REC-ERC-83-11

# SEEPAGE ANALYSIS USING THE BOUNDARY ELEMENT METHOD

May 1984

Engineering and Research Center


U. S. Department of the Interior

Bureau of Reclamation

7-2090 (4-81)
Bureau of Reclamation

# TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.
REC-ERC-83-11

2. GOVERNMENT ACCESSION NO.

3. RECIPIENT'S CATALOG NO.

4. TITLE AND SUBTITLE
**Seepage Analysis Using the Boundary Element Method**

5. REPORT DATE
May 1984

6. PERFORMING ORGANIZATION CODE

7. AUTHOR(S)
**Ashok K. Chugh and Henry T. Falvey**

8. PERFORMING ORGANIZATION REPORT NO.
**REC-ERC-83-11**

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Engineering and Research Center
Bureau of Reclamation
Denver CO 80225

10. WORK UNIT NO.

11. CONTRACT OR GRANT NO.

12. SPONSORING AGENCY NAME AND ADDRESS
Same

13. TYPE OF REPORT AND PERIOD COVERED

14. SPONSORING AGENCY CODE
DIBR

15. SUPPLEMENTARY NOTES
Microfiche or hard copy available at the Engineering and Research Center, Denver, Colorado.

Ed: RNW

16. ABSTRACT
A simple and effective extension of the boundary element method for solving basic equation of steady flow in isotropic soil is presented to solve steady confined or steady unconfined flow problems or both, in zoned anisotropic mediums. A computer program listing in FORTRAN IV and a set of user's instructions are included to facilitate the use of the computer program. Only steady-state seepage problems in two-dimensional domains with general distribution of piecewise homogeneous material zones are considered. Sample problems illustrating the use of the computer program and the accuracy of results are given.

17. KEY WORDS AND DOCUMENT ANALYSIS
*a. DESCRIPTORS*--/computer models/ mathematical models/ boundary conditions/ computer programming/ groundwater movement/ steady flow/ dams/ foundations/ seepage/ *boundary element method (2011).

*b. IDENTIFIERS*-- Foss Dam

*c. COSATI Field/Group* 08M *COWRR*: 2011 *SRIM*:

18. DISTRIBUTION STATEMENT
*Available from the National Technical Information Service, Operations Division, 5285 Port Royal Road, Springfield, Virginia 22161.*
Microfiche or hard copy available from NTIS.

19. SECURITY CLASS (THIS REPORT)
*UNCLASSIFIED*

20. SECURITY CLASS (THIS PAGE)
*UNCLASSIFIED*

21. NO. OF PAGES
55

22. PRICE

REC-ERC-83-11

# SEEPAGE ANALYSIS USING THE BOUNDARY ELEMENT METHOD

by

Ashok K. Chugh

and

Henry T. Falvey

May 1984

Dams Branch — Division of Design
Hydraulics Branch — Division of Research and Laboratory Services
Engineering and Research Center
Denver, Colorado

SI METRIC

UNITED STATES DEPARTMENT OF THE INTERIOR ★ BUREAU OF RECLAMATION

# CONTENTS



## TABLES



## FIGURES

## CONTENTS – Continued

## LETTER SYMBOLS AND QUANTITIES

| | |
|---|---|
| $b$ | Constant |
| $d$ | Distance from a base point to a field point |
| $g$ | Hydraulic gradient |
| $h$ | Total head |
| $i$ | Hydraulic gradient |
| $i, j, k$ | Orthogonal directions for unit vectors |
| $k_x$ | Permeability in $x$ direction |
| $k_y$ | Permeability in $y$ direction |
| $k_z$ | Permeability in $z$ direction |
| $m$ | Number of interface boundary elements |
| $n$ | Direction normal to the boundary of the flow domain |
| $q$ | Boundary flux; rate of seepage flow |
| $r$ | Radial index from center of drain envelope |
| $u$ | Piezometric head or potential head |
| $v$ | Any differentiable vector |
| $x, y, z$ | Orthogonal coordinates |
| $A$ | Coefficient matrix |
| $A_s$ | Cross sectional area |
| $B$ | Vector of knowns |
| $B_1$ | Prescribed head boundary |
| $B_2$ | Prescribed gradient boundary |
| $C$ | Circumferential path |
| $D$ | Flow domain |
| $F_1$ | Free surface boundary |
| $F_2$ | Seepage face boundary |
| $G, H$ | Matrices obtained through integration of equation (27) |
| $I$ | Interface boundary element |
| $M$ | Number of zones |
| $N$ | Number of node points |
| $P$ | Singular point or "base point" |
| $Q$ | Point on the boundary or "field point" |
| $R$ | Radial line |
| $S$ | Surface surrounding the domain |
| $U, V$ | General functions which satisfy Laplace equation $\nabla^2 U = \nabla^2 V = 0$ |
| $X$ | Vector of unknowns |
| $Y$ | Elevation head |
| $Z$ | Elevation of free surface |
| $\alpha$ | Angle between boundary segments, alpha |
| $\delta$ | Dirac delta function, delta |
| $\epsilon$ | Radius of circle, epsilon |
| $\pi$ | 3.141 592 65, pi |
| $\tau$ | Volume, tau |
| $\underset{\sim}{\nabla}\cdot$ | Vector operator $\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$ |
| $\nabla^2$ | Laplace operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ |
| $\theta$ | Angle index of radial line with vertical |

### Subscripts

| | |
|---|---|
| 1,2 | Zone number |
| $ij$ | Second order tensor |
| $i, j$ | Dummy indices |
| $x, y, z$ | Orthogonal coordinate directions |
| $T$ | Transformed coordinates |
| ln | Natural logarithm |
| ~ | Vector notation |
| { } | Vector notation |
| [ ] | Matrix notation |
| $\Omega$ | Boundary of the flow domain |
| • | Dot product |

### Superscript

| | |
|---|---|
| e | Boundary element |

## INTRODUCTION

The *boundary element method* for solving boundary value problems in engineering sciences is gaining acceptance by the practicing engineers because of simplicity, effectiveness, and accuracy offered by the method as compared with other numerical methods. This popularity and acceptance of the boundary element method is evidenced by the publication of several books and research papers on the subject [1, 2, 3, 4, 5, 6, 7, 8][1]. Listings of several computer programs to solve simple engineering problems illustrate the simplicity of computer implementation of the boundary element method [2, 3]. Availability of computer programs for solving relatively more complex problems is limited.

The boundary element method is especially well adapted in applications in which the Laplace equation needs to be solved in an irregular region but results are needed primarily on the boundary. One class of problems which is described by the Laplace equation is the phenomenon of steady flow in isotropic (same permeability in all directions) and homogeneous (same permeability at all points) soil. This problem is mathematically described by:

$$\nabla^2 u = 0 \tag{1}$$

where

$\nabla^2$ is the Laplace operator

In three dimensions,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

In two dimensions,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$u$ is the potential head.

The governing differential equation for steady flow in an anistropic, homogeneous soil is:

$$k_x \frac{\partial^2 u}{\partial x^2} + k_y \frac{\partial^2 u}{\partial y^2} + k_z \frac{\partial^2 u}{\partial z^2} = 0 \tag{2}$$

in three dimensions, and

$$k_x \frac{\partial^2 u}{\partial x^2} + k_y \frac{\partial^2 u}{\partial y^2} = 0 \tag{3}$$

in two dimensions

where

$k_x$, $k_y$, and $k_z$ are the soil permeabilities in orthogonal directions $x$, $y$, and $z$.

A few practical examples of flow in porous media are:

- Steady-confined seepage through the foundation zones of an impervious dam,
- Steady-unconfined seepage through a pervious earth dam on an impervious foundation, and
- The combined case of seepage through a pervious earth dam on a pervious foundation.

The potential and the gradient on interface boundary, the potential or the gradient on exterior boundary, and the potential on specified set of interior points are generally desired.

The objectives of this report are to:

- Present a simple and effective extension of the available boundary element method for solving the Laplace's equation ($\nabla^2 u = 0$) in a bounded region to study seepage problems in zoned anisotropic soil deposits.
- Present a computer program listing in FORTRAN IV and a set of user's instructions.
- Illustrate the use of the computer program and accuracy of numerical results with sample problems.

## CONCLUSIONS

1. An effective extension of existing procedures was devised to use the *boundary element method* for the solution of seepage problems in a piecewise homogeneous (zoned) anisotropic medium. An iterative procedure was developed to locate the phreatic path through the body of an embankment dam subjected to a reservoir head on the upstream face and to a tailwater head on the downstream face.

2. A computer procedure was modified to solve planar seepage problems under confined and (or) unconfined steady-state flow conditions through a zoned anisotropic medium. The method uses constant boundary elements.

3. The computer procedure included here not only reproduces the results of known analytic solutions but it also gives results which compare favorably with the measured response.

4. Use of the boundary element method provides an effective and efficient numerical procedure for performing seepage studies in zoned anisotropic mediums. The

---

[1] Numbers in brackets refer to the bibliography.

Bibliographical references are representative, but not a complete list, of the works on the subject.

greater savings lie in data preparation for the boundary element analyses as compared with the conventional finite element analyses. The savings in total computational cost can also be appreciable.

5. The boundary element method also avoids a number of difficulties—that plague the finite element method—in relation to adjustment of mesh size and shape during iteration for unconfined flow problems.

## THEORY

### Introduction

In the boundary element method, use is made of approximating functions that satisfy the governing equations in the domain but not the boundary conditions. The approximating functions can be the fundamental solution to the governing equation or other simple functions that satisfy the governing equation. These functions have some unknown coefficients which then are determined by enforcing the boundary conditions at a number of points on the boundary.

The boundary element method—through its formulation strategy—reduces the dimensionality of the problem by one. Thus, a two-dimensional problem, in effect, is treated as if it were one dimensional; that is, two-dimensional problems are solved by integration over a line. All problem data required for a complete solution pertain to the boundary of the numerical model. The results at any point(s) within the domain can be obtained by identifying its location.

As an example, consider a typical two-dimensional, free-surface problem of flow through an earth dam as shown on figure 1. Neglecting the capillary and surface tension, the flow domain $D$ has the following types of boundaries [5]:

- A prescribed head boundary, $B_1$
- A prescribed gradient boundary, $B_2$
- A free-surface, $F_1$
- A seepage face, $F_2$

This boundary value problem can be described by the following set of equations:

$$\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial u}{\partial x_j}\right) = 0 \qquad \text{on } D \tag{4}$$

$$u = h \qquad \text{on } B_1 \tag{5}$$

$$k_{ij}\frac{\partial u}{\partial x_j} n_i = -b \qquad \text{on } B_2 \tag{6}$$

$$u = Z \tag{7}$$

$$k_{ij}\frac{\partial u}{\partial x_j} n_i = 0 \tag{8}$$

(7) and (8) on $F_1$

$$u = Y \qquad \text{on } F_2 \tag{9}$$

where

$u$ is the piezometric or the potential head[2]

$k_{ij}$ is the permeability tensor

$Z$ is the elevation of the free surface above the horizontal datum from which head is measured.

Differential equation (4) can be transformed into an integral equation by means of the fundatmental Green's functions and Green's second formula [9]. A brief description of the procedure is included here and is based on references [5, 10, 11, 12].

### Basic Formulation

An expression of continuity in a volume is the *divergence theorem.* It states that the volume integral of the divergence of a vector field, taken throughout a bounded domain $D$, equals the surface integral of the normal component of the vector field taken over the boundary of $D$. The mathematical description of the divergence theorem is [10, 11, 12]:

$$\int_D (\underset{\sim}{\nabla}\cdot\underset{\sim}{v})\, d\tau = \int_\Omega \underset{\sim}{v}\cdot\underset{\sim}{n}\; dS \tag{10}$$

in which

$\underset{\sim}{\nabla}$ is the vector operator $\frac{\partial}{\partial x}\underset{\sim}{i} + \frac{\partial}{\partial y}\underset{\sim}{j} + \frac{\partial}{\partial z}\underset{\sim}{k}$

$\underset{\sim}{v}$ is any differentiable vector

$D$ is the domain of integration, a volume in three dimensions, an area in two dimensions

$d\tau$ is the element of volume

$\Omega$ is the boundary of $D$

$\underset{\sim}{n}$ is the unit outward vector normal to $D$ on $\Omega$

$dS$ is the element of surface area

[2] The energy per unit force has the dimension of length. Thus, the terms making up the total energy are characterized as heads. The total head is the summation of a velocity head, a pressure head, and an elevation head. The velocity head in soils is negligible. Because the pressure head and elevation head represent potential energy, their sum is called the potential head.

First, $\underset{\sim}{v}$ is defined as $U \underset{\sim}{\nabla} V$, where $U$ and $V$ are any two functions, twice differentiable in $D$.

Thus

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{v} = \underset{\sim}{\nabla} \cdot U \underset{\sim}{\nabla} V$$

$$= \left[\frac{\partial}{\partial x}\underset{\sim}{i} + \frac{\partial}{\partial y}\underset{\sim}{j} + \frac{\partial}{\partial z}\underset{\sim}{k}\right] \left[U \frac{\partial V}{\partial x}\underset{\sim}{i} + U \frac{\partial V}{\partial y}\underset{\sim}{j} + U \frac{\partial V}{\partial z}\underset{\sim}{k}\right]$$

$$= \frac{\partial U}{\partial x}\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}\frac{\partial V}{\partial y} + \frac{\partial U}{\partial z}\frac{\partial V}{\partial z} + U\frac{\partial^2 V}{\partial x^2} + U\frac{\partial^2 V}{\partial y^2} + U\frac{\partial^2 V}{\partial z^2}$$

$$= \underset{\sim}{\nabla} U \cdot \underset{\sim}{\nabla} V + U \nabla^2 V \tag{11}$$

and equation (10) becomes:

$$\int_D (\underset{\sim}{\nabla} U \cdot \underset{\sim}{\nabla} V + U \nabla^2 V)\, d\tau = \int_\Omega U \underset{\sim}{\nabla} V \cdot \underset{\sim}{n}\, dS \tag{12}$$

Secondly, $\underset{\sim}{v}$ is defined as $V \underset{\sim}{\nabla} U$, so that

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{v} = \underset{\sim}{\nabla} V \cdot \underset{\sim}{\nabla} U + V \nabla^2 U \tag{13}$$

and equation (10) becomes:

$$\int_D (\underset{\sim}{\nabla} V \cdot \underset{\sim}{\nabla} U + V \nabla^2 U)\, d\tau = \int_\Omega V \underset{\sim}{\nabla} U \cdot \underset{\sim}{n}\, dS \tag{14}$$

Substracting equation (14) from equation (12) yields:

$$\int_D (U \nabla^2 V - V \nabla^2 U)\, d\tau = \int_\Omega (U \underset{\sim}{\nabla} V - V \underset{\sim}{\nabla} U) \cdot \underset{\sim}{n}\, dS \tag{15}$$

Equations (12) and (14) are known as the first form of Green's identity. Equation (15) is known as the second form of Green's identity [11].

