MODELING JOINT RESPONSE UNDER CONSTANT OR VARIABLE NORMAL STIFFNESS BOUNDARY CONDITIONS

by

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INTRODUCTION

The response of a rock joint to shear loading in-situ depends on the joint surface properties but also on the boundary conditions that are applied across the joint surfaces. These boundary conditions can take multiple forms and vary as the rock mass is subject to cycles of loading and unloading. For instance, in rock slope stability, the moving block above a critical joint surface is free to move upward. In this case, the normal stress across the joint remains essentially constant. On the other hand, a block constrained between dilatant joints in the roof or sidewalls of an underground excavation does not move as freely as in the previous case. As the block moves, joint dilation is restricted by the surrounding rock and is controlled by the deformability (or stiffness) of the rock mass. Hence, the normal stress across the joint planes along which sliding takes place, is no longer constant but increases. In general, joint shear strength under increasing normal stress will be different from its shear strength under constant normal stress.

The range of joint normal loading conditions in-situ and the importance of properly modeling rock joint behavior have been emphasized by Goodman (1976), Heuze (1979), Leichnitz (1985) and Goodman and Boyle (1985) among others. Lam and Johnston (1982) also recognized the importance of properly modeling the joint interface between concrete and rock when assessing the side resistance induced in concrete piles in rough rock sockets as the pile is loaded and displaced vertically.

The range of joint normal loading conditions can best be represented by assuming that the deformability of the surrounding rock mass is modelled by a spring with normal stiffness $K = d\sigma_n/dv$ where $d\sigma_n$ and dv are the changes in joint normal stress and displacement, respectively. The stiffness

K varies between zero for a joint under constant normal stress (as in slope stability problems) and infinity if the rock mass is very stiff for which no change in joint normal deformation is allowed. The stiffness is constant if the change in joint normal stress remains proportional to the change in normal displacement. The stiffness may also vary with the load history of the rock mass as it undergoes cycles of loading and unloading.

A large amount of experimental work is available in the literature on the behavior of rock joints under normal loading and unloading and on their shear response under constant normal stress. Knowing this behavior, Goodman (1980) proposed a method to predict the shear response of rock joints under constant normal displacement. Although constant or variable normal stiffness boundary conditions are more likely to exist across joint surfaces in situ rather than constant normal stress or displacement, joint response under constant or variable normal stiffness has not received much attention in the literature. Test data are limited (Leichnitz,1985; Hutson, 1987; Lam and Johnston, 1982) since constant normal stiffness shear tests on rock joints are difficult and require complicated computer controlled equipment. The purpose of this note is to present a simple and complete method to predict the shear response of a dilatant rock joint under constant or variable normal stiffness knowing its behavior under constant normal stress.

PRESENTATION OF THE METHOD

Consider the behavior of a joint under increasing normal stress (with zero shear stress) and its behavior under increasing shear stress (with constant normal stress). The idealized joint response curves shown in Figure 1a, 1b and 1c illustrate this behavior. These curves were proposed by Goodman and Boyle (1985) and are used to introduce the present method.

Figure 1a shows a typical hyperbolic joint closure versus normal stress curve for a joint tested in the laboratory. The joint has a maximum closure V_{mc}. Figure 1b shows the shear stress versus shear displacement curves for the joint under different constant normal stresses varying between A and 20A where A is an arbitrary number. The peak and residual joint shear strengths increase as the joint normal stress increases. Finally, Figure 1c shows the normal displacement versus shear displacement curves for the shear tests of Figure 1b. These curves show a decrease in joint dilatancy as the normal stress increases and assume that, for each level of normal stress, there is no change in joint normal displacement once the peak shear strength has been reached for a shear displacement equal to u_4.

The method consists of using the curves in Figures 1a-1c to plot the variation of the joint normal stress σ_n versus the joint normal displacement v for different values of the shear displacement u. This is shown in Figure 2a for the data of Figure 1. Each curve $u = u_i$ (i=0,4) in Figure 2a is constructed using the values of σ_n and v at the points of intersection between each line $u = u_i$ and the normal displacement versus shear displacement curves in Figure 1c. The following remarks can be made about Figure 2a: (1) The curve $u = u_o$ is identical to the joint closure versus normal stress curve in Figure 1a, (2) All the curves $u = u_i$ (i=1,4) become closer to the curve $u = u_o$ as σ_n increases since joint dilatancy decreases as the joint normal stress increases, (3) For the joint response shown in Figure 1c, for which there is no further dilatancy for values of u larger than u_4, all curves $u = u_i$ ($i > 4$) coincide with the curve $u = u_4$, hence, the joint response is admissible if it is contained in the domain limited by the curves $u = u_0$ and $u = u_4$, (4) Each curve $u = u_i$ represents the behavior of the joint under normal loading after being mismatched by a shear displacement equal to u_i.

Figures 2a and 1b can be used to predict the shear strength of the joint for any load path. Starting with zero normal stress, assume that a normal stress $\sigma_{no} = 4A$ is first applied across the joint without any shearing. The joint closes and follows path OA in Figure 2a. Then, as shearing occurs, the joint can follow different paths depending on the boundary conditions across its surfaces. Under constant normal stress, the joint follows path ABCDE. It follows path AFGHI under constant normal stiffness K and path AJKLM when no change in joint normal displacement is allowed. In Figure 2a, the constant normal stress path (K=0) and the constant displacement path ($K=\infty$) clearly appear as two special cases of the constant normal stiffness path.

The shear stress versus shear displacement curves for the three paths mentioned above can be constructed by first recording in Figure 2a the values of σ_n and u at the points of intersection of each path with the curves $u = u_i$ (i=0,4). Then, these values are transferred to the family of shear stress versus shear displacement curves in Figure 1b. For the present example, the shear stress versus shear displacement curve for the constant normal stress path corresponds to the curve labelled 4A in Figure 1b. The curves for the constant normal displacement and stiffness paths are shown as dashed lines in Figure 2b. The method illustrated in Figure 2a can also be used to predict the variation of the joint normal displacement and normal stress versus shear displacement for each one of the three paths. This is shown as dashed lines in Figures 2c and 2d, respectively.

In view of Figures 2a-2d, the constant normal stress path leads to the lowest peak shear strength (point E) whereas the constant normal displacement path leads to the highest peak shear strength (point M) due to an increase in joint normal stress. For the constant normal stiffness path, both joint normal displacement and normal stress increase during shearing. This results in a peak shear strength lying in between the two previous extremes (point I).

Figure 2a is not limited to joints with constant normal stiffness. The stiffness may vary and the path in Figure 2a may be stepwise or non-linear such as path ANPQR corresponding to a joint in a rock mass with an increasing normal stiffness. The corresponding shear stress versus shear displacement curve, the normal displacement versus shear displacement curve and the normal stress versus shear displacement curve for σ_{no} = 4A are also shown in Figures 2b,2c and 2d, respectively.

The previous example shows that joint shear response is controlled by its dilatant behavior. The same method was applied for a more dilatant joint for which the normal displacement versus shear displacement curves in Figure 3 were substituted for those in Figure 1c. Figure 2a is now replaced by Figure 4a. Compared to Figure 1c, the new set of dilation curves is such that joint dilation continues beyond the shear displacement u_4 necessary to mobilize the peak shear strengths in Figure 1b. Also, for shear displacements larger than u_{11}, the joint ceases to dilate and contracts. Because of this behavior, the response curves $u = u_i$ ($i > 11$) in Figure 4a will always be below the curve $u = u_{11}$. Indeed, for the present example, curves $u = u_7$ and $u = u_{13}$ coincide. The same applies to curves $u = u_9$ and $u = u_{12}$.

