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DESIGNING COST-EFFECTIVE HABITAT MANAGEMENT PLANS USING OPTIMIZATION METHODS

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16. ABSTRACT

This report describes the use of optimization methods for designing cost-effective wildlife management plans. Many management problems require decisions about which management activities, and how much of each, to implement to achieve a prescribed wildlife response at minimum possible cost. An example wildlife management problem involving wetland and grassland habitats is used to introduce concepts of linear programming. Following this example, separable programming techniques are introduced as a means of handling nonlinear wildlife-habitat relationships. An actual separable programming model for the Garrison Diversion Unit is described in detail. This model is used to develop management plans for three species (blue-winged teal, gadwall, and hungarian partridge) based on several activities including land acquisition (fee title, easement), planting vegetation (dense nesting cover, native grasses, tame grasses), livestock grazing (regulation of AUM's, fence construction), and wetland development (wetland construction and restoration). Guidance is provided on the organization and computer coding of a separable programming model.

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INTRODUCTION

Habitat plays a key role in wildlife management, and habitats in North America are undergoing rapid change because of man's manipulation of the environment to meet his needs. A growing recognition of the importance of habitat has led to the development of planning methods such as the *Habitat Evaluation Procedures* [1][1], wherein habitat information plays a dominant role in decisionmaking.

While habitat is receiving more emphasis in planning, cost-effectiveness of habitat management has been neglected. However, one of the biggest problems faced by wildlife managers is funding for management because wildlife resource agencies typically operate within tight budgets. Thus, it is surprising that the economics of management has not received more emphasis in wildlife planning. Although this situation may be changing, Cutler [2] argues for additional emphasis on the economic aspects of wildlife management.

This report describes a method for designing cost-effective habitat management plans, following the approach of Matulich et al. [3]. This report outlines the analytical steps of optimization, but is not intended as a strict procedural guide. Rather, we have tried to emphasize concepts and a basic understanding of how optimization methods can be used to solve habitat management problems.

Frequently, the description of optimization methods is accompanied by mathematically complex arguments that obscure the simplicity of the concepts. This is understandable because optimization is a mathematical topic. While we also have used mathematics in this report, this use has been limited to the essentials, which include college level algebra and a working knowledge of Cartesian coordinate systems.

This report is written for the wildlife biologist responsible for the design of wildlife management plans who has had little, if any, prior exposure to optimization methods. Consequently, the third section is devoted to an introduction to basic concepts and terminology of linear programming, a type of optimization. The fourth section builds on the basic concepts of linear programming by providing guidelines for developing a habitat management model using examples from an actual mitigation study and, as such, comes as close as anything in this report to being a "cookbook" for applying optimization methods. The fifth section provides guidelines for using available computer programs for solving optimization problems. The sixth and seventh sections provide additional planning concepts pertinent to habitat management problems, and the eighth section is devoted to a brief narrative of the major assumptions and limitations of optimization techniques for habitat management.

The concepts outlined in this report are applicable to a broad range of habitat management problems encountered in mitigation studies, refuge planning, and forest planning. These problems are similar because in many instances they require comparable analytical approaches. Identifying this common analytical approach is the subject of the next section and provides the foundation upon which this report is based.

AN ANALYTICAL APPROACH TO HABITAT MANAGEMENT PLANNING

Many habitat management problems lend themselves to solution by optimization techniques. A common feature of these problems is to decide which management actions to implement to achieve specified wildlife objectives. Of course, there is never free reign with respect to the choices because budget and manpower limitations impose severe constraints on which strategies are feasible. Moreover, the recommended plans must be consistent with social, institutional, and legal constraints. Thus, the management plans must be simultaneously feasible with respect to several different perspectives.

The balance between different and competing perspectives is the essence of an optimization problem. By their nature, optimization problems tend to require relatively complex solution approaches. There are ways to minimize this complexity, however, by specifying the problem in a certain format. This section describes that format and, in so doing, introduces some basic concepts on which later sections rely.

Defining the Problem

A crucial aspect of planning is developing a clear statement of the problem. In wildlife habitat management planning, the first aspect of specifying the problem is a statement about the habitat to be managed. This may be a habitat for a single species, multiple species, or a group of species; e.g., waterfowl. Second, the desired consequence of management for each species must be clearly stated. A statement such as "improve the habitat for blue-winged teal" is too general, and often ambiguous. A habitat response goal such as "increase teal brood-rearing habitat by 10 acres" is more useful because it provides a way to measure progress toward achieving the goal (i.e., acres of habitat), and it defines a stopping place (i.e., after creating 10 acres of habitat).

[1]Numbers in brackets refer to entries in the Bibliography.

Another part of the problem statement is the time horizon or interval of future time over which the management plan is to be relevant. The time horizon for wildlife habitat management is usually relatively long because many habitat management activities require several years to become effective. Also, habitat management plans developed in conjunction with many projects (e.g., Federal water projects) require a time horizon identical to that used in the project benefit/cost analysis. Using the same time horizon, sometimes called the project life or period of analysis, ensures that wildlife information can be combined with other project data for planning purposes.

The third part of the problem statement is identifying constraints on achieving the prescribed habitat response for the selected species. Constraints designate what is acceptable from a political perspective or what is possible based on physical limits. For example, it may be technically possible to convert cropland to wetland habitat; however, it may not be politically acceptable to do so. Similarly, it may be possible to create cropland by draining existing wetland habitat, but such conversion might be counter to achieving habitat goals and thus deemed inappropriate. Political and physical constraints are inherent to all habitat management problems.

Given the constraints on individual management activities, the number of ways that feasible management activities can be combined to form management plans may still be very large. Selecting from among the feasible alternatives is never a trivial task and requires some way of determining the best or optimum alternative; however, "optimum" has no meaning unless the criteria for optimality is specified. There are several alternative criteria available, but this report is based upon cost minimization. Given this criterion, the habitat management problem described above can be restated as follows:

> Which management actions and how much of each should be implemented to achieve a specified wildlife habitat response for a specified period at *minimum overall costs*?

The emphasis in this statement is on the last three words because the criterion for an optimum solution, as used in this report, is minimum cost. In other words, among all alternatives that would achieve the desired habitat response and that are feasible from political and other perspectives, the optimum plan is the alternative requiring the smallest amount of money to implement.[2]

[2]Other possible criteria for optimality in habitat management also exist, e.g., maximizing wildlife production. Optimization methods can usually be applied when the problem can be stated in terms of maximizing or minimizing some quantity.

The complexity of this problem becomes more apparent by referring to certain aspects of a typical situation. Several species, or several groups of species, may be targeted for management, and these will differ in their habitat preferences. Any single management action, such as prescribed burning in grassland, may cause positive habitat responses for some species and negative responses for others. Thus, any given management action may simultaneously contribute to and detract from achieving the desired habitat responses. Even if a single species is considered, both positive and negative aspects of a management activity are possible. For example, constructing wetlands for waterfowl may enhance brood-rearing habitat but only at the expense of upland nesting cover because the uplands are converted to wetlands.

Another complicating factor is the widely differing costs of individual management activities. The best management activity to use in a given situation is not only a function of habitat changes but also of costs relative to other possible management activities; however, it is not always easy to choose the most cost-effective activity. For example, some management actions, such as livestock grazing allotments and timber harvests, may generate income rather than deplete budgets. In some situations, revenue generating activities, in and of themselves, may not enhance habitat suitability. However, the revenues they generate may be spent on other management activities, thereby improving the cost effectiveness of the overall management plan.

The preceding points may be summarized by stating that the complexity of the typical habitat management problem is mostly due to the many interactions that must be analyzed simultaneously. Single habitat relationships or management effects are relatively well known. The difficulty lies in the fact that designing a plan requires the understanding of hundreds of simultaneous interactions between management activities and wildlife species. Thus, it would be desirable to have a means of systematically organizing the problem such that this complexity can be overcome.

Simplifying the Problem

The complexity of the habitat optimization problem is principally related to the many causal relationships rather than to a lack of biological information about any single interaction. Warfield [4], Miller [5], and Simon [6] have stated that the span of control of the human mind is limited to about seven items. Thus, a system involving the potential interaction of only 10 management activities with 10 wildlife species requires the ability to simultaneously analyze at least 100 possible management interactions, not including

other information such as management costs and constraints. Clearly, the human mind can become quickly overwhelmed by a habitat management problem when taken as a whole.

There are methods of dealing with this type of complexity. The first, and simplest, method is to base decisions on a small subset of data related to the problem. However, this method has the risk of limiting the scope of the problem to such a degree as to be unrealistic. The second method, which is used in this report, is a fundamental idea in systems analysis. Assuming that a phenomenon can be divided into small parts, these parts can then be divided into even smaller subparts until the individual subparts at the lowest level are small enough to easily comprehend and describe. This disaggregation idea can be used to help solve a habitat management problem by systematically writing down the cause and effect relationships between management activities and habitat responses so that the effects of each activity can be traced. Simplifying the habitat management problem in this way is accomplished by an initial disaggregation into two components: (1) habitat models, and (2) management activity functions.

Habitat Models.—Habitat models relate environmental conditions, such as vegetation, physical, and chemical attributes, to the food, cover, water, and space requirements of a particular wildlife species. Recent efforts have yielded several approaches to habitat modeling that are also useful for management planning. The approach around which this report was based is the *Habitat Evaluation Procedures* [1].

The HEP (Habitat Evaluation Procedures) were developed for use in impact assessment and planning (Schamberger and Farmer [7]). The HEP and associated modeling approaches are discussed in considerable detail in reference [1], only a brief overview is provided here.

The HEP describe habitat using an index called HU (habitat units), which are computed by taking the product of HSI (Habitat Suitability Index) and the area of habitat. The HSI is a dimensionless value between 0.0 and 1.0, and can be thought of as a ratio between two other numbers (Inhaber [8]). The two numbers making up the HSI ratio are a measure of the actual habitat conditions in an area and a measure of the best possible habitat conditions that could be obtained through management:

$$\text{HSI} = \frac{\text{actual habitat conditions}}{\text{optimum habitat conditions}}$$

In the above equation, the numerator and denominator must have the same units of measurement. Optimum habitat is defined such that the HSI does not exceed 1.0; i.e., optimum habitat is the maximum value on a specified scale of measurement. The scale used for the HSI must be defined by the user for each specific application because "optimum habitat" does not have a universal meaning.

The HSI is an index of carrying capacity as conceptually defined by Giles [9]. In this sense, HSI is an expression of the population limits with respect *only* to the habitat resources, such as food or nest sites, that are included in the model (Farmer et al., [10]). Each model may define the limits differently, therefore the need to clearly state the measurement scale in each case.

The framework of a typical habitat suitability model is illustrated on figure 1. The HU (level 1) are a function of HSI and habitat area, levels 2 and 4, respectively. Habitat suitability (level 2) depends on the suitability of life requisites (level 3), e.g., food and cover. In turn, the suitability of life requisites is a function of several habitat variables (level 4). The habitat variables at level 4 are measurable characteristics of vegetation, soils, landform, or water. All relationships between the four levels can be described mathematically, thus providing a means of quantifying habitat availability (i.e., HU's) in terms of measurable attributes of the environment. More importantly, however, is that by using the HU relationships, the effects of habitat management, as described in the next section, can be readily quantified.

Management Activity Functions. – The HU serves as the basis for describing the effects of habitat management. Expansion of the HU framework to level 5 (fig. 2) allows the relationship of management activities to a change in habitat units. The management activities must be well-defined actions that alter the habitat in some predictable way. For convenience, management activities can be classed into one of two general categories:

(1) Activities that are perceived to cause a change in the HSI (not AREA) of specific habitats by modifying vegetation, soils, water, or other habitat variables. This would include activities such as fencing, grazing control, and prescribed burning, the purpose of which is to enhance the suitability of a particular cover type with respect to resources (e.g., food and cover) used by a wildlife species; and

(2) Activities that are perceived to cause a change in the AREA of specific habitats by converting one surface cover type to another. This would include activities that alter the relative amount

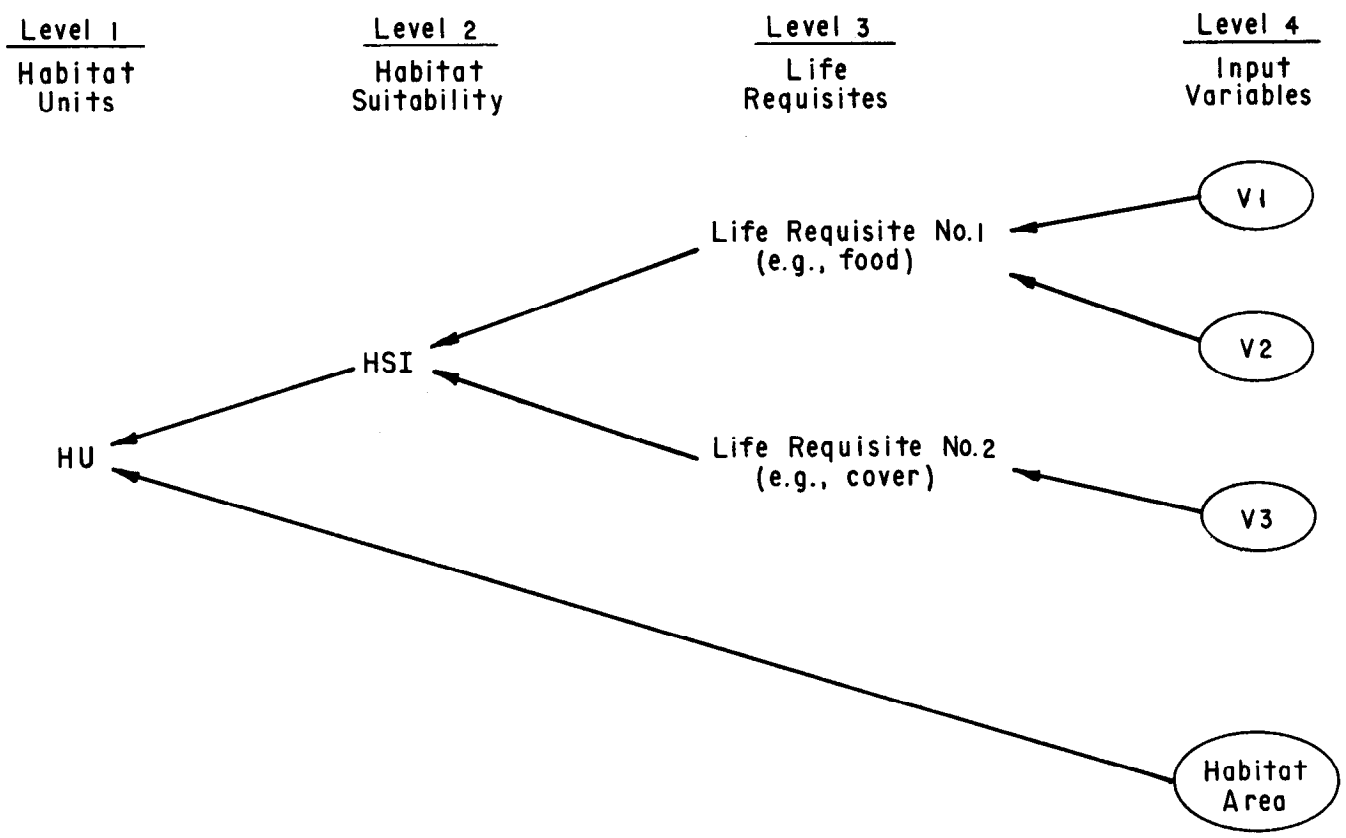

Figure 1. – Framework of a typical habitat suitability model.

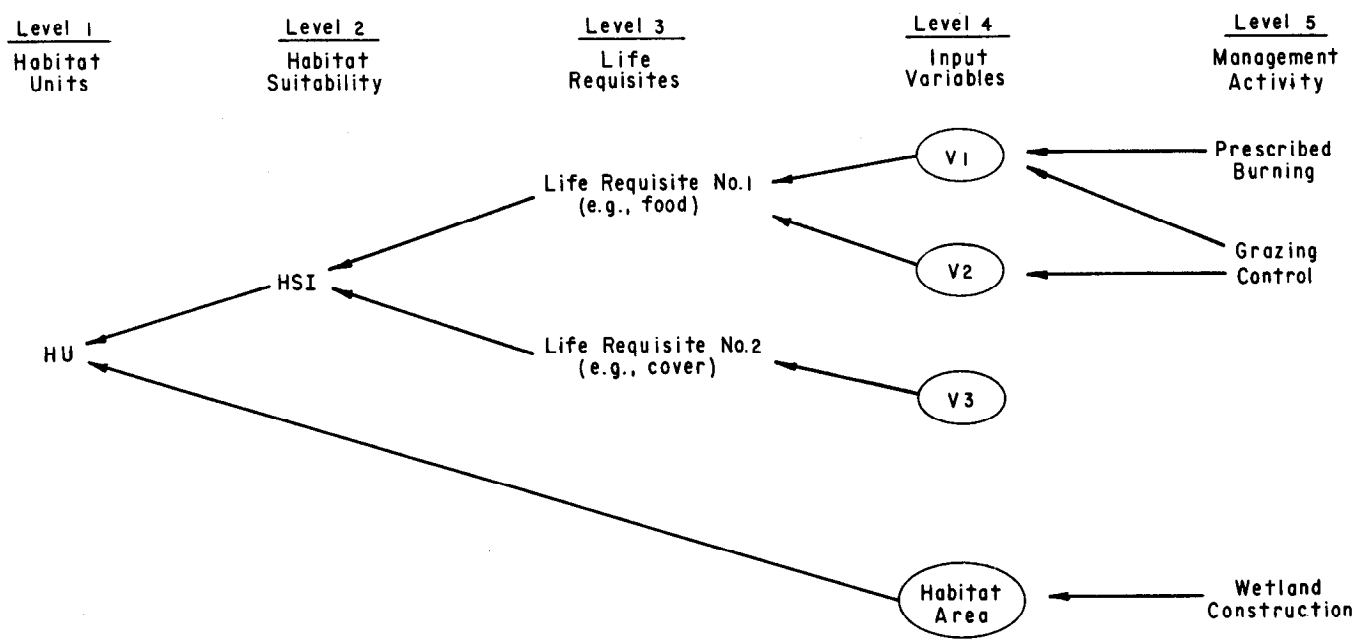

Figure 2. – Habitat framework extended to level 5 to incorporate management actions.

and interspersion of surface cover types within a given area. The conversion of one surface cover type to another usually requires some form of "construction" activity; e.g., constructing ponds or water impoundments, although conversion of surface cover types by seeding or planting new vegetation species is also possible.

The distinction between the previous two categories is somewhat arbitrary and can be made only on the basis of the particular surface cover classification used for a study. Every habitat management activity has an effect on vegetation, soils, or water, but it is the degree of the effect that is important in making the distinction. Management activities in the first category have a relatively slight effect on the habitat, and the surface cover designation of the affected area does not change. Activities in the second category have such a large effect on the habitat that the resulting conditions would direct changing the affected area to one of a different surface cover type.

Each management activity must be defined in units that are sensible and easily applied to a planning scenario. For example, wetland construction can be defined as: (1) number of wetland basins constructed (of a given size), or (2) number of acres constructed. The choice of units depends on the problem setting and preferences of the wildlife planner, but the units for all activities should be defined to be compatible with one another. For example, if "wetlands construction" is defined as the number of basins constructed, then "wetland maintenance" activities should be defined in the same units.

Extension of the HU framework to the management activity level requires developing mathematical relationships between each activity and the habitat variables. Each relationship should specify how a given habitat variable changes in response to that management activity. There are specific rules for developing these relationships, and these rules will be discussed in later sections. At this time, there are several things to note about figure 2. First, a management activity may affect either HSI or its independent variables, or AREA, as previously discussed. Second, a given input variable, such as *V1*, may be simultaneously affected by more than one management activity. For example, prescribed burning and grazing control both have an effect on density of grasses. However, some variables (e.g., *V3*) will not be affected by any defined management activity. This is typically the case for landform variables, such as slope and aspect, that are constant with respect to any conceivable management activity.

Relationships of management activities to habitat units have additional advantages. First, existing tools (e.g., HSI models) can be used to formulate management solutions. Second, there is a long-term economy in relating management activities to habitat variables rather than directly to wildlife species. Habitat models for many sympatric species can be linked to one set of management relationships. Thus, successive studies can make maximum use of management activity information, even though the emphasis may be on different wildlife species.

Solving the Problem

The reason for developing functional relationships between management activities and habitat units is to find the minimum cost alternative for achieving a desired habitat response. In solving this problem, it is desirable to compare many alternatives so that the absolute minimum cost management strategy can be reasonably assured of being found.

One obvious way to find the minimum cost alternative would be to translate habitat models, management functions, and management costs into computer coding. A computer program could then be written to mimic the flow of information shown on figure 3. The amount to spend on each management activity would be specified, and the computer would be instructed to output the expected habitat unit response and the total costs of the specified management activities. Different amounts of management activities would be tried and, eventually, combinations that meet or exceed the desired habitat response for each species would be found. The most economical alternatives would then be selected for more detailed study, or possible implementation.

The "trial-and-error" approach described above to find the minimum cost management alternative has certain advantages in that computer programs required to calculate total costs and habitat response are relatively simple to construct. Programs could even be developed to provide information that would guide the next "trial" toward an overall better mix of management information. Evans [11] and Andrews et al. [12] proposed similar trial-and-error computer programs, and they describe other advantages of this approach. However, the trial-and-error approach also has major disadvantages. There are an infinite number of possible management alternatives and, using trial-and-error, many feasible solutions could be identified; however, the absolute least costly solution would still not be known. Also, even if planning time was not important, it still might be impossible to find the optimum solution because of the theoretical flaws in the trial-and-error approach.

The optimization methods described in this report are based on what might be called a "smart trial-and-error" method of finding the minimum cost management alternative. The methods that will be described

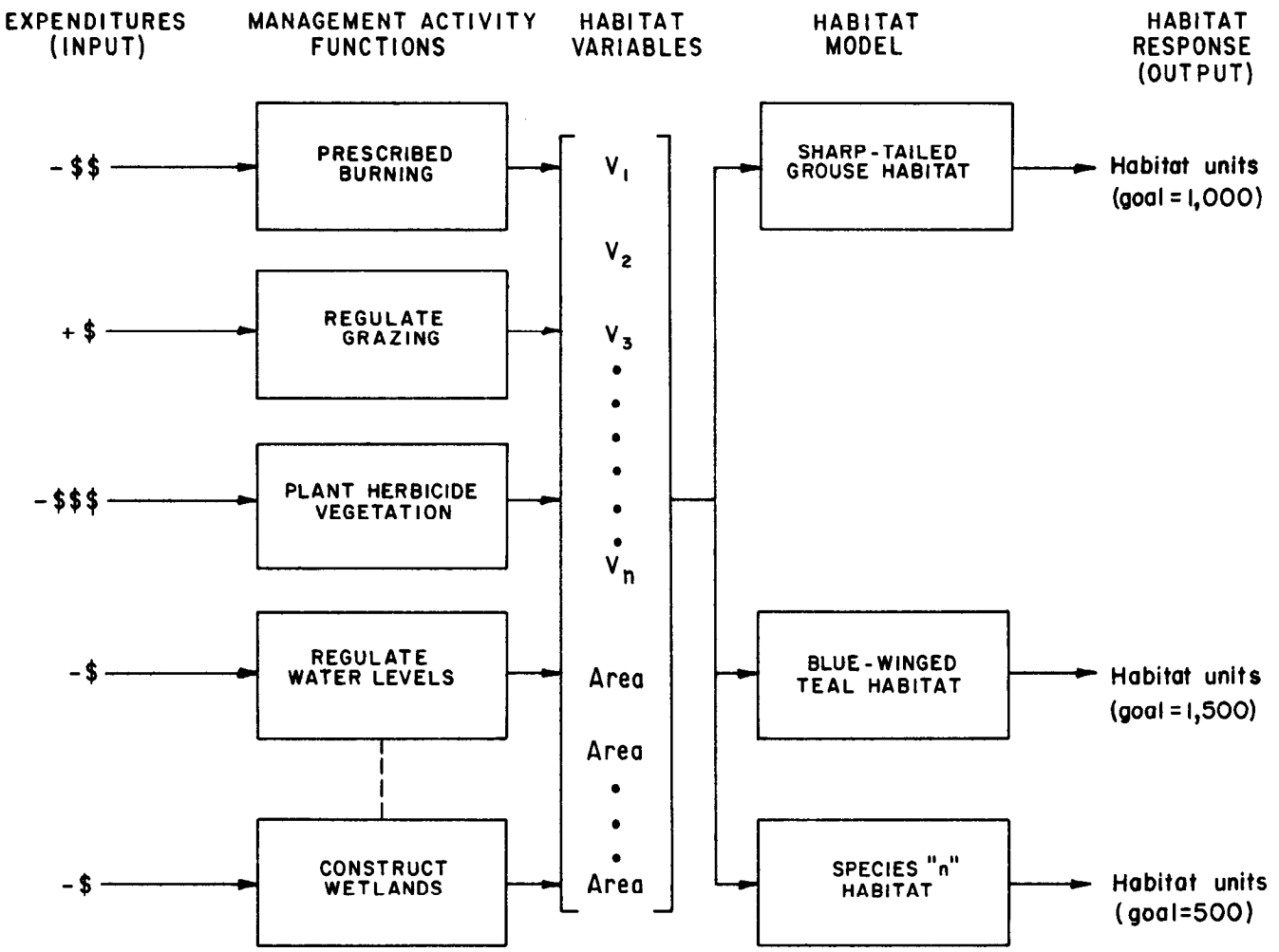

Figure 3. – Habitat management framework.

belong to a group called "mathematical programming" methods, which provide a systematic means of finding the minimum cost alternative. The associated mathematical computations are rather complex, but the underlying principles are relatively simple. You don't have to understand the mathematics to apply the methods because computerized solution procedures are readily available. The following section provides an introduction to the basic concepts and terminology of mathematical programming methods, and an example habitat management problem.

AN INTRODUCTION TO MATHEMATICAL PROGRAMMING

Mathematical programming problems require decisions about how to allocate limited resources to meet prescribed objectives. These problems involve situations where resources, such as people, machines, materials, and land, must be utilized in such a manner as to maximize or minimize some quantity, such as costs or profits (Hadley [13]).

Linear programming is the most basic type of mathematical programming. The mathematics of linear programming are relatively simple, the solution techniques are easy to learn and, in this sense, linear programming is a good introduction to mathematical programming concepts. The following habitat management example was developed principally to introduce the terminology and concepts of linear programming. The example is simplistic, but the concepts that emerge can be directly applied to larger, real-life management problems.

An Example Linear Programming Solution to Habitat Management

Assume the task of managing a tract of land for two species, sharp-tailed grouse and blue-winged teal. The available land is limited to 2,000 acres of agricultural land that currently provides no suitable habitat for either species. Any management plan must

produce at least 200 habitat units for each species, and can be comprised of varying amounts of two management alternatives: (1) constructing wetlands, and (2) planting and maintaining herbaceous vegetation.

Defining Objective Function and Constraints. – The problem is determining how many acres each of wetland construction and vegetation planting will provide the specified habitat units at minimum costs, given that wetland construction costs $100 per acre and grassland planting costs $75 per acre. The total cost of management is expressed:

$$C = 100W + 75V \tag{1}$$

where:
C = cost in dollars,
W = acres of wetland constructed, and
V = acres of grassland planted.

