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FREQUENCY-RESPONSE ANALYSIS OF AUTOMATED CANALS

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16. ABSTRACT

Two methods are available for determining control parameters on automated canals. These are, the transient-response method and the frequency-response method. Since a typical canal system is influenced predominantly by transient inputs, the transient-response method has been extensively used in the analysis of automated canal systems. The main advantage of the frequency-response method is that the effects of varying the various control parameters can be readily visualized and evaluated. The presently used transient-response method and parameter selection are reviewed. The basic concepts of the frequency-response method are presented and their application to an automated reach is illustrated with an example. Two computer programs are given which can be used to determine canal response characteristics. The state-of-the-art in process control has progressed sufficiently so that both the transient-response and frequency-response methods of analysis can be used in a complementary fashion. Employment of only one method in the design of automated systems is, in general, wasteful of both computer time and engineering effort.

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UNITED STATES DEPARTMENT OF THE INTERIOR * BUREAU OF RECLAMATION

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INTRODUCTION

The application of automation to water systems has been well summarized by a "state-of-the-art" report compiled by the USBR Water Systems Automation Team [10] [1]. Additional details concerning controller equipment have been reported by Buyalski [2]. Methods for determining the parameters for specific controllers have been developed by Shand [9] and Wall [11]. These reports all use analysis methods which are derived from control theory; however, the advantages and limitations that are inherent with a particular method are not always apparent from the individual reports. Ogata [7], on the other hand, has very carefully delineated the various analysis methods and their respective areas of applicability. The purpose of this contribution is to put the methods in perspective with respect to water systems automation and to illustrate the applicability of the frequency-response analysis method.

The performance characteristics of a system can be considered as having two components: a transient response and a steady-state response. The transient response refers to changes that occur in the output from a system as a result of the input passing from one state or level to another state. The steady-state response refers to the behavior of the output as time becomes infinite. Normally, step, ramp, and impulse inputs are analyzed using transient-response methods; whereas, frequency-response methods are used to analyze sinusoidal inputs.

The choice of the analysis method to be used is frequently indicated by the expected shape of the signals input into the system. With a single reach of canal, the water demands change by fixed increments. In this case, the typical operation consists of a fixed amount of water being turned out or diverted from the canal. The length of time from the beginning of the diversion until the final discharge quantity is reached is so short that the change in discharge can be considered to be a step change. This, in turn, produces a momentary step change in the water surface level which actuates the controller. Therefore, the use of the transient-response methods is appropriate.

With multiple reaches of canal, the choice of an analysis method is no longer trivial. As the disturbances pass further from the site of the turnout, they become more and more sinusoidal. Therefore, for these cases, the use of the frequency-response method is a reasonable approach.

[1] Numbers in brackets identify references contained in the bibliography.

The main advantage of the frequency-response method is that the absolute and relative stabilities of a linear, closed-loop system can be determined from the open-loop frequency-response characteristics. Another way of stating this is that the stability of the control system can be determined by knowing the frequency-response characteristics of each part. Since these characteristics can be determined experimentally, the stability analysis is facilitated. In contrast, the transient-response analysis requires a knowledge of the mathematical expression for the entire system in order to investigate the system's stability. For complicated systems, such as canals, the mathematical expression is almost impossible to determine. Even with simplifying assumptions, the resulting expression is usually very complicated.

A secondary advantage of the frequency-response method is that the analysis and design can be extended to certain nonlinear control systems. For instance, the effect of a dead band on a controller can be investigated without the need for trial and error computer runs.

The main disadvantage of the frequency-response method is that for complex systems (higher order systems in control systems terminology) the correlation between frequency response and transient response is not direct. For canal systems, this difference can be significant since the transient-response characteristics are frequently the most critical with respect to the safety of the structure and the speed with which demands on the system can be met. However, this disadvantage can be overcome; through the use of specific design criteria applied to the frequency-response methods, acceptable transient-response characteristics can be achieved.

In canals, discharge is normally the parameter which is to be controlled although discharge is not a quantity which can be determined directly. Instead some other quantity must be measured and then discharge is related to that quantity through a mathematical relationship. The quantity most frequently used in canals as the significant parameter is the water surface elevation or the difference in the water surface elevation between two points. Therefore, in the discussions that follow, the water surface elevation is the controlled parameter.

The present methods use terms to describe the control parameters which are based on the behavior of a second order system. The general form of a second order system can be expressed by the following differential equation:

$$\frac{d^2y}{dt^2} + K_1 \frac{dy}{dt} + K_2 y + K_3 = 0 \qquad (1)$$

Shand [9] derived an approximate model for the canal behavior which has the form of this equation; he retained only the first two terms. In his derivation, y refers to the change in the downstream water depth. The canal geometry, initial water depths, friction factors, etc., are simulated by different values of the coefficient K_1. With Shand's simulation, all disturbances decay exponentially.

The addition of a controller (feedback) significantly alters the behavior of the downstream water surface. Shand [9] showed, for certain control parameters, a unit step change in water elevation at the upstream end of the canal produced a damped oscillation, figure 1. With other control parameters, sustained oscillations will result. The shape of the transient-response curve is essentially determined by five parameters, figure 1. These parameters can be described numerically by either time intervals or amplitudes. The conventional nomenclature which is used to describe the parameters and the transient-response characteristics, is:

t_d = Delay time
t_r = Rise time
t_p = Peak time
t_s = Settling time
M_1 = Maximum overshot (percent)

Ideally, each element of the controller and of the water surface fluctuations in the canal can be described by a mathematical function. These functions and the signal flow path through the system can be represented pictorially by a block diagram. For instance, the canal system shown in figure 2 can be represented by the block diagram of figure 3. The capital letter by each block represents a mathematical function which gives

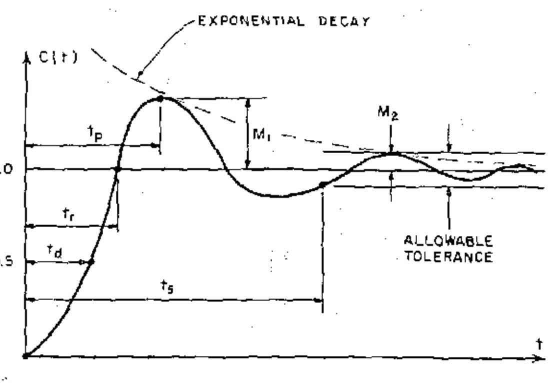

Figure 1.—Response of a second order system to a unit step input.

the relationship between signals input into the block and the signals which are output from the block; in other words, the letter represents the transfer function for the block.

The figure 2 diagram represents downstream control; this is called downstream control because disturbances are detected downstream of the controlling element. These disturbances are sent through a controller to an element which affects the desired remedial action to eliminate the disturbance. In this case, the controlling element is the upstream gate.

It should be noted at this point that the frequency-response analysis methods will work equally well for upstream control. For the purposes of illustrating the application of frequency-analysis techniques to canal automation, a downstream control scheme was selected.

The system to be controlled in a single canal reach is the water prism contained between the upstream and downstream gates (fig. 2). Both upstream and downstream disturbances can effect the water levels in the canal reach. This system is regulated by a controller that attempts to control the water surface disturbances within certain prescribed limits.

Typical controllers contain filter, proportional, and integral elements. The purposes of the filter are to compensate for the wave-travel time in the canal reach and to prevent passage of all trivial disturbances from the water level sensor signal. These extraneous disturbances generally have high-frequency components and include, among other things, wind waves. The purpose of the proportional element of the controller is to make the inflow rate through the upstream gate equal to the outflow rate from the canal reach. To achieve this end, the upstream gate is moved in proportion to the change in water level at the sensor. This part of the controller function is based upon the assumption that changes in water level at the sensor and changes in the gate position are both directly proportional to changes in discharge. The purpose of the integral element is to move the upstream gate in such a fashion that the water level at the sensor is always returned to its original position.

The flow of disturbances through the system and the controller can be followed on the block diagram, figure 3. In this diagram, G refers to an element being controlled and H refers to an element on the feedback path. For instance, a turnout at the downstream gate produces two signals that travel upstream simultaneously. One signal passes through the canal itself; the transformation of this signal as it passes

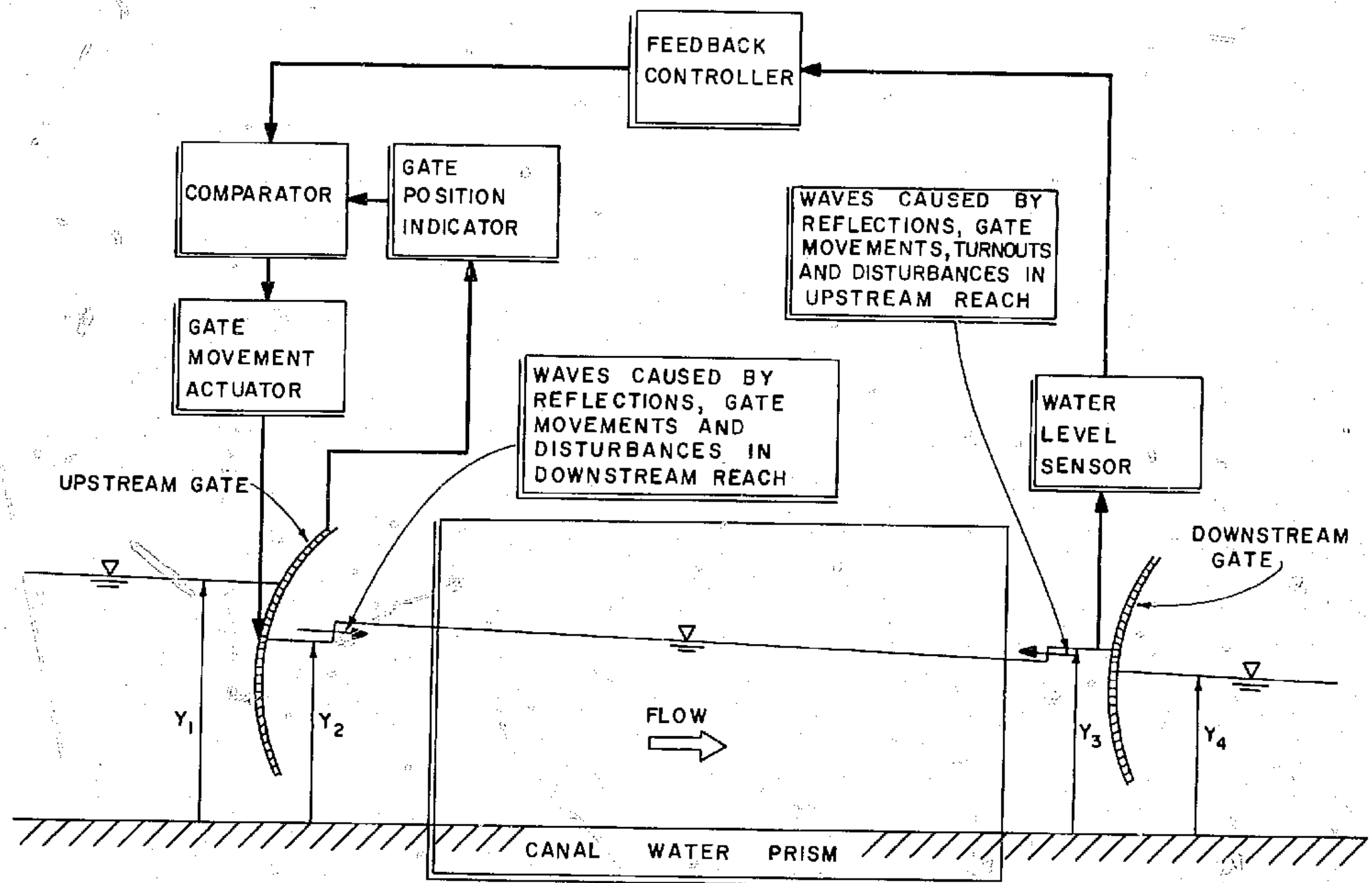

Figure 2.—Schematic representation of a canal reach.

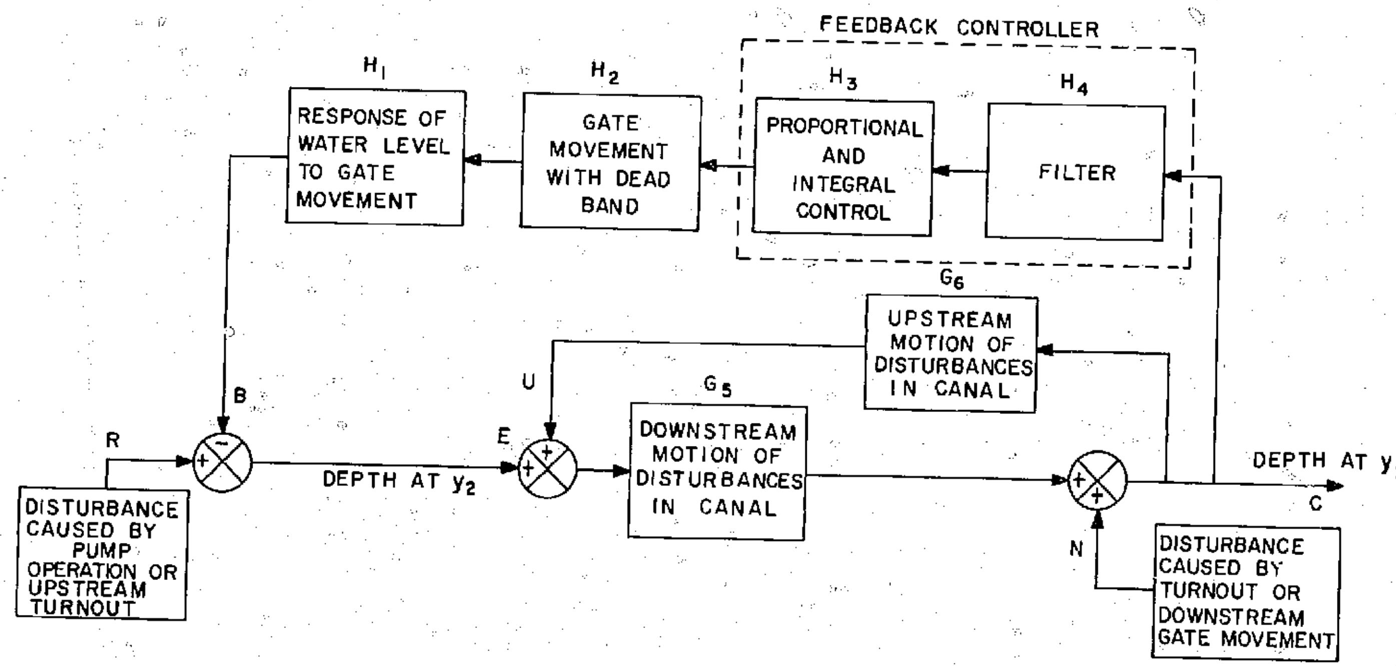

Figure 3.—Block diagram of individual canal reach.

through the canal is represented by block G_6. The other signal passes through the controller elements H_4 and H_3 to actuate the upstream gate. The effect of the current gate position, gate dead band, and gate movement actuator are represented by block H_2. When the gate moves, it creates a wave which enters the canal reach. The height of the wave as a function of the gate movement is represented by block H_1. The cycle is completed when the wave passes back downstream through the canal G_5 to produce a new disturbance at the downstream end of the canal reach. The transit of disturbances originating at the upstream end of the canal reach can be followed in a similar manner.

The method outlined by Shand [9] is applicable to disturbances due to turnouts. The frequency-response method outlined here is applicable to disturbances at the upstream end of the reach. Since the canal must be stable to all disturbances arising anywhere in the canal system, both Shand's method and the frequency-response method should be used to develop the control parameters.

The signal flow paths through the controllers have been studieds extensively. Various flow paths and combinations of the functions performed by the separate parts of the system have specific names. For instance, some of these terms are:

The *Feedback signal* (B) is an element which modifies the input signal (R). Two feedback signals are present in a single canal reach; one goes through the controller and the other passes through the canal itself. In terms of the transfer functions, these are:

$$B = C(H_1 H_2 H_3 H_4) \qquad (2)$$

$$U = CG_6 \qquad (3)$$

The *Open-loop transfer* function is the ratio of the feedback signal (B) to the actuating signal (E). This is given by:

$$\frac{B}{E} = H_1 H_2 H_3 H_4 G_k \qquad (4A)$$

where

$$G_k = \frac{G_5}{1 - G_5 G_6} \qquad (4B)$$

when the downstream disturbance $N = 0$.

The *Feedforward transfer function* is the ratio of the output signal (C) to the actuating signal (E), so that:

$$\frac{C}{E} = \frac{G_5}{1 - G_5 G_6} + \frac{N}{E} \qquad (5)$$

The *Closed-loop transfer function* is the ratio of the output to the input. If the output is influenced only by the downstream disturbance, then:

$$\frac{C}{N} = \frac{1}{1 + H_1 H_2 H_3 H_4 G_5 - G_5 G_6} \qquad (6)$$

If the output is influenced only by the upstream disturbance, then:

$$\frac{C}{R} = \frac{G_5}{1 + H_1 H_2 H_3 H_4 G_5 - G_5 G_6} \qquad (7)$$

If the output is influenced by both the disturbance and the reference input, it is possible to write the output as:

$$C = \frac{N + G_5 R}{1 + H_1 H_2 H_3 H_4 G_5 - G_5 G_6} \qquad (8)$$

In general, it can be seen that a delivery of water at an intermediate reach in a series of canal reaches will cause N-type disturbances to propagate in the upstream direction and R-type disturbances to propagate in the downstream direction. When the N-type disturbances reach the upper end of the canal, either a pump is turned on or a gate is opened. These actions cause R-type disturbances which propagate in a downstream direction. These in turn cause new N-type disturbances to form as a result of the downstream gate movements. The process continues until a steady-state condition is achieved. The desired steady-state condition is one in which the water surface in the entire canal does not vary with time. However, the most usual steady-state condition is one in which there are low-amplitude, low-frequency fluctuations in the water surface elevation.

SUMMARY AND CONCLUSIONS

Two methods are available for determination of control parameters on automated canals. These are the transient-response method and the frequency-response method. Since a typical canal system is influenced predominantly by transient inputs, the transient-response method has been extensively used in the analysis of automated canal systems. The main advantage of the frequency-response method is that the effects of varying the various control parameters can be readily visualized and evaluated.

The presently used transient-response method and parameter selection is reviewed. The basic concepts of the frequency-response method are presented and their application to an automated reach are illustrated with an example. Two computer programs are given which can be used to determine canal response characteristics.

It would seem axiomatic that a good description of the process to be controlled would be specified before the design of a controller would be undertaken. In the case of canals, a clear-cut description of the process is lacking. Although the simplified mathematical response model developed by Shand [9] appears to yield relatively good results, no systematic studies of the canal frequency response have been made. In particular, studies describing the effects of supercritical flow beneath gates, friction factors, and changes in canal cross section need to be performed.

The state-of-the-art in process control has progressed sufficiently far so that both the transient-response and frequency-response methods of analysis can be used in a complimentary fashion. Employment of only one method in the design of automated systems is, in general, wasteful of both computer time and engineering effort.

Circuits other than those in the EL-FLO controller are feasible for compensation. These include lead, lag, and lag-lead compensation, Ogata [7]. Once the canal response has been determined, these circuits should be investigated to test their frequency- and transient-response characteristics. In addition, the availability of electrical components to achieve the desired compensation on canals should be studied.

REVIEW OF CURRENT METHODS FOR CONTROLLER PARAMETER SELECTION

With the exception of the integral function, the selection of the parameters for a canal controller is based primarily on the work of Shand [9]. The parameters to be considered are the:

- Water level offset—the difference between the water level depth at zero flow and the water level depth with flow,

- Proportional gain—the ratio of gate opening to offset,

- Filter time constant—to govern the stability of the control system, and

- Integral function—the value needed in the controller to make the offset equal to zero.

Proportional Gain

Shand [9] has developed parameters for the value of the proportional gain such that water level disturbances above the downstream gate are not transmitted upstream. Since the method is somewhat involved and requires several computer runs of a program developed by Shand, the reader is referred to the original report for more details.

The Water Systems Automation Team [10] defines the proportional gain such that the discharge out of the canal reach equals that entering. Also the proportional gain in their report is referenced to changes in the upstream water depth, ΔY_2. The two definitions of proportional gain are referenced to the same depth only when the constant volume concept of canal operation is used.

The definition of proportional gain with constant volume canal operation is:

$$\text{GAIN} = \frac{\Delta G_o}{\Delta Y_3} = \frac{T(V+C)}{BC_d\ 2g\Delta H} \tag{9}$$

where

G_o = gate opening
T = mean width of water surface
V = mean water velocity
C = mean wave celerity
B = gate width
C_d = discharge coefficient of gate
g = acceleration of gravity
ΔH = differential head across the gate

This equation is useful in obtaining initial estimates of gain to be used in first-run simulation programs.

Filter Time Constant

The canal and controller are a stable system if disturbances applied to the system do not increase without limit. Practically speaking, disturbances that are amplified so much that physical limits are exceeded, such as gates being overtopped, also represent an unstable condition. A procedure for preventing unlimited amplification of disturbances was developed by Shand [9]. Through trial and error, Buyalski [2] found that acceptable results are obtained if the time constant is set equal to the time it takes a wave to transverse the length of the canal, or:

$$t_f \approx L/C_w \tag{10}$$

where

t_f = filter time constant
L = canal length
C_w = wave velocity = $\sqrt{g\bar{A}/T}$
g = acceleration of gravity
$\bar{A}$ = mean cross sectional area of canal prism
T = mean width of water surface

This is essentially the same criterion recommended by the Water Systems Automation Team [10].

These criteria, while preventing unlimited amplification of disturbances, do not ensure that some limiting values are not exceeded. Therefore, the parameters must be tested in the mathematical simulation model developed by Shand [8] to verify the system stability and performance.

Offset

Shand [9] defines the target depth as the depth at the downstream end of the reach with zero flow. The maximum offset is the difference between the target depth and the normal or uniform flow depth with maximum discharge. If proportional gain is the only component of a controller, then the maximum gate opening for the maximum gain is determined from:

$$G_{max} = (\text{GAIN})\,(\text{OFFSET}_{max}) \tag{11}$$

The maximum gate opening remains essentially constant with various water surface elevations when the maximum discharge is constant. Therefore, it seems tempting, from the above equation, to reduce the maximum offset by increasing the gain. However, increases in gain over the values recommended by Shand will result in disturbances being amplified upstream and in unstable flow conditions.