For the dilation curves in Figure 3, the shear stress versus shear displacement curves for the constant normal displacement path (AJKLM...), the constant normal stiffness path (AFGHI...) and the variable stiffness path (ANPQR...) of Figure 4a are shown in Figure 4b. Compared to the shear response for the constant normal stress path ABCDE... (curve σ_n = 4A in Figure 4b), these curves show two distinct peak shear strengths. The second peak is associated with joint dilation for shear displacements less than u_{11} followed by joint contraction. The normal displacement versus shear displacement and the normal stress versus shear displacement response curves for the different paths are shown in Figures 4c and 4d, respectively.

SUMMARY

A simple method is presented in this technical note. It can be used to predict the response of a dilatant rock joint under a wide variety of loading conditions such as constant normal stress, constant normal displacement and constant or variable normal stiffness. Constant normal stiffness tests on rock joints that require complicated servo-controlled equipment do not have to be conducted since, using the proposed method, the response of joints under constant or variable stiffness can be derived from the results of constant normal stress tests.

The response of a rock joint under constant or variable stiffness is strongly dependent on its dilatant behavior. It is shown that a second peak in the shear stress versus shear displacement curve can occur under such boundary conditions. This second peak strength takes place beyond the peak strength for the constant normal stress path. This has been observed experimentally by Lam and Johnston (1982) in the testing of artificial joints under constant normal stiffness (Figure 5). The existence of two peak shear strengths suggests that more than one set of strength parameters is needed to describe the shear strength of dilatant rock joints under constant or variable normal stiffness boundary conditions.

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LIST OF FIGURES

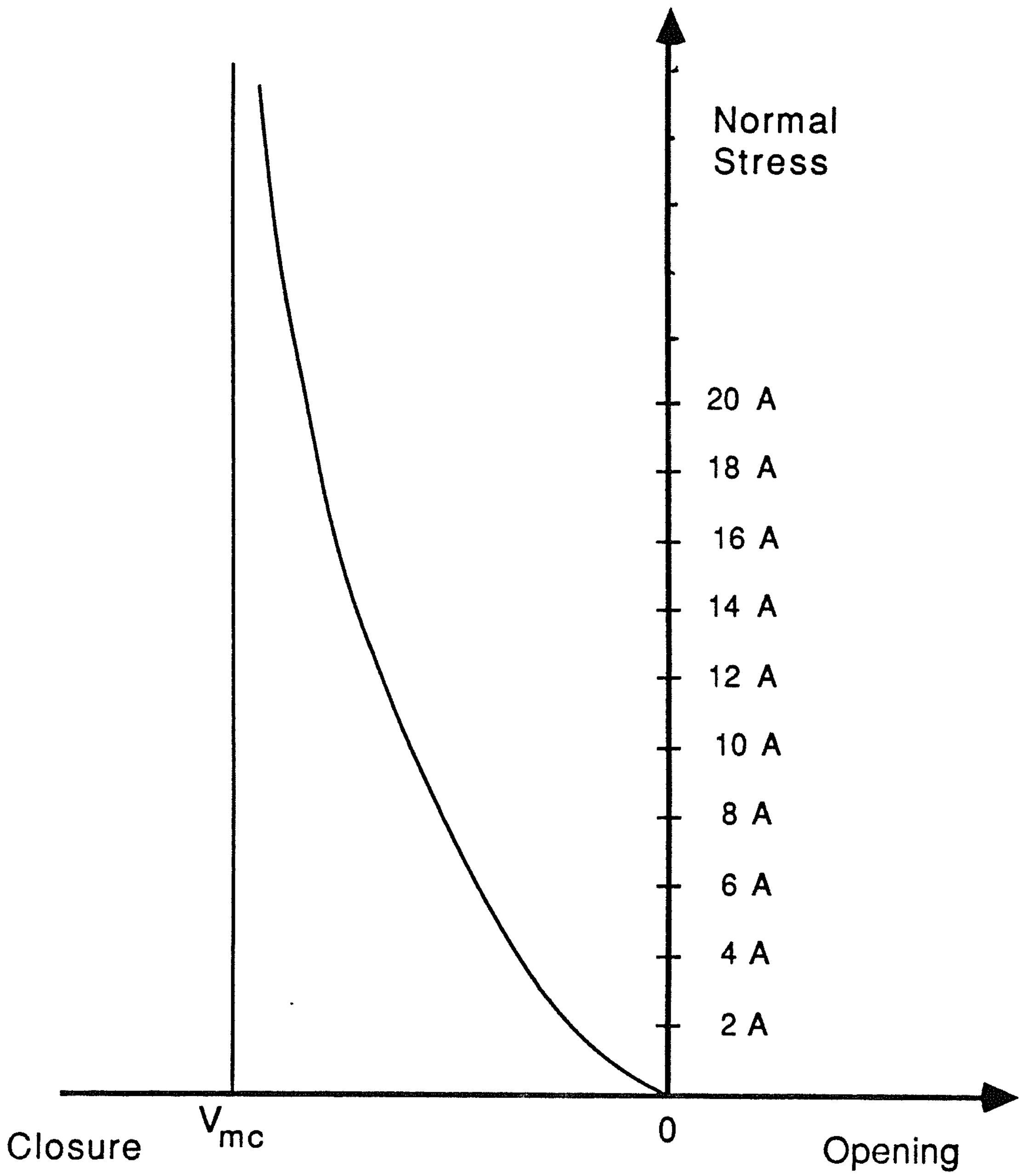
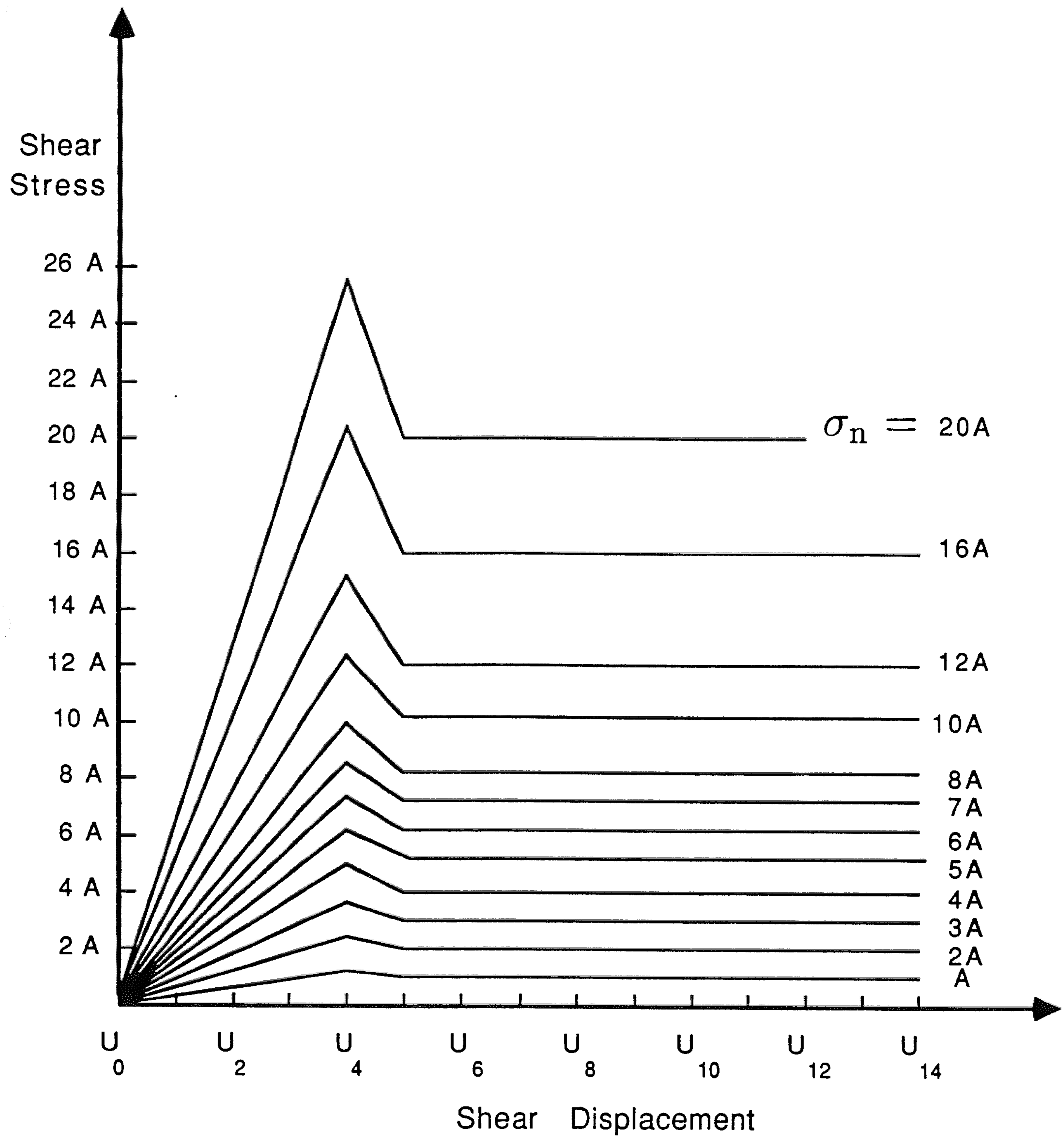
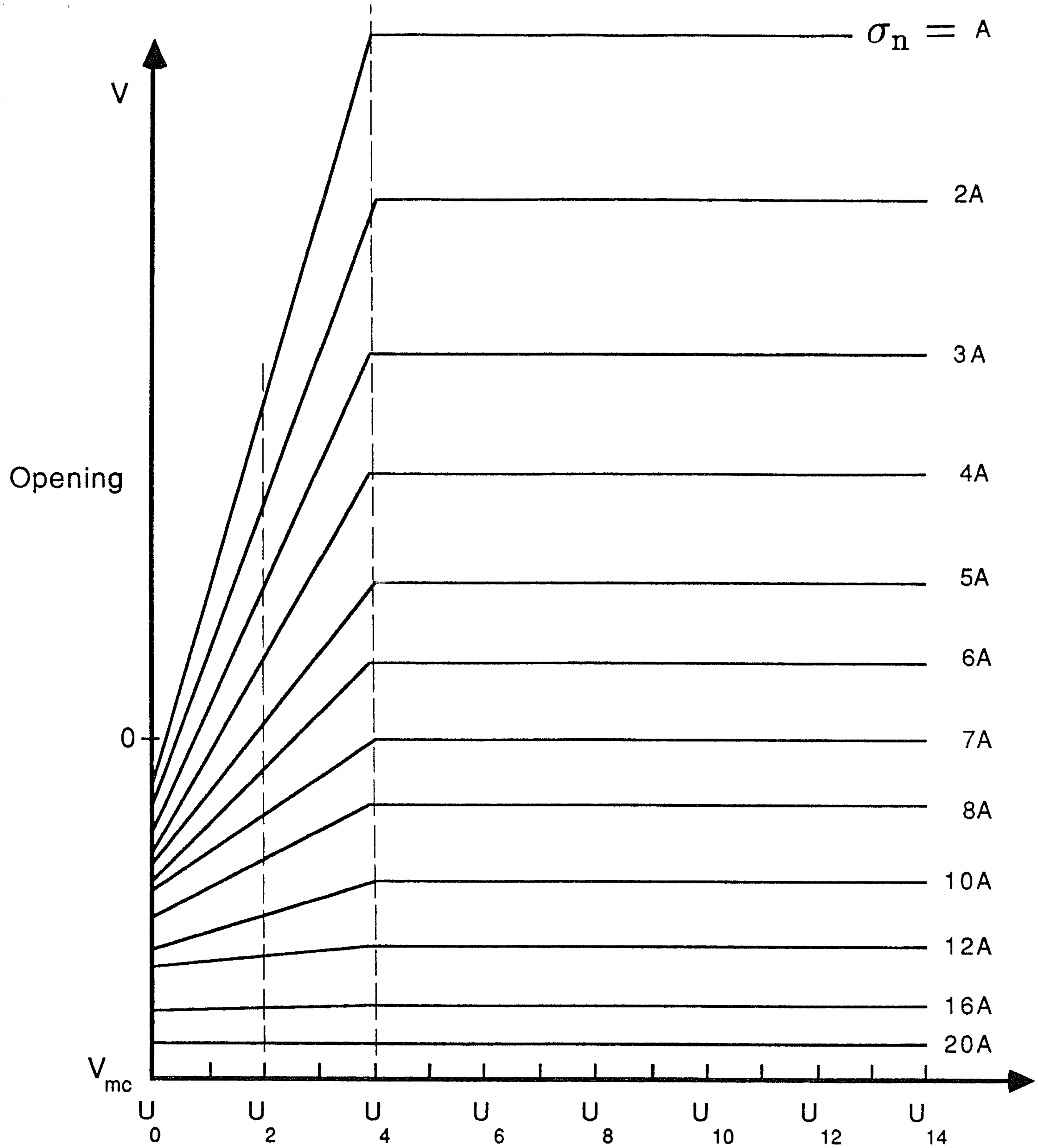