Minimizing the management cost means that the quantity $100W + 75V$, called the "objective function", should be as small as possible. The smallest possible nonnegative value of this function would be for $W = V = 0$; however, this is not realistic because no habitat improvement would occur and such a plan would be meaningless. There are only certain values of W and V that are realistic, and they are called "feasible" values. The set of feasible values is defined by "constraints." Therefore, to find feasible solutions to the problem, each constraint must be converted to mathematical form.

Response Constraint. – The response constraint is a mathematical statement that any plan must produce at least 200 habitat units each for sharp-tailed grouse and blue-winged teal. Assuming that no habitat exists for either species, prior to management, the response constraints would be:

$$HU(BWT) \geq 200 \tag{2}$$

$$HU(STG) \geq 200 \tag{3}$$

where:
$HU(BWT)$ = symbolic name for blue-winged teal habitat units, and
$HU(STG)$ = symbolic name for sharp-tailed grouse habitat units.

Management Constraint. – The response equations (2) and (3) only specify the desired end result of management. The relationship of the habitat units $HU(BWT)$ and $HU(STG)$ to management or, more specifically, to wetland construction W and vegetation planting V must be found. Following the arguments given in the previous section, the relationships between HU's and management can be described in two components: (1) HU's as a function of habitat variables, and (2) habitat variables as a function of management activities.

The relationship of HU's to habitat variables is given by the following equations:

$$HU(BWT) = 0.3A_W + 0.1A_V \tag{4}$$

$$HU(STG) = 0.3A_V \tag{5}$$

where:
$HU(BWT)$, $HU(STG)$ = as previously defined,
A_W = area of wetland habitat in acres, and
A_V = area of grassland habitat in acres.

Equations (4) and (5) are arbitrary and were fabricated to simplify the mathematics of this example. Both equations state the functional relationships between HU's and habitat variables, which in this case is the area of two surface cover types. In actual practice, the algebraic relationships between HU's, HSI, and area would require several equations rather than just one for each species. More realistic examples will be shown later in this report, this example is to demonstrate the concept.

Regarding the habitat variables as a function of management activities in this example, the management activities affect the area of specific cover types, i.e., they convert agricultural land to either wetlands or grasslands. The relationship between wetland area and wetland construction is as follows:

$$A_W = IW + W \tag{6}$$

where:
A_W and W = as previously defined, and
IW = initial area of wetland constructed in acres.

Equation (6) was written in the most generally applicable form; however, in this specific example, IW is zero and equation (6) reduces to the simpler form:

$$A_W = W \tag{7}$$

Following a similar line of reasoning, the equation relating grassland area to the grassland planting activity would be:

$$A_V = V \tag{8}$$

Equations (7) and (8) are trivial because the example has been made as simple as possible to facilitate an understanding of linear programming. Normally, the

algebraic relationships between habitat variables and management activities will require several equations rather than just one.

At this point, equations (4), (5), (7), and (8) completely specify the management constraint. Although not necessary for computer solutions, the example can be further simplified by combining the four constraint equations into two equivalent equations. Of the four equations, two involve A_V and two involve A_W. These pairs can be combined to eliminate A_V and A_W as follows:

$$HU(BWT) = 0.3W + 0.1V \quad (9)$$
$$HU(STG) = 0.3V \quad (10)$$

Thus, the four equations have been reduced to two equivalent equations expressing HU's only in terms of management activities.

Land Constraint. – Now assume that any management plan is limited to no more than 2,000 acres. Since there is no initial suitable habitat and habitat can be created only by constructing wetlands or planting grasslands, the land (resource) constraint can be written as follows:

$$W + V \leq 2{,}000 \quad (11)$$

Equation (11) shows that any amount of wetland construction and grassland planting can be implemented as long as the total number of units of both activities does not exceed the amount of available land. The equation is written as a sum of the activities because wetlands and grasslands cannot occupy the same space.

Nonnegativity Constraint. – Although not explicitly stated earlier, there are other constraints on the values of W and V. Since a negative amount of wetlands or grasslands is not realistic, the following constraints exist:

$$W \geq 0 \quad (12)$$
$$V \geq 0 \quad (13)$$

These nonnegativity equations are written here only for completeness of the example. In this case, the constraints are obvious and do not need to be written. In fact, most computer programs for solving linear programming problems assume nonnegativity and do not require the constraint equations.

At this point, the constraint equations for the example have been completed. Before proceeding to a solution, however, it would be helpful to further simplify the set of constraint equations to eliminate as many variables as possible. Although this is not necessary for the computer, it will make the remainder of the example easier to present. The two constraint equations (2) and (9) involve *HU(BWT)*, and equations (3) and (10) involve *HU(STG)*. These pairs can be combined to completely eliminate *HU(BWT)* and *HU(STG)*, and to reduce the four equations to two as follows:

$$0.3W + 0.1V = HU(BWT) \geq 200$$

$$0.3V = HU(STG) \geq 200$$

or

$$0.3W + 0.1V \geq 200 \quad (14)$$

$$0.3V \geq 200 \quad (15)$$

The example problem can now be summarized into standard linear programming form by using equations (14) and (15):

$$\begin{aligned} \text{Minimized cost } C = {} & 100W + 75V \\ \text{Subject to: } & 0.3W + 0.1V \geq 200 \\ & 0.3V \geq 200 \\ & W + V \leq 2{,}000 \\ & W \geq 0,\ V \geq 0 \end{aligned}$$

A *feasible* solution to this problem consists of values of W and V that simultaneously satisfy all the constraints, and there are many feasible solutions. An optimal solution is the one feasible solution that also minimizes costs. The goal is to select an optimal solution from the many feasible ones. The trial-and-error approach could be used to find the optimal solution by trying different feasible values for W and V, while always trying to reduce the value of the objective function. In actual practice, however, trial-and-error would be terribly inefficient owing to the infinite number of feasible values of W and V from which to choose. Instead, the problem can be solved using a graphical technique that is relatively efficient because it rapidly narrows down the set of choices to a few. In actual practice, this graphical technique would have only limited utility for reasons that will become obvious. However, graphical solutions are intuitively appealing because they clearly demonstrate concepts that are applicable to all linear programming problems. The graphical technique is composed of two steps: (1) determining the set of feasible solutions, and (2) identifying the optimal solution from the set of feasible solutions.

Determining Feasible Solutions. – The set of feasible solutions is determined by plotting all the constraints on a graph of W versus V. Initially, plot the nonegativity constraints. Nonnegativity simply requires that all values of W and V be nonnegative. This set of nonnegative values is the first quadrant in the W versus V plane, and is represented by the

shaded area on figure 4. Next, plot the land constraint. Figure 5 shows a plot of the line $W + V = 2{,}000$; values of W and V along this line total 2,000 acres. The shaded area below this line represents the set of W and V values totaling less than 2,000 acres. Combinations of W and V values lying either on the line or in the shaded area will total less than or equal to 2,000 acres and satisfy the land constraint. Similarly, the habitat response/management constraints shown by equations (14) and (15) were plotted on figure 6 for blue-winged teal and on figure 7 for sharp-tailed grouse. The habitat response goals for each species can be obtained by combinations of W and V within or on the boundary of the shaded areas.

The set of feasible solutions consists of the set of values for W and V that simultaneously satisfy all the constraints. This set of feasible solutions can be determined by combining all the previous plots. The feasible set is the area overlapped by all the shaded areas on figures 4, 5, 6, and 7. This set is shown on figure 8 by the darkly shaded triangle. Only combinations of values for W and V lying inside this triangle satisfy all the constraints; combinations outside the triangle will violate at least one constraint and will not produce feasible solutions.

Identifying Optimal Feasible Solution. – In the previous section, the set of feasible solutions by graphing the constraints was determined (fig. 8). In this section, a point in the feasible set that minimizes total management cost (i.e., the optimal solution) needs to be selected. To do this, plot the cost function for different total management costs. Three example cost lines are plotted on figure 9. Note on this figure that there are potentially an infinite number of cost lines, each corresponding to a different cost of management. All of these cost lines are parallel because the per unit cost of each management activity is constant. Furthermore, all combinations of W and V producing a certain cost lie on the same line.

Only one cost line is of interest, which is the lowest cost line that intersects the set of feasible solutions. Note that costs decrease moving to the left on the graph (fig. 9). Given this trend, the lowest cost line that intersects the feasible set corresponds to a cost of about $94,350 (fig. 10). This intersection point corresponds to the optimal solution because all criteria for optimality are met; i.e., the minimum cost solution that is also feasible with respect to the constraints. The optimal solution is represented by the coordinates associated with the intersection of the cost line and the lower left corner of the feasible set triangle, which results in a $W = 444$ acres and a $V = 666$ acres. The cost associated with this level of management can be verified by substituting these values for W and V into the cost function, equation (1):

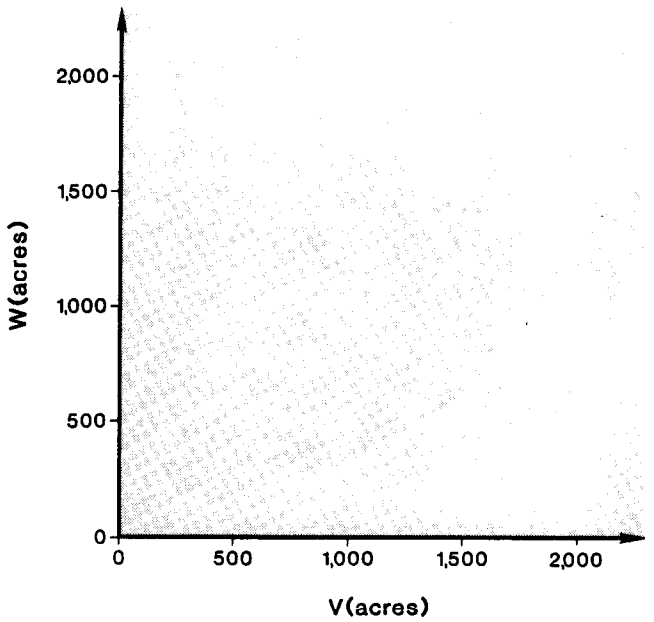

Figure 4. – Feasible space (entire first quadrant) for nonnegativity constraint.

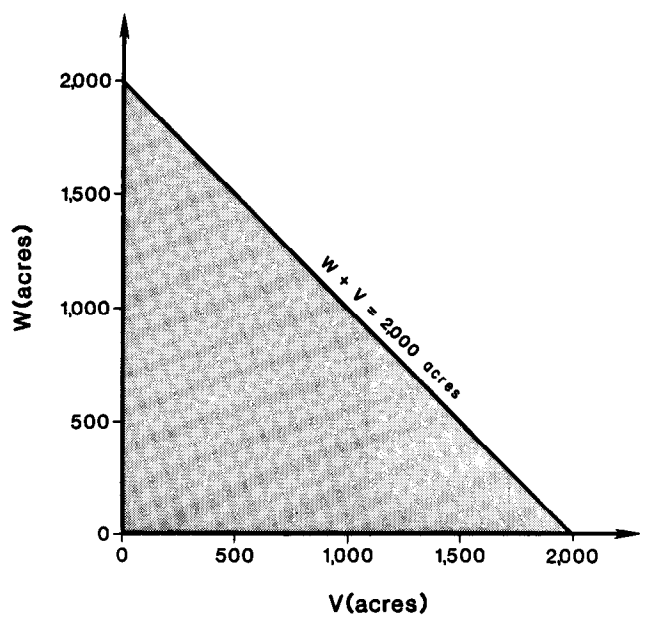

Figure 5. – Feasible space (shaded area) for land constraint.

$$C = 100W + 75V$$
$$= 100(444) + 75(666) = \$94{,}350$$

Solution Concepts

The preceding graphic example illustrates concepts that hold for all linear programming problems regardless of size or complexibility. The most fundamental concept is that an optimal solution will always be a point lying on the boundary of the feasible set.

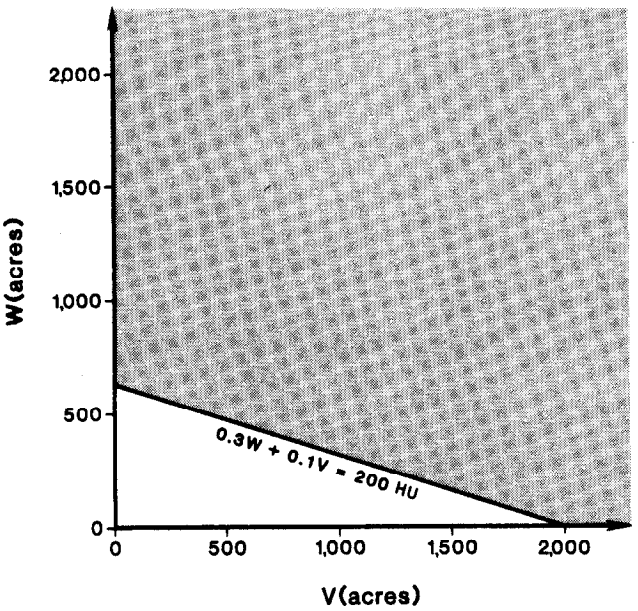

Figure 6. – Feasible space (shaded area) for blue-winged teal habitat response constraint.

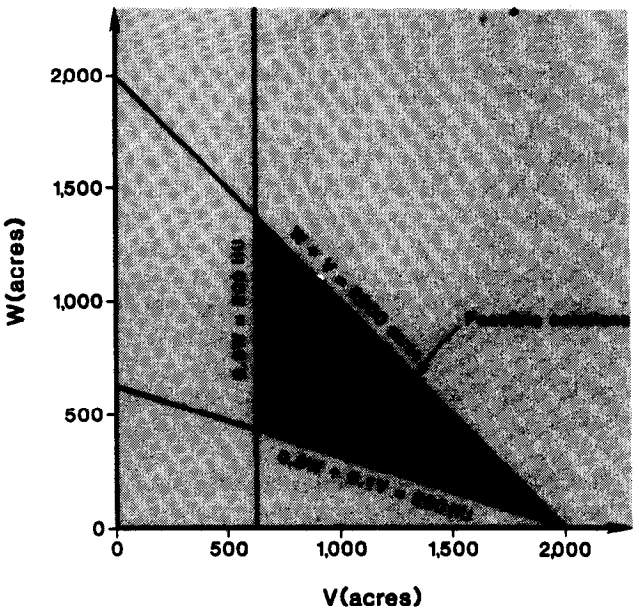

Figure 8. – Feasible space (darkly shaded area) for all constraints.

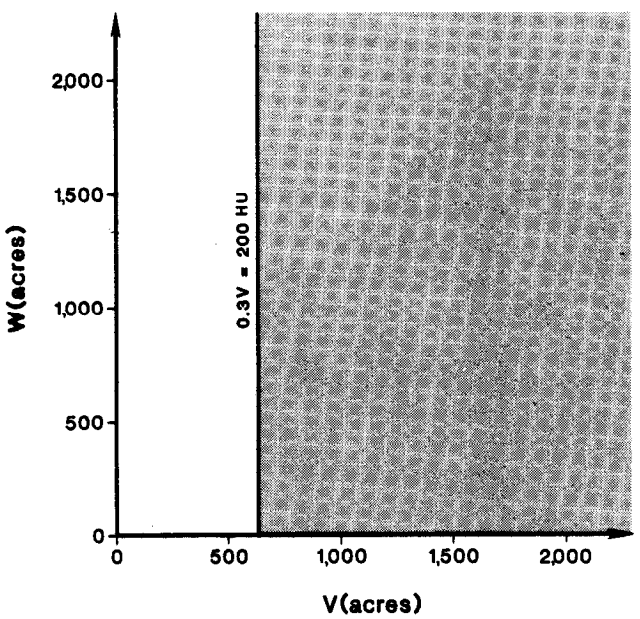

Figure 7. – Feasible space (shaded area) for sharp-tailed grouse habitat response constraint.

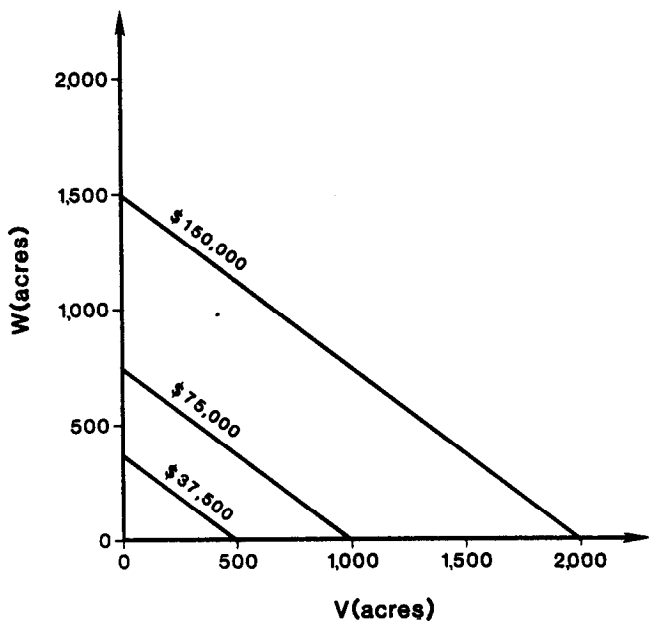

Figure 9. – Example cost lines. Each line defines all possible combinations of *W* and *V* producing that particular cost.

If this is not intuitively obvious, select an arbitrary point inside the boundary of the feasible set shown on figure 10; e.g., $W = 500$ acres and $V = 1{,}300$ acres. The costs associated with this interior point are about $150,000. Using equations (14) and (15), the habitat response from this solution would be:

$$\begin{aligned} HU(BWT) &= 0.3(500) + 0.1(1{,}300) \\ &= 280 \text{ habitat units} \end{aligned}$$

$$\begin{aligned} HU(STG) &= 0.3(1{,}300) \\ &= 390 \text{ habitat units} \end{aligned}$$

The magnitude of habitat unit gains associated with interior points is considerably higher than the specified minimum goal of at least 200 habitat units for each species. Therefore, management activities and costs could be reduced and not violate the 200-habitat unit minimum for each species. However, the

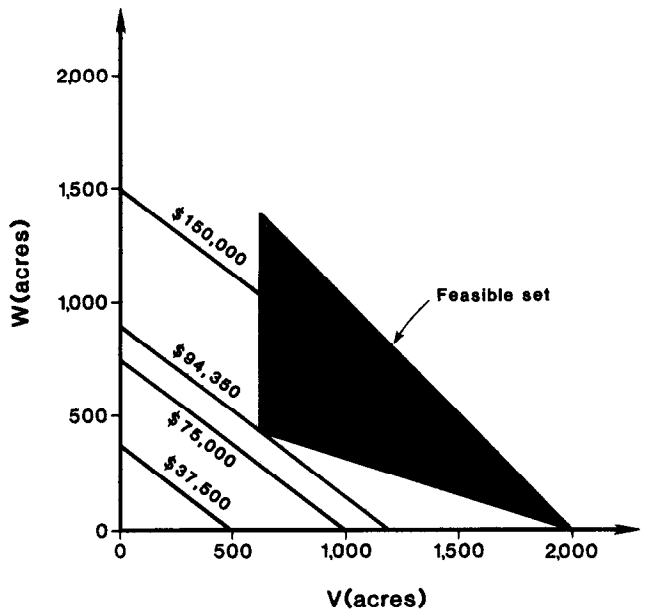

Figure 10. – Lowest cost line that intersects feasible set is optimal solution.

maximum reduction would be to the previously identified optimal point, which lies on the boundary of the feasible set and has an associated cost of $94,350. If costs were reduced below this point, say to $75,000, there would be no solution lying in the feasible set and one or more of the constraints would be violated. This balance between simultaneously minimizing the cost function and obtaining the desired habitat response is central to an understanding of the habitat management problem.

It should also be obvious that, in addition to lying on the boundary, the optimal solution also occurs at a corner point of the feasible set. The corner points are fundamental to the theory of linear programming and are called "basic feasible solutions". The occurrence of optimal solutions at corner points also has considerable practical significance. To find an optimal solution, only the corner points need to be considered, not the infinite number of feasible points. Since corner points satisfy all the constraints, the optimal solution is simply the corner point with the lowest associated cost in a cost minimization problem.

The graphic method for identifying and comparing corner points is limited to problems involving three or less decision variables, which also means three or less dimensions. Fortunately, there is an algebraic technique that works for any size problem and that mimics the concepts outlined by the two-decision variable example. This technique, known as the Simplex method, solves a linear programming problem by initially finding any arbitrary corner point, and then successively proceeding to other corner points that improve the objective function (e.g., reduce costs) until the optimum is found. The algebraic calculations involved are rather complex and beyond the scope of this report; however, numerous computer programs are available for use in solving linear programming problems by the Simplex method. The details of the Simplex method are described in many texts, those recommended would be Hadley [13], Hillier and Lieberman [14], or Wagner [15].

Linear Approximations of Nonlinear Systems

An assumption of linear programming is that all mathematical functions, including the objective function and the constraint equations, are linear in form. Linear functions are those that contain only terms that are either a constant or a constant multiple of a single variable to the first power. The general form of a linear function is:

$$y = C_0 + C_1X_1 + C_2X_2 + \ldots + C_nX_n \qquad (16)$$

where:

C_i = constants, and
X_i = variables to the first power.

Linear functions plotted in two dimensions form straight lines, as shown in the previous example where both the objective function and the constraints were linear. The geometric properties of linear functions ensure that optimal solutions occur at corner points in graphic solutions. The Simplex method is based on the special properties of linear functions.

Unfortunately, many wildlife habitat relationships are nonlinear in form. This would appear to limit the application of linear programming techniques for solving habitat management problems. Fortunately, there are ways around this apparent limitation.[3] In some cases, it may be initially assumed that certain relationships are linear when, in fact, they are nonlinear. Considerable judgement may be required to determine under what circumstances an assumption of linearity is reasonable. Obviously, the more curvilinear a function is, the more error that is introduced by assuming linearity. In many cases, however, it is possible to assume linearity and introduce little or no error into the overall solution. Also, there is a systematic way to approximate nonlinear functions; this approximation method is called "separable programming".

[3]There are "nonlinear" mathematical programming methods available when either the objective function or one (or more) of the constraints is not linear in form. These methods are not covered in this report, but are described in Wagner [15], and Hillier and Lieberman [14].

Separable programming is a way of approximating a nonlinear function by an equivalent linear form. The essence of separable programming is that a nonlinear function is represented by a series of linear segments (i.e., straight lines), as shown on figure 11. The number of linear segments can be selected so that there is a reasonably close approximation to the original functional form, although it is common to use no more than five or six segments for computational efficiency. In mathematical terms, the nonlinear function is represented by a piecewise linear approximation. Appendix A shows detailed examples on how to separate and represent a nonlinear function using piecewise approximations.

Computer programs are also available for solving separable programming problems. To do this, the computer program must have a feature called "restricted basis entry", which assures that interpolation along the straight line segments is done sequentially.

The major disadvantage of separable programming is that the linear approximations result in more equations than the original formulations. If many relationships need to be separated, a significant investment in time is required to write out all of the linear approximations. Also, not all functions are easily separated, see appendix A. The major advantage of separable programming is that the Simplex method, which is highly efficient and can be applied to virtually any size problem, can be used. Also, much wildlife habitat data, by nature of the way they are calculated, are amenable to separable programming. Wildlife studies frequently report compilations of single observations on habitat preferences. When a modeler uses these data, a common approach is to represent the isolated observations as single points connected by straight lines on a graphic function. Thus, many habitat model relationships are piecewise linear approximations initially and no additional transformations are required.

DEVELOPMENT OF A HABITAT MANAGEMENT MODEL

The linear programming example in the previous section was used to introduce basic concepts. To accomplish this, the example had to be simple and two-dimensional (involving only two-decision variables). Because of the simplicity of the example, it was possible to define the objective function and constraint equations and produce a graphic solution. The example was hypothetical, however, and consequently subject to the limits imposed by oversimplification. It is frequently difficult for novices to bridge the gap between simple graphic examples and actual applications, although in this case the concepts are iden-

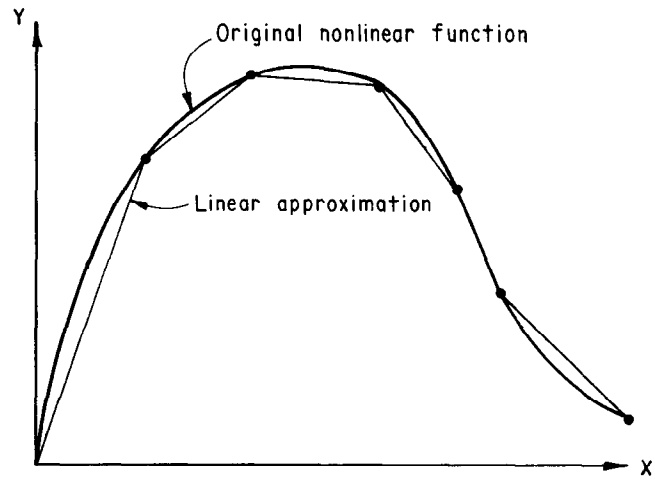

Figure 11. – Linear approximation of a curvilinear function.

tical. However, actual problems may involve hundreds of dimensions, and therefore are impossible to visualize in graphic form. To solve these larger problems, algebraic approaches and computers must be used to solve the corresponding linear or separable programming problems.

This section is developed around an actual example, the Bureau's Garrison Diversion Project in North Dakota. This example should be an excellent guide for developing linear programming models for habitat management. However, the Garrison model is comprised of constraint equations too numerous to describe in detail in this report and, even if done so, it is doubtful that this would have been more instructive. Many of the equations are similar in form, and listing all of them would not serve any useful purpose. Rather, only those equations that are representative of major portions of a typical programming model, or that illustrate a particular "trick," are presented.

Equations that were actually used for the Garrison study were frequently revised for this example to make a more straightforward presentation. Also, several new equations were necessary to provide a more complete example. All of these changes were made with a single objective in mind—the construction of an actual example for instructional purposes. Rigid adherence to the original Garrison model was not desirable to accomplish this objective.

The example problem is organized in the sequence that should be followed when developing equations for other habitat management problems. However, when constructing a model, it would be necessary to iterate through this sequence of steps many times. In each step, emphasis has been placed on the unique problems likely to be encountered in that phase of model building. There are also suggested conventions or "tricks" that are useful in solving the problems.

Problem Setting

Construction of the Garrison Diversion Project in North Dakota will result in the drainage of many acres of prairie potholes, the most significant waterfowl breeding habitat in the contiguous United States. The Federal Government, being mandated to mitigate losses, was charged with the task of developing an appropriate mitigation plan. In an effort to make this a cooperative study, private, State, and Federal entities were involved in the process.

Three issues confronted the planning team: (1) to what extent were habitat losses to be mitigated?, (2) how were habitat losses to be mitigated?, and (3) where were habitat losses to be mitigated?

For the first of these issues, extent of mitigation, there were two fundamental options. One option was in-kind mitigation for each of the target species: blue-winged teal, gadwall, and Hungarian partridge [1]. The second option was for out-of-kind mitigation, by which excess habitat units for one species could be used to offset habitat losses for a second species. These two options were addressed by developing two different sets of habitat response constraint equations, both of which will be described later.

