tables.

Table 2. – System A data and calculations

Test No.	R_{L1} (Raw)	R_{L1} (Corrected)	V_{L1}	V_{IN1}	R_{L2} (Raw)	R_{L2} (Corrected)	V_{L2}	V_{IN2}
1	2.20	2.13	110.52	119.50	4.27	4.20	114.35	119.02
2	2.20	2.13	110.59	119.68	4.26	4.19	114.27	119.00
3	2.22	2.15	110.60	119.57	4.26	4.19	114.10	118.82
4	2.22	2.15	110.57	119.53	4.25	4.18	114.14	118.87
5	2.23	2.16	110.61	119.66	4.26	4.19	114.03	118.74
6	2.23	2.16	110.57	119.55	4.26	4.19	114.00	118.68
7	2.22	2.15	110.51	119.41	4.25	4.18	114.15	118.77
8	2.23	2.16	110.42	119.34	4.25	4.18	114.19	118.86
9	2.19	2.12	110.28	119.18	4.25	4.18	114.15	118.89
10	2.20	2.13	110.18	119.15	4.25	4.18	114.17	118.83
Average		2.144	110.458	119.457		4.186	114.155	118.848

From equations (10), (11), and (12):

$R_{ST} = 0.1698031$ ohm, $X_S = 0.1411568$ ohm, and $L_S = 0.37443$ mH; then,

$R_S = R_{ST} - 0.04$ ohm = 0.13 ohm; and, from equation (14):

$$S = \frac{(118.848)^2}{\sqrt{(0.13)^2 + (0.14)^2}} = 75 \text{ KVA}$$

Table 3. – System B1 data and calculations

Test No.	R _{L1} (Raw)	R _{L1} (Cor-rected)	V _{L1}	V _{IN1}	R _{L2} (Raw)	R _{L2} (Cor-rected)	V _{L2}	V _{IN2}
1	24.77	24.70	110.15	121.77	49.61	49.54	115.96	121.88
2	24.73	24.66	110.46	122.03	49.60	49.53	115.81	121.77
3	24.77	24.70	110.28	121.82	49.62	49.55	115.84	121.83
4	24.80	24.73	110.10	121.65	49.67	49.60	115.84	121.65
5	24.81	24.74	110.17	121.73	49.70	49.63	115.94	121.86
Average		24.706	110.232	121.80		49.57	115.878	121.798

From equations (10), (11), and (12);

R_{ST} = 2.466353 ohm, X_S = 2.62345 ohm, and L_S = 6.95892 mH; then,
R_S = R_{ST} – 0.04 ohm = 2.4 ohm; from equation (14);

$$S = \frac{(121.798)^2}{\sqrt{(2.46)^2 + (2.62)^2}} = 4.2 \text{ KVA}$$

Table 4. – System B2 data and calculations

Test No.	R _{L1} (Raw)	R _{L1} (Cor-rected)	V _{L1}	V _{IN1}	R _{L2} (Raw)	R _{L2} (Cor-rected)	V _{L2}	V _{IN2}
1	24.77	24.70	115.73	122.32	49.15	49.08	118.91	122.30
2	24.79	24.72	115.58	122.19	49.12	49.05	118.89	122.23
3	24.82	24.75	115.54	122.15	49.18	49.11	118.88	122.23
4	24.85	24.78	115.56	122.15	49.20	49.13	118.77	122.10
5	24.86	24.79	115.66	122.36	49.22	49.15	118.71	122.03
Average		24.748	115.614	122.234		49.104	118.832	122.178

From equations (10), (11), and 12;

R_{ST} = 1.34569167 ohm, X_S = 1.931197 ohm, and L_S = 5.12266 mH;

R_S = R_{ST} – 0.04 ohm = 1.3 ohm .

From equation (14):

$$S = \frac{(122.178)^2}{\sqrt{(1.35)^2 + (1.93)^2}} = 6.4 \text{ KVA}$$

Systems C and D

Systems C and D included variable autotransformers (figs. 4 and 5). The nonlinear V-I (volt-ampere) characteristics of the transformer brushes violate the assumptions upon which equations (10) and (11) are based. The initial attempt to use these equations directly resulted in the consistent calculation of imaginary reactances. It was therefore determined that systems C and D must be linearized prior to applying the two-load method.

The general linearization procedure for systems with variable autotransformers is:

- 1) Determine the a-c V-I characteristics for the brushes of all variable autotransformers in the system;
- 2) Model the V-I curves as current dependent resistances;
- 3) Reflect the measured load current back through all the variable autotransformers;
- 4) Using the V-I curves, find the brush voltage of each autotransformer;
- 5) Reflect all brush voltages to the load;
- 6) Find the total equivalent brush resistance, which is

$$R_B = \frac{V_B}{I_L}$$

where R_B = Total equivalent brush resistance,
 V_B = Total brush voltage reflected to the load, and
 I_L = Load current; and

- 7) Move the brush characteristics out of the system and to the load by adding the total equivalent brush voltage to the measured load voltage, and adding the equivalent brush resistance to the measured load resistance.