Normal
Stress
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
Closure
V_{mc}
0
Opening

FIGURE 1a



Shear
Stress
26 A
24 A
22 A
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
σ_n = 20A
16A
12A
10A
8A
7A
6A
5A
4A
3A
2A
A
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}
Shear Displacement

FIGURE 1b



$\sigma_n =$ A
2A
3A
4A
5A
6A
7A
8A
10A
12A
16A
20A
V
Opening
0
V_{mc}
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}

FIGURE 1c

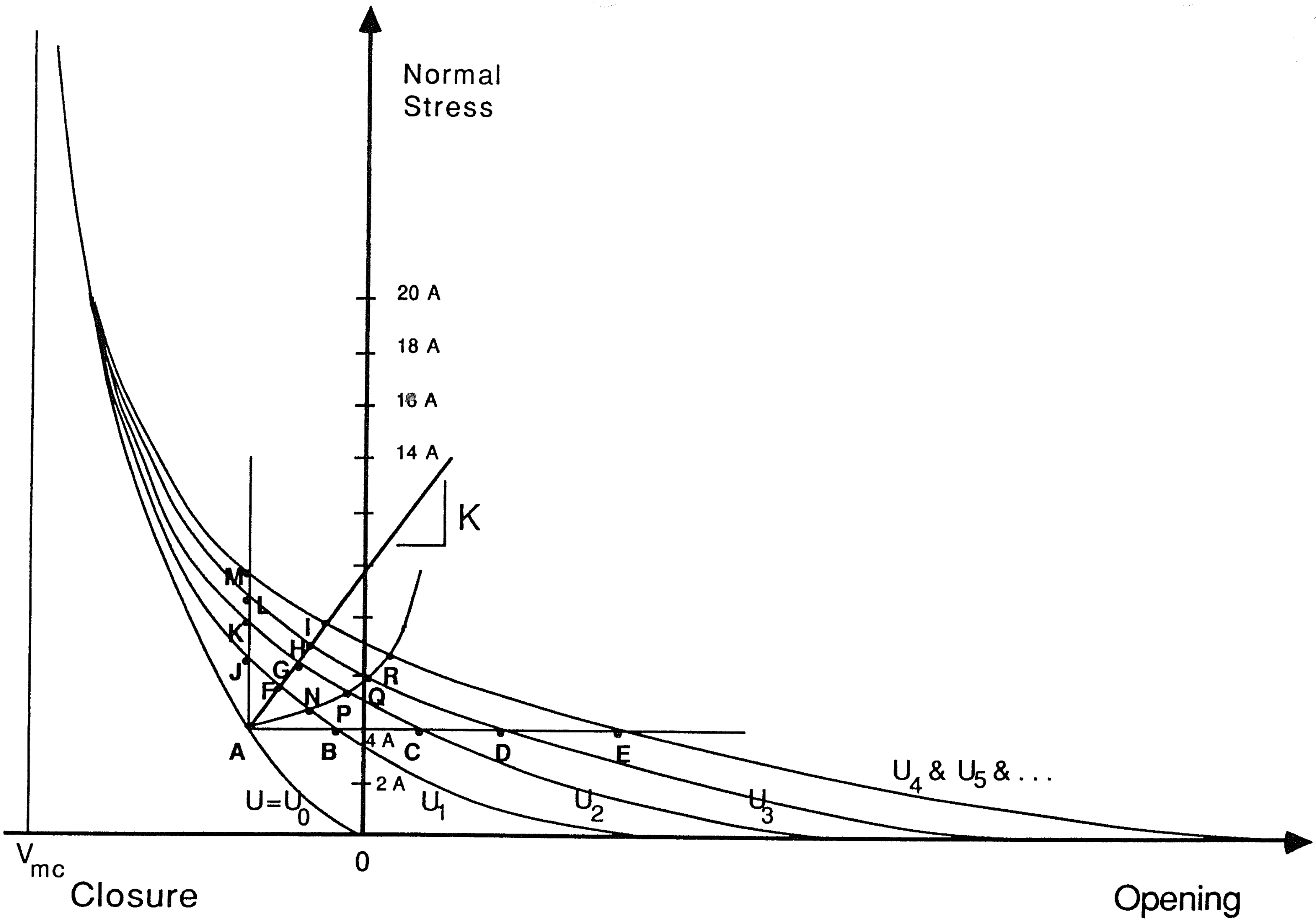

Normal
Stress
20 A
18 A
16 A
14 A
K
M
L
K
J
I
H
G
F
N
P
Q
R
A
B
4 A
C
D
E
2 A
U=U0
U1
U2
U3
U4 & U5 & . . .
Vmc
Closure
0
Opening