The second issue, how mitigation was to be achieved, involved two problems. The first problem was to determine which management activities would be feasible. The final derived list of feasible management activities is given in appendix B. Other management activities, such as predator control, were considered, but were excluded. The second problem was that among the feasible activities, some included relatively large initial costs while others had relatively large O&M (operation and maintenance) costs. Participants in the study were concerned that any plan that relied heavily on O&M intensive activities would have a high probability of failure due to uncertainties of future year funding. Thus, several "runs" of the model included constraints on the maximum amount of annual O&M costs. These constraints are described later in the section on policy constraints.

The third issue, where mitigation should take place, or more specifically, should mitigation take place on lands in private ownership or on public lands, was the foremost issue in the planning effort. The State of North Dakota clearly voiced opposition to increased Federal land ownership. Also, conversion of private lands to the public sector was not viewed favorably by the conservation districts and farmers. Thus, several mitigation alternatives were developed using constraints on the maximum amount of private land acquisition. These constraints are described later in the section on policy constraints.

The following examples characterize how the management model for the Garrison study was constructed to integrate the three issues into the solution. Formulation of the objective function is presented first, followed by the constraint portion of the model. The constraints are by far the largest and most tedious part of the model, in terms of the number of equations. Consequently, only example constraint equations are included in this report. It will be helpful to explain the overall structure of the model to show how the examples relate to the entire problem.

The Garrison study area was characterized as containing three land types that were believed to be appropriate for management: (1) existing wetland tracts, (2) drained wetland tracts, and (3) public land. The "existing" wetland tracts were private lands on which wetlands made up at least 10 percent of the surface area. The drained wetland tracts were private lands on which drained, and restorable, wetlands made up at least 10 percent of the surface area. Public lands were owned in fee by either a State or Federal agency. The analyses were based on a typical tract of each land type, which contained an average proportion of the surface cover types.

An optimal solution specified how much of each land type was included in the design and how many units of activities were implemented on each land type. Each land type differed in its initial habitat condition; therefore, some means to keep them separate was necessary. This was done by duplicating the equations for each land type using different symbolic names. Thus, there were actually three sets of equations, one for each land type. Also, existing wetland tracts could be acquired through fee title or easement. The easements provided limited management rights in that the only feasible activity was the maintenance of existing wetlands (i.e., preservation by preventing drainage). A fourth set of equations was used to represent "easement lands" to explore the relative cost-effectiveness of easements versus fee title.

The example equations shown are mostly incomplete because they are for one land type and do not include all the duplicates for other land types and easements. Certain equations linking the land types are also not given; for example, equations that sum habitat units for each species across all land types. However, a general set of equations that cover most of the situations are given. The final portion of the example characterizes the optimal solution in the context of the Garrison issues.

Developing Objective Function

The objective function is a mathematical way to describe the quantity to be optimized; in this case, the

cost of management is to be minimized. To write an objective function, symbolic names must be assigned to each management activity (see app. B for all the symbolic names used in the Garrison example). Also, the unit cost of each management activity must be computed. The symbolic names of the activities and the unit costs are then combined to make a mathematical statement of the objective function.

Mathematical Statement of Objective Function . – The objective function must be a linear function for the management activities. The general form of an objective function is as follows:

$$Z = C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n \quad (17)$$

where:

Z = total cost of management including all n activities,
C_i = unit cost of activity i,
X_i = number of units of management activity i included in management plan, and
n = total number of discrete management activities.

To construct an objective function for a particular application, substitute for C_i a number representing the unit cost of that activity, and for X_i a symbolic name for that activity. The term X_i is a perfectly acceptable symbolic name; however, more meaningful symbolic names are usually used (e.g., LA to represent Land Acquisition).

In addition to the requirement that the objective function be linear in form, there is one other requirement; the C_i and X_i for a given management activity must be defined in compatible units. For example, if wetland construction is defined in acres, then the costs of wetland construction must be defined in dollars per acre. This compatibility is required because every term in the objective function must be in units of dollars; therefore, the units on the individual C_i and X_i, when multiplied, must combine to yield dollar units. Also, it may save time later in the model construction if all the X_i units are defined at this early stage because the management activities may later be combined algebraically in the constraint equations. For example, fence construction and grazing regulation are two management activities that interact; i.e., a fence must be constructed to effectively regulate grazing. To mimic this interaction, the activities have to be combined algebraically in several constraint equations. Suffice it now to say that, in the case of fencing and grazing, the constraint equations are easier to construct if both activities are defined in the same units—acres. Grazing regulation is usually described in acres; however, to describe fence construction in acres, a fixed amount of linear feet of fence per acre had to be assumed. The final decision about units for each X_i cannot be made logically until after all the constraint equations are formulated; however, as many units as possible should be standardized at this point in model construction.

As previously mentioned, the symbolic names for the management activities X_i are picked only as a matter of convenience; however, the cost coefficients C_i must be calculated according to specific rules, which are discussed in the next section.

Calculating Cost Coefficients. – The cost coefficients are the total costs for one unit of each management activity. The unit cost of a given activity can include initial capital costs as well as maintenance costs, and is a function of four specific items: (1) initial capital costs, (2) periodic replacement costs (for constructed structures needing replacement), (3) annual maintenance costs, and (4) periodic (nonannual) maintenance costs.

The cost items must be summed into a single cost coefficient by incorporating the time value of money. All wildlife habitat management plans are implemented over an extended time interval. However, the value of money changes over an interval of time and, since individual management cost items occur at different points in time, they are not on the same value basis. To put the different cost items on the same basis, they must be annualized. The annualized costs are then used as the cost coefficients in the objective function. Glenn and Barbour [17] provide an excellent description, including examples, of the time value of money. Two steps are necessary to calculate annualized cost coefficients: (1) determine present value of each cost item for a given management activity, and (2) total the present values for all cost items and amortize this total.

Computing present value. – The present value of a dollar amount to be spent in a future year is computed by the following equation:

$$PV = \frac{1}{(1 + r)^t}(FV) \quad (18)$$

where:

PV = present value of one dollar,
r = discount rate,
t = future year in which dollar amount is to be spent, and
FV = dollar amount to be spent in future year t.

Using equation (18), the present value of each relevant cost item for a given activity can be computed. For example, compute the present value of fence

construction, which includes several cost items. Assume the discount rate is 3.125 percent and the period of analysis is 100 years, to be consistent with the cost information given in appendix B:

Initial Costs

The initial cost of three-strand barbed wire fence is $48,600 per square mile, assuming 6 linear miles of fence, including materials and labor. The discounted value is given by:

$$PV = \frac{1}{(1 + 0.03125)^1} \$48,600$$
$$= (0.9697)\ \$48,600$$
$$= \$47,127.42$$

In this example, we assumed that the fence was not fully constructed until the end of the first year (i.e., $t=1$). If initial cost was assumed to occur at the beginning of the time interval (i.e., $t=0$), the initial cost would not be discounted and the present value would have been $48,600, the same as the initial cost of the fence. The rules for discounting the initial cost vary, but are often set through policy of a particular agency. If no other guidance is available, make your own decision about first year discounting (use $t=0$ or $t=1$); and apply it consistently for all management activities.

Replacement Costs

The assumed life expectancy of barbed wire fence is 25 years. At the end of this time, the fence would be replaced with new posts and wire. The replacement costs are $48,600 per square mile every 25 years so, for a 100-year period of analysis, three replacement cycles, at 25, 50, and 75 years, are required:

$$PV = \frac{1}{(1 + 0.03125)^{25}} (48,600) = \$22,518.39$$
$$PV = \frac{1}{(1 + 0.03125)^{50}} (48,600) = 10,433,70$$
$$PV = \frac{1}{(1 + 0.03125)^{75}} (48,600) = 4,834.36$$

Total replacement costs = $37,786.45

Annual O&M Costs

Annual O&M costs that are constant from year to year fall into a special category because they do not have to be discounted; therefore, the costs represent present worth. This may seem counterintuitive, but a level annual series is not affected by the annualization calculations. For this example, the annual O&M costs are $200 per mile of fence, or $1,200 per square mile based on an assumed 6 miles of fence per square mile.

Periodic Maintenance Costs

Included in these costs are O&M costs that either vary in magnitude from year to year or that do not occur every year. There are none of these costs for fence construction in the Garrison model; however, for activities associated with this item, the present value would be computed the same as periodic replacement costs:

$$PV = \frac{A_1}{(1+r)^1} + \frac{A_2}{(1+r)^2} + \ldots \frac{A_n}{(1+r)^n} \qquad (19)$$

where:

A_i = O&M costs in year i,
r = discount rate, and
n = period of analysis or time interval over which management is to be analyzed.

The present value amounts are totaled for all cost items (except the constant annual O&M cost) to find the total present value of the activity, which must be amortized to an average annual amount, or *AAEV* (average annual equivalent value). The *AAEV* is the cost coefficient being sought.

Computing AAEV. – The *AAEV* is computed as follows:

$$AAEV = (PVT)(AR) + AOM \qquad (20)$$

where:

AAEV = average annual equivalent value,
PVT = present value of all cost items except constant annual O&M cost,
AR = annuity rate $= \left(\frac{r}{1 - \frac{1}{(1+r)^n}}\right)$,
r = discount rate,
n = project life, and
AOM = constant annual O&M cost.

Continuing with the fence construction example:

$$PVT = 47,127.42 + 37,786.45 = \$84,913.87$$

$$AAEV = 84,913.87 \left(\frac{0.03125}{1 - \frac{1}{(1 + 0.03125)^{100}}}\right) + 1,200.00$$
$$= (84,913.87)(0.0328) + 1,200.00$$
$$= \$3,985.18 \text{ per square mile, or}$$
$$= \$6.23 \text{ per acre}$$

Cost coefficients for the other management activities are computed following the same steps and are given in apprendix B. When these

computations have been completed for each activity, the objective function is written following the form of equation (17) using the calculated values for *AAEV* and the algebraic symbol for each activity.

Developing Response Constraints

The response constraints specify the minimum number of habitat units to be achieved by management. These constraints do not specify how to calculate the habitat response, they establish the desired goal of management. There are two fundamental choices for the response constraints: (1) in-kind, and (2) out-of-kind.

In-kind constraints are used when it is appropriate to set response goals independently for each species. There would be one equation for each species as follows:

$$\begin{aligned} (HU_1 - IHU_1) &\geq R_1 \\ (HU_2 - IHU_2) &\geq R_2 \\ &\vdots \\ (HU_m - IHU_m) &\geq R_m \end{aligned} \qquad (21)$$

where:

HU_i = mean annual number of habitat units for species i with management,

IHU_i = mean annual number of habitat units for species i without management,

R_i = desired mean annual habitat unit response for species i due to management, and

m = number of species considered.

In equation (21), the habitat resource goals R_i are defined as a number that represents a *change* in habitat conditions. These numbers are derived from statements such as "the goal is to increase blue-winged teal habitat units by x." The initial conditions IHU_i are also set at a constant value based on an initial inventory of the management area. When writing these constraints for an actual study, substitute the actual numbers for IHU_i and R_i. The post management conditions HU_i are variables, and their value is determined by a linear programming solution. The Garrison model was set up for in-kind mitigation for three species as follows:

$$\begin{aligned} (HU_1 - IHU_1) &\geq 13{,}336 \\ (HU_2 - IHU_2) &\geq 9{,}421 \\ (HU_3 - IHU_3) &\geq 0 \end{aligned}$$

where:

HU_1 = mean annual number of habitat units for blue-winged teal with management,

IHU_1 = mean annual number of habitat units for blue-winged teal without management,

HU_2 = mean annual number of habitat units for gadwall with management,

IHU_2 = mean annual number of habitat units for gadwall without management,

HU_3 = mean annual number of habitat units for Hungarian partridge with management,

IHU_3 = mean annual number of habitat units for Hungarian partridge without management,

13,336 = desired habitat unit response for blue-winged teal,

9,421 = desired habitat unit response for gadwall, and

0 = desired habitat unit response for Hungarian partridge. There were no losses for which to mitigate, but this constraint was defined to avoid selection of a management plea that would negatively impact partridge.

The constraints on the right-hand side of the above inequalities were set equal to the number of habitat units lost as a result of the Garrison Project [18].

Out-of-kind constraints are used when it is appropriate to consider all targeted species collectively, rather than independently. Typically, only one equation is required to specify this constraint, which generally could be stated in the following form:

$$(HU_1 - IHU_1)T_1 + (HU_2 - IHU_2)T_2 + (HU_3 - IHU_3)T_3 + \ldots + (HU_m - IHU_m)T_m \geq CR \qquad (22)$$

where:

HU_i = mean annual number of habitat units for species i with management,

IHU_i = mean annual number of habitat units for species i without management,

T_i = relative human preference value associated with habitat units for species i,

m = total number of species considered, and

CR = desired total habitat unit response, measured in commensurated habitat units.

When using this type of constraint, there should be no concern with how many habitat units are gained for any one species, but rather that the total units, weighted by relative values, equals or exceeds some overall amount. Gains for one species offset losses for another and, in principle, the desired response could be achieved by gains in habitat units for only one of the species while units for the others decline. Out-of-kind constraints were not actually used in the Garrison model; however, examples for developing out-of-kind constraint equations in the context of mitigation are provided by the U.S. Fish and Wildlife Service [1]. Also, there are techniques available for systematically developing the relative value coefficients based on judgements of each species' social value, biological importance, or scarcity [19].

Developing Management Constraints

The next set of equations specifies the relationships between habitat units and management activities, which is the heart of the linear programming model and also the most difficult to develop. The difficulty is not due to the complexity of any single equation, but rather to the relatively large number of equations required. We have tried to simplify the development of these equations, but this section is still somewhat tedious and requires patience by the reader. The management constraints are of the following general form:

$$HU = f(MA_1, MA_2, \ldots, MA_n) \qquad (23)$$

where:

HU = number of habitat units for a given species,

MA_i = number of units of management activity i, and

n = number of discrete management activities.

Equation (23) is only an implicit statement of the cause and effect between the habitat units and management. To be useful, this implicit equation must be turned into explicit equations describing how various amounts of management activities alter the habitat variables, and ultimately the number of habitat units. In the previous example, only one equation was required for each of the wildlife species, equation (2) for blue-winged teal and equation (3) for grouse; however, numerous equations are generally required to trace the relationships between management and habitat units.

Management constraint equations can be written in any order; however, since many equations may be required, a systematic approach is desirable to keep track of the equations that have been completed. Recalling earlier discussions, this effort can be simplified by subdividing the problem and writing the equations in two different sets: (1) all equations describing habitat suitability; these equations relate an index of habitat response (i.e., HU's) to vegetation, landform, and water attributes of the habitat (i.e., a habitat model); and (2) a set of equations that describes how a given amount of each of the management activities will change the value of the habitat variables. The two sets of constraint equations describe the presumed cause and effect relationships between management and wildlife response. The equations for the blue-winged teal habitat model are developed next, followed by the equations for management activities that affect teal habitat.

Blue-Winged Teal Habitat. — The first set of equations describes the number of habitat units as related to the value of selected habitat variables. The first of these equations relates habitat units to two components, the habitat suitability index and habitat area.

$$HU(BWT) = AREA\ (BWT) \times HSI \qquad (24)$$

where:

$HU\ (BWT)$ = habitat units for blue-winged teal, in acres;

$AREA\ (BWT)$ = area of blue-winged teal habitat, in acres, and

HSI = Habitat Suitability Index for blue-winged teal, unitless.

The product of *HSI* and *AREA(BWT)* is a nonlinear function and must be reformulated using piecewise linear approximations as described in appendix A. The habitat area, *AREA (BWT)*, for the blue-winged teal model is simply the total amount of land acquired for management, including all surface cover types. The blue-winged teal *HSI* is described in detail by Sousa [20], and summarized below with brief explanations.

The blue-winged teal *HSI* equations were developed to assess the suitability of the breeding habitat, which is assumed to be related to three habitat components: pair habitat, brood habitat, and nesting habitat. These components are related to other habitat variables. The structure of the model is shown on figure 12.

Many equations are required to completely represent the *HSI* model, so some systematic way to write them is beneficial. By following the structure of the model (fig. 12), an equation can be developed for *HSI*, and then for the first life requisite, which is nesting habitat. By starting at the top of figure 12 and going left to right, formulate all the equations for the first life requisite before proceeding to the second requisite. Many of the equations will be nonlinear and will require separation and reformulation as piecewise linear approximations using separable programming. To condence this example as much as possible, all of the piecewise approximations will not be developed herein.

The *HSI* is a function of three life requisites[4]:

$$HSI = (NESTI \times PAIRI \times BROODI)^{1/3} \qquad (25)$$

where:

HSI = Habitat Suitability Index for blue-winged teal,

$NESTI$ = nesting habitat suitability index,

$PAIRI$ = pair habitat suitability index, and

$BROODI$ = brood habitat suitability index.

[4] The *HSI* model in Sousa [20] uses a different function; *HSI* in his model is the minimum of the three indices.

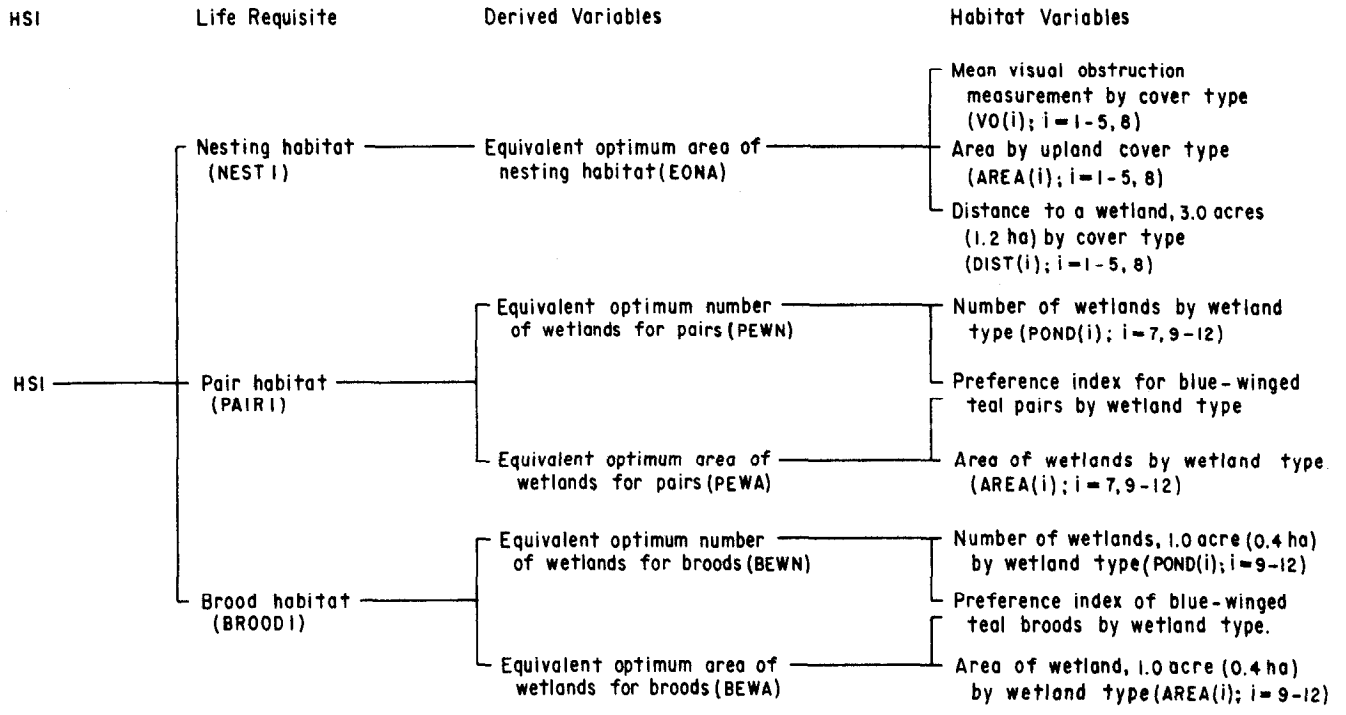

Figure 12. – Structure of the Habitat Suitability Index model for blue-winged teal. The index numbers, *i*, identified for each habitat variable are the cover types within which that variable is measured. All cover type index numbers are defined in appendix B.

The life requisite indices in equation (25) are based on a concept called "equivalent optimum," a concept that attempts to take into account the fact that certain cover types, or wetland types, seem to be preferred habitat. The model incorporates preference factors, with a factor of 1.0 being the most preferred condition. For example, the standard of comparison for brood habitat is defined as "a minimum of six wetlands of the most preferred type per square mile." If an actual area contained eight wetlands per square mile, but had a preference value of 0.5, this condition would be expressed as an equivalent optimum of (8) (0.5) = 4.0. The value 4.0, when divided by the standard of comparison, which is 6.0, produces an overall index for brood habitat of 0.67.

Nesting habitat. – The suitability of nesting habitat within an entire management area is a function of the equivalent optimum area of nesting habitat. Nesting habitat has a maximum suitability (*NESTI* = 1.0) when there are at least 480 acres of equivalent optimum habitat per square mile (*EONA* ≥ 480 acres). This functional relationship is shown on figure 13. The graphic relationship shown on this figure must be represented by equations following the separable programming procedure in appendix A. Since the function shown on figure 13 is piecewise linear initially (i.e., grid points are already specified), only the last step (i.e., the Delta method) in appendix A needs to be done. The Delta method is shown below for example purposes, but will not be shown for the remaining nonlinear functions. The equations resulting from the Delta method for figure 13 are:

$$NESTI = (1/480)\,(NA_1) \tag{26}$$

$$EONA = NA_1 + NA_2 \tag{27}$$

$$NA_1 \leq 480 \tag{28}$$

$$NA_2 \leq 160 \tag{29}$$

where:

NESTI = index of nesting habitat suitability;
EONA = equivalent optimum area of nesting habitat, in acres per square mile; and
NA_1, NA_2 = special linearizing variables, in acres per square mile.

In turn, the *EONA* is the summation of the equivalent optimum nesting habitat provided by all pertinent cover types, in this case, three different types:

$$EONA = EOGA + EODA + EOW2A \tag{30}$$

where:

EONA = equivalent optimum area of nesting habitat;
EOGA = equivalent optimum area of nesting habitat in grassland cover types, except cover type "dense nesting cover";

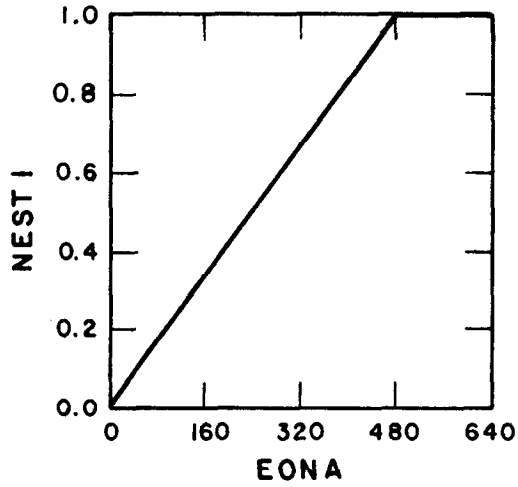

Figure 13. – Suitability index relationship for equivalent optimum area of nesting habitat.

EODA = equivalent optimum area of nesting habitat in DNC (dense nesting cover), a grassland type planted for purposes of waterfowl management; and

EOW2A = equivalent optimum area of nesting habitat in type 2 wetlands.

All four areas are in acres per square mile.

The equivalent optimum area for the individual cover type is a function of three variables: (1) area of that cover type, (2) distance to nearest brood wetland and visual obstruction, and (3) a measure of the height and density of residual vegetation [21]. The equations are:

$$EOGA = AREA(3) \times DISTI(3) \times VOI(3) \quad (31)$$
$$EODA = AREA(5) \times DISTI(5) \times VOI(5) \quad (32)$$
$$EOW2A = AREA(8) \times DISTI(8) \times VOI(8) \quad (33)$$

where:

EOGA, *EODA*, and *EOW2A* as previously defined,

AREA(3) = area of grassland, in acres;

DISTI(3) = index of mean distance from within grassland to nearest wetland greater than 3 acres;

VOI(3) = index of mean visual obstruction measurement for grassland;

AREA(5) = area of DNC, in acres;

DISTI(5) = index of mean distance from within DNC to nearest wetland greater than 1 acre;

VOI(5) = index of mean visual obstruction for DNC;

AREA(8) = area of wetland type 2, in acres;

DISTI(8) = index of mean distance from within wetland type 2 to nearest wetland greater than or equal to 1 acre; and

VOI(8) = index of mean visual obstruction measurement for wetland class 2.

Equations (31), (32), and (33) are nonlinear and must be separated using the separable programming procedures in appendix A. Also, the indices *DISTI(i) and VOI(i)* are determined from the functional relationships on figures 14 and 15. These functions relate the indices to actual measures of mean distance and mean visual obstruction, and these relationships must be discussed alphabetically using the Delta method (app. A).

Pair habitat. – Blue-winged teal pairs use wetlands for feeding, loafing, and courtship prior to nesting. The pair habitat is a function of the area of wetlands and the number of wetlands per unit area as follows:

$$PAIRI = (EOWAPI \times EOWNPI)^{0.5} \quad (34)$$

where:

PAIRI = suitability index for pair habitat,

EOWAPI = index of equivalent optimum area of wetlands,

EOWNPI = index of equivalent optimum number of wetlands, and

0.5 = a constant, the value of which was selected to get a geometric mean of the two variables.

In turn, the indices *EOWAPI* and *EOWNPI* are functions of the equivalent optimum area and number of wetlands. With respect to the number of wetlands, the best condition for pair habitat is a minimum of 150 equivalent optimum wetlands per square mile. The functional relationship between number of wetlands per square mile and *EOWNPI* is shown on figure 16. The Delta method must be used to represent this relationship algebraically.

With respect to the area of wetlands, the best condition for pair habitat is a minimum of 160 equivalent optimum acres per square mile of wetlands. The functional relationship between *EOWAPI* and pair habitat is shown on figure 17. This piecewise linear graphic relationship also must be converted to equation form using the Delta method.

The *PEWN* (equivalent optimum number of wetlands) for pairs is a function of the number of basins and pair preference values for each wetland class:

$$PEWN = 0.72\ POND(7) + 1.0\ POND(9) + 0.93\ POND(10) + 0.44\ POND(11) + 0.02\ POND(12) \quad (35)$$

where:

PEWN = equivalent optimum number of wetlands for pairs,

POND(7) = number of type 1 wetlands,

POND(9) = number of type 3 wetlands,

POND(10) = number of type 4 wetlands,

POND(11) = number of type 5 wetlands,

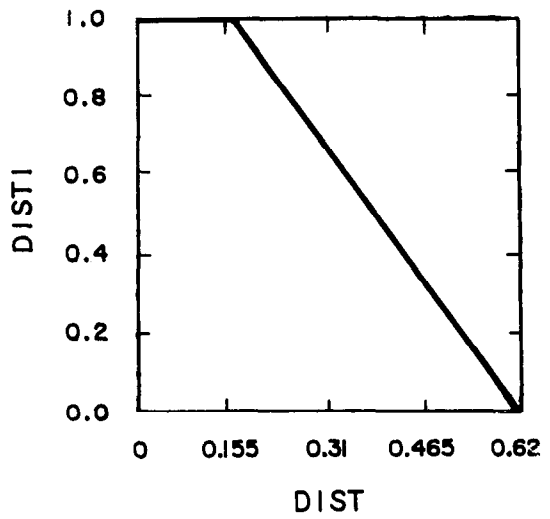

Figure 14. – Suitability index relationship for distance to a wetland.