The above procedure resulted in a linear system to which equations (10) and (11) could be applied. The procedure for systems A and B above may then be followed to find R_S and X_S . It must be remembered that the value of R_S *does not include the effective brush resistance, which is a function of load current*. For system strength calculations, an iterative procedure must be used, continuing the general linearization procedure begun above:

- 8) Calculate the short-circuit current from R_s and X_s ;
- 9) Find the equivalent brush voltage based upon this current;
- 10) Subtract this voltage from the source voltage;
- 11) Using this new source voltage, repeat steps 8 through 10 until the short-circuit current has been calculated to sufficient accuracy; and
- 12) Multiply the source voltage by the short-circuit current to obtain the system strength.

This linearization procedure, while theoretically very accurate, is quite cumbersome to use. It requires an estimate of the system short circuit current so that the a-c V-I characteristics of the transformers may be experimentally determined up to a large enough current to be useful. This V-I determination at high a-c current levels is itself non-trivial and time consuming. The following procedure, while not as accurate, is much simpler and proved sufficient in this investigation:

- 1) Using a d-c source, determine the V-I characteristic of the brushes beyond the main knees of the curves;
- 2) Use the curves obtained from step 1) to approximate the brushes as constant voltages (V_{BN} where N is the transformer number) in series with fixed resistances;
- 3) Reflect the constant voltages to the load. The fixed resistances are linear and become part of the system for analysis purposes;
- 4) Determine the load current by dividing the measured load voltage by the measured load resistance;
- 5) Determine the equivalent brush resistance by dividing the brush voltage by the calculated load current;
- 6) Add the equivalent brush resistance to the measured load resistance to form an equivalent load resistance, R_{LEQ} ;
- 7) Add the brush voltage to the measured load voltage to form an equivalent load voltage, V_{LEQ} . The brush characteristics have been removed from the system, which is now linearized;

8) Using the equivalent parameters, follow the procedure of systems A and B to determine the system R_s and X_s *exclusive of the brush characteristics*;

9) Subtract the brush voltage from the source voltage to obtain the effective open circuit voltage, V_{OCEFF} . This compensates for the brush characteristics in the system strength calculations; and

10) Using the effective open circuit voltage, R_s and X_s , calculate the system strength using equation (14).

See Appendix B for the measurement of the brush V-I characteristics and the determination of the brush V_B 's.

Tables 5, 6, 7, 8, 9, and 10 pertain to the system strength measurements involving one variable auto transformer.

Table 5. – System C1 measured data.

Test No.	R_{L1} (Raw)	R_{L1} (Corrected)	V_{L1}	V_{IN1}	R_{L2} (Raw)	R_{L2} (Corrected)	V_{L2}	V_{IN2}
1	2.19	2.12	104.58	120.02	4.23	4.16	111.34	119.90
2	2.17	2.10	104.58	120.08	4.25	4.18	111.24	120.00
3	2.17	2.10	104.55	120.11	4.26	4.19	111.19	119.90
4	2.16	2.09	104.36	119.96	4.26	4.19	111.16	119.95
5	2.16	2.09	104.36	119.95	4.27	4.20	111.19	120.04

Sample calculations:

With $V_B = 1.7$ V, find R_{BV1} and R_{BV2} :

Example: Test No. 1

$$I_1 = \frac{104.58 \text{ V}}{2.12 \text{ ohms}} = 49.33 \text{ A}$$

$$I_2 = \frac{111.34 \text{ V}}{4.16 \text{ ohms}} = 26.76 \text{ A}$$

$$R_{BV1} = \frac{1.7 \text{ V}}{49.33 \text{ A}} = 0.0345 \text{ ohm}$$

$$R_{BV2} = \frac{1.7 \text{ V}}{26.76 \text{ A}} = 0.0635 \text{ ohm}$$

Determine R_{BV1} and R_{BV2} similarly for every test number. Then, correct the data to linearize the system as follows:

$$R_{LEQ1} = R_{L1} + R_{BV1};$$

$$R_{LEQ2} = R_{L2} + R_{BV2};$$

$$V_{LEQ1} = V_{L1} + 1.7 \text{ V}; \text{ and}$$

$$V_{LEQ2} = V_{L2} + 1.7 \text{ V}.$$

Table 6. – System C1 (linearized)

Test No.	R_{LEQ1}	V_{LEQ1}	V_{IN1}	R_{LEQ2}	V_{LEQ2}	V_{IN2}
1	2.154	106.28	120.02	4.223	113.04	119.90
2	2.134	106.26	120.08	4.243	112.94	120.00
3	2.134	106.25	120.11	4.253	112.89	119.94
4	2.124	106.06	119.96	4.253	112.86	119.95
5	2.124	106.06	119.95	4.263	112.89	120.04
Average	2.134	106.182	120.024	4.247	112.924	119.966