Integral Function

In most canals, satisfactory delivery of water cannot be made with even small offset values. For these cases, a function is added to the controller which returns or "resets" the offset to zero. In the following discussion, the terms reset and integration are used interchangeably. The output of a proportional controller with the reset function is given by:

$$\text{OUTPUT} = K_1\,(\text{INPUT}) + K_2 \int \text{INPUT}\,dt \tag{12A}$$

or

$$\text{OUTPUT} = K_1\,(\text{INPUT}) + \frac{K_1}{T_i} \int \text{INPUT}\,dt \tag{12B}$$

where:

K_1 = gain factor
K_2 = reset factor = K_1/T_i
T_p = period of time for reset action
T_i = integral time
= time for proportional action to be duplicated
$\frac{1}{T_i}$ = reset rate
= number of times per unit time that the proportional part of the control action is duplicated.

The action of the reset function can be seen in figure 4.

In practice, the integrator runs continuously. That is, the time T_p, is infinite. No guidelines presently exist to estimate the magnitude of the reset rate for canals. The reset rate is determined solely on the basis of experiment on a mathematical model of the canal system. A satisfactory value for the reset rate is one which reduces the offset by 90 percent in 2 to 3 hours.

Dead Band

The signal from the controller is introduced into the circuitry which controls the motion of a gate. A position indicator is located somewhere on the gate

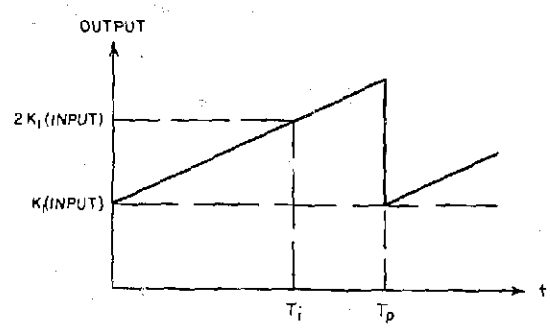

Figure 4.—Proportional plus reset output from a controller with a step input.

mechanism. The output from the position indicator is compared with the signal from the controller. If the difference between the two is not zero, the gate moves so that the difference becomes equal to zero. In order to keep the gate from moving continuously, a dead band is employed. With a dead band, the controller signal must exceed some preset value before the gate receives a signal to move. For example, a typical dead band value on a canal system is 30 mm of gate movement. That is, the gate must have a command from the controller to move more than 30 mm before a gate movement is initiated.

In the field of process engineering, the number of control parameters in a controller is referred to as level of the mode for the controller. Thus, a controller with gain only is a "one-mode" controller. Whereas, a controller with gain plus reset is a "two-mode" controller, etc. Buckley [1] recommends the following principles in the design of controllers:

- If at all possible, one-mode controllers should be used for process control.
- If one-mode control is unsuitable, the engineer should try to specify a field adjustment of only one mode of a two-mode controller.
- The engineer should avoid three-mode control or at least having three modes adjusted in the field.

The original EL-FLO controller is an example of a two-mode controller. The addition of the reset has turned it into a three-mode controller. The use of multimode controllers coupled with the nonlinear effects introduced by the dead bands has made the parameter selection for a given canal a matter of trial and error. Due to the complexities of the canal and controller system, the parameter selection for a wide range of flow conditions requires an engineer with both field and automation experience. Since this combination is rather rare, some techniques that provide additional insight into the effects of variations to the parameters seems to be required. Frequency-response analysis appears to hold promise as a method of providing the additional insight. To date, this method has not been extensively exploited in the analysis of canal systems.

Frequency-Response Methods

Fundamental Concepts

Frequency-response methods describe the steady-state response of a system to a sinusoidal input. One of the most common ways to describe a sinusoidal input is by a vector which rotates with a constant angular speed. This can be represented in either complex or in real notation, figure 5.

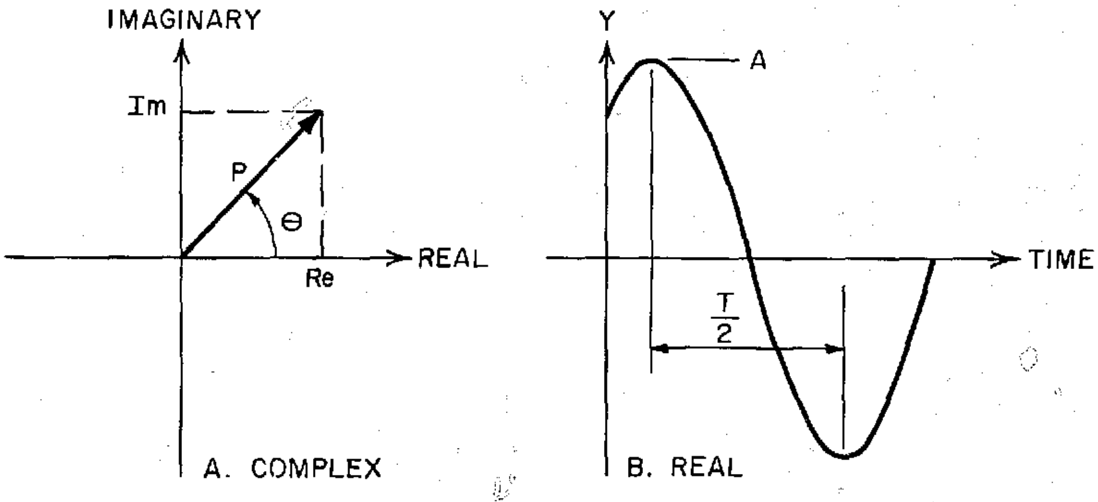

Figure 5.–Representation of sinusoidal wave.

The real representation of a sinusoid can be described as:

$$Y = A \sin\left(\frac{2\pi t}{T} + \alpha\right) \quad \text{sine} \qquad (13A)$$

$$= A \cos\left(\frac{2\pi t}{T} + \beta\right) \quad \text{cosine} \qquad (13B)$$

$$= a \cos\left(\frac{2\pi t}{T}\right) + b \sin\left(\frac{2\pi t}{T}\right) \quad \text{sine and cosine} \qquad (13C)$$

where

$a = A \sin \alpha = A \cos \beta$
$b = A \cos \alpha = -A \sin \beta$
$\alpha = 90 - \beta$

Several possible ways are used to describe the sinusoid in complex notion.

These are:

$$\vec{H} = Re + j\,Im \quad \text{rectangular} \qquad (14A)$$

$$= Pe^{j\theta} \quad \text{exponential} \qquad (14B)$$

$$= P(\cos\theta + j \sin\theta) \quad \text{trigonometric} \qquad (14C)$$

$$= P \;\; \theta \quad \text{polar} \qquad (14D)$$

where

$P = \sqrt{Re^2 + Im^2}$ = amplitude

$\theta = \frac{2\pi t}{T} + \theta_o$ for $t = nT$ with $n = 0, 1, 2,$ etc.

$\theta_o = \tan^{-1}(Im/Re)$ = phase angle

$j = \sqrt{-1}$

The relationship between the complex representation and the real representation is given by Lee [5]. This relationship is simply:

$$\theta_o = \tan^{-1}(-b/a) \qquad (15)$$

The frequency-response method utilizes the complex representation of a series of sinusoids. Specific names have been given to the graphical plots associated with the complex representation. Each type of plot has its own advantages with respect to the analysis. The most commonly used plots are:

A **Bode diagram** which is actually two separate plots. One gives magnitude on a logarithmic scale versus frequency; the other gives phase angle versus frequency. Essentially, this diagram uses the exponential type of complex representation. The usual unit used on the amplitude scale is the decibel. Mathematically

$$dB = 20 \log |P|$$

where

dB = decibels
P = amplitude
log = logarithm to the base 10

However, the decibel scale is not used in this report because none of the frequency-response amplitudes are computed in terms of decibels.

A **Polar or Nyquist diagram** is a plot of the imaginary component versus the real component. The diagram uses the rectangular form of the complex representation. Magnitudes are usually plotted on a linear scale.

A **Nichols diagram** is a plot of the log-magnitude versus phase. Very frequently a Nichols diagram includes curves of amplification and phase shift for a closed-loop system with unity-feedback. If the system has nonunity-feedback, it can always be converted to a unity-feedback, figure 6. This conversion is only one of the rules of block diagram algebra summarized by Ogata [7].

The relative usefulness of these various diagrams can be seen when they are compared for the same function, figure 7. From the Bode diagram, the magnitude and phase shift of the output with respect to the input signal is represented as a function of frequency. In addition, the maximum magnification M_r is clearly indicated.

The Nyquist plot shows the frequency-response characteristics over the entire frequency range (zero to infinity) in a single plot. The natural frequency (ω_n) of the system is easily determined. The stability of the system is also easily determined from this plot. The system stability is examined using the open-loop transfer function. The behavior of the locus of this function with respect to the point ($Re = -1$, $Im = 0$). With a stable system, the point (−1,0) is not enclosed by the Nyquist plot.[2] That is the point (−1,0) must lie outside of the shaded area. However, this rule does not

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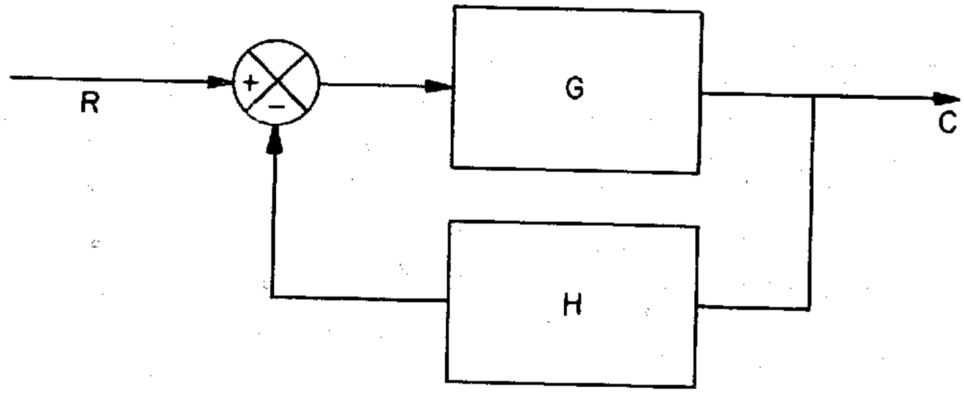
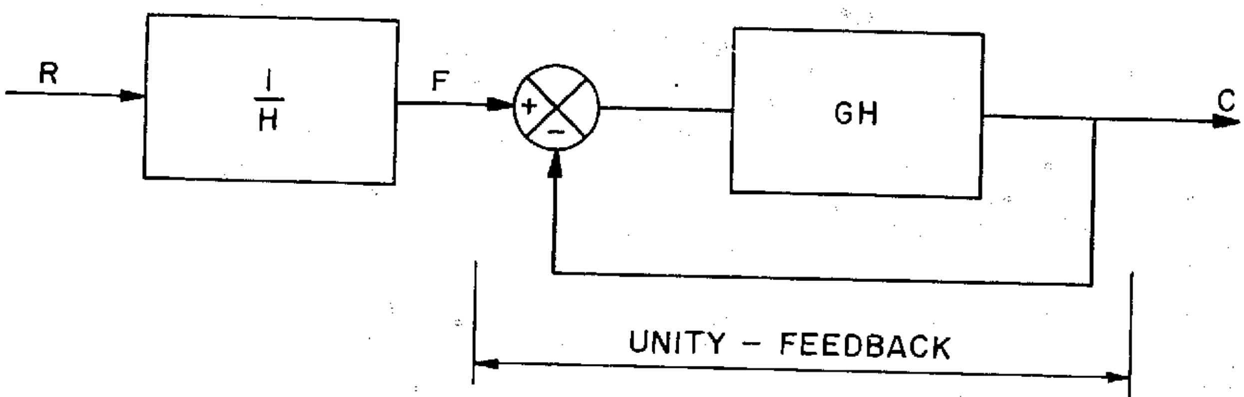
[2] All points lying to the right hand side of the curve from $\omega = 0$ to $\omega = \infty$ are said to be enclosed by the curve.



A. NONUNITY – FEEDBACK SYSTEM



B. EQUIVALENT SYSTEM

Figure 6.–Reduction of nonunity feedback to unity-feedback system.

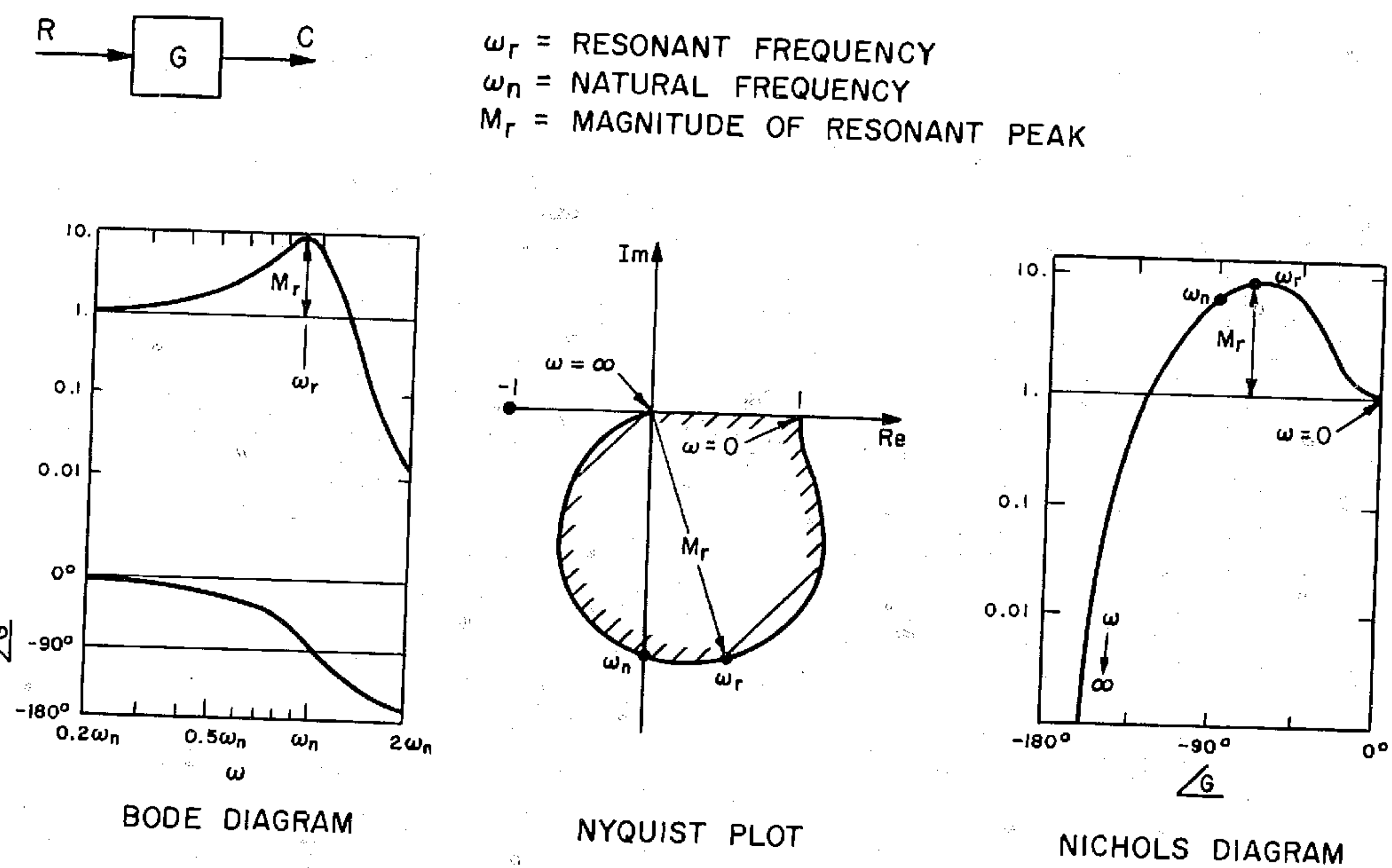


Figure 7.—Three representations of the frequency response from Ogata [7].

apply for systems with multiple feedback loops. Although the Nyquist plot can pass through the stability point (−1,0), theoretically, it should be avoided in practical control systems.

The Nichols plot has the advantage that the frequency response of the closed-loop system can be determined easily. In addition, the effect of changing the frequency can be quickly estimated. For instance, if the open-loop response of figure 7 is superimposed upon magnitude and phase curves for a unity-feedback system, the resulting intersecting points are the desired closed-loop response characteristics, figure 8. In this case, the magnitude of the resonant peak has changed from a value of 10 with the open-loop system to about 1.02 for a closed-loop unity-feedback system. If the closed-loop gain is changed, the open-loop curve is merely shifted up (gain increased) or down (gain decreased).

In the sections that follow, the response characteristics for each controller element are discussed in detail and illustrated with an example. Then a method for estimating the response characteristics of a canal reach is described. A graphical method is given which combines these individual response elements into the open-loop transfer function. From this open-loop function the closed-loop response of the example canal reach is obtained. Finally, conclusions regarding stability and the adequacy of the compensation are discussed.

Response Characteristics of Control Elements

Basic considerations.—The output of the controller consists of the summation of several transfer functions. If each of these is linear, the overall gain of the sum is equal to the product of the individual gains, Jenkins and Watts [5]. The overall phase shift is the sum of the individual phase shifts. Therefore, the effect of varying parameters in the controller on the overall response characteristics can be readily seen by examining the response characteristic of each part.

As an example, the EL-FLO canal controller consists of a filter, a gain, and usually an integration (or reset) element. Therefore, only these three types of systems will be considered. The response characteristics of other types of systems can be found in Jenkins and Watts [5]. The inclusion of elements or systems, other than gain, into a controller in order to make the overall behavior of the entire system have some desired characteristics is called "providing compensation".

Filter response.—The filter response is given by a simple exponential function, Shand [9]. The equation is:

$$\frac{\text{OUTPUT}}{\text{INPUT}} = \frac{e^{-t/T}}{T} \tag{17}$$

where

T = filter time constant.

The amplification or gain of a sinusoidal signal input into the filter is given by:

$$P_f = [1 + (2\pi fT)^2]^{-1/2} \tag{18}$$

where f and T are in consistent units, such as f in cycles per minute and T in minutes.

The corresponding phase shift is given by:

$$\phi_f = -\tan^{-1}(2\pi fT) \tag{19}$$

A Bode diagram of the filter response is given in figure 9. It is evident that the equations for the two asymptotes are:

$$P_f = 1 \text{ for small frequency values and} \tag{20A}$$

$$P_f = \frac{1}{2\pi fT} \text{ for large frequency values} \tag{20B}$$

The two asymptotes intercept at:

$$2\pi f = T^{-1} \tag{21}$$

Proportional gain response.—Setting the proportional gain is one of the first steps in adjusting the system for satisfactory performance. Normally the proportional gain is set to produce some specified result of steady-state conditions. For instance, an example was given earlier in which the proportional gain was determined so that the discharge out of a canal reach was equal to that entering the reach. In many practical situations, increasing the proportional gain above some value will result in instabilities. Therefore, it is necessary to provide compensation to eliminate the undesirable operating characteristics.

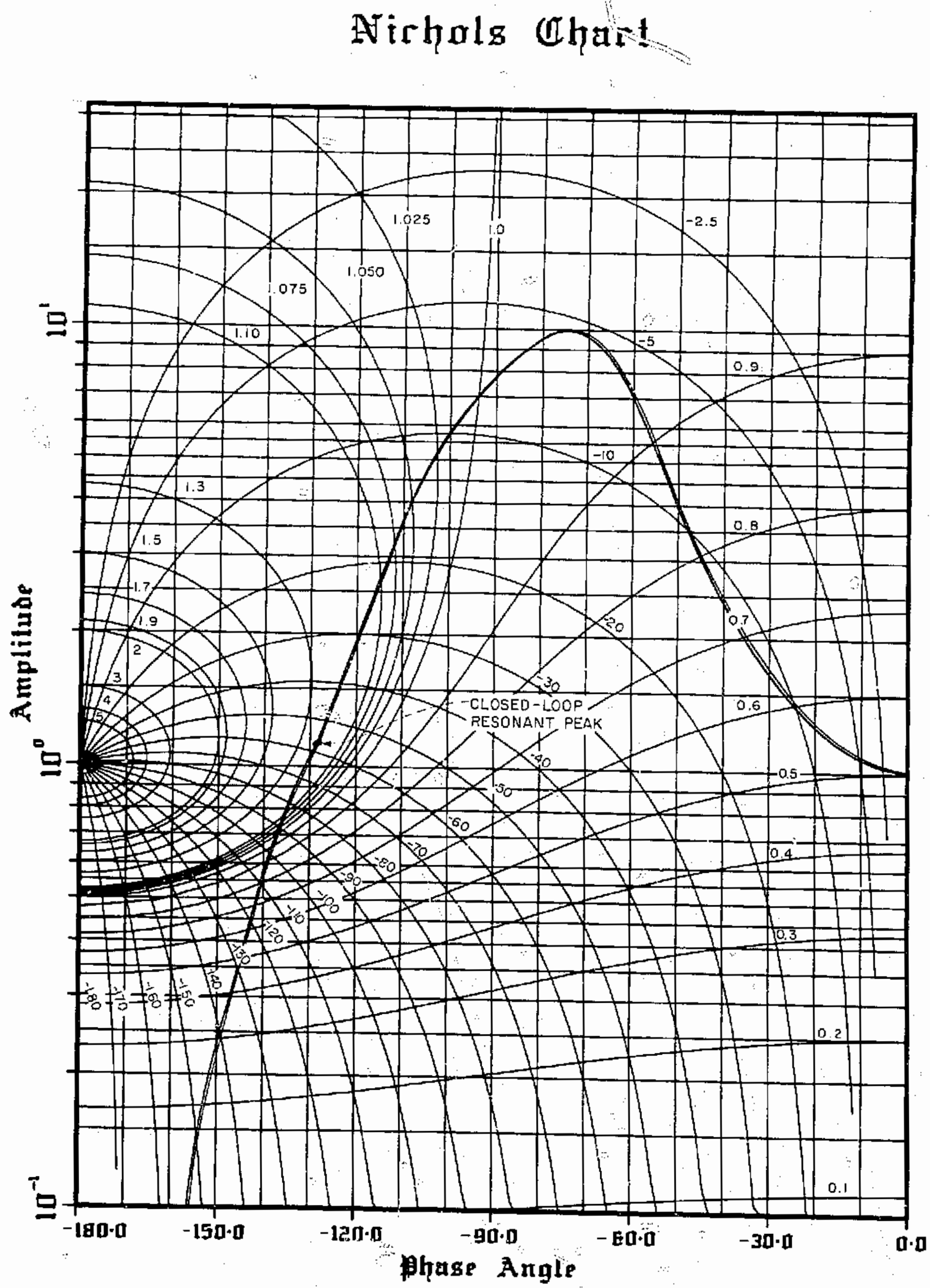


Figure 8.—Determination of closed-loop response with a Nichols plot.