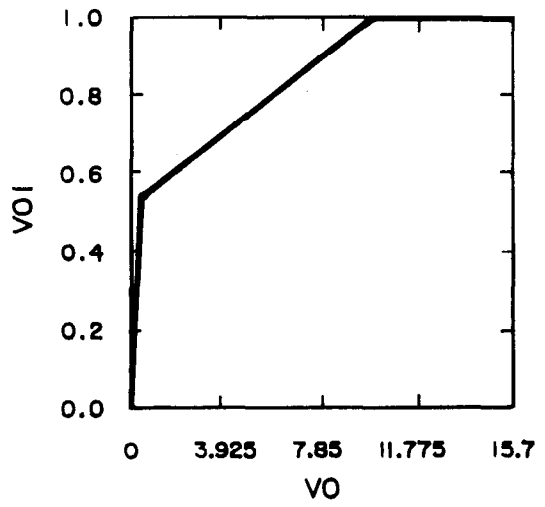

Figure 15. – Suitability index relationship for mean visual obstruction measurement.

POND(12) = number of saline wetlands,
0.72 = pair preference value for wetland type 1,
1.0 = pair preference value for wetland type 3,
0.93 = pair preference value for wetland type 4,
0.44 = pair preference value for wetland type 5, and
0.02 = pair preference value for saline wetlands.

The equivalent optimum area of wetlands for pairs is a function of the area of each wetland class and the pair preference values for each wetland class. The equation, which is similar to equation (35), is as follows:

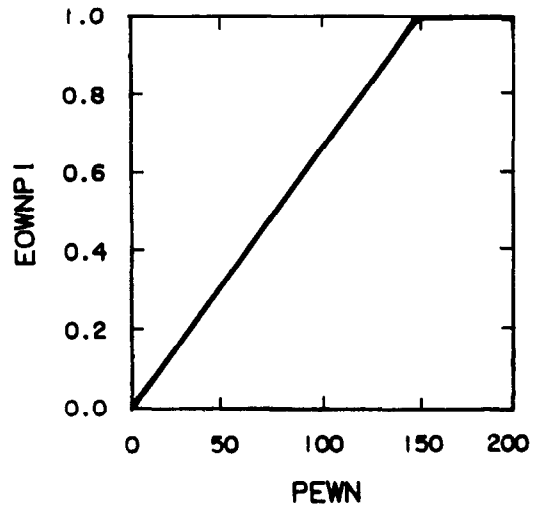

Figure 16. – Suitability index relationship for equivalent optimum number of wetlands for pairs.

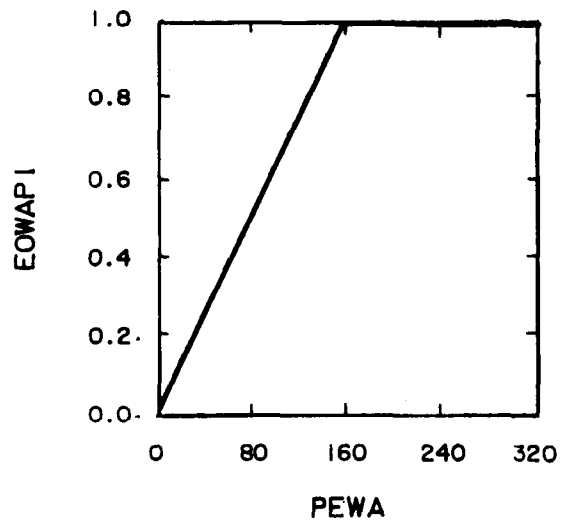

Figure 17. – Suitability index relationship for equivalent optimum area of wetlands for pairs.

$$PEWA = 0.72\ AREA(7) + 1.0\ AREA(9) + 0.93\ AREA(10) + 0.44\ AREA(11) + 0.02\ AREA(12) \quad (36)$$

where:
PEWA = equivalent optimum area of wetlands for pairs, 0.72, 1.0, 0.93, 0.44, and 0.02 as previously defined;
AREA(7) = surface area of wetland type 1,
AREA(9) = surface area of wetland type 3,
AREA(10) = surface area of wetland type 4,
AREA(11) = surface area of wetland type 5, and
AREA(12) = surface area of saline wetlands.

All the above areas are in acres.

Brood habitat. – The brood habitat component of the *HSI* model is similar to the pair component. Brood habitat suitability is related to indices of the equivalent optimum number and area of wetlands as follows:

$$BROODI = (EOWNBI \times EOWABI)^{0.5} \qquad (37)$$

where:

BROODI = suitability index for brood habitat,
EOWNBI = index of equivalent optimum number of wetlands,
EOWABI = index of equivalent optimum area of wetlands, and
0.5 = a constant, the value of which was selected to get a geometric mean of the two variables.

In turn, the indices *EOWNBI* and *EOWABI* are functions of the *BEWN* (equivalent optimum number) and *BEWA* (equivalent optimum area). With respect to the number of wetlands, the best condition for brood habitat is at least six equivalent optimum wetlands per square mile. The functional relationship (fig. 18) must be converted to an algebraic equation by the Delta method.

With respect to the area of wetlands, the best condition is a minimum of 50 equivalent optimum acres per square mile of wetlands. The functional relationship is shown on figure 19, and representative equations must be derived by the Delta method.

The equivalent optimum number and area of wetlands for brood rearing habitat differ from the equivalent optimum values for pair habitat in two ways. First, where types 1, 3, 4, 5, and saline wetlands provide pair habitat, broods utilize only types 3, 4, 5, and saline wetlands. Second, the preference values for a given wetland type are different for broods and pairs. The relationship for equivalent number of wetlands for broods is:

$$BEWN = 0.5\ POND(9) + 1.0\ POND(10) + 0.5\ POND(11) + 0.15\ POND(12) \qquad (38)$$

where:

BEWN = equivalent optimum number of wetlands for broods,
POND(9), *POND*(10), *POND*(11), and *POND*(12) are previously defined wetland numbers for equation (35),
0.5 = brood preference value for wetland types 3 and 5,
1.0 = brood preference value for wetland type 4, and
0.15 = brood preference value for saline wetlands.

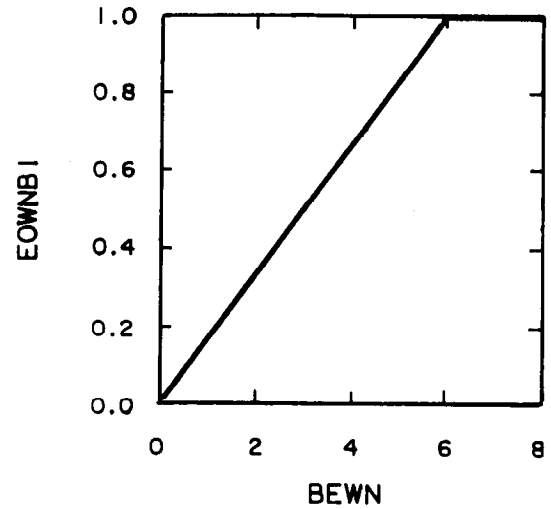

Figure 18. – Suitability index relationship for equivalent optimum number of wetlands for broods.

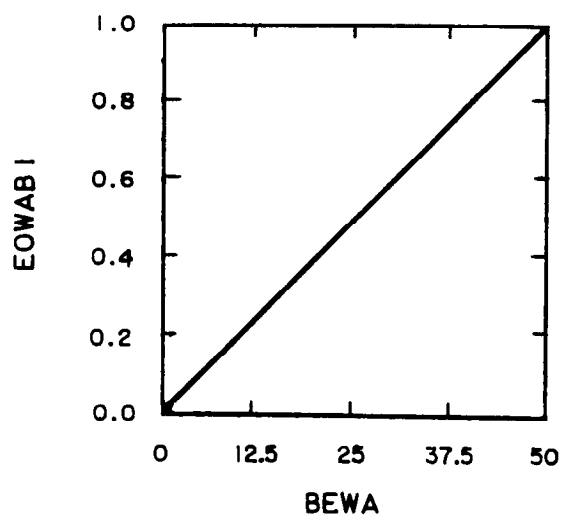

Figure 19. – Suitability index relationship for equivalent optimum area of wetlands for broods.

Equivalent optimum area for broods is a function of the area and the brood preference values for each wetland class as follows:

$$BEWA = 0.15\ AREA(9) + 1.0\ AREA(10) + 0.5\ AREA(11) + 0.15\ AREA(12) \qquad (39)$$

where:

BEWA = equivalent optimum area for broods,
0.5, 1.0, and 0.15 are previously defined brood preference values for equation (38); and

AREA(9), *AREA*(10), *AREA*(11), and *AREA*(12) are previously defined wetland areas for equation (36).

This completes the specifications for the first set of management constraint equations, those pertaining

to the habitat model for blue-winged teal. Habitat model equations for the other species would normally be completed at this point before moving on to the second set of management constraint equations, which are the relationships between management activities and habitat model input variables.

Management Activities Affecting Blue-Winged Teal Habitat. — The second set of constraint equations describes the relationships between management activities and the habitat model input variables. These equations can be thought of as an extension of the habitat model, as shown on figure 2.

There usually are many ways that management activities can interact with habitat model variables, so a systematic way of identifying and keeping track of the interactions is desirable. A matrix format is useful for this purpose. The matrix showing the interactions between management activities and the input variables[5] for the blue-winged teal model is shown in table 1.

The row headings in table 1 are the input variables for the blue-winged teal habitat model, these variables were used in several of the model's equations. The variables are grouped into five categories, two related to upland cover types, two related to wetland classes, and one pertaining to the interspersion of uplands and wetlands.

The column headings in table 1 identify the management activities considered in the Garrison study, and have also been grouped into categories. The first category, "vegetation development," consists of all the activities that affect the amount of area in specific upland cover types. The second category, "vegetation manipulations," consists of all the activities that affect the structure of vegetation within a given cover type. The final category, "wetland development," consists of all the activities that affect the abundance of wetlands.

The individual activities within the three categories require further explanation. First, some of the activities are entitled to "maintain" a specific cover type; e.g., maintain native grassland. These activities are synonymous with preservation of that cover (or wetland) type because, without the activity, land conversions or drainage would have occurred. Maintenance activities were identified explicitly to take into account the habitat unit increase associated with preservation, and also to include certain maintenance costs associated with the activity. However, not all acquired upland cover types could be maintained in their existing condition. A political decision was made that cropland and alfalfa must be converted to some other cover or wetland type. Furthermore, neither existing woodlands nor existing dense nesting cover could be converted to other upland types or wetlands. The remaining upland cover types could all be converted to wetlands either through restoration or construction of new wetlands. Cropland, tame grasslands, and alfalfa could also be converted to other upland vegetation types; i.e., to native grassland, woodland, or dense nesting cover; however, no cropland, tame grassland, or alfalfa could be planted.

Vegetation manipulation was to consist of fencing and livestock grazing control. The fencing option was an all or nothing activity, either the entire mitigation area was to be fenced or no fence could be built. Grazing control was allowed only with the existence of a fence, in which case, grazing leases were feasible. However, Federal regulations stipulate that no revenue could be used to offset mitigation costs.

Five wetland types were considered, ranging from seasonal marshes to open-water lakes and saline basins; i.e., types 1, 3, 4, 5, and a combination of 9, 10, or 11, as categorized by Shaw and Fredine [22]. Varying amounts of all five wetland types are found throughout North Dakota, all may be "maintained," but none can be drained. It was assumed that only four of the five types (1, 3, 4, and 5) could be restored, while wetland construction was limited to types 3 and 4. Each existing wetland type was assumed to have fixed size and shape characteristics, as determined from field inventory studies; restorable and constructed wetlands each differed in size from existing wetlands. Wetland maintenance impacts and associated costs for constructed wetlands were incorporated into the construction activity definition. It was assumed that no maintenance occurs on existing or restored wetlands.

A final comment is in order regarding the symbolic names used in table 1 and the equations. Some of the activities in table 1 are generic in the sense that there are several cover types within which the activity can be applied. The symbolic name for these activities is identified by the variable subscript *i*, which is the cover type identification number. For example, planting native grassland *VD(i,2)* is actually three discrete activities: (1) plant in cropland *VD*(1,2), (2) plant in alfalfa *VD*(2,2), and (3) plant in dense nesting cover *VD*(4,2). The complete list of cover types and corresponding subscript numbers is given in appendix B.

The X's in the matrix shown in table 1 indicate management activities that affect a given habitat variable. A habitat variable may be affected by more than one

[5]In this example, only variables for the blue-winged teal are listed; in practice, variables for all HSI models would be included in the matrix.

Table 1. – Cross reference of management activities to habitat variables for the blue-winged teal.

Habitat variables (blue-winged teal)	Vegetation Development			Vegetation Manipulations	
	Plant DNC VD(*i*, 1)	Plant native grassland VD(*i*, 2)	Maintain native grassland VD(3, 3)	Grazing regulations VM(*i*, 1)	Fence construction VM(*I*, 2)
Visual Obstruction					
VO(3)				X	X
VO(4)				X	X
VO(8)				X	X
Area of Upland Cover Types					
AREA(3)		X	X		
AREA(4)	X	X	X		
AREA(5)	X				
AREA(1)	X	X	X		
AREA(2)	X	X			
Number of Wetlands					
POND(7)					
POND(9)					
POND(10)					
POND(11)					
POND(12)					
Area of Wetland Types					
AREA(7)					
AREA(8)					
AREA(9)					
AREA(10)					
AREA(11)					
AREA(12)					
Distance to Nearest Wetland (J1)					
DIST(1)					
DIST(2)					
DIST(3)					
DIST(4)					
DIST(5)					

activity, and a given activity may affect more than one habitat variable. The X's were placed in the matrix based on judgement, experience with the particular habitats, and knowledge of study area conditions. Constraint equations must be developed for each row of the matrix that contains one or more X's, and these equations should be of the following general form:

$$V(NEW) = V(OLD) + (CHANGE\ DUE\ TO\ MANAGEMENT) \quad (39)$$

where:

V(NEW) = value of habitat variable with management, and

V(OLD) = value of habitat variable without management.

Equation (39) is called a "difference" equation, it simply increments the variable value by an amount proportional to the amount of management performed. Difference equations have the property that if no management is done, the variable is equal to its "without management" value. The value of the variable changes from its baseline value by a prescribed amount dependent only on the number of units of management applied. Difference equations for all activities follow the same general format; however, some variables may be affected by more than one management activity and, in those cases, the equations can be more complex than equation (39). Also, some variables may not be affected by any management activity and will not require difference equations. For example, the distance variables in table 1 were assumed to be constant with respect to management.

The constraint equations for the blue-winged teal variables will be developed by row, starting at the

Table 1. – Cross reference of management activities to habitat variables for the blue-winged teal.—Continued

Habitat variables (blue-winged teal)	Wetland Development											
	Maintain type 1 wetland WD(7,1)	Maintain type 2 wetland WD(8,1)	Maintain type 3 wetland WD(9,1)	Maintain type 4 wetland WD(10,1)	Maintain type 5 wetland WD(11,1)	Maintain saline wetland WD(12,1)	Construct type 4 WD(i,3)	Construct type 3 WD(i,2)	Restore type 1 WD(i,4)	Restore type 3 WD(i,5)	Restore type 4 WD(i,6)	Restore type 5 WD(i,7)
Visual Obstruction												
VO(3)												
VO(4)												
VO(7)												
Area of Upland Cover Types												
AREA(3)							X	X	X	X	X	X
AREA(4)							X	X	X	X	X	X
AREA(5)												
AREA(1)	X	X	X	X	X	X	X	X	X	X	X	X
AREA(2)							X	X	X	X	X	X
Number of Wetlands												
POND(7)	X								X			
POND(9)			X					X		X		
POND(10)				X			X				X	
POND(11)					X							X
POND(12)						X						
Area of Wetland Types												
AREA(7)	X								X			
AREA(8)		X										
AREA(9)		X										
AREA(10)			X					X		X		
AREA(11)					X							X
AREA(12)						X						
Distance to Nearest Wetland (J1)												
DIST(1)			X	X	X	X	X	X		X	X	X
DIST(2)			X	X	X	X	X	X		X	X	X
DIST(3)			X	X	X	X	X	X		X	X	X
DIST(4)			X	X	X	X	X	X		X	X	X
DIST(5)			X	X	X	X	X	X		X	X	X

top in table 1. The following equations are not comprehensive because all activities were not considered. Rather, we have tried to show examples that can be followed in developing a similar model.

Visual obstruction. – The equations relating management to visual obstruction are similar for each of the three affected cover types: native grassland, tame grassland, and type 2 wetlands. The following equations are given only for native grassland and would be similarly developed for the other cover types. The mean visual obstruction measurement for native grassland is affected by two management activities: AUM (animal unit month) regulation and fencing. For the Garrison study, the following relationship was assumed:

$$VO(3) = BVO(3) + \Delta VO(3) \qquad (40)$$

where:

$VO(3)$ = mean visual obstruction measurement for native grassland with management, in decimeters;

$BVO(3)$ = value of mean visual obstruction measurement without management, in decimeters; and

$\Delta VO(3)$ = change in mean visual obstruction due to management, in decimeters.

The change in mean visual obstruction $\Delta VO(3)$ is, in turn, a function of grazing regulation and fence construction, and involves the interaction of both activities:

$$\Delta VO(3) = \Delta VOG(3) \times FENCEI \qquad (41)$$

where:

$\Delta VO(3)$ = change in mean visual obstruction with respect to grazing regulation and fence construction, in decimeters;

$\Delta VOG(3)$ = change in mean visual obstruction with respect to grazing regulation, in decimeters; and

$FENCEI$ = an index relating mean visual obstruction measurement to fence construction.

Grazing regulation impacts vegetation height and density; i.e., the visual obstruction measurement. The relationship between grazing, *VM(i,2)*, and changes in the grassland visual obstruction measurement, *VO*(3), was derived from field observation and professional judgement (fig. 20). Both positive and negative changes in the baseline visual obstruction are possible, depending on whether grazing decreased or increased. If grazing is maintained at baseline levels, there will be no change in the mean visual obstruction; i.e., $\Delta VOG(3) = 0$. If grazing is increased, $\Delta VOG(3)$ will be negative. The maximum sustained grazing pressure, 0.45 AUM, will cause reduction of the Robel Index by an estimated 0.189 units. Conversely, elimination of grazing results in an increase of the Robel Index by an estimated 0.79 units. Equation (42) is an algebraic representation of the change in mean visual obstruction as a function of grazing:

$$\Delta VOG(3) = 0.79 - 2.175\ VM(3,1) \qquad (42)$$

where:

$\Delta VOG(3)$ = change in mean visual obstruction measurement for grassland, in decimeters;

0.79 = maximum value of ΔVOG occurring when AUM = 0.0;

−2.175 = slope of the function; i.e., the change in *VO*(3) for every additional AUM per acre; and

VM(3,1) = AUM per acre in grassland.

However, grazing regulations cannot be considered independent of the fencing action because fence construction provides the means of regulating grazing. The fencing action is defined in terms of the percent of habitat protected by a fence. As defined for the Garrison study, fencing protection was an all or nothing action, either the entire study area perimeter was to be fenced, or no fence could be constructed. Also, no interior fencing was permitted within a perimeter. Thus, when the fencing action *VM(i,2)* is at 100 percent, the entire management unit perimeter is fenced (6 miles of fence per square mile).

Admittedly, fencing need not be an all or nothing action, and additional interior fencing may be a desirable management alternative; however, the low resolution nature of the Garrison mitigation study allowed these simplifying assumptions. The appearance of the graphic function relating fence construction to *FENCEI* is a consequence of the all or nothing assumption regarding the fencing activity (fig. 21). The step function shown on figure 21 could not have a completely vertical face because the Delta method for using piecewise linear approximations would not have worked; i.e., a vertical segment has an infinite slope, which the Delta method cannot handle.

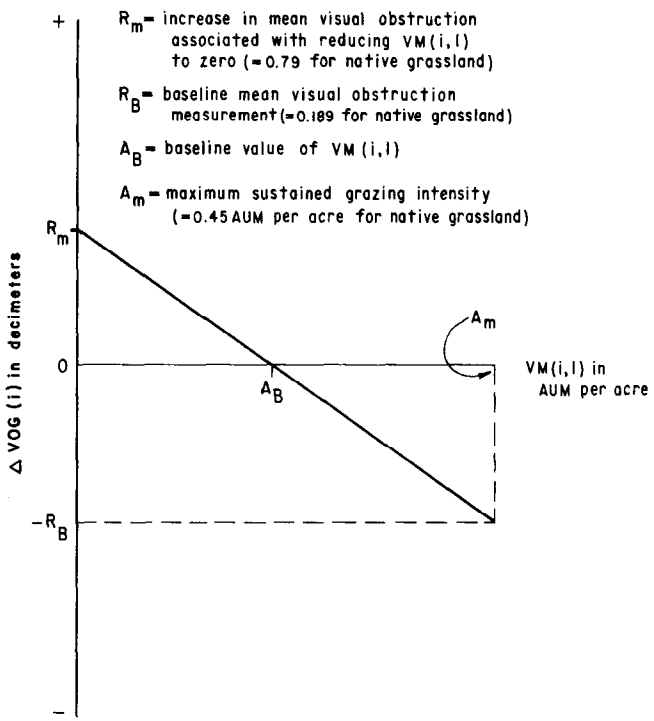

Figure 20. – Relationship between grazing and change in mean visual obstruction measurement.

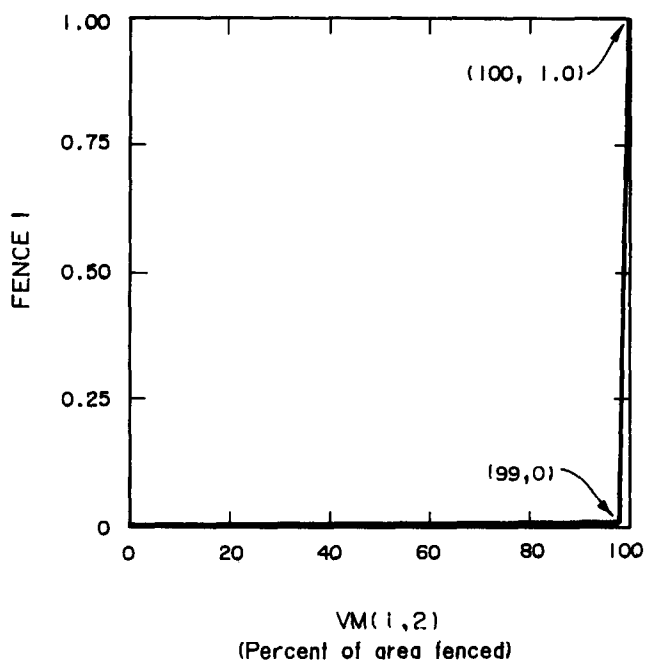

Figure 21. – Relationship between percent of habitat fenced and change in mean visual obstruction measurement.

If equation (42) is substituted into equation (41), the resulting equation for grazing and fencing is:

$$\Delta VO(3) = [0.79 - 2.175\ VM(3,1)] \times FENCEI \qquad (43)$$

Equation (43) contains a product term and is nonlinear. In this study, the logarithmic transformation was used for separating the product term into an equivalent linear expression. Unfortunately, the range of the product relationship is from −0.189 to 0.79, the range of $\Delta VOG(3)$, and the log of a negative number does not exist. To remedy this problem, figure 20 and equation (42) must be reindexed so that the range of $\Delta VOG(3)$ includes only nonnegative values. This was done by adding 0.189 to both sides of equation (42), and the entire curve on figure 20 is shifted into the positive quadrant. The revised equation then becomes:

$$VOG(3) + 0.189 = 0.979 - 2.175\ VM(3,1) \quad (44)$$

If equation (44) is then substituted into equation (41) as before, the result is:

$$\Delta VO(3) = [0.979 - 2.175\ VM(3,1)] \times FENCEI - 0.189\ FENCEI \quad (45)$$

Equation (45) can now be separated into an equivalent linear expression. Also, equation (45) meets the other requirements, which can be verified as follows: If fence is not constructed ($FENCEI = 0.0$), the value of $\Delta VO(3)$ is zero; i.e., visual obstruction cannot change because grazing intensity cannot be regulated. If fence is constructed ($FENCEI = 1.0$), the value of $\Delta VO(3)$ is between +0.79 and −0.189. The product term is never less than zero because $VM(3,1)$ is never larger than 0.45. Thus, all desired characteristics are met. As previously stated, the equations for tame grassland and type 2 wetlands would be similarly developed using their unique values for A_B, A_M, R_B, and R_M.

Area of upland cover types. – The area of upland cover types is affected over time by several management activities. The equations for area of native grassland are given below:

$$AREA(3) = BAREA(3) + \Delta AREA(3) \quad (46)$$

where:

$AREA(3)$ = mean annual area of native grassland with management, in acres;

$BAREA(3)$ = mean annual area of native grassland without management, in acres; and

$\Delta AREA(3)$ = change in mean annual area caused by management, in acres.

There are preservation credits for maintaining existing native grassland because a certain amount of the grassland area would otherwise be converted to cropland. In this study area, it was estimated that by future year 100, 80 percent of the existing grassland would be converted to cropland if not protected; i.e., only 20 percent would remain at the end of 100 years. Further, it was assumed that the area versus time relationship was linear and that the mean annual grassland area would therefore be 60 percent of the initial area. Based on these simplifying assumptions, only 40 out of every 100 maintained acres, on the average, could be credited to management because the remaining 60 percent of the baseline area would be available whether or not any management is undertaken. Consequently, if no existing area is maintained and no new area is planted, the 100-year average area of grassland would be only effectively 0.6 times the initial area. Rewriting equation (46) to reflect this preservation percentage:

$$AREA(3) = 0.6\ IAREA(3) + \Delta AREA(3) \quad (47)$$

where:

$AREA(3)$ and $\Delta AREA(3)$ are as previously defined, and

$IAREA(3)$ = initial (first year) area of native grassland, in acres.

The change in area due to management, $\Delta AREA(3)$, is a function of several activities: maintenance of existing grassland, planting of grassland, and wetland development, which decreases the amount of grassland:

$$\Delta AREA(3) = 0.4\ VD(3,3) + \sum_{i=1,2,5} VD(i,2) - \sum_{k=2,7} [A(k)\ WD(3,k)] \quad (48)$$

where:

$\Delta AREA(3)$ is as previously defined,

$VD(3,3)$ = number of units of grassland to be maintained, in acres;

$VD(i,2)$ = number of units of grassland to be planted in host cover type i, in acres;

$WD(3,k)$ = number of wetlands of type k constructed or restored in grassland;

$A(k)$ = mean size of wetland type k, in acres; see table 2 for values of $A(k)$.