Based on the average data in table 5, and equations (10) and (11), R_{ST} , R_{SW1} , and X_S are:

$$R_{ST} = 0.249652 \text{ ohm};$$

$$R_{SW1} = 0.04 \text{ ohm}; \text{ and}$$

$$X_S = 0.36995 \text{ ohm}.$$

Then,

$$R_S = R_{ST} - 0.04 \text{ ohm} = 0.21 \text{ ohm}, \text{ and from equation (12), } L_S = 980 \mu\text{H},$$

and

$$S = \frac{(120.024 - 1.7)^2}{\sqrt{(0.21)^2 + (0.37)^2}} = 33 \text{ KVA}$$

Table 7. – System C2 measured data

Test No.	R_{L1} (Raw)	R_{L1} (Correc- ted)	V_{L1}	V_{IN1}	R_{L2} (Raw)	R_{L2} (Correc- ted)	V_{L2}	V_{IN2}
1	2.23	2.16	114.56	139.89	4.32	4.25	125.20	139.75
2	2.23	2.16	114.52	139.83	4.31	4.24	125.22	139.72
3	2.23	2.16	114.42	139.69	4.31	4.24	125.27	139.81
4	2.23	2.16	114.51	139.77	4.31	4.24	125.61	140.19
5	2.24	2.17	114.35	139.68	4.31	4.24	125.64	140.31

Following the procedure used for system C1 results in the data shown in table 8.

Table 8. – System C2 (linearized)

Test No.	R_{LEQ1}	V_{LEQ1}	V_{IN1}	R_{LEQ2}	V_{LEQ2}	V_{IN2}
1	2.19205	116.26	139.89	4.30770	126.90	139.75
2	2.19206	116.22	139.83	4.29754	126.92	139.72
3	2.19201	116.12	139.69	4.29738	126.97	139.81
4	2.19206	116.21	139.77	4.29737	127.31	140.19
5	2.20226	116.05	139.68	4.29951	127.34	140.31
Average:	2.19409	116.172	139.772	4.29990	127.088	139.956

From equations (10), (11), and (12):

$$R_{ST} = 0.4224317 \text{ ohm};$$

$$R_{SW1} = 0.04 \text{ ohm};$$

$$X_S = 0.34990 \text{ ohm};$$

$$L_S = 930 \text{ } \mu\text{H};$$

$$R_S = R_{ST} - 0.04 = 0.38 \text{ ohm}; \text{ and}$$

$$S = \frac{(139.956 - 1.7)^2}{\sqrt{(0.37)^2 + (0.35)^2}} = 37 \text{ KVA}$$

System D

System D consists of two variable autotransformers in series (fig. 5). The brush voltage of transformer 1 must be reflected to the load and added to the brush voltage of transformer 2. The data calculations and procedure follow.

Table 9. – System D measured data

Test No.	R_{L1} Raw	R_{L1} Cor-rected	V_{L1}	V_{IN1}	R_{L2} Raw	R_{L2} Cor-rected	V_{L2}	V_{IN2}
1	2.21	2.14	119.76	155.24	4.26	4.19	134.29	155.64
2	2.20	2.13	119.45	155.60	4.26	4.19	134.50	155.81
3	2.18	2.11	119.31	155.65	4.26	4.19	134.54	155.75
4	2.20	2.13	119.65	155.74	4.26	4.19	134.51	155.76
5	2.19	2.12	119.63	155.66	4.26	4.19	134.53	155.72

Sample Calculations:

$$V_{B1} = 1.7 \text{ V and } V_{B2} = 0.3 \text{ V}$$

$$V'_{B2} = 0.3 \left(\frac{1.184}{1} \right) = 0.3552 \text{ V}$$

$$V_{BEQ} = V_{B1} + V'_{B2} = 1.7 + 0.3552 = 2.0552 \text{ V}$$

Following the linearization procedure using V_{BEQ} at the load gives the data shown in table 10.

Table 10. – System D (linearized)

Test No.	R_{LEQ1}	V_{LEQ1}	V_{IN1}	R_{LEQ2}	V_{LEQ2}	V_{IN2}
1	2.17672	121.8152	155.24	4.2541	136.3452	155.64
2	2.16664	121.5052	155.60	4.2540	136.5552	155.81
3	2.14635	121.3652	155.65	4.2540	136.5952	155.75
4	2.16659	121.7052	155.74	4.2540	136.5652	155.76
5	2.15642	121.6852	155.66	4.2540	136.5852	155.72
Average	2.16254	121.6152	155.578	4.2540	136.5292	155.736

From equations (10), (11), and (12):

$$R_{ST} = 0.59122 \text{ ohm;}$$

$$R_{SW1} = 0.04 \text{ ohm;}$$

$$X_S = 0.26480 \text{ ohm;}$$

$$L_S = 700 \text{ } \mu\text{H;}$$

$$R_S = R_{ST} - 0.04 \text{ ohm} = 0.55 \text{ ohm; and}$$

$$S = \frac{(155.736 - 2.0552)^2}{\sqrt{(0.55)^2 + (0.26)^2}} = 39 \text{ KVA.}$$

APPENDICES

APPENDIX A

This appendix gives comparison of system strength calculations based on different system impedance element assumptions.