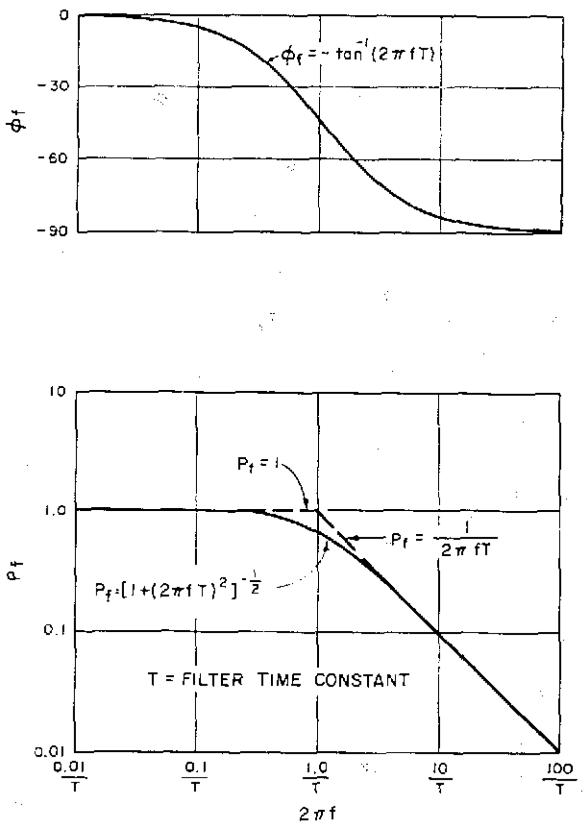


Figure 9.—Filter response.

The amplification (P_g) of a sinusoidal signal input into the proportional gain element is given by:

$$P_g = \text{GAIN} \tag{22}$$

The phase shift is given by:

$$\varphi_g = 0^{\circ} \tag{23}$$

The Bode diagram for the gain response is given in figure 10.

Reset response.—The reset parameter has the characteristics of integration. Its response characteristics are given by:

$$P_r = K_2/2\pi f \tag{24}$$

and

$$\phi_r = -\pi/2 \tag{25}$$

The Bode diagram of the reset response is given in figure 11.

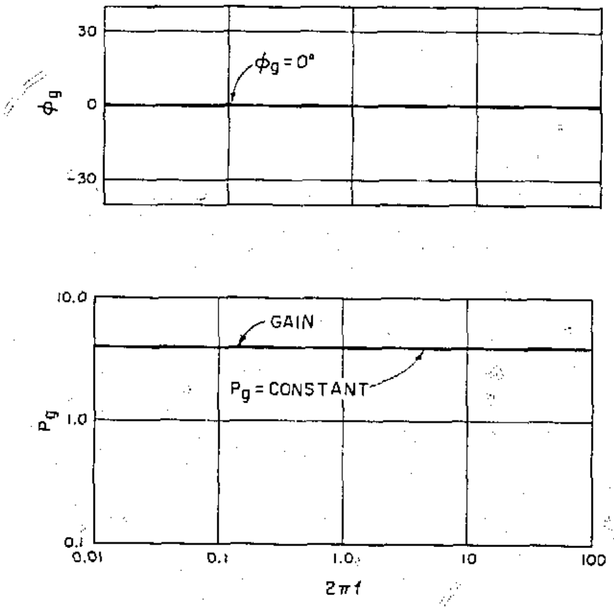


Figure 10.—Gain response.

Since the gain is high at low frequencies, the reset function can be regarded as a low-pass filter. This tends to create instabilities near steady state. The reduced gain at high frequencies results in slow response characteristics for transient conditions. Thus, reset has undesirable characteristics with respect to feedback controllers.

Combined response characteristics of the EL-FLO controller.—Knowing the response characteristics of each element in the controller it is possible then to determine the overall response characteristics for the controller. For instance, on the Coalinga Canal, the control parameters for the first reach are:

Gain = 2.50
Filter time constant = 1033 s
= 17.2 min
Reset coefficient, K_2 = 0.019 min^{-1}

If all elements of the controller are in series, the combined response is obtained by multiplying the amplifications (P values) and summing the phase angles (ϕ values) of all the components which make up the controller. Since the amplifications are plotted on a logarithmic scale, the amplifications can be determined graphically by a simple summation of all the components. This graphical summation must be done relative to the $P = 1$ line.

The proportional control and the filter are connected in series on the Coalinga Canal. However, the integral

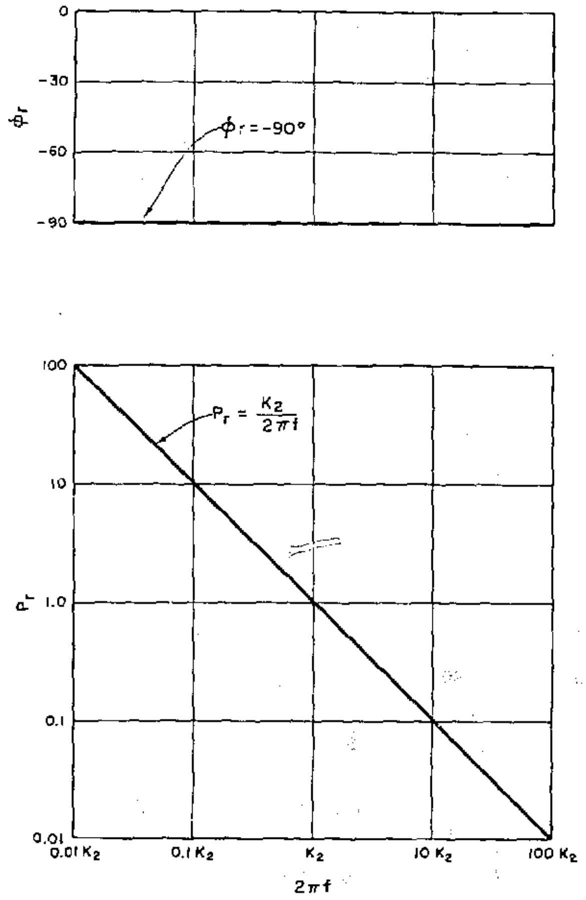


Figure 11.—Reset response.

control is connected in parallel, figure 12. Therefore, a modified method must be used to determine the combined response. Using the rules for block diagram algebra, it is possible to reform the integral controller hookup into an equivalent block that can be combined according to the rules previously mentioned. The equivalent block transfer function is given by:

$$(H_r) \text{ equivalent} = 1 + H_r$$

In terms of amplitudes and phase angles:

$$(P_r) \text{ equivalent} = \sqrt{1 + P_r^2}$$

$$(\phi_r) \text{ equivalent} = -\tan^{-1}(P_r)$$

The response of the individual elements and of their combination is presented in figure 12.

The preceding values are not the only ones which result in the same characteristics for the controller. For instance, the following values give almost identical overall characteristics with those given by the values previously given:

Gain = 4.5
Filter time constant = 2066 s
Reset coefficient, K_2 = 0.01 min^{-1}

Other combinations of controller parameters which result in identical overall controller characteristics can be found by trial and error. Since an infinite number of combinations can yield the identical controller response, attention should be focused on desirable combined characteristics and not on the characteristics of each controller component. Current parameter selection methods are based upon this latter approach.

The controller response depicted in figure 12 represents the combined response of blocks H_3 and H_4 in figure 3.

Response Characteristics of Gate Movement with Dead Band

The response characteristics of the gate movement segment of the feedback loop cannot be expressed as a simple function as was possible for the controller elements. The reason that no functional relationship exists is due to the dead band which introduces nonlinear elements into the feedback loop. The effect of these nonlinearities on the operating system are considered in this section.

The gate movement segment consists of a comparator, a gate position indicator, and a gate movement actuator, figure 2. Signals from the feedback controller are compared with the actual gate position. If the difference between the desired position and the actual position exceeds some value, known as the dead band, the gate moves in such a manner as to reduce the difference to zero. The motors that move the gates are generally a-c devices. Thus, they are either on or off. As a result, the gate moves at a constant rate which is not a function of the magnitude of the input signal. The direction of movement is controlled by the sign of the input signal, however.

The time required to move the gate a specified distance is given by:

$$\Delta t = \frac{\Delta G}{r} \tag{26}$$

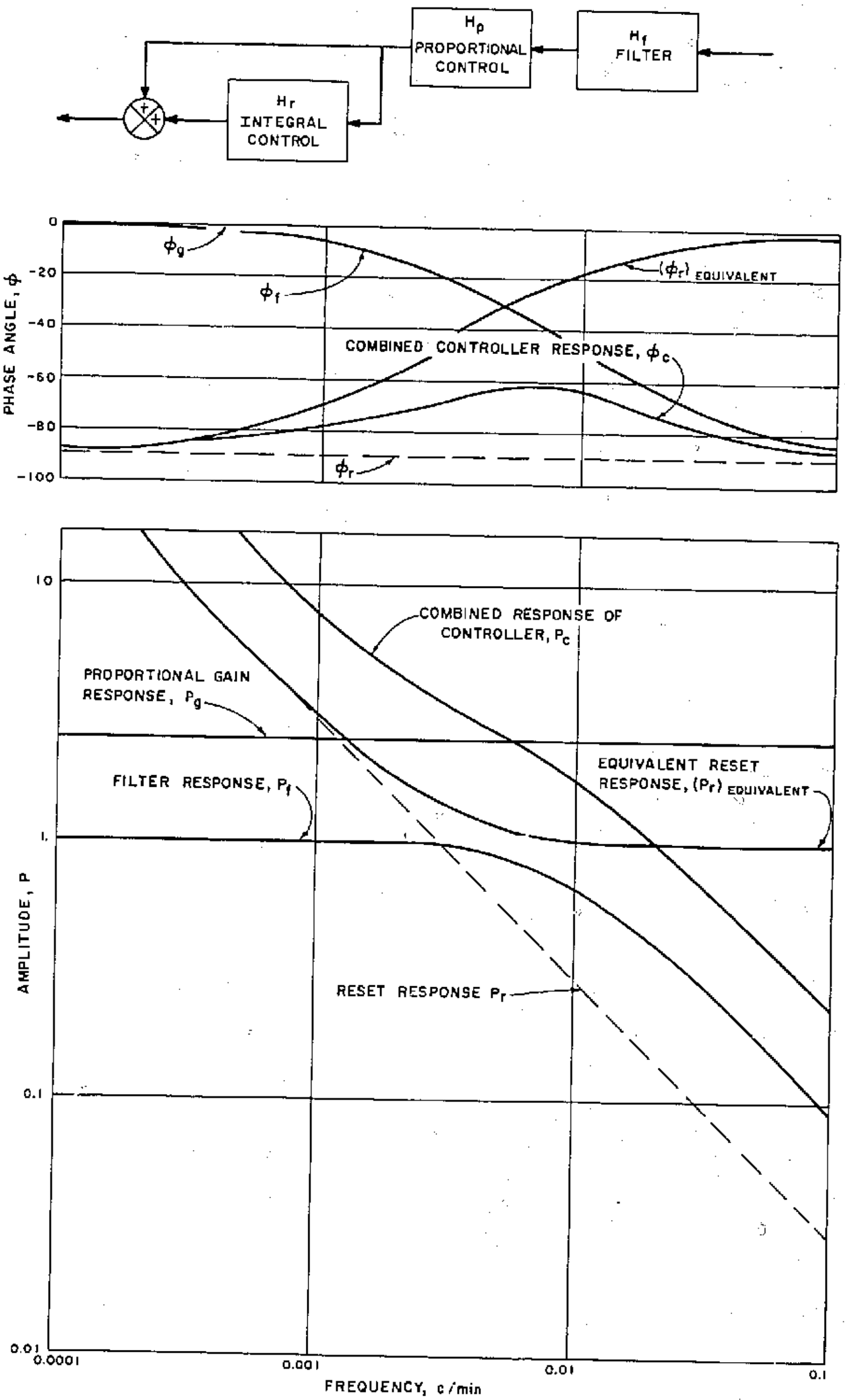


Figure 12.—Response of controller on Coalinga Canal, second reach.

where

Δt = time interval required
ΔG = change in gate opening
r = rate of gate movement

The change in gate opening corresponds roughly with the magnitude of the dead band since the time interval to complete a gate movement is generally small with respect to the time interval for changes in the signal from the controller. This fact permits instabilities caused by the dead band action to be estimated.

At steady state, a specific flow rate of water is delivered out of a canal reach. However, due to the dead bands, the inflow to the reach changes in increments. Only under the most unusual conditions does the number of incremental discharges equal the delivered discharge. In the more usual case, the inflow discharge fluctuates about the delivered or set discharge, figure 13. It can be shown that the time interval for a cycle to repeat is given by:

$$t_m = \left(\frac{2dLT_w}{\mathrm{GAIN}(Q_{set})} \right) \times \left(\frac{(d/G_o)}{[((n+1)d/G_o) - 1]\ [1 - nd/G_o]} \right) \quad (27)$$

where

d = dead band
L = canal length
T_w = average top width of water surface
GAIN = proportional gain of controller
Q_{set} = delivery discharge
G_o = gate opening required to deliver Q_{set}
n = truncated value of G_o/d

The frequency of the cyclical variations about the steady-state flow is simply the reciprocal of the time interval.

To facilitate the computations, the right-hand side of equation 27 is factored into two fractions. The first fraction consists of the canal parameters. The second fraction consists of dimensionless values. The reciprocal of the second fraction is called the frequency parameter (ϕ). Through this procedure the frequency of the steady-state cyclical operation can be easily computed, figure 14. The frequencies for any range of discharges will range between some maximum

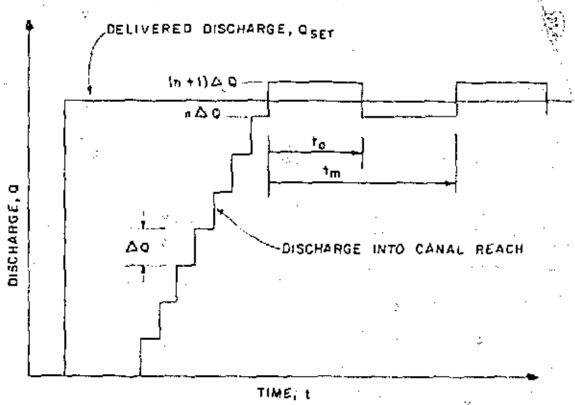


Figure 13.—Discharge relationships in a canal reach.

value and zero. The maximum possible frequency is given approximately by:

$$f_{max} = \frac{\mathrm{GAIN}(Q_{set})}{8LT_w G_o} \quad (28)$$

It has thus been shown that a dead band introduces cyclical fluctuations into a canal reach because the sum of the discharges from the incremental gate movements does not precisely equal the desired delivery discharge. These fluctuations can have frequencies between zero and some maximum value given by equation 28.

In addition to knowing the magnitude of the cyclical frequency introduced by the dead band, the designing engineer also needs to know how the gate control element reacts to inputs of any random frequency. In systems technology, this general problem class is known as *"on-off nonlinearity with dead zone and hysteresis."* The method of analysis is called *"describing function analysis."* The problem solution involves a couple of very gross assumptions.

The first of these assumptions is that the linear and nonlinear elements of a nonlinear system can be separated into individual elements, figure 15.

A nonlinear element can generally be recognized by one of two characteristics. The frequency-response characteristics vary with the amplitude of the input signal or the output is not directly proportional to the input. A dead band element is nonlinear because a sinusoidal input signal does not produce a sinusoidal output. The describing function analysis treats only the characteristics of the nonlinear element. Its effect on the overall system performance is considered later.

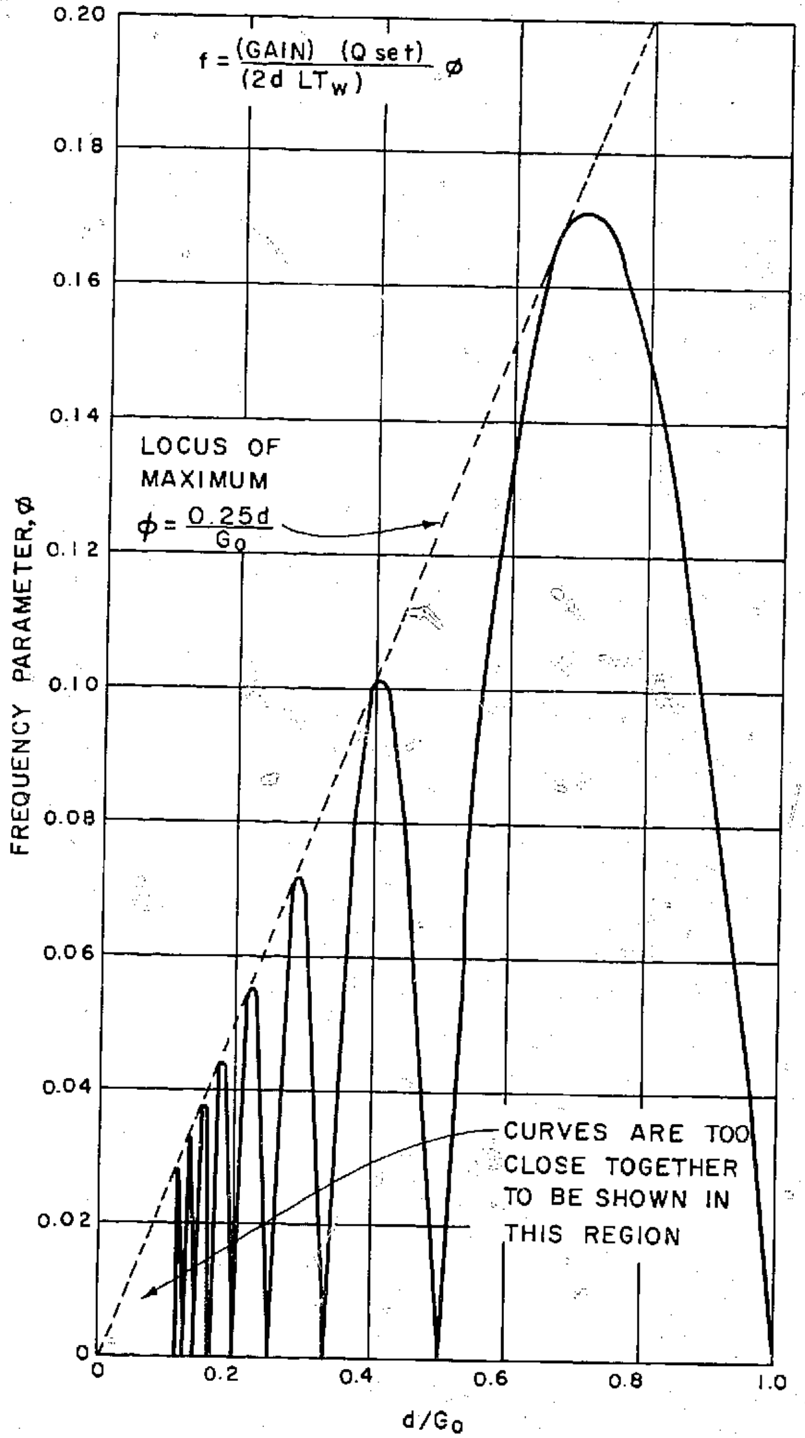


Figure 14.—Dimensionless frequency parameter.

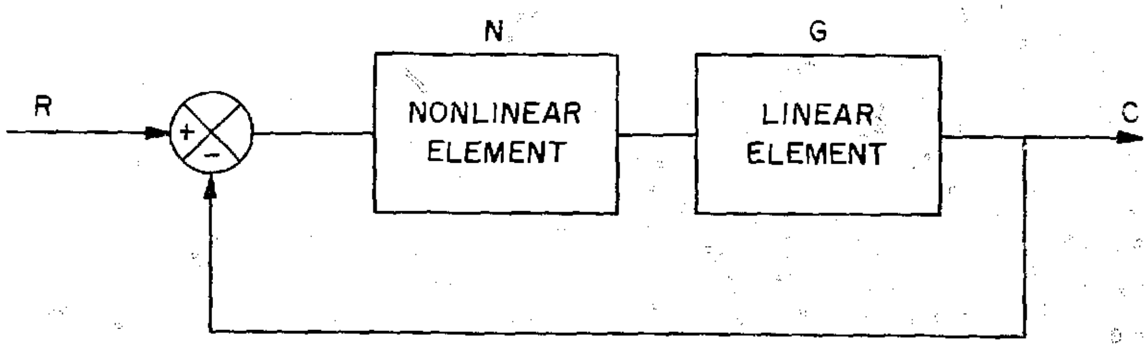


Figure 15.—Nonlinear control system.

The second major gross assumption is that the stepwise fluctuations of the output can be approximated by a sine wave. This implies that the input fluctuations are symmetric. In addition, this assumption implies that, with respect to stability, only the fundamental component of the output is significant.

The input and output characteristics of the comparator are shown in figure 16. The response of the dead band is defined by ratio of the fundamental harmonic component of the output to the input. This is expressed in complex numbers as:

$$N = \frac{Y}{X} \angle\phi \qquad (29)$$

where

N = describing function
X = amplitude of input sinusoid
Y = amplitude of fundamental component of output sinusoid
ϕ = phase shift of fundamental component of output sinusoid

The solution of equation 29 for the general case is shown in figure 17. The parameter d_z is a descriptor of the dead zone and the parameter h is a descriptor of the hysteresis. The dead zone refers to a controller characteristic in which the output has a zero value when the input value lies within the dead zone. Hysteresis refers to a controller characteristic in which the output depends upon the direction in which the input changes. For instance, the output can suddenly go from a negative value to a positive value as the input exceeds some value h. The output will then remain positive until the input decreases below the negative of the h value. A gate position controller generally contains both of these parameters. The comparator turns on when $d_z + h$ exceed a value which is known as the dead band setting, (d). It then turns off when $d_z - h$ equals zero. In addition, the output magnitude (M) is usually set equal to the dead band setting, d. These relationships can be expressed in dimensionless terms as:

$$\alpha = \frac{h}{d_z} = 1,$$

$$\frac{d}{d_z} = \frac{h}{d_z} + 1 = 2,$$

and

$$\beta = \frac{M}{d_z} = \frac{d}{d_z} = 2$$

For example, if the input signal to the comparator has a maximum amplitude (X) of 100 mm and the comparator output (M) is set at 45 mm then $d_z/X = d/2X = 45/200 = 0.225$.