By combining equations (47) and (48), the overall expression for the area of native grassland as a function of management is:

$$AREA(3) = 0.6\ IAREA(3) + 0.4\ VD(3,3) + \sum_{i=1,2,5} VD(i,2) - \sum_{k=2,7} [A(k) \times WD(3,k)] \quad (49)$$

Equation (49) can be verified as follows: If no management is done, all but the first term on the right side of the equation are zero, and grassland area is 60 percent of its initial value. If all grassland is maintained; i.e., $VD(3,3) = IAREA$, then 100 percent of the initial area of grassland is available into the future.

Table 2. – Mean size of constructed, restored, and baseline wetlands by wetland type.

Wetland type	Constructed, acres	Restored, acres	Baseline, acres
1	ND	0.5	0.59
2	ND	ND	3.29
3	1.0	5.0	18.45
4	5.0	25.0	136.15
5	ND	25.0	30.17
Saline (9, 10, 11)	ND	ND	164.60

ND indicates not defined.

The grassland area is increased by additional planting, $VD(i,2)$, or decremented by wetland creation, $WD(3,k)$.

The equations for the other upland cover types are similar to native grassland, except for DNC (dense nesting cover). The area of DNC is simply a function of the amount planted:[6]

$$AREA(5) = \sum_{i=1,2,4} VD(i,1) \qquad (50)$$

where:

$AREA(5)$ = area of dense nesting cover, in acres; and

$VD(i, 1)$ = amount of dense nesting cover planted in host cover type i, in acres.

There was no initial dense nesting cover in the Garrison study area. Consequently, there was no activity for maintenance of existing DNC, nor was there a historic loss rate without management, as was the case with grassland.

Number of wetlands. – The number of wetlands is affected by maintenance, construction, and restoration of drained wetlands. The number of type 1 wetlands is a function of the initial number, maintenance, and restoration of type 1 wetlands:

$$POND(7) = 0.89\ IPOND(7) + 0.11\ WD(7,1) + \sum_{i=1}^{5} WD(i,4) \qquad (51)$$

where:

$POND(7)$ = mean annual number of type 1 wetlands with management,

$IPOND(7)$ = initial number of type 1 wetlands,

$WD(7,1)$ = activity, maintain type 1 wetlands,

$WD(i,4)$ = activity, restore drained type 1 wetlands, and

i = host cover type within which drained wetlands are restored.

The coefficients 0.89 and 0.11 in equation (51) define the preservation credit for maintaining baseline wetlands.[7] Based on historic drainage rates, it was assumed that in the absence of management, 22 percent of both the number and area of wetlands would be drained and converted to cropland by future year 100. It also was assumed that the wetland decline over time would be linear; thus, the average annual number of wetlands would be 11 percent (i.e., half of 22%) lower than the initial value. Consequently, in the absence of any management, the number of type 1 wetlands would be only 89 percent (100% – 11%) of the baseline value. The balance, 11 percent, is the preservation value of maintenance.

The number of saline wetlands (types 9, 10 and 11) was treated differently than other wetland types. The number of saline wetlands is a function only of the initial number for two reasons: (1) there were no management activities that had any effect on the number of saline wetlands, and (2) saline wetlands typically are not drained because their associated soils are not arable and there was no preservation due to maintenance. Thus, it was assumed that the number of saline wetlands would stay constant at the baseline value:

$$POND(12) = IPOND(12) \qquad (52)$$

where:

$POND(12)$ = mean annual number of saline wetlands with management, and

$IPOND(12)$ = initial number of saline wetlands.

In principle, equation (52) was not needed because $POND(12)$ was a constant. However, it was explicitly included so that a nonzero drainage rate could be assumed and thus show a credit for maintenance if this was later determined to be a more likely depiction of reality.

Area of wetland types. – The functions for the area of wetland types were derived from the equations for wetland number by factoring in values for the mean size (in acres) of each wetland type.[8] The values used for mean size of each wetland type are given in table 2. Note that in this table there is more than one value for the mean size of a given wetland type as the value depended on whether a particular

[6] Negligible baseline DNC was assumed, so that maintenance of existing DNC was not a defined activity. It was further assumed that DNC planted as a management activity would not be converted to other cover types or wetlands.

[7] These same coefficients were used for all wetland types; consequently, the same future without drainage rate was assumed for all wetland types, except saline wetlands, which were assumed to have a zero drainage rate.

[8] There was no equation for the number of type 2 wetlands because this was not a variable in the blue-winged teal HSI model.

wetland basin existed at baseline or was to be constructed or restored through management. The values for mean size were multiplied by the appropriate variable in the equations for wetland number to produce equations for wetland area. Using equations (51) and (52), the area of wetlands for type 1 and saline wetlands, respectively, are:

$$AREA(7) = 0.89[0.59\ IPOND(7)] + 0.11[0.59\ WD(7,1)] + \sum_{i=1}^{5} 0.5\ WD(i,4) \quad (53)$$

$$AREA(12) = 164.6\ IPOND(12) \quad (54)$$

Distance to nearest wetland. – The maintenance, construction, and restoration of wetlands has the effect of reducing the distance between nesting habitat (upland cover types) and brood rearing areas. In the Garrison study, however, the effects of management on the interspersion distance could be ignored because it was never a limiting factor. The initial (i.e., pre-management) distances were near optimum and constructing new wetlands did not cause a significant increase in the HSI for blue-winged teal. Consequently, the knowledge of local conditions allowed simplification of the model substantially by treating interspersion distance as a constant.

For the sake of completeness, a distance function has been included below. This is just one way of relating wetland construction to interspersion distance. The spatial relationships between wetlands and uplands is very complex because the effect of constructing a single wetland is dependent on the location of the wetland. Since the specific locations for wetland construction and restoration may not be known, a simpler and less accurate approach may be more appropriate. The function used to relate management to the distance variables is:

$$DIST(i) = FDIST(i) + \Delta DIST(i) \quad (55)$$

where:

$DIST(i)$ = mean distance, in miles, from cover type i to a wetland that is greater than or equal to 1.0 acre in size;

$FDIST(i)$ = new distance, in miles, from within cover type i to a wetland that is greater than or equal to 1.0 acre in size assuming a future without management scenario (i.e, an 11 percent reduction from baseline); and

$\Delta DIST(i)$ = change in distance, in miles, due to maintenance, construction, and restoration of wetlands that are greater than or equal to 1.0 acre in size.

The term $\Delta DIST(i)$ in equation (55) is defined as the following function of wetland development activities:

$$\Delta DIST(i) = \frac{MDIST(i) - FDIST(i)}{WMAX} \sum_{i=1}^{5} \sum_{k=2}^{7} WD(i, k) + 0.11 \sum_{j=9}^{11} WD(j,1) \quad (56)$$

where:

$MDIST(i)$ = distance, in miles, to a wetland that is greater than or equal to 1.0 acre in size, given that all wetlands were maintained, constructed, and restored;

$FDIST(i)$ = as previously defined;

$WMAX$ = maximum number of wetlands that can be maintained, constructed, and restored. This is a parameter computed by summing the maximum number that could be constructed and restored with 0.11 times the baseline number of wetlands;

$WD(i, k)$ = number of constructed and restored wetlands in cover type i;

i = host cover type for wetland construction and restoration;

$WD(j,1)$ = number of maintained baseline wetlands;

j = cover type index for wetland types only; and

k = index value indicating wetland development activity.

The first term on the right side of equation (56) contains three parameters that are computed for each upland cover type (i = 1 through 5) using a map sampling procedure. The two parameters $MDIST(i)$ and $FDIST(i)$ are mean distances determined on a map by selecting random points in cover type i, and measuring the distance to the closest wetland that is greater than or equal to 1.0 acre in size. Parameter $MDIST(i)$ is measured for each cover type, given all baseline constructed and restorable wetlands. If the precise locations of the constructed and restored wetlands is not known, a uniform distribution pattern can be assumed. Parameter $FDIST(i)$ also is measured for each cover type after randomly "removing" 11 percent of the wetlands. Several replicates, each using a different 11-percent sample, can be performed to get a representative estimate of $FDIST(i)$.

The parameter $WMAX$ is equal to the maximum number of wetlands that can be constructed and restored in all cover types (not including type 1 wetlands), plus 11 percent of the number of baseline wetlands (not including types 1 and 2).

There are several key assumptions implicit to equation (56):

- Each restored or constructed wetland has the same effect on the change in distance, $\Delta DIST(i)$. Also, each maintained wetland has the same effect as all other maintained wetlands.

- Each wetland constructed or restored within a given host cover type has the same effect on $\sum DIST(i)$ for all cover types (i = 1 through 5).

- The relationship between wetland development and distance was assumed to be linear; i.e., equation (56) is linear. In reality, this relationship is nonlinear; the first few wetlands have a greater effect on a reduction in distance. The linear equation was used because it was simpler and because it provided a reasonable approximation.

This completes the management constraints for the blue-winged teal and related management activities. All of the management constraints were not developed; rather, only illustrative examples. Many of the developed equations remain in nonlinear form and must be separated and converted to their linear approximations using the procedures described in appendix A.

Developing Resource Constraints

The resource constraints define physical limits on application of the individual management activities. In general, these constraints fall into one of two categories: (1) constraints that ensure that optimum solution does not require more land than acquired for management, and (2) constraints that ensure that optimum solution does not require acquisition of more of a particular category of land than is conceivably available.

Land Acquisition Constraints on Maintenance. – The amount of area maintained for each cover type cannot exceed the initial amount acquired. Within a given acquisition category, this constraint was expressed for each cover type for which there was a maintenance activity:

$$VD(3,3) \leq LA(3,j) \qquad (57)$$

$$WD(i,1)\ BA(i) \leq LA(i,j);\ i = 7 \text{ through } 12 \qquad (58)$$

where:

$VD(3,3)$ = maintenance of baseline grassland area, in acres;
$LA(3,j)$ = area of grassland in acquisition category j, in acres;
$WD(i,1)$ = maintenance of baseline wetland area, in acres;
$BA(i)$ = mean size of a wetland type at baseline conditions, in acres;
$LA(i,j)$ = area of cover type i in acquisition category j, in acres; and
j = acquisition category: 1 = fee title, 2 = easement.

For each acquisition, there are seven equations representing the maintenance constraints because equation (58) is actually six different equations, one for each value of i from 7 through 12.

Land Acquisition Constraints on Creating New Habitat. – As with the maintenance constraints, there are upper limits on the amount of new habitat that can be created; i.e., planting of DNC, grassland and wetland construction, and restoration. The upper limit is defined by the acquired area of the host cover types within which the activities are performed. The constraint equations are defined by host cover type:

$$VD(i,1) + VD(i,2) + \sum_{k=2}^{7} A(k)\ WD(i,k) \leq LA(i,j);\ i = 1,2,4;\ j=1 \qquad (59)$$

where:

$VD(i,1)$ = amount of DNC planted in cover type i, in acres;
$VD(i,2)$ = amount of native grassland planted in cover type i, in acres;
$A(k)$ = mean size of a constructed or restored wetland, in acres;
$WD(i, k)$ = number of wetlands constructed or restored in cover type i; and
$LA(i, j)$ = amount of cover type i in acquisition category j, in acres.

Equation (59) actually expands into three separate equations, one each for cropland (i=1), alfalfa (i=2), and tame grassland (i=4). Also, these constraints are identified only for fee title acquisition (j=1) since creation of new habitat is not possible through easements.

Potential Site Constraints on Wetland Development. – Wetland development is constrained by the number of physical sites on which wetlands can be constructed (based on terrain features) and on the number of drained wetlands that can be restored. These constraints are defined through an inventory of the land category using a combination of on-site and map data. The general form of the constraint equation is:

$$WD(i,k) \leq MAXP(i,k);\ i = 1,2,3,4;\ k = 2 \text{ through } 7 \qquad (60)$$

where:

$WD(i,k)$ = number of k type wetlands constructed in cover type i, and

$MAXP(i,k)$ = maximum number of wetlands that can be constructed or restored in cover type i (determined through inventory).

There are 24 separate equations implicit to equation (60), one for each combination of wetland development activity k and host cover type i. There are also 24 values of $MAXP(i,k)$ that must be determined through an inventory.

Total Resource Supply. – There is always an upper limit on the total amount of land available for acquisition. In many cases, the upper limit may be so large that, for all practical purposes, it can be ignored. Frequently, another upper bound constraint (e.g., available dollars) may be the controlling factor on land acquisition. These constraints were identified for the Garrison study because there were some perceived upper limits. One equation was developed for each acquisition category as follows:

$$\sum_{i=1}^{12} LA(i,1) \leq WETMAX \qquad (61)$$

$$\sum_{i=1}^{12} LA(i,2) \leq DRYMAX \qquad (62)$$

$$\sum_{i=1}^{12} LA(i,3) \leq EASEMAX \qquad (63)$$

$$\sum_{i=1}^{12} LA(i,4) \leq PUBMAX \qquad (64)$$

where:

$LA(i,1)$ = amount of cover type i acquired by fee title purchase of existing wetland tracts, in acres;

$LA(i,2)$ = amount of cover type i acquired by fee title purchase of drained wetland tracts, in acres;

$LA(i,3)$ = amount of cover type i acquired by easements on existing wetland tracts, in acres;

$LA(i,4)$ = amount of cover type i acquired by use of existing public land, in acres;

$WETMAX$ = total supply of existing wetland tracts that can be acquired by fee title, in acres;

$DRYMAX$ = total supply of drained wetland tracts that can be acquired by fee title, in acres;

$EASEMAX$ = total supply of existing wetland tracts on which wetland easements can be acquired, in acres; and

$PUBMAX$ = total amount of existing public land that can be acquired for management, in acres.

Developing Policy Constraints

The policy constraints define limits on the application of individual management activities from the legal, social, and political perspectives. As a general rule, these constraints restrict the feasible activities to a narrower range of possibilities than the resource constraints. Policy constraints for the Garrison Project were developed to represent several major issues.

Legal Constraints. – Mitigation for the Garrison Project was part of the Federal authorizing legislation, in which mitigation for project-associated losses was allowed as long as the total fee title land acquisition did not exceed 143,000 acres. This legal constraint was expressed as follows:

$$\sum_{i=1}^{12} LA(i,1) + \sum_{i=1}^{12} LA(i,2) \leq 143{,}000 \qquad (65)$$

where:

$LA(i,1)$ and $LA(i,2)$ = as previously defined in equations (61) and (62).

Acquisition Constraints. – Federal acquisition of land in North Dakota was the most significant political concern during the Garrison study. The State of North Dakota, conservation districts, and individual farmers were opposed to additional Federal purchase of land within the State. To address these concerns, the constraints were defined on the maximum amount of land acquisition. These political constraints on land acquisition were included by changing the right sides of equations (61) and (62) such that $WETMAX$ and $DRYMAX$ represented not what was physically available, but how much fee title acquisition was allowed politically. The political constraints on acquisition were set at several different levels to provide alternative solutions to the problem.

Operation and Maintenance Constraints. – The U.S. Fish and Wildlife Service was concerned that any mitigation plan relying heavily on O&M intensive management activities would have a high probability of failure due to the uncertainties of continued funding for a 100-year period. Therefore, several alternative solutions were developed wherein O&M costs were constrained at different levels.

The O&M cost constraint was not handled as a constraint equation per se. The objective function was redefined by substituting O&M rather than total unit costs for each of the cost coefficients. Consequently, the entire problem was redefined to minimize O&M costs subject to the same constraints as before. An

additional constraint equation was added to constrain total cost at or below some maximum amount (defined from the political perspective).

COMPUTER CODING AND FORMATTING THE MODEL

After being initially written out, the objective function and all constraint equations must be organized into the proper format for solution by the Simplex method. The precise format, in part, is dependent on which particular Simplex algorithm and computer is to be used; however, there are some general guidelines that are valid for most situations.

Most Simplex computer codes require that the objective function and constraints be organized into a matrix, or "tableau," which consists of rows and columns. Each row of the tableau corresponds to a single equation, and each column corresponds to a single variable or a constant. There are several formats for organizing the rows and columns, some of which are merely for convenience, while others are mandatory.

The rows, or individual equations, can be arranged in any order; however, it is convenient to arrange them in some systematic way to make the individual constraints easier to locate for possible later modification of the model. Figure 22 shows a tableau that was organized in the general row sequence that was used for this report. The constraint equations are grouped in the same sequence they were developed in the previous section. Within any of the groups of constraints shown on figure 22, additional organization may be desirable. For example, the management constraint group may first list the HSI equations, followed by equations for management activities. The HSI equations could be organized by presenting the life requisites separately, etc.

The columns in the tableau must be organized according to a fairly rigid format. Most of the constraint equations for Garrison were initially written in the following form:

$$Y = k + b_1X_1 + b_2X_2 + \ldots + B_nX_n \qquad (66)$$

where:

Y = variable to be derived (e.g., HSI),
k = a scalar constant,
b_i = coefficient of variable i,
X_i = a decision variable, and
n = number of variables.

These constraint equations must be rewritten by transposing all terms involving variables to the left side and all constant terms to the right side. Each constraint equation will then be of the following form:

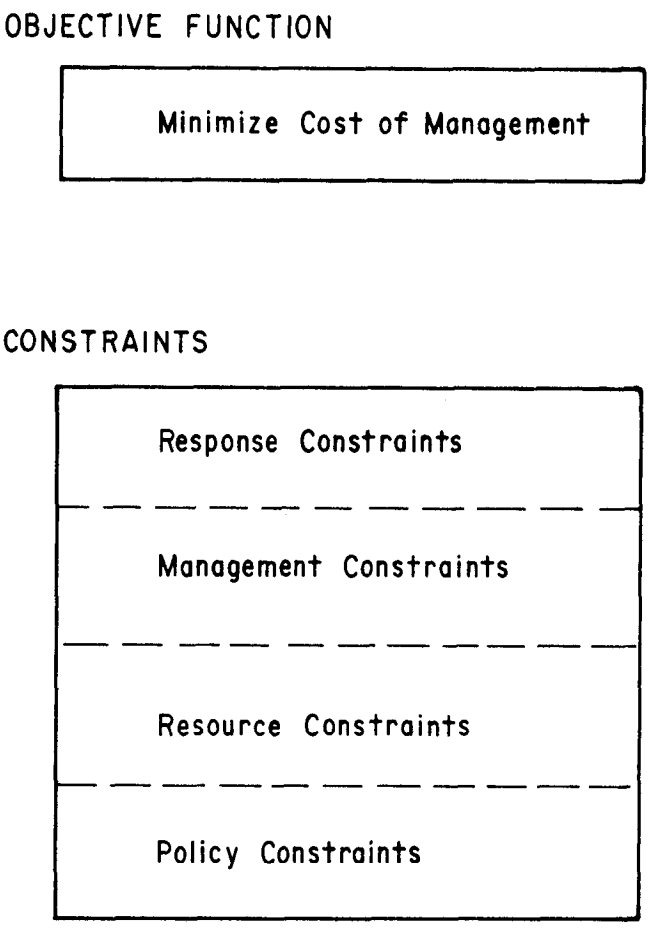

Figure 22. – Organization of tableau for computer solution using Simplex method.

$$Y - b_1X_1 - b_2X_2 - \ldots - b_nX_n = k \qquad (67)$$

After the equations are written in the form of equation (67), the tableau is constructed by placing the coefficients of the variables in a matrix; i.e., the coefficients 1, b_1, b_2, and b_3 would be placed in the first, second, third, and fourth columns respectively. The difficulty in doing this is that a given column in the tableau must always be devoted to a given variable, and any single equation may contain only a few of the total number of variables. When an equation does not contain a variable, the column entry would be zero. This tableau-building exercise will result in a matrix containing a large proportion of zeros because most equations do not contain all possible variables.

The tableau, or matrix of coefficients, is the principal data input file for the computer solution using the Simplex method. Manual creation of a tableau as described can be very tedious for large problems; fortunately, many Simplex computer programs offer some assistance in tableau construction. Some programs even offer a tableau "generator" that requires the user only to enter the equations in the proper

form. Then, the computer constructs the matrix of coefficients.

The computer program used for this report, the IBM-MPSX code, does not have a tableau generator, so a separate program was developed to build the tableau in the correct format. The FORTRAN code and documentation for the IBM-MPSX tableau generator are provided in appendix C; however, it has not been used extensively and, consequently, is not warranted to be error free.

SOLVING THE GARRISON MITIGATION PROBLEM

The optimal solution to the Garrison mitigation problem was a statement of which management activities were required, and how many units of each to employ, to produce the desired number of habitat units at the minimum possible cost. In somewhat more detail, the computer solution provided the following information:

(1) number of acres that should be acquired within each acquisition category: fee title purchase of existing wetland tracts, fee title purchase of drained wetland tracts, easements on existing wetland tracts, and acquisition of existing Federal land;

(2) which management activities and how many units of each to employ on each acquisition category;

(3) average total annual management costs and individual cost for each land acquisition category; and

(4) total number of habitat units generated and number of each land acquisition category.

Optimal solutions were developed for several different planning scenarios. This may sound contradictory because we have consistently described the "optimum" as the least costly plan for achieving the desired number of habitat units. However, the least costly plan is based on a given set of constraints, including the response constraints, and by changing some of the constraints, the optimum strategy may change. This is precisely what happened on the Garrison study. Most of the management and resource constraints were held constant, while the response and policy constraints were varied to reflect different assumptions about the project. Each time constraints were varied, the model was executed to determine the optimal solution with respect to the new constraints. This approach produced several solutions, based on slightly different assumptions, developed for consideration by decisionmakers.

Some of the alternatives contrasted extreme views with respect to a particular issue. For example, minimum land acquisition and minimum O&M costs are at opposite ends of a continuum. Alternatives based on minimum land acquisition must require more intensive management, including O&M intensive activities, to achieve the desired habitat units on a smaller land base. The inverse is true also; solutions based on minimum O&M costs were slanted toward land-intensive activities, such as restoring and constructing wetlands.

The final decision concerning the design of the Garrison study was based not on blind faith in one computer-generated optimum plan, but rather on human judgement considering several possible "optimum solutions." The separable programming mitigation model played a key role in shaping thinking processes and organizing information such that decisions could be made in a more orderly fashion.

POST OPTIMALITY CONSIDERATIONS

The principal emphasis in this report has been on the use of separable programming to find an optimal solution; however, the optimal solution is only part of the information that can be obtained from the Simplex method. Frequently, it may be desired to know how drastically the optimal solution might shift with a change in the value of a given input parameter. Sensitivity analysis provides this type of information.

For the habitat management problem, several types of questions can be answered by sensitivity analysis. First, how would a given change in the unit cost of an activity affect the optimal solution? Second, how would a given change in the right side of a constraint equation affect the solution? Finally, how would the incorporation of additional management activities or constraints affect the solution?

Some of these questions could be answered by using the brute force method; i.e., changing a parameter value, solving the problem from scratch, and then comparing the new solution to the original one. Fortunately, this is not required because information generated in the Simplex tableau during the original solution can be used to answer these questions directly. To illustrate what is possible with sensitivity analysis, refer to the simple example in the third section of this report, "An Introduction to Mathematical Programming," and explore the consequences of changing the cost coefficients in the objective function. Table 3 shows a printout from a computer solution that was produced by a Simplex program called LINDO (Schrage [23]).

Table 3. – Sensitivity analysis information for cost coefficients.

Variable	Objective Function Coefficient Ranges		
	Current coefficient	Allowable increase	Allowable decrease
W	100.000000	125.000000	100.000000
V	75.000000	INFINITY	41.666670

The first two columns in table 3 list the symbolic names and original cost coefficients for the variables composing the objective function. The third and fourth columns provide extreme ranges for values of the cost coefficients, within which the optimal solution does not change. The cost coefficient for wetland construction *W* could be as high as $225 ($100 + $125) or as low as $0.0 ($100 – $100), and the initial optimum solution would still be optimum. Likewise, the cost coefficient for vegetation planting *V* could vary between $34 ($75 - $41) and INFINITY, and the optimal solution would still hold. To summarize, the original optimal solution of 444 acres of wetland construction and 666 acres of vegetation planting is the minimum cost solution provided the following conditions apply:

$0.0 < (WETLAND COSTS) < $225
Vegetation costs = $75

and

$34 < (VEGETATION COSTS) < INFINITY
Wetland costs = $100

As long as the costs stay within these ranges, the optimal solution will remain valid, although the minimum cost value will change. The new costs can be recalculated by substituting the new cost coefficients into the objective function with $W = 444$ acres and $V = 666$ acres.

It may seem curious that the optimal solution remains valid over such a wide range and does not change gradually in response to changing cost coefficients. A previous graphic solution (fig. 10) demonstrates what happens. Remember that the optimal solution will be at one of the corner points of the feasible space. The lower left corner on figure 10 is the optimal solution in this case due to the slope of the cost lines in relationship to the shape of the feasible space. If the slope of the cost lines were to change, however, the minimum cost line might contact the feasible space at either of the other corner points; i.e., the optimal solution would change. The slope can be modified by changing either of the cost coefficients. The slope of the cost lines is equal to the ratio of the cost coefficients and, on figure 22, the slope is 0.75; i.e., 75/100, or vegetation costs divided by wetland costs. Based on the geometry, the slope would have to be reduced to less than 0.33 before the lower right corner (2000,0) would be the optimal solution. The upper left corner could never become the optimal solution because the slope of the cost function cannot go to the right of vertical. Thus, the optimal solution shifts only when the slope of the cost lines exceeds the critical values that are defined by the geometry of the problem.

Now examine the effects of a variation on the right side of constraint equations. Table 4 is a sensitivity analysis printout for the same problem. The first column is the constraint number that was assigned according to order that constraints were listed in the third section of this report [equations (12), (13), (14), and (15)]. The second colum is the original right-side values, equations (2), (3), and (11), and the third and fourth columns are the allowable changes for the right sides. The fifth column, "dual prices," is explained in the next paragraph.

The allowable increase and decrease columns in table 4 must be interpreted differently than were the objective function cost coefficients. If any of the right sides are changed, the optimal solution also will change. The allowable range of values defines the range within which the previously optimal solution remains feasible, rather than optimum as before. As long as the solution remains feasible, new values of the objective function, associated with new optimal solutions, can be readily calculated using the "dual prices." The dual prices represent the change in the value of the objective function (i.e., change in total management cost) resulting from a unit change in the right-side value. Thus, if the response constraint on blue-winged teal was relaxed to 199 habitat units, the total cost would decrease by $333.33 or, if response constraint was increased to 201 habitat units, the total cost would increase by $333.33. This relationship for recomputing costs is valid within the allowable ranges specified in table 4. Outside of these ranges, the problem may no longer have a feasible solution.

The dual price for the resource (land availability) constraint has a value of zero, which means that changing the land constraint within the specified allowable ranges has no affect on total management costs. The

Table 4. – Sensitivity analysis information for right sides of constraint equations.

Row	Current right side	Allowable increase	Allowable decrease	Dual Prices
2	200.000000	266.666700	133.333300	−333.333300
3	200.000000	400.000000	200.000000	−138.888900
4	2000.000000	INFINITY	888.889000	0.000000

reason for this was that all the land was not required for an optimal solution—only 1,111.111 acres are required to produce 200 habitat units for each species. Thus, there is "excess" land. Commodities that have a dual price of zero are in abundant supply and do not exert any control on the optimal solution.

