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HYDRAULIC AND ELECTRICAL ANALOGY TESTS OF GRAVEL ENVELOPES FOR SUBSURFACE DRAINS

*Hydraulics Branch
Division of Research
Engineering and Research Center
Bureau of Reclamation*

*cooperative study with the
Agricultural Research Service
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16. ABSTRACT <p>Present Bureau practice requires that a layer of gravel (gravel envelope) be placed around subsurface agricultural drainpipes. Hydraulic laboratory tests (phase one of a three-phase study) were made on gravel envelopes to measure discharge and head loss, to check for the presence of and the effects of turbulent flow approaching the tubing perforations, and to compare these results with electric analogy tests. The pipe used for this study was corrugated plastic tubing with perforations. Turbulent flow occurred, but with larger discharges than normal per unit length of tubing for corrugated plastic field drains. Flow function values showing inflow capacity per unit length of drain tubing were used for comparing test results. The hydraulic flow values were 25 to 30 times larger than the analog values. This difference was due to nonhomogeneity of the gravel envelope in the hydraulic tests. The gravel did not completely fill the corrugations of the tubing, and larger rocks which randomly agglomerated during placement formed less dense stratifications in the envelope. Water flowed more readily through these less dense portions of the envelope than did the current through the uniformly resistive conductor of the electrical analogy tests. Two more phases of this study are planned.</p>		
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by

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Hydraulics Branch
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Denver, Colorado
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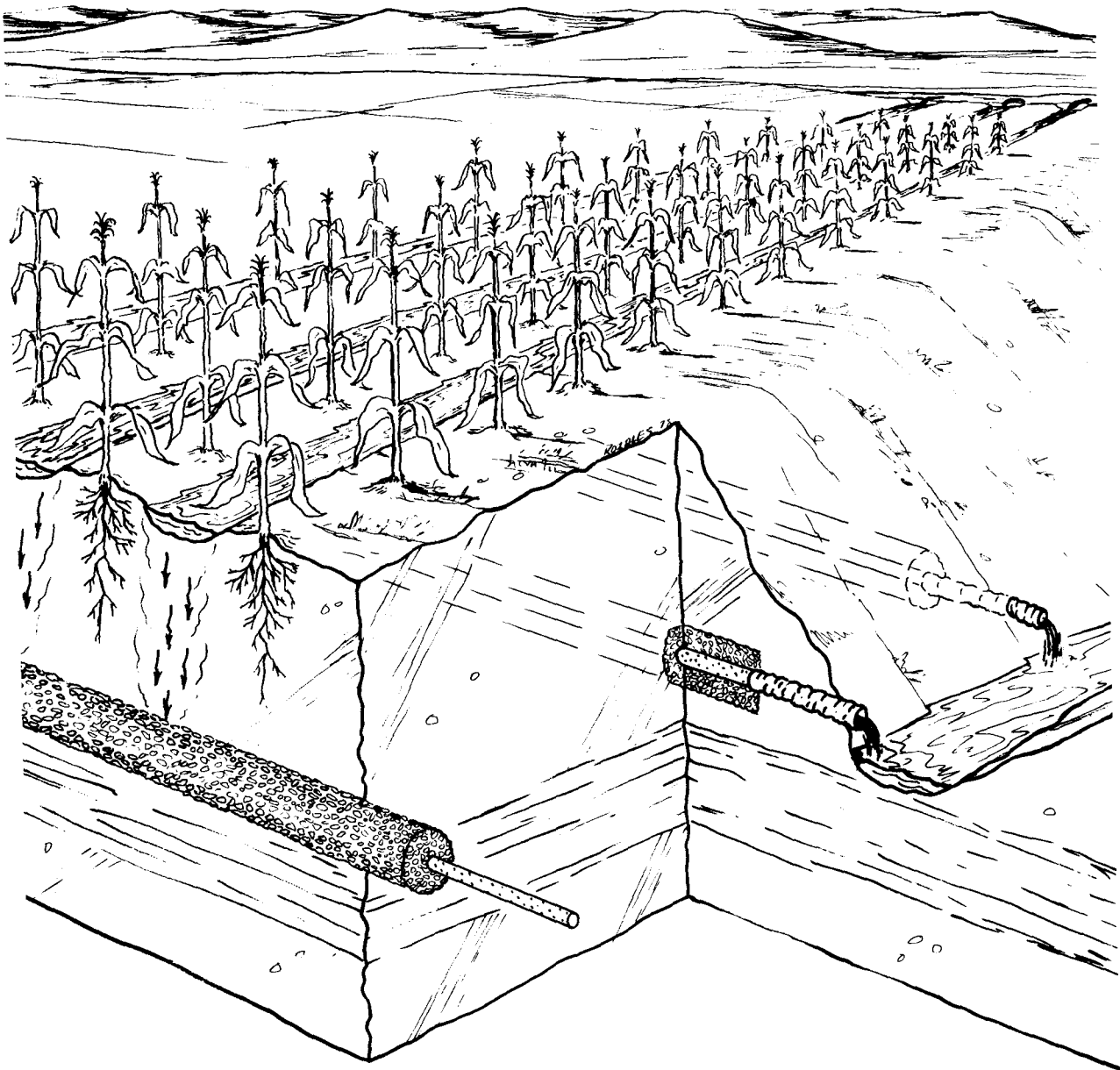


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¹ ARS has been redesignated as Science and Education
Administration — Federal Research

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Gravel envelopes for subsurface agricultural drains.

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INTRODUCTION

When installing subsurface agricultural drains, Bureau practice requires that a layer of gravel (called the gravel envelope) be placed around the perforated drain tubing. The well-graded gravel provides adequate permeability for the converging waterflow into the drain, reduces head loss, prevents movement of the base soil material into the drain, and serves as a suitable bedding for the plastic drain tubing. The purpose of this study was to make hydraulic tests of the flow through a gravel envelope, and compare the test results to those of an electrical analogy study [1].²

The gravel envelope is designed with a gradation and permeability compatible to the base soil material being drained [2], and head loss for water flow through the envelope is determined from the results of an electrical analogy study [1]. However, the electrical analogy simulates only laminar flow, not turbulent flow. It is possible that turbulent flow, resulting in larger head loss, could occur in the gravel envelope. If this were the case, the head loss indicated by the electrical analogy would not be precise. Another question was whether the electrical analogy accurately simulated boundary conditions for the gravel envelope adjacent to the corrugated drain tubing.

The ARS (Agricultural Research Service) of the Department of Agriculture and USBR (Bureau of Reclamation) of the Department of the Interior, started a three-phase research study on drain envelopes. Testing will be done in the Hydraulics Branch laboratory, Division of Research, Bureau of Reclamation, at the Engineering and Research Center in Denver, Colo. This report describes phase one of the research study. For phases two and three, tests will be made in a laboratory simulated field drain. A large tank has been constructed, containing a full size drain with gravel envelope that is surrounded by a fine sand base material.

² Numbers in brackets refer to literature cited in the bibliography.

CONCLUSIONS

1. Turbulent flow occurred in the hydraulic tests of the gravel envelope. The transition to turbulent flow started at about 0.9 to $1.4 \times 10^{-4} \text{ (m}^3/\text{s)/m}$ [0.0010 to $0.0015 \text{ (ft}^3/\text{s)/ft}$] discharge.
2. The hydraulic tests showed that when turbulent flow occurred, the head loss was greater than the linear relationship of discharge versus head loss of the electrical analogy study.
3. Field drains normally operate at inflow discharges lower than those producing turbulent flow; therefore, turbulence need not be considered in agricultural drain design when using corrugated plastic tubing similar to that tested in this study.
4. Hydraulic test results of the gravel envelope compared poorly with the electrical analogy test results. Values of hydraulic ϕ (flow function per unit length of drain) were 25 to 32 times greater than the electrical analogy ϕ values, indicating a greater envelope inflow capacity.
5. Uniformity of the envelope mediums differed between the hydraulic and electrical analogy tests. For the hydraulic tests, the gravel did not completely fill corrugations of the plastic tubing, and large gravel particles formed stratifications in the envelope. For the electric analogy tests the envelope medium was completely uniform.
6. Open spaces of the gravel envelope in the tubing corrugations and coarse gravel stratifications provided less flow resistance than that simulated by the electrical analogy.
7. The vertical positioning of drain tubing in the hydraulic tests contributed to, but was not wholly the cause of, incomplete filling of the corrugations.

8. With horizontally laid field drains and large gravel particles in the envelope material, bridging and incomplete filling of the corrugations can occur. Flow properties through the gravel adjacent to the field installed drain tubing can, and probably will, be different from those simulated in the hydraulic and the electrical analogy study.
9. The effects of the different boundary conditions could cause field drain ϕ values to be higher than those of the electrical analogy tests.
10. There was a closer agreement between hydraulic and electrical-analogy ϕ values when a fine sand $200\mu\text{m}$ (No. 70) (mean particle size) envelope material was used in the hydraulic tests.
11. The electrical analogy ϕ value should be used for design purposes. Although the hydraulic tests indicated high ϕ values for gravel envelope material similar to that used for field drains, these hydraulic tests did not provide a conclusive ϕ value. Additional testing will be needed before design ϕ values should be changed.
12. The boundary condition of the gravel envelope adjacent to the soil base material was not tested in this study.

APPLICATION

Hydraulic test results were compared directly with the electric analogy study, and the results support the continued use of the electrical analogy ϕ value for design purposes. However, the gravel envelope head loss for a field drain may be less than that indicated by the electrical analogy study. The test results of this first-phase study will be compared with data obtained from the phase two tests.

THE ELECTRICAL ANALOGY STUDY

Six different envelope configurations were tested in the electrical analogy study [1], but only the first configuration was selected for the hydraulic tests. Hydraulic flow

conditions simulated in the electrical analogy tests were simplified compared to those in the field; the drain was assumed flowing full (fig. 1), and the water table was at the top of the envelope. Although field drains sometimes operate with a free water surface inside the tubing, the full tube flow condition was the more conservative type test used because there was less head available for forcing flow through the envelope. For the flow conditions of figure 1 there is one piezometric head acting within the drain, and a larger piezometric head acting on the entire outer edge of the envelope; the head loss for the gravel envelope is the difference between the two piezometric heads.

In the electrical analogy apparatus (fig. 2), current flowed from the outside edge of the gravel envelope (envelope boundary electrode) to the drain tubing perforations (drain tubing electrode). The electrical analogy simulated a perfectly homogeneous porous medium for the gravel envelope. Different envelope thicknesses and diameters of drain tubing were tested. The test data were used to obtain values of a flow function, defined by the following equation:

$$\phi = \frac{I}{bE\sigma} \quad (1)$$

where,

ϕ = flow function per unit length of drain

I = current flow (A) (per unit length of drain)

b = outside radius of drain tubing m (ft)

E = voltage (V), producing current flow

σ = conductivity of electrolyte 1/ohm-m (1/ohm-ft)

Electrical flow properties in a conducting medium are similar to fluid flow properties in a porous medium. The corresponding hydraulic flow function is defined by the following equation:

$$\phi = \frac{q}{b\bar{H}k} \quad (2)$$

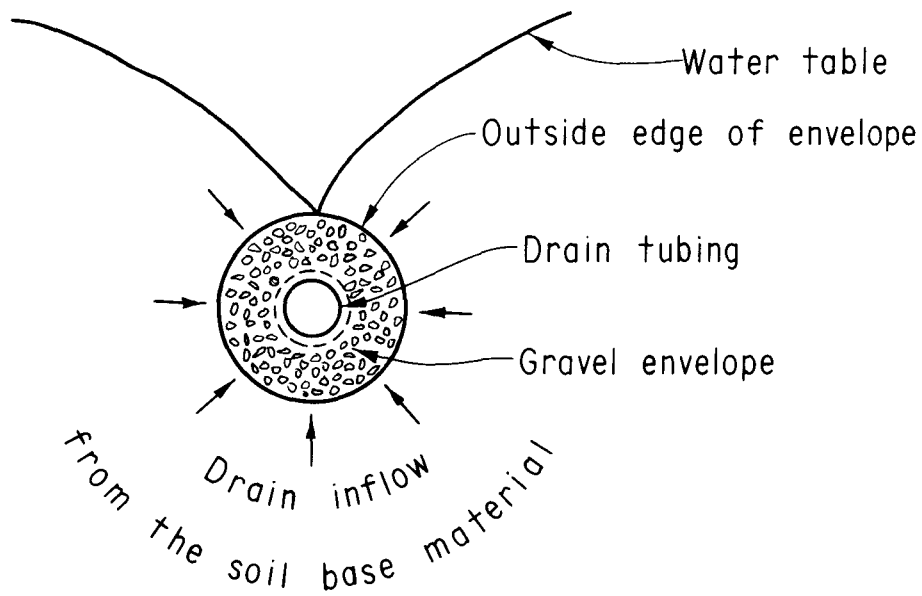


Figure 1.—Assumed flow conditions for the electrical analogy study.