The most common assumption made concerning power system impedance is that the impedance is sufficiently reactive that resistive contributions can be neglected. To test the validity of this supposition, the system strength of System A was calculated based on this assumption, and the result compared with that calculated from the "two load" method.

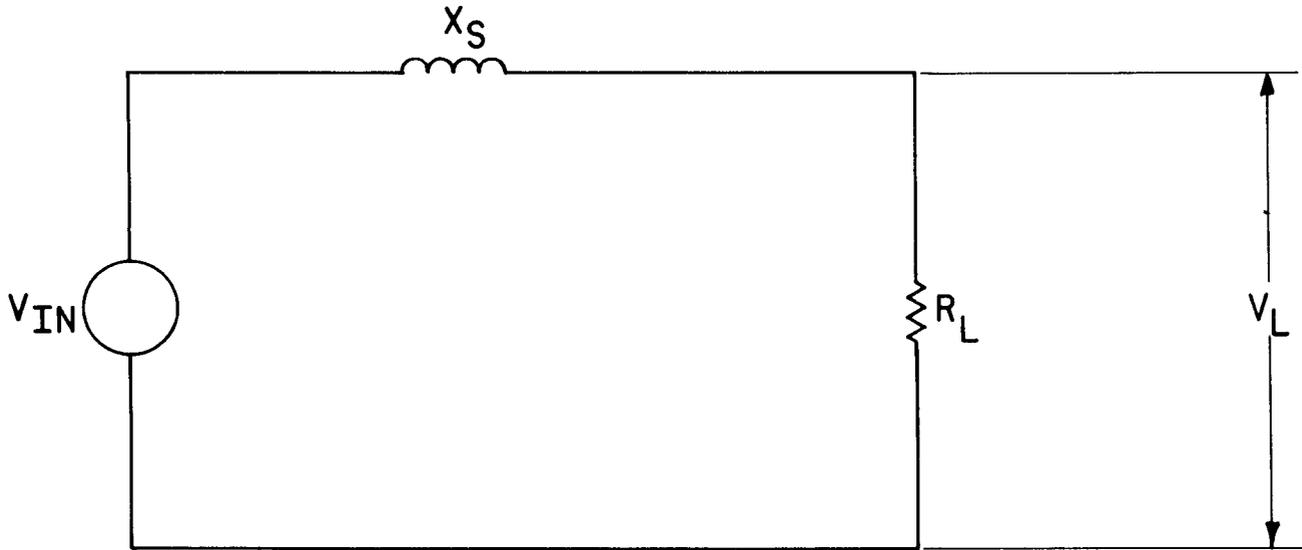


Figure A.1. – Series reactance power system model.

From figure A.1.:

$$V_L = \frac{R_L}{\sqrt{R_L^2 + X_S^2}} V_{IN} \quad (A1)$$

or

$$\left(\frac{V_L}{V_{IN}}\right)^2 = \frac{R_L^2}{R_L^2 + X_S^2}$$

From which:

$$R_L^2 \left(\frac{V_{IN}}{V_L}\right)^2 = R_L^2 + X_S^2$$

$$R_L^2 \left[\left(\frac{V_{IN}}{V_L}\right)^2 - 1 \right] = X_S^2$$

$$X_s = R_L \sqrt{\left(\frac{V_{IN}}{V_L}\right)^2 - 1} \quad (A2)$$

The short circuit current for the circuit in figure A.1 is:

$$I_{SC} = \frac{V_{IN}}{X_s} \quad (A3)$$

so the system strength, S, is:

$$S = \frac{V_{IN}^2}{X_s} \quad (A4)$$

Using the data for System A, test 1, and R_{L1}

$$\begin{aligned} \text{where } R_{L1} \text{ (corr.)} &= 2.13, \\ V_{L1} &= 110.52, \text{ and} \\ V_{IN1} &= 119.50, \end{aligned}$$

and substituting these values into equations (A2) and (A4):

$$X_s = (2.13) \sqrt{\left(\frac{119.50}{110.52}\right)^2 - 1} = 0.876 \text{ ohm}$$

$$S = \frac{(119.5)^2}{0.876} = 16.3 \text{ KVA}$$

Using the data for System A, test 1, and R_{L2}

$$\begin{aligned} \text{where } R_{L2} \text{ (corr.)} &= 4.20, \\ V_{L2} &= 114.35, \text{ and} \\ V_{IN2} &= 119.02 \end{aligned}$$

and substituting these values into equations (A2) and (A4):

$$X_s = 4.20 \sqrt{\left(\frac{119.02}{114.35}\right)^2 - 1} = 1.21 \text{ ohm}$$

$$S = \frac{(119.02)^2}{1.21} = 11.7 \text{ KVA}$$

These results are considerably less than the 75 KVA obtained from the "two load" method. Significant error occurs if the system strength is calculated based on the usual assumption that the resistive component of system impedance is negligible, when in fact it is significant.