This concludes the brief introduction to post optimality, or sensitivity, analysis. There are many other types of analyses that are possible, although some are more complex than the examples discussed here. However, the basic idea is always the same, exploring the consequences of different assumptions and parameter values included (or not included) in the initial problem formulation. The ability to explore potential consequences in a rapid and efficient manner is a tremendous benefit during a typical planning study.

LIMITATIONS OF MATHEMATICAL PROGRAMMING

For many, mathematical programming is a new approach to wildlife planning. This report has attempted to help wildlife planners visualize potential applications of the techniques. In this regard, we offer one bit of advice—the use of mathematical programming should not be approached in a casual manner. The potential benefits derived from the analyses require a concentrated effort on making the approach work.

A major problem that many encounter in their initial attempt is learning to express management activities in a mathematically consistent form. The type of activity must be very specific, along with where it can be performed, and constraints on its application. The effects of a management activity on habitat are frequently not well known. This lack of information requires relying heavily on judgement, and this compounds the difficulty of developing the mathematics. Finally, the costs associated with many activities are not well documented, and obtaining these data in a correct form can be difficult at times.

As a wildlife planning tool, mathematical programming has certain technical limitations to be aware of:

- Mathematical programming does not easily allow time-series simulation of changes. Moreover, programming does not provide a simple means of handling change in habitat conditions; therefore, mean annual values of variables must be used. As a result, representing management activities where temporal changes are important requires innovative techniques, which is a limitation in two respects: (1) some management activities cannot easily be represented by collapsing temporal variations into an average condition, and (2) even if it is possible to do so, it is often at the expense of realism or accuracy in the mathematical representation.
- It is often difficult to represent the spatial distribution of landscape features in a highly realistic manner. The wetland construction in the Garrison study is a good example of this. The precise location of each wetland could not be prescribed in the model, so we had to assume that all newly constructed wetlands were uniformly distributed on the landscape.
- The separable programming techniques used in this report require that the wildlife system be represented by linear functions or functions that can be separated into a sum of functions of single variables. This is a limitation that may be too restrictive for some situations.

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APPENDIX A

SEPARABLE PROGRAMMING TECHNIQUES

APPENDIX A. SEPARABLE PROGRAMMING TECHNIQUES

Separable programming is a technique for solving constrained optimization problems that contain either a nonlinear objective function, one or more nonlinear constraints, or both (Hadley [13], Beale [24], Simmons [25], Wagner [15], Phaffenberger and Walker [26], and Hillier and Lieberman [14].

In using separable programming, the original nonlinear function with piecewise linear segments must be approximated, and the approximations require using special linearizing variables. Approximating the nonlinear function increases the number of equations; however, since a version of the Simplex Method can be used as a solution algorithm, the separable programming technique has considerable practical significance for solving nonlinear programming problems.

The essence of separable programming is to replace curvilinear relationships with piecewise linear approximations. Given a curvilinear relationship, as shown on figure A-1, the objective is to represent this nonlinear curve as a piecewise linear approximation using several variables. Four steps are involved: (1) separating nonlinear function, (2) defining relevant functional range, (3) selecting approximating grid points, and (4) formulating mathematical approximation using special variables.

Separating the Function

A function is said to be separable if it can be written as the sum of n functions, one for each single variable X_i:

$$f(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} f(X_i) \tag{A1}$$

where:

X_i = a variable associated with programming problem,
$f(X_1)$ = a function in the single variable X_i (e.g., $3X_i$, ln X_i, X_i^2), and
n = number of single variable functions.

All functions fall into two categories regarding separability: (1) an equation that is already separated or is easily separated, and (2) an equation that must be transformed to be separated (e.g., products). It should be noted that not all equations are easily separated. For example, equations involving a complex interaction, as in equation (A2), are not easily separated.

$$f(X,Y) = X^2 + \frac{2\,XY}{(X-2)^2} + Y^2 \tag{A2}$$

There are two techniques commonly used to transform a function: (1) difference of two squares transformation, and (2) logarithmic transformation.

Difference of Two Squares Transformation. – The product of two linear variables can be represented as follows:

$$Z = XY = \frac{X+Y^2}{2} - \frac{X-Y^2}{2} \tag{A3}$$

The equality in equation (A3) is not obvious; however, by multiplying out the squared terms and collecting like terms, it becomes obvious. Given this equality, it is possible to generate a series of equations and identities that, when taken together, represent the original function. By defining equations (A4) and (A5) as follows:

$$P = \frac{X+Y}{2} \tag{A4}$$

$$Q = \frac{X-Y}{2} \tag{A5}$$

equation (A3) can be written as

$$Z = P^2 - Q^2 \tag{A6}$$

Equation (A6) is the sum of two nonlinear functions, each of which is comprised of a single variable. Thus, equation (A6) satisfies the definition of a separable function. All three equations, (A4), (A5), and (A6), satisfy the separability conditions, and must be substituted for equation (A3) to represent the product of X and Y.

Logarithmic Transformation. – The logarithmic transformation is an alternative method for achieving separability. Separation of a product such as

$$Z = XY \tag{A7}$$

can be accomplished using logarithms. Taking the log of both sides of equation (A7) yields

$$\ln Z = \ln X + \ln Y \tag{A8}$$

At this point, equation (A8) satisfies the separability criterion; however, the original dependent variable was Z, not $\ln Z$. Thus, the antilog of Z must be calculated:

$$Z = e^{\ln Z} \tag{A9}$$

Also, the independent variables were X and Y, thereby requiring calculation of their log equivalents:

$$\ln X = f(X) \tag{A10}$$

$$\ln Y = f(Y) \tag{A11}$$

Collectively, equations (A8) through (A11) must be substituted to represent the original function, equation (A7). As with the difference of two squares approach, several equations are needed to replace the original function.

Several factors must be considered in deciding which transformation technique to use. First, and foremost, is computation efficiency. The method that is easiest to use, and results in the fewest number of equations, should be used. Second, if zero is a feasible value for the function, it may be best to use the difference of two squares approach because the log of zero is undefined. However, since the log of zero may be closely approximated by selecting an extremely large negative number as a proxy, this is not a serious limitation of the log transformation method. Finally, if a constant appears in the original function, additional specification is required when using log transformation to accommodate the problem with approximating zero. One method is illustrated in the following example. Assume the nonlinear function is given by

$$Z = 3\ XY \tag{A12}$$

Using log transformation,

$$\ln Z = \ln 3 + \ln X + \ln Y \tag{A13}$$

$$Z = e^{\ln Z} \tag{A14}$$

Since ln 3 is a constant, ln Z will equal zero only when ln 3 + ln X + ln Y = 0. However, when X or Y equals zero, Z is zero. Thus, equation (A13) can be written:

$$\ln Z = \ln W + \ln Y \tag{A15}$$

where W is defined:

$$W = 3X \tag{A16}$$

By using equations (A14), (A15), and (A16), the problem with constants appearing in the original equation is circumvented.

Defining Relevant Functional Range

The second step in formulating a separable programming problem is defining the relevant functional range for each nonlinear relationship. This means defining lower and upper bounds on the value of X; e.g., the suitability index on figure A-1. The extreme lower and upper bounds (0.0 and 1.0) were selected on this figure; however, it may not always be so easy to define these bounds because some variables can take on values between $-\infty$ and $+\infty$. All other things being equal, the narrower the range, the better will be the approximation. Determining the range depends on where the solution to the problem is expected to occur, and the specific attributes of the problem.

Selecting Grid Points

The third step involves division of the relevant curvilinear relationship into linear segments over the range defined in the second step. This process is facilitated by initially graphing the relationship and then selecting the segments. Each endpoint of a segment is termed a "grid point." The nonlinear relationship graphed on figure A-1 relates the SI (suitability index) to its logarithmic equivalent; i.e., ln $SI = f(SI)$, where the relevant range on SI has been defined as from 0.0 to 1.0.

Selection of grid points, in part, determines how accurately the piecewise linear approximation will approximate the original nonlinear relationship. Consequently, great care should be used when selecting these points. Two factors influence the selection accuracy: (1) number of points selected, and (2) location of points with regard to overall range of function. As a general rule, five to seven points should be selected (Beale [24]), thereby defining four to six linear segments. Seven points is considered to be the upper limit to avoid excessive increases in the computer solution time of the problem.

Selection of the specific location for the grid points is critical. In principal, the nonlinear function should be approximated with shorter line segments in the area where the optimum value of X is likely to occur. However, this rule is difficult to implement for complex actual problems because optimum values simply cannot be accurately predicted in advance. Simple observation has proved to be the best strategy; the points should be spaced closer together in the more curvilinear portions of the graph. Having selected the number and location of grid points, it is possible to define each point in relationship to the horizontal and vertical axis; i.e., X, $f(X)$, where the first grid point becomes X_1, $f(X_1)$, the second X_2, $f(X_2)$, etc.

Formulating Mathematical Approximation

The fourth and final step in formulating the separable programming problem is to use the selected grid points to explicitly approximate the nonlinear function. There are two methods available: (1) Miller Method, and (2) Delta Method, which is used in IBM's MPSX computer package. Either method is equally useful, the choice being solely dependent upon the available computer package. For purposes of illustration, the Delta Method is used here.

Example Using Delta Method. – A common occurrence in the Garrison study examples, shown in the "Development of a Habitat Management Model" section, was to have constraint equations containing products of two or more variables. For example, equation (25) for the blue-winged teal was similar to:

$$HSI = (SI1 \times SI2 \times SI3)^{1/3} \tag{A17}$$

This equation can be separated using the logarithmic method:

$$\ln(HSI) = \frac{\ln(SI1)}{3} + \frac{\ln(SI2)}{3} + \frac{\ln(SI3)}{3} \tag{A18}$$

Each of the logarithmic terms on the right side of equation (A18) must be represented by a linear approximation. Rather than do this for all three terms, a linear approximation for the general logarithmic expression was developed:

$$\ln(SI) = f(SI) \tag{A19}$$

In solving for the logarithmic equivalent of a suitability index, the relevant range for the index is between 0.0 and 1.0. The grid points selected on figure A-1 have the following coordinates: (0, −300), (0.05, −2.996), (0.1, − 2.3), (0.2, −1.61), (0.3, −1.2), (0.5, −0.69), and (1.0,0). In this case, the −300 coordinate was used to approximate the log of zero.[1] Table A-1 lists the grid points in the first two rows, and the slope components, that were computed from the grid points, are listed in rows three and four. The first slope component Δ *SI* is calculated as the difference between the first and second *SI* grid (0.0 to 0.05). The second Δ *SI* is the difference between the second and third *SI* grid coordinates (0.05 to 0.1, etc.). The fifth row in this table is simply the ratios between slope components, as indicated.

The values in table A-1 provide all the information needed to construct a set of equations that are the linear equivalent of equation (A19), and which would be substituted for this equation in the set of constraint equations. The replacement equations will be functions of special linearizing variables NA_i. There are two equations plus a set of upper bound constraints for each linearizing variable. The equations and constraints are as follows:

$$\ln SI = -300.0 + 5940.09\, NA_1 + 13.86\, NA_2 + 6.93\, NA_3 + 4.05\, NA_4 + 2.55\, NA_5 + 1.39\, NA_6 \tag{A20}$$

$$SI = NA_1 + NA_2 + NA_3 + NA_4 + NA_5 + NA_6 \tag{A21}$$

$$NA_1 \leq 0.05 \tag{A22}$$

$$NA_2 \leq 0.05 \tag{A23}$$

$$NA_3 \leq 0.01 \tag{A24}$$

$$NA_4 \leq 0.01 \tag{A25}$$

$$NA_5 \leq 0.02 \tag{A26}$$

$$NA_6 \leq 0.05 \tag{A27}$$

Equation (A20) is called the "polygonal approximation", and equation (A21) is called the "reference row". The upper bound constraints, equations (A22) through (A27), in conjunction with the restricted basis entry criteria ensure that interpolations along any linear segment are performed sequentially. For example, interpolations along the first segment on figure A-1 should be based on the coordinates of grid points "a" and "b", not "a" and "c" or any other combination of grid points.

The separable programming approach has several disadvantages that should be noted (Simmons [25]). First, the solutions are only approximations and, therefore, there is a loss in accuracy. However, the

[1]In cases where a function has both dependent and independent variables defined as log relationships, the value chosen to approximate the log of zero for the independent variable must be a multiple of the number of logged dependent variables and the value used to approximate the log of zero for each dependent variable. For example in equation (A9), if ln (−300) = 0 for both *X* and *Y*, then *Z* must equal −600 when ln *Z* = 0.

Table A-1. — Linearizing variable coefficient derivation for ln *SI* = *f(SI)*.

Variables	Values						
Grid coordinates:							
SI	0.0	0.05	0.1	0.2	0.3	0.5	1.0
ln *SI*	−300.0	−2.996	−2.30	−1.61	−1.20	−0.69	0.0
Slope components:							
Δ *SI*	0.05	0.05	0.1	0.1	0.2	0.5	
Δ ln *SI*	297.004	0.69	0.69	0.41	0.51	0.69	
Linearizing variable coefficients:							
Δ ln *SI*/Δ *SI*	5940.09	13.86	6.93	4.05	2.55	1.39	

degree of inaccuracy can be minimized through careful use of the method. Second, the solutions are not necessarily global optima, but rather local optima. This means that the plan computed to have minimum costs may only be one of several minimum cost solutions rather than the absolute minimum cost solution.

Finally, the number of equations is larger than in the original problem. The computational efficiency of the Simplex method is negated if a large number of linear approximations are required. If this is the case, available nonlinear algorithms may be more cost effective to use.

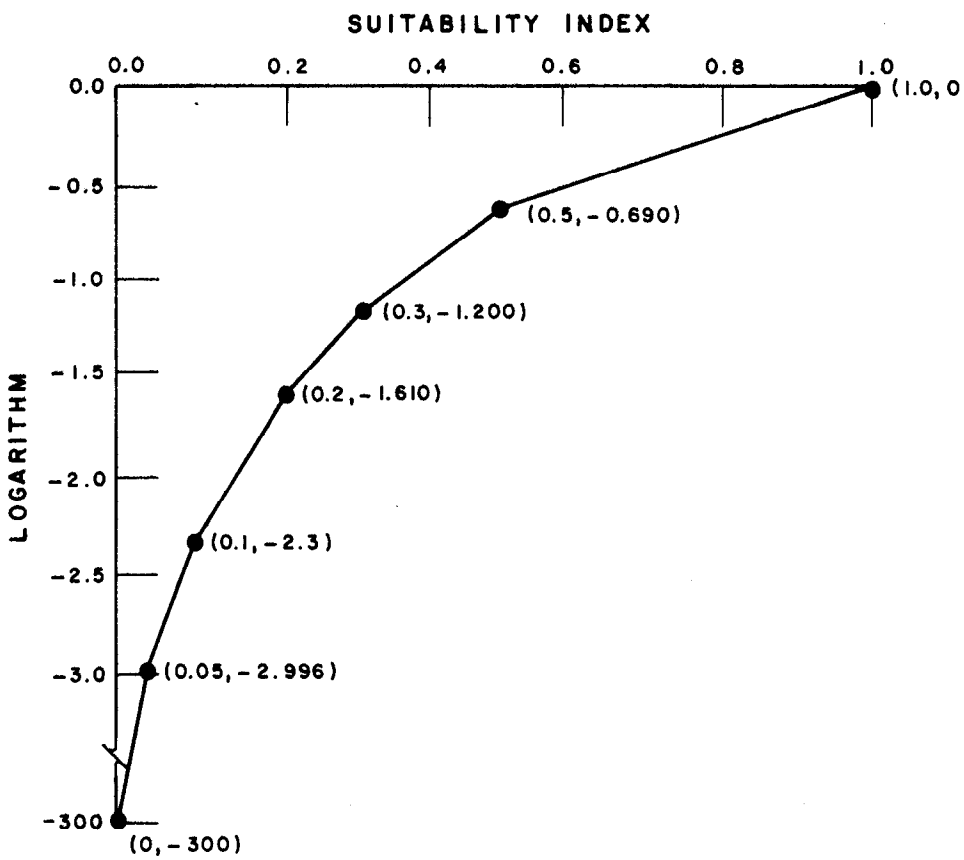

Figure A-1. – Logarithmic transformation of a suitability index.

APPENDIX B

HABITAT MANAGEMENT ACTIVITIES

APPENDIX B. HABITAT MANAGEMENT ACTIVITIES[1]

This appendix contains brief narrative descriptions and data for some of the management activities developed for the Garrison study. The symbolic names for each management activity are given for cross reference to habitat management examples in the text. The symbols used for some management activities contain subscripts used to denote the vegetation cover types within which the activity can be employed. The following subscript values[2] used are:

Cover Type Name	*Cover Type No. (i)*
Cropland (nonalfalfa)	1
Alfala	2
Native grassland	3
Tame grassland	4
DNC (dense nesting cover)	5
Woodland	6
Type 1 wetlands	7
Type 2 wetlands	8
Type 3 wetlands	9
Type 4 wetlands	10
Type 5 wetlands	11
Types 9, 10, and 11 (saline) wetlands	12

The management actions were placed into five groups for ease of presentation: (1) land acquisition, (2) upland vegetation development, (3) upland vegetation manipulation, (4) regulation of grazing, and (5) wetland development.

Land Acquisition

Management for wildlife may occur on either privately or publicly owned lands. For modeling purposes, management rights must be acquired before habitat management activities can occur on either private or public lands. For purposes of the Garrison study, management rights on private lands could be acquired either through purchase (fee title), or limited rights could be acquired through long-term easements. Management rights on existing public lands could be obtained through an agreement between the land holding agency and the wildlife management agency. Land acquisition action is denoted by the following symbol:

$$LA(i,j) \tag{B1}$$

where:

i = cover type number, and

j = category of acquisition (1 = fee title purchase of existing wetland tracts, 2 = fee title purchase of drained wetland tracts, 3 = easements on existing wetland tracts, and 4 = use of existing public land).

Fee Title. — The fee title, *LA* (*i*,1) and *LA* (*i*,2), involves purchase of lands to be used for management. All other management activities defined here are allowed on lands acquired by fee title. The cost to obtain these management rights depends on many factors such as location, access, type of soil, topography, and wetland types (density and size). To obtain an accurate cost for a parcel of land, a specific piece of land should be delineated and appraised based on comparable land sales in the vicinity. The estimated values, excluding appraisal fees, presented in tables B-1 and B-2 are based on typical tracts of land offered for sale by willing sellers in the vicinity of the Garrison study area.

It is frequently easier to develop land acquisition costs for general categories of land, rather than just for specific cover types. During the Garrison study, it was felt that there were two categories of land that had mitigation potential and could be acquired by fee title. Table B-3 shows the deviation of costs for these

[1] This appendix was compiled and written by Rodney Olson and Chuck Solomon of the U.S. Fish and Wildlife Service, Western Energy and Land Use Team, Ft. Collins, CO, for use on the Garrison study. Only those management activities of major consequence to the blue-winged teal are included.

[2] The list of cover types does not include all types "identified" in the Garrison study. Only those cover types most pertinent for the blue-winged teal example in this report are included.

Table B-1. – Land costs by cover type.

Land type	Range of cost, dollars per acre	Average cost, dollars per acre
Native Grassland	200 - 400	275
Tame Grassland	300 - 500	400
Alfalfa	400 - 1,000	550
Cropland	400 - 1,000	550
Woodland (associated with grassland)	200 - 400	275
Woodland (associated with cropland)	400 - 1,000	550

Table B-2. – Land costs by wetland type.

Wetland types	Associated with cropland, dollars per acre	Associated with pasture, dollars per acre
Type 1	550	275
Type 3	330	250
Type 4	220	140
Type 5	140	100
Types 9, 10, and 11	80	50

two land categories. The costs were computed using cover type figures from tables B-1 and B-2 and an inventory of cover types on a sample of lands within the category.

The annual costs described in table B-3 include only the purchase price to the landowner, and do not include other costs associated with the acquisition process; e.g., appraisal fees. These additional costs were ignored in the Garrison study, but they may be significant in other situations.

Easements. — The easements, *LA*(*i*,3), involve the purchase of limited management rights, and are designed to protect specific types of land. The easements defined here were used to protect wetlands from being drained, leveled, or filled for a given period, in this case, for the life of the Garrison Project. The purchase of easements permitted only one management activity, that of maintaining existing wetlands, *WD*(*i*,1). Landowners would retain overall use and control of the wetland areas subject only to the burn, drain, level, and fill restrictions and the right to inspect the wetlands. Easements were defined only for one land category, that of existing wetland tracts.

To minimize administrative costs, a single lump payment would be made for the easement, and this payment was calculated from the appraised fee title value of the property containing the wetlands. Depending on land values and the wetland type, size, and density, the estimated cost per wetland acre for an easement would vary considerably. Larger type 4, 5, and saline wetlands, which are usually impractical to drain for agricultural purposes, are generally less costly to lease. Applying these criteria to the typical existing wetland tract from the willing seller list, table B-3, indicates an estimated easement cost per wetland acre of about 55 percent of the fee value of the property (total property value per acre times the percent of wetland acreage equals easement payment). Using the cost figures for fee title purchase, the cost per wetland acre can be computed as follows:

$$\text{Easement cost} = (WA)\,(C)\,(0.55)\,(0.0328) \qquad \text{(B2)}$$

where:

WA = area of the wetland, in acres;
C = cost of wetland type from table B-2, in dollars per acre;
0.55 = average cost of easement expressed as a proportion of fee title costs, no units; and
0.0328 = amortization factor for 3⅛ percent for 100 years, no units.

Use of Public Lands. — The use of public lands, *LA*(*i*,4), would consist of the use of currently owned public lands for mitigation. Public lands are defined as "those lands currently in Federal or State ownership,

Table B-3. – Land costs by land acquisition category.

LA(i,1) — purchase existing wetland tracts (i.e., 10 to 100% existing wetlands)								
Alfalfa	3.6 acres	×	550.00		=	1,980		
Cropland	93.7 acres	×	550.00		=	51,535		
Tame grass	15.7 acres	×	400.00		=	6,280		
Native grass	66.3 acres	×	275.00		=	18,235		
Type 1	0.5 acre	×	550.00	(in crop)	=	275		
Type 2	5.5 acres	×	200.00		=	1,100		
Type 3	27.8 acres	×	330.00	(in crop)	=	9,175		
Type 4	27.4 acres	×	140.00	(in pasture)	=	3,835		
Type 5	3.2 acres	×	100.00	(in pasture)	=	320		
Saline	11.6 acres	×	50.00	(in pasture)	=	580		
Woodlands	2.3 acres	×	550.00	(planted)	=	1,265		
Other	3.7 acres	×	550.00		=	2,035		
	261.3 acres					96,615	=	$369.75/acre
	Amortization factor (3⅛% for 100 years)						x	0.0328
						Annual cost	=	$ 12.13/acre
LA(i,2) — purchase drained wetland tracts (i.e., ≥ 10% drained wetlands)								
Alfalfa	0.0 acre							
Cropland	140.8 acres	×	550.00		=	77,440		
Tame grass	7.0 acres	×	400.00		=	2,800		
Type 1	0.4 acre	×	550.00		=	220		
Type 2	0.5 acre	×	200.00		=	100		
Type 3	12.7 acres	×	330.00		=	4,190		
Type 1 (drained)	0.4 acre	×	550.00	(currently cropland)	=	220		
Type 3 (drained)	43.9 acres	×	550.00	(currently cropland)	=	24,145		
Type 4 (drained)	35.0 acres	×	550.00	(currently cropland)	=	19,250		
Other	3.3 acres	×	550.00		=	1,815		
	244.0 acres					130,180	=	$533.52/acre
	Amortization factor (3⅛% for 100 years)						x	0.0328
						Annual cost	=	$ 17.50/acre

and on which wildlife management rights currently exist." Agreements between public agencies may be required to implement management, and there may be some associated costs to the management agency. In the Garrison study, there were no acquisition costs associated with public lands because the lands were limited to project lands and previously acquired mitigation lands. Also, all other management activities were permitted on public land, although in other situations the activities which are permitted may be constrained by policy of the land holding agency.

Upland Vegetation Development

Upland vegetation development encompasses the preservation and development of two types of vegetative cover: (1) NG (native grassland), and (2) DNC (dense nesting cover). The symbol for upland vegetation development is:

$$VD(i,k) \tag{B3}$$

where:

i = cover type number within which activity can occur, i = 1, 2, 3, 4; and
k = development action (1 = planting DNC, 2 = planting NG, and 3 = maintaining existing grassland).

Each of these activities would provide habitat with different suitabilities for wildlife at different costs. Seedbed preparation, seed, seeding, fertilization, and herbicide spraying costs were estimated for DNC and native grass. Woodland costs included land preparation and planting costs.

The specifications for upland vegetation development for the Garrison study were as follows:

1. Native grassland and DNC may be planted *only* in DNC, cropland, tame grassland, or alfalfa (i.e., i = 1, 2, 4, or 5 only).

2. Existing native grassland must be maintained; i.e., not converted to other cover types, except wetlands may be restored or constructed in native grassland.

3. It was assumed that adequate cropland, alfalfa, and tame grassland would be available to meet wildlife needs in adjacent areas. Therefore, there are no identified management activities for the preservation or development of cropland, alfalfa, or tame grassland.

The unit of measure for this activity is the number of acres. The minimum application is 1 acre, and the maximum is constrained to the area of host cover types.

Planting Dense Nesting Cover. — This activity, *VD*(i,1), would provide cover for wildlife by seeding a PLS (Pure Live Seed) mixture of 4.5 pounds of tall wheatgrass, 4.0 pounds of intermediate wheatgrass, 1.0 pound of alfalfa, and 0.5 pound of yellow sweet clover or equivalent. Under this activity, DNC may be planted in croplands, alfalfa, or tame grass (i.e., i = 1, 2, or 4 only).

This activity assumes DNC would be replanted with a growth cycle of 7 years. Planting operations including seedbed preparation, seed acquisition, seeding, reseeding, fertilization, and initial and maintenance herbicide spraying for weed control would precede the 7-year growth cycle of DNC. Only the initial planting of DNC would be fertilized.

Preparing a seedbed from alfalfa requires more tillage than cropland. The added costs to the farmer of plowing (breaking) alfalfa were assumed to be offset by income from haying the alfalfa prior to plowing and harvesting a nursery crop planted with DNC seed. The annual cost per acre for planting DNC is $3.97; the details pertaining to these costs are presented in table B-4.

Planting Native Grassland. — This activity, *VD*(i,2), would provide cover for wildlife by seeding native grasses. Grassland can be planted in cropland, DNC, and alfalfa tame gass (i.e., i = 1, 2, or 5 only). The blend of grass species would vary according to soil, mositure, and other conditions. The cost of the seeding operation would be higher than for DNC seeding because a grass drill could be used (DNC can be planted using a commonly available grain drill). However, only one planting of NG would be required. The costs of herbicide spraying on NG would be the same as for DNC, except that DNC costs are lower during the first years of each cycle due to tillage. When DNC is replanted, the soil is tilled, thus reducing weed infestation and lowering herbicide spraying costs. Both fertilization and reseeding operations were assumed for establishing NG.

Preparing a seedbed from alfalfa requires more tillage than cropland. The added costs of plowing (breaking) alfalfa were assumed to be offset by income from haying the alfalfa prior to plowing and harvesting a nursery crop planted with NG seed.