Using the following notation for the normal component of a gradient with respect to the boundary:

$$\underset{\sim}{\nabla} V \cdot \underset{\sim}{n} = \frac{\partial V}{\partial n} \tag{16}$$

$$\underset{\sim}{\nabla} U \cdot \underset{\sim}{n} = \frac{\partial U}{\partial n} \tag{17}$$

the Green's second identity, equation (15), becomes:

$$\int_D (U \nabla^2 V - V \nabla^2 U)\, d\tau = \int_\Omega \left(U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n}\right) dS \tag{18}$$

If $U$ and $V$ are both chosen such that they satisfy the Laplace equation, $\nabla^2 U = \nabla^2 V = 0$, then equation (18) becomes:

$$\int_\Omega \left(U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n}\right) dS = 0 \tag{19}$$

The present application makes direct use of equation (19) in which $U$ is chosen as a variable potential head $u$, and $V$ is chosen as a fundamental solution (also called free-space Green function [10]), which satisfies the Laplace equation in an infinite space except at the source point $P$, then:

$$\nabla^2 V = \delta(P) \quad \text{in } D \tag{20}$$

where $\delta(P)$ is a Dirac delta function representing a unit source at point $P$ and equal to zero everywhere else. The fundamental solution $V$ has different forms for two-dimensional and three-dimensional problems. For a two-dimensional problem:[3]

$$V = \ln (d) \tag{21}$$

where $d$ denotes the distance from an arbitrary but singular point $P$ (where $d = 0$) to another point $Q$ on the boundary of the two-dimensional domain.

In applying equation (19), it is essential to exclude the point $P$ by a small circle of radius $\epsilon$ as shown on figure 2. Equation (19) can then be expressed as two terms integrated along the outer boundary $S$ and the pole circle. Thus, equation (19) becomes:

$$\int_S \left[u \frac{\partial}{\partial n}(\ln d) - \ln d \frac{\partial u}{\partial n}\right] dS + \lim_{\epsilon \to 0} \int_{S_\epsilon} \left[u \frac{\partial}{\partial n}(\ln d) - \ln d \frac{\partial u}{\partial n}\right] dS_\epsilon = 0 \tag{22}$$

in which $S$ is the boundary contour of $D$, and $S_\epsilon$ is a small circle of radius $\epsilon$ around $P$ (fig. 2). The portions of the integral along the two lines connecting the circle to $S$ will cancel and, thus, they do not appear in equation (22).

[3] ln $(d)$ and $-$ln $(1/d)$ are the two basic forms of the fundamental solution of Laplace's equation in an infinite space. However, these two forms are equivalent as per the rules of logarithms.

On the circle $S_\epsilon$, the outward normal from $D$ points inward toward $P$, thus:

$$\frac{\partial}{\partial n}(\ln d) = \frac{1}{d}\frac{\partial d}{\partial n} = \frac{1}{d}(\underset{\sim}{\nabla} d \cdot \underset{\sim}{n}) = \frac{-1}{d} \qquad (23)$$

and the integral around the circle becomes:

$$\lim_{\epsilon \to 0} \int_{\theta=0}^{2\pi} \left[ u\left(\frac{-1}{\epsilon}\right) - \ln \epsilon \frac{\partial u}{\partial n} \right] \epsilon \, d\theta = -2\pi \; u(P) \qquad (24)$$

The potential at any point $P$ is defined in terms of the boundary integral:

$$2\pi \; u(P) = \int_S \left[ u(Q) \frac{\partial}{\partial n} (\ln d) - \ln d \frac{\partial}{\partial n} u(Q) \right] dS \qquad (25)$$

If $u$ and $\partial u/\partial n$ are known every where on the boundary $S$, the solution for $u$ at any interior point can be found by a line integration indicated in equation (25). In general, $u$ and $\partial u/\partial n$ are not known on $S$. In a well-posed problem, either $u$ or $\partial u/\partial n$ or a relation between them is known at all points on the boundary. The integral equation can be used to find $\partial u/\partial n$ or $u$ on the boundary.

To complete the boundary data, the point $P$ can be moved to the boundary as shown in figure 3. It is still excluded from $D$ by a circular arc. The same considerations hold as before except that the integration indicated in equation (24) takes place over an angle which is less than $2\pi$. For the case (fig. 3a), in which $P$ is on a smooth part of the boundary:

$$\pi \; u(P) = \int_S \left[ \frac{u(Q)}{d} \; \frac{\partial d}{\partial n} - \ln d \; \frac{\partial u(Q)}{\partial n} \right] dS \qquad (26)$$

and (fig. 3b) in the general case:

$$\alpha \; u(P) = \int_S \left[ \frac{u(Q)}{d} \; \frac{\partial d}{\partial n} - \ln d \; \frac{\partial u(Q)}{\partial n} \right] dS \qquad (27)$$

in which $\alpha$ is the angle between the boundary segments at $P$. Equation (27) can be discretized and solved numerically to obtain the "missing data" on the boundary. A subsequent use of equation (25) gives the potential $u$ at any specified interior point.

This development assumes that the flow is through a homogeneous and isotropic soil medium and described by Laplace equation. If the soil is anisotropic or nonhomogeneous, the describing equations must be transformed or zoned. These cases are considered separately.

**Anisotropic Soil**

The Laplace equation for a two-dimensional flow through anisotropic soil with permeabilities $k_x$ and $k_y$ in the orthogonal directions $x$ and $y$ is:

$$k_x \frac{\partial^2 u}{\partial x^2} + k_y \frac{\partial^2 u}{\partial y^2} = 0 \qquad (28)$$

Equation (28) can be written [13] for a transformed section with $x_T = \sqrt{\frac{k_y}{k_x}}\, x$ and $y_T = y$ as:

$$\frac{\partial^2 u}{\partial x_T^2} + \frac{\partial^2 u}{\partial y_T^2} = 0 \qquad (29)$$

**Nonhomogeneous Soil**

A nonhomogeneous soil medium can often be divided into a series of subregions or zones each having homogeneous soil; that is, soil properties in each zone are the same from point-to-point—horizontally or vertically—but may differ in different zones. By treating each zone separately, it becomes possible to study seepage through a single homogeneous, anisotropic material. The boundary element method can be applied to each zone of a nonhomogeneous soil medium. The individual zones can be coupled by adding the appropriate compatibility and equilibrium equations on the interfaces between zones to the algebraic equations created for the uncoupled zones. These boundary conditions shown on figure 4 at the interface for flow conditions are:

- Compatibility: $u_{I_1} = u_{I_2}$ (30)

  where

  $u_{I_1}$ is the potential at node I considering it belongs to zone 1.

  $u_{I_2}$ is the potential at node I considering it belongs to zone 2.

- Equilibrium: $q_{I_1} = -q_{I_2}$ (31)

  where

  $q_{I_1}$ is the rate of seepage flow out of zone 1 at node I

  $q_{I_2}$ is the rate of seepage flow out of zone 2 at node I

The rate of seepage flow by the Darcy's law is $q = kiA_s$ where $k$ is the soil permeability in the direction of flow, $i$ is the hydraulic gradient in the direction of flow, and $A_s$ is the cross-sectional area through which flow occurs.

The geometric transformation used between equation (28) and equation (29) gives an effective isotropic permeability of $(k_x\ k_y)^{1/2}$ for each zone in the transformed space. Applying Darcy's law for flow rate across the interface element between two zones (fig. 4), equation (31) becomes:

$$\left[\sqrt{k_{x_1}\ k_{y_1}}\ \frac{\partial u}{\partial n}\right]_{I_1} = -\left[\sqrt{k_{x_2}\ k_{y_2}}\ \frac{\partial u}{\partial n}\right]_{I_2} \qquad (32)$$

where $n$ is the direction of seepage flow assumed normal to the interface.

Application of equations (30) and (32) to the interface boundary elements gives the necessary equations to couple the heretofore uncoupled zones in the transformed space.

**Solution Procedure**

By using Green's second identity, the differential equation (4)—assuming $k_{ij}$ constant—has been transformed into the boundary integral equation (27) which involves only the variables on the boundary. Equation (27) can be solved numerically by discretizing the boundary into a finite number of straight line segments or elements as shown on figure 5. The points where the unknown values are considered or "nodes" are taken to be in the middle of each segment (fig. 5a), or at the intersection between two elements (fig. 5b). The boundary could be discretized by the use of curved elements (fig. 5c).

The procedure described in this paper uses constant elements. With this procedure, the values of $u$ and $\partial u/\partial n$ are assumed to be constant on each element and equal to the value at the midnode of the element. Letting $\{u\}^e$ and $\{\partial u/\partial n\}^e$ denote the values of the variables $u$ and $\partial u/\partial n$ on the boundary elements and locating the arbitrary point $P$ at each of the $N$ nodal points, a set of simultaneous equations can be formed from equation (27) as follows:

$$\{0\} = [H]\ \{u\} - [G]\ \left\{\frac{\partial u}{\partial n}\right\} \qquad (33)$$

where [H] and [G] are two matrices obtained through integration of equation (27). Equation (33) may be rewritten

$$[H]\{u\} = [G]\{q\} \qquad (34)$$

where $q$ represents $\partial u/\partial n$:

Equation (34) relates the value of $u$ at midpoint $i$ (called node $i$) of each boundary element with the values of $u$ and $q$ at all the nodes on the boundary, including $i$. There is one equation for each boundary element. If there are $N_1$ values of $u$ and $N_2$ values of $q$ prescribed on an exterior boundary ($N = N_1 + N_2$), a set of $N$ unknowns exists in equation (34). Rearranging the equations in such a way that all the unknowns are on the left-hand side, equation (34) can be written as:

$$[A]\{X\} = \{B\} \qquad (35)$$

where $\{X\}$ contains all the unknowns $u$ and $q$ for the boundary elements.

The above development is for flow through a homogeneous region with isotropic permeability.

For a nonhomogeneous material body, zones of piecewise homogeneity are identified—each zone is geometrically scaled to obtain an equivalent isotropic zone and the above procedure applied to each transformed zone. This produces sets of linear algebraic equations, one set per zone, which are submatrices of $[H]$ and $[G]$ in equation (34). These sets of equations are coupled by imposing constraint equations (30) and (32) for the interface boundary elements (fig. 4). If these constraint equations are appended at the end of matrices $[H]$ and $[G]$, the number of equations becomes $N + 2m$ where $m$ is the number of interface boundary elements. Alternatively, the interface constraint equations could be enforced by judiciously positioning the entries in $[H]$ and $[G]$ matrices for the interface boundary elements [14]. This way, the order of $[G]$ and $[H]$ matrices equals $N$.

For a multizone material body, $N = \sum_{j=1}^{M} N_j$ where $M$ is the number of zones, $N_j$ is the number of boundary elements in the zone $j$.

The above developments imply that the boundary was specified *a priori* and remains fixed. However, with a phreatic line (the free-water surface) in an embankment, the location of the free-water surface is not known. Therefore, this case must be considered separately.

**Phreatic Line Location**

The location of the phreatic line of seepage through an embankment dam is determined in the following steps [15, 16]:

1. Assume a path for the phreatic line. This path is used as a boundary for discretization of the problem into boundary elements. The embankment cross section above the assumed phreatic line is of no consequence and therefore is not included in the problem definition.

2. The boundary condition along the phreatic line is specified to be $\partial u/\partial n = 0$.

3. Calculate the potential head for the boundary elements including the elements representing the phreatic line.

4. Because the pressure head along the phreatic line is zero, the calculated potential head in step 3 in effect, provides the calculated estimate for the location of phreatic line element by element.

5. Using this revised estimate for the phreatic line location, calculations for the step 3 are repeated. When the difference between phreatic line locations in two consecutive calculations is within a specified tolerance, the solution is assumed to have converged.

The preceding step-by-step procedure is effective in adjusting the path for the phreatic line within the embankment section. The entry point on the phreatic line on the upstream face of the dam is defined by the headwater elevation and is assumed fixed in the iterative procedure. All other points on the phreatic line including the exit point on the downstream face of the dam are adjusted at the end of each iteration cycle.

The boundary conditions along the downstream face of the dam, above the tailwater elevation, are in terms of the total potential (being equal to the elevation head) and nonzero gradient; that is $\partial u/\partial n \neq 0$. The procedure in step 3 provides calculated potentials or gradients for all of the exterior boundary elements. If the gradient(s) for the boundary elements on the downstream face of the dam imply flow into the dam rather than out of the dam, these boundary elements on the downstream face of the dam should be considered part of the phreatic line definition.

## COMPUTER IMPLEMENTATION

### Program Description

The computer program in appendix 1 named BIE2DCP (Boundary Integral Equation, 2–Dimensions, Constant Potential) was developed in FORTRAN IV for the CDC CYBER 170-730 computational system. The program implements the ideas presented in the **Theory** section to perform seepage studies in two-dimensional zoned anisotropic mediums. In its present form, the program can be used to analyze seepage problems for up to 10 zones. The total number of boundary elements cannot exceed 200. If more elements are needed, it is only necessary to change the size of the array in the DIMENSION statement of the main program. All subroutines are designed to be dimensionally compatible with the main program. The program is self-complete; however, it uses some of the online functions available on the computational system.