APPENDIX B

Figure B.1 shows the circuit arrangement for determining the transformer brush V-I characteristics.

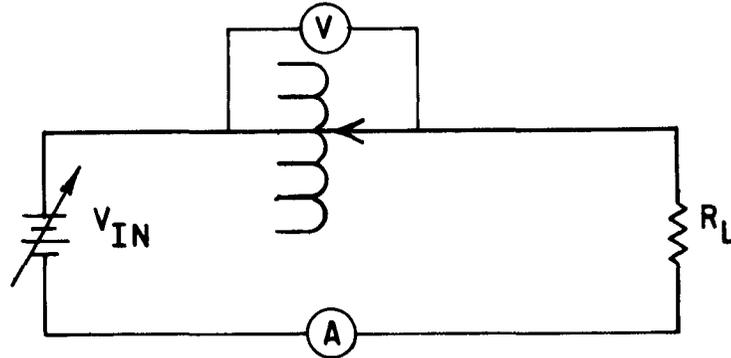


Figure B.1. – Transformer brush V-I test circuit.

Procedure: The V_{IN} was varied and the voltage and current values recorded. The transformers had fuses. The test lead and fuse resistances were measured and found to total 0.05 ohm. The brush voltage was the measured voltage minus the fuse and lead drop. Table B.1 and B.2 present the measurements and calculated brush voltages, while figures B.2 and B.3 show the V-I curves constructed from the measurements.

Table B.1. – Brush characteristics for variable autotransformer #1

Current (A)	Voltage (V)	(Fuse + Lead) Drop (V)	Voltage Corrected (V)
0.1	0.063	0.005	0.058
0.2	0.121	0.01	0.111
0.5	0.286	0.025	0.261
1.0	0.531	0.050	0.481
2.0	0.927	0.10	0.827
5.0	1.66	0.25	1.41
10.0	2.30	0.50	1.80
20.0	3.70	1.0	2.70
30.0	4.60	1.5	3.10
40.0	5.50	2.0	3.50
50.0	6.10	2.5	3.60

Table B.2. – Brush characteristics for variable autotransformer #2

Current (A)	Voltage (V)	(Fuse + Lead) Drop (V)	Voltage Corrected (V)
0.1	0.010	0.001	0.009
0.2	0.018	0.002	0.016
0.5	0.04	0.005	0.035
1.0	0.079	0.01	0.069
2.0	0.154	0.02	0.134
5.0	0.383	0.05	0.333
10.0	0.855	0.1	0.755
20.0	1.66	0.2	1.46
30.0	2.34	0.3	2.04
40.0	2.91	0.4	2.51
50.0	3.67	0.5	3.17

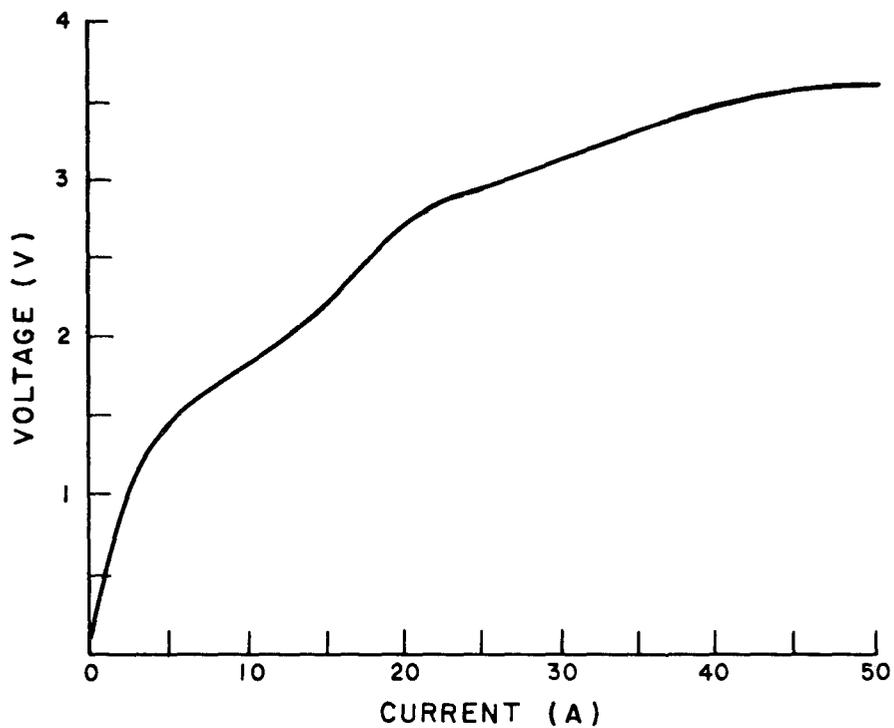


Figure B.2. – Variable autotransformer #1: V-I characteristics.