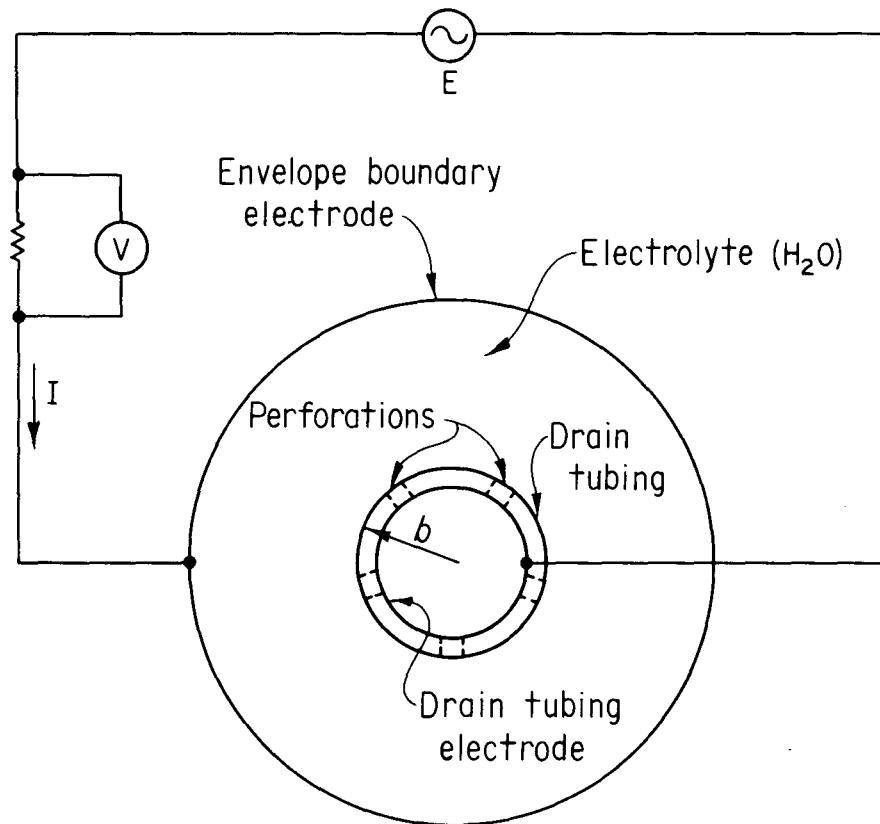


Figure 2.—Diagram of the electrical analogy test apparatus.
(Reprinted by permission, fig. 5 of reference [1])

where,

ϕ = flow function per unit length of drain, 1/m (1/ft)

q = discharge per unit length of drain, (m³/s)/m [(ft³/s)/ft]

b = radius of the drain to the outside corrugation, m (ft)

\bar{H} = head producing the discharge, m (ft)

k = coefficient of permeability for the gravel envelope material, m/s (ft/s)

Values of ϕ are given in graph form (fig. 3) for three different drain tubing diameters and for various envelope thicknesses.

WATER FLOW THROUGH THE GRAVEL ENVELOPE

A close examination of the envelope flow field will be helpful in understanding the reasons for making the hydraulic tests, and also in understanding the test results. Each of the five symmetrically spaced openings in the drain tubing has a flow field (fig. 4a). Enlarged sectional and longitudinal views (figs. 4b, c) show two types of converging flow: from the outer perimeter of the gravel envelope to an inner concentric perimeter, and sharply constricted convergence into the small drain openings. The outer corrugated surface of the drain tubing (fig. 4c) forms flow boundaries that cause increased convergence. Thus, the shape of the drain tubing surface affects the geometry of the flow field.

The velocity variation in the flow field is important and is examined here by using the flow continuity principle. For a given discharge, the velocity increases with a decrease in the flow area, and flow area (figs. 4b and c) is indicated by the distance between flow lines. Notice the decrease in flow area (fig. 4c) from point O at the outside edge of the envelope to point I inside the drain tubing. By comparison of area $AB \times d_s$ (fig. 4b) at the outside edge of the envelope and the area of the 4.8-mm (3/16-in) diameter hole, the water velocity is about 400 times greater at the drain opening than at the outer edge of the envelope. Turbulent flow was suspected at the high velocity region near the drain openings.

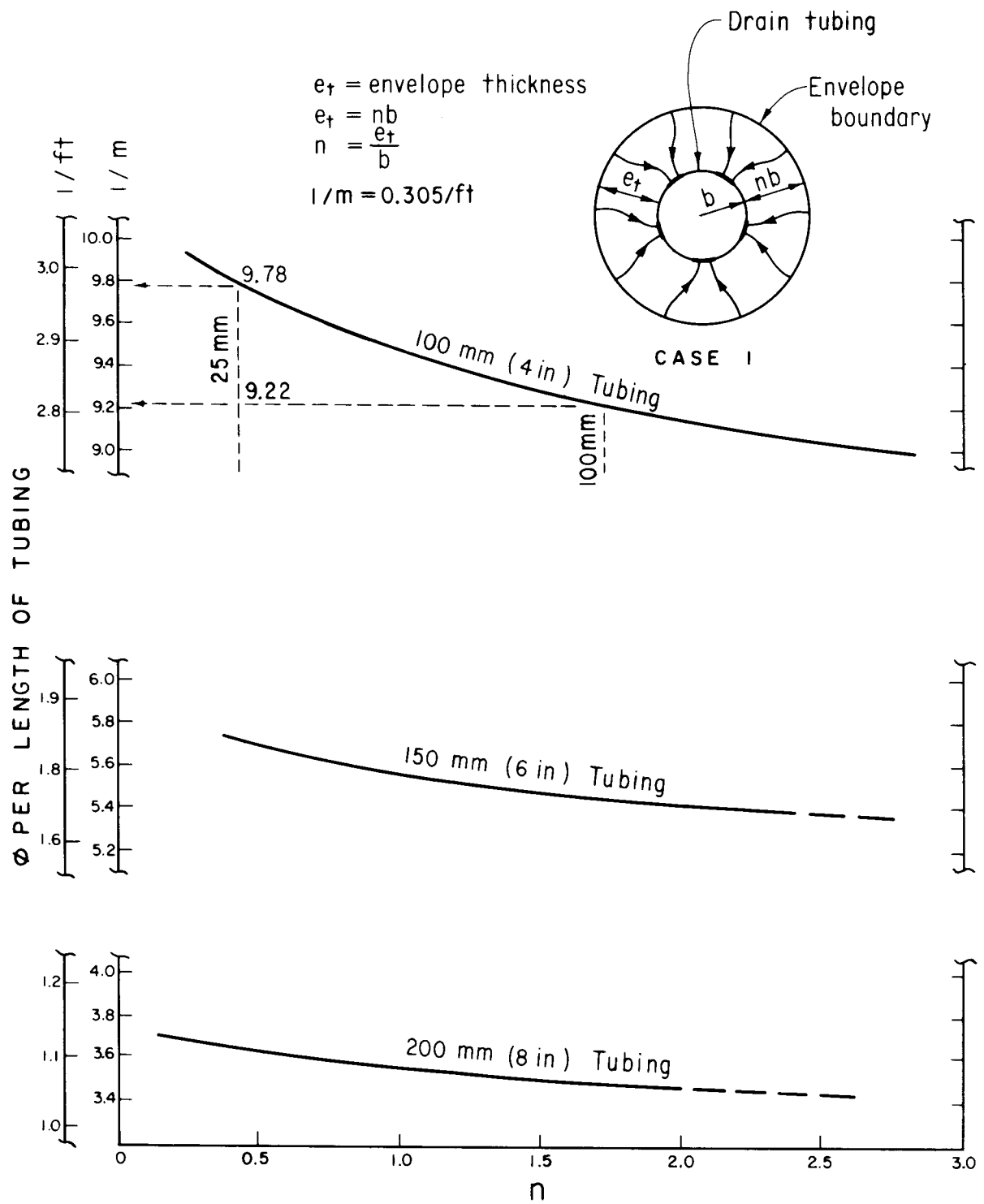
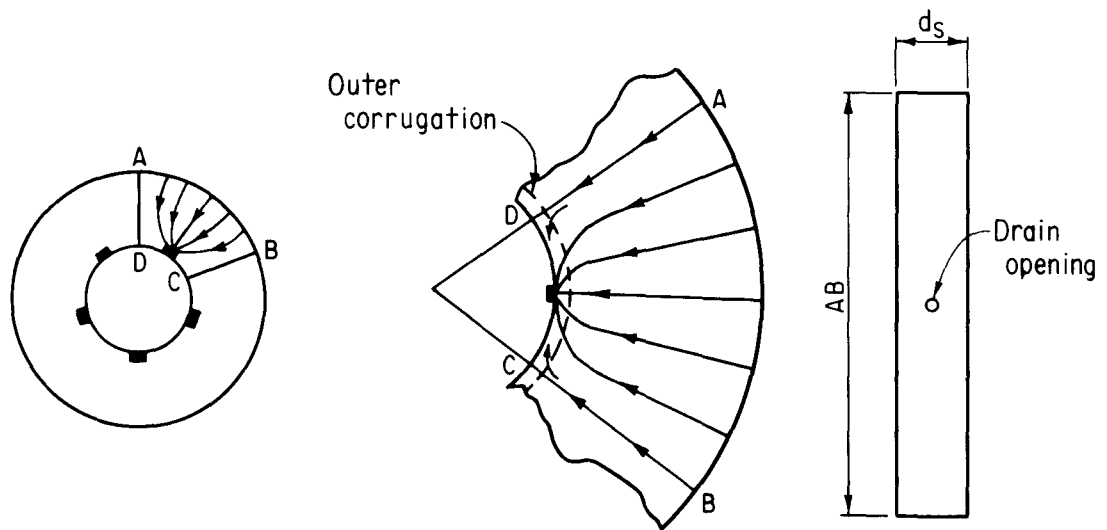
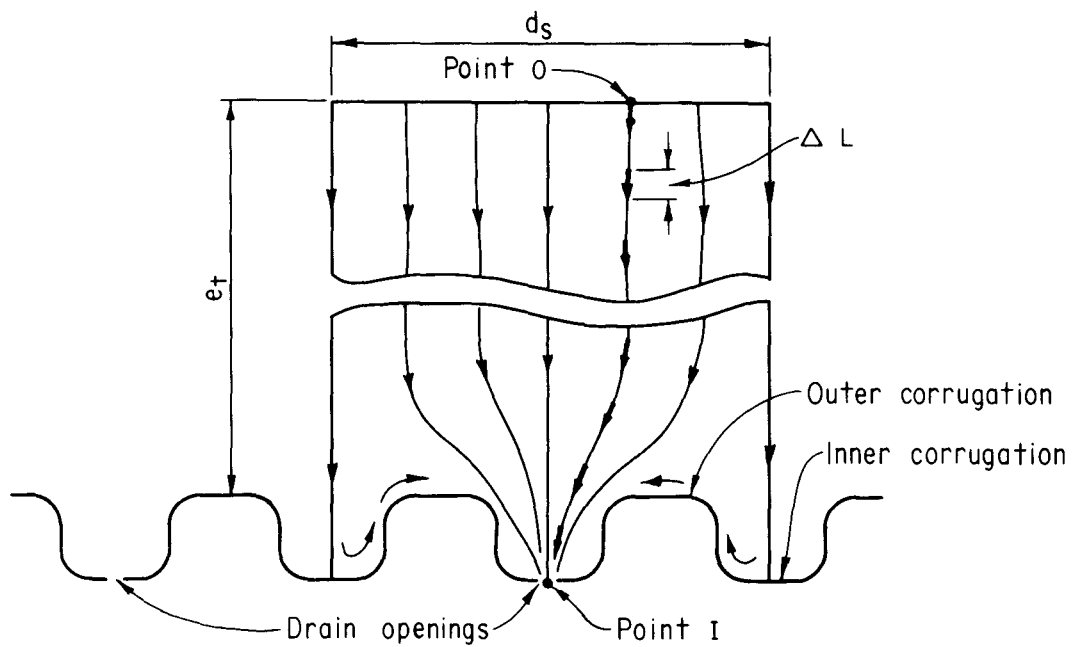


Figure 3.—Flow function ϕ versus n , from test results of the electrical analogy study (fig. 2 of reference [1]).



(a) Symmetrical segment for one drain hole opening. (b) Sectional view of flow convergence from the outside edge of the envelope into the drain opening.