The numbering of the boundary elements for each zone, and connectivity between zones through the interface boundary elements follow a fixed convention as shown on figure 6. For the exterior boundary of the numerical model, either the potential or its derivative normal to the boundary (but not both) are prescribed. At least one value of gradient $\partial u/\partial n$ or one value of potential $u$ must be given along with the rest of the boundary conditions data for the set of equations (35) to have a unique solution [16]. For the interface boundary elements, neither the potential nor its gradient is defined. The constraint equations (30) and (32) for the interface elements are enforced automatically by the computer program to ensure completeness of the numerical model. These constraint equations are implemented in the computer program by judiciously positioning the entries in $[H]$ and $[G]$ matrices for the interface boundary elements. Thus, the order of $[G]$ and $[H]$ matrices in equation (34) equals the total number of boundary elements used to discretize piecewise homogeneous zones of an anisotropic material body.

### Interzonal Connectivity

Each zone in a multizone body is given a numerical ID (identity) number. The ID numbers begin with 1, and are incremented by 1 for the complete problem. The relationship of each zone with all other zones in a multizone body is identified by specifying the ID number of other zones and giving the serial numbers of the interface boundary elements (fig. 6). This procedure allows for accountability of general planar distributions of materials and the resulting interfaces between material zones.

It is necessary to divide a material deposit into zones when the material is nonhomogeneous. It is desirable to divide a material deposit into zones when the aspect ratio of the body is large—that is, long but thin seams of material generally encountered in foundation zones of dams—to avoid numerical inaccuracies. In general, the ratio of the longest to shortest dimension should not exceed ten.

### Input Requirements

*General Considerations*—All control and input data, with the exception of the identification card, must be in either integer or floating point numerical format. Each card is divided into 16 fields of 5 columns each which end with columns 5, 10, 15, . . . 80. Integers must be right justified in their fields and all fields may contain no more characters than the number of columns in each field. Right justified means that all numbers must end at rightmost character position (i.e., in columns 5, 10, 15, . . . etc.) in the field. All real numbers must carry an explicit decimal point and may appear anywhere in their fields. A blank field implies zero for its variable, real or integer. Thus, leading zeros may be suppressed.

In the following, an expression starting with *I, J, K, L, M,* or *N* and containing any alphanumeric characters indicates an integer field. All others are real fields.

For convenience in checking a problem, all input data are printed in a convenient format with appropriate identifications so the information used by the program can be verified. This is especially useful when the results are apparently or obviously in error; the input can be scanned quickly to see if faulty input data must be corrected.

The input data must follow a specified format as shown in table 1. There are 13 different types of input cards to the program. The card type number is created for the convenience of the user in coding data for a problem. It is not made a part of the data and must not appear in the data file to be used by the computer program. A description of the variable names used for the input definition follows. The input for an array of variables may require more than one card depending on the size of the array.

*Description of Card Types* — The input data may be in either the inch-pound or SI system.

1. * Job identification name. This card is necessary for all problems, one card per problem.

2. N: Number of boundary elements (equal to the number of nodes in this case of constant elements).

   NRGNS: Number of regions. One region can have only one set of permeability values in the plane of the problem.

3. For $I$ = 1 to NRGNS, the following information:

   NL ($I$): Number of internal points in region $I$, where the potential $u$ need to be calculated.

4. For $J$ = 1 to NL ($I$), the following information:

   CX ($I,J$): The $x$ coordinate for internal point $J$ in region $I$ where the value of $u$ is required.

   CY ($I,J$): The $y$ coordinate for internal point $J$ in region $I$ where the value of $u$ is required.

5. For $I$ = 1 to $N$, the following information:

   X($I$): The $x$ coordinate of the extreme point of boundary element $I$.

   Y($I$): The $y$ coordinate of the extreme point of boundary element $I$.

6. For $I$ = 1 to $N$, the following information:

   KODE($I$): Code for the boundary conditions at the element nodes.

   KODE($I$) = 0 implies that potential $u$ for the element node $I$ is known and specified in the data set.

   KODE($I$) = 1 implies that the derivative of the potential $\partial u/\partial n$ for the element node $I$ is known and specified in the data set.

   KODE($I$) = 2 implies that the element node $I$ is on an interface and neither the potential nor the derivative of the potential is known.

   FI($I$): Prescribed value of the boundary condition for element node $I$ corresponding to the value of KODE ($I$). For KODE ($I$) = 2, FI($I$) must be left blank or specified a value of 0.0.

7. For $I$ = 1 to NRGNS, the following information:

   PERMX($I$): Permeability in the $x$ direction for region $I$.

   PERMY($I$): Permeability in the $y$ direction for region $I$.

8. ISTART($I$): Starting node number for the boundary elements for region $I$.

   IEND($I$): Ending node number for the boundary elements for region $I$.

9. NINTF($I,J$): Number of boundary elements on interface between regions $I$ and $J$.

10. ID($I,J,K$): Node number of interface boundary element $K$ between regions $I$ and $J$.

11. NPHREL: Number of boundary elements used to define the phreatic line.

    ITRMAX: Maximum number of iterations permitted to seek the phreatic line to a desired accuracy.

    ACC: Accuracy desired for the location of the phreatic line. If the difference between the calculated elevation of phreatic line in two consecutive iteration cycles is less than ACC, the desired convergence is assumed to have been achieved.

    HWE: Headwater elevation.

    TWE: Tailwater elevation.

12. IDPHR ($I$): Node numbers of boundary elements on the phreatic line.

13. IDPPHR: Node number of the boundary element past the exit of the phreatic line.

The card types No. 1 through 13 must be stacked in the above order.

The boundary element numbering convention and the coupling of the interface boundary elements are shown in figure 6. With this numbering scheme, the inward normal to a boundary element is positive. Thus, a positive gradient implies flow into the region and a negative gradient implies flow out of the region. The numbering for the phreatic line elements must be continuous.

**Computer Output**

All output has complete headings and should be self-explanatory. An itemized synopsis of the computer output follows:

1. A listing of the input data in a reformated structure. All blank input items will be output as 0.0.

2. The total potential and gradient (gradient or potential is input for all exterior boundary elements as in a mixed boundary value problem) and pressure head for all boundary nodes including the interface elements.

3. The total potential and pressure head for the specified interior points.

4. The location of the phreatic line for steady unconfined flow problems.

**Program Messages**

The computer program BIE2DCP is designed to generate one information message during execution. Additional messages—if any—shall be those caused by the computational system; therefore, a reference should be made to the computational system manuals.

The program generated message is:

"Error Indicators from The Equation Solver = " IER1, IER2

This message is generated to indicate the condition of the system of simultaneous linear algebraic equations. The error IER1 is generated in subroutine FACTR; IER2 is generated in subroutine RSLMC [17].

IER1 = 0 implies there was no error in the factorization of the matrix $[A]$, equation (35), into a product of a lower triangular matrix and an upper triangular matrix.

IER1 = 3 implies there was error in factorization of the matrix $[A]$, equation (35), into a product of a lower triangular matrix and an upper triangular matrix.

IER2 = 0 implies each component of the computed solution vector $\{X\}$, equation (35), meets the precision of $1 \times 10^{-6}$.

IER2 = 1 implies only the norm of the computed solution vector $\{X\}$, equation (35), meets the precision of $1 \times 10^{-6}$.

IER2 = 2 implies the precision in the norm of the computed solution vector $\{X\}$, equation (35), is lower than $1 \times 10^{-6}$.

IER2 = 3 implies the computed solution vector $\{X\}$, equation (35), has no meaning at all.

IER2 = 4 implies a diagonal term of the upper triangular factor is zero.

IER1 = 0 and IER2 = 0, 1, or 2 should be interpreted to mean that the set of algebraic equations is well conditioned and its computed solution is stable.

IER1 = 3 and IER2 = 3 or 4 should be interpreted to mean that the computed results are not good.

User Action: Check input data.

This message will not cause a termination of the program execution. These checks were built into the program during its development, and are being left in the program as they serve useful checks on the program and the problem definition.

## SAMPLE PROBLEMS

Two types of sample problems are presented. In one, the sample problems have known analytical or numerical solutions or both. In the other, the sample problem simulates a laboratory model in which the pressures were measured. The objectives are to demonstrate:

(1) the accuracy of the boundary element method with known solutions which use other techniques, and

(2) the usefulness of the boundary element method in predicting the response of physical model studies.

Example problems are described below.

**Problems With Known Analytical and/or Numerical Solutions**

*Examples 1 and 2.*—These problems have known analytical solutions [1]. Both problems could be considered as heat flow in a plate.

Problem 1, on figure 7, has a single medium with homogeneous and isotropic thermal conductivity. The exact solution to this problem is $\partial u/\partial n = 0.5$ and $u = (6 - x)/2$. The agreement is excellent between the numerical solution from the computer program and the analytical solution (fig. 7).

Problem 2, on figure 8, has two zones with homogeneous and isotropic thermal conductivity for each zone. The exact solution to this problem is a linear distribution from $u = 5$ to $u = 1$ in the left half zone and from $u = 1$ to $u = 0$ in the right half zone. The results of analysis of this problem using the computer program and the analytical solution show excellent agreement (fig. 8).

*Examples 3 and 4.*—These problems have known graphical solutions [13]. Both problems could be considered as confined flow through foundation zones.

Problem 3, on figure 9, has a sheet pile wall driven into a silty soil having a permeability of $10^{-6}$ ft/min ($0.3 \times 10^{-6}$ m/min). The sheet pile wall runs for a considerable length in a direction perpendicular to the page as shown on figure 9. The flow underneath the sheet pile wall is two-dimensional. Line b e represents an impervious cutoff wall (sheet pile). The water pressure distribution on the sheet pile wall is of interest. This problem provides a comparison of results obtained by the use of the boundary element computer program and the graphical solution. The agreement between the two methods is good (fig. 9).

Problem 4, on figure 10, represents a concrete spillway resting on an isotropic soil [13]. Lines AB and GH represent impervious cutoff walls (sheet piles). The piezometric head along the base of the spillway and around the sheet piles are of interest. This problem provides a comparison of results obtained by the use of the boundary element computer program and the known graphical solution. Again, the agreement between the methods is excellent (fig. 10).

Table 2 shows the input data file listing for this problem. The information in this table has been annotated to relate it to the input variable listing of table 1. The inclusion of details of the input data file for this problem rather than for some other sample problems included in this report is coincidental.

*Example 5.*—This problem, on figure 11, is an illustration of the use of the boundary element method for steady unconfined flow through a homogeneous and isotropic embankment dam resting on an impervious foundation. The location of the phreatic line is of interest. This problem is taken from reference [1] where a numerical solution is given. The discretization of the problem for the boundary element procedure and the appropriate boundary conditions are shown in figure 11. The iterative procedure used in the computer program BIE2DCP to locate the phreatic line is quite effective in achieving its objective (fig. 11). The comparison of the results obtained by the two numerical procedures is excellent.

*Example 6.*—This problem, on figure 12, is an illustration of the use of the boundary element method for a combination of confined and unconfined flow through an embankment dam and its foundation materials. The embankment soil and the foundation soils are assumed to be piecewise homogeneous with different permeabilities in two orthogonal directions. This sample problem is a simplified version of Bureau of Reclamation's Foss Dam seepage analysis studies. The results of the analysis obtained through the use of the BIE2DCP computer program are shown on figure 12. The results of this problem are not compared with Foss Dam studies because the simplified configuration was not recomputed using the finite element analysis.

**Problem With Measured Response**

*Example 7.*—The objective of this example, on figure 13, is to compare the results of the numerical model based on the boundary element method with those of a physical model. The laboratory model consists of a subsurface agricultural drain surrounded by an isotropic sand medium [18]. Briefly, the drain was constructed of a 4-inch-diameter (100-mm) corrugated plastic tubing surrounded by a 4-inch-thick gravel drain envelope. A symmetrically placed annulus of sand base material having an 86.4-inch diameter (2195 mm) was installed around the envelope. Unrestricted flow into the sand base was achieved by providing a coarse gravel pack at the periphery of the sand base. The tank containing the drain, envelope, sand base, and gravel pack was 9 feet long, 8 feet high, and 2.5 feet deep (2743 by 2438 by 762 mm). One side was constructed from an acrylic plastic face so that flow lines could be observed and to facilitate pressure measurements. Pressures were measured with one transducer systematically connected to all of the piezometric measurement points. All tests were made at a flow depth of 1 inch (25 mm) in the drain tubing.

The gravel envelope surrounding the drain pipe is about 18 times more permeable than the sand, and flow of water from gravel pocket to drain pipe is caused essentially by a stationary steady-state pool in the gravel envelope. There is a geometric and prescribed boundary conditions symmetry in the physical model about the vertical centerline. Thus, it is only necessary to numerically model one-half of the physical model and to treat the gravel envelope as a free-flow boundary, see figure 13. Results of the numerical model, in terms of water elevation at several interior points compared with those observed in the laboratory test for identical conditions of applied potentials for steady-state conditions, are shown on figure 14. Water elevations in these results are referenced from the steady-state pool elevation of 0.0 in the gravel envelope which is arbitrarily assigned an elevation of 0.0. The nomenclature for referring to the interior points is $C_r - R_\theta$; where $C_r$ = the circumferential path located at radius $r$ and $R_\theta$ = the radial line located at an angle $\theta$ with respect to vertical. Specifically, radii $r$ has values of 6.48, 8.16, 10.32, 12.96, 16.44, 20.76, 26.16, 33.0, and 41.64 *inches* (165, 207, 262, 329, 418, 527, 665, 838, 1058 mm), and angles $\theta$ have values of 15, 30, 45, 60, 75, 90, and 105 degrees measured counterclockwise from the vertical.

The original gravel envelope diameter in the model was 12.72 inches (323 mm). However, during pretesting operations at high heads, sand material entered the gravel envelope. Therefore, the interface between the sand and the gravel envelope could not be located precisely. The results of the numerical model compared favorably with laboratory measurements when an effective gravel envelope diameter of 12.8 inches (325 mm) was assumed.