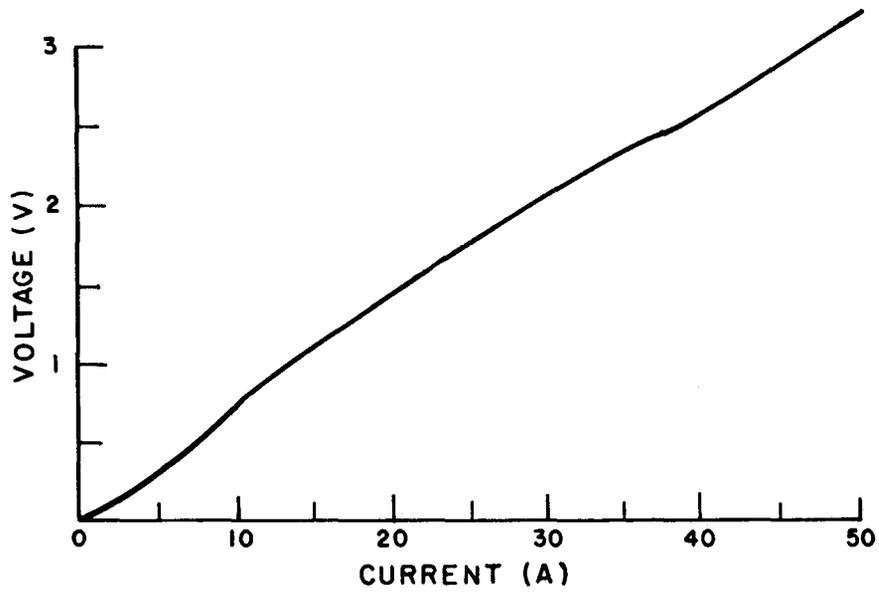


Figure B.3. – Variable autotransformer #2: V-I characteristics.

Figures B.4 and B.5 show the approximation of the brush characteristics as constant voltages V_B in series with fixed resistances.

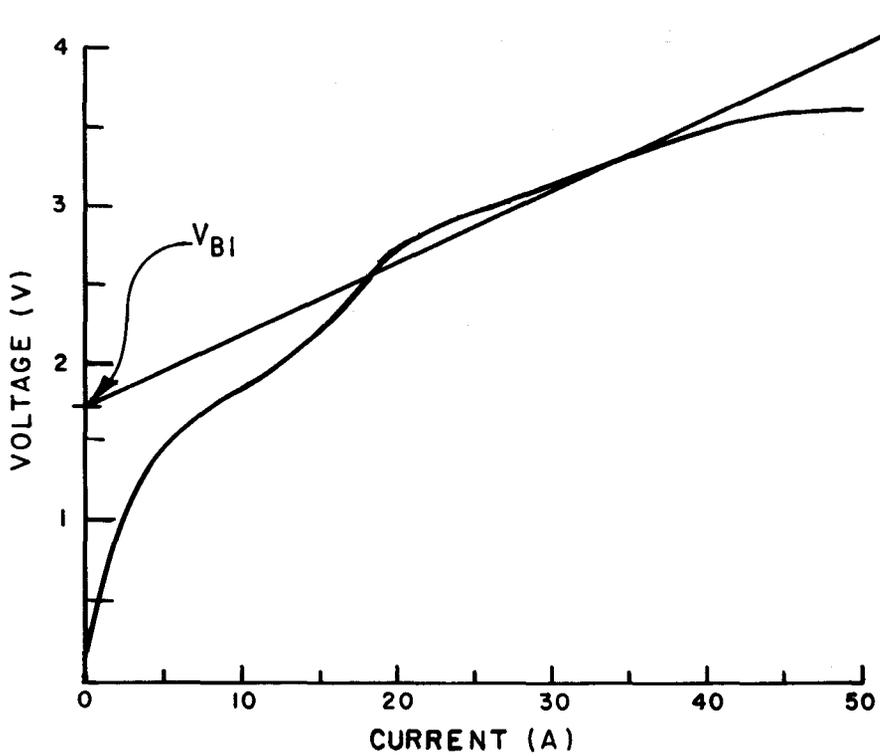


Figure B.4. – Variable autotransformer #1: brush modeling.