The annual cost per acre for planting NG is $2.70; cost breakdowns are provided in table B-5.

Maintain Existing Native Grassland. – This activity, *VD* (i,3), is similar to the prior activity of planting native grassland, except that initial planting is not required for those areas initially supporting native grasses. Seedbed preparation, seed acquisition, seeding, reseeding, and fertilizing would not be needed, but herbicide spraying would be performed annually on 10 percent of the area to control weed encroachment onto adjacent croplands.

The annual cost per acre for maintaining existing NG would be $0.90; the cost breakdown is provided in table B-6.

Table B-4. – Cost of planting dense nesting cover.

Year	Activity	Cost per acre, dollars	×	Discount factor	=	Cost (present value), dollars
1	Fallow (tillage cost offset by nursery crop)	0		0.9697		0
2	Seedbed preparation, seed, and seeding	19.00[a]		.9403		17.87
	Fertilization	14.00[b]		.9403		13.16
	Initial herbicide spraying	7.00[c]		.9403		6.58
3	Reseeding (20% of area)	8.00[d]		.9118		7.29
	Follow-up herbicide spraying (15% of area)	0.83[e]		.9118		0.76
4	Herbicide spraying (10% of area)	0.90[f]		.8842		.80
5	Herbicide spraying (10% of area)	.90		.8574		.77
6	Herbicide spraying (10% of area)	.90		.8314		.75
7	Herbicide spraying (10% of area)	.90		.8062		.73
8	Herbicide spraying (10% of area)	.90		.7818		.70
9	Herbicide spraying (10% of area)	.90		.7581		.68
10	Herbicide spraying (10% of area)	.90		.7351		.66
				Cost of first cycle (present value)		$50.75
20	Cost of subsequent cycles	37.59[g]		0.5404		20.31
30		37.59		.3972		14.93
40		37.59		.2920		10.98
50		37.59		.2147		8.07
60		37.59		.1578		5.93
70		37.59		.1160		4.36
80		37.59		.0853		3.21
90		37.59		.0627		2.36
100	Fallow (tillage cost offset by nursery crop)	0		.0461		0
		Total cost of first and subsequent cycles				$120.90
		Amortization factor (3⅛% for 100 years)			×	0.0328
				Annual cost per acre		$3.97

[a]Personal communication from D. Schmidt on June 18, 1982.
[b]Cost of applying ammonium nitrate fertilizer at the rate of 50 pounds per acre. Nitrogen typically would be applied for both NG and DNC. Phosphate would only be applied under exceptional conditions where soil tests show very low phosphate. Personal communication from H. Goetz on August 26, 1982.
[c]Personal communication from R. Shupe on July 13, 1982.
[d]Assume that 20 percent of area would be replanted. Costs would be:

Seedbed preparation, seed, and seeding	$19.00
Fertilization	14.00
Initial herbicide spraying	7.00
Total initial cost	$40.00 (0.20) = $8.00

[e]Personal communication from R. Shupe on July 13, 1982.
[f]This cost is calculated at 10 percent of the $9.00 cost per acre including $2.00 labor, $5.00 chemical (2.4-D), and $2.00 equipment.
[g]Assume only first planting is fertilized. Therefore, cost of each subsequent cycle is cost of first cycle, $50.75, minus fertilization, $13.16, = $37.59.

Table B-5. – Costs of planting native grasses.

Year	Activity	Cost per acre, dollars	×	Discount factor	=	Cost (present value), dollars
1	Fallow (tillage cost offset by nursery crop)	0		0.9697		0
2	Seedbed preparation, seed, and seeding	30.00[a]		.9403		28.21
	Fertilization	14.00[b]		.9403		13.16
	Initial herbicide spraying	7.00[c]		.9403		6.58
3	Reseeding (20% of area)	10.20[d]		.9118		9.30
	Followup herbicide spraying (15% of area)	0.83[e]		.9118		0.76
		Cost of establishing NG (present value)				$58.01
4-100	Annual herbicide spraying at $0.90/acre[f] × 30.3825 (discount factor at 1 per period for 97 years)	$27.34		.8842[g]		24.18
		Total cost (establishing and maintaining)				$82.19
		Amortization factor (3⅛% for 100 years)				× 0.0328
				Annual cost per acre		$ 2.70

[a]Personal communication from D. Schmidt on June 18, 1982.
[b]Soil Conservation Service, Technical Guide, Section V-A, p.14, January 1, 1982.
[c]Personal communication from R. Shupe on July 13, 1982.
[d]Assume that 20 percent of area would be replanted. Costs would be:

Seedbed preparation, seed, and seeding	$30.00
Fertilization	14.00
Initial herbicide spraying	7.00
Total initial cost	$51.00 (0.20) = $10.20

[e]Personal communication from R. Schupe on July 13, 1982.
[f]Fourth year discount factor. This factor converts the $27.34 value, which was calculated from the $0.90 annual cost for 97 years, to present value.

Table B-6. – Cost of maintaining existing native grassland.

Year	Activity	Cost per acre, dollars	×	Discount factor	=	Cost (present value), dollars
1-100	Herbicide spraying	0.90[a]		N/A[b]		0.90[a]

[a]Personal communication from R. Shupe on July 13, 1982. The $0.90 cost was calculated at 10 percent of the $9.00 cost per acre including $2.00 labor, $5.00 chemical (2, 4-D), and $2.00 equipment because 10 percent of the area would be sprayed annually.
[b]Since the $0.90 cost occurs annually without change during the life of the project, neither discounting nor amortization are applied.

Upland Vegetation Manipulation

This activity provides for the periodic manipulation of upland vegetation:

$$VM(i,k) \tag{B4}$$

where:
i = cover type number (i = 1 through 12), and
k = action taken (1 = regulation of grazing, 2 = fence construction and management).

Regulate Animal Unit Months. – This revenue-generating activity, $VM(i,1)$, is management of the level or extent of grazing by domestic livestock, which is measured in AUM's (animal unit months) per acre. For purposes of the Garrison study, it was assumed that any generated revenues would revert to the Federal general fund and would not be available for wildlife management in North Dakota. It was also assumed that grazing could occur on all cover types.

Regulation of AUM's is related to the following activity, which is fence construction and management $VM(i,2)$, because grazing cannot be regulated unless fences are present. Thus, both activities are required to effect a change in wildlife habitat. Regulation of AUM's and fencing are defined separately because of their cost structure; fencing has a definite cost whereas additional AUM's may actually generate revenues that can be used to pay for other management activities. Table B-7 provides estimates of the revenue generation potential for cover types in the Garrison study area. Since it was assumed that any revenues were not available for management, these revenue values were not actually used in the programming model.

Fence Construction and Maintenance. – This activity , $VM(i,2)$, consists of constructing fence to protect vegetation from grazing animals. A perimeter (boundary) fence would be constructed for each parcel. This fence would protect all cover types within grazing by animals from external origins. It was assumed that each section would require 2 miles of additional internal fencing. Thus, a typical section would require 6 miles of fence. The fence would be a 4-wire barbed fence with a 25-year life. The annual cost of the 6-mile fence would be $3,985.18, including $200.00 per mile annual O&M cost. The annual cost per acre of the 6-mile fence would be $6.23. Cost breakdowns are provided in table B-8.

Wetland Development

Wetland development consists of using existing wetlands, construction of new wetlands, and restoration of drained wetlands. The purpose of this activity is to enhance productivity for waterfowl, shorebirds, and other wetland dependent wildlife species. The symbolic name for wetland development is:

$$WD(i,k) \tag{B5}$$

where:

i = cover type number within which methods are developed (i = 1 through 12), and
k = action taken (1 = maintain existing wetland, 2 = construct type 4 wetlands, 3 = construct type 3 wetlands, 4 = restore type 1 wetland, 5 = restore type 3 wetland, 6 = restore type 2 wetland, and 7 = restore type 4 wetland).

Maintain Existing Wetlands. — All wetland types can be maintained in their initial state. Thus, this is really six discrete activities, $WD(i,1)$, i = 7, 8, 9, 10, 11, and 12. This activity consists of maintaining currently existing wetlands, thereby preventing future drainage. No management activities or costs, except acquisition,

Table B-7. – Revenues for grazing on various vegetation cover types.

Cover type[a]	AUM per acre	Revenue per acre ($6.18 per AUM)[b]
3	0.45[c]	$ 2.78
4	0.53[d]	3.24
6	0.15[e]	0.93
7	2.40[c]	14.83
8	1.90[c]	11.74
9	0.60[c]	3.71

[a]Cover types not listed because they would not be used for wildlife mitigation (e.g., cropland and alfalfa), or would not produce significant forage for grazing (e.g., Types 9, 10, and 11 wetlands).
[b]Personal communication from R. Shupe on July 29, 1982.
[c]Personal communication from E. Podell on July 14, 1982.
[d]Personal communication from J. Peterson on September 23, 1982.
[e]Personal communication from J. Peterson on August 24, 1982.

Table B-8. – Costs of fence construction and maintenance.

Year	Activity	Cost of 6-mile fence, dollars	×	Discount factor	=	Cost (present value), dollars
1	Fence construction, ($8,100 per mile)	$48,600		0.9697		$47,127.42
25		48,600		.4633		$22,518.39
50				.2147		$10,433.70
75				.0995		4,835.36
		Total fence cost (present value)				$84,913.87
		Amortization (3⅛% for 100 years)			×	0.0328
		Annual cost per section (ex. O&M)				$ 2,785.18
		O&M: $200 per mile[1] × 6 miles		=		+ 1,200.00
						$ 3,985.18
		Annual cost per acre: $3,985.18/640		=		$ 6.23

[1]Personal communication from K. Weber on August 25, 1982. The estimated cost of fence construction is based on current contract costs including $2,100 per mile administrative overhead, boundary surveys, and inspection during construction.

are involved. Existing wetlands are defined as those areas that have been categorized as wetland types 1-5, and 9-11 (Shaw and Fredine [22]). No costs other than acquisition are involved with this activity.

Construct Type 4 Wetlands. — This action, *WD*(*i*,2), entails construction of type 4 wetlands. The wetland construction sites should be large enough to be efficiently managed for waterfowl pair and brood-rearing requirements (i.e., 2 to 8 acres).

The following constraints apply to construction of type 4 wetlands in the Garrison study area:

1. Type 4 wetlands should be constructed only on sites in native grassland, alfalfa, cropland, and dense nesting cover; i.e., i = 1, 2, 3, and 5.

2. There is an average of only one potential construction site per square mile, and these sites consist of natural ravines or catch basins where earthfill dams can readily be constructed.

There are many possible shapes and sizes of ponds that represent suitable habitat for blue-winged teal and gadwall. It was not possible to model all of these, therefore, only one constructed pond size was considered. The construction activity consists of building an earth dam with an average height of 5 feet, average top width of 8 feet, average bottom width of 25 feet, and an average length of 75 feet. Generally, the pond will have an oblong configuration, and will range from 2 to 8 acres in size. The average size pond is 5 acres with an average depth of 3 feet (range of 2 to 5 feet). No natural islands will be present in the constructed wetlands. Water to fill the wetland will come from snowmelt and rain (i.e., natural runoff).

The cost for constructing and maintaining one type 4 wetland is $5,000, which includes design, construction of the dam and emergency spillway, earth moving, riprap, and planting DNC around the periphery during the first year.[4,5] A 2-percent ($100) annual maintenance cost also is included.[4] Thus, the annual cost is $52.80 per acre.

Construct Type 3 Wetlands. — This activity, *WD*(*i*,3), will provide pair and brood-rearing habitat, and consists of constructing type 3 wetlands on native grassland, tame grassland, alfalfa, and cropland, and DNC; i.e., i = 1, 2, 3, or 5 only. Similar to activity *WD*(*i*,2), many possible sizes and shapes of wetland can

[4]Personal communication from S. Hoetzer on August 25, 1982.
[5]Personal communication from R. Shupe on August 24-26, 1982.

facilitate pair and brood-rearing habitat; however, only one constructed pond size and shape will be considered here. Construction of type 3 wetlands has similar constraints to the type 4 wetlands. An average of 0.5 site per square mile is available within the study area.

Construction would consist of building an earthen dike 2 feet high with an average top width of 4 feet and an average length of 75 feet. The shape of the wetland will conform to the natural shape of the catchment basin, generally an oblong configuration that will range from 0.1 to 2 acres. The average size pond would be 1 acre with an average depth of 6 inches. Snowmelt and rain will fill the wetland.

The construction of one wetland costs $2,000, which includes design, construction of dike, earth moving, and planting DNC around the periphery during the first year.[4,5] A 2-percent ($40) annual maintenance cost also is included. Thus, the annual cost for constructing and maintaining one type 3 wetland is $105.60 per acre.

Restoration of Drained Wetlands. — Restoration of previously drained wetlands, $WD(i,k)$, will enhance pair and brood-rearing habitat for waterfowl, and consists of constructing plugs or dikes to improve previously drained wetlands:

1. Wetland restoration is restricted to wetland types 1, 3, 4, and 5 (i.e., $k = 4, 5, 6$, and 7, respectively). Overall, wetland types 9, 10, and 11 are not drained since the alkaline soils are generally not suitable for agricultural purposes.

2. Wetlands can be restored in cropland, alfalfa, native grassland, and DNC (i.e., $i = 1, 2, 3$, or 5).

3. The size, type, and number of restorable wetlands on a square mile (i.e., section) basis are shown in table B-9.

The process of draining wetlands consists of digging a ditch from the wetland and allowing the water to drain. The average size ditch in a type 1 and 3 wetland is 3 feet deep and 5 feet wide, and 5 feet deep and 8 feet wide in a type 4 and 5 wetland. After the soils have dried, the wetlands are planted with crops or grasses and used for agricultural purposes.

Wetland restoration is accomplished by plugging or filling the ditch so that water can be impounded, thus recreating a wetland habitat again. Construction requirements are minor, often requiring only hand labor. However, a backhoe or small bulldozer is used to plug larger ditches. The time and labor required to plug a type 1 or 3 wetland is 3 hours, including travel. An average of 5 hours is required to plug a type 4 or 5 wetland.

The costs for wetland restoration, per wetland type, include construction of plugs or dikes, labor, equipment, and travel time, and are shown in table B-10.

Table B-9. – Number of restorable wetlands per section.

Type	Average number per section[1]	Total acres per section[1]
1	8	4
3	20	100
4	2	50
5	0.5	12.5

[1] Mike McEnroe, U.S. Fish and Wildlife Service, Bismarck, ND, July 30, 1982.

Table B-10. – Costs of wetland restoration.

Wetland type	Average cost to plug ditch[1], dollars	Amortization factor	Amortized cost per ditch, dollars	Amortized cost per acre, dollars
1	150	0.0328	4.92	9.84
3	150	0.0328	4.92	0.98
4	250	0.0328	8.20	0.32
5	250	0.0328	8.20	0.32

[1] Personal communications from R. Antonette on July 16, 1982, and from J. Foster on July 15, 1982.

APPENDIX C

IBM-MPSX DATA SET GENERATOR

APPENDIX C. IBM-MPSX DATA SET GENERATOR

This generator (GENERATE) was designed to create an IBM-MPSX coded data set by transforming the system of equations/inequalities that define a mathematical programming problem. There are four elements to the program. Lines 1 through 4 and 107 comprise the JCL (Job Control Language) that is system dependent. Lines 5 through 104 are the Fortran program, lines 105 and 106 are the system specific output disk space commands, and lines 108 through 211 are a sample input data set.

After all equations/inequalities are written, they must be keypunched into a semi-coded format for data set input into GENERATE. For example, the equation

$$\text{ATENI} = .101*\text{ACAREA} \tag{C1}$$

would be semi coded for input into the generator as in line 126

AT9 E (RHS) ATENI 1.0 ACAREA .100

where:
AT9 = row name or designator,
E = an equality, and
(RHS) = intercept value expressed as a right hand side, in this example, it is zero.

The following card format is used to input the data:

Column	GENERATE Input Card Format Description
1-8	Row designation - left justified
9	Constraint type code E → =; ≤; G → ≥.
11-20	RHS Value (constant term in equation, leave blank if zero)
21-28	1st column designation (name)
31-40	Value of 1st column
41-48	2nd column designation
51-60	Value of 2nd column
61-68	3rd column designation
71-80	Value of 3rd column

Notes:
1. If more than three variables (column designation) appear in an equation, repeat as above except columns 9-20 may be left blank.
2. If a "RHS" value is not given, leave 11-20 blank.
3. If a "column value" is one (1), you may punch a 1., or leave appropriate columns blank.
4. All names and values are to be left justified, values *must* contain a decimal.

Although the program GENERATE is commented, special features warrant additional discussion. First, because lines 1-4 are system dependent, they generally would not be the same on any other machine. The following JCL description is offered for completeness:

Line	JCL Description of Lines 1-4
1	Standard JOB card – IBM system MVS
2	PRØCLIB card – The compiler FØRGI, that resides in the procedure library SYS4.PROCS. This card is computer system specific, i.e., it is installation dependent, if needed at all.
3	IBM system – involves the FORGIC compiler FØRGICLG
	FØORGIC – FORTRAN 66 with IBM enhancement compiler

L – invokes the linkage editor

G – began execution if the program following was error free.

REGION=600K – reserve 600K bytes for compilation, linkage, and execution.

TIME = (,20) – set upper limit at 20 CPU seconds for execution.

4 FORT .SYSIN DD * Inform the system that a FORTRAN program follows.

Secondly, the generator contains a work matrix X(599,100), a "column name" vector COL(1000), and a constraint vector CON(599). These matrices are used in creation of the MPSX code. Dimensions on "X" dictate any given problem may contain up to 599 rows, and each row (100-2)/2 unique column names, but the total number of column names cannot exceed 1000. "CON" contains the constraint of its corresponding "X" row. If these limits are too constraining, simply change the dimensions on these three matrices and the data statement #7 reflecting these new dimensions.

Thirdly, lines 105 and 106 direct the machine to store the generated data set. This JCL is entirely installation dependent, i.e., a programmer should be consulted for changes. The logical unit number for output by GENERATE is 1. Line 107 denotes that data input for GENERATE follows. This logical unit is the universal "card reader" unit 5.

Lastly, this package was written for a system with terminal interaction; in this case, WYLBUR was the text editing package used. The generator does not produce an executable LP matrix, however, it does allow the user a fast method of inputting equations, rows, and RHS values. Special considerations such as "BOUNDS" or separable variables and their markers must be input in their appropriate locations by the user, as well as any reordering to fit the given LP package.

```
1.   // JOB (),GENERATE
2.   // PROCLIB DD DSN=SYS4.PROCS,DISP=OLD
3.   // EXEC FORGICLG,REGION=600K,TIME=(,20)
4.   // FORT.SYSIN DD *
5.           REAL*8 XTRX(599,100),CNAME,CARD(9),BLK,COL(1000)
6.           DIMENSION CON(599)
7.           DATA N/0/, XTRX/59900*' '/, BLK/' '/,COL/100*' '/
8.   C
9.   C  WORK VECTOR 'CARD'
10.  C
11.  C  INPUT AND SET UP ONE ROW
12.  C
13.  C  XTRX(I,J) L P MATRIX CONSTRUCTION WORK AREA
14.  C            ROW BY ROW CONSTRUCTION
15.  C  CONTENTS OF XTRX
16.  C     XTRX(I,1) = ROW NAME
17.  C     XTRX(I,2) = RHS VALUE
18.  C  (1)    XTRX(I,3) = 1-ST COLUMN NAME
19.  C  (2)    XTRX(I,4) = COEFFICIENT (NON-ZERO) OF FIRST COLUMN NAME
20.  C
21.  C         (1) AND (2) ARE RESPECTIVELY REPEATED UNTIL ALL NON ZERO
22.  C         COEFFICIENTS AND THEIR COLUMN NAMES HAVE BEEN PROCESSED
23.  C         FOR THIS ROW.
24.  C
25.        READ (5,300,END=20) CARD
26.    300 FORMAT (A8,A1,1X,F10.0,3(A8,2X,F10.0))
27.      6 N = N + 1
28.        CARD(5) = -CARD(5)
29.        CON(N) = CARD(2)
30.        XTRX(N,1) = CARD(1)
31.      7 K = -1
32.  C
33.      9 DO 9 J=3,9
34.      9 XTRX(N,J+K) = CARD(J)
35.     10 READ (5,300,END=20) CARD
36.        IF (XTRX(N,1) .NE. CARD(1)) GO TO 6
37.        K = K + 6
38.        DO 11 J=4,9
39.     11 XTRX(N,J+K) = CARD(J)
40.        GO TO 10
41.  C
42.     20 CONTINUE
43.  C
44.  C  START MPSX FORMATTED OUTPUT
45.  C     ROWS SECTION, CONTAINS ALL ROWS AND THEIR CONSTRAINT
46.  C
47.        WRITE (1,400)
48.    400 FORMAT ('NAME',10X,'TEAL'/'ROWS')
```

```
49.   C
50.         DO 25 I=1,N
51.         WRITE (1,401) CON(I),XTRX(I,1)
52.      401 FORMAT (1X,A1,2X,A8)
53.       25 CONTINUE
54.   C
55.   C  RIGHT HAND SIDE (RHS) SECTION  ONLY NON-ZERO VALUES
56.   C    CONTAINS:
57.   C       RHS SECTION NAME COL 5-12  (MPSX ALLOWS MORE THAN ONE
58.   C                                  RHS SECTION PER PROBLEM SETUP)
59.   C       ROW NAME         COL 15-22
60.   C       RHS VALUE        COL 25-36 DECIMAL MUST BE PUNCHED
61.   C
62.           WRITE (1,402)
63.       402 FORMAT ('RHS')
64.           DO 30 I=1,N
65.           XTRX(I,2) = XTRX(I,2)
66.           IF (XTRX)(I,2) .NE. 0) WRITE (1,403) XTRX(I,1),XTRX(I,2)
67.       403 FORMAT (4X,'RHS',7X,A8,F14.6)
68.        30 CONTINUE
69.   C
70.   C  COLUMNS SECTION - NOTE MPSX IS COLUMN ORIENTED INPUT IE.
71.   C     WHEN A COLUMN IS STARTED ALL ROWS REFERENCED BY THAT COLUMN
72.   C     MUST BE OUTPUT.  ROW/COLUMN INTERSECTIONS WITH ZERO
73.   C     COEFFICIENTS NEED NOT BE PUNCHED.
74.   C
75.           WRITE (1,404)
76.      404 FORMAT ('COLUMNS')
77.          MC = 0
78.          DO 40 I=1,N
79.          DO 35 J=3,50,2
80.          IF (XTRX(I,J) .EQ. BLK) GO TO 40
81.          DO 32 K=1 MC
82.          IF (XTRX(I,J) .EQ. COL(K)) GO TO 35
83.       32 CONTINUE
84.          MC = MC + 1
85.          COL(MC) = XTRX(I,J)
86.       35 CONTINUE
87.       40 CONTINUE
88.   C
89.          DO 50 I=1,MC
90.          CNAME = COL(I)
91.          DO 45 J=1,N
92.          DO 42 K=3,50,2
93.          IF (XTRX(J,K) .EQ. BLK) GO TO 45
94.          IF (XTRX(J,K) .NE. CNAME) GO TO 42
95.          WRITE (1,405) CNAME,XTRX(J,1),XTRX(J,K+1)
96.      405 FORMAT (4X,A8,2X,A8,F14.6)
97.       42 CONTINUE
98.       45 CONTINUE
99.       50 CONTINUE
```

```
C
C       REWIND 3
        RETURN
        END
/*
// GO.FT01F001 DD DSN=USER.Y4107.TEAL,DISP=(NEW,CATLG),UNIT=CDISK),
// SPACE=(TRK,(200,1),RLSE),DCB=(LRECL=80,RECFM=FB,BLKSIZE=6160)
// GO.SYSIN DD*
AT1   E        ATHSIL    1.        ATNIL     .333      ATPIL     .333
AT1              ATBIL     .333
AT2   E        ATHSI     1.        ATA1      .00017    ATA2      .07214
AT2              ATA3      .1443     ATA4      .2467     ATA5      .3915
AT2              ATA6      .7214
AT3   E -300   ATHSIL    1.        ATA1      1.        ATA2      1.
AT3              ATA3      1.        ATA4      1.        ATA5      1.
AT3              ATA6      1.
AT4   E -300   ATNIL     6.93      ATB4      4.05      ATB5      2.55
AT4              ATB3      6.93      ATB4      4.05      ATB5      2.55
AT4              ATB6      1.39
AT5   E        ATNESTI   1.        ATB1      1.        ATB2      1.
AT5              ATB3      1.        ATB4      1.        ATB5      1.
AT5              ATB6      1.
AT6   E        ATNESTI   1.        ATC1      .0021
AT7   E        ATEONA    1.        ATC1      1.        ATC2      1.
AT8   E        ATEONA    1.        ATEN1     1.        ATEN2     1.
AT8              ATEN5     1.        ATEN13    1.        ATEN12    1.
AT9   E        ATEN1     1.        ACAREA    .100
AT10  E        ATEN2L    1.        ATIRNL    1.        ANTAREAL  1.
AT11  E        ATEN2     1.        ATD1      .0033     ATD2      5.1697
AT11             ATD3      44.8052   ATD4      144.2793  ATD5      268.0486
AT11             ATD6      411.5226  ATD7      556.1349
AT12  E -300.  ATEN2L    1.        ATD1      1.        ATD2      1.
AT12             ATD3      1.        ATD4      1.        ATD5      1.
AT12             ATD6      1.        ATD7      1.
AT13  E -300.  ATIRNL    1.        ATE1      5940.09   ATE2      13.86
AT13             ATE3      6.93      ATE4      4.05      ATE5      2.55
AT13             ATE6      1.39
AT14  E        ATIRN     1.        ATE1      1.        ATE2      1.
AT14             ATE3      1.        ATE4      1.        ATE5      1.
AT14             ATE6      1.
AT15  E .09    ATIRN     1.        ATF1      .92       ATF2      .52
AT15             ATE3      .18       ATF4      .16       ATF5      .04
AT16  E        AROBELN   1.        ATF1      1.        ATF2      1.
AT16             ATF3      1.        ATF4      1.        ATF5      1.
AT16             ATF6      1.
AT17  E        ATEN5     1.        AAAREA    .1608
AT18  E        ATEN13L   1.        ATIRDL    1.        ADAREAL   1.
AT19  E        ATEN13    1.        ATG1      .0033     ATG2      5.1697
AT19             ATG3      44.8052   ATG4      144.2793  ATG5      268.0486
AT19             ATG6      411.5226  ATG7      556.1349
AT20  E -300.  ATEN13L   1.        ATG1      1.        ATG2      1.
```