## FUTURE DEVELOPMENTS

This report presents results of a study that demonstrate the practical application of the *boundary element method* to steady seepage problems through zoned anisotropic soils. A need exists to extend the method for the analysis of the following flow problems:

- The inclusion of internal sources or sinks within the two-dimensional flow problem
- Two-dimensional transient flow problems in soils
- Three-dimensional steady flow problems in soils
- Three-dimensional transient flow problems in soils

## BIBLIOGRAPHY


[1] Baker, R., "Use of Micro-Computers for the Solution of Seepage Problems by the Boundary Element Method," *Proceedings: Symposium on Implementation of Computer Procedures and Stress-Strain Laws in Geotechnical Engineering,* C. S. Desai and S. K. Saxena, Eds., pp. 463-475, 1981.

[2] Banerjee, P. K. and R. Butterfield, *Boundary Element Methods in Engineering Science,* McGraw Hill Book Company, 1981.

[3] Brebbia, C. A., *The Boundary Element Method for Engineers,* Pentech Press, pp. 58-70, 1978.

[4] Brebbia, C. A., and O. V. Chang, "Boundary Elements Applied to Seepage Problems in Zoned Anisotropic Soils," *Advances in Engineering Software,* vol. 1, No. 3, pp. 95-105, 1979.

[5] Chang, Ching S., "Boundary Element Method in Seepage Analysis With a Free Surface," *Proceedings: Symposium on Implementation of Computer Procedures and Stress-Strain Laws in Geotechnical Engineering,* C. S Desai and S. K. Saxena, Eds., pp. 421-430, 1981.

[6] Tomlin, G. R., *Numerical Analysis of Continuum Problem in Zoned Anisotropic Media,* Ph. D. Thesis, Southampton University, 1972.

[7] Banerjee, P. K. and R. Butterfield, "Boundary Element Methods in Geomechanics," ch. 16,: *Finite Elements in Geomechanics,* G. Gudehus, Ed., John Wiley and Sons, 1976.

[8] Banerjee, P. K., "Nonlinear Problems of Potential Flow," ch. 2,: *Developments in Boundary Element Methods,* vol. 1, P. K. Banergee and R. Butterfield, Eds., Applied Science Publishers, 1979.

[9] Bear, J., *Dynamics of Fluids in Porous Media,* American Elsevier, 1972.

[10] Liggett, James A. and Philip L-F. Liu, *The Boundary Integral Equation Method for Porous Media Flow,* George Allen and Unwin, 1983.

[11] Pipes, Louis A., *Applied Mathematics for Engineers and Physicists,* McGraw Hill Book Company, 1958.

[12] Davis, Harry F., *Introduction to Vector Analysis,* Allyn and Bacon, Inc., 1967.

[13] Lambe, T. W. and R. V. Whitman, *Soil Mechanics,* John Wiley and Sons, 1969.

[14] Chugh, A. K. and H. T. Falvey, "A Computer Program for Planar Seepage Analysis in a Zoned Anisotropic Medium by the Boundary Element Method," *Advances in Engineering Software,* vol. 5, No. 4, pp. 196-201, 1983.

[15] Liggett, J., "Location of Free Surface in Porous Media," *Journal of the Hydraulics Division,* American Society of Civil Engineers, vol. 103, No. HY4, pp. 353-365, 1977.

[16] Chugh, A. K. and H. T. Falvey, "Seepage Analysis in a Zoned Anisotropic Medium by the Boundary Element Method," *International Journal for Numerical and Analytical Methods in Geomechanics,* in press:

[17] *System/360 Scientific Subroutine Package,* Version III, 5th ed., International Business Machines, 1970.

[18] Zeigler, E. R. and J. N. Christopher, "Laboratory Test for a Drain With Gravel Envelope, "*American Society of Civil Engineers Proceedings of Specialty Conference—Water Forum* 81, vol. 2, pp. 765-772, 1981.

Table 1.—Input data field format for the computer program BIE2DCP

| Card Type No. / Column No. | 5 | 10 | 15 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|---|
| 01 | *72 Columns of identifying information | | | | | | |
| 02 | N | NRGNS | | | | | |
| | For I=1 to NRGNS | | | | | | |
| 03 | NL(I) | | | | | | |
| | For J=1 to NL(I) | | | | | | |
| 04 | | CX(I,J) | | CY(I,J) | | | |
| | Repeat card type 04 for J=1 to NL(I) | | | | | | |
| | Repeat card type 03 for I=1 to NRGNS | | | | | | |
| | For I=1 to N | | | | | | |
| 05 | | X(I) | | Y(I) | | | |
| | Repeat card type 05 for I=1 to N | | | | | | |
| | For I=1 to N | | | | | | |
| 06 | KODE(I) | | FI(I) | | | | |
| | Repeat card type 06 for I=1 to N | | | | | | |
| | For I=1 to NRGNS | | | | | | |
| 07 | | PERMX(I) | | PERMY(I) | | | |
| 08 | | ISTART(I) | | IEND(I) | | | |
| | For J=1 to NRGNS | | | | | | |
| 09 | | NINTF(I,J) for J≠I | | | | | |
| 10 | (ID(I,J,K), K=1, NINTF(I,J)) in fields of 10 columns each. | | | | | | |
| | Repeat card types 09 and 10 for J=1 to NRGNS | | | | | | |
| | Repeat card types 07, 08, 09 and 10 for I=1 to NRGNS | | | | | | |
| 11 | | NPHREL | | ITRMAX | ACC | HWE | TWE |
| 12 | (IDPHR(I), I=1, NPHREL) in fields of 10 columns each. | | | | | | |
| 13 | | IDPPHR | | | | | |

- 5, 10, 15, 20, etc. indicate the ending column of a field on the data card.
- All input is either in 5- or 10-column field format.
- The input for an array of variables may require more than one card, depending upon the size of the array.
- All real numbers should carry an explicit decimal point and may appear anywhere in their fields. All integers must be right justified in their own fields. All user created data must be in explicit real or integer format, except for the identification card which has an alphanumeric format.
- Unused portion of cards should be left blank. A blank field implies zero value for its variable, real or integer.
- The card type number is created for convenience of reference in this report. It is not to be included in the data set.

**Table 2.—Annotated input data file for the sample problem No. 4**

```
SAMPLE PROBLEM - PRESSURE DISTRIBUTION UNDER TWO PILED CONCRETE SPILLWAY
 82    3
  9
     8.0         14.0
    35.0         14.0
    58.0         14.0
     8.0         34.0
    35.0         34.0
    58.0         34.0
     8.0         54.0
    35.0         54.0
    58.0         54.0
 11
    78.0         14.0
    98.0         14.0
   118.0         14.0
   138.0         14.0
    88.0         34.0
   108.0         34.0
   128.0         34.0
    78.0         54.0
    98.0         54.0
   118.0         54.0
   138.0         54.0
  9
   158.0         14.0
   183.0         14.0
   208.0         14.0
   158.0         34.0
   183.0         34.0
   208.0         34.0
   158.0         54.0
   183.0         54.0
   208.0         54.0
```

Location of interior points where potentials are desired. card types 3 and 4.

```
     0.0          0.0
     8.0          0.0
    18.0          0.0
    28.0          0.0
    38.0          0.0
    48.0          0.0
    58.0          0.0
    66.0          0.0
    66.0         14.0
    66.0         24.0
    66.0         34.0
    66.0         43.0
    66.0         51.5
    66.0         60.0
    66.0         64.0
    58.0         64.0
    48.0         64.0
    38.0         64.0
    28.0         64.0
    18.0         64.0
     8.0         64.0
     0.0         64.0
     0.0         54.0
     0.0         44.0
     0.0         34.0
     0.0         24.0
     0.0         14.0
    66.0          0.0
    78.0          0.0
    88.0          0.0
    98.0          0.0
   108.0          0.0
   118.0          0.0
   128.0          0.0
   138.0          0.0
   151.0          0.0
   151.0         14.0
   151.0         24.0
   151.0         34.0
```

Location of end points of boundary elements. card type 5.

Table 2.—Annotated input data file for the sample problm No. 4 — Continued

```
150.92      43.0
150.92      51.5
150.92      60.0
 138.0      60.0
 128.0      60.0
 118.0      60.0
 108.0      60.0
  98.0      60.0
  88.0      60.0
  78.0      60.0
 66.08      60.0
 66.08      51.5
 66.08      43.0
  66.0      34.0
  66.0      24.0
  66.0      14.0
 151.0       0.0
 158.0       0.0
 168.0       0.0
 178.0       0.0
 188.0       0.0
 198.0       0.0
 208.0       0.0
 217.0       0.0
 217.0      14.0
 217.0      24.0
 217.0      34.0
 217.0      44.0
 217.0      54.0
 217.0      64.0
 208.0      64.0
 198.0      64.0
 188.0      64.0
 178.0      64.0
 168.0      64.0
 158.0      64.0
 151.0      64.0
 151.0      60.0
 151.0      51.5
 151.0      43.0
 151.0      34.0
 151.0      24.0
 151.0      14.0
1
1
1
1
1
1
1
2
2
2
2
1
1
1
0      94.0
0      94.0
0      94.0
0      94.0
0      94.0
0      94.0
0      94.0
1
1
1
1
1
1
1
1
```

Boundary conditions for exterior and interface boundary elements, card type 6.

Table 2.—Annotated input data file for the sample problem No. 4 — Continued

```
1
1
1
1
1
1
2
2
2
2
1
1
1
1
1
1
1
1
1
1
1
1
2
2
2
2
1
1
1
1
1
1
1
1
1
1
1
1
1
0       68.0
0       68.0
0       68.0
0       68.0
0       68.0
0       68.0
0       68.0
1
1
1
2
2
2
2
   0.1         0.1
     1          27
     4
     8           9         10          11
     0
   0.1         0.1
    28          55
     4
    55          54         53          52
     4
    36          37         38          39
   0.1         0.1
    56          82
     0
     4
    82          81         80          79
     0                               94.0        68.0
```

Zone identification and interzonal connectivity card types 7, 8, and 9.

Table 3.—Computer output for the sample problem No. 4

************************************************************************************************************************

* SAMPLE PROBLEM - PRESSURE DISTRIBUTION UNDER TWO PILED CONCRETE SPILLW

DATA

NUMBER OF BOUNDARY ELEMENTS= 82
NUMBER OF REGIONS = 3

COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

| POINT | X | Y |
|---|---|---|
| 1 | 0. | 0. |
| 2 | .8000000E+01 | 0. |
| 3 | .1800000E+02 | 0. |
| 4 | .2800000E+02 | 0. |
| 5 | .3800000E+02 | 0. |
| 6 | .4800000E+02 | 0. |
| 7 | .5800000E+02 | 0. |
| 8 | .6600000E+02 | 0. |
| 9 | .6600000E+02 | .1400000E+02 |
| 10 | .6600000E+02 | .2400000E+02 |
| 11 | .6600000E+02 | .3400000E+02 |
| 12 | .6600000E+02 | .4300000E+02 |
| 13 | .6600000E+02 | .5150000E+02 |
| 14 | .6600000E+02 | .6000000E+02 |
| 15 | .6600000E+02 | .6400000E+02 |
| 16 | .5800000E+02 | .6400000E+02 |
| 17 | .4800000E+02 | .6400000E+02 |
| 18 | .3800000E+02 | .6400000E+02 |
| 19 | .2800000E+02 | .6400000E+02 |
| 20 | .1800000E+02 | .6400000E+02 |
| 21 | .8000000E+01 | .6400000E+02 |
| 22 | 0. | .6400000E+02 |
| 23 | 0. | .5400000E+02 |
| 24 | 0. | .4400000E+02 |
| 25 | 0. | .3400000E+02 |
| 26 | 0. | .2400000E+02 |
| 27 | 0. | .1400000E+02 |
| 28 | .6600000E+02 | 0. |
| 29 | .7800000E+02 | 0. |
| 30 | .8800000E+02 | 0. |
| 31 | .9800000E+02 | 0. |
| 32 | .1080000E+03 | 0. |
| 33 | .1180000E+03 | 0. |
| 34 | .1280000E+03 | 0. |
| 35 | .1380000E+03 | 0. |
| 36 | .1510000E+03 | 0. |
| 37 | .1510000E+03 | .1400000E+02 |
| 38 | .1510000E+03 | .2400000E+02 |
| 39 | .1510000E+03 | .3400000E+02 |
| 40 | .1509200E+03 | .4300000E+02 |

| | | |
|---|---|---|
| 41 | .1509200E+03 | .5150000E+02 |
| 42 | .1509200E+03 | .6000000E+02 |
| 43 | .1380000E+03 | .6000000E+02 |
| 44 | .1280000E+03 | .6000000E+02 |
| 45 | .1180000E+03 | .6000000E+02 |
| 46 | .1080000E+03 | .6000000E+02 |
| 47 | .9800000E+02 | .6000000E+02 |
| 48 | .8800000E+02 | .6000000E+02 |
| 49 | .7800000E+02 | .6000000E+02 |
| 50 | .6608000E+02 | .6000000E+02 |
| 51 | .6608000E+02 | .5150000E+02 |
| 52 | .6608000E+02 | .4300000E+02 |
| 53 | .6600000E+02 | .3400000E+02 |
| 54 | .6600000E+02 | .2400000E+02 |
| 55 | .6600000E+02 | .1400000E+02 |
| 56 | .1510000E+03 | 0. |
| 57 | .1580000E+03 | 0. |
| 58 | .1680000E+03 | 0. |
| 59 | .1780000E+03 | 0. |
| 60 | .1880000E+03 | 0. |
| 61 | .1980000E+03 | 0. |
| 62 | .2080000E+03 | 0. |
| 63 | .2170000E+03 | 0. |
| 64 | .2170000E+03 | .1400000E+02 |
| 65 | .2170000E+03 | .2400000E+02 |
| 66 | .2170000E+03 | .3400000E+02 |
| 67 | .2170000E+03 | .4400000E+02 |
| 68 | .2170000E+03 | .5400000E+02 |
| 69 | .2170000E+03 | .6400000E+02 |
| 70 | .2080000E+03 | .6400000E+02 |
| 71 | .1980000E+03 | .6400000E+02 |
| 72 | .1880000E+03 | .6400000E+02 |
| 73 | .1780000E+03 | .6400000E+02 |
| 74 | .1680000E+03 | .6400000E+02 |
| 75 | .1580000E+03 | .6400000E+02 |
| 76 | .1510000E+03 | .6400000E+02 |
| 77 | .1510000E+03 | .6000000E+02 |
| 78 | .1510000E+03 | .5150000E+02 |
| 79 | .1510000E+03 | .4300000E+02 |
| 80 | .1510000E+03 | .3400000E+02 |
| 81 | .1510000E+03 | .2400000E+02 |
| 82 | .1510000E+03 | .1400000E+02 |