(c) Longitudinal view showing corrugation influence on the flow field entering the drain opening.

Figure 4.—Flow field within the gravel envelope.

An important insight about gravel envelope head losses can be obtained from the velocity variation. For laminar flow through a porous media the head loss is directly proportional to the velocity

$$v = ki = k \frac{\Delta h}{\Delta L} \quad (3)$$

$$\Delta h = \frac{\Delta L v}{k}$$

where,

v = particle velocity along the streamline, m/s (ft/s)

i = hydraulic gradient, m/m (ft/ft)

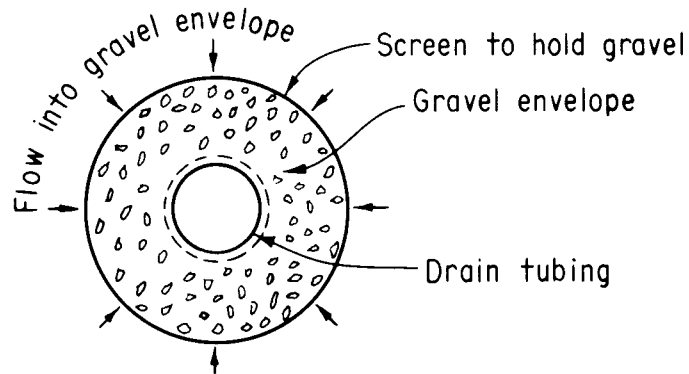
Δh = differential head loss, m (ft)

ΔL = differential length, m (ft) over which the differential head loss occurs

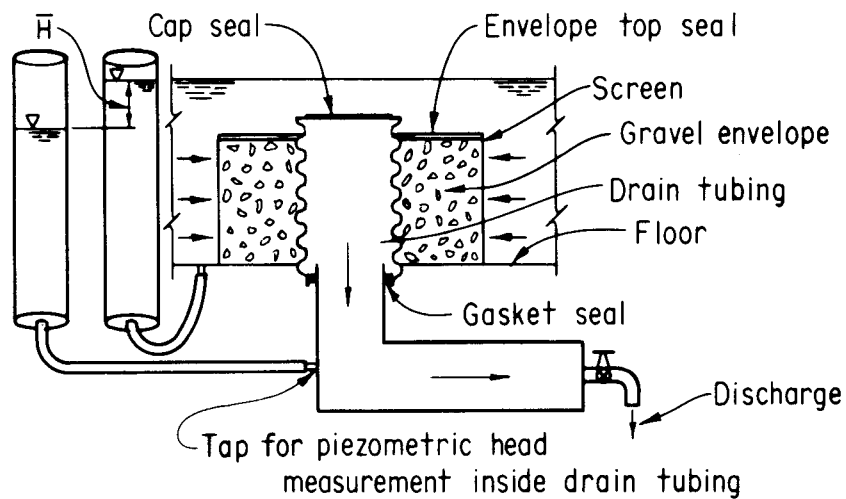
Consider the head loss for a water particle traveling along a flow line (fig. 4c) from point O to point I. The flow line is divided into numerous differential lengths, each with a corresponding differential head loss. A summation of these differential head losses along the flow line will equal \bar{H} of equation (2). Near the drain opening the flow velocities are the highest, and thus it shows a substantial head loss occurrence in the region near the drain openings.

Equation (2) is a mathematical statement of water flow through the gravel envelope. The following derivation (although not complete) is an attempt to show the relationship of the equation elements to gravel envelope flow properties. Simplifications are made, convergence of flow is neglected, and only uniform flow is assumed through the envelope thickness, e_1 (fig. 4c). Then the velocity, V , through the envelope is

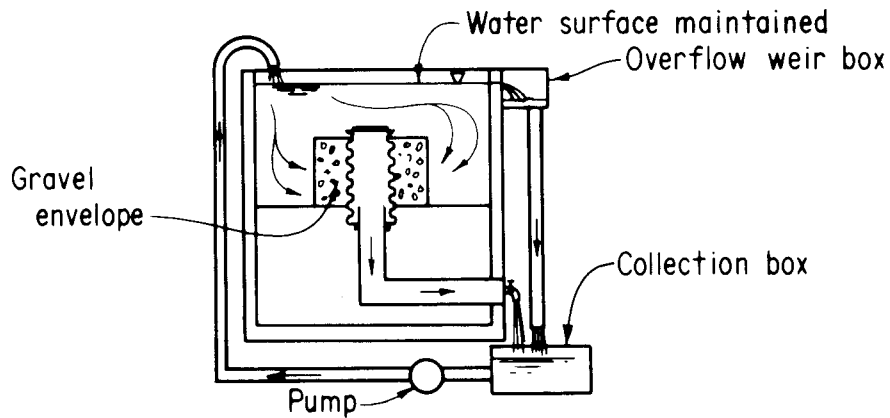
$$V = ki = \frac{k\bar{H}}{e_1}$$



(a) Plan view of drain tubing and envelope.



(b) Elevation of the gravel envelope test apparatus.



(c) Diagram of the test box.

Figure 5.—Gravel envelope test apparatus.

and the flow area, A , per unit length of the envelope is a cylindrical surface area ($2\pi b$). Thus the discharge per unit length of drain is

$$q = VA = \frac{k\bar{H}(2\pi b)}{e_t}$$

and rearranged becomes

$$q = \frac{2\pi k\bar{H}b}{e_t} \quad (4)$$

Equation (4) is similar to equation (2), with common elements, q , k , \bar{H} , and b . The element k indicates resistance of the gravel material to water flow, \bar{H} the head needed to force the discharge q through the envelope, and b an indication of envelope size. Comparing equations (2) and (4) ϕ appears equal to $2\pi/e_t$, but this is untrue. Instead, the significance is that ϕ is influenced by e_t , which is also shown by figure 3 where e_t is included in n .

The electrical analogy simulated flow field complexities of convergence, geometry of the tubing corrugations, and velocity variations with the corresponding head loss. These complexities were absorbed in the electrical analogy flow function values of ϕ . Thus the flow function provides an empirical relationship between parameters q , b , \bar{H} , and k , providing a mathematical description of the flow field. From this the drainage engineer has a simple formula to find gravel envelope head loss for a given discharge.

HYDRAULIC TEST APPARATUS

The intent was to hydraulically test the same conditions that were tested in the electrical analogy study, except that prototype gravel was used for the envelope, which allowed for development of turbulent flow. Thus, the hydraulic test apparatus (fig. 5) was similar to that of the electrical analogy test apparatus. A hydraulic potential acted on the outer envelope surface; another acted on the inside surface of the drain tube, and a discharge flowed through the gravel envelope into the drain tubing. The boundary condition of the envelope adjoining the soil base material was not tested in either study.

A length of 100-mm (4-in) diameter drain tubing, surrounded by a gravel envelope, was placed vertically in a test box. The gravel was contained in a cylinder formed with a layer of 9.5 mm (3/8 in) and a layer of 2.0 mm (No. 10) screen size material. The effective test length of drain tubing was 216 mm (8.5 in), with the bottom boundary formed by the floor; the top boundary formed by a sealant. Valve-controlled flow through the drain tube was directed into a collection box, and pumped back into the test box. When making tests, the pump discharge was adjusted to maintain water flow through the overflow weir. A nearly constant water surface elevation could thus be maintained in the test box. Drain discharges were measured at the outlet valve with a stopwatch and graduated cylinder.

Piezometer taps were placed in the test box and outlet pipe for measuring the piezometer head acting on the outside gravel envelope surface and inside the drain tube. Tubing led from the piezometer taps to stilling wells where water surface elevations were measured with point gages. The water flow head loss \bar{H} from the outer envelope surface to inside the drain tube was the difference in the elevation between the two wells. Later in the study a pressure transducer and other electronic equipment were used for making head loss measurements.

An effort was made to prevent excessive segregation of the gravel during placement of the envelope. A funnel was used to place gravel between the drain tube and screen (fig. 6) and also shown is the finished gravel placement and the flexible sealant on top of the gravel. The small tube in the cap seal was installed to bleed air out of the drain tubing while filling the test box with water, but was closed during the tests.

THE HYDRAULIC TESTS

Two different envelope materials (gravel A and B) were tested which are within the upper and lower size limits of envelope material recommended by Winger and Ryan [2]³

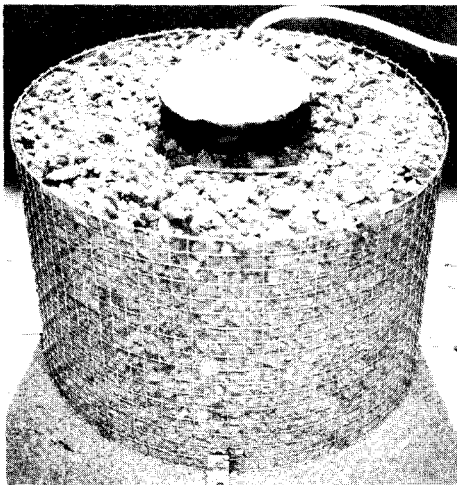
³ Plot reproduced by permission, for figure 7.

Gravel B points plotted on "lower limit" curve.

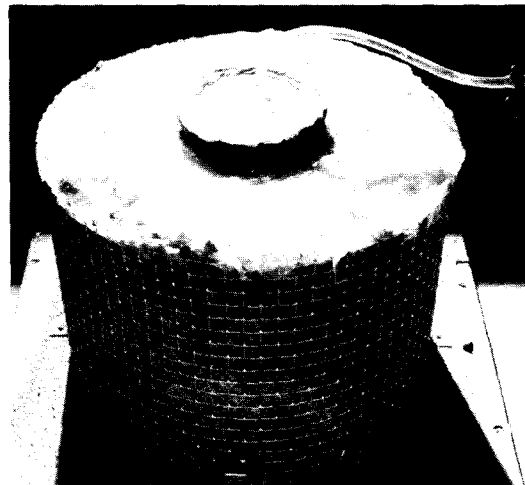
Gravel A points and curve added to figure.



(a) Placing the gravel. Photo P801-D-79027



(b) Finished placement of gravel.
Photo P801-D-79028



(c) Top seal on the gravel envelope.
Photo P801-D-79029

Figure 6.—Gravel envelope in the test box.

SILT	COARSE SILT	VERY FINE SAND	FINE SAND	MED. SAND	COARSE SAND	VERY COARSE SAND	FINE GRAVEL	COARSE GRAVEL	COBBLES
GRADATION TEST									

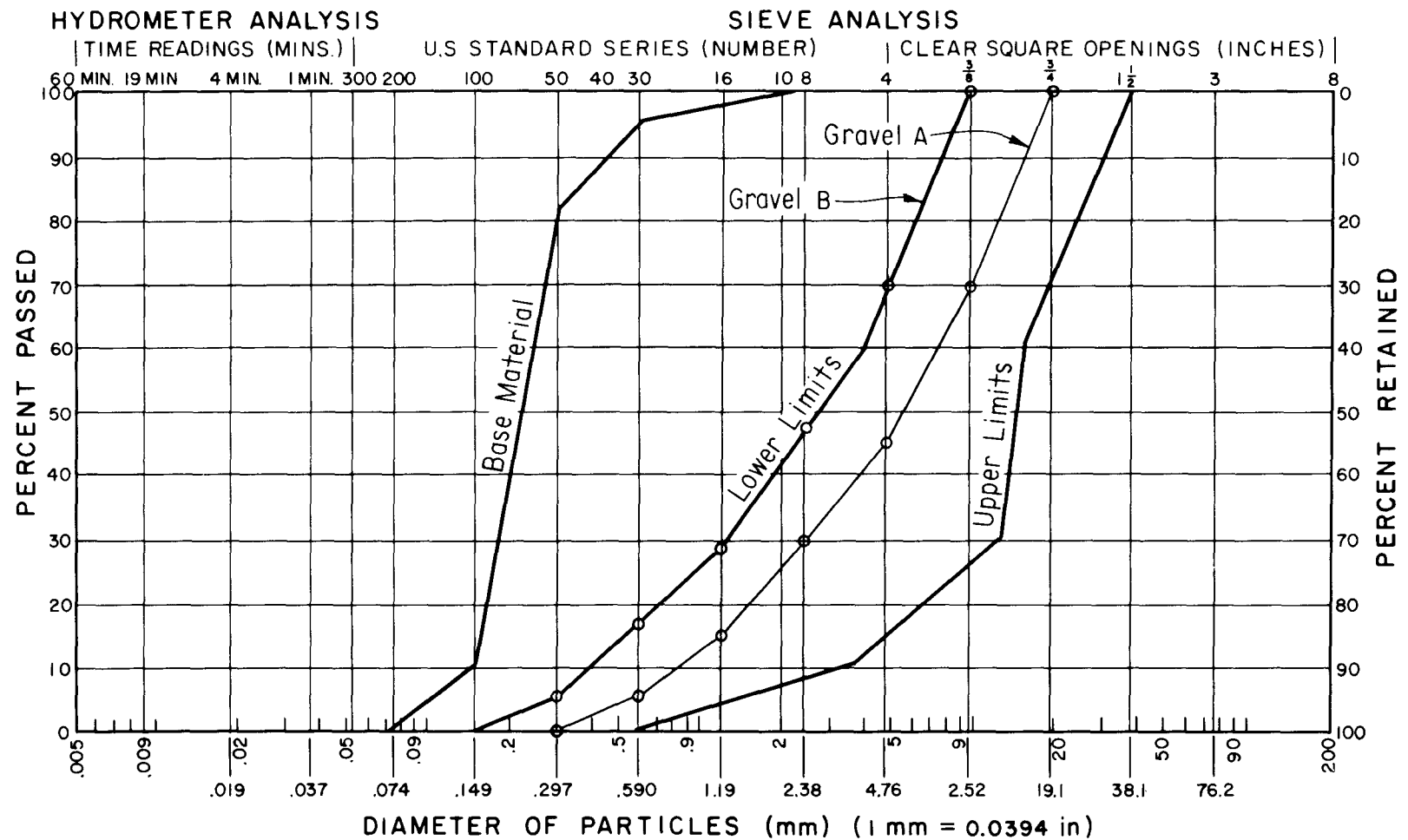


Figure 7.—Envelope gravel used in the tests, and envelope criteria for fine sand base material. (from fig. 6 [2])