FIGURE 2a

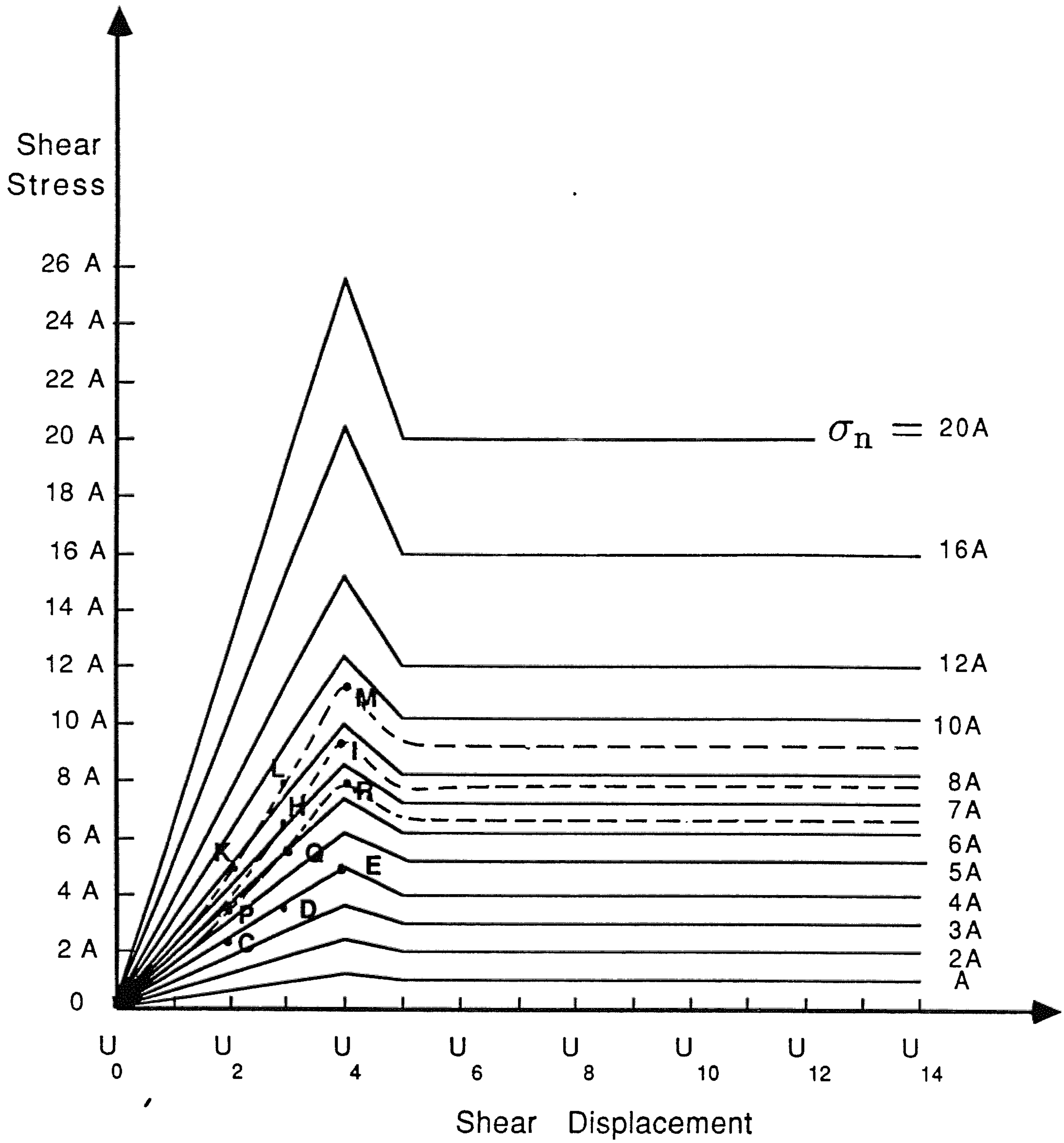
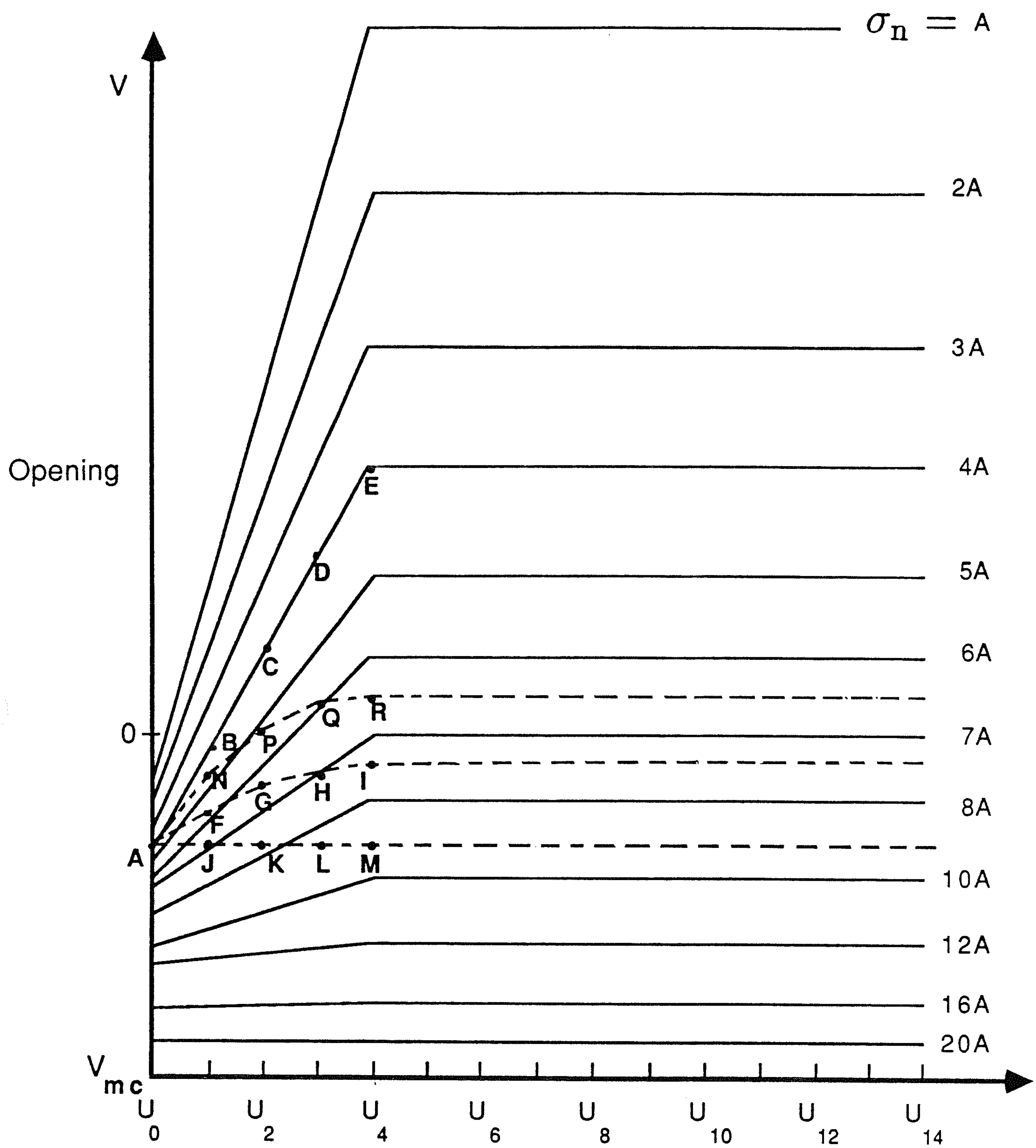
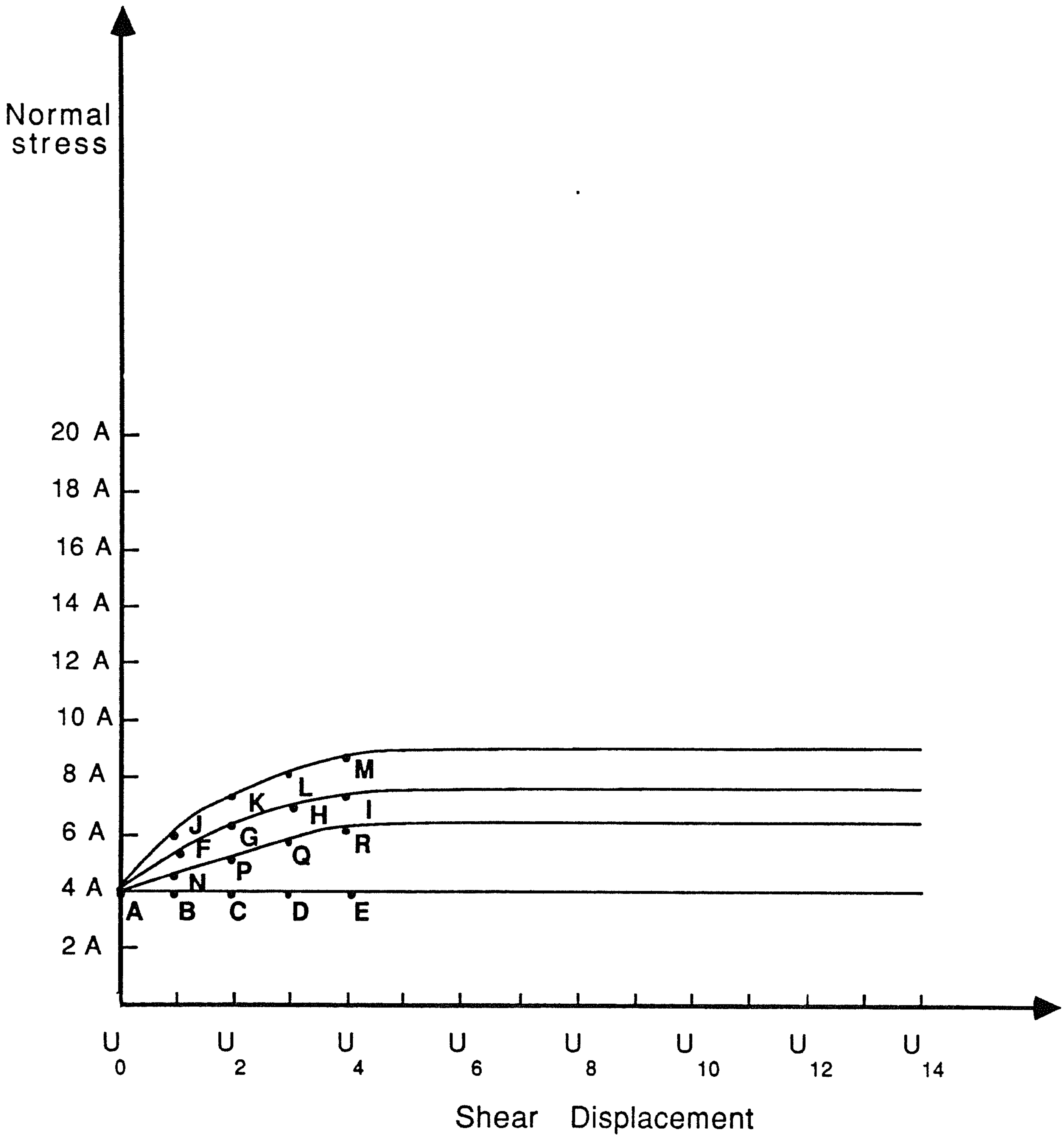

Shear Stress
26 A
24 A
22 A
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
0
σ_n = 20A
16A
12A
10A
8A
7A
6A
5A
4A
3A
2A
A
M
I
R
L
H
K
Q
E
P
D
C
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}
Shear Displacement

FIGURE 2b



$\sigma_n = A$
2A
3A
4A
5A
6A
7A
8A
10A
12A
16A
20A
V
Opening
0
V_{mc}
A
B
C
D
E
F
G
H
I
J
K
L
M
N
P
Q
R
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}

FIGURE 2c



Normal stress
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
M
L
K
J
H
I
G
F
R
Q
P
N
A
B
C
D
E
U 0
U 2
U 4
U 6
U 8
U 10
U 12
U 14
Shear Displacement