The gain and phase angle from figure 17 are:

$$\text{GAIN} = \beta(0.279) = 0.56$$

$$\phi = -13^{\circ}$$

In terms of the describing function:

$$N = 0.56 \angle -13^{\circ}$$

The stability of the overall system can be examined by considering the locii of the $-1/N$ and G curves on a Nyquist plot, figure 18. If the $-1/N$ and the G curves do not intersect, the system is stable. If the $-1/N$ curve is enclosed by the G curve, the disturbances will increase to some limiting value as determined by a safety stop. Finally, if the curves just intersect, the system will exhibit sustained oscillations that can be approximated by a sinusoid. The frequency is determined from the G curve at the point of intersection. The amplitude of the sustained oscillations is determined from the $-1/N$ curve at the

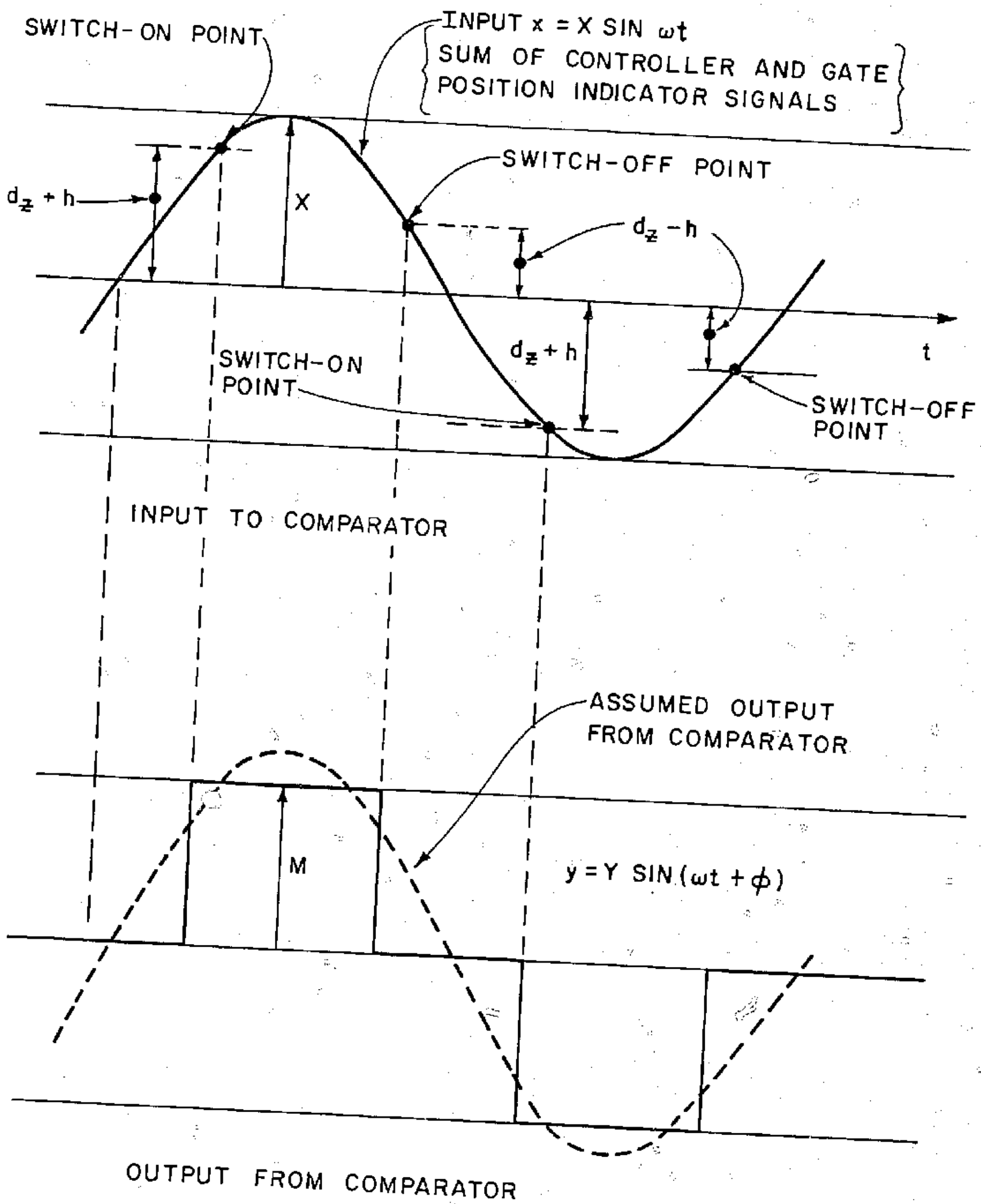


Figure 16.—Input-output characteristics for an on-off nonlinearity with dead band and hysteresis.

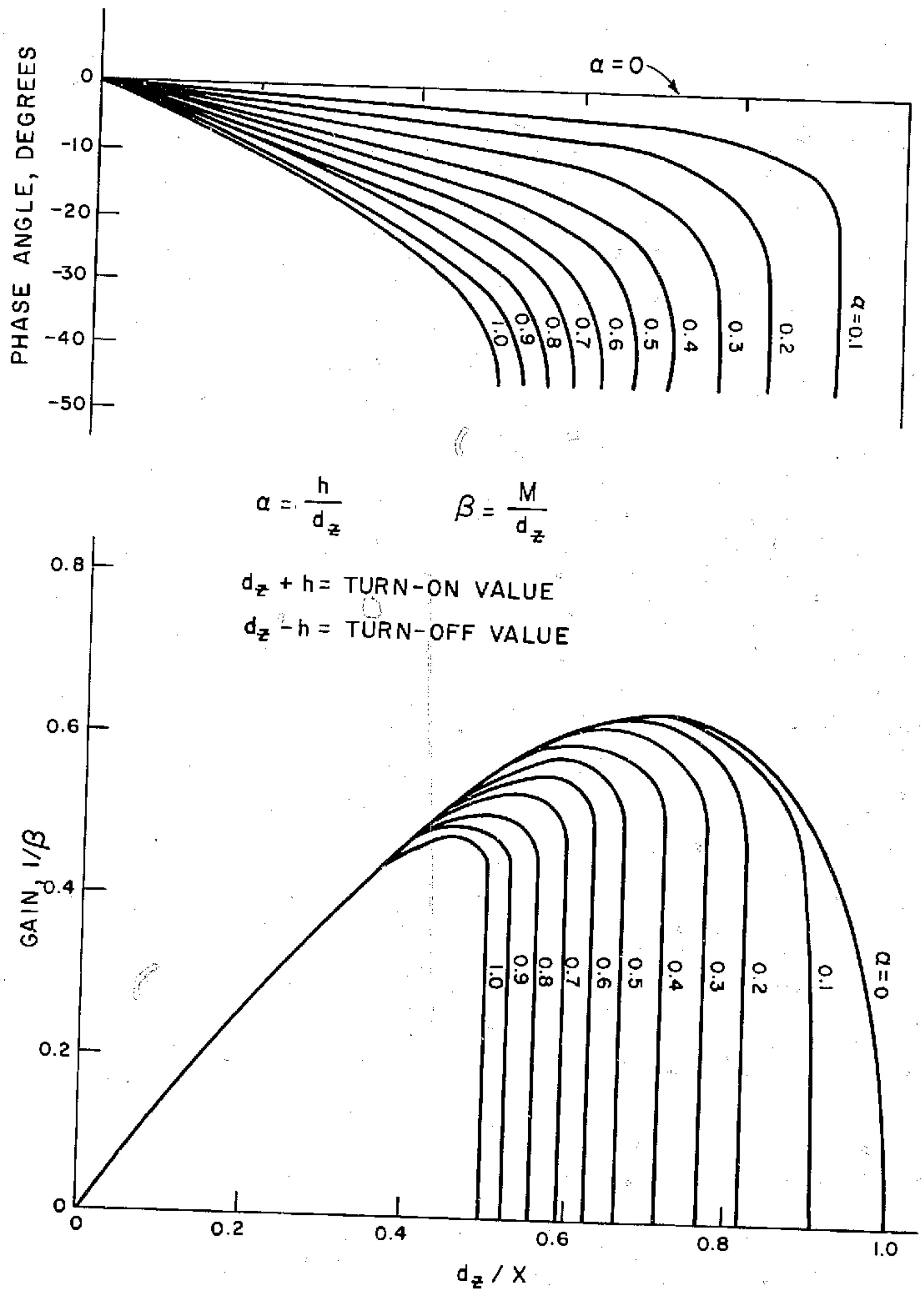


Figure 17.—Frequency response characteristics of an on-off nonlinearity with dead band and hysteresis.

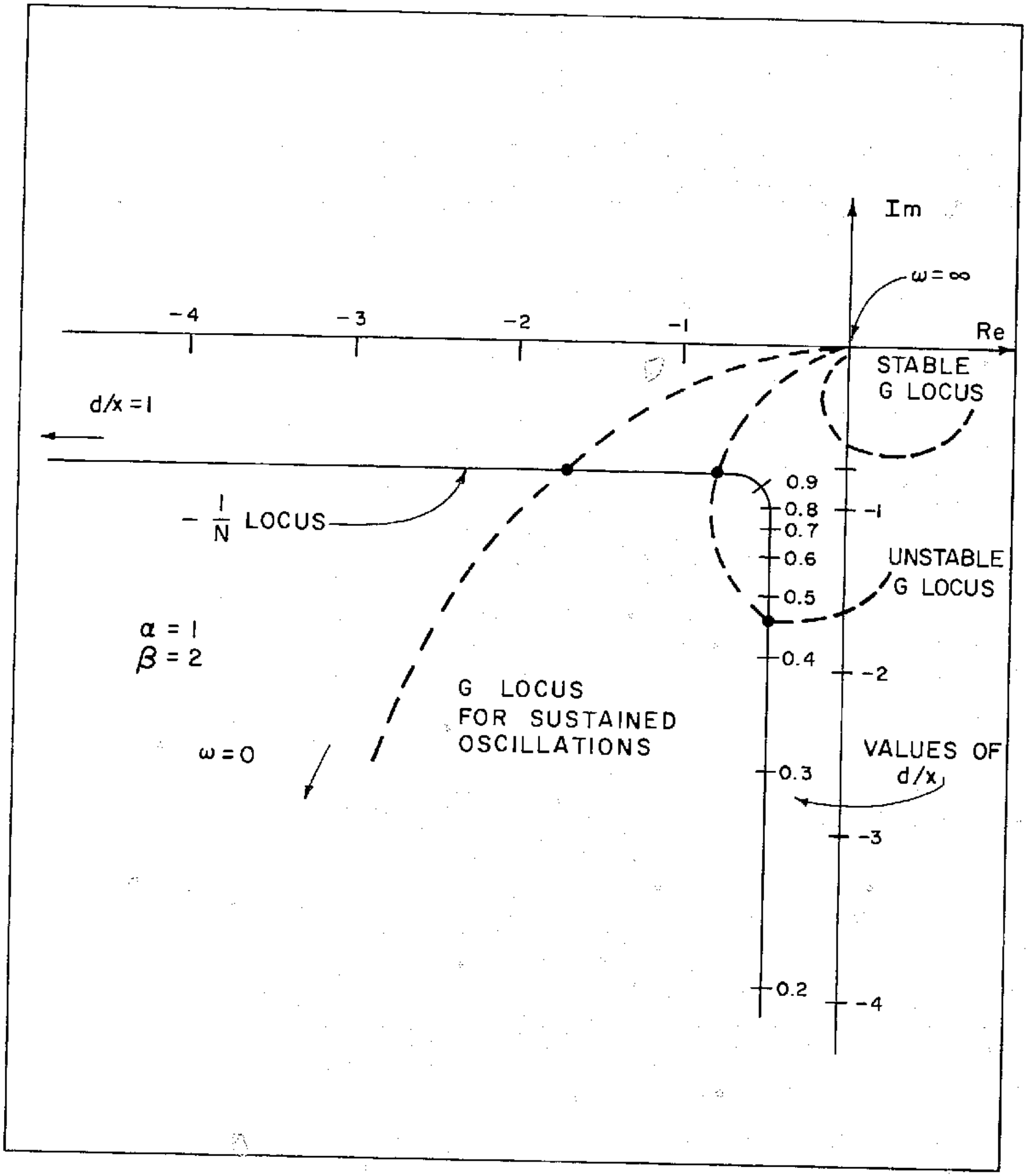


Figure 18.—Dead band stability criteria.

same point. The application of these considerations to the Coalinga Canal is given in the succeeding System Resonance and Stability section.

Response of Water Level to Gate Movement

The response of the water level below the gate to gate movements can be approximated by a gain function. Determination of the gain function is a trial and error procedure which requires a rather detailed knowledge of the hydraulic properties of both the gate and the canal. The gain function is defined as:

$$H_1 = \frac{\Delta Y_2}{\Delta G_o} \qquad (30)$$

where

ΔY_2 = change in canal depth downstream of gate, and
ΔG_o = change in gate opening

The phase shift for a simple gain is equal to zero.

The governing equations, in incremental form are:

$$\frac{\Delta Q}{Q} = \frac{\Delta G_o}{G_o} + \frac{\Delta C_d}{C_d} + \frac{\Delta Y_1 - \Delta Y_2}{2(Y_1 - Y_2)} \qquad (31)$$

$$\Delta Y_1 = \frac{\Delta Q}{T_1(V_1 - C_1)} \qquad (32)$$

$$\Delta Y_2 = \frac{\Delta Q}{T_2(V_2 + C_2)} \qquad (33)$$

and $$Y_1 - Y_2 = \frac{Q^2}{2gC_d^2 W^2 G_o^2} \qquad (34)$$

where

$\bar{A}$ = mean cross-sectional area of wetted prism
C_d = discharge coefficient
C_1, C_2 = wave celerity
$= \sqrt{\frac{g\bar{A}}{T}}$
G_o = gate opening
ΔG_o = change in gate opening, approximates the dead band
g = local acceleration of gravity
Q = discharge
T_1, T_2 = top width of wetted prism
V = mean water velocity in canal
$= Q/\bar{A}$
W = gate width
Y_1, Y_2 = water depth in canal

The subscripts 1 and 2 refer to locations upstream and downstream of the gate, respectively.

In addition to these equations, the variation of discharge coefficient as a function of gate opening must also be known in order to compute ΔC_d. This can usually be determined graphically, figure 19.

Equation 31 can be solved for ΔQ by direct substitution of equations 32, 33, and 34. However, this procedure is cumbersome; a somewhat easier method is to approximate ΔQ by:

$$\Delta Q = \frac{Qd}{G_o} \qquad (35)$$

where

d = dead band

This value can be used in equations 32 and 33 to determine ΔY_1 and ΔY_2. Through iteration with equation 31, the value of ΔQ can be obtained to any desired accuracy. This procedure automatically determines the value of ΔY_2 which can be substituted into equation 30 to determine the gain.

As an example, for the second reach of the Coalinga Canal the significant parameters are:

$\bar{A}$ = 30.5 m^2	T_2 = 13.56
G_o = 0.168 m	Y_1 = 3.45 m
T_1 = 14.02 m	W = 5.182
V_2 = 0.070 m/s	C_2 = 4.54 m/s
d = 0.046 m	Q = 1.98 m^3/s
C_1 = 4.62 m/s	V_1 = 0.064 m/s
g = 9.81 m/s^2	Y_2 = 3.45 m

Employing the trial and error procedure results in a frequency response function of:

$$H_1 = 0.180 \qquad (36A)$$

$$\phi_1 = 0^{\circ} \qquad (36B)$$

Response Characteristics of Canals

Surprisingly little work has been done to determine the response characteristics of canals. In fact, most textbooks on open channel hydraulics do not address this topic. One of the few references that can be found

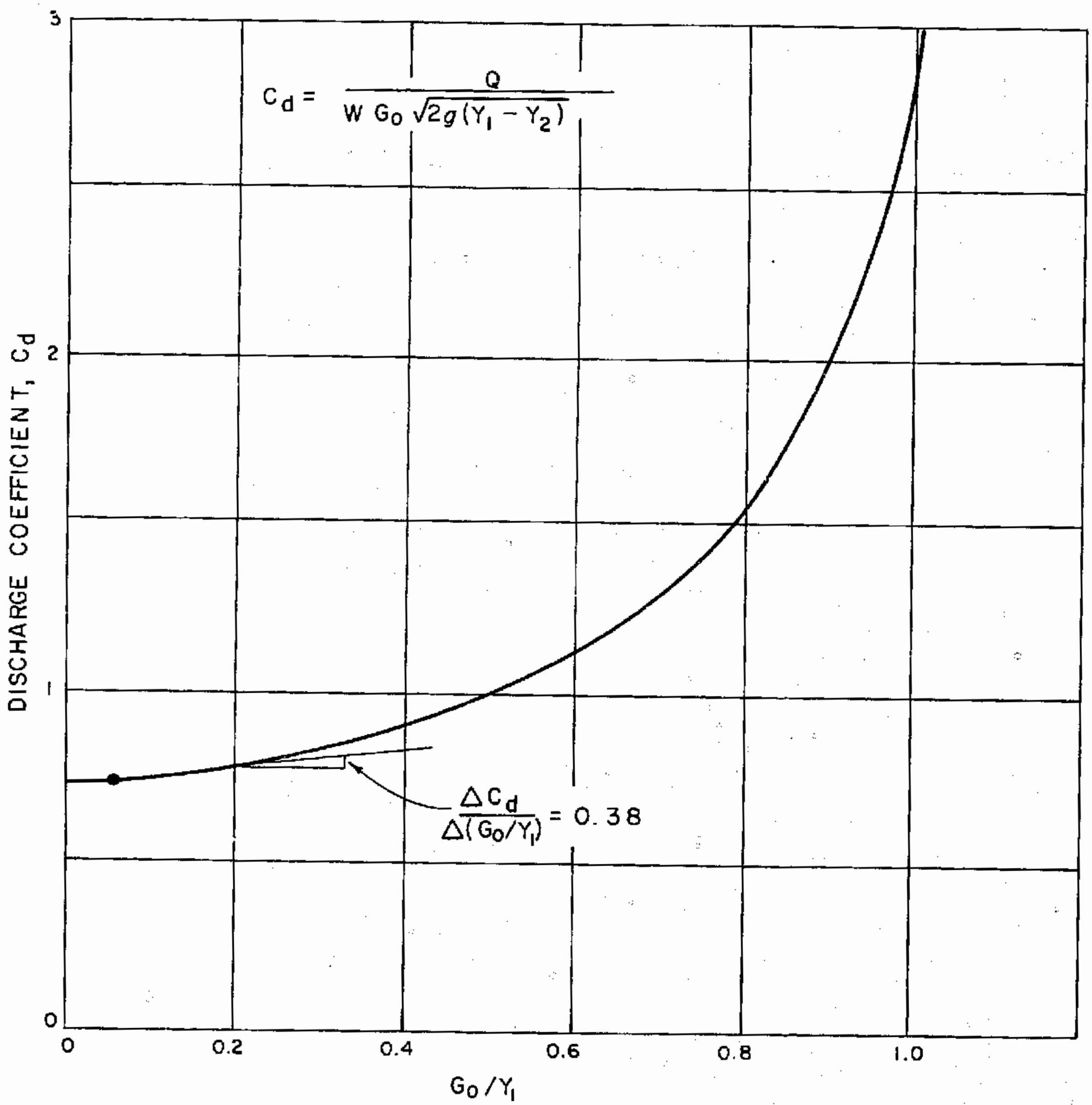


Figure 19.—Discharge coefficient, radial gate No. 1, Coalinga Canal.

concerns harbor reasonance by Ippen [4]. Although his studies were for harbors, they provide trends that are applicable to canals.

The desired response characteristics relate the motion of the water surface at the downstream end of a reach to sinusoidal motions at the upstream end of the reach. Due to reflections from the downstream section it is very difficult to maintain a sinusoidal variation in the upstream water surface. One method which can be used to achieve a sinusoidal variation in the upstream water depth is to place a large reservoir upstream from the reach. Sinusoidal waves generated in the reservoir are almost unaffected by what happens in the canal reach. Therefore, the waves in the reservoir tend to create a sinusoidal forcing function at the upstream end of the canal which is unaffected by reflections within the canal.

The change in water surface elevation in the canal with respect to water surface changes in the reservoir can be determined from the equation of continuity. It can be shown that, neglecting resonance in the canal, the change in elevation of the upstream water surface is equal to the change in reservoir elevation, appendix C. Therefore, creating sinusoidal variations in the reservoir water surface is equivalent to a sinusoidal forcing function at the upstream end of the canal reach.

The ratio of the wave height actually observed in the canal to that created in the reservoir is called the amplification factor. Experimental determinations of the amplification factors are usually performed with waves of various lengths in the reservoir. The period of these waves in the canal is given by:

$$T = \frac{L_w}{V_d} = \frac{1}{f} \tag{37}$$

where

L_w = wave length = $2\pi/K$
K = wave number
V_d = velocity of disturbance
f = wave frequency

In canals, the velocity of the disturbance is given by:

$$V_d = \sqrt{\frac{gA}{T}} + V \tag{38A}$$

for waves traveling in the downstream direction, and

$$V_d = \sqrt{\frac{gA}{T}} - V \tag{38B}$$

for waves traveling in the upstream direction

where

g = acceleration of gravity
A = cross-sectional area of canal
T = top width of water surface prism
V = mean velocity in cross section
V_d = velocity of disturbance

In many canals the mean velocity is small enough to be neglected in this approximate analysis.

The results of Ippen indicate that the amplification factors increase as the restriction to flow into the canal increases (smaller gate openings), figure 20. The maximum amplification occurs when:

$$kL = 1$$

This is equivalent to a frequency of:

$$f_o = \overline{V}_d/(2\pi L) \tag{39}$$

where

L = length of canal
f = frequency in cycles per unit time
$\overline{V}_d$ = average velocity of disturbance

The next highest amplification occurs at a frequency of:

$$f_1 = 3.5\,\overline{V}_d/(2\pi L) \tag{40}$$

The frequency response for a canal will not be exactly like that given in figure 20. The frictional effects will reduce the magnitude of the maximum amplifications and will cause the resonant frequencies to decrease. In addition, a total reflection of the wave does not occur at the downstream end of the canal due to the downstream gate being partially open. This too will decrease the maximum amplification. However, as a first approximation to the resonance characteristics of canals, figure 20 and equation 39 can be used. Either a mathematical model or physical model of a canal system can be used to determine the exact frequency response by application of the method outlined in appendix A.