(fig. 7) for use with a sand base material. The second phase of this study will include a large laboratory test box where a drain with gravel envelope will be surrounded by a similar sand base material. Test data from this study should prove beneficial for comparison to that of the second phase.

Envelopes were constructed with 25- and 100-mm (1- and 4-in) thicknesses of gravel A and tested with various discharges. Measurements were made of the discharge and head loss, converted to a linear equivalent of drain tubing, adjusted to a common water temperature of 15.6 °C (60 °F) and plotted (fig. 8). The method of temperature correction is shown in appendix A. The 100-mm-thick envelope was expected to have a greater head loss than the 25-mm-thick envelope; however, this was only noticeable at the higher discharges and additional tests were run on envelope gravels with a larger head loss. Envelopes were then constructed with 25-, 100-, and 150-mm (6-in) thickness of gravel B. Head loss for the 100-mm-thick envelope was distinguishable from the 25-mm-thick envelope (fig. 9). However, the 150-mm-thick envelope head loss fell between that of the 25- and 100-mm-thick envelopes.

The 150-mm-thick envelope was the only one in which the funnel was not used for gravel placement. Because of the 150-mm width between drain tubing and screen, gravel could be placed directly into the envelope with a small scoop. Therefore, believing this change might have been the cause of the head loss differences, the gravel was removed and a second placement was made using the funnel (fig. 10); however, this did not create a significant change. The gravel envelope head losses were small and the differences of head loss among the three envelope thicknesses smaller; thus, no additional gravel envelope tests were made.

TURBULENT FLOW IN THE GRAVEL ENVELOPE

Generally, velocities of ground-water flow are slow and laminar within a fine-grained porous media. However, in large gravel with sufficient velocity, the flow can be turbulent, or partially turbulent. Previously, it was shown that velocities near the drain openings

were about 400 times greater than at the outer edge of the envelope. Thus, turbulent flow could occur near the drain openings, causing a deviation in flow characteristics from that of the purely laminar electrical analogy flow. This difference can be seen from the respective formulas. Head loss for completely turbulent flow is proportional to the second power of velocity:

$$h \propto \frac{V^2}{2g} \quad (5)$$

where,

h = head loss m (ft)

V = flow velocity m/s (ft/s)

g = gravity m/s² (ft/s²)

Head loss for laminar flow is linearly proportional to the velocity:

$$V \propto k \frac{\Delta h}{\Delta L} \quad (6)$$

If turbulent flow is a factor in envelope operation, then the laminar flow simulation of the electrical analogy would indicate a head loss lower than the true head loss.

The q versus \bar{H} relationship was used to check for turbulent flow. The q of equation (2) transposed, ($q = \phi b \bar{H} k = VA$) from the electrical analogy study can be considered as a discharge flowing through a given area, and laminar flow is implied because the velocity varies linearly with \bar{H} .

Thus, for each test gravel and envelope thickness, the parameters ϕ , b , and k are constants and q then varies linearly with \bar{H} . If the flow is turbulent or partially turbulent, the relationship between q and \bar{H} will no longer be linear. Lines were drawn through the linear trend of the lower discharge data points for figures 8, 9, and 10. A bias was used, which raised the data points slightly above the line at low discharge because the

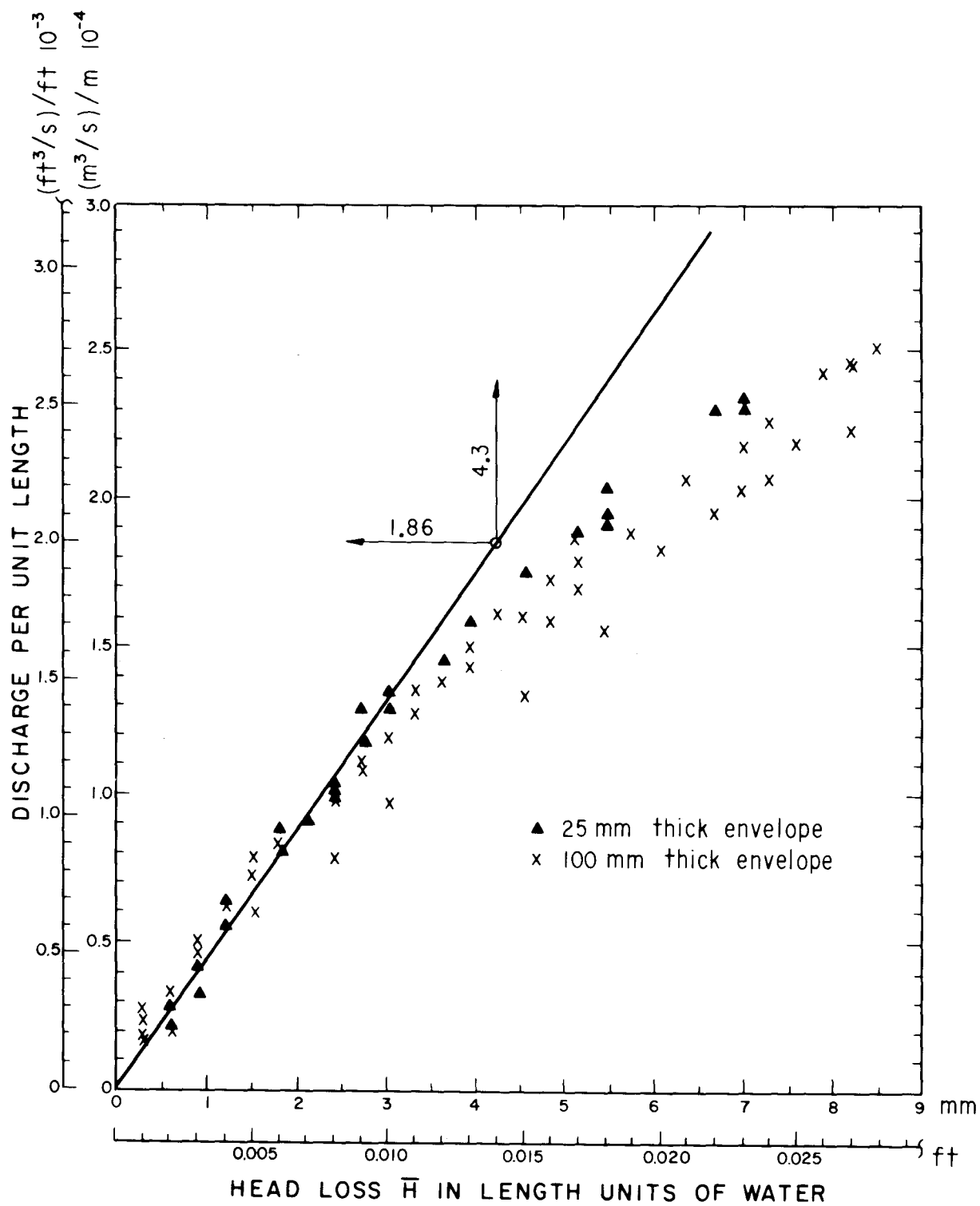


Figure 8.—Discharge versus head loss for gravel A.

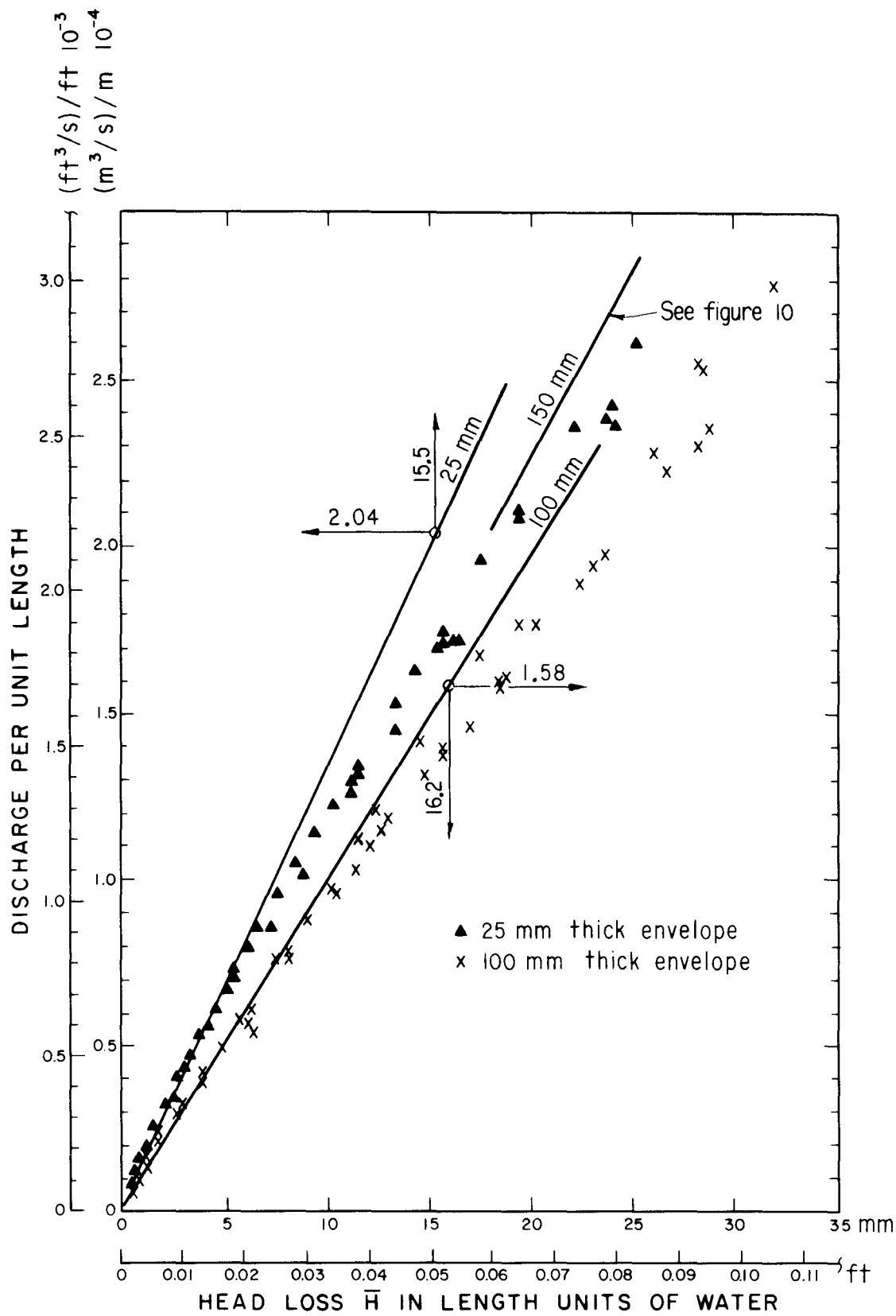


Figure 9.—Discharge versus head loss for gravel B.