BOUNDARY CONDITIONS

| NODE | CODE | PRESCRIBED VALUE |
|---|---|---|
| 1 | 1 | 0. |
| 2 | 1 | 0. |
| 3 | 1 | 0. |
| 4 | 1 | 0. |
| 5 | 1 | 0. |
| 6 | 1 | 0. |
| 7 | 1 | 0. |

Table 3.—Computer output for the sample problem No. 4 — Continued

| | | |
|---|---|---|
| 8 | 2 | 0. |
| 9 | 2 | 0. |
| 10 | 2 | 0. |
| 11 | 2 | 0. |
| 12 | 1 | 0. |
| 13 | 1 | 0. |
| 14 | 1 | 0. |
| 15 | 0 | .9400000E+02 |
| 16 | 0 | .9400000E+02 |
| 17 | 0 | .9400000E+02 |
| 18 | 0 | .9400000E+02 |
| 19 | 0 | .9400000E+02 |
| 20 | 0 | .9400000E+02 |
| 21 | 0 | .9400000E+02 |
| 22 | 1 | 0. |
| 23 | 1 | 0. |
| 24 | 1 | 0. |
| 25 | 1 | 0. |
| 26 | 1 | 0. |
| 27 | 1 | 0. |
| 28 | 1 | 0. |
| 29 | 1 | 0. |
| 30 | 1 | 0. |
| 31 | 1 | 0. |
| 32 | 1 | 0. |
| 33 | 1 | 0. |
| 34 | 1 | 0. |
| 35 | 1 | 0. |
| 36 | 2 | 0. |
| 37 | 2 | 0. |
| 38 | 2 | 0. |
| 39 | 2 | 0. |
| 40 | 1 | 0. |
| 41 | 1 | 0. |
| 42 | 1 | 0. |
| 43 | 1 | 0. |
| 44 | 1 | 0. |
| 45 | 1 | 0. |
| 46 | 1 | 0. |
| 47 | 1 | 0. |
| 48 | 1 | 0. |
| 49 | 1 | 0. |
| 50 | 1 | 0. |
| 51 | 1 | 0. |
| 52 | 2 | 0. |
| 53 | 2 | 0. |
| 54 | 2 | 0. |
| 55 | 2 | 0. |
| 56 | 1 | 0. |
| 57 | 1 | 0. |
| 58 | 1 | 0. |
| 59 | 1 | 0. |
| 60 | 1 | 0. |
| 61 | 1 | 0. |

Table 3.—Computer output for the sample problem No. 4 — Continued

| | | |
|---|---|---|
| 62 | 1 | 0. |
| 63 | 1 | 0. |
| 64 | 1 | 0. |
| 65 | 1 | 0. |
| 66 | 1 | 0. |
| 67 | 1 | 0. |
| 68 | 1 | 0. |
| 69 | 0 | .6800000E+02 |
| 70 | 0 | .6800000E+02 |
| 71 | 0 | .6800000E+02 |
| 72 | 0 | .6800000E+02 |
| 73 | 0 | .6800000E+02 |
| 74 | 0 | .6800000E+02 |
| 75 | 0 | .6800000E+02 |
| 76 | 1 | 0. |
| 77 | 1 | 0. |
| 78 | 1 | 0. |
| 79 | 2 | 0. |
| 80 | 2 | 0. |
| 81 | 2 | 0. |
| 82 | 2 | 0. |

| REGION NO. | STARTING ELEM NO. | ENDING ELEM NO. | PERM-^X^ | PERM-^Y^ |
|---|---|---|---|---|
| 1 | 1 | 27 | .1000E+00 | .1000E+00 |
| 2 | 28 | 55 | .1000E+00 | .1000E+00 |
| 3 | 56 | 82 | .1000E+00 | .1000E+00 |

| ZONE NO. | INTERFACED ZONE NO. | INTERFACE ELEMENT NOS. | | | |
|---|---|---|---|---|---|
| 1 | 2 | 8 | 9 | 10 | 11 |
| 2 | 1 | 55 | 54 | 53 | 52 |
| 2 | 3 | 36 | 37 | 38 | 39 |
| 3 | 2 | 82 | 81 | 80 | 79 |

| HWE | TWE |
|---|---|
| 94.00 | 68.00 |

TOTAL NUMBER OF EQUATIONS = 82

ERROR INDICATORS FROM THE EQUATION SOLVER = 0 0

*******************************************************************************************************************************************************************************************************************

RESULTS

BOUNDARY NODES

| NODE | X | Y | POTENTIAL | POTENTIAL DERIVATIVE | PIEZOMETRIC RISE | NODE |
|---|---|---|---|---|---|---|
| 1 | .4000000E+01 | 0. | .9088655E+02 | 0. | .9088655E+02 | 1 |
| 2 | .1300000E+02 | 0. | .9073832E+02 | 0. | .9073832E+02 | 2 |
| 3 | .2300000E+02 | 0. | .9040193E+02 | 0. | .9040193E+02 | 3 |
| 4 | .3300000E+02 | 0. | .8986830E+02 | 0. | .8986830E+02 | 4 |
| 5 | .4300000E+02 | 0. | .8912651E+02 | 0. | .8912651E+02 | 5 |
| 6 | .5300000E+02 | 0. | .8817576E+02 | 0. | .8817576E+02 | 6 |
| 7 | .6200000E+02 | 0. | .8715997E+02 | 0. | .8715997E+02 | 7 |
| 8 | .6600000E+02 | .7000000E+01 | .8671488E+02 | -.1291348E+00 | .7971488E+02 | 8 |
| 9 | .6600000E+02 | .1900000E+02 | .8691162E+02 | -.1360755E+00 | .6791162E+02 | 9 |
| 10 | .6600000E+02 | .2900000E+02 | .8719951E+02 | -.1604466E+00 | .5819951E+02 | 10 |
| 11 | .6600000E+02 | .3850000E+02 | .8759198E+02 | -.3436234E+00 | .4909198E+02 | 11 |
| 12 | .6600000E+02 | .4725000E+02 | .9056792E+02 | 0. | .4331792E+02 | 12 |
| 13 | .6600000E+02 | .5575000E+02 | .9246932E+02 | 0. | .3671932E+02 | 13 |
| 14 | .6600000E+02 | .6200000E+02 | .9366061E+02 | 0. | .3166061E+02 | 14 |
| 15 | .6200000E+02 | .6400000E+02 | .9400000E+02 | .1866031E+00 | .3000000E+02 | 15 |
| 16 | .5300000E+02 | .6400000E+02 | .9400000E+02 | .1608211E+00 | .3000000E+02 | 16 |
| 17 | .4300000E+02 | .6400000E+02 | .9400000E+02 | .1310757E+00 | .3000000E+02 | 17 |
| 18 | .3300000E+02 | .6400000E+02 | .9400000E+02 | .1073537E+00 | .3000000E+02 | 18 |
| 19 | .2300000E+02 | .6400000E+02 | .9400000E+02 | .9097605E-01 | .3000000E+02 | 19 |
| 20 | .1300000E+02 | .6400000E+02 | .9400000E+02 | .7988426E-01 | .3000000E+02 | 20 |
| 21 | .4000000E+01 | .6400000E+02 | .9400000E+02 | .8339566E-01 | .3000000E+02 | 21 |
| 22 | 0. | .5900000E+02 | .9363921E+02 | 0. | .3463921E+02 | 22 |
| 23 | 0. | .4900000E+02 | .9287888E+02 | 0. | .4387888E+02 | 23 |
| 24 | 0. | .3900000E+02 | .9219748E+02 | 0. | .5319748E+02 | 24 |
| 25 | 0. | .2900000E+02 | .9163254E+02 | 0. | .6263254E+02 | 25 |
| 26 | 0. | .1900000E+02 | .9121452E+02 | 0. | .7221452E+02 | 26 |
| 27 | 0. | .7000000E+01 | .9093563E+02 | 0. | .8393563E+02 | 27 |
| 28 | .7200000E+02 | 0. | .8596615E+02 | 0. | .8596615E+02 | 28 |
| 29 | .8300000E+02 | 0. | .8448887E+02 | 0. | .8448887E+02 | 29 |
| 30 | .9300000E+02 | 0. | .8312199E+02 | 0. | .8312199E+02 | 30 |
| 31 | .1030000E+03 | 0. | .8175196E+02 | 0. | .8175196E+02 | 31 |
| 32 | .1130000E+03 | 0. | .8038281E+02 | 0. | .8038281E+02 | 32 |
| 33 | .1230000E+03 | 0. | .7901297E+02 | 0. | .7901297E+02 | 33 |
| 34 | .1330000E+03 | 0. | .7764547E+02 | 0. | .7764547E+02 | 34 |
| 35 | .1445000E+03 | 0. | .7609868E+02 | 0. | .7609868E+02 | 35 |
| 36 | .1510000E+03 | .7000000E+01 | .7529175E+02 | -.1293815E+00 | .6829175E+02 | 36 |
| 37 | .1510000E+03 | .1900000E+02 | .7508809E+02 | -.1358809E+00 | .5608809E+02 | 37 |
| 38 | .1510000E+03 | .2900000E+02 | .7479936E+02 | -.1603418E+00 | .4579936E+02 | 38 |
| 39 | .1509600E+03 | .3850000E+02 | .7440669E+02 | -.3435008E+00 | .3590669E+02 | 39 |
| 40 | .1509200E+03 | .4725000E+02 | .7642458E+02 | 0. | .2917458E+02 | 40 |
| 41 | .1509200E+03 | .5575000E+02 | .7712550E+02 | 0. | .2137550E+02 | 41 |
| 42 | .1444600E+03 | .6000000E+02 | .7732592E+02 | 0. | .1732592E+02 | 42 |
| 43 | .1330000E+03 | .6000000E+02 | .7820563E+02 | 0. | .1820563E+02 | 43 |
| 44 | .1230000E+03 | .6000000E+02 | .7928099E+02 | 0. | .1928099E+02 | 44 |
| 45 | .1130000E+03 | .6000000E+02 | .8045818E+02 | 0. | .2045818E+02 | 45 |
| 46 | .1030000E+03 | .6000000E+02 | .8165982E+02 | 0. | .2165982E+02 | 46 |
| 47 | .9300000E+02 | .6000000E+02 | .8283130E+02 | 0. | .2283130E+02 | 47 |
| 48 | .8300000E+02 | .6000000E+02 | .8388977E+02 | 0. | .2388977E+02 | 48 |

Table 3.—Computer output for the sample problem No. 4 — Continued

| | | | | | | |
|---|---|---|---|---|---|---|
| 49 | .7204000E+02 | .6000000E+02 | .8469687E+02 | 0. | .2469687E+02 | 49 |
| 50 | .6608000E+02 | .5575000E+02 | .8487832E+02 | 0. | .2912832E+02 | 50 |
| 51 | .6608000E+02 | .4725000E+02 | .8557413E+02 | 0. | .3832413E+02 | 51 |
| 52 | .6604000E+02 | .3850000E+02 | .8759198E+02 | .3436234E+00 | .4909198E+02 | 52 |
| 53 | .6600000E+02 | .2900000E+02 | .8719951E+02 | .1604466E+00 | .5819951E+02 | 53 |
| 54 | .6600000E+02 | .1900000E+02 | .8691162E+02 | .1360755E+00 | .6791162E+02 | 54 |
| 55 | .6600000E+02 | .7000000E+01 | .8671488E+02 | .1291348E+00 | .7971488E+02 | 55 |
| 56 | .1545000E+03 | 0. | .7490398E+02 | 0. | .7490398E+02 | 56 |
| 57 | .1630000E+03 | 0. | .7393254E+02 | 0. | .7393254E+02 | 57 |
| 58 | .1730000E+03 | 0. | .7296071E+02 | 0. | .7296071E+02 | 58 |
| 59 | .1830000E+03 | 0. | .7219740E+02 | 0. | .7219740E+02 | 59 |
| 60 | .1930000E+03 | 0. | .7164319E+02 | 0. | .7164319E+02 | 60 |
| 61 | .2030000E+03 | 0. | .7128763E+02 | 0. | .7128763E+02 | 61 |
| 62 | .2125000E+03 | 0. | .7111910E+02 | 0. | .7111910E+02 | 62 |
| 63 | .2170000E+03 | .7000000E+01 | .7106525E+02 | 0. | .6406525E+02 | 63 |
| 64 | .2170000E+03 | .1900000E+02 | .7078614E+02 | 0. | .5178614E+02 | 64 |
| 65 | .2170000E+03 | .2900000E+02 | .7036814E+02 | 0. | .4136814E+02 | 65 |
| 66 | .2170000E+03 | .3900000E+02 | .6980326E+02 | 0. | .3080326E+02 | 66 |
| 67 | .2170000E+03 | .4900000E+02 | .6912211E+02 | 0. | .2012211E+02 | 67 |
| 68 | .2170000E+03 | .5900000E+02 | .6836281E+02 | 0. | .9362814E+01 | 68 |
| 69 | .2125000E+03 | .6400000E+02 | .6800000E+02 | -.8243230E-01 | .4000000E+01 | 69 |
| 70 | .2030000E+03 | .6400000E+02 | .6800000E+02 | -.8083644E-01 | .4000000E+01 | 70 |
| 71 | .1930000E+03 | .6400000E+02 | .6800000E+02 | -.9234625E-01 | .4000000E+01 | 71 |
| 72 | .1830000E+03 | .6400000E+02 | .6800000E+02 | -.1093966E+00 | .4000000E+01 | 72 |
| 73 | .1730000E+03 | .6400000E+02 | .6800000E+02 | -.1338401E+00 | .4000000E+01 | 73 |
| 74 | .1630000E+03 | .6400000E+02 | .6800000E+02 | -.1638532E+00 | .4000000E+01 | 74 |
| 75 | .1545000E+03 | .6400000E+02 | .6800000E+02 | -.1879669E+00 | .4000000E+01 | 75 |
| 76 | .1510000E+03 | .6200000E+02 | .6833780E+02 | 0. | .6337796E+01 | 76 |
| 77 | .1510000E+03 | .5575000E+02 | .6953007E+02 | 0. | .1378007E+02 | 77 |
| 78 | .1510000E+03 | .4725000E+02 | .7143137E+02 | 0. | .2418137E+02 | 78 |
| 79 | .1510000E+03 | .3850000E+02 | .7440669E+02 | .3435008E+00 | .3590669E+02 | 79 |
| 80 | .1510000E+03 | .2900000E+02 | .7479936E+02 | .1603418E+00 | .4579936E+02 | 80 |
| 81 | .1510000E+03 | .1900000E+02 | .7508809E+02 | .1358809E+00 | .5608809E+02 | 81 |
| 82 | .1510000E+03 | .7000000E+01 | .7529175E+02 | .1293815E+00 | .6829175E+02 | 82 |