FIGURE 2d

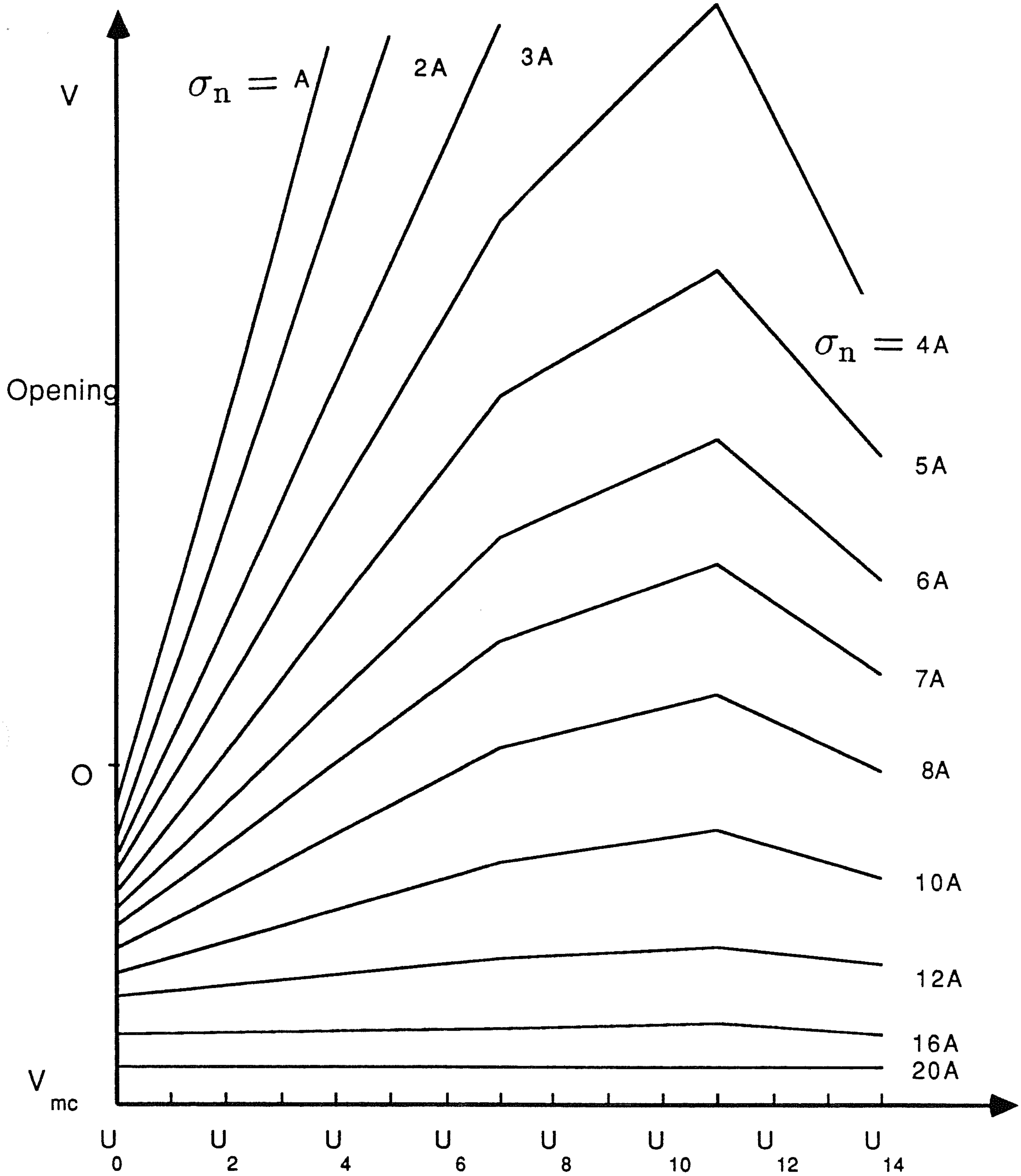

V
$\sigma_n =$ A
2A
3A
$\sigma_n =$ 4A
Opening
5A
6A
7A
8A
O
10A
12A
16A
20A
V_{mc}
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}

FIGURE 3

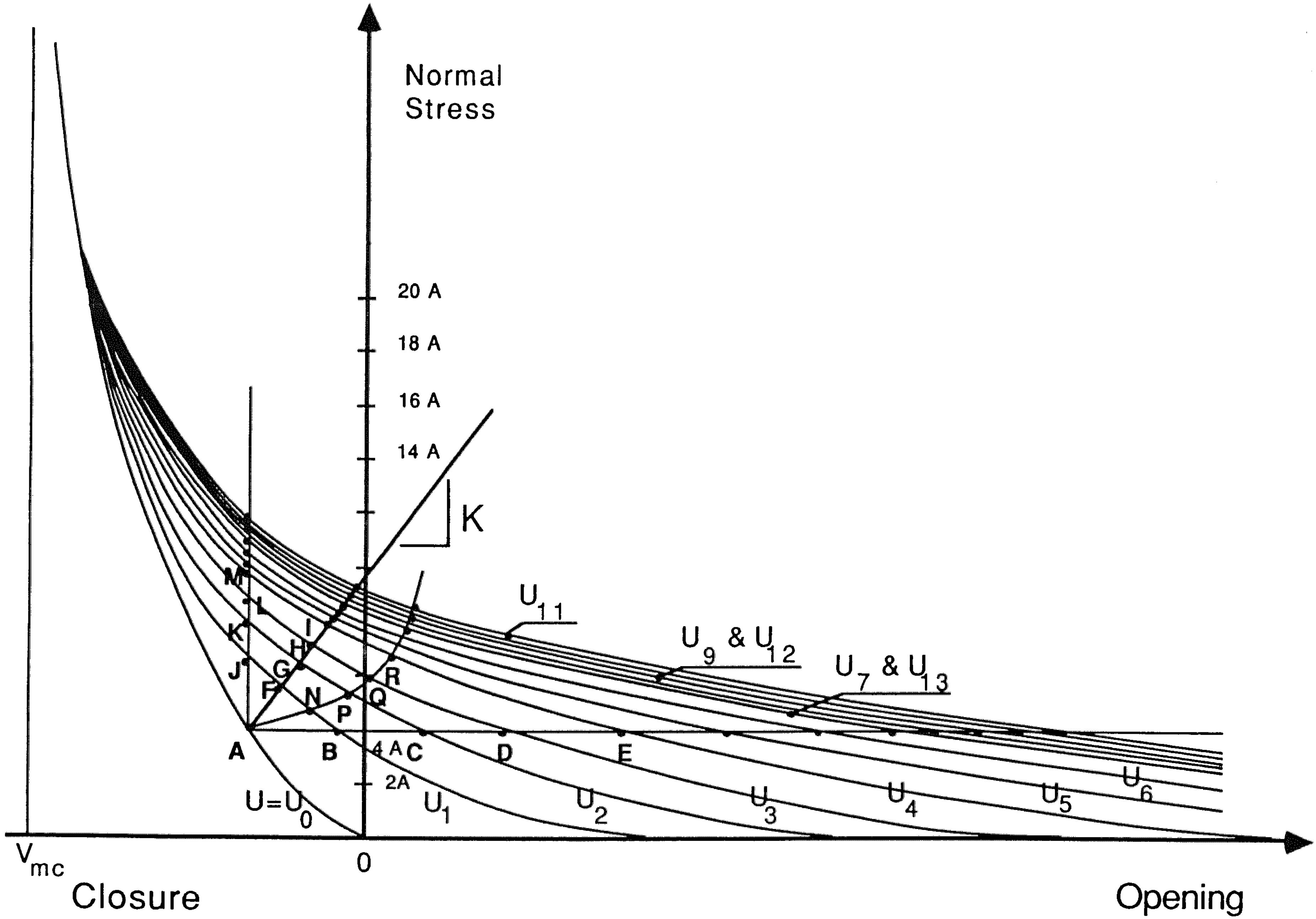

Normal
Stress
20 A
18 A
16 A
14 A
K
U_{11}
U_9 & U_{12}
U_7 & U_{13}
M
L
K
J
I
H
G
F
N
P
Q
R
A
B
4 A
C
D
E
2A
$U=U_0$
U_1
U_2
U_3
U_4
U_5
U_6
V_{mc}
0
Closure
Opening