151.	AT20		ATG3	1.	ATG4	1.	ATG5	1.
152.	AT20		ATG6	1.	ATG7	1.		
153.	AT21	E -300.	ATIRDL	1.	ATH1	5940.09	ATH2	13.86
154.	AT21		ATH3	6.93	ATH4	4.05	ATH5	2.55
155.	AT21		ATH6	1.39				
156.	AT22	E	ATIRD	1.	ATH1	1.	ATH2	1.
157.	AT22		ATH3	1.	ATH4	1.	ATH5	1.
158.	AT22		ATH6	1.				
159.	AT23	E .09	ATIRD	1.	ATI1	.92	ATI2	.52
160.	AT23		ATI3	.18	ATI4	.16	ATI5	.04
161.	AT24	E	AROBELD	1.	ATI1	1.	ATI2	1.
162.	AT24		ATI3	1.	ATI4	1.	ATI5	1.
163.	AT24		ATI6	1.				
164.	AT25	E	ATEN12L	1.	ATIRRL	1.	ARAREAL	1.
165.	AT26	E	ATEN12	1.	ATJ1	.0033	ATJ2	5.1697
166.	AT26		ATJ3	44.8052	ATJ4	144.2793	ATJ5	268.0486
167.	AT26		ATJ6	411.5226	ATJ7	556.1349		
168.	AT27	E -300.	ATEN12L	1.	ATJ1	1.	ATJ2	1.
169.	AT27		ATJ3	1.	ATJ4	1.	ATJ5	1.
170.	AT27		ATJ6	1	ATJ7	1.		
171.	AT28	E -300.	ATIRRL	1.	ATK1	5940.09	ATK2	13.86
172.	AT28		ATK3	6.93	ATK4	4.05	ATK5	2.55
173.	AT28		ATK6	1.39				
174.	AT29	E	ATIRR	1.	ATK1	1.	ATK2	1.
175.	AT29		ATK3	1.	ATK4	1.	ATK5	1.
176.	AT29		ATK6	1.				
177.	AT30	E .09	ATIRR	1.	ATL1	.92	ATL2	.52
178.	AT30		ATL3	.18	ATL4	.16	ATL5	.04
179.	AT31	E	AROBELR	1.	ATL1	1.	ATL2	1.
180.	AT31		ATL3	1.	ATL4	1.	ATL5	1.
181.	AT31		ATL6	1.				
182.	AT32	E -300.	ATPIL	1.	ATM1	5940.09	ATM2	13.86
183.	AT32		ATM3	6.93	ATM4	4.05	ATM5	2.55
184.	AT32		ATM6	1.39				
185.	AT33	E	ATPAIRI	1.	ATM1	1.	ATM2	1.
186.	AT33		ATM3	1.	ATM4	1.	ATM5	1.
187.	AT33		ATM6	1.				
188.	AT34	E	ATPAIRI	1.	ATPEWNI	.5	ATPEWAI	.5
189.	AT35	E	ATPEWNI	1.	ATN1	.0067		
190.	AT36	E	ATPEWN	1.	ATN1	1.	ATN2	1.
191.	AT37	E	ATPEWN	1.	AEPOND	.72	ASPOND	1.
192.	AT37		AFPOND	.93	APPOND	.44	ABPOND	.02
193.	AT38	E	ATPEWAI	1.	ATO1	.0067		
194.	AT39	E	ATPEWA	1.	ATO1	1.	ATO2	1.
195.	AT40	E	ATPEWA	1.	AEAREA	.72	ASAREA	1.
196.	AT40		AFAREA	.93	ABAREA	.02	APAREA	.44
197.	AT41	E -300.	ATBIL	1.	ATP1	5940.09	ATP2	13.86
198.	AT41		ATP3	6.93	ATP4	4.05	ATP5	2.55
199.	AT41		ATP6	1.39				
200.	AT42	E	ATBROODI	1.	ATP1	1.	ATP2	1.
201.	AT42		ATP3	1.	ATP4	1.	ATP5	1.

```
202.   AT42            ATP6      1.
203.   AT43  E         ATBROODI  1.        ATBEWNI  .5        ATBEWAI  .5
204.   AT44  E         ATBEWNI   1.        ATQ1     .1667
205.   AT45  E         ATBEWN    1.        ATQ1     1.        ATQ2     1.
206.   AT46  E         ATBEWN    1.        ASPOND   .5        AFPOND   1.
207.   AT46            APPOND    .5        ABPOND   .15
208.   AT47  E         ATBEWAI   1.        ATR1     .02
209.   AT48  E         ATBEWA    1.        ATR1     1.        ATR2     1.
210.   AT49  E         ATBEWA    1.        ASAREA   .5        AFAREA   1.
211.   AT49            APAREA    .5        ABAREA   .15
```

An example of the MPSX data set generated from the input data (liens 1 through 327) is listed below. It is preceded by an example listing of JCL and program control statements required to execute a complete separable programming model.

```
 1.    // JOB (,,45),'TEALPLAN'
 2.    // PROCLIB DD DSN=SYS3.PPROCS,DISP=OLD
 3.    // EXEC MPSX,TIME=(,59),REGION=1000K
 4.    // MPSCOMP.SYSIN DD *
 5.           PROGRAM
 6.    *      THIS MODEL FORMULATES THE COST MINIMIZING PLAN FOR
 7.    *      MITIGATING TEAL HABITAT LOSSES IN THE GARRISON
 8.    *      DIVERSION IRRIGATION PROJECT AREA, NORTH DAKOTA.
 9.           TITLE ('TEAL MITIGATION MODEL - RUN 1')
10.           INITIALZ
11.           MOVE (XDATA,'TEAL')
12.           MOVE (XPRNAME,'CASE1')
13.           MOVE (XRHS,'RHS')
14.           MOVE (XOBJ,'MINCOST')
15.           CONVERT ('SUMMARY')
16.           SETUP ('BOUND','BOUNDS')
17.           PRIMAL
18.           SOLUTION
19.           EXIT
20.           PEND
21.    // MPSEXEC.SYSIN DD DSN=USER.Y3560.GARR.TEA,DISP=OLD
```

```
 1.   NAME                    TEAL
 2.   ROWS
 3.    E     AT1
 4.    E     AT2
 5.    E     AT3
 6.    E     AT4
 7.    E     AT5
 8.    E     AT6
 9.    E     AT7
10.    E     AT8
11.    E     AT9
12.    E     AT10
13.    E     AT11
14.    E     AT12
15.    E     AT13
16.    E     AT14
17.    E     AT15
18.    E     AT16
19.    E     AT17
20.    E     AT18
21.    E     AT19
22.    E     AT20
23.    E     AT21
24.    E     AT22
25.    E     AT23
26.    E     AT24
27.    E     AT25
28.    E     AT26
29.    E     AT27
30.    E     AT28
31.    E     AT29
32.    E     AT30
33.    E     AT31
34.    E     AT32
35.    E     AT33
36.    E     AT34
37.    E     AT35
38.    E     AT36
39.    E     AT37
40.    E     AT38
41.    E     AT39
42.    E     AT40
43.    E     AT41
44.    E     AT42
45.    E     AT43
46.    E     AT44
47.    E     AT45
48.    E     AT46
49.    E     AT47
```

```
 E    AT48
 E    AT49
RHS
        RHS              AT3        300.000000
        RHS              AT4        300.000000
        RHS              AT12       300.000000
        RHS              AT13       300.000000
        RHS              AT15        -0.090000
        RHS              AT20       300.000000
        RHS              AT21       300.000000
        RHS              AT23        -0.090000
        RHS              AT27       300.000000
        RHS              AT28       300.000000
        RHS              AT30        -0.090000
        RHS              AT32       300.000000
        RHS              AT41       300.000000
COLUMNS
        ATHSIL           AT1         -1.000000
        ATHSIL           AT3         -1.000000
        ATNIL            AT1          0.333000
        ATNIL            AT4         -1.000000
        ATPIL            AT1          0.333000
        ATPIL            AT32        -1.000000
        ATBIL            AT1          0.333000
        ATBIL            AT41        -1.000000
        ATHSI            AT2         -1.000000
        ATA1             AT2          0.000070
        ATA1             AT3          1.000000
        ATA2             AT2          0.072140
        ATA2             AT3          1.000000
        ATA3             AT2          0.144300
        ATA3             AT3          1.000000
        ATA4             AT2          0.246700
        ATA4             AT3          1.000000
        ATA5             AT2          0.391500
        ATA5             AT3          1.000000
        ATA6             AT2          0.721400
        ATA6             AT3          1.000000
        ATB1             AT4       5940.090000
        ATB1             AT5          1.000000
        ATB2             AT4         13.860000
        ATB2             AT5          1.000000
        ATB3             AT4          6.930000
        ATB3             AT5          1.000000
        ATB4             AT4          4.050000
        ATB4             AT5          1.000000
        ATB5             AT4          2.550000
        ATB5             AT5          1.000000
        ATB6             AT4          1.390000
        ATB6             AT5          1.000000
```

100.	ATNESTI	AT5	-1.000000
101.	ATNESTI	AT6	-1.000000
102.	ATC1	AT6	0.002100
103.	ATC1	AT7	1.000000
104.	ATEONA	AT7	-1.000000
105.	ATEONA	AT8	-1.000000
106.	ATC2	AT7	1.000000
107.	ATEN1	AT8	1.000000
108.	ATEN1	AT9	-1.000000
109.	ATEN2	AT8	1.000000
110.	ATEN2	AT11	-1.000000
111.	ATEN5	AT8	1.000000
112.	ATEN5	AT17	-1.000000
113.	ATEN13	AT8	1.000000
114.	ATEN13	AT19	-1.000000
115.	ATEN12	AT8	1.000000
116.	ATEN12	AT26	-1.000000
117.	ACAREA	AT9	0.101000
118.	ATEN2L	AT10	-1.000000
119.	ATEN2L	AT12	-1.000000
120.	ATIRNL	AT10	1.000000
121.	ATIRNL	AT13	-1.000000
122.	ANTAREAL	AT10	1.000000
123.	ATD1	AT11	0.003300
124.	ATD1	AT12	1.000000
125.	ATD2	AT11	5.169700
126.	ATD2	AT12	1.000000
127.	ATD3	AT11	44.805200
128.	ATD3	AT12	1.000000
129.	ATD4	AT11	144.279300
130.	ATD4	AT12	1.000000
131.	ATD5	AT11	268.048600
132.	ATD5	AT12	1.000000
133.	ATD6	AT11	411.522600
134.	ATD6	AT12	1.000000
135.	ATD7	AT11	556.134900
136.	ATD7	AT12	1.000000
137.	ATE1	AT13	5940.090000
138.	ATE1	AT14	1.000000
139.	ATE2	AT13	13.860000
140.	ATE2	AT14	1.000000
141.	ATE3	AT13	6.930000
142.	ATE3	AT14	1.000000
143.	ATE4	AT13	4.050000
144.	ATE4	AT14	1.000000
145.	ATE5	AT13	2.550000
146.	ATE5	AT14	1.000000
147.	ATE6	AT13	1.390000
148.	ATE6	AT14	1.000000
149.	ATIRN	AT14	-1.000000
150.	ATIRN	AT15	-1.000000

151.	ATF1	AT15	0.920000
152.	ATF1	AT16	1.000000
153.	ATF2	AT15	0.520000
154.	ATF2	AT16	1.000000
155.	ATF3	AT15	0.180000
156.	ATF3	AT16	1.000000
157.	ATF4	AT15	0.160000
158.	ATF4	AT16	1.000000
159.	ATF5	AT15	0.040000
160.	ATF5	AT16	1.000000
161.	AROBELN	AT16	-1.000000
162.	ATF6	AT16	1.000000
163.	AAAREA	AT17	0.160800
164.	ATEN13L	AT18	-1.000000
165.	ATEN13L	AT20	-1.000000
166.	ATIRDL	AT18	1.000000
167.	ATIRDL	AT21	-1.000000
168.	ADAREAL	AT18	1.000000
169.	ATG1	AT19	0.003300
170.	ATG1	AT20	1.000000
171.	ATG2	AT19	5.169700
172.	ATG2	AT20	1.000000
173.	ATG3	AT19	44.805200
174.	ATG3	AT20	1.000000
175.	ATG4	AT19	144.279300
176.	ATG4	AT20	1.000000
177.	ATG5	AT19	266.048600
178.	ATG5	AT20	1.000000
179.	ATG6	AT19	411.522600
180.	ATG6	AT20	1.000000
181.	ATG7	AT19	556.134900
182.	ATG7	AT20	1.000000
183.	ATH1	AT21	5940.090000
184.	ATH1	AT22	1.000000
185.	ATH2	AT21	13.860000
186.	ATH2	AT22	1.000000
187.	ATH3	AT21	6.930000
188.	ATH3	AT22	1.000000
189.	ATH4	AT21	4.050000
190.	ATH4	AT22	1.000000
191.	ATH5	AT21	2.550000
192.	ATH5	AT22	1.000000
193.	ATH6	AT21	1.390000
194.	ATH6	AT22	1.000000
195.	ATIRD	AT22	-1.000000
196.	ATIRD	AT23	-1.000000
197.	ATI1	AT23	0.920000
198.	ATI1	AT24	1.000000
199.	ATI2	AT23	0.520000
200.	ATI2	AT24	1.000000

201.	ATI3	AT23	0.180000
202.	ATI3	AT24	1.000000
203.	ATI4	AT23	0.160000
204.	ATI4	AT24	1.000000
205.	ATI5	AT23	0.040000
206.	ATI5	AT24	1.000000
207.	AROBELD	AT24	-1.000000
208.	ATI6	AT24	1.000000
209.	ATEN12L	AT25	-1.000000
210.	ATEN12L	AT27	-1.000000
211.	ATIRRL	AT25	1.000000
212.	ATIRRL	AT28	-1.000000
213.	ARAREAL	AT25	1.000000
214.	ATJ1	AT26	0.003300
215.	ATJ1	AT27	1.000000
216.	ATJ2	AT26	5.169700
217.	ATJ2	AT27	1.000000
218.	ATJ3	AT26	44.805200
219.	ATJ3	AT27	1.000000
220.	ATJ4	AT26	144.279300
221.	ATJ4	AT27	1.000000
222.	ATJ5	AT26	268.048600
223.	ATJ5	AT27	1.000000
224.	ATJ6	AT26	411.522600
225.	ATJ6	AT27	1.000000
226.	ATJ7	AT26	556.134900
227.	ATJ7	AT27	1.000000
228.	ATK1	AT28	5940.090000
229.	ATK1	AT29	1.000000
230.	ATK2	AT28	13.860000
231.	ATK2	AT29	1.000000
232.	ATK3	AT28	6.930000
233.	ATK3	AT29	1.000000
234.	ATK4	AT28	4.050000
235.	ATK4	AT29	1.000000
236.	ATK5	AT28	2.550000
237.	ATK5	AT29	1.000000
238.	ATK6	AT28	1.390000
239.	ATK6	AT29	1.000000
240.	ATIRR	AT29	-1.000000
241.	ATIRR	AT30	-1.000000
242.	ATL1	AT30	0.920000
243.	ATL1	AT31	1.000000
244.	ATL2	AT30	0.520000
245.	ATL2	AT31	1.000000
246.	ATL3	AT30	0.180000
247.	ATL3	AT31	1.000000
248.	ATL4	AT30	0.160000
249.	ATL4	AT31	1.000000
250.	ATL5	AT30	0.040000
251.	ATL5	AT31	1.000000

252.	AROBELR	AT31	-1.000000
253.	ATL6	AT31	1.000000
254.	ATM1	AT32	5940.000000
255.	ATM1	AT33	1.000000
256.	ATM2	AT32	13.860000
257.	ATM2	AT33	1.000000
258.	ATM3	AT32	6.930000
259.	ATM3	AT33	1.000000
260.	ATM4	AT32	4.050000
261.	ATM4	AT33	1.000000
262.	ATM5	AT32	2.550000
263.	ATM5	AT33	1.000000
264.	ATM6	AT32	1.390000
265.	ATM6	AT33	1.000000
266.	ATPAIRI	AT33	-1.000000
267.	ATPAIRI	AT34	-1.000000
268.	ATPEWNI	AT34	0.500000
269.	ATPEWNI	AT35	-1.000000
270.	ATPEWAI	AT34	0.500000
271.	ATPEWAI	AT38	-1.000000
272.	ATN1	AT35	0.006700
273.	ATN1	AT36	1.000000
274.	ATPEWN	AT36	-1.000000
275.	ATPEWN	AT37	-1.000000
276.	ATN2	AT36	1.000000
277.	AEPOND	AT37	0.720000
278.	ASPOND	AT37	1.000000
279.	ASPOND	AT46	0.500000
280.	AFPOND	AT37	0.930000
281.	AFPOND	AT46	1.000000
282.	APPOND	AT37	0.440000
283.	APPOND	AT46	0.500000
284.	ABPOND	AT37	0.020000
285.	ABPOND	AT46	0.150000
286.	ATO1	AT38	0.006700
287.	ATO1	AT39	1.000000
288.	ATPEWA	AT39	-1.000000
289.	ATPEWA	AT40	-1.000000
290.	ATO2	AT39	1.000000
291.	AEAREA	AT40	0.720000
292.	ASAREA	AT40	1.000000
293.	ASAREA	AT49	0.500000
294.	AFAREA	AT40	0.930000
295.	AFAREA	AT49	1.000000
296.	ABAREA	AT40	0.020000
297.	ABAREA	AT49	0.150000
298.	APAREA	AT40	0.150000
299.	APAREA	AT49	0.500000
300.	ATP1	AT41	5940.090000

301.	ATP1	AT42	1.000000
302.	ATP2	AT41	13.860000
303.	ATP2	AT42	1.000000
304.	ATP3	AT41	6.930000
305.	ATP3	AT42	1.000000
306.	ATP4	AT41	4.050000
307.	ATP4	AT42	1.000000
308.	ATP5	AT41	2.550000
309.	ATP5	AT42	1.000000
310.	ATP6	AT41	1.390000
311.	ATP6	AT42	1.000000
312.	ATBROODI	AT42	-1.000000
313.	ATBROODI	AT43	-1.000000
314.	ATBEWNI	AT43	0.500000
315.	ATBEWNI	AT44	-1.000000
316.	ATBEWAI	AT43	0.500000
317.	ATBEWAI	AT47	-1.000000
318.	ATQ1	AT44	0.166700
319.	ATQ1	AT45	1.000000
320.	ATBEWN	AT45	-1.000000
321.	ATBEWN	AT46	-1.000000
322.	ATQ2	AT45	1.000000
323.	ATR1	AT47	0.020000
324.	ATR1	AT48	1.000000
325.	ATBEWA	AT48	-1.000000
326.	ATBEWA	AT49	-1.000000
327.	ATR2	AT48	1.000000

BACKGEN

A comparison program to GENERATE, named BACKGEN, was developed to transform an IBM-MPSX data set back into the original set of equations underlying that data set. Thus, BACKGEN is a back generator. The program listing follows. Comments within this program instruct the user on its application.

```
 1.   // JOB (), 'BACKGEN'
 2.   //PROCLIB DD DSN=SYS4.PPROCS,DISP=OLD
 3.   // EXEC FORG1CLG,REGION=600K,TIME=(,20)
 4.   //FORT.SYSIN DD *
 5.           REAL*8 XTRX(500,50),RHS,VAL,MARK,ROW,BLK,COL,SIGN,
 6.           1 MI,M2,M3,M4,BAD,E,NN,G,LL,EQUAL,GTHAN,LTHAN
 7.           INTEGER TOTAL,VAR(500),POPT
 8.           DATA BLK/' '/,XTRX/25000*' '/,VAR/500*0/,M1/'ROWS'/,
 9.           1 M2/'R'/,M3/'C'/,M4/'B'/,BAD/'''MARKER'''/,E/'E'/,
10.           2 POPT/4/,LF/0/LCOUNT/0/,NN/'N'/,G/'G'/,LL/'L'/,EQUAL/' ='/,
11.           3 GTHAN/'>='/,LTHAN/'<='/
12.   C
13.   C  DEFAULT IS PRINTING 4 COLUMNS PER PAGE.  IF YOU WANT TO
14.   C  CHANGE THIS, ENTER A LINE THAT SAYS POPT=3 AFTER THIS
15.   C  SECTION OF COMMENTS
16.   C
17.           POPT=3
18.   C
19.   C  INPUT:  MPS DATA CARDS FOR LP.  ROWS SHOULD BE FIRST SET
20.   C  OF INPUTS.  PROGRAM CHECKS FOR KEYWORDS STARTING IN COLUMN 1
21.   C  (ROWS, COLUMNS, RHS, OR BOUNDS) AND SENDS TO APPROPRIATE
22.   C  AREA OF CODE TO BUILD MATRIX.
23.   C
24.   C  BEFORE RUNNING:
25.   C       TITLE - IF YOU WANT TO CHANGE THE TITLE, MODIFY LINES
26.   C       151/152 BY PUTTING THE NEW TITLE YOU WANT WITHIN THE QUOTES.
27.   C       ALSO, CHANGE THE SPACING (VALUE BEFORE "X") TO THE
28.   C       VALUE OF 55 - LENGTH/2, WHERE LENGTH IS THE LENGTH OF
29.   C       YOUR TITLE.  THIS WILL CENTER THE TITLE.  IF YOU DON'T
30.   C       WANT A TITLE, JUST PUT IN ' '
31.   C
32.   C       PAGE - IF YOU WANT THE PAGING TO START AT OTHER THAN
33.   C       PAGE #1, CHANGE TH IPAGE VARIABLE JUST BELOW THIS
34.   C       TO THE NEW LINE NUMBER.  IF YOU DON'T WANT ANY PAGING,
35.   C       PUT A "C" IN COLUMN ONE OF LINES 130 & 148 TO COMMENT
36.   C       THEM OUT OF THE PROGRAM.
37.   C
38.   C  TO RUN, TYPE:
39.   C            RUN UNN TER
40.   C  WHEN JOB IS COMPLETE, TYPE:
41.   C
42.   C            FET * DDB 6 CC
43.   C  THIS WILL FETCH ONLY THE OUTPUT WITH THE CARRIAGE CONTROL
44.   C  CHARACTERS.  LOOK AT THIS AN PUT IN ANY EXTRA LINES YOU MAY
45.   C  WANT OR MAKE ANY CHANGES.  THEN DO,
46.   C       COP FRO #DOC2 TO .1
47.   C  TO GET THE DOCUMENT PRINTER JCL TO THE BEGINNING
48.   C  SAVE THIS FOR FUTURE REFERENCE.
49.   C
```

```
C  TO GET YOUR PRINTOUT, TYPE:
C            PRINT UNN CC NOJ DEST=DOCUMENT
C  THEN YOU CAN PICK UP YOUR OUTPUT ON THE HILL.
C
C       IPAGE=1
2       READ (5,100,END=54) MARK
100     FORMAT(A8)
1       IF (MARK .EQ. M1) GO TO 50
4       IF (MARK .EQ. M2) GO TO 51
        IF (MARK .EQ. M3) GO TO 52
        IF (MARK .EQ. M4) GO TO 53
        GO TO 2
50      N=0
C
C  BUILD ROWS
C
C  READ IN A LINE, CHECK FOR KEYWORD.  IN COL 1 OF XTRX, KEEP
C  ROW NAME.  IN COL 3 KEEP CONSTRAINT E, L, G, OR N.  KEEP
C  TRACK OF TOTAL NUMBER OF EQNS.
C
3       READ (5,101,END=54) MARK,SIGN, ROW
101     FORMAT(2A1,2X,A8)
        IF (MARK .NE. BLK) GO TO 4
        N=N+1
        XTRX(N,1)=ROW
        IF (SIGN .EQ. NN .OR. SIGN .EQ. E) XTRX(N,3)=EQUAL
        IF (SIGN .EQ. G) XTRX(N,3)=GTHAN
        IF (SIGN .EQ. LL) XTRX(N,3)=LTHAN
        TOTAL=N
        GO TO 3
C
C  BUILD RHS'S
C
C  READ IN A LINE, CHECK FOR KEYWORD.  SEARCH XTRX FOR MATCHING
C  ROW NAME THEN PUT NEGATIVE OF RHS IN COL 4 OF XTRX.
C
51      N=0
6       READ (5,102,END=54) MARK,ROW,RHS
10      FORMAT(A1,13X,A8,F14.0)
        IF (MARK .NE. BLK) GO TO 4
5       N=N+1
        IF (ROW .NE. XTRX(N,1)) GO TO 5
        XTRX(N,4)=-RHS
        GO TO 51
C
C  BUILD COLUMNS
C
```

```
C  READ IN A LINE, CHECK FOR KEYWORD.  IF COLUMN NAME IS 'MARKER',
C  READ ANOTHER LINE.  SEARCH XTRX FOR MATCHING ROW NAME.  IN
C  EQUALITY CONSTRAINTS, FIRST COL NAME WITH COEFF OR -1 IS
C  LEFTHAND VARIABLE.  ALL OTHERS ARE REGULAR VARIABLES.  ARRAY
C  VAR HOLDS NUMBER OF COLUMNS FOR EACH ROW.
C
52      CONTINUE
8       READ (5,103,END-54) MARK,COL,ROW,VAL
103     FORMAT(A1,3X,A8,2X,A8,F14.0)
        IF (MARK .NE. BLK) GO TO 4
        IF (COL .EQ. BAD) GO TO 8
        N=0
7       N=N+1
        IF (N .GT. TOTAL) GO TO 8
        IF (N .EQ. TOTAL .AND. ROW .NE. XTRX(N,1)) GO TO 8
        IF (ROW .NE. XTRX(N,1) .AND. N .T. TOTAL) GO TO 7
        IF (XTRX(N,2) .NE. BLK .OR. VAL .NE. -1.0) GO TO 10
        IF (XTRX(N,3) .NE. EQUAL) GO TO 10
        XTRX(N,2)=COL
        GO TO 8
10      L=2*VAR(N)+5
        XTRX(N,L)=VAL
        XTRX(N,L+1)=COL
        VAR(N)=VAR(N)+1
        GO TO 8
C
C  DO NOTHING WITH BOUNDS
C
53      GO TO 2
C
C  PRINT OUT RESULTS
C
54      CONTINUE
        WRITE(6,201) IPAGE
        WRITE(6,202)
        WRITE(6,203)
        WRITE(6,204)
        DO 9 I=1,TOTAL
        L=2*VAR(I)+4
        IF (XTRX(I,4) .EQ. BLK) XTRX(I,4)=0.0
        IF (POPT .EQ. 4) WRITE(6,104) (XTRX(I,J),J=1,LO
        IF (POPT .EQ. 3) WRITE(6,105) (XTRX(I,J),J=1,L)
105     FORMAT('0',A8,2X,A8,2X,A2,F15.4,2X,3(' +',F13.5,1X,A8)/
        1 (40X,3(' +',F13.5,1X,A8)))
        LF=0
        LCOUNT=1+LCOUNT+(VAR(I)+POPT-1)/POPT
        IF (LCOUNT .LT. 42) GO TO 9
        LCOUNT=0
        WRITE(6,106)
106     FORMAT('1')
        IPAGE=IPAGE+1
```

```
148.            WRITE(6,201) IPAGE
149.    201     FORMAT(108X,I3)
150.            WRITE(6,202)
151.    202     FORMAT('0',31X,'MALLARD ENHANCEMENT EXPLICIT PROGRAMMING',
152.            1'EQUATIONS'/)
153.            WRITE(6,203)
154.    203     FORMAT('OEQN NAME')
155.            WRITE(6,204)
156.    204     FORMAT('+       ')
157.    104     FORMAT('0',A8,2X,A8,2X,A1,F15.4,2X,4(F13.5,1X,A8)/
158.            1 (39X,4(F13.5,1X,A8)))
159.    9       CONTINUE
160.            RETURN
161.            END
162.    //GO.SYSIN DD*
```

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