INTERNAL POINTS

REGION NO = 1

| X | Y | POTENTIAL | PIEZOMETRIC RISE |
|---|---|---|---|
| .8000000E+01 | .1400000E+02 | .9101019E+02 | .7701019E+02 |
| .3500000E+02 | .1400000E+02 | .8994448E+02 | .7594448E+02 |
| .5800000E+02 | .1400000E+02 | .8780835E+02 | .7380835E+02 |
| .8000000E+01 | .3400000E+02 | .9185572E+02 | .5785572E+02 |
| .3500000E+02 | .3400000E+02 | .9100222E+02 | .5700222E+02 |
| .5800000E+02 | .3400000E+02 | .8885028E+02 | .5485028E+02 |
| .8000000E+01 | .5400000E+02 | .9322642E+02 | .3922642E+02 |
| .3500000E+02 | .5400000E+02 | .9289315E+02 | .3889315E+02 |
| .5800000E+02 | .5400000E+02 | .9222898E+02 | .3822898E+02 |

Table 3.—Computer output for the sample problem No. 4 — Continued

REGION NO = 2

| | | | |
|---|---|---|---|
| .7800000E+02 | .1400000E+02 | .8519423E+02 | .7119423E+02 |
| .9800000E+02 | .1400000E+02 | .8243293E+02 | .6843293E+02 |
| .1180000E+03 | .1400000E+02 | .7970131E+02 | .6570131E+02 |
| .1380000E+03 | .1400000E+02 | .7694071E+02 | .6294071E+02 |
| .8800000E+02 | .3400000E+02 | .8372176E+02 | .4972176E+02 |
| .1080000E+03 | .3400000E+02 | .8106429E+02 | .4706429E+02 |
| .1280000E+03 | .3400000E+02 | .7841302E+02 | .4441302E+02 |
| .7800000E+02 | .5400000E+02 | .8440132E+02 | .3040132E+02 |
| .9800000E+02 | .5400000E+02 | .8226228E+02 | .2826228E+02 |
| .1180000E+03 | .5400000E+02 | .7985457E+02 | .2585457E+02 |
| .1380000E+03 | .5400000E+02 | .7767969E+02 | .2367969E+02 |

REGION NO = 3

| | | | |
|---|---|---|---|
| .1580000E+03 | .1400000E+02 | .7431235E+02 | .6031235E+02 |
| .1830000E+03 | .1400000E+02 | .7199062E+02 | .5799062E+02 |
| .2080000E+03 | .1400000E+02 | .7100539E+02 | .5700539E+02 |
| .1580000E+03 | .3400000E+02 | .7330633E+02 | .3930633E+02 |
| .1830000E+03 | .3400000E+02 | .7094329E+02 | .3694329E+02 |
| .2080000E+03 | .3400000E+02 | .7015672E+02 | .3615672E+02 |
| .1580000E+03 | .5400000E+02 | .6979725E+02 | .1579725E+02 |
| .1830000E+03 | .5400000E+02 | .6908601E+02 | .1508601E+02 |
| .2080000E+03 | .5400000E+02 | .6877955E+02 | .1477955E+02 |

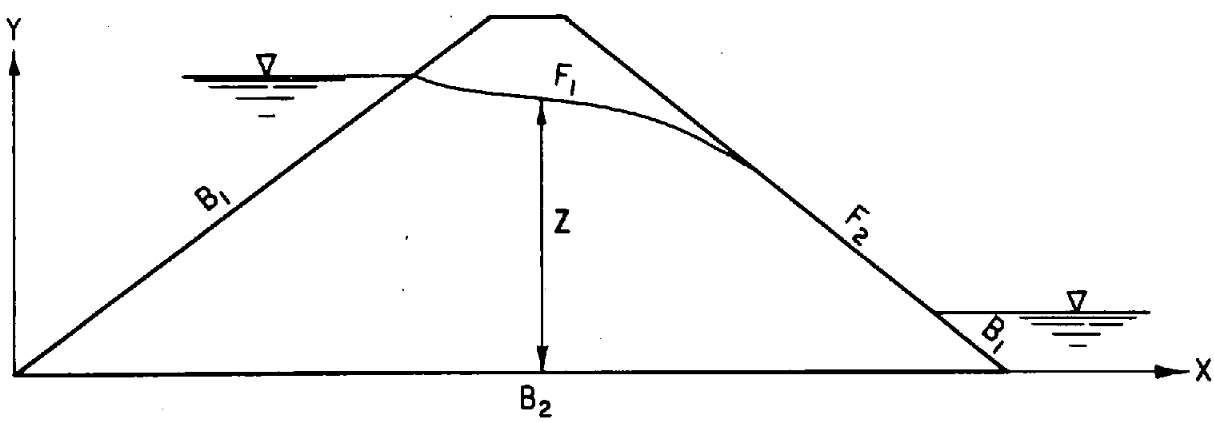



Figure 1.—General description of a typical free-surface seepage problem.

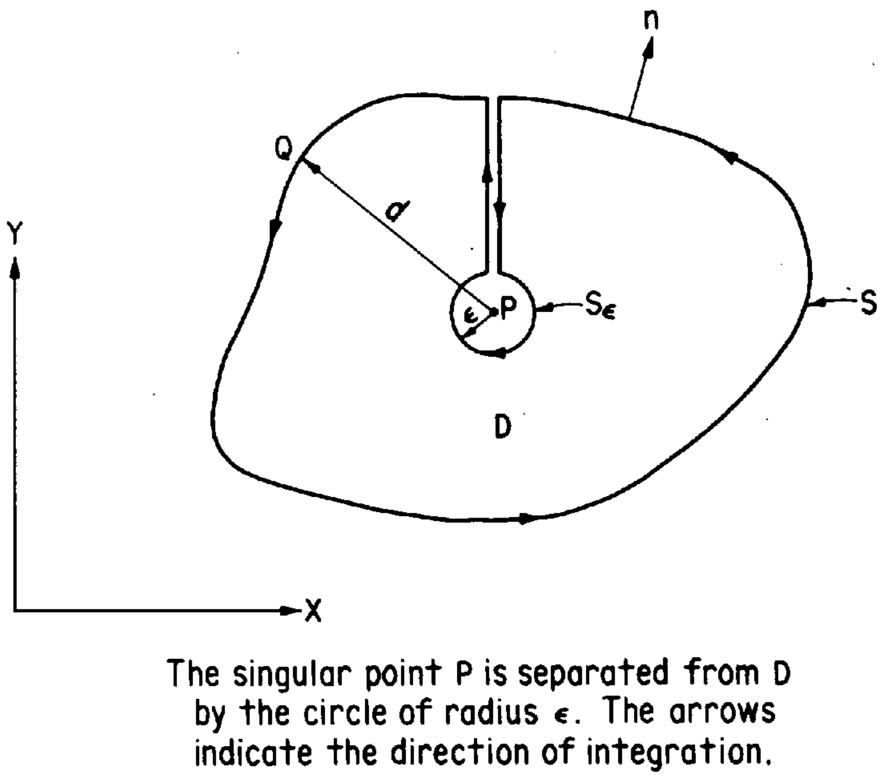



Figure 2.—Two-dimensional domain D surrounded by the boundary curve S.

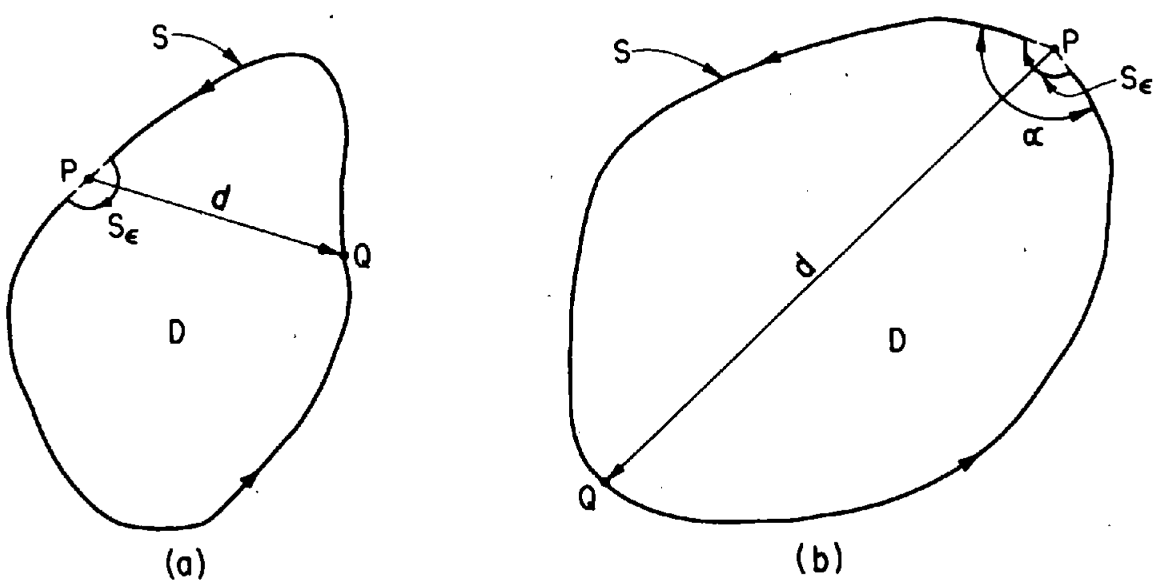
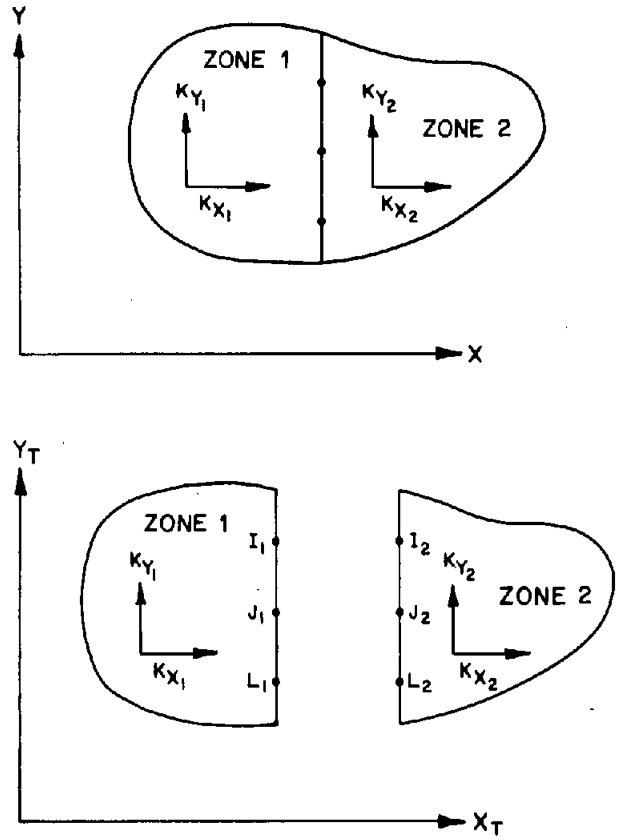



Figure 3.—Point P on the boundary, (a) at a smooth part of the boundary, (b) where the boundary contour forms an angle.




In the transformed space with $X_T = \sqrt{\frac{K_Y}{K_X}}\,X$; $Y_T = Y$, the boundary conditions at the interface are:

1. Compatibility conditions

$$u_{I_1} = u_{I_2}$$

$$u_{J_1} = u_{J_2}$$

$$u_{L_1} = u_{L_2}$$

2. Equilibrium condition

$$\left[\sqrt{K_{X_1}\;K_{Y_1}}\;\frac{\partial u}{\partial n}\right]_{I_1} = -\left[\sqrt{K_{X_2}\;K_{Y_2}}\;\frac{\partial u}{\partial n}\right]_{I_2}$$

$$\left[\sqrt{K_{X_1}\;K_{Y_1}}\;\frac{\partial u}{\partial n}\right]_{J_1} = -\left[\sqrt{K_{X_2}\;K_{Y_2}}\;\frac{\partial u}{\partial n}\right]_{J_2}$$

$$\left[\sqrt{K_{X_1}\;K_{Y_1}}\;\frac{\partial u}{\partial n}\right]_{L_1} = -\left[\sqrt{K_{X_2}\;K_{Y_2}}\;\frac{\partial u}{\partial n}\right]_{L_2}$$

Figure 4.—Boundary conditions at interface between zoned anisotropic regions.