FIGURE 4a

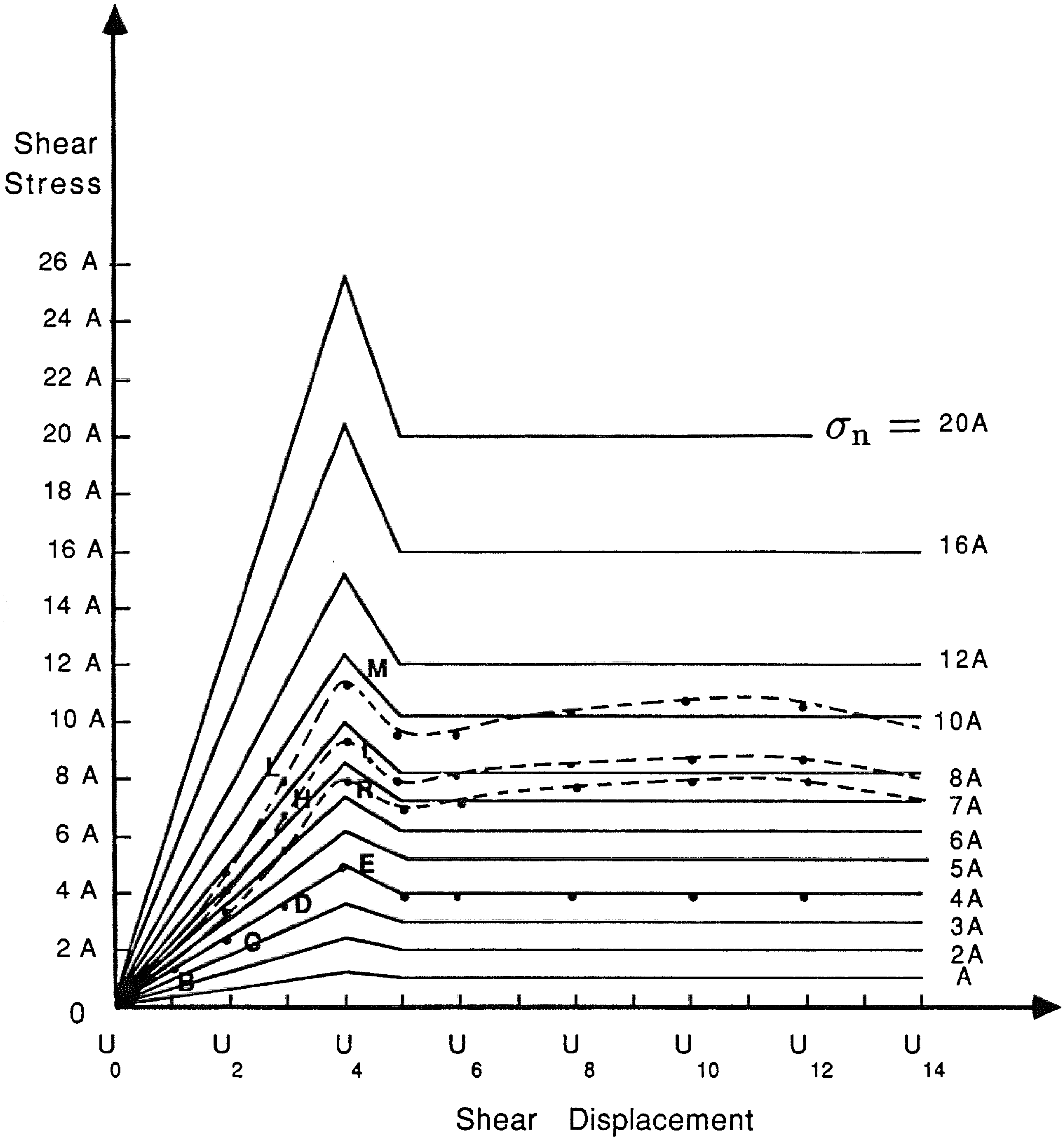
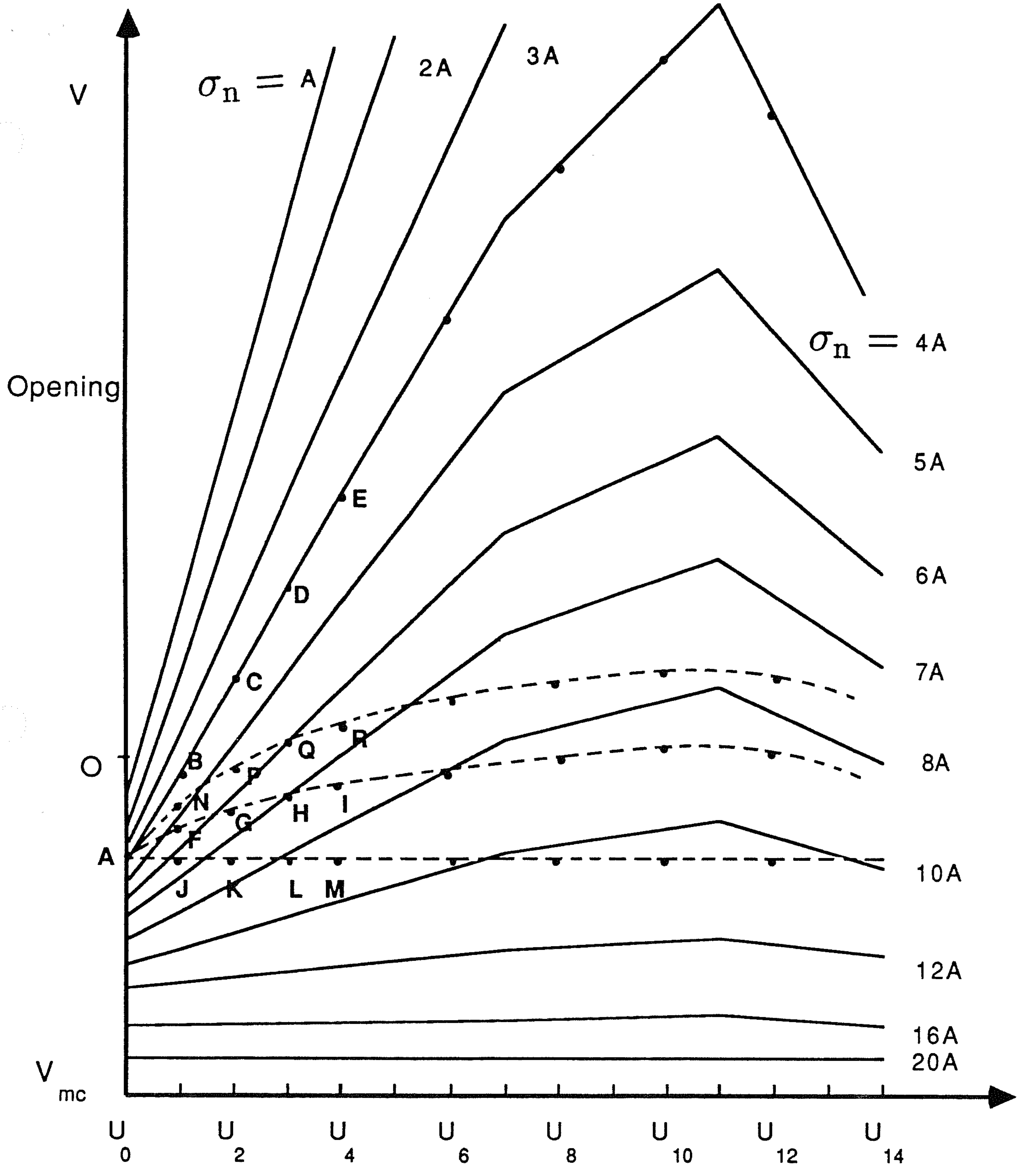

Shear Stress
26 A
24 A
22 A
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
0
σ_n = 20A
16A
12A
10A
8A
7A
6A
5A
4A
3A
2A
A
M
L
H
R
E
D
C
B
U 0
U 2
U 4
U 6
U 8
U 10
U 12
U 14
Shear Displacement

FIGURE 4b



V
$\sigma_n = A$
2A
3A
$\sigma_n = 4A$
5A
6A
7A
8A
10A
12A
16A
20A
Opening
O
A
B
C
D
E
F
G
H
I
J
K
L
M
N
P
Q
R
V_{mc}
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}