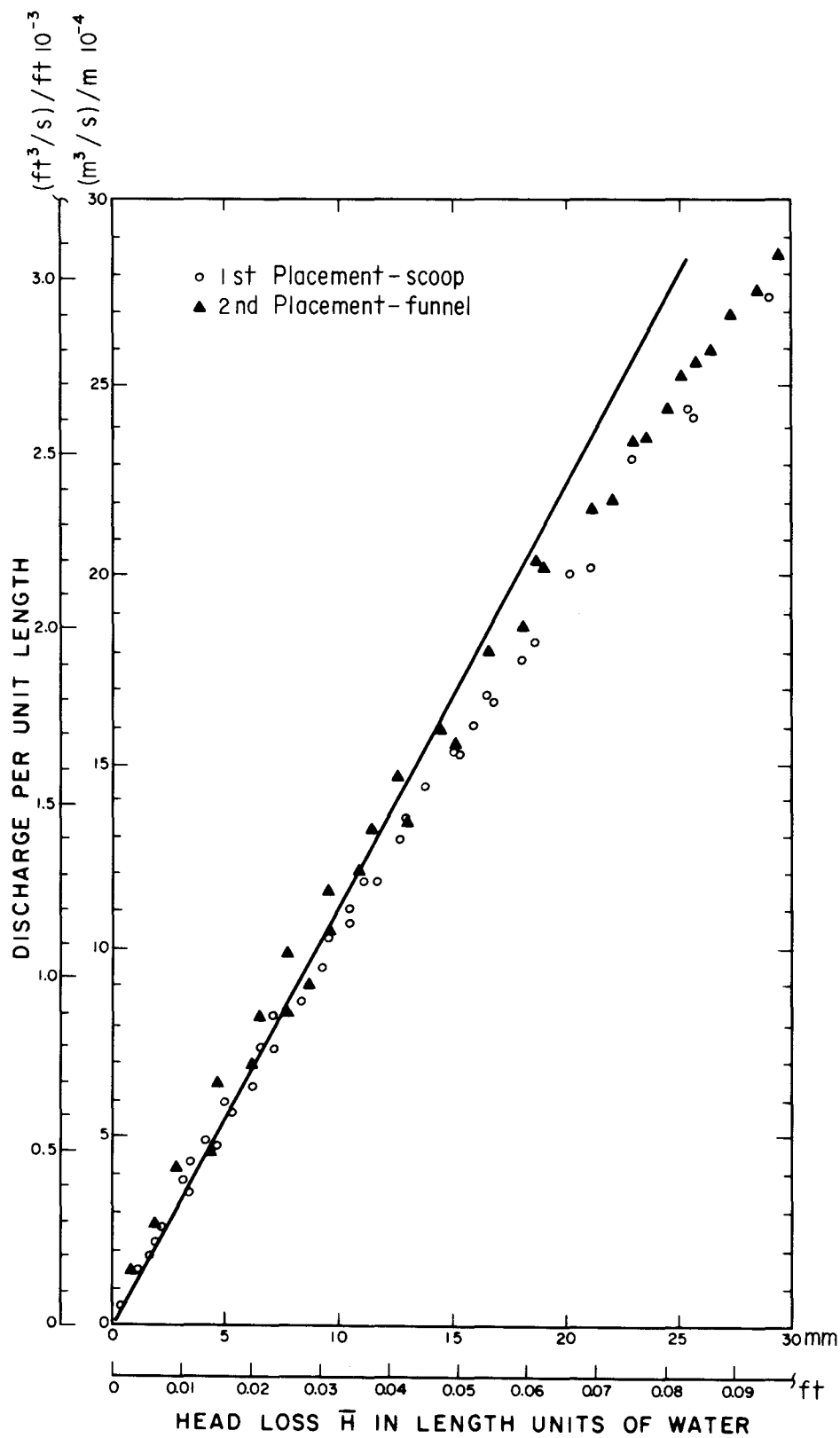


Figure 10.—Discharge versus head loss for gravel B for two placements of a 150-mm (6-in) thick envelope.

\bar{H} was small and more susceptible to error. The rate of increase of head loss \bar{H} increased at the higher discharges, which indicated turbulent flow in this area. At low q values the relationship appears linear, but with an increasing q , there is a slight deviation and with a further increase the deviation becomes greater. This tendency is consistent for a gradual transition which occurs from laminar to turbulent flow in porous media. These tests show the transition occurring between 0.9 to 1.4×10^{-4} (m³/s)/m [0.0010 to 0.0015 (ft³/s)/ft] discharge. Design discharges for drains are normally less than 0.2×10^{-4} (m³/s)/m [0.00022 (ft³/s)/ft]; thus, there is little probability of turbulent flow in field installations.

PERMEABILITY TESTS

Before comparisons could be made between the hydraulic and electrical analogy tests, the permeability values of gravels A and B were needed. Permeability tests were made for the two test gravels using a 140-mm (5.5-in) diameter plastic tube. A 0.3-m (1-ft) depth of gravel was placed in the tube, and head loss measurements were made for various discharges flowing through the gravel. Measurements of discharge were made with a graduated cylinder and stopwatch. Straight lines were drawn through the average location of the test result data points (fig. 11). The slope of the lines indicate permeability, k , and deviation of the data points from the line indicate turbulent flow. Turbulent flow was indicated for the larger gradation gravel A but not for the smaller gravel B.

Permeability measurements were also made using a small can from a method developed for fast field measurements, as described in appendix B. For gravel A, the k values compare favorably; however, the k value for gravel B tested in the can (table 1) was 2.4 times greater than for the plastic tube.

Table 1.—Permeability k measurements

	Plastic tube, mm/s (in/s)	Can, mm/s (in/s)
Gravel A	2.5 (0.10)	3.1 (0.12)
Gravel B	0.8 (0.03)	1.8 (0.07)

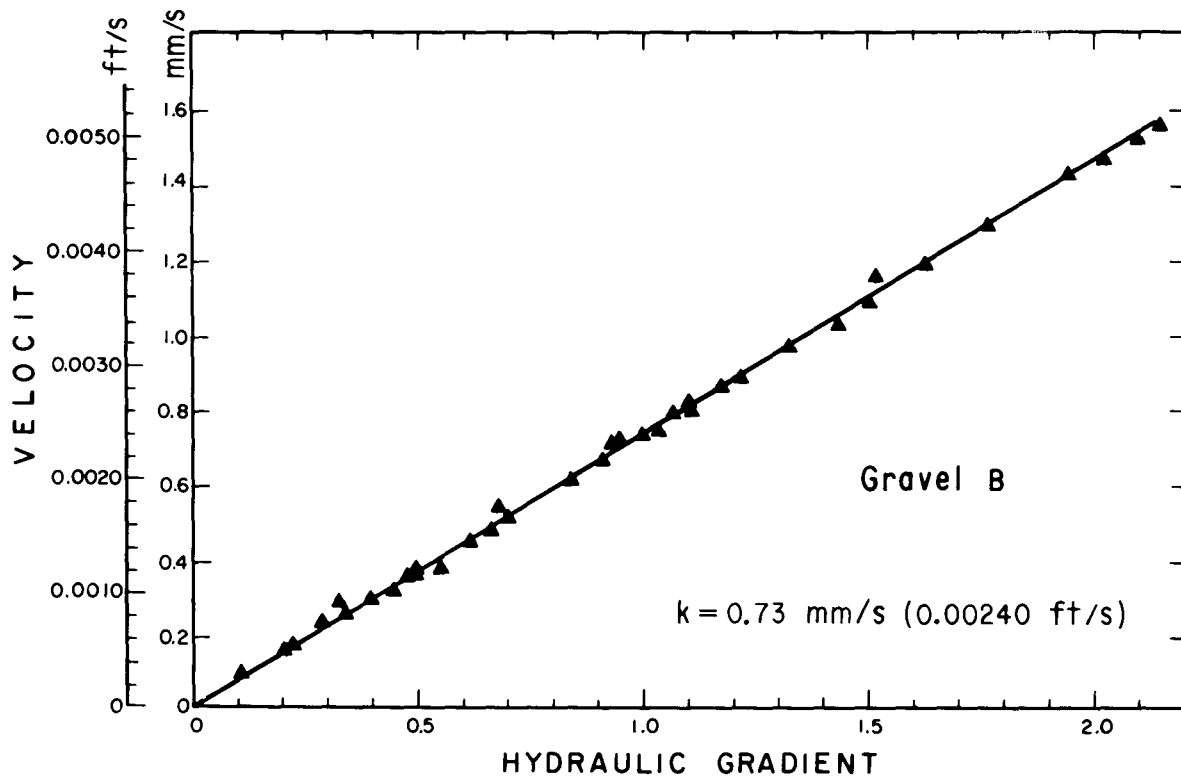
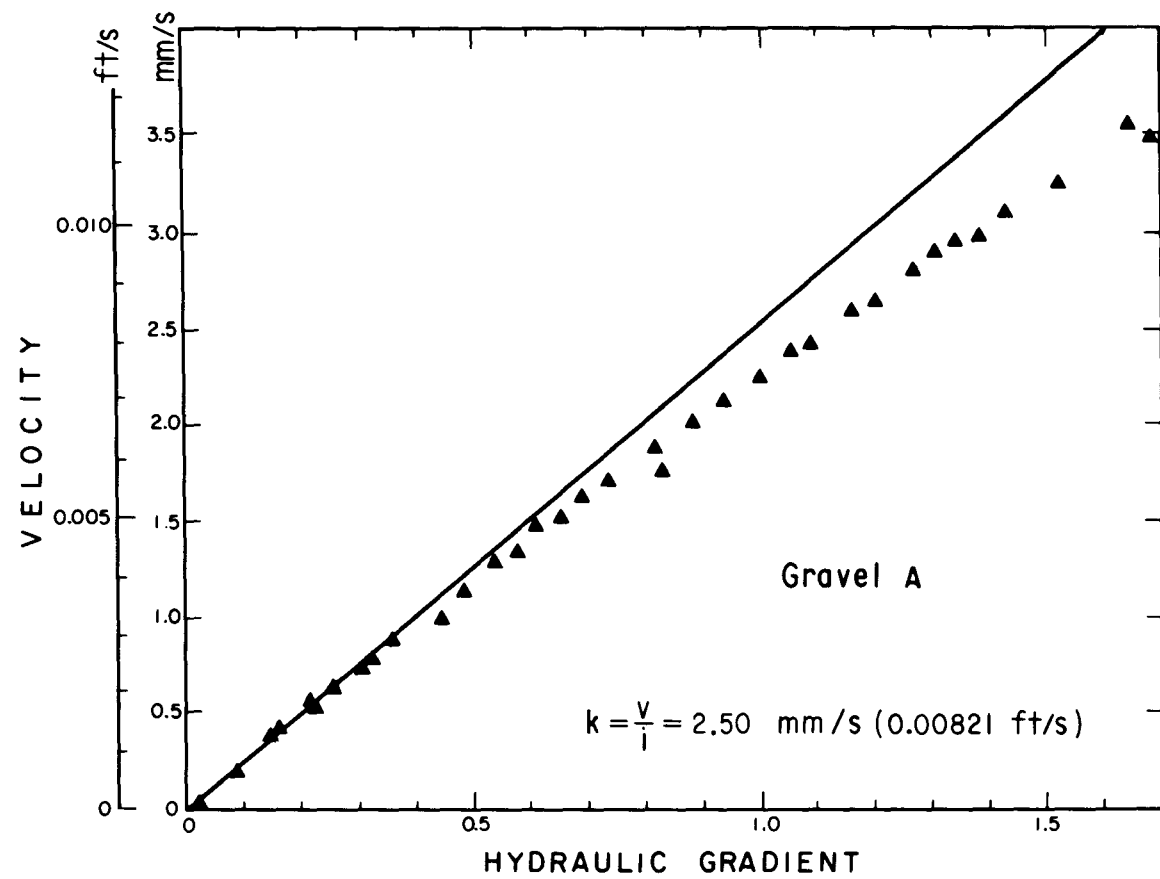


Figure 11.—Velocity versus hydraulic gradient; permeability data for gravels A and B.

When the plastic tube was filled with gravel and then water, air bubbles appeared that had been trapped in the gravel. The water was allowed to stand in the apparatus for several days to allow the air bubbles to dissolve; however, during this time algae formed. The algae was undesirable and hindered water flow through the gravel; thereafter, chlorine was added to the water.

Also undesirable was the segregation of gravel sizes which occurred in the plastic tube, where some areas had strata of coarse, and others of fine, gravel. The gravel was poured through a funnel and a 25-mm (1-in) diameter tube. The funnel and tube were raised and moved around while gravel flowed into the 140-mm (5.5-in) plastic tube; however, even this did not produce a homogeneous gravel mixture.