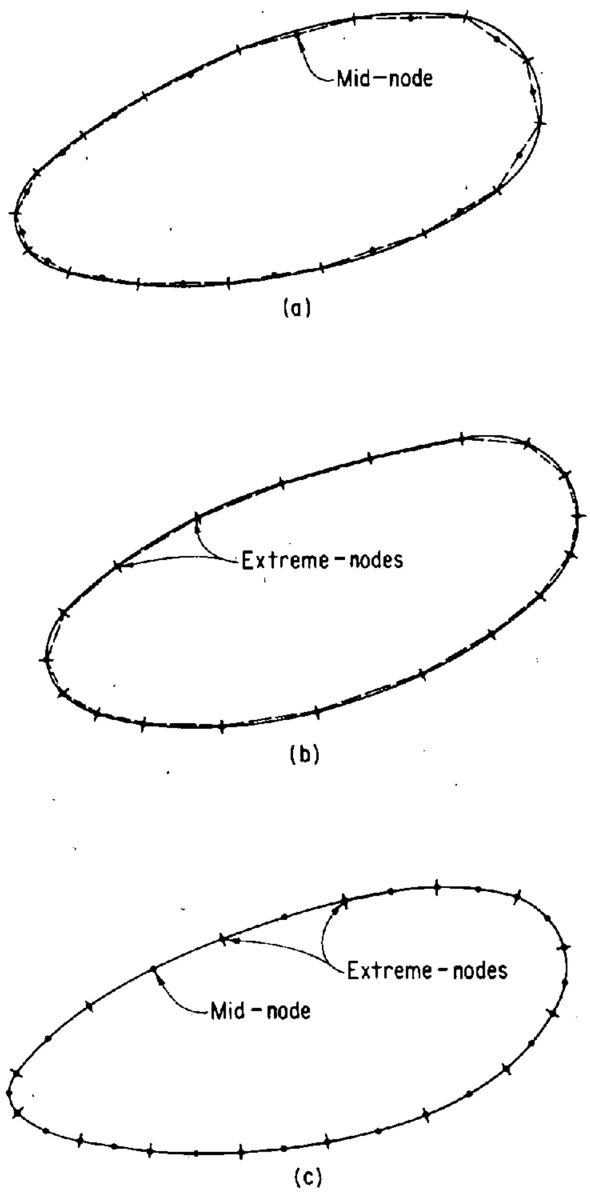
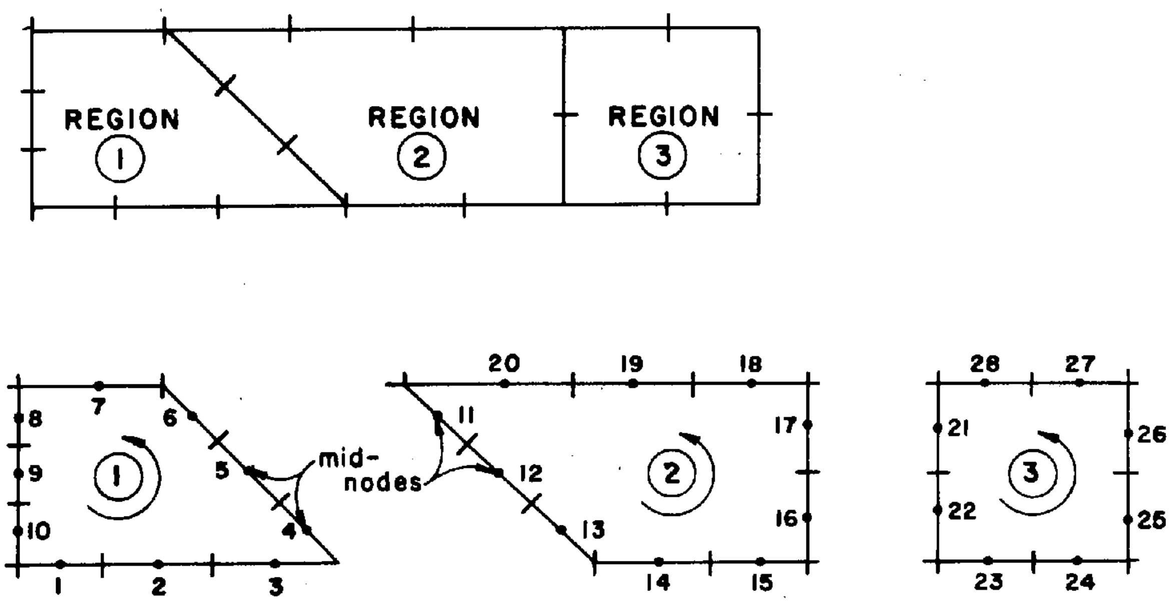



Figure 5.—Different types of boundary elements. (a) constant elements, (b) linear elements, and (c) quadratic elements.




Element numbering is counterclockwise.

Interface elements for Region 1 with Region 2 are 4, 5, 6

Interface elements for Region 1 with Region 3 are none

Interface elements for Region 2 with Region 1 are 13, 12, 11

Interface elements for Region 2 with Region 3 are 16, 17

Interface elements for Region 3 with Region 1 are none

Interface elements for Region 3 with Region 2 are 22, 21

Figure 6. Element numbering and ordering sequence of nodes for interface elements.

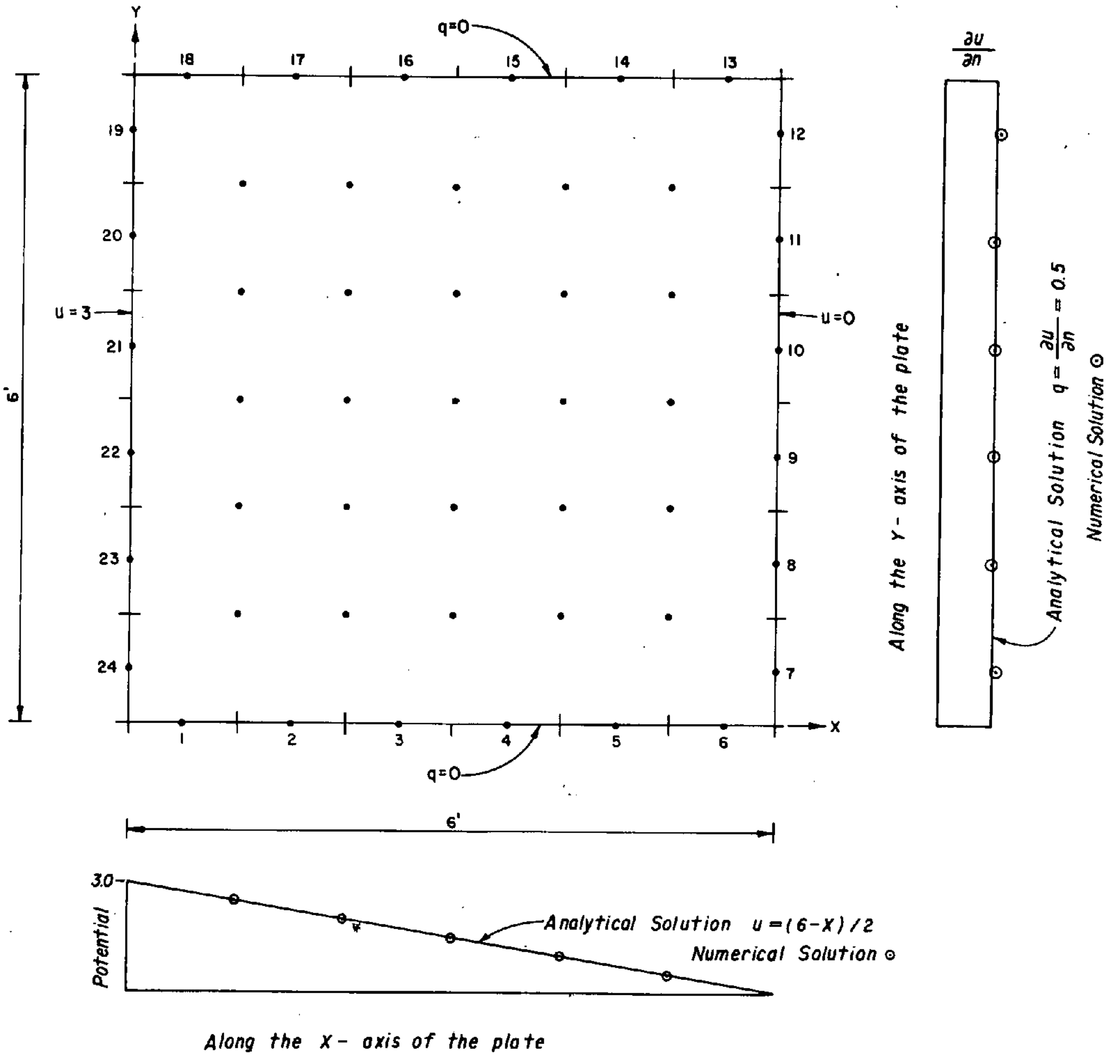



Figure 7.—Sample problem No. 1 — Comparison of boundary element solution with the known analytic solution.

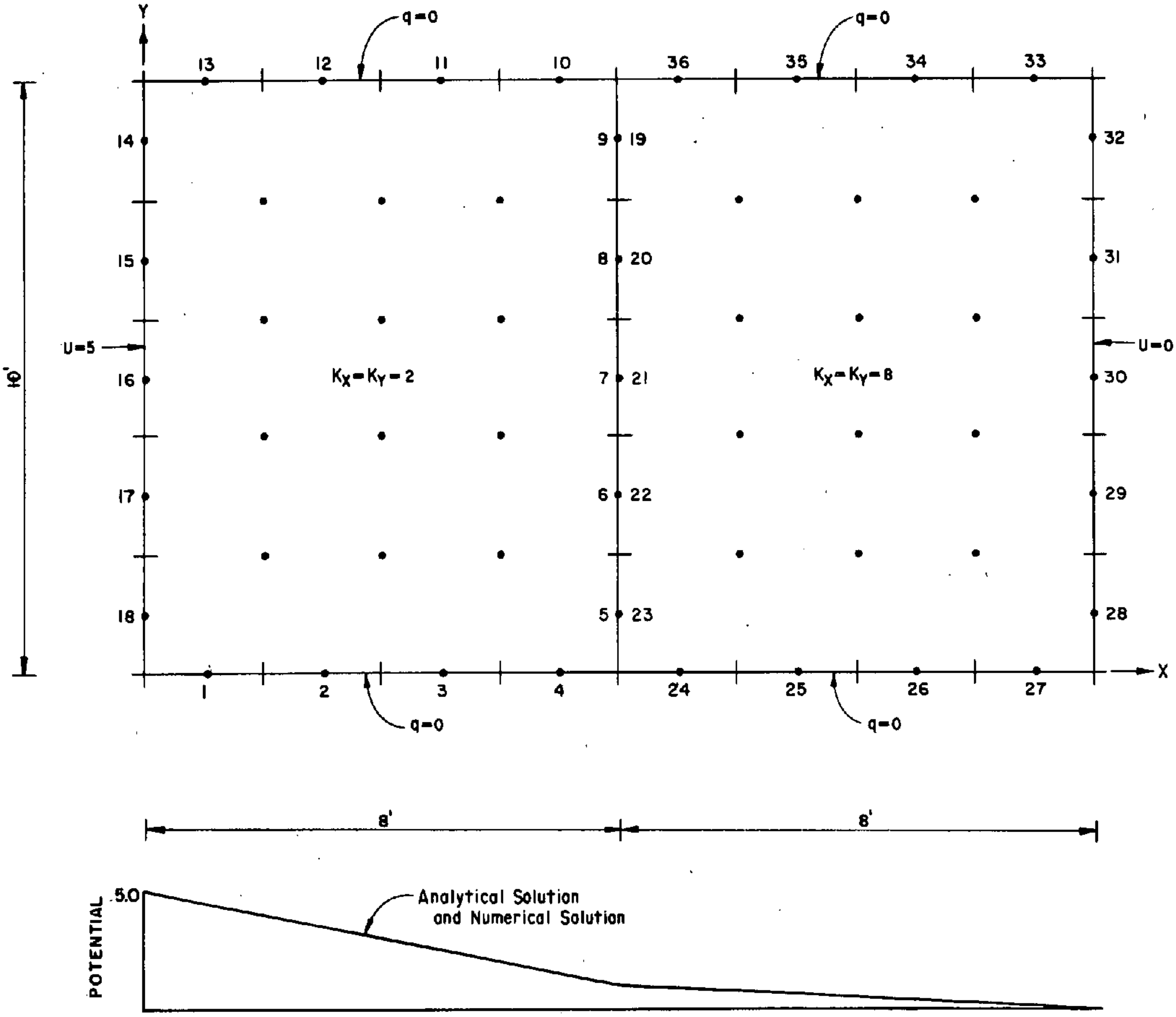



Figure 8.—Sample problem No. 2 – Comparison of boundary element solution with the known analytic solution.