For example, on Coalinga Canal the significant parameters on the second reach are:

Reach length	2536.8 m
Bottom width	3.66 m
Side slopes	1.0 vertical to 1.5 horizontal (3:2)

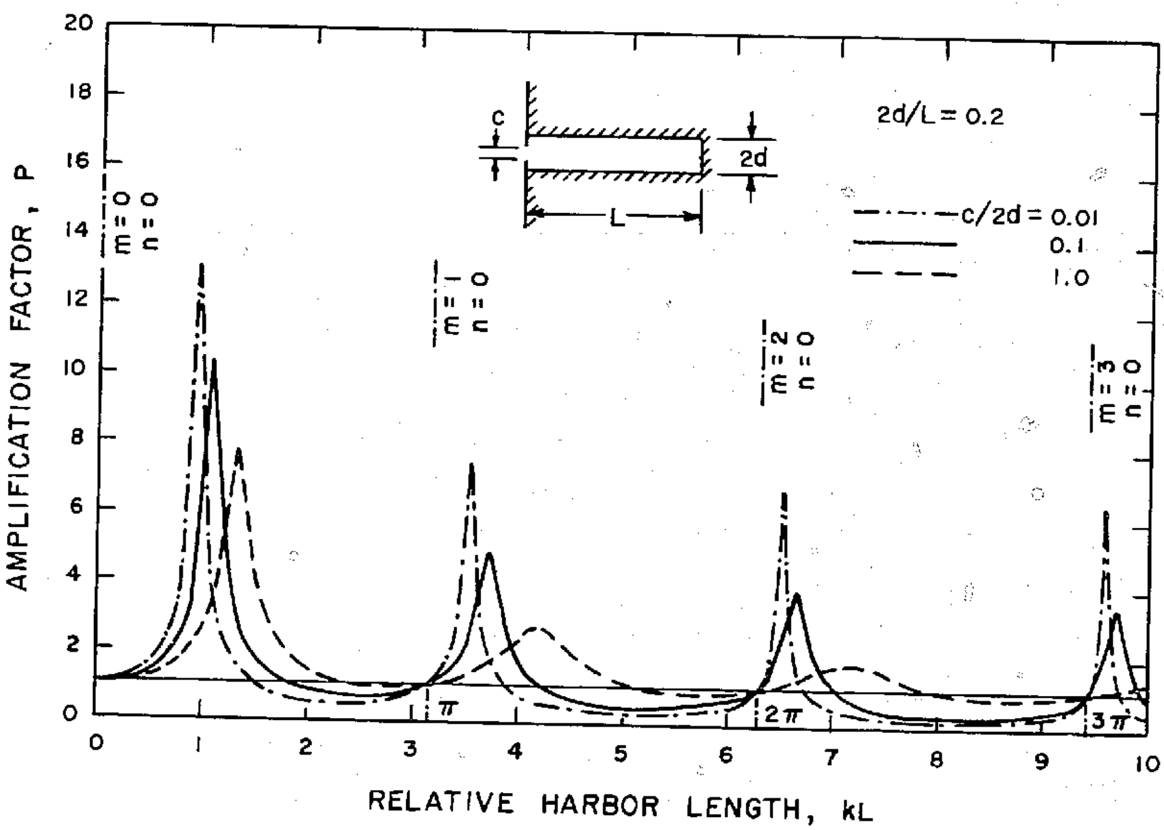


Figure 20.—Frequency response of harbors, from Ippen [4].

Target depth	3.45 m
Discharge	9.34 m³/s

Approximate top width of wetted prism:

$$3.66 + \left[(2)\ (1.5)\ (3.45)\right] = 14\ \text{m}$$

Average cross-sectional area:

$$\frac{(3.66 + 14)}{2}\ (3.45) = 30.5\ \text{m}^2$$

Mean velocity:

$$V = \frac{9.34}{30.5} = 0.31\ \text{m/s}$$

Wave celerity:

$$C = \left[\frac{9.81 \times 30.5}{14}\right]^{0.5} = 4.62\ \text{m/s}$$

Celerity of disturbance:

$$(V_d)_1 = 4.62 - 0.31 = 4.31\ \text{m/s}$$

$$(V_d)_2 = 4.62 + 0.31 = 4.93\ \text{m/s}$$

Fundamental resonant frequency:

$$f_o = \frac{60(V_d\ (\text{up}) + V_d\ (\text{down}))}{4\pi L}$$

$$= \frac{60(4.31 + 4.93)}{(4)\ (3.14)\ (2536.8)} = 0.017\ \text{c/min}$$

Second harmonic:

$$f_1 = \left(\frac{3.4}{4\pi}\right) \frac{(9.24)\ (60)}{2536.8} = 0.061\ \text{c/min}$$

The response characteristics for the second canal reach can be approximated based on figure 20 and the preceding computations; see figure 21. Referring back to figure 3, these characteristics are the response of elements G_5 and G_6 to sinusoidal variations of E. It is assumed that no disturbances enter at N.

Recent studies by Buyalski and Falvey [3] indicate that the general shape of figure 21 is correct. However, the higher frequency components are attenuated more in an actual canal system. As a rule of thumb, the amplitude of the peak at a given frequency is roughly half as large as the peak at the next lower resonant frequency.

Determination of System Response Functions

Open-loop response.—The open-loop response function gives the amplitude of the feedback signal in terms of sinusoidal variations at the upstream end of the reach. Neglecting nonlinear effects introduced by the dead band action at the gate, the open-loop response can be determined from the response characteristics of each element. The gain is obtained by multiplication of the gains of each element. The phase angles are obtained by summing the phase angles. The results of this process for the second reach of the Coalinga Canal are given in table 1.

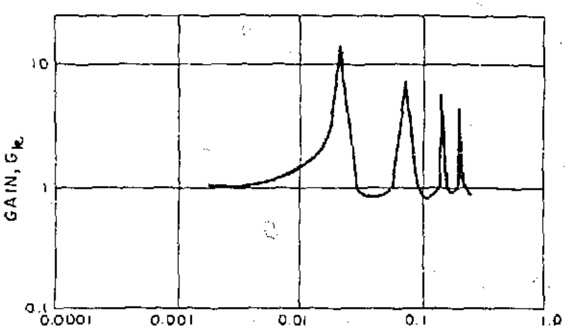


Figure 21.—Approximate frequency response, second reach, Coalinga Canal.

Closed-loop response.—The closed-loop response function gives the magnitude of the output (C) in terms of the input signal (R). The closed-loop function can be obtained from the Nichols diagram in the following manner. First, the values of the open-loop function $|GH|$ and $\angle GH$ are plotted on the Nichols diagrams, figure 22.

A series of light lines are superimposed on the Nichols diagram. The closed-loop frequency-response curves can be constructed by reading the magnitudes and phase angles on the light lines at those points where the frequency is known. The results of this process for the second reach of the Coalinga Canal are given in the second and third columns of table 2.

The mathematics upon which this procedure is based results in frequency characteristics of a unity feedback system, figure 6. However, in most canal controllers, the response characteristics are not equal to unity for all frequencies. Therefore, to obtain the actual closed-loop response, it is necessary to multiply the

Table 1.—*Response Characteristics, Second Reach, Coalinga Canal*

Frequency c/min	Controller[1]		Gate movement[2]		Water level[3]		Canal[4]		Open-loop[5]	
	P_c	[6] ϕ_c	P_2	ϕ_2	P_1	ϕ_1	P_k	ϕ_k	$\|GH\|$	$\angle GH$
0.0001	75.7	−89	1.0	0	0.18	0	1.0	0	13.6	−89
.001	7.9	−78	1.0	0	.18	0	1.0	0	1.42	−78
.01	1.8	−64	1.0	0	.18	0	1.4	0	0.45	−64
.016	1.3	−71	1.0	0	.18	0	2.5	0	0.58	−71
.021	1.0	−74	1.0	0	.18	0	12.0	0	2.20	−74
.04	0.56	−81	1.0	0	.18	0	0.8	0	0.081	−81
.06	0.38	−84	1.0	0	.18	0	1.6	0	0.109	−84
.075	0.30	−35	1.0	0	.18	0	7.9	0	0.434	−85
.10	0.23	−86	1.0	0	.18	0	0.8	0	0.033	−86

[1] Combined response, P_c, from figure 12.
[2] Effect of dead band is neglected.
[3] From equation 35.
[4] From figure 20.
[5] Amplitude $= |H_c\ H_2\ H_1\ G_k|$
Phase angle $= \phi_c + \phi_2 + \phi_1 + \phi_k$
[6] All angles are in degrees.

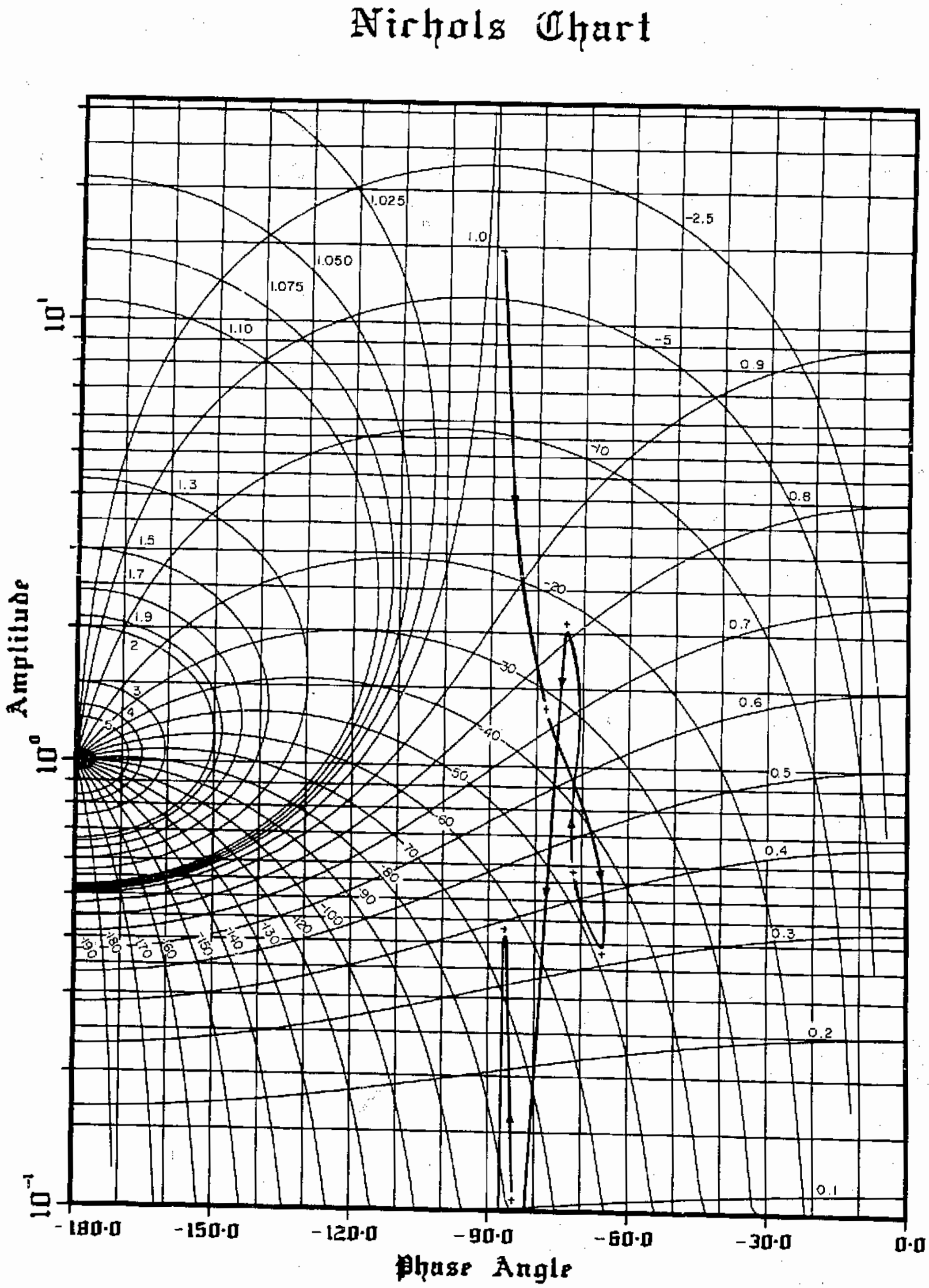


Figure 22.—Open-loop response, second reach, Coalinga Canal.

Table 2.—*Closed-Loop Response, Second Reach, Coalinga Canal*

Frequency c/min	Closed-loop response unity feedback[1]		Reciprocal of feedback response[2]		Closed-loop response nonunity feedback[3]	
	$\lvert F/C \rvert$	[4] $\angle(F/C)$	$\lvert 1/H \rvert$	$-\angle H$	$\lvert C/R \rvert$	$\angle(C/R)$
0.0001	1.0	–4	0.013	89	0.013	89
.001	0.75	–32	0.127	78	0.095	43
.01	0.33	–48	0.565	64	0.19	16
.016	0.44	–47	0.781	71	0.34	24
.021	0.82	–22	0.980	74	0.80	52
.04	0.08	–80	1.77	81	0.14	1
.06	0.11	–78	2.63	84	0.29	6
.075	0.38	–63	3.28	85	1.25	22
.10	0.03	–86	4.35	86	0.13	0

[1] From figure 22.
[2] $1/H = (H_c H_2 H_1)^{-1}$
$-\angle H = -\phi_c - \phi_2 - \phi_1$
[3] $\lvert C/R \rvert = \lvert F/C \rvert / H$
$\angle C/R = \angle F/C - \angle H$
[4] All angles are in degrees.

results just obtained by the reciprocal of the feedback response, figure 6.

Application of this procedure to the second reach of the Coalinga Canal results in the true closed-loop response, table 2 and figure 23. It can be seen that the canal is sensitive to disturbances having a frequency of about 0.075 cycles per minute (4.5 c/h). The amplitude of the resonant peak is a result of the method used to estimate the response characteristics of the canal. This peak probably is of a lower amplitude in the actual canal.

System Resonance and Stability

Very frequently the concepts of system resonance and system stability are used interchangeably. Strictly speaking, resonance refers to sustained oscillations that occur at the maximum value of the closed-loop frequency response. Instability, on the other hand, refers to oscillations which increase in magnitude until something in the real system fails. The stability of a system depends upon the system itself and is not a function of the amplitude of the input to the system. In analyzing a system, both the resonance characteristics and the overall system stability must be considered.

System resonance.—To examine resonance, the entire system must be considered with regard to disturbances which could have frequencies near the resonant frequency. The most likely sources of disturbances are turning off and on of deliveries at regular intervals,

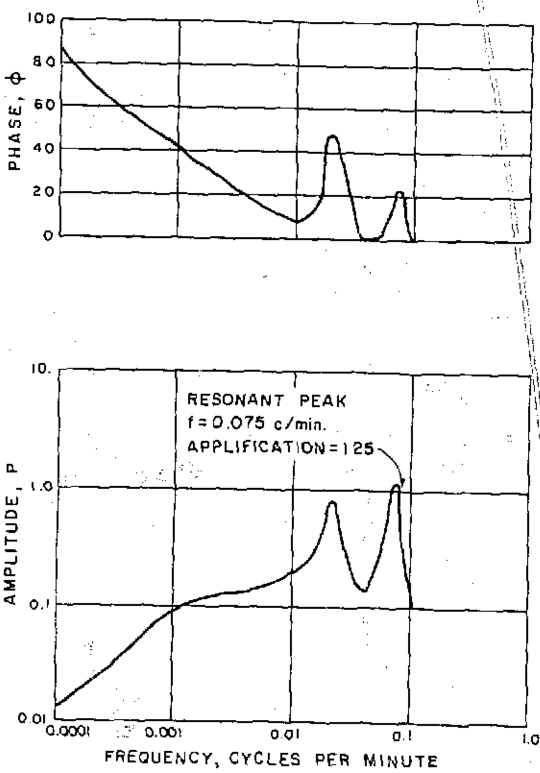


Figure 23.—Closed-loop response, second reach, Coalinga Canal.

incremental gate operations as a result of dead band action, and fluctuations originating outside the reach under consideration. Of these three, the dead band action could be the most critical. Its effect is relatively constant at all discharge values.

The effect of dead band on the system resonance can be illustrated by the second reach of the Coalinga Canal. Using the previously given canal parameters and equation 28, the maximum frequency induced by the dead band is 0.0063 c/min for a discharge of 2 m^3/s. Increasing the discharge to 7 m^3/s only increases the maximum frequency to 0.0065 c/min. From figure 23, it is obvious that this and all lower frequencies will not excite the reach into resonance. If, however, the controller GAIN had been increased to 29, the dead band action would have caused the reach to resonate at 0.075 c/min. This could have created a resonance problem.

System stability.—The system stability can be conveniently analyzed with a Nyquist plot. The real and imaginary values are computed from the open-loop characteristics using

$$Re = |GH| \cos \angle(GH)$$

and

$$Im = |GH| \sin \angle(GH),$$

table 3. It is clear for the second reach of the Coalinga Canal that the locus of the open-loop transfer function neither encloses the stability point (−1.0) nor intersects the locus of the dead band response function, figure 24. Therefore, the second reach is stable and it will not exhibit sustained oscillations due to nonlinearities introduced by the dead band action.

Table 3.—*Stability of Closed-Loop, Second Reach, Coalinga Canal*

Frequency, c/min	[1] Re	[2] Im
0.0001	0.237	−13.6
.001	.295	−1.39
.01	.197	−0.404
.016	.189	−0.548
.021	.606	−2.115
.04	.013	−0.080
.06	.011	−0.108
.075	.038	−0.432
.10	.002	−0.033

[1] Re = |GH| cos ∠ (GH)
[2] Im = |GH| sin ∠ (GH)
Values of |GH| and ∠ (GH) from table 1.

Design Recommendations

In many cases, the designer is concerned not with the frequency response, but in the response of a system to transients. Ogata [7] recommends that the amplification factor for the closed-loop response should lie between 1.0 and 1.4 for satisfactory transient response. This corresponds with an effective damping ratio of 0.4 to 0.7. For amplifications greater than 1.5, several overshoots will occur when a step change is input into the system. Additional topics on design and compensation techniques can be found in the appropriate chapter of Ogata [7].

In the case of the Coalinga Canal, the closed-loop gain at the resonant peak is 1.25, figure 23. According to the design recommendation, this gain is acceptable. In general, the determination of acceptable values for the amplification, resonant frequency, and dead bands is a trial and error process. For each trial, parameters are chosen to develop a controller response curve as in figure 12. Then the required computations as outlined in the previous sections are performed to determine the frequency response and stability characteristics of the system. When acceptable control parameters are developed, they must be tested in a mathematical model which simulates the operation of the entire canal system.

For the second reach of the Coalinga Canal, the chosen values of gain, reset, and filter time constant apparently resulted in satisfactory transient response characteristics. This indicates that the canal frequency response characteristics may have been estimated accurately enough for these computations. An accurate evaluation of the frequency response characteristics of any canal reach is very difficult to perform in the field because the output of the controller is influenced both by changes in the reference input and by disturbances, see figure 3.

In general, two means are available for determining the canal frequency response characteristics. The first method involves changing the water surface elevation upstream of the reach and observing water level changes in the reach. This method is oriented primarily for use with mathematical simulations. For this method the controller must be disconnected and all gates positioned so that they represent some steady-state flow condition. Usually a step change is employed. The duration of the step must be chosen to produce all of the frequencies of interest. In general, the duration of the step should be less than $(2f_{max})^{-1}$. If the maximum frequency expected is about 0.1 c/min, then the step duration must be shorter than

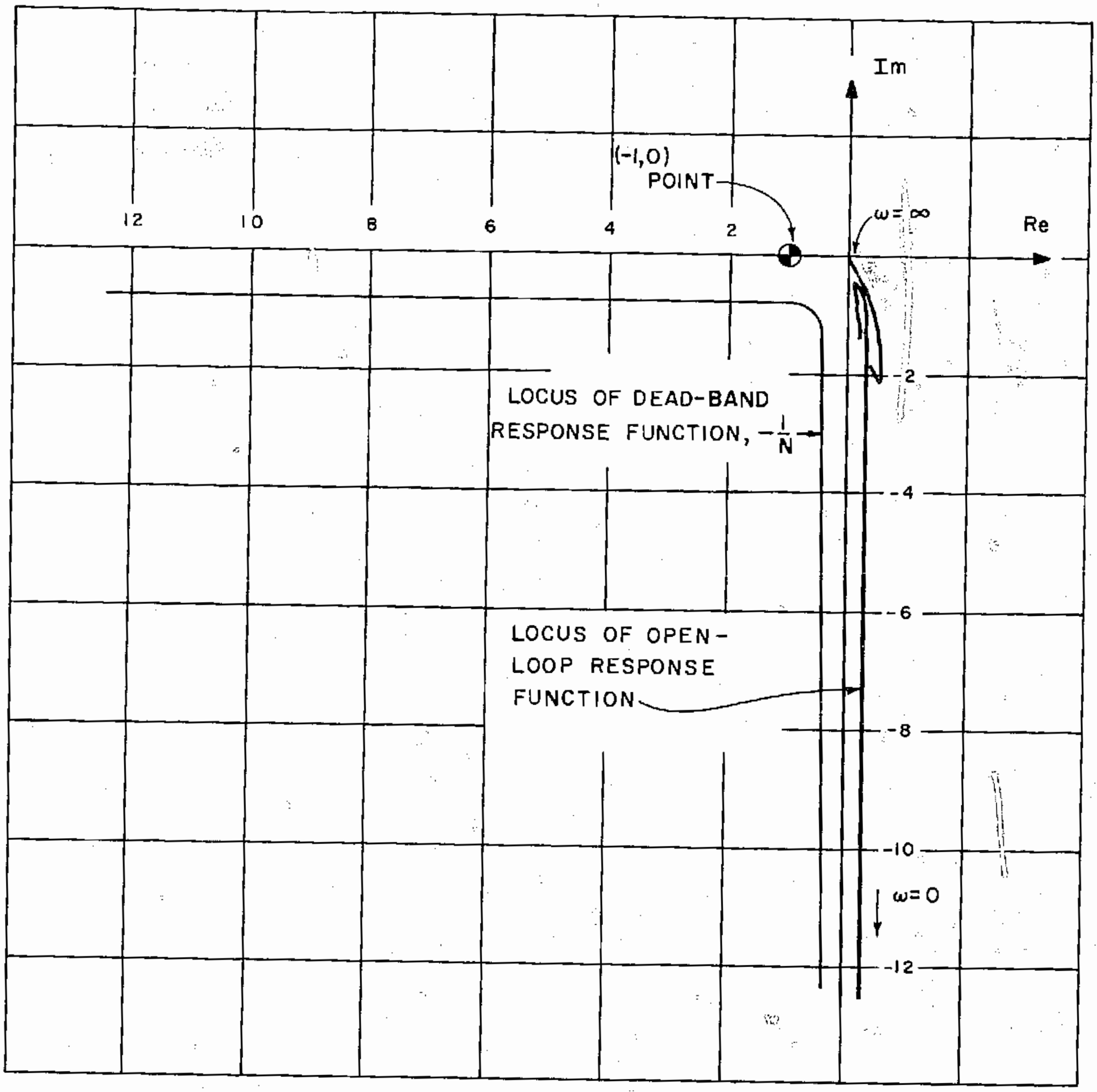


Figure 24.—Stability analysis, second reach, Coalinga Canal.