COMPARISON OF THE HYDRAULIC AND ELECTRICAL ANALOGY ϕ VALUES

The ϕ values, as defined by equation (2), $\phi = \frac{q}{b\bar{H}k}$, were used for making comparisons. For the hydraulic gravel envelope tests, ϕ values were computed directly by use of this equation. The b value was 58 mm (2.3 in), k values used were those from figure 11, and q and \bar{H} values were from figures 8 and 9. For the electrical analogy study, ϕ values were obtained from figure 3. Using envelope thicknesses of 25 and 200 mm, radius b to the outer corrugations of 58 mm (2.3 in), n values of 0.42 and 1.72 were obtained for entering the plot to find ϕ (fig. 3 — 100 mm tubing). The comparison between the hydraulic and electrical analogy studies was not favorable.

Table 2.—Comparative values of ϕ per unit length of drain tube

	Envelope thickness,		Hydraulic ϕ ,				Electric ϕ ,		Concentric converging ϕ_c ,	
	mm	(in)	1/m	(1/ft)	1/m	(1/ft)	1/m	(1/ft)	1/m	(1/ft)
Gravel A	25	(1.0)	*[247	(75.3)]	295	(89.9)	9.8	(2.99)	301	(91.8)
	100	(4.0)	[247	(75.3)]	295	(89.9)	9.2	(2.80)	107	(32.6)
Gravel B	25	(1.0)	[125	(38.1)]	305	(93.0)	9.8	(2.99)	301	(91.8)
	100	(4.0)	[98	(29.9)]	230	(70.1)	9.2	(2.80)	107	(32.6)

* Hydraulic ϕ values in brackets are those for the "can" method of permeability measurements, appendix B.

The differences between hydraulic and electrical analogy ϕ values were too large to accept without finding a reason for the discrepancy. A critical review was made of the hydraulic study to determine whether there was a logical explanation.

Difference in Permeability Measurements

Permeability was a factor affecting hydraulic ϕ values, and there were differences in the permeability measurements (Permeability Tests). The hydraulic values for the can-permeability measurements were included in the preceding tabulation to show permeability influence on ϕ . There was only a slight difference with gravel A, but a significantly better agreement with gravel B. However, differences in permeability measurements could only account for some of the discrepancy between the ϕ values for the hydraulic and electrical analogy.

Test Apparatus Leakage

The hydraulic ϕ values indicated the gravel envelope was 25 to 32 times more efficient than that of the electrical analogy study. For example, compare discharges ($q = \phi b k \bar{H}$) between hydraulic and electrical values for a given envelope condition. The parameters b , k , and \bar{H} would be the same and thus q would be 25 to 32 times greater with the hydraulic ϕ values. If the test apparatus had leakage, and water could readily bypass the gravel envelope into the drain tubing, then ϕ for the hydraulic model would be higher. One such possibility was the gasket seal between the drain tubing and metal pipe in the bottom of the test box.

The screen and gravel were removed from the test box and all the drain opening holes of the tubing were sealed. The test apparatus was filled with water, both in the box and inside the drain tubing. With the discharge valve slightly open, there was a small steady drip of water. Measurements showed a 5.4×10^{-7} (m³/s)/m (58×10^{-7} (ft³/s)/ft) discharge for an \bar{H} value of 0.783 m (2.57 ft). This very small leakage quantity, even under an exaggerated head, could not explain the larger hydraulic ϕ values.

Head Loss Near the Drain Openings

In the section “Water Flow Through the Gravel Envelope”, a high head loss was indicated in the region near the drain openings. Also, electrical analogy results indicate a similar condition. Consider 25- and 100-mm thick envelopes, each envelope made of the same gravel, each envelope passing the same discharge, and then obtain ϕ values from figure 3,

$$s = \text{slim, 25 mm, } \phi = 9.78$$

$$t = \text{thick, 100 mm, } \phi = 9.22$$

Each envelope would have the same b and k values, and the envelope discharges can be equated to show comparison of envelope head loss:

$$\begin{aligned} q &= \phi \bar{H} b k \\ [\phi \bar{H} b k]_s &= [\phi \bar{H} b k]_t \\ 9.78 \bar{H}_s &= 9.22 \bar{H}_t \\ \frac{\bar{H}_s}{\bar{H}_t} &= \frac{9.22}{9.78} = 0.94 \end{aligned}$$

The head loss for the 25-mm envelope is 94 percent of that for the 100-mm envelope, showing a substantial head loss occurring within a 25-mm distance of the drain openings. Hydraulically, it is the large flow velocities near the drain openings that produce this high head loss.

Both electrical analogy and hydraulic considerations show the importance of the envelope medium near the drain openings. If flow resistance characteristics were different between envelope mediums of the electrical analogy and hydraulic tests, then the ϕ values could be disparately different. Therefore, further thought was given to the envelopes of the hydraulic tests. The gravel positioning around the drain tubing was suspected of providing a different envelope medium for the hydraulic tests, figure 12a.

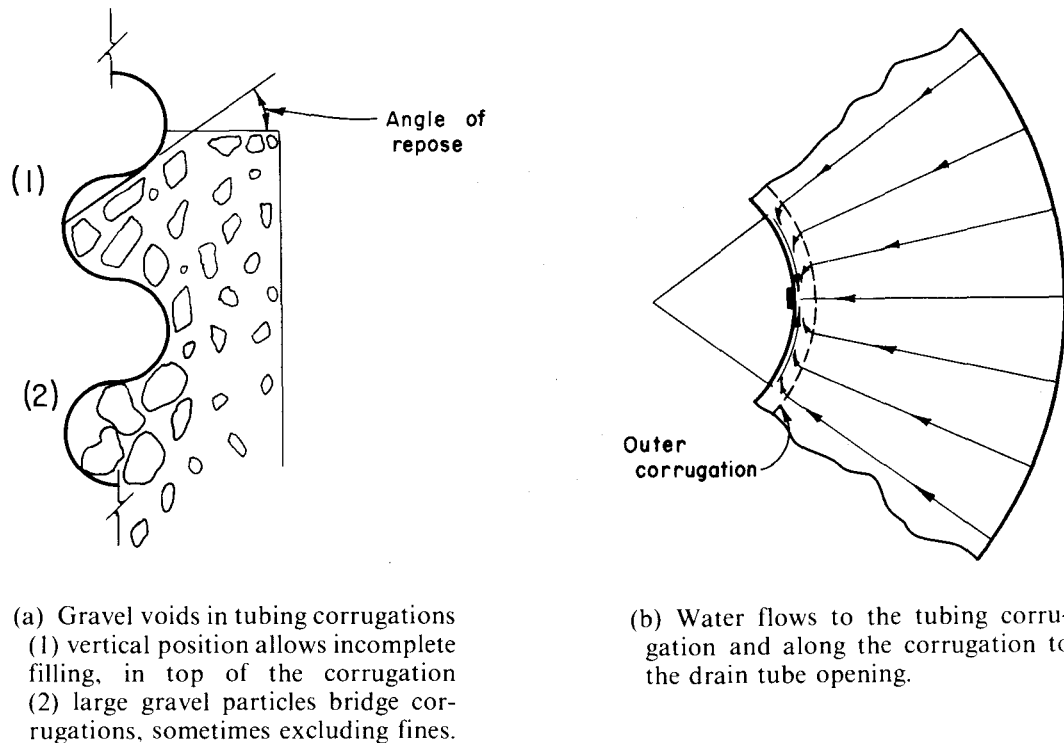


Figure 12.—Differences in the gravel envelope of the hydraulic study.

To visually check boundary conditions of the gravel envelope adjacent to the drain tubing, a small cross-sectional model (fig. 13) was constructed. A piece of drain tubing was cut in half and placed against the transparent plastic side of a box. In the same manner as the hydraulic tests, a gravel envelope was placed around the drain tubing, gravel A on the left side and gravel B on the right.

An examination showed voids in the corrugations that were not filled with gravel. Some of the drain openings had gravel particles protruding into them and others had different size void spaces extending back away from the openings, into the gravel envelope. These voids in the tubing corrugations, and near the drain openings would provide less flow resistance for the hydraulic tests, and therefore produce large ϕ values.

Concentric Converging Flow

Voids in the gravel along the corrugations could allow water to flow readily along corrugations to the drain openings. The flow field would be changed from that of

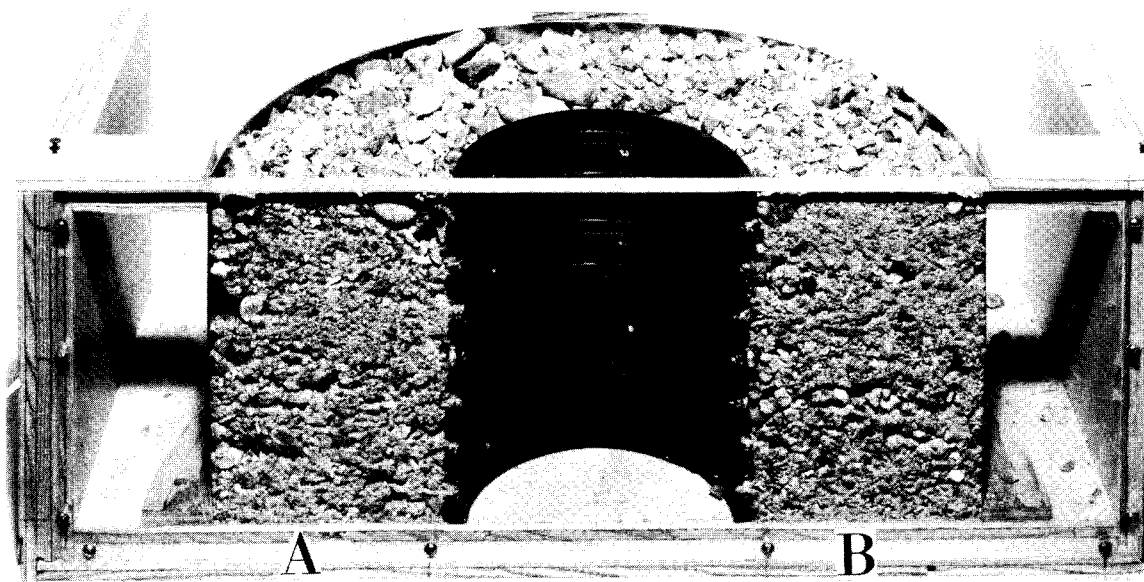


Figure 13.—Cross-sectional model showing gravel A (left) and gravel B (right) adjoining the corrugations. Photo P801-D-79030

figure 4b to the more concentric-type converging flow of figure 12b. For concentric converging flow ϕ values can be derived mathematically, appendix C, and may provide information for explaining differences between electrical and hydraulic ϕ values. However, the concentric convergence calculations were for flow from the outer edge of the envelope to a concentric inner circle of the outer tube corrugation, and exclude head losses for flow along corrugations, into and through the drain openings.

The concentric converging ϕ_c values (table 2) have very good agreement with the hydraulic values for the 25-mm envelopes and some agreement for the 100-mm envelopes. Thus, there is theoretical support showing the reasons for the higher hydraulic ϕ values. However, void spaces in the gravel along the corrugations that could produce concentric converging flow were thought to be only a partial explanation for the higher hydraulic ϕ values.

Nonhomogeneity of the Gravel Envelope

Nonhomogeneity of the gravel appeared to be a valid explanation of the reasons for the higher hydraulic ϕ values. The electrical analogy simulated a perfectly homogeneous

medium from the outside edge of the envelope to the drain openings. To check validity of the nonhomogeneity effect in the hydraulic test apparatus it was decided to use fine sand for the envelope. The fine sand would provide a nearly homogeneous envelope medium similar to the electrical analogy study.

TESTS WITH FINE SAND

A fine, uniform size sand with a 200 μm (No. 70) mean particle size was used for the 100-mm-thick envelope. Small 5-mm (1/4-in) square pieces of 0.18 mm (No. 80 screen) were placed over the drain opening holes to prevent the fine sand from flowing into the drain tubing. The dry sand was placed in 20- to 50-mm (1- to 2-in) layers and tamped with a wood block to completely fill the drain tube corrugations with the sand.

Two test series were made with the fine sand envelope. Discharges were progressively increased to the maximum, then decreased; discharge and head loss measurements were made throughout. Flow resistance of the fine sand envelope changed during both test series. With a constant valve opening, the discharge decreased while the head loss increased. This condition is shown (fig. 16) by the time of day marked adjacent to the data points of the constant valve opening condition.

The sand apparently compacted around the drain openings. During operation there were relatively large \bar{H} values from 0.3 to 0.6 m (1 to 2 ft). A very large portion of this head loss probably occurred within 25 mm of the drain openings. Thus, it may have been possible that the local high velocity and force, shifted the sand particles to block some of the 180 μm (No. 80 screen) openings. Also, the shifting particles may have reduced the sand pore spaces, decreasing the sand permeability and increasing flow resistance.