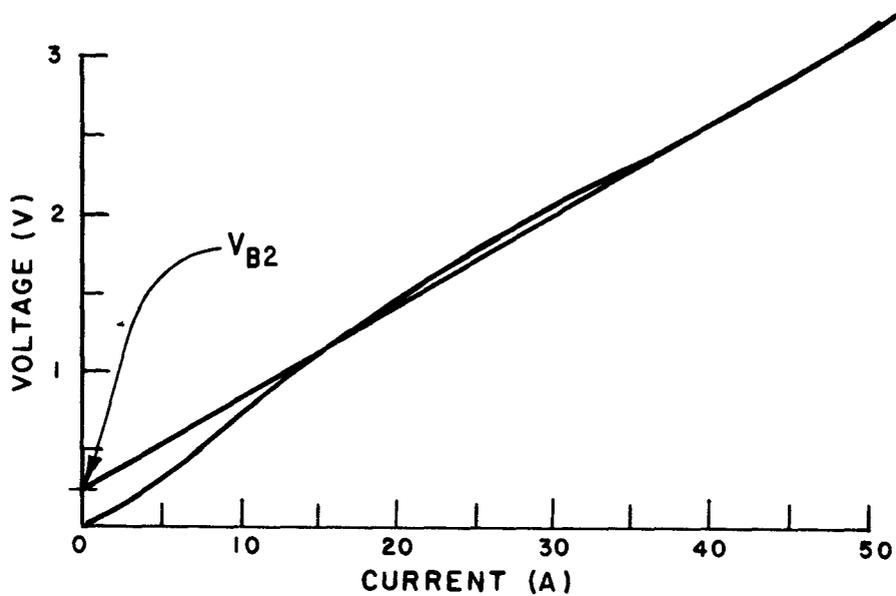


Figure B.5. – Variable autotransformer #2: brush modeling.

The results indicated that for

variable autotransformer #1, $V_B = 1.7$ V; and for

variable autotransformer #2, $V_B = 0.3$ V.

Error is introduced in the process of linearizing the wiper brush characteristics. For example, referring to figure B.4, it is not clear where the best tangent line should be drawn. Figures B.6 and B.7 show a range of brush voltages obtainable from different decisions for drawing the tangent to the curve. These results indicated that for figure B.6, $1.3 \leq V_{B1} \leq 2.0$; and for figure B.7, $0.1 \leq V_{B2} \leq 0.5$.

Using the data for system C1, test number 1, calculating the system strength twice and using the extremes of 1.3 V and 2.0 V for V_B produced the following:

$$R_{L1} = 2.12 \text{ ohms};$$

$$V_{L1} = 104.58 \text{ V};$$

$$V_{IN1} = 120.02 \text{ V};$$

$$R_{L2} = 4.16 \Omega ;$$

$$V_{L2} = 111.34 \text{ V}; \text{ and}$$

$$V_{IN2} = 119.90 \text{ V}.$$

$$\text{Then, } I_1 = \frac{104.58 \text{ V}}{2.12 \text{ ohm}} = 49.33 \text{ A}$$

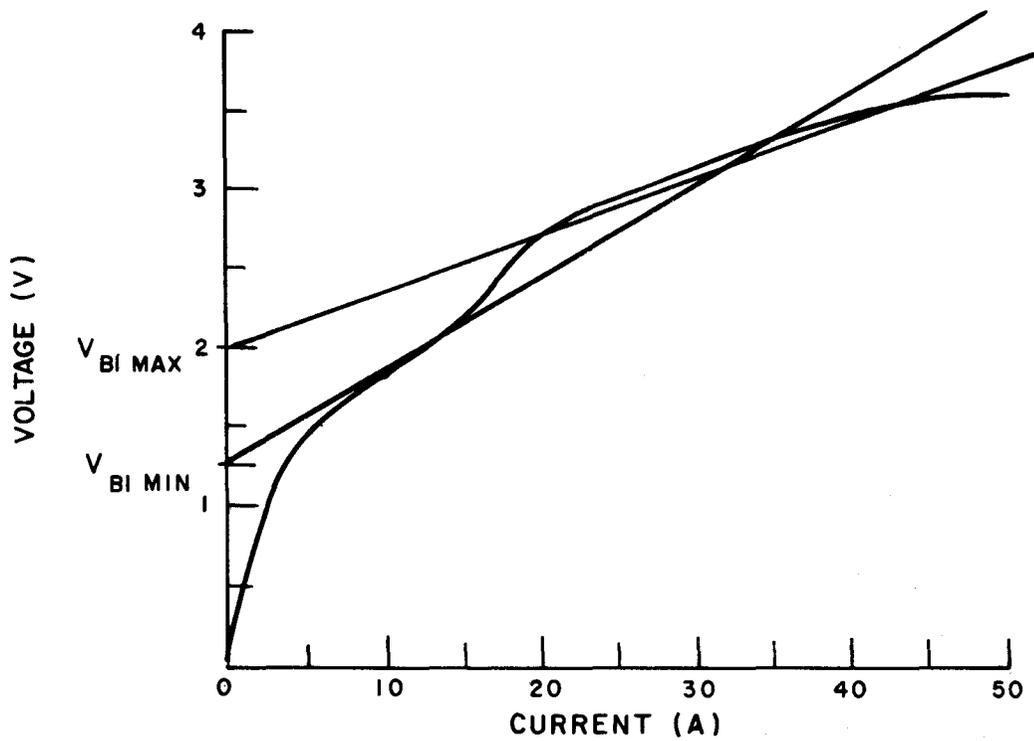


Figure B.6. – Variable autotransformer #1: modeling error.