5 minutes. A computer program called HFRES has been developed to calculate the frequency response of a canal using this method, see appendix A.

The second method involves the superposition of sinusoidal variations in the upstream water surface on the normal operation of the system. The response is determined for each frequency through the use of cross correlations between the sinusoidal input and the water surface changes in the reach. This method can be applied to operating systems without interrupting the process. Since only phase and amplitude vary as the impressed frequency varies, the process of cross correlation can be applied successively to each canal reach even though only the most upstream pool is varied. The disadvantage of this method is that only one frequency at a time can be investigated. A computer program called HFCOV has been developed to determine the amplitude and phase characteristics for any inputted frequency, see appendix B.

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APPENDIX A

APPENDIX A

DETERMINATION OF SYSTEM FREQUENCY RESPONSE CHARACTERISTICS FROM INPUT AND OUTPUT SIGNALS

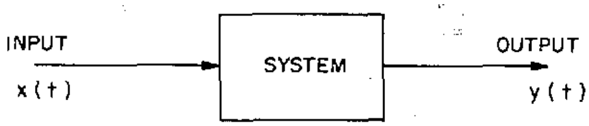
The frequency response of a system is defined as the ratio of the Fourier Transform of the output from the system to the Fourier Transform of the input to the system. Mathematically:

$$G(f) = \frac{\overline{Y}(f)}{\overline{X}(f)}$$

where

$G(f)$ = frequency response
$\overline{Y}(f)$ = Fourier transform of output
$\overline{X}(f)$ = Fourier transform of input

The system can be represented schematically by a block diagram.



For analysis with a digital computer a special form of the Fourier Transform must be used. This form is called a discrete finite Fourier Transform and it is defined as:

$$\overline{X}(m\Delta f) = \frac{\Delta T}{T} \sum_{n=o}^{n=N-1} x(n\Delta T)\, e^{-j2\pi(m\Delta f)(n\Delta T)}$$

$m = 0, 1, \ldots, N\text{-}1$

Since $\Delta f = \frac{1}{N\Delta T}$ and $T = N\Delta T$, the expression can also be written as:

$$\overline{X}\left(\frac{m}{N\Delta T}\right) = \frac{1}{N} \sum_{n=o}^{n=N-1} x(n\Delta t)\, e^{-j2\pi mn/N}$$

From this expression it can be seen that the lowest frequency that can be determined is equal to:

$$f_{min} = \frac{1}{N\Delta T} = T^{-1}$$

Therefore, in any analysis, the sample time must be long enough to determine the lowest frequencies in the signal.

The highest frequency that can be determined is:

$$f_{max} = \left(\frac{N}{2}\right)\Delta f = (2\Delta T)^{-1}$$

Therefore, the time interval between samples must be chosen so that:

$$\Delta T = (2f_{max})^{-1}$$

For example, if one wishes to determine the frequency response of the first reach of the Coalinga Canal, it is necessary to determine the Fourier Transforms of the input and outputs to the canal. Using the computer requires that the input and output signals be digitized.

The frequency range of interest is:

$$f_{min} = 0.001 \text{ c/min}$$
$$f_{max} = 0.5 \text{ c/min}$$

For this range, the sample time T must be:

$$T = f_{min}^{-1} = 1000 \text{ min}$$

and the interval between samples must be:

$$\Delta T = (2f_{max})^{-1} = 1 \text{ min}$$

The number of samples is therefore:

$$N = \frac{T}{\Delta T} = 1000$$

The computer program works only on radix-2 numbers of input, that is, 2 to an integer power. Therefore, the number of samples must be adjusted to:

$$N = 2^{10} = 1024$$

This adjustment causes the lowest frequency analyzed to become:

$$f_{min} = \frac{1}{N\Delta T} = \frac{1}{(1024 \times 1)} = 0.00098 \text{ c/min}$$

To determine the frequency response of a system any type of input signal can be used. That is, it can be periodic, random, or a transient. However, it must contain all frequencies of interest. Jenkins and Watts [5] show that the frequency response is equal to the Fourier Transform of an impulse applied to the system.

Therefore, a simple means of determining the frequency response is to input an impulse into the system and then transform the output. An impulse is created by setting all input signals equal to the same value except for the first value.

If the system being analyzed is linear, the frequency response is unique. For nonlinear systems the response is a function of the input. Since canals are nonlinear systems, the frequency response can be expected to vary as the discharge in the canal is varied.

Therefore, the recommended method of determining the canal response is:

- Establish a discharge in the canal
- Cause the depth upstream of the reach under study to undergo a step change at $t = 0$
- Set a new discharge.

The latter two steps are repeated for the full range of discharges anticipated.

To facilitate the determination of the frequency-response characteristics of a system, a computer program called HFRES was developed. The listing which follows includes instructions for its use.

```
      PROGRAM HFRES(HFSIG,PLFILE,OUTPUT,TAPE3=HFSIG,TAPE5=OUTPUT)
C
C
C     THIS PROGRAM DETERMINES THE FREQUENCY RESPONSE OF A SYSTEM.
C
C     FREQUENCY RESPONSE IS DEFINED AS THE RATIO OF THE FOURIER
C     TRANSFORM OF THE OUTPUT OF A SYSTEM TO THE FOURIER TRANSFORM
C     OF THE INPUT.  THE FREQUENCY RESPONSE OF A SYSTEM IS UNIQUE
C     IF THE SYSTEM IS LINEAR.  IF THE SYSTEM IS NON-LINEAR, THE
C     RESPONSE DEPENDS UPON THE INPUT.  THE FREQUENCY RESPONSE GIVEN
C     BY THIS PROGRAM IS THE CONTINUOUS DOUBLE-SIDED INTEGRAL
C     TRANSFORM.   ALTHOUGH, ONLY POSITIVE FREQUENCIES ARE PRESENTED IN
C     IN THE OUTPUT.  TO OBTAIN THE SINGLE-SIDED DISCRETE SPECTRUM
C     (AMPLITUDES AND PHASES OF A SINE SERIES) IT IS NECESSARY
C     TO MULTIPLY THE AMPLITUDES BY 2./(NOUT*DELT).
C
C     TO DETERMINE THE FREQUENCY RESPONSE OF A SYSTEM, THE INPUT
C     SIGNAL MUST HAVE A FREQUENCY SPECTRUM IN THE RANGE OF INTEREST.
C     ONE GOOD INPUT SIGNAL IS AN IMPULSE.  THE IMPULSE HAS ONE
C     FIXED VALUE AT ZERO TIME AND ANOTHER FIXED VALUE FOR ALL
C     OTHER TIMES.  AN IMPULSE WHICH PRODUCES A UNIT AMPLITUDE
C     AT ALL FREQUENCIES CAN BE ACHIEVED BY MAKING THE ZERO TIME
C     VALUE EQUAL TO 1./DELT AND ALL OTHER AMPLITUDES EQUAL TO
C     ZERO. IF THE INPUT IS AN IMPULSE,YOU MUST INFORM THE COMPUTER
C     THROUGH THE -IPULSE- PARAMETER.
C
C     IF BOTH THE INPUT AND OUTPUT SIGNALS HAVE A NOISE COMPONENT,
C     THE TRANSFORMS WILL INCLUDE A TRANSFORM OF THE NOISE.  TO
C     MINIMIZE THIS, A NUMBER OF RECORDS CAN BE TAKEN AND THE ESTIMATES
C     OF THE INPUT AND OUTPUT TRANSFORMS AVERAGED BEFORE THE FREQUENCY
C     RESPONSE IS DETERMINED.
C
C     THE INPUT CONSISTS OF THE FOLLOWING;
C         NIN-  THE NUMBER OF DATA POINTS DEFINING THE INPUT
C         NOUT- THE NUMBER OF DATA POINTS DEFINING THE OUTPUT
C               BOTH OF THESE MUST BE RADIX-2 NUMBERS, THAT IS,
C               2 RAISED TO AN INTEGER POWER GREATER THAN 1.
C         NAVG- THE NUMBER OF TIMES THE SIGNAL WILL BE REPEATED
C               TO GET AN AVERAGE
C         DELT- THE TIME INCREMENT BETWEEN DATA POINTS
C         IPULSE- IF THE INPUT IS AN IMPULSE,SET THIS =1
C                 OTHERWISE SET =0.
C         IPLOT-  IF NO PLOT IS DESIRED SET = 0
C                 IF A PLOT FILE IS DESIRED SET = 1
C                 IF A MICROFILM PLOT IS DESIRED SET = 2 .
C
C     THIS INPUT MUST BE READ IN A 3I4,F8.4,2I4 FORMAT.
C
C     THE NEXT DATA IS THE INPUT SIGNAL, FOLLOWED BY THE OUTPUT SIGNAL.
C     BOTH OF THESE ARE READ WITH A 10F8.4 FORMAT.  EACH ADDITIONAL
C     SET OF INPUT AND OUTPUT DATA MUST START ON A NEW LINE.  THE DATA
C     IS INPUT FROM A FILE CALLED HFSIG.
C
C     THE OUTPUT CONSISTS OF
C         THE INPUT DATA
C         THE FREQUENCY RESPONSE DATA
C         A 3-D BODE PLOT OF THE OUTPUT USING DISPLA SOFTWARE
```

```
C
C
C
C      THE CONTROL CARD STRING TO RUN THE PROGRAM WITHOUT A PLOT ON
C      THE NOS SYSTEM IS AS FOLLOWS;
C          /JOB.
C
C            .
C            .
C            .
C           FTN(I=HFRES)
C           ATTACH(DISSPLA/UN=LIBRARY)
C           LDSET(LIB=DISSPLA)
C           LOAD(LGO)
C           EXECUTE.
C            .
C            .
C            .
C          /EOF
C
C      THE CONTROL CARD STRING TO CREATE A PLOT FILE ON THE NOS
C      SYSTEM IS AS FOLLOWS;
C          /JOB.
C            .
C            .
C            .
C           PURGE(HF1)
C           DEFINE(PLFILE=HF1)
C           FTN(I=HFRES)
C           ATTACH(DISSPLA/UN=LIBRARY)
C           LDSET(LIB=DISSPLA)
C           LOAD(LGO)
C           EXECUTE.
C            .
C            .
C            .
C          /EOF
C
C      THE CONTROL CARD STRING TO DISPOSE THE PLOT TO MICROFILM
C      USING THE NOS SYSTEM IS AS FOLLOWS;
C          /JOB.
C            .
C            .
C            .
C           FTN(I=HFRES)
C           ATTACH(DISSPLA/UN=LIBRARY)
C           LDSET(LIB=DISSPLA)
C           LOAD(LGO)
C           EXECUTE.
C           DISPOSE(FR80=P8/ID=37)
C            .
C            .
C            .
C          /EOF
C
C      THESE STRINGS MUST BE SUBMITTED ON A PROCEDURE FILE.
C
C
```

```
C
C
      DIMENSION X(1024),Y(1024),CSR(1024),CSI(1024),ASR(1024)
     1,MESG(4)
      DATA(MESG(1)=6HFALVEY)
      DATA(MESG(2)=7HMC 1530)
      DATA(MESG(3)=9HEXT. 3760)
      DATA(MESG(4)=4HPLOT)
C
C     DATA INITIALIZATION
C
      DATA((CSR(J),J=1,1024)=1024*0.),((CSI(J),J=1,1024)=1024*0.),
     1((ASR(J),J=1,1024)=1024*0.)
      K=0
C
C     INPUT OF DATA
C
      READ(3,1)NIN,NOUT,NAVG,DELT,IPULSE,IPLOT
    1 FORMAT(3I4,F8.4,2I4)
      IF(IPLOT.EQ.1)CALL COMPRS
      IF(IPLOT.EQ.2)CALL DISSID(MESG)
      DO 14 I=1,NAVG
      READ(3,2)(X(J),J=1,NIN)
      READ(3,2)(Y(J),J=1,NOUT)
    2 FORMAT(10F8.4)
      IF(EOF(3))3,4
    3 K= 1
      GO TO 22
C
C     OUTPUT OF INPUT DATA
C
    4 IF(I.EQ.1)WRITE(5,5)NIN,NOUT,NAVG,DELT
    5 FORMAT(1H1,39X,10HINPUT DATA/
     126X,37HNUMBER OF DATA POINTS IN INPUT SIGNAL,I4/
     226X,38HNUMBER OF DATA POINTS IN OUTPUT SIGNAL,I4/
     333X,23HNUMBER OF DATA AVERAGES,I4/
     428X,33HTIME INTERVAL BETWEEN DATA POINTS,F8.4/)
      WRITE(5,6)I,(X(J),J=1,NIN)
    6 FORMAT(30X,24HDATA FOR DATA SET NUMBER,I4//
     137X,12HINPUT SIGNAL //
     2(4X,10F8.4))
      WRITE(5,25)(Y(J),J=1,NOUT)
   25 FORMAT(37X,13HOUTPUT SIGNAL//
     1(4X,10F8.4))
      PULSE= 1.
      IF(IPULSE.EQ.1)PULSE= ABS(X(1)-X(2))*DELT
C
C     FILLING OUT DATA WITH ZEROS IF NIN NOT EQUAL TO NOUT
C
      N= NIN
      IF(NOUT.EQ.NIN)GO TO 11
      IF(NOUT.GT.NIN)GO TO 7
      N= NIN
      NF= NOUT+1
      GO TO 8
    7 N= NOUT
      NF= NIN+1
```

```
      8 DO 10 J=NF,N
        IF(NOUT.GT.NIN)GO TO 9
        Y(J)= 0.
        GO TO 10
      9 X(J)= 0.
     10 CONTINUE
C
C       FORWARD TRANSFORM OF BOTH X AND Y SIMULTANEOUSLY
C
     11 CALL HFFFT(X,Y,N,1)
C
C       UNSCRAMBLING OF TRANSFORMED VALUES INTO CROSS SPECTRA
C       AND  AUTO SPECTRA
C
        NP2= N+2
        DO 13 J=1,N
        IF(J.EQ.1)GO TO 12
        CSR(J)= CSR(J)+(X(J)*Y(NP2-J)+X(NP2-J)*Y(J))*FLOAT(N)/2.
        CSI(J)= CSI(J)-(X(J)*X(J)-X(NP2-J)*X(NP2-J)+Y(J)*Y(J)
       1-Y(NP2-J)*Y(NP2-J))*FLOAT(N)/4.
        ASR(J)= ASR(J)+(X(J)*X(J)+2.*X(J)*X(NP2-J)+X(NP2-J)*X(NP2-J)
       1+Y(J)*Y(J)-2.*Y(J)*Y(NP2-J)+Y(NP2-J)*Y(NP2-J))*FLOAT(N)/4.
        GO TO 13
     12 CSR(1)= CSR(1)+X(1)*Y(1)*FLOAT(N)
        CSI(1)= 0.
        ASR(1)= ASR(1)+X(1)*X(1)*FLOAT(N)
     13 CONTINUE
     14 CONTINUE
C
C       COMPUTATION OF FREQUENCY RESPONSE
C
        PRO= DELT*FLOAT(N)
        NO= N/2+1
        DO 17 J=1,NO
        IF(ASR(J).LE.0.)X(J)= 1000000.
        IF(ASR(J).GT.0.)X(J)= PULSE*SQRT(CSR(J)*CSR(J)+CSI(J)*
       1CSI(J))/ASR(J)
        DELA= 0.0001
        IF(ABS(CSI(J)).GT.DELA.AND.ABS(CSR(J)).GT.DELA)GO TO 15
        Y(J)= 0.
        IF(CSR(J).GT.DELA)Y(J)= 0.
        IF(CSI(J).GT.DELA)Y(J)= 90.
        IF(CSR(J).LT.-DELA)Y(J)= 180.
        IF(CSI(J).LT.-DELA)Y(J)= 270.
        GO TO 16
     15 Y(J)= 57.296*ATAN2(CSI(J),CSR(J))
        IF(Y(J).LT.0.)Y(J)= Y(J)+360.
     16 CSR(J)= FLOAT(J-1)/PRO
     17 CONTINUE
C
C       OUTPUT OF RESULTS
C
        NTT= N/90+1
        WRITE(5,18)NTT
     18 FORMAT(1H1,1X,1H+,78X,1H+///
       161X8HSHEET  1,3H OF,I3//
       230X,28HFREQUENCY RESPONSE OF SYSTEM/
```

```
     337X,14H(BODE DIAGRAM)////)
      DO 21 J=1,NTT
      NL= 40
      IF(NL.GE.NO)NL= NO
      IF(J.EQ.1)WRITE(5,19)(CSR(I),X(I),Y(I),I=1,NL)
   19 FORMAT(26X,37HTHE FREQUENCY RESPONSE COMPONENTS ARE //
     129X,32H   FREQ      AMPLITUDE     PHASE  /
     233X,2HHZ,20X,3HDEG//
     3(29X,F8.3,3X,E10.3,2X,F8.3))
      NH= (J-1)*40+1
      NL= NH+39
      IF(NL.GE.NO)NL= NO
      IF(J.GT.1)WRITE(5,20)J,NTT,(CSR(I),X(I),Y(I),I=NH,NL)
   20 FORMAT(1H1,1X,1H+,78X,1H+///
     161X,5HSHEET,I3,3H OF,I3//
     226X,37HTHE FREQUENCY RESPONSE COMPONENTS ARE //
     329X,32H   FREQ      AMPLITUDE     PHASE  /
     433X,2HHZ,20X,3HDEG//
     5(29X,F8.3,3X,E10.3,2X,F8.3))
   21 CONTINUE
C
C     FLOT OF  BODE DIAGRAM
C
      IF(IPLOT.GE.1) CALL BODEPL(NO,CSR,X,Y)
C
C     DEFAULT BECAUSE OF INSUFFICIENT DATA
C
   22 IF(K.EQ.1)WRITE(5,23)
   23 FORMAT(1H1,22HPLEASE ENTER MORE DATA  )
      CALL EXIT
      END
```

```
      SUBROUTINE HFFFT(X,Y,N,IT)
C
C
C        THIS SUBROUTINE COMPUTES FORWARD AND INVERSE FOURIER
C     TRANSFORMS USING FFT TECHNIQUES.
C
C     MODIFIED BY  G.A. TETER   2/5/ 75
C     AND BY  H.T. FALVEY  3/14/75
C
C
C     THIS TRANSFORM USES THE COOLEY-TUKEY ALGORITHM TO DETERMINE
C     THE REAL AND IMAGINARY COEFFICIENTS IN THE FREQUENCY-DOMAIN
C     AND THE AMPLITUDE AND TIME VALUES IN THE TIME-DOMAIN.  THE
C     FORTRAN PROGRAM IS A CONVERSION OF A BASIC PROGRAM PUBLISHED
C     IN
C          DICE P.K., SPEED UP THE FAST FOURIER TRANSFORM
C          ELECTRONIC DESIGN 9
C          APR 26, 1970,  4PP.
C
C     THE  TRANSFORM REQUIRES RADIX-2 DATA (2 TO AN INTEGER POWER).
C     BOTH FORWARD (TIME-DOMAIN TO FREQUENCY-DOMAIN) AND INVERSE
C     (FREQUENCY-DOMAIN TO TIME-DOMAIN) TRANSFORMS CAN BE PERFORMED.
C     THE DATA IS ASSUMED TO BE EQUALLY SPACED AND REPETITIVE AFTER
C     A PERIOD OF N DATA POINTS.  THE DATA INPUT AND OUTPUT FOR THE
C     TWO DIRECTIONS OF TRANSFORMATION ARE AS FOLLOWS:
C
C     FORWARD-TRANSFORM.  THE REAL DATA IS INPUT THROUGH ARRAY X.  THE
C     IMAGINARY DATA (IF ANY) IS INPUT THROUGH ARRAY Y.  THE
C     REAL COEFFICIENTS (COSINE TRANSFORM) OF THE FREQUENCY-DOMAIN
C     SERIES ARE OUTPUT IN THE ARRAY X.  THE IMAGINARY COEFFICIENTS
C     (SINE TRANSFORM) OF THE FREQUENCY-DOMAIN SERIES ARE OUTPUT IN
C     THE ARRAY Y.  THE OUTPUT IS ORDERED FROM ZERO FREQUENCY TO THE
C     MOST POSITIVE FREQUENCY (ARRAY VALUES 1 THRU N/2+1) AND FROM
C     THE MOST NEGATIVE FREQUENCY TO THE ZERO FREQUENCY MINUS ONE
C     (ARRAY VALUES N/2+1 THRU N).  THE NUMBER OF DATA POINTS MUST
C     BE INPUT THRU THE PARAMETER N.  FOR THE FORWARD TRANSFORM
C     SET IT=1.  THE ORIGINAL DATA IS LOST.
C
C     INVERSE-TRANSFORM.   THE DATA IS INPUT THROUGH THE ARRAYS X AND
C     Y.  THE REAL TERMS (COSINE TRANSFORMS) ARE INPUT IN X. THE
C     IMAGINARY TERMS (SINE TRANSFORMS) ARE INPUT IN THE ARRAY Y.
C     THE INPUT MUST BE ORDERED IN THE IDENTICAL SEQUENCE AS GIVEN
C     ABOVE FOR THE OUTPUT.  THE REAL VALUES OF THE TIME-DOMAIN
C     ARE STORED IN THE X ARRAY AND THE IMAGINARY VALUES IN THE
C     Y ARRAY.  THESE VALUES ARE ORDERED IN TERMS OF INCREASING TIME.
C     FOR THE INVERSE TRANSFORM SET IT=-1.
C
C
      DIMENSION X(1),Y(1)
      AN=FLOAT(N)
      IG = ALOG(AN)/ALOG(2.0)+0.01
      P=8.*ATAN(1.)/AN
      IF(IT .EQ. -1)GO TO 180
      DO 170 I=1,N
      X(I)=X(I)/AN
  170 Y(I)=Y(I)/AN
  180 DO 370 L=1,IG
```

```
      IG1=2**(IG-L)
      M=0
      K4=2**(L-1)
      DO 360I=1,K4
      K1=M/IG1
      CALL IMAGE (K1,K2,IG)
      AK=FLOAT(K2)
      Y1=COS(P*AK)
      Y2=-SIN(P*AK) * FLOAT(IT)
      DO 330 J=1,IG1
      M1=M+IG1+1
      Y3=X(M1)*Y1-Y(M1)*Y2
      Y4=X(M1)*Y2+Y(M1)*Y1
      X(M1)=X(M+1)-Y3
      Y(M1)=Y(M+1)-Y4
      X(M+1)=X(M+1)+Y3
      Y(M+1)=Y(M+1)+Y4
  330 M=M+1
      M=M+IG1
  360 CONTINUE
  370 CONTINUE
      DO 480 I=1,N
      K1=I-1
      CALL IMAGE (K1,K2,IG)
      IF (K2-I+1) 420,480,480
  420 Y3=X(I)
      X(I)=X(K2+1)
      X(K2+1)=Y3
      Y3=Y(I)
      Y(I)= Y(K2+1)
      Y(K2+1)=Y3
  480 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE IMAGE(K1,K2,IG)
C
C     THIS PROGRAM IS USED IN CONJUNCTION WITH HFFFT TO PERFORM
C     THE BIT REVERSAL OF BINARY NUMBERS.  THROUGH THIS PROCESS
C     THE REAL AND IMAGINARY VALUES ARE PLACED IN THEIR PROPER
C     LOCATIONS IN THEIR RESPECTIVE ARRAYS.  A BIT REVERSAL
C     CREATES THE MIRROR IMAGE OF THE NUMBER.  FOR INSTANCE,
C     THE BINARY NUMBER 001010 BECOMES 010100 AFTER BIT REVERSAL.
C
      K2= 0
      DO 10 K=1,IG
      K3= K1-K1/2*2
      K1= K1/2
      IF(K3.EQ.0)GO TO 10
      K2= K2+2**(IG-K)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BODEPL(NPTS,X,Y,Z)
C
C     THIS SUBROUTINE PLOTS A THREE DIMENSIONAL BODE DIAGRAM
C     AND ITS PROJECTIONS ON THE PHASE, AMPLITUDE, AND FREQUENCY
C     PLANES.
C
      DIMENSION X(1024),Y(1024),Z(1024),XX(1024),YY(1024)
      NL= NPTS-2
      XMAX= X(2)
      XMIN= X(2)
      YMAX= Y(2)
      YMIN= Y(2)
      DO 1 J=1,NL
      XX(J)= ALOG10(X(J+1))
      YY(J)= ALOG10(Y(J+1))
      X(J)= X(J+1)
      Y(J)= Y(J+1)
      Z(J)= Z(J+1)
C
C     SELF SCALING ROUTINE
C
      IF(X(J).GE.XMAX)XMAX= X(J)
      IF(X(J).LE.XMIN)XMIN= X(J)
      IF(Y(J).GE.YMAX)YMAX= Y(J)
      IF(Y(J).LE.YMIN)YMIN= Y(J)
    1 CONTINUE
      IX= IFIX(ALOG10(XMIN)-1)
      XORIG= FLOAT(IX)
      IY= IFIX(ALOG10(YMIN)-1)
      YORIG= FLOAT(IY)
      IXM= IFIX(ALOG10(XMAX)+1)
      XMAX= FLOAT(IXM)
      IYM= IFIX(ALOG10(YMAX)+1)
      YMAX= FLOAT(IYM)
      FMIN= 10.**XORIG
      AMIN= 10.**YORIG
      IDX= IXM-IX
      IDY= IYM-IY
      XCY= 6./FLOAT(IDX)
      YCY= 6./FLOAT(IDY)
C
C     BEGINNING OF PLOT
C
      CALL BGNPL(-1)
      CALL SCMPLX
C
C     3-D PLOT
C
      CALL TITL3D("3-D BODE DIAGRAM$",100,6.5,8.)
      CALL HEADIN("FREQUENCY RESPONSE OF SYSTEM$",-100,1.5,1)
      CALL MIXALF("GREEK")
      CALL BASALF("STANDARD")
      CALL AXES3D(0,0,0,0,0,0,6.,6.,3.)
      CALL VUABS(20.,-30.,20.)
      CALL GRAF3D(XORIG,1.,XMAX,YORIG,1.,YMAX,0.,120.,350.)
      CALL CURV3D(XX,YY,Z,NL,0)
C
```

```
C      ROUTINES TO HEAVY UP 3-D PLOT CJRVE
C
       DO 2 J=1,NL
     2 YY(J)= 1.010*YY(J)
       CALL CUR/3D(XX,YY,Z,NL,0)
       DO 3 J=1,NL
     3 XX(J)= 1.010*XX(J)
       CALL CURV3D(XX,YY,Z,NL,0)
       DO 4 J=1,NL
     4 Z(J)= 1.01*Z(J)
       CALL CURV3D(XX,YY,Z,NL,0)
       DO 5 J=1,NL
     5 Z(J)= Z(J)/1.01
       CALL HEIGHT(0.19)
       CALL GRFITI(0.,0.,0.,6.,0.,0.,0.,3.,0.)
C
C      PROJECTION ON XY PLANE
C
       CALL TITLE(0,0,"FREQUENCY (H)$",100,"AMPLITUDE A$",100,6.,6.)
       CALL LOGLOG(FMIN,XCY,AMIN,YCY)
       CALL GRID(1,1)
       CALL CURVE(X,Y,NL,0)
       CALL END3GR(1)
       CALL GRFITI(0.,6.,0.,6.,6.,0.,6.,6.,3.)
C
C      PROJECTION ON XZ PLANE
C
       CALL INTAXS
       CALL TITLE(0,0,0,0,"PHASE (Q)$",100,6.,3.)
       CALL XLOG(FMIN,XCY,0.,120.)
       CALL GRID(1,4)
       CALL CURVE(X,Z,NL,0)
       CALL END3GR(2)
       CALL ENDPL(0)
       CALL DONEPL
       RETURN
       END
```