FIGURE 4c

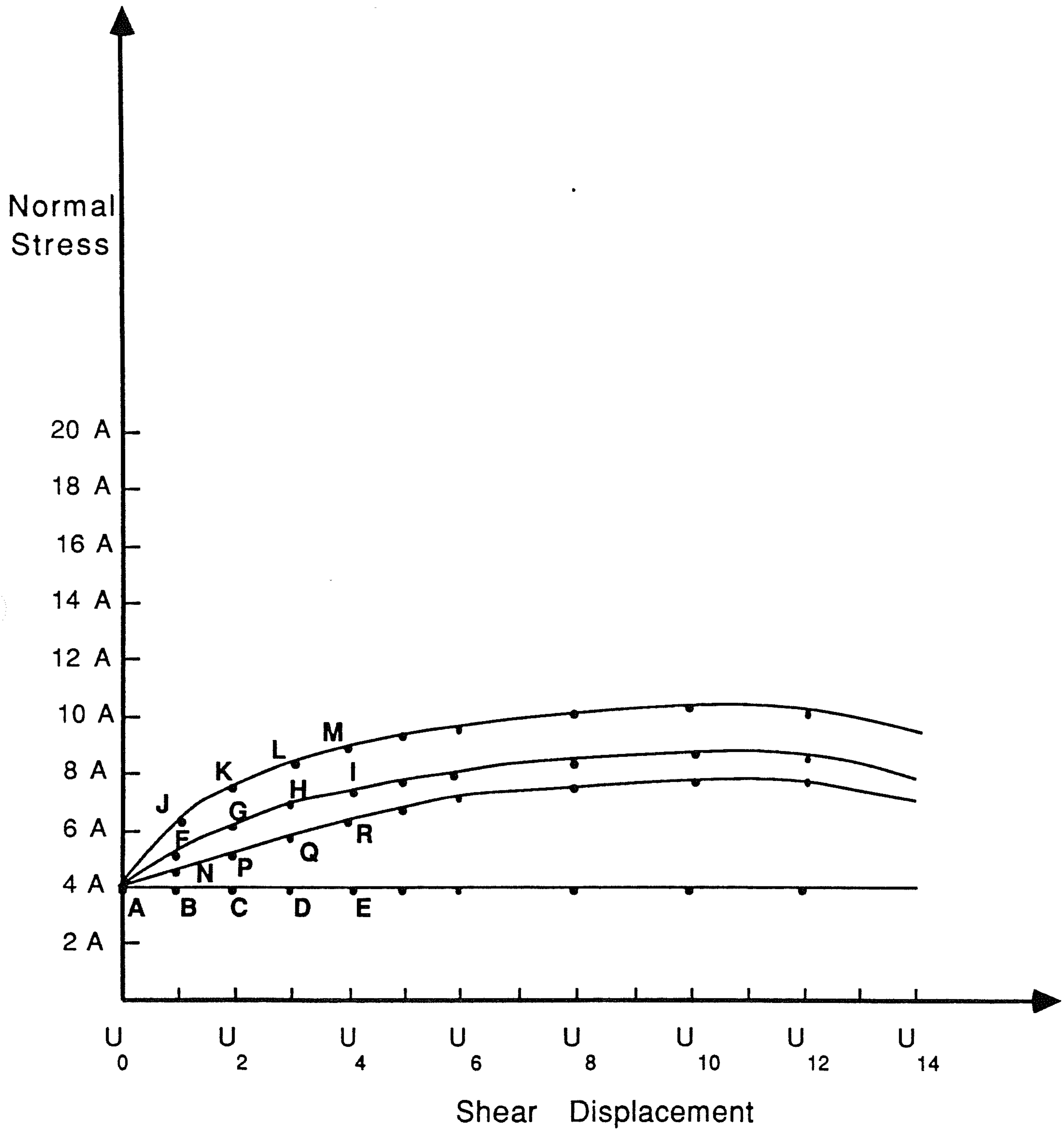
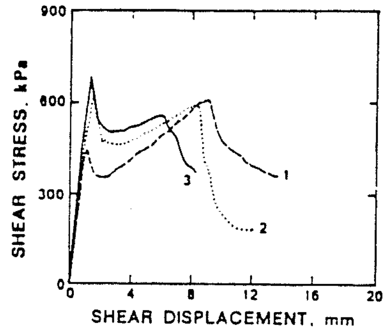
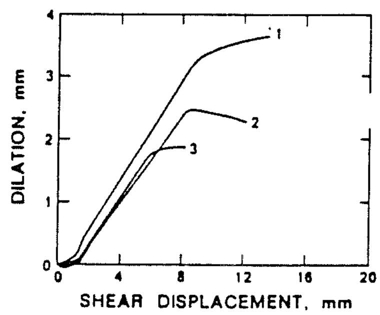

Normal
Stress
20 A
18 A
16 A
14 A
12 A
10 A
8 A
6 A
4 A
2 A
A
B
C
D
E
F
G
H
I
J
K
L
M
N
P
Q
R
U_0
U_2
U_4
U_6
U_8
U_{10}
U_{12}
U_{14}
Shear Displacement

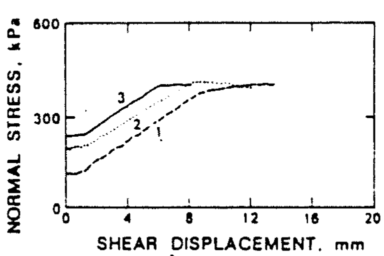
FIGURE 4d


SHEAR STRESS, kPa
900
600
300
0
1
2
3
0
4
8
12
16
20
SHEAR DISPLACEMENT, mm

a.


DILATION, mm
4
3
2
1
0
1
2
3
0
4
8
12
16
20
SHEAR DISPLACEMENT, mm

b.


NORMAL STRESS, kPa
600
300
0
3
2
1
0
4
8
12
16
20
SHEAR DISPLACEMENT, mm

c.

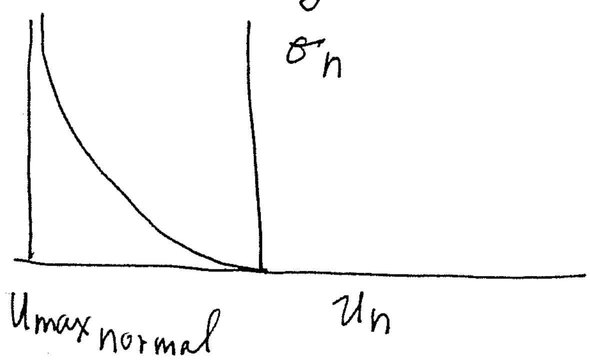
FIGURE 5

Slightly Rough		Starting Normal Stress
E1	78.7	250
E1	291.7	500
E1	301.9	1000
E1	304.2	1500

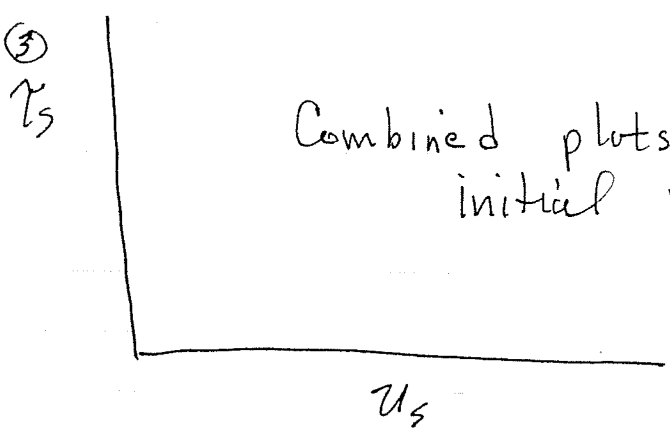
Tests

① Profile each joint

②



③



Combined plots by groups but initial normal stress values only.

④