$$i_2 = \frac{111.34 \text{ V}}{4.16 \text{ ohm}} = 26.76 \text{ A}$$

$$R_{BV1A} = \frac{1.3 \text{ V}}{49.33 \text{ A}} = 0.02635 \text{ ohm}$$

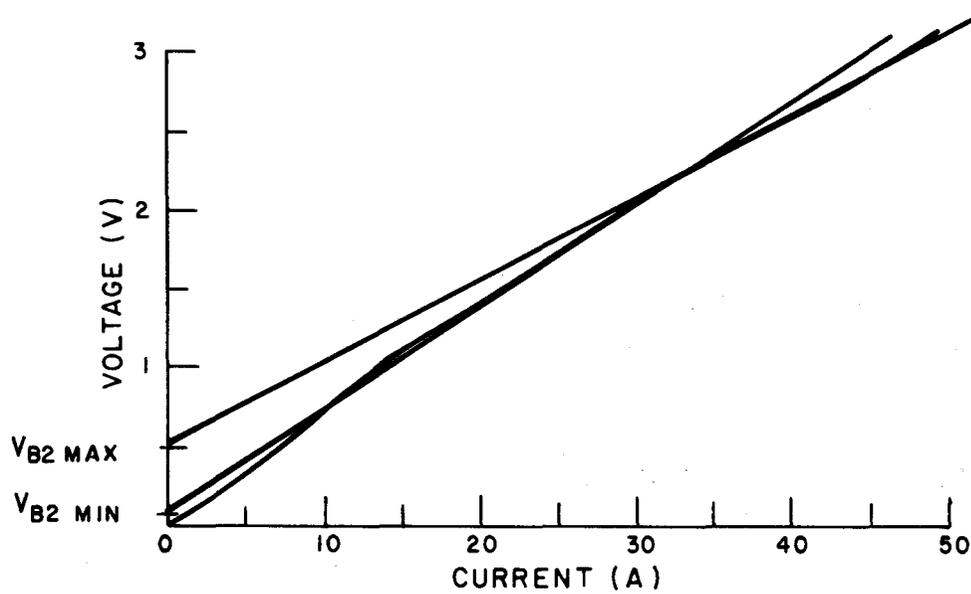


Figure B.7. – Variable autotransformer #2: modeling error.

$$R_{BV2A} = \frac{1.3 \text{ V}}{26.76 \text{ A}} = 0.04857 \text{ ohm}$$

$$R_{BV1B} = \frac{2.0 \text{ V}}{49.33 \text{ A}} = 0.0405 \text{ ohm}$$

$$R_{BV2B} = \frac{2.0 \text{ V}}{26.76 \text{ A}} = 0.074726 \text{ ohm}$$

The corrected data for $V_B = 1.3 \text{ V}$ would be:

$$R_{L1} + R_{BV1} = 2.14635 \Omega;$$

$$V_{L1} + 1.3 = 105.88\text{V};$$

$$V_{IN1} = 120.02\text{V};$$

$$R_{L2} + R_{BV2} = 4.20857 \Omega;$$

$$V_{L2} + 1.3 = 112.64\text{V};$$

$$V_{IN2} = 119.9\text{V};$$

$$R_S + R_V = 0.25316296\Omega; \text{ and}$$

$$L_S + L_V = 1.0669 \mu\text{H}$$

The corrected data for $V_B = 2.0 \text{ V}$ would be:

$$R_{L1} + R_{BV1PV} = 2.1605\Omega;$$

$$V_{L1} + 2.0 = 106.58\text{V};$$

$$V_{IN1} = 120.02\text{V};$$

$$R_{L2} + R_{BV2} = 4.234726\Omega;$$

$$V_{L2} + 2.0 = 113.34\text{V};$$

$$V_{IN2} = 119.9\text{V};$$

$$R_S + R_V = 0.21321013\Omega; \text{ and}$$

$$L_S + L_V = 1.41539\mu\text{H}$$

Finally,

$$S = \frac{(119.9 - 1.3)^2}{\sqrt{(0.25316296)^2 + (120 \times 1.0669 \times 10^{-3})^2}} = 30 \text{ KVA (for } V_B = 1.3 \text{ V)}$$

$$S = \frac{(119.90 - 2.0)^2}{\sqrt{(0.21321013)^2 + (120 \times 1.41539 \times 10^{-3})^2}} = 24 \text{ KVA (for } V_B = 2.0 \text{ V)}$$

This example shows the nature of the error introduced by approximating the brush characteristics as a constant voltage in series with a fixed resistance.

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