The curves for the three ϕ values and the curve for the electrical analogy ϕ value (fig. 16) vary somewhat, but are in close agreement. Permeability of the sand used was

0.17 mm/s (0.0066 in/s) as measured in a previous study. The change in the hydraulic ϕ was attributed to this changing permeability near the drain openings.

INTERPRETATION OF NONHOMOGENEITY EFFECTS

In the hydraulic study the fine sand envelope was much more homogeneous than were the gravels. Flow properties of the homogeneous sand envelope in the hydraulic study approached those simulated in the electrical analogy envelope, and test results showed good agreement of hydraulic and electrical analogy ϕ values. Therefore, the nonhomogeneity of the gravel envelopes was considered a valid explanation for the differences in ϕ values between the hydraulic and electrical analogy tests. Nonhomogeneity provided less flow resistance in the envelope, thus producing much higher ϕ values for the hydraulic tests.

Two conditions of nonhomogeneity were noted for the gravel envelopes: (1) incomplete filling of the tubing corrugations and (2) horizontal stratifications of coarser gravel particles. Each condition can have varying influences upon the ϕ values, especially when trying to relate hydraulic test and electrical analogy test ϕ values to field drain ϕ values.

Incomplete filling of the tubing corrugations was caused by vertical position of the drain tubing, figure 12a. In the field the drain tubing is horizontal, and better filling of the corrugations may be expected. However, it is questionable that corrugations at the bottom portion of the tubing will be completely filled. Bridging of gravel particles that occurred in the hydraulic tests (figs. 12a and 15) can also occur for field drain envelopes. Bridging would be dependent on size and quantity of the large gravel particles present in the envelope material. Thus, envelope material with smaller size gravels would reduce bridging, and permit better filling of the tubing corrugations.

Horizontal stratifications, similar to the large particle stratifications appearing in figures 13 and 15, were observed in the plastic tube permeability apparatus. In the plastic tube

permeability tests, water flow was perpendicular to the stratifications. However, for the hydraulic envelope tests, water flow was parallel to the coarse gravel stratifications and with less head loss than indicated by the permeability measurement in the plastic tube. A greater quantity of water could be supplied to flow through the coarse gravel (fig. 14) under the hydraulic test conditions than for a field drain. The fine base material surrounding a field drain envelope would prevent a high discharge from approaching and flowing through the coarse particle stratification.

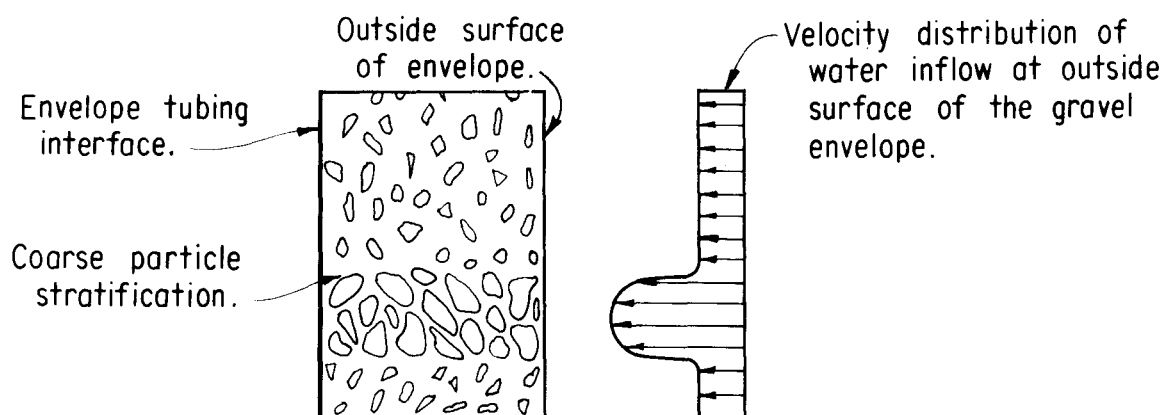


Figure 14.—Schematic of water flow through the gravel envelope.

Hydraulic test ϕ values are probably higher than those of a field drain. Horizontal stratifications of coarse gravel particles that occurred and which influenced the hydraulic tests would have small influence on a field drain envelope. However, incomplete filling of the drain tubing corrugations can occur in a field drain. Thus, the hydraulic test results indicate field drain ϕ values could be higher than shown by the electrical analogy tests.

RECOMMENDED DESIGN ϕ VALUE

The electrical analogy ϕ values should be used for design purposes even though field drains could have higher ϕ values. Field drain gravel envelopes will undoubtedly vary considerably, which will cause difficulty in predicting the correct ϕ value. Therefore, the electrical analogy ϕ value is recommended because it will give a more conservative head loss. The head loss through a coarse gravel envelope will not be significant, and through a fine gravel, where the head loss is greater and more critical to the design, it will be more similar to that of the electrical analogy study.

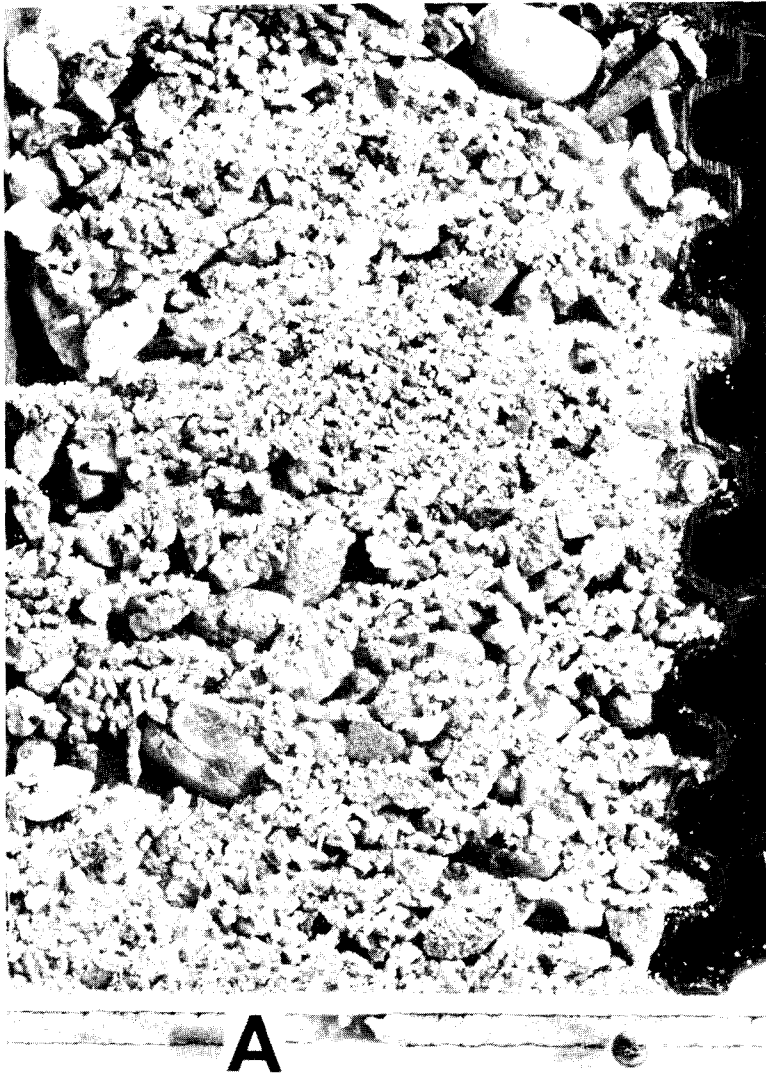


Figure 15.-Voids adjoining the corrugations. Photo P801-D-79031

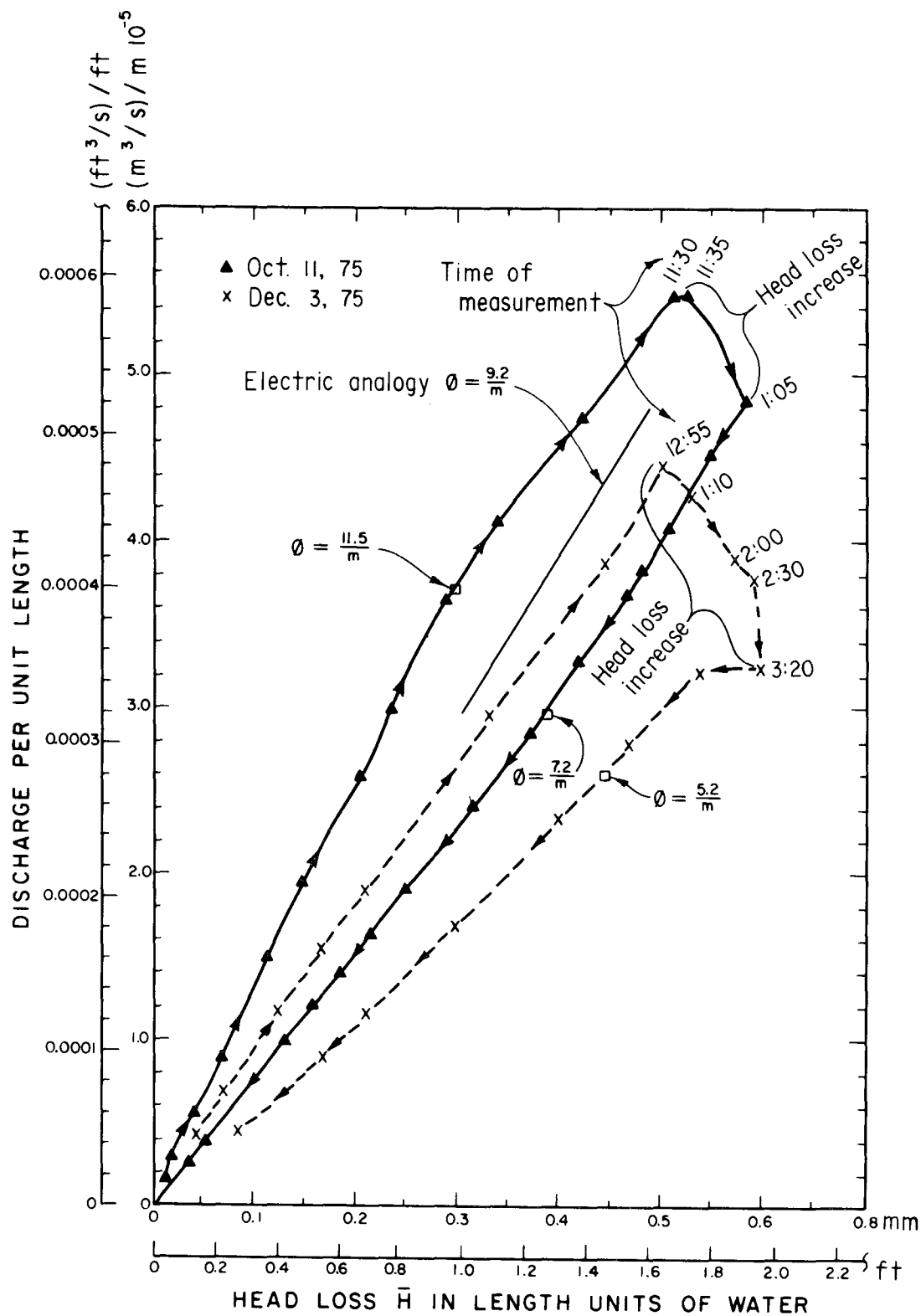


Figure 16.—Discharge versus head loss for the 100-mm (4-in) thick fine sand envelope. Note head loss increase with time of measurement.

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- [3] Terzaghi, Karl and Ralph B. Peck, "Soil Mechanics in Engineering Practice," John Wiley & Sons, Inc., N.Y., N.Y. p. 43, 1948.

- [4] Winger, Ray J., "A Simple Method for Selecting Gravel Envelopes for Agricultural Pipe Drains," Bureau of Reclamation, Water O&M Bulletin No. 88, Denver, Colo., June 1974.