INPUT DATA

NUMBER OF DATA POINTS IN INPUT SIGNAL 128
NUMBER OF DATA POINTS IN OUTPUT SIGNAL 128
NUMBER OF DATA AVERAGES 1
TIME INTERVAL BETWEEN DATA POINTS .2000

DATA FOR DATA SET NUMBER 1

INPUT SIGNAL

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		

OUTPUT SIGNAL

0.0000	1.0764	.8365	-.2197	-.4693	.5878	1.6356	1.3583	.2565	-.0462
.9511	1.9333	1.5858	.4102	.0312	.9511	1.8559	1.4321	.1827	-.2665
.5878	1.4328	.9559	-.3391	-.8257	0.0000	.8257	.3391	-.9559	-1.4328
-.5878	.2665	-.1827	-1.4321	-1.8559	-.9511	-.0312	-.4102	-1.5858	-1.9333
-.9511	.0462	-.2565	-1.3583	-1.6356	-.5878	.4693	.2197	-.8365	-1.0764
0.0000	1.0764	.8365	-.2197	-.4693	.5878	1.6356	1.3583	.2565	-.0462
.9511	1.9333	1.5858	.4102	.0312	.9511	1.8559	1.4321	.1827	-.2665
.5878	1.4328	.9559	-.3391	-.8257	0.0000	.8257	.3391	-.9559	-1.4328
-.5878	.2665	-.1827	-1.4321	-1.8559	-.9511	-.0312	-.4102	-1.5858	-1.9333
-.9511	.0462	-.2565	-1.3583	-1.6356	-.5878	.4593	.2197	-.8365	-1.0764
0.0000	1.0764	.8365	-.2197	-.4693	.5878	1.6356	1.3583	.2565	-.0462
.9511	1.9333	1.5858	.4102	.0312	.9511	1.8559	1.4321	.1827	-.2665
.5878	1.4328	.9559	-.3391	-.8257	0.0000	.8257	.3391		

FREQUENCY RESPONSE OF SYSTEM
(BODE DIAGRAM)

THE FREQUENCY RESPONSE COMPONENTS ARE

FREQ HZ	AMPLITUDE	PHASE DEG
0.000	.341E+01	0.000
.039	.397E+01	3.961
.078	.831E+01	8.190
.117	.809E+01	193.039
.156	.189E+01	199.081
.195	.833E+00	207.411
.234	.428E+00	22[illegible].360
.273	.241E+00	242.647
.313	.171E+00	277.020
.352	.175E+00	308.967
.391	.207E+00	328.260
.430	.245E+00	339.481
.469	.282E+00	346.706
.508	.319E+00	351.855
.547	.357E+00	355.789
.586	.398E+00	358.986
.625	.443E+00	1.691
.664	.496E+00	4.053
.703	.560E+00	6.149
.742	.641E+00	8.078
.781	.749E+00	9.847
.820	.902E+00	11.496
.859	.114E+01	13.046
.898	.155E+01	14.516
.938	.249E+01	15.903
.977	.652E+01	17.231
1.016	.962E+01	198.501
1.055	.270E+01	199.718
1.094	.155E+01	200.893
1.133	.107E+01	202.001
1.172	.812E+00	203.075
1.211	.648E+00	204.104
1.250	.536E+00	205.087
1.289	.454E+00	206.034
1.328	.392E+00	206.908
1.367	.343E+00	207.753
1.406	.303E+00	208.546
1.445	.270E+00	209.285
1.484	.243E+00	209.960
1.523	.220E+00	210.608

THE FREQUENCY RESPONSE COMPONENTS ARE

FREQ HZ	AMPLITUDE	PHASE DEG
1.563	.200E+00	211.164
1.602	.182E+00	211.657
1.641	.167E+00	212.177
1.680	.153E+00	212.424
1.719	.141E+00	212.642
1.758	.131E+00	212.804
1.797	.121E+00	212.851
1.836	.112E+00	212.778
1.875	.104E+00	212.571
1.914	.971E-01	212.236
1.953	.906E-01	211.716
1.992	.847E-01	210.936
2.031	.793E-01	210.206
2.070	.743E-01	209.100
2.109	.698E-01	207.777
2.148	.657E-01	206.196
2.188	.621E-01	204.347
2.227	.589E-01	202.213
2.266	.560E-01	199.735
2.305	.536E-01	197.044
2.344	.516E-01	194.053
2.383	.500E-01	190.794
2.422	.489E-01	187.324
2.461	.482E-01	183.701
2.500	.480E-01	180.000

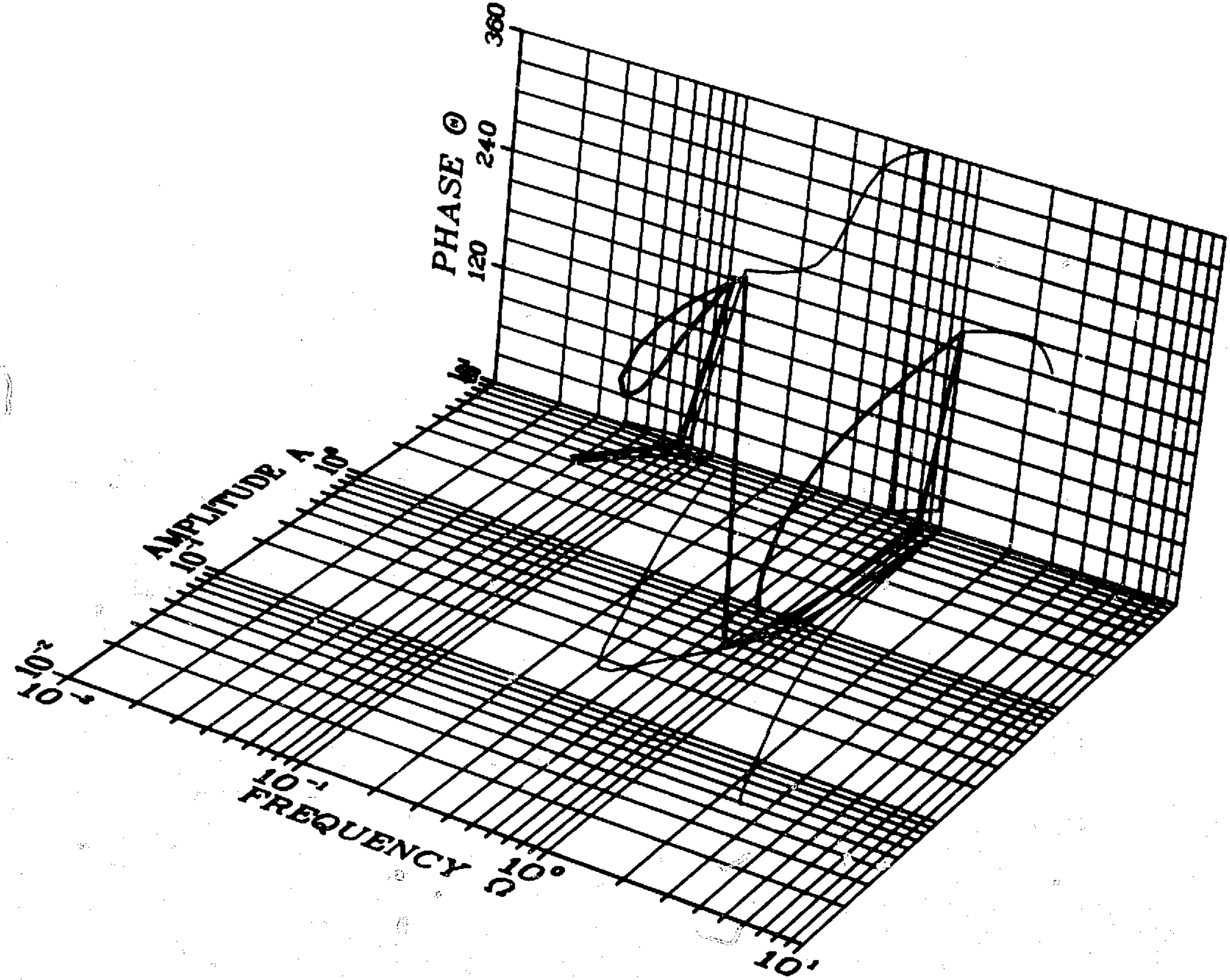


Figure 25.—Three-dimensional Bode diagram.

APPENDIX B

APPENDIX B

DETERMINATION OF FREQUENCY-RESPONSE CHARACTERISTICS AT SPECIFIC FREQUENCIES BY SUPERIMPOSING A SINUSOIDAL SIGNAL ON THE PROCESS SIGNAL

The principal behind this method involves inputting a sine wave into the system and measuring the output from the system. From these measurements, the amplitude and phase shifts for the inputted frequency can be determined. If sufficient frequencies are tested, it is possible to construct the frequency-response characteristics on a Bode diagram.

The amplitude of the input signal must be chosen carefully. If it is too large, the system will saturate. That is, the output will not be proportional to the input near the peaks of the sinusoidal wave. In fact, in some cases the output can be constant as the input continues to follow the sinusoidal variation near the peak.

On the other hand, if the input signal is too small, the dead bands in the controller will completely eliminate the effect to the sinusoidal input.

A sinusoidal variation can be input into a reach in two ways. The most practical method is to mix the electrical output of a sinusoidal generator with the signal being input into the comparator element of the gate positioner. The second method is to introduce sinusoidal variation in the water surface upstream from the reach. If certain conditions are met, this method will produce a sinusoidal input that is not influenced by disturbances that occur within the reach, appendix C. The upstream variations can be created by proper cycling of a supply pump, etc.

A system under operation does not have to be stopped to perform this type of test. In fact, this method is designed to be used with operating systems. The theory is based on the assumption that the sinusoidal variations and the process signals simply add together to produce the resultant output signal. Through the use of cross correlations, it is possible to determine the amplitude and phase shift of the output signal.

Assume that the input and output sinusoids are given by:

$$S_{in}(t) = E_{in} \sin(\omega\tau + \theta_i)$$

$$S_{out}(t) = E_{out} \sin(\omega\tau + \theta_2)$$

The output signal, which is a water depth in the case of a canal, consists of the output sinusoid and the process signal ($C = S_{out} + N$).

The cross correlation ϕ between the input sinusoid and the output signal equal:

$$\phi = (E_{in})(E_{out})(\omega\tau + \theta)$$

where

$$\theta = \theta_2 - \theta_1$$

If this function is made dimensionless by dividing by E_{in}^2, then

$$\phi_o = \left(\frac{E_{out}}{E_{in}}\right) \cos(\omega\tau + \theta)$$

The E_{out}/E_{in} ratio is the desired amplitude ratio for use in the Bode diagram and θ is the corresponding phase angle.

The E_{out}/E_{in} ratio is the maximum value of the cross correlation function and the phase angle is determined from the first point ($\tau = 0$) by:

$$\theta = \cos^{-1}\left(\frac{E_{in}}{E_{out}}\phi\right)$$

A computer program called HFCOV has been developed to perform the cross correlation. The listing which follows includes instructions for its use.

```
      PROGRAM HFCOV(HFSIG,OUTPUT,TAPE3=HFSIG,TAPE5=OUTPUT)
C
C
C     THIS IS A PROGRAM TO DETERMINE THE CO-VARIANCE FUNCTION AND THE
C     CO-SPECTRUM OF TWO SIGNALS X AND Y.  EACH SIGNAL IS DEFINED BY A
C     SERIES OF DATA POINTS BEGINNING AT TIMES A AND B FROM SOME
C     REFERENCE TIME. THE TOTAL NUMBER OF DATA POINTS DEFINING BOTH
C     SERIES MUST BE LESS THAN 2048.  THE PROGRAM IS BASED ON METHODS
C     OUTLINED IN THE BOOK:
C         BRIGHAM, E. O.
C         THE FAST FOURIER TRANSFORM
C         PRENTICE HALL,INC.
C         1974, 252PP.
C
C     THE INPUT CONSISTS OF THE FOLLOWING:
C     A- THE TIME DISPLACEMENT FROM A REFERENCE FOR THE X-SERIES OF DATA
C     B- THE TIME DISPLACEMENT FROM A REFERENCE FOR THE Y-SERIES OF DATA
C     NP- THE NUMBER OF DATA POINTS DEFINING THE X-SERIES
C     NQ- THE NUMBER OF DATA POINTS DEFINING THE Y-SERIES
C     DELT- THE TIME INCREMENT BETWEEN DATA POINTS
C     THIS INPUT MUST BE READ ACCORDING TO THE FORMAT
C            2F8.4,2I4,F8.4
C
C     THREE TYPES OF COMPUTATIONS CAN BE MADE
C         1) THE CO-VARIANCE FUNCTION
C         2) THE CROSS-CORRELATION FUNCTION
C              (CO-VARIANCE AVERAGED AND MADE DIMENSIONLESS
C         3) SIGNAL DETECTION WHERE THE X-DATA ARE A SINUSOIDAL INPUT
C               AND THE Y-DATA ARE THE OUTPUT (SINUSOID PLUS NOISE)
C     FOR DETAILS OF 3), SEE
C
C                LEE Y.W.
C     STATISTICAL THEORY OF COMMUNICATION
C           JOHN WILEY AND SONS
C             1960, 509 PP.
C
C     CHOOSE THE TYPE OF ANALYSIS DESIRED BY INPUTTING 1, 2, OR 3
C     FOR NTYP WITH AN I4 FORMAT.
C
C     THE X-SERIES AND Y-SERIES DATA ARE READ IN A 10F8.4 FORMAT WITH
C     ALL OF THE X-SERIES DATA IS READ FIRST, FOLLOWED ALL OF THE
C     Y-SERIES DATA.
C     THE DATA IS INPUT FROM A FILE CALLED HFSIG.
C
C     THE OUTPUT CONSISTS OF
C         THE INPUT DATA
C         THE CO-SPECTRUM AND
C         THE CO-VARIANCE OR CROSS-CORRELATION.
C
C
C
      DIMENSION X(2048),Y(2048),ZR(2048),ZI(2048),AMP(2048),PHI(2048)
C
C     INPUT OF DATA
C
      READ(3,1)A,B,NP,NQ,DELT
    1 FORMAT(2F8.4,2I4,F8.4)
```

```
      READ(3,15)NTYP
   15 FORMAT(I4)
      READ(3,2)(X(I),I=1,NP),(Y(I),I=1,NQ)
    2 FORMAT(10F8.4)
      K= 0
      IF(EOF(3))29,3
   29 K=1
      GO TO 27
C
C     OUTPUT OF INPUT DATA
C
    3 WRITE(5,4)A,B,NP,NQ,DELT,NTYP,NP,(X(I),I=1,NP),(Y(I),I=1,NQ)
    4 FORMAT(1H1,39X,10HINPUT DATA/
     127X,27HTIME OFFSET OF FIRST SIGNAL ,F8.4/
     226X,28HTIME OFFSET OF SECOND SIGNAL ,F8.4/
     324X,37HNUMBER OF DATA POINTS IN FIRST SIGNAL ,I4/
     423X,38HNUMBER OF DATA POINTS IN SECOND SIGNAL ,I4/
     537X,7HDELTA T ,F8.4 /
     A36X,13HANALYSIS TYPE ,I3/
     B//27X,29HANALYSIS TYPE 1 = CO-VARIANCE/
     C27X,35HANALYSIS TYPE 2 = CROSS-CORRELATION/
     D27X,34HANALYSIS TYPE 3 = SIGNAL DETECTION ///
     622X,9HTHE FIRST,I4,33H DATA POINTS ARE THE FIRST SIGNAL /
     727X,35HTHE REMAINDER ARE THE SECOND SIGNAL /
     8(8X,10F8.3))
C
C     COMPUTATION OF N
C
      N=NP+NQ-1
      DO 5 I=1,12
      NCK= 2**I
      IF(NCK.GE.N)GO TO 6
      IF(NCK.GT.2048)GO TO 7
    5 CONTINUE
    6 N=NCK
      GO TO 9
    7 WRITE(3,8)
    8 FORMAT(1H1,31X,27HNP+NQ-1 CANNOT EXCEED 2048. /
     124X,41HPLEASE VERIFY YOUR NUMBERS OF INPUT DATA. )
      CALL EXIT
C
C     COMPUTATION OF RMS VALUES IF NTYP = 2
C
    9 IF (NTYP.EQ.1)GO TO 17
      SUMX= 0.
      SUMXX= 0.
      SUMY= 0.
      SUMYY= 0.
      DO 31 I=1,N
      SUMX= SUMX+X(I)
      SUMXX= SUMXX+X(I)*X(I)
      SUMY= SUMY+Y(I)
      SUMYY= SUMYY+Y(I)*Y(I)
   31 CONTINUE
      SX= SQRT(SUMXX/FLOAT(NP))
      SY= SQRT(SUMYY/FLOAT(NQ))
C
```

```
      READ(3,15)NTYP
   15 FORMAT(I4)
      READ(3,2)(X(I),I=1,NP),(Y(I),I=1,NQ)
    2 FORMAT(10F8.4)
      K= 0
      IF(EOF(3))29,3
   29 K=1
      GO TO 27
C
C     OUTPUT OF INPUT DATA
C
    3 WRITE(5,4)A,B,NP,NQ,DELT,NTYP,NP,(X(I),I=1,NP),(Y(I),I=1,NQ)
    4 FORMAT(1H1,39X,10HINPUT DATA/
     127X,27HTIME OFFSET OF FIRST SIGNAL ,F8.4/
     226X,28HTIME OFFSET OF SECOND SIGNAL ,F8.4/
     324X,37HNUMBER OF DATA POINTS IN FIRST SIGNAL ,I4/
     423X,38HNUMBER OF DATA POINTS IN SECOND SIGNAL ,I4/
     537X,7HDELTA T ,F8.4 /
     A36X,13HANALYSIS TYPE ,I3/
     9//27X,29HANALYSIS TYPE 1 = CO-VARIANCE/
     C27X,35HANALYSIS TYPE 2 = CROSS-CORRELATION/
     D27X,34HANALYSIS TYPE 3 = SIGNAL DETECTION ///
     622X,9HTHE FIRST,I4,33H DATA POINTS ARE THE FIRST SIGNAL /
     727X,35HTHE REMAINDER ARE THE SECOND SIGNAL /
     8(8X,10F8.3))
C
C     COMPUTATION OF N
C
      N=NP+NQ-1
      DO 5 I=1,12
      NCK= 2**I
      IF(NCK.GE.N)GO TO 6
      IF(NCK.GT.2048)GO TO 7
    5 CONTINUE
    6 N=NCK
      GO TO 9
    7 WRITE(3,8)
    8 FORMAT(1H1,31X,27HNP+NQ-1 CANNOT EXCEED 2048. /
     124X,41HPLEASE VERIFY YOUR NUMBERS OF INPUT DATA. )
      CALL EXIT
C
C     COMPUTATION OF RMS VALUES IF NTYP = 2
C
    9 IF (NTYP.EQ.1)GO TO 17
      SUMX= 0.
      SUMXX= 0.
      SUMY= 0.
      SUMYY= 0.
      DO 31 I=1,N
      SUMX= SUMX+X(I)
      SUMXX= SUMXX+X(I)*X(I)
      SUMY= SUMY+Y(I)
      SUMYY= SUMYY+Y(I)*Y(I)
   31 CONTINUE
      SX= SQRT(SUMXX/FLOAT(NP))
      SY= SQRT(SUMYY/FLOAT(NQ))
C
```

```
C      REMOVAL OF DC COMPONENT IF NTYP = 3
C
       IF(NTYP.EQ.2)GO TO 17
       XMEAN= SUMX/FLOAT(N)
       YMEAN= SUMY/FLOAT(N)
       DO 42 I=1,N
       X(I)= X(I)-XMEAN
       Y(I)= Y(I)-YMEAN
    42 CONTINUE
C
C      DISPLACEMENT OF X SIGNAL TO LEFT HAND SIDE OF THE N POINTS
C
    17 NL= NP+1
       XMAX= 0.
       DO 10 I=1,N
       IF(X(I).GE.XMAX)XMAX= X(I)
       IF(I.GE.NL)X(I)= 0.
    10 CONTINUE
C
C      PLACEMENT OF Y SIGNAL TO GET CORRECT PHASE ANGLE
C
       TL= (B-A)/DELT+0.0001
       M= IFIX(TL)
       NMIN= M/N
       NP1= NMIN+1
       MTST= NMIN*N
       NMAX= NP1*N
       MMAX= M+NQ
       IF(M.GE.MTST.AND.MMAX.LE.NMAX)GO TO 11
       NO1= MMAX-NMAX+1
       NOMN= NQ-NO1+1
       NOTT= N-NOMN+1
       DO 38 I=1,N
       J= NO1+I-1
       IF(I.LE.NOMN)ZR(I)= Y(J)
       IF(I.GT.NOMN)ZR(I)= 0.
       J= I-NOTT+1
       IF(I.GE.NOTT)ZR(I)= 0.
    38 CONTINUE
       GO TO 40
    11 DO 39 I=1,N
       J= I-M
       JL= M+NQ
       IF(I.LE.M)ZR(I)= 0.
       IF(I.GT.M)ZR(I)= Y(I)
       IF(I.GT.JL)ZR(I)= 0.
    39 CONTINUE
    40 DO 41 I=1,N
       Y(I)= ZR(I)
    41 CONTINUE
C
C      FORWARD TRANSFORMATION TO COMPUTE CO-SPECTRUM IN POLAR FORM
C
       CALL HFFFT(X,Y,N,1)
       NP2= N+2
       DO 13 I=1,N
       IF(I.EQ.1)GO TO 30
```

```
      ZR(I)= (X(I)*Y(NP2-I)+X(NP2-I)*Y(I))/2.*FLOAT(N)
      ZI(I)= -(X(I)*X(I)-X(NP2-I)*X(NP2-I)+Y(I)*Y(I)-
     1Y(NP2-I)*Y(NP2-I))/4.*FLOAT(N)
      GO TO 13
   30 ZR(1)= X(1)*Y(1)*FLOAT(N)
      ZI(1)= 0.
   13 CONTINUE
C
C
C     COMPUTATION OF TIME DISPLACEMENT FROM REFERENCE TIME
C
      T1= B-A
C
C     COMPUTATION OF CO-SPECTRA FREQUENCY, AMPLITUDE, AND PHASE
C
      PRD= DELT*FLOAT(N)
      NO= N/2+1
      DO 16 I=1,NO
      AMP(I)= SQRT(ZR(I)*ZR(I)+ZI(I)*ZI(I))
      IF(I.GE.2)AMP(I)= 2.*AMP(I)
      DELA= 0.0001
      IF(ABS(ZI(I)).GT.DELA.AND.ABS(ZR(I)).GT.DELA)GO TO 14
      PHI(I)= 0.
      IF(ZR(I).GT.DELA)PHI(I)= 0.
      IF(ZI(I).GT.DELA)PHI(I)= 90.
      IF(ZR(I).LT.-DELA)PHI(I)= 180.
      IF(ZI(I).LT.-DELA)PHI(I)= 270.
      ZI(I)= -ZI(I)
      GO TO 16
   14 PHI(I)= 57.296*ATAN2(ZI(I),ZR(I))
      IF(PHI(I).LT.0.)PHI(I)= PHI(I)+360.
   16 CONTINUE
C
C     COMPUTATION OF CO-VARIANCE TIME AND SPECTRAL DENSITY FREQUENCY
C
      DO 18 I=1,N
      X(I)= T1+DELT*FLOAT(I-1)
      Y(I)= FLOAT(I-1)/PRD
   18 CONTINUE
C
C     COMPUTATION OF CO-VARIANCE
C
      CALL HFFFT(ZR,ZI,N,-1)
C
C     COMPUTATION OF  CROSS-CORRELATION
C
      IF(NTYP.NE.2)GO TO 32
      RNOP= NQ+NP
      DO 33 I=1,N
      ZR(I)= 2.*ZR(I)/(RNOP*SX*SY)
   33 CONTINUE
C
C     SIGNAL DETECTION
C
   32 IF(NTYP.NE.3)GO TO 34
      ZRMAX= 0.
      DO 35 I=1,N
```

```
      ZR(I)= ZR(I)/(XMAX*XMAX)
      IF(ZR(I).GE.ZRMAX)ZRMAX= ZR(I)
   35 CONTINUE
      A= ZR(1)/ZRMAX
      PHAS= 57.296*ACOS(A)
C
C     OUTPUT OF RESULTS
C
   34 NT= N/40+1
      NTT= N/80+1
      WRITE(5,19)NTT
   19 FORMAT(1H1,1X,1H+,78X,1H+///
     161X, 8HSHEET  1,3H OF,I3//
     236X,20HCO-SPECTRUM ANALYSIS ///)
      DO 22 J=1,NTT
      NL= 40
      IF(NL.GE.NO)NL= NO
      IF(J.EQ.1)WRITE(5,20)(Y(M),AMP(M),PHI(M),M=1,NL)
   20 FORMAT(32X,27HTHE SPECTRAL COMPONENTS ARE //
     129X,30H   FREQ       AMPLITUDE      PHASE /
     233X,2HHZ,20X,3HDEG//
     3(29X,F8.3,3X,E10.3,2X,F9.3))
      NM= (J-1)*40+1
      NL= NM+39
      IF(NL.GE.NO)NL= NO
      IF(J.GT.1)WRITE(5,21)J,NTT,(Y(M),AMP(M),PHI(M),M=NM,NL)
   21 FORMAT(1H1,1X,1H+,78X,1H+///
     161X,5HSHEET,I3,3H OF,I3 //
     231X,27HTHE SPECTRAL COMPONENTS ARE //
     329X,30H   FREQ       AMPLITUDE      PHASE /
     433X,2HHZ,20X,3HDEG//
     5(29X,F8.3,3X,E10.3,2X,F9.3))
   22 CONTINUE
      IF(NTYP.EQ.1)WRITE(5,36)NT
   36 FORMAT(1H1,1X,1H+,78X,1H+///
     161X, 8HSHEET  1,3H OF,I3//
     236X,20HCO-VARIANCE ANALYSIS ///)
      IF(NTYP.EQ.2)WRITE(5,23)NT
   23 FORMAT(1H1,1X,1H+,78X,1H+///
     161X,8HSHEET  1,3H OF,I3//
     234X,26HCROSS-CORRELATION ANALYSIS ///)
      IF(NTYP.EQ.3)WRITE(5,37)NT,ZRMAX,PHAS
   37 FORMAT(1H1,1X,1H+,78X,1H+///
     161X,8HSHEET  1,3H OF,I3//
     236X,16HSIGNAL DETECTION ///,
     338X,6HGAIN =,F6.2/
     437X,8HPHASE = ,F6.1////)
      DO 26 J=1,NT
      NL= 40
      IF(NL.GE.N)NL= N
      IF(J.EQ.1)WRITE(5,24)(X(M),ZR(M),ZI(M),M=1,NL)
   24 FORMAT(29X,32H    TIME          REAL      IMAGINARY   //
     1(23X,F8.3,3X,F8.3,4X,F8.3))
      NM= (J-1)*40+1
      NL= NM+39
      IF(NL.GE.N)NL= N
      IF(J.GT.1)WRITE(5,25)J,NT,(X(M),ZR(M),ZI(M),M=NM,NL)
```

```
      25 FORMAT(1H1,1X,1H+,78X,1H+///
        161X,5HSHEET,I3,3H OF,I3,  //
        229X,32H   TIME        REAL      IMAGINARY   //
        3(29X,F8.3,3X,F8.3,4X,F6.3))
      26 CONTINUE
      27 IF(K.EQ.1)WRITE(5,28)
      28 FORMAT(1H1,33X,22HPLEASE ENTER MORE DATA )
         CALL EXIT
         END
```

```
      SUBROUTINE HFFFT(X,Y,N,IT)
C
C
C        THIS SUBROUTINE COMPUTES FORWARD AND INVERSE FOURIER
C     TRANSFORMS USING FFT TECHNIQUES.
C
C     MODIFIED BY  G.A. TETER   2/5/ 75
C     AND BY  H.T. FALVEY  3/14/75
C
C
C
C     THIS TRANSFORM USES THE COOLEY-TUKEY ALGORITHM TO DETERMINE
C     THE REAL AND IMAGINARY COEFFICIENTS IN THE FREQUENCY-DOMAIN
C     AND THE AMPLITUDE AND TIME VALUES IN THE TIME-DOMAIN.  THE
C     FORTRAN PROGRAM IS A CONVERSION OF A BASIC PROGRAM PUBLISHED
C     IN
C          RICE P.K., SPEED UP THE FAST FOURIER TRANSFORM
C          ELECTRONIC DESIGN 9
C          APR 26, 1970,  4PP.
C
C     THE  TRANSFORM REQUIRES RADIX-2 DATA (2 TO AN INTEGER POWER).
C     BOTH FORWARD (TIME-DOMAIN TO FREQUENCY-DOMAIN) AND INVERSE
C     (FREQUENCY-DOMAIN TO TIME-DOMAIN) TRANSFORMS CAN BE PERFORMED.
C     THE DATA IS ASSUMED TO BE EQUALLY SPACED AND REPETITIVE AFTER
C     A PERIOD OF N DATA POINTS.  THE DATA INPUT AND OUTPUT FOR THE
C     TWO DIRECTIONS OF TRANSFORMATION ARE AS FOLLOWS:
C
C     FORWARD-TRANSFORM.  THE REAL DATA IS INPUT THROUGH ARRAY X.  THE
C     IMAGINARY DATA (IF ANY) IS INPUT THROUGH ARRAY Y.  THE
C     REAL COEFFICIENTS (COSINE TRANSFORM) OF THE FREQUENCY-DOMAIN
C     SERIES ARE OUTPUT IN THE ARRAY X.  THE IMAGINARY COEFFICIENTS
C     (SINE TRANSFORM) OF THE FREQUENCY-DOMAIN SERIES ARE OUTPUT IN
C     THE ARRAY Y.  THE OUTPUT IS ORDERED FROM ZERO FREQUENCY TO THE
C     MOST POSITIVE FREQUENCY (ARRAY VALUES 1 THRU N/2+1) AND FROM
C     THE MOST NEGATIVE FREQUENCY TO THE ZERO FREQUENCY MINUS ONE
C     (ARRAY VALUES N/2+1 THRU N).  THE NUMBER OF DATA POINTS MUST
C     BE INPUT THRU THE PARAMETER N.  FOR THE FORWARD TRANSFORM
C     SET IT=1.  THE ORIGINAL DATA IS LOST.
C
C     INVERSE TRANSFORM.   THE DATA IS INPUT THROUGH THE ARRAYS X AND
C     Y.  THE REAL TERMS (COSINE TRANSFORMS) ARE INPUT IN X, THE
C     IMAGINARY TERMS (SINE TRANSFORMS) ARE INPUT IN THE ARRAY Y.
C     THE INPUT MUST BE ORDERED IN THE IDENTICAL SEQUENCE AS GIVEN
C     ABOVE FOR THE OUTPUT.  THE REAL VALUES OF THE TIME-DOMAIN
C     ARE STORED IN THE X ARRAY AND THE IMAGINARY VALUES IN THE
C     Y ARRAY.  THESE VALUES ARE ORDERED IN TERMS OF INCREASING TIME.
C     FOR THE INVERSE TRANSFORM SET IT=-1.
C
C
      DIMENSION X(1),Y(1)
      AN=FLOAT(N)
      IG = ALOG(AN)/ALOG(2.0)+0.01
      P=8.*ATAN(1.)/AN
      IF(IT .EQ. -1)GO TO 180
      DO 170 I=1,N
      X(I)=X(I)/AN
  170 Y(I)=Y(I)/AN
  180 DO 370 L=1,IG
```

```
      IG1=2**(IG-L)
      M=0
      K4=2**(L-1)
      DO 360I=1,K4
      K1=M/IG1
      CALL IMAGE (K1,K2,IG)
      AK=FLOAT(K2)
      Y1=COS(P*AK)
      Y2=-SIN(P*AK) * FLOAT(IT)
      DO 330 J=1,IG1
      M1=M+IG1+1
      Y3=X(M1)*Y1-Y(M1)*Y2
      Y4=X(M1)*Y2+Y(M1)*Y1
      X(M1)=X(M+1)-Y3
      Y(M1)=Y(M+1)-Y4
      X(M+1)=X(M+1)+Y3
      Y(M+1)=Y(M+1)+Y4
  330 M=M+1
      M=M+IG1
  360 CONTINUE
  370 CONTINUE
      DO 480 I=1,N
      K1=I-1
      CALL IMAGE (K1,K2,IG)
      IF (K2-I+1) 420,480,480
  420 Y3=X(I)
      X(I)=X(K2+1)
      X(K2+1)=Y3
      Y3=Y(I)
      Y(I)= Y(K2+1)
      Y(K2+1)=Y3
  480 CONTINUE
      RETURN
      END
```

```
SUBROUTINE IMAGE       74/74    OPT=1                     FTN 4.6+428        77/05/27. 14.12.30        PAGE     1

              SUBROUTINE IMAGE(K1,K2,IG)
        C
        C     THIS PROGRAM IS USED IN CONJUNCTION WITH HFFFT TO PERFORM
        C     THE BIT REVERSAL OF BINARY NUMBERS.  THROUGH THIS PROCESS
        C     THE REAL AND IMAGINARY VALUES ARE PLACED IN THEIR PROPER
        C     LOCATIONS IN THEIR RESPECTIVE ARRAYS.  A BIT REVERSAL
        C     CREATES THE MIRROR IMAGE OF THE NUMBER.  FOR INSTANCE,
        C     THE BINARY NUMBER 001010 BECOMES 010100 AFTER BIT REVERSAL.
        C
              K2= 0
              DO 10 K=1,IG
              K3= K1-K1/2*2
              K1= K1/2
              IF(K3.EQ.0)GO TO 10
              K2= K2+2**(IG-K)
           10 CONTINUE
              RETURN
              END
```

```
                    INPUT DATA
        TIME OFFSET OF FIRST SIGNAL  0.0000
       TIME OFFSET OF SECOND SIGNAL  0.0000
      NUMBER OF DATA POINTS IN FIRST SIGNAL  11
     NUMBER OF DATA POINTS IN SECOND SIGNAL  11
                  DELTA T  1.0000
                ANALYSIS TYPE  1


        ANALYSIS TYPE 1 = CO-VARIANCE
        ANALYSIS TYPE 2 = CROSS-CORRELATION
        ANALYSIS TYPE 3 = SIGNAL DETECTION


   THE FIRST  11 DATA POINTS ARE THE FIRST SIGNAL
        THE REMAINDER ARE THE SECOND SIGNAL
  0.000   1.000   2.000   3.000   4.000   5.000   6.000   7.000   8.000   9.000
 10.000   0.000   1.000   2.000   3.000   4.000   5.000   6.000   7.000   8.000
  9.000  10.000
```

CO-SPECTRUM ANALYSIS

THE SPECTRAL COMPONENTS ARE

FREQ HZ	AMPLITUDE	PHASE DEG
0.000	.949E+02	0.000
.031	.150E+03	0.000
.063	.719E+02	0.000
.094	.211E+02	0.000
.125	.952E+01	0.000
.156	.919E+01	0.000
.188	.577E+01	0.000
.219	.371E+01	0.000
.250	.381E+01	0.000
.281	.299E+01	0.000
.313	.222E+01	0.000
.344	.238E+01	0.000
.375	.210E+01	0.000
.406	.170E+01	0.000
.438	.189E+01	0.000
.469	.182E+01	0.000
.500	.156E+01	0.000

CO-VARIANCE ANALYSIS

TIME	REAL	IMAGINARY
0.000	385.000	-.000
1.000	330.000	-.000
2.000	276.000	-.000
3.000	224.000	-.000
4.000	175.000	.000
5.000	130.000	.000
6.000	90.000	.000
7.000	56.000	.000
8.000	29.000	-.000
9.000	10.000	.000
10.000	.000	-.000
11.000	.000	.000
12.000	.000	.000
13.000	.000	.000
14.000	.000	.000
15.000	.000	.000
16.000	.000	.000
17.000	.000	.000
18.000	.000	.000
19.000	.000	.000
20.000	.000	-.000
21.000	.000	-.000
22.000	.000	.000
23.000	10.000	.000
24.000	29.000	-.000
25.000	56.000	-.000
26.000	90.000	-.000
27.000	130.000	.000
28.000	175.000	-.000
29.000	224.000	-.000
30.000	276.000	-.000
31.000	330.000	-.000

APPENDIX C

APPENDIX C

RELATIONSHIP BETWEEN WAVE HEIGHTS IN A RESERVOIR AND WAVE HEIGHTS IN A CONNECTING CANAL

The change in water surface elevation in a canal with respect to water surface changes in a connecting reservoir can be determined from the equation of continuity.

The discharge through a gate is given by:

$$Q = C_d BH \sqrt{2g(Y_{res} - Y_c)}$$

where

C_d = discharge coefficient
B = width of gate
H = gate opening
g = acceleration of gravity
Y_{res} = reservoir elevation
Y_c = canal water surface elevation

Differentiating this equation in finite terms gives:

$$\begin{aligned}\Delta Q = {} & BH[2g(Y_{res} - Y_c)]^{1/2}\, \Delta C_d \\ & + gC_d BH[2g(Y_{res} - Y_c)]^{-1/2}\, \Delta Y_{res} \\ & + C_d H[2g(Y_{res} - Y_c)]^{1/2}\, \Delta B \\ & + C_d B[2g(Y_{res} - Y_c)]^{-1/2}\, \Delta H \\ & - C_d BHg(2g(Y_{res} - Y_c)]^{-1/2} \Delta Y_c\end{aligned}$$

For fixed gate opening and width:

$$\begin{aligned}\Delta Q = {} & Q\frac{\Delta C_d}{C_d} \\ & + (\Delta Y_{res} - \Delta Y_c)\frac{C_d BHg}{[2g(Y_{res} - Y_c)]^{1/2}} \\ = {} & Q\left[\frac{\Delta C_d}{C_d} + \frac{(\Delta Y_{res} - \Delta Y_c)}{2(Y_{res} - Y_c)}\right]\end{aligned}$$

In the canal, the change in discharge due to a sudden gate movement is:

$$\Delta Q = \Delta Y_c B(V_c + C)$$

where

V_c = canal velocity
C = wave celerity in canal

Equating the changes in discharge gives:

$$Q\left[\frac{\Delta C_d}{C_d} + \frac{\Delta Y_{res} - \Delta Y_c}{2(Y_{res} - Y_c)}\right] = \Delta Y_c B(V_c + C)$$

Solving for ΔY_c gives:

$$\Delta Yc = \frac{\left[\dfrac{\Delta C_d}{C_d} + \dfrac{\Delta Y_{res}}{2(Y_{res} - Y_c)}\right]}{\left[\dfrac{1}{2(Y_{res} - Y_c)} + \dfrac{B(V_c + C)}{Q}\right]}$$

If $B(V_c + C)\,\Delta Y_{res} << Q$, and $\dfrac{\Delta C_d}{C_d} \approx 0$; then, $\Delta Y_c = \Delta Y_{res}$.

Therefore, changes in the reservoir water surface elevation can be employed to provide a forcing function to a canal reach which is connected to the reservoir through a gate. The only two restrictions are:

- The head loss across the gate must be much less than the change in reservoir elevation.
- The percent change in discharge coefficient must be negligible with respect to the percent change in relative reservoir elevation.

GPO 843-018

ABSTRACT

Two methods are available for determining control parameters on automated canals. These are the transient-response method and the frequency-response method. Since a typical canal system is influenced predominantly by transient inputs, the transient-response method has been extensively used in the analysis of automated canal systems. The main advantage of the frequency-response method is that the effects of varying the various control parameters can be readily visualized and evaluated. The presently used transient-response method and parameter selection are reviewed. The basic concepts of the frequency-response method are presented and their application to an automated reach is illustrated with an example. Two computer programs are given which can be used to determine canal response characteristics. The state-of-the-art in process control has progressed sufficiently so that both the transient-response and frequency-response methods of analysis can be used in a complementary fashion. Employment of only one method in the design of automated systems is, in general, wasteful of both computer time and engineering effort.

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