APPENDIX A. – CONVERTING TEST RESULTS TO COMMON WATER TEMPERATURE

The equation commonly known as “Darcy’s law” is:

$$v = ki$$

where v = velocity m/s (ft/s)
 k = coefficient of permeability, m/s (ft/s)
 i = hydraulic gradient, m/m (ft/ft)

Generally k is used as a constant in this equation for ground-water computations; however, k is not a constant and the following [3] equation shows the variation of k .

$$k = \underline{K} \frac{\gamma}{\mu}$$

where \underline{K} = permeability, m² (ft²)
 γ = specific weight of water, kg/m³ (lb/ft³)
 μ = dynamic viscosity of water, Pa·s (pdl·s/ft²)

The permeability \underline{K} is a constant for a given permeable material and the units are descriptive of porosity properties for the material. The variation of k occurs because the properties of water vary with temperature. For the gravel envelope hydraulic tests and permeability tests the temperature ranged from 16.7 to 26.7 °C (62 to 80 °F). The variation of γ for this temperature range was insignificant. Only variation of μ was considered for k and the following equation was used for converting the test results to a common water temperature of 15.6 °C (60 °F),

$$k = \frac{\mu_{15.6} k_{15.6}}{\mu}$$

where k = coefficient of permeability at test temperature
 $\mu_{15.6}$ = viscosity of 15.6 °C (60 °F)
 $k_{15.6}$ = coefficient of permeability at 15.6 °C (60 °F)
 μ = viscosity at test temperature

APPENDIX B. - CAN METHOD PERMEABILITY MEASUREMENTS

WATER OPERATION AND MAINTENANCE BULLETIN No. 88 (see [4]) June 1974

A SIMPLE METHOD FOR SELECTING GRAVEL ENVELOPE FOR AGRICULTURAL PIPE DRAINS¹

Specialized personnel are not always available to select envelope material to be placed around subsurface pipe drains. Therefore, contractors, irrigation district construction personnel, and farmers should be acquainted with a simple, but reasonable reliable method for determining the suitability of available material. Suitability of material for an envelope depends, for the most part, on rate of flow of ground water from the in-place soil to the drains, permeability of the envelope material, and gradation of the material.

While the permeability of sand-gravel mixtures can be quite simply determined, many physical and chemical soil characteristics not readily or easily measured must be known to determine the rate of flow from the soil, making this determination one to be performed by specialists when high accuracy is necessary. However, field experience and many carefully performed soil permeability tests have indicated that a reasonable relationship usually exists between rates of flow in a given soil and its texture and structure. Soil texture can be determined in the field within acceptable accuracy for this purpose by relatively inexperienced personnel if they carefully follow standard descriptions of soil texture characteristics.

Table 1 on the next page for determining minimum envelope permeability was developed on the basis of this measured relationship between soil permeability and texture. This table shows the minimum envelope permeability requirements for the most common soil textures for an envelope 4 inches thick surrounding the pipe drain. If a plastic or asphalt-saturated felt sheet is placed over the top half of the pipe drain, the permeability values should be doubled.

To use Table 1, compare the feel and appearance of a sample of soil taken at about the depth of the proposed drains with the various soil textures described. Select the texture that fits best and read the minimum envelope permeability in inches per hour. If the drain is constructed in coarse sand or gravel, the excavated material can be used as the envelope, care being taken that none of the top soil is mixed with the sand or gravel.

To test for permeability of the envelope material, follow these simple steps:

¹Winger, R. J., Jr., Chief, Drainage and Groundwater Branch, Engineering and Research Center, Denver Federal Center, Denver, Colorado.

Table 1

MINIMUM ENVELOPE PERMEABILITY FOR VARIOUS SOIL TEXTURES

Soil texture	General Description	Minimum envelope permeability inches/hour
Medium Sand	Sand is loose. Individual grains can be seen readily. No cast forms when a dry or moist sample is squeezed in the hand.	50
Loamy sand	Sand is loose. Individual grains can be seen or felt readily. Contains small amount of silt and clay. No cast forms when a dry sample is squeezed. Cast forms in a moist sample that crumbles when touched.	35
Sandy loam	Contains much sand. Individual sand grains can be seen and felt. Sand grains tend to stick together because of the amount of silt and clay present. Squeezed when dry, cast forms that crumbles readily. Moist cast will bear careful handling.	25
Loam	Contains about equal amounts of sand, silt, and clay. Feels somewhat gritty yet fairly smooth and plastic. Squeezed when dry, a cast forms that will bear careful handling. Moist cast can be handled freely.	15
Silt loam	Smooth feel when wet. Contains some fine grades of sand, and a small amount of clay which gives a slight plastic feel. When dry it may appear quite cloddy but lumps can be readily broken and when pulverized it feels soft and floury. When wet, the soil readily runs together. Either dry or moist, it will form casts that can be freely handled without breaking but when moistened and squeezed between thumb and finger, it will not "ribbon" but will give a broken appearance.	10
Clay loam	Plastic when moist. Dry sample usually breaks into hard clods. Squeezed when moist, cast forms that will bear much handling. Can be kneaded into heavy compact mass.	10

1. Place 4 inches of the pit run material, free of vegetable matter, clays, or other deleterious substance in any nontapered gallon can from which the bottom has been removed and a copper window screen soldered over the bottom.
2. Drop can on ground from about 1 inch above ground 10 times to eliminate large voids.
3. Refill can to 4-inch mark and slowly lower it into a larger pail of water until 3 inches of water stands above the upper surface of the test sample.
4. Lift the gallon can above the water surface in the larger pail to provide for free drainage, and pour water through the material for about 1 minute maintaining the 3 inches head of water over the material.
5. Stop pouring water into the can and determine the time in minutes and seconds for the water level in the can to fall the 3 inches to the surface of the material being tested. (The stopwatch should be started when the water level in the can is on a mark 3 inches above the surface of the 4-inch-thick envelope material and stopped as the last free water disappears from the surface.)
6. Repeat the test at least three times to obtain an average time.

The permeability of the envelope material can then be estimated from Table 2 below.

Table 2

Permeabilities of test sample 4 inches thick
based on time required for water level
to drop 3 inches to level of soil.

<u>Time</u>		<u>Estimated permeability</u>
<u>Min:Sec.</u>		<u>Inches/Hour</u>
Less Than	2:00	70 +
	2:41	50
	3:50	35
	5:23	25
	8:58	15
	13:26	10

* * * * *

Gravels A and B were tested by Mr. Winger's method [4], and found to be more permeable than those of table 2. Considerably less than 2 minutes time elapsed as the water drained through the gravel. The equation (the derivation of which follows) was used for computing the coefficient of permeability.

$$k = \frac{L}{T} \ln \left(\frac{h_2}{h_1} \right) \quad (\text{B1})$$

where: k = coefficient of permeability, m/s (ft/s)
 L = depth of gravel in the can, flow length, m (ft)
 T = elapsed time for water surface to drop from h_2 to h_1 , s
 h_2 = water depth acting on gravel sample at beginning of test, m (ft)
 h_1 = water depth acting on gravel sample at end of test, m (ft)

Gravel A: $T = 21.5$ s, $L = 102$ mm, $h_2 = 190$ mm, and $h_1 = 102$ mm

$$k = \frac{L}{T} \ln \left(\frac{h_2}{h_1} \right) = \frac{102}{21.5} \ln \left(\frac{190}{102} \right) = 2.95 \text{ mm/s (0.0098 ft/s)}$$

Gravel B: $T = 36$ s

$$k = \frac{102}{36} \ln \left(\frac{190}{102} \right) = 1.76 \text{ mm/s (0.0058 ft/s)}$$

As the water level in the can (fig. B1) drops from h_2 to h_1 , the velocity of the falling water surface varies. The velocity may be defined $v = dh/dt$, where during a short time increment, dt , the water drops a small distance, dh . Resistance of the gravel determines how fast the water level drops, and velocity in the gravel is governed by permeability and hydraulic gradient.

Equating the two velocities gives equation (B1).

$$v = \frac{dh}{dt}$$

$$v = ki = k \frac{h}{L}$$

$$\frac{dh}{dt} = k \frac{h}{L}$$

$$\int_{h_1}^{h_2} \frac{dh}{h} = \frac{k}{L} \int_{t_1}^{t_2} dt$$

$$\ln h_2 - \ln h_1 = \frac{k}{L} (t_2 - t_1)$$

where: $T = (t_2 - t_1)$

$$k = \frac{L}{T} \ln \left(\frac{h_2}{h_1} \right) \quad (B1)$$

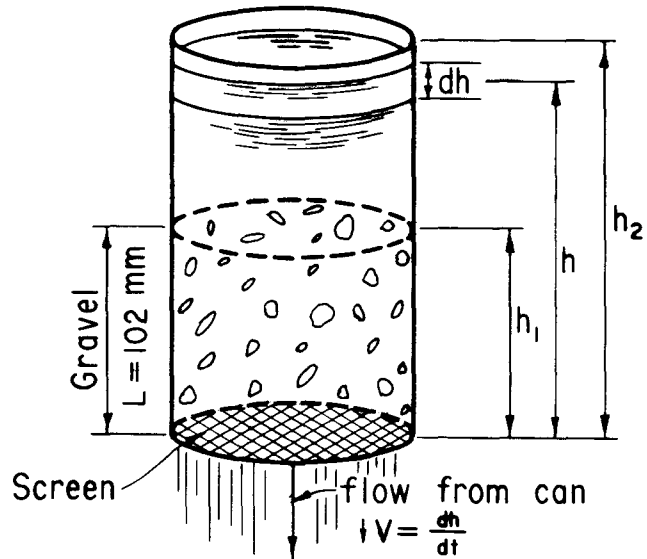


Figure B1.—Dimensions of can for permeability test calculations.

APPENDIX C. – VALUES OF ϕ_c FOR CONCENTRIC CONVERGING FLOW

A formula can be obtained for converging concentric flow that is in the same form as equation (2).

$$q = \phi b \bar{H} k \quad (C1)$$

Consider a discharge q , for a unit length, flowing from the outer cylinder, in toward the inner cylinder (fig. C1). The flow velocity v is the discharge q divided by the area $2\pi r$.

$$v = \frac{q}{2\pi r} \quad (C2)$$

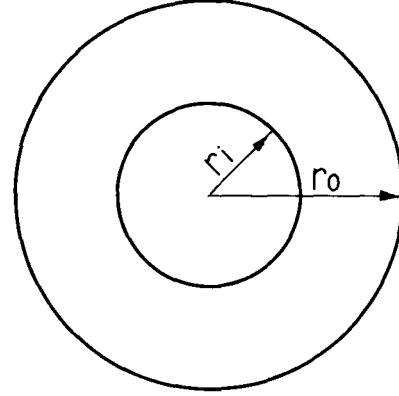


Figure C1.—Section diagram of porous media.

The flow velocity through porous media is also a function of the hydraulic gradient i

$$v = ki = k \frac{\Delta h}{\Delta r} \quad (C3)$$

where the differential head loss Δh occurs over the differential radial distance Δr .

Equating velocities of equations C2 and C3 gives

$$k \frac{\Delta h}{\Delta r} = \frac{q}{2\pi r}$$

Therefore

$$\Delta h = \frac{q \Delta r}{k 2\pi r} \quad (C4)$$

This equation combines properties of concentric converging flow, and head loss properties of porous media flow. The head loss \bar{H} can be obtained for the flow from the outer r_o to the inner radius r_i .

A summation is made of the differential head losses occurring from r_o to r_i .

$$\int_{h_i}^{h_o} dh = \frac{q}{k2\pi} \int_{r_i}^{r_o} \frac{dr}{r}$$

Integration gives

$$[h]_{h_i}^{h_o} = \frac{q}{k2\pi} [\ln r]_{r_i}^{r_o}$$

The limits of integration h_o and h_i are the piezometric heads acting on r_o and r_i .

$$\bar{H} = h_o - h_i = \frac{q}{k2\pi} (\ln r_o - \ln r_i) = \frac{q}{k2\pi} \ln \left(\frac{r_o}{r_i} \right) \quad (C5)$$

Rearranging equation C5

$$q = \frac{2\pi\bar{H}k}{\ln \left(\frac{r_o}{r_i} \right)}$$

and multiplying numerator and denominator by b ; ϕ_c for a converging concentric flow is obtained

$$\phi_c = \frac{2\pi}{b \ln \left(\frac{r_o}{r_i} \right)} \quad (C6)$$

Then

$$q = \frac{2\pi b \bar{H} k}{b \ln \left(\frac{r_o}{r_i} \right)} = \phi_c b \bar{H} k \quad (\text{C1 restated})$$

For the hydraulic gravel envelope tests described in this report the value b is 59 mm, r_i is 59 mm, and r_o equals b plus the envelope thickness.

Therefore:

for the envelope, $e_t = 25$ mm

$$\phi_c = \frac{2\pi}{(0.059) \ln \left(\frac{0.084}{0.059} \right)} = 301.4/\text{m} \text{ (91.9/ft)} \quad (\text{C6})$$

and for the envelope, $e_t = 100$ mm

$$\phi_c = \frac{2\pi}{(0.059) \ln \left(\frac{0.159}{0.059} \right)} = 107.4/\text{m} \text{ (32.7/ft)} \quad (\text{C6})$$