| Boundary Element No. | Pressure head, $h_p$ feet | Gradient |
|---|---|---|
| 1 | 85.84 | 0 |
| 2 | 85.55 | 0 |
| 3 | 84.94 | 0 |
| 4 | 83.99 | 0 |
| 5 | 82.64 | 0 |
| 6 | 80.83 | 0 |
| 7 | 78.61 | 0 |
| 8 | 72.50 | -.253 |
| 9 | 62.50 | -.271 |
| 10 | 52.50 | -.629 |
| 11 | 47.97 | 0 |
| 12 | 41.26 | 0 |
| 13 | 33.85 | 0 |
| 14 | 30.00 | .251 |
| 15 | 30.00 | .211 |
| 16 | 30.00 | .182 |
| 17 | 30.00 | .152 |
| 18 | 30.00 | .130 |
| 19 | 30.00 | .113 |
| 20 | 30.00 | .114 |
| 21 | 34.49 | 0 |
| 22 | 43.44 | 0 |
| 23 | 52.50 | 0 |
| 24 | 61.73 | 0 |
| 25 | 71.19 | 0 |
| 26 | 80.90 | 0 |
| 27 | 76.38 | 0 |
| 28 | 74.17 | 0 |
| 29 | 72.36 | 0 |
| 30 | 71.01 | 0 |
| 31 | 70.06 | 0 |
| 32 | 69.46 | 0 |
| 33 | 69.16 | 0 |
| 34 | 64.10 | 0 |
| 35 | 53.82 | 0 |
| 36 | 43.27 | 0 |
| 37 | 32.50 | 0 |
| 38 | 21.56 | 0 |
| 39 | 10.51 | 0 |
| 40 | 5.00 | -.114 |
| 41 | 5.00 | -.114 |
| 42 | 5.00 | -.130 |
| 43 | 5.00 | -.153 |
| 44 | 5.00 | -.182 |
| 45 | 5.00 | -.211 |
| 46 | 5.00 | -.250 |
| 47 | 11.14 | 0 |
| 48 | 23.74 | 0 |
| 49 | 37.03 | 0 |
| 50 | 52.50 | .629 |
| 51 | 62.50 | .271 |
| 52 | 72.50 | .253 |

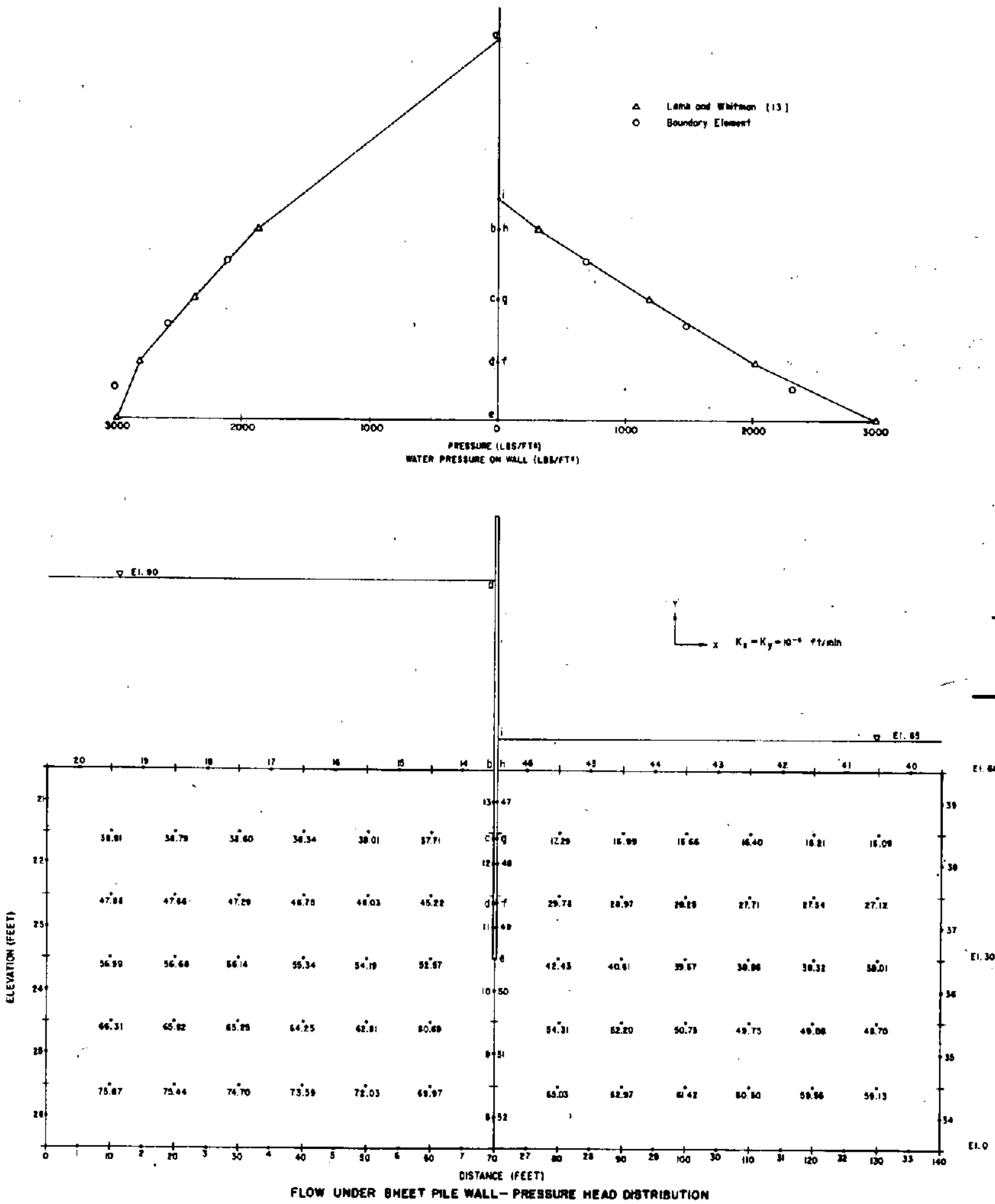
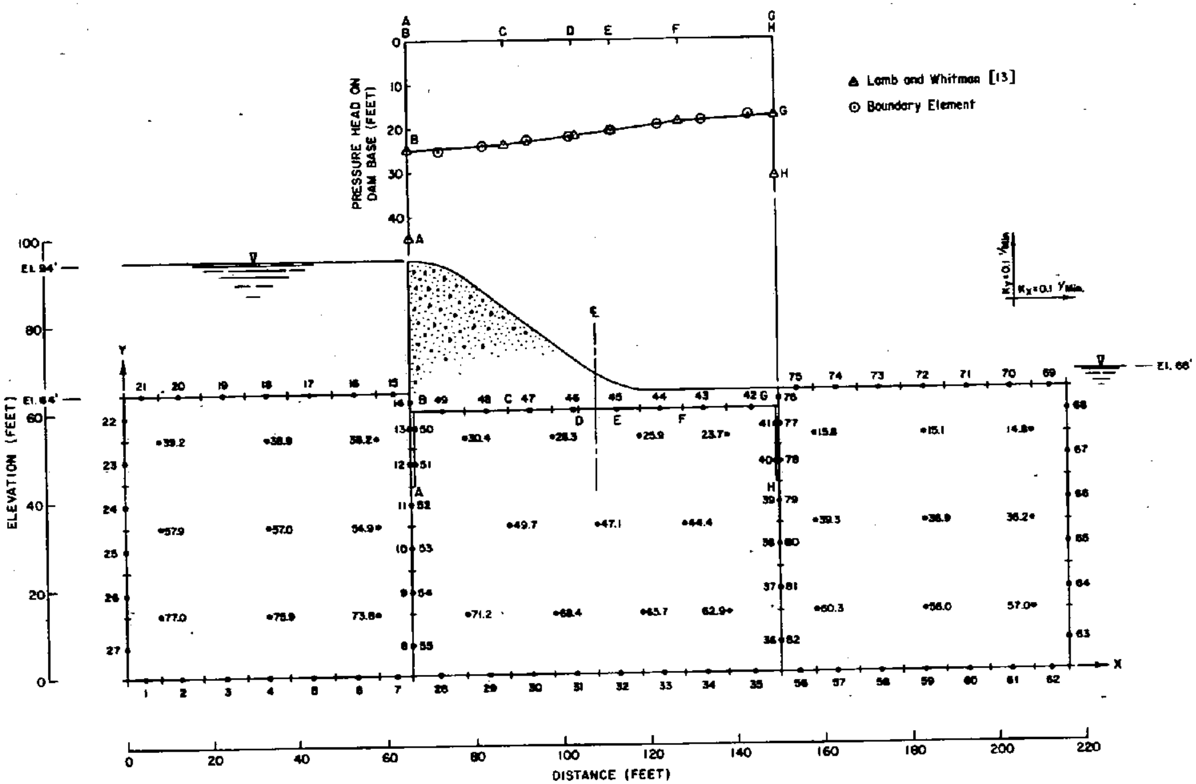



Figure 9.—Sample problem No. 3 — Pressure head distribution under sheet pile well.




| Boundary Element No. | Pressure head, $h_p$ ft | Gradient $\frac{\partial U}{\partial n}$ |
|---|---|---|
| 1 | 90.88 | 0 |
| 2 | 90.74 | 0 |
| 3 | 90.40 | 0 |
| 4 | 89.87 | 0 |
| 5 | 89.13 | 0 |
| 6 | 88.18 | 0 |
| 7 | 87.16 | 0 |
| 8,55 | 79.71 | -0.129 |
| 9,54 | 67.91 | -0.136 |
| 10,53 | 58.20 | -0.160 |
| 11,52 | 49.09 | -0.344 |
| 12 | 43.32 | 0 |
| 13 | 36.72 | 0 |
| 14 | 31.67 | 0 |
| 15 | 30.0 | 0.186 |
| 16 | 30.0 | 0.161 |
| 17 | 30.0 | 0.131 |
| 18 | 30.0 | 0.107 |
| 19 | 30.0 | 0.091 |
| 20 | 30.0 | 0.079 |
| 21 | 30.0 | 0.085 |
| 22 | 34.64 | 0 |
| 23 | 43.88 | 0 |
| 24 | 53.20 | 0 |
| 25 | 62.63 | 0 |
| 26 | 72.22 | 0 |
| 27 | 83.94 | 0 |
| 28 | 85.96 | 0 |
| 29 | 84.49 | 0 |
| 30 | 83.12 | 0 |
| 31 | 81.75 | |
| 32 | 80.38 | |
| 33 | 79.01 | |
| 34 | 77.65 | |
| 35 | 76.10 | |
| 36,82 | 68.29 | -0.129 |
| 37,81 | 56.09 | -0.136 |
| 38,80 | 45.60 | -0.160 |
| 39,70 | 35.91 | -0.344 |
| 40 | 29.18 | 0 |
| 41 | 21.37 | 0 |
| 42 | 17.33 | 0 |
| 43 | 18.21 | 0 |
| 44 | 19.28 | 0 |
| 45 | 20.46 | 0 |
| 46 | 21.66 | 0 |
| 47 | 22.83 | 0 |
| 48 | 23.88 | 0 |
| 49 | 24.70 | 0 |
| 50 | 29.12 | 0 |
| 51 | 38.32 | 0 |
| 56 | 74.91 | 0 |
| 57 | 73.93 | 0 |
| 58 | 72.96 | 0 |
| 59 | 72.20 | 0 |
| 60 | 71.64 | 0 |
| 61 | 71.23 | 0 |
| 62 | 71.12 | 0 |
| 63 | 64.06 | 0 |
| 64 | 51.79 | 0 |
| 65 | 41.37 | 0 |
| 66 | 30.80 | 0 |
| 67 | 20.12 | 0 |
| 68 | 9.36 | 0 |
| 69 | 4.0 | -0.082 |
| 70 | 4.0 | -0.081 |
| 71 | 4.0 | -0.092 |
| 72 | 4.0 | -0.109 |
| 73 | 4.0 | -0.134 |
| 74 | 4.0 | -0.164 |
| 75 | 4.0 | -0.188 |
| 76 | 6.34 | 0 |
| 77 | 13.78 | 0 |
| 78 | 24.18 | 0 |

Figure 10.—Sample problem No. 4 — Pressure head distribution under a concrete spillway.

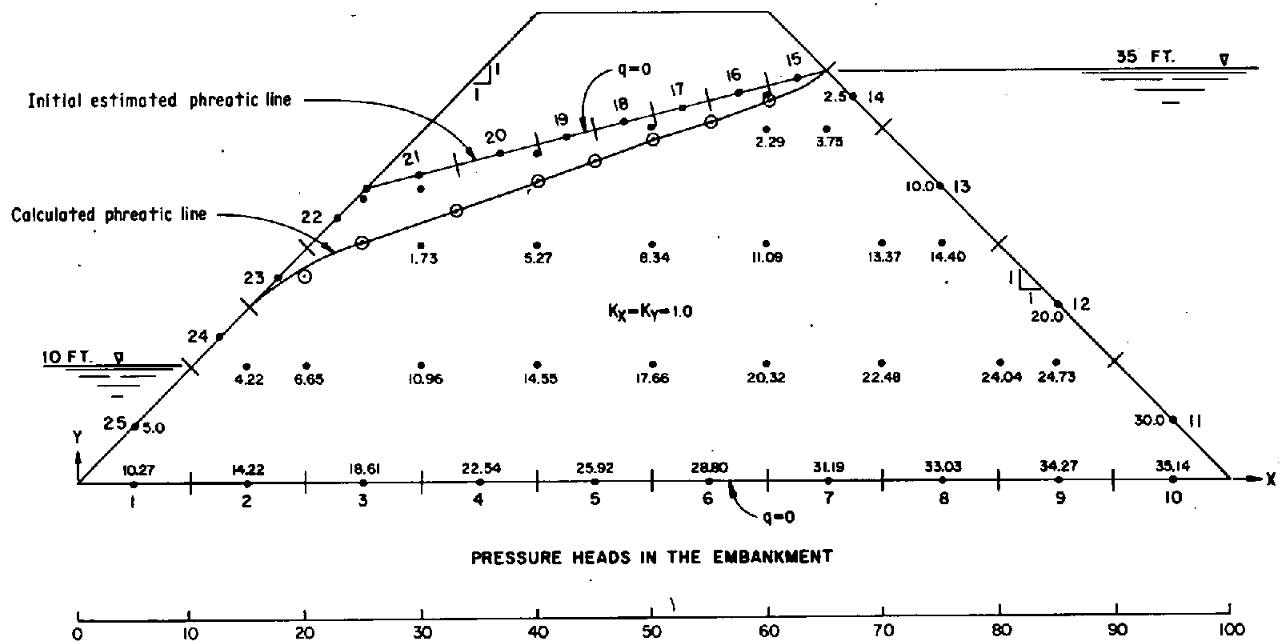



Figure 11.—Sample problem No. 5 – Pressure head distribution in an embankment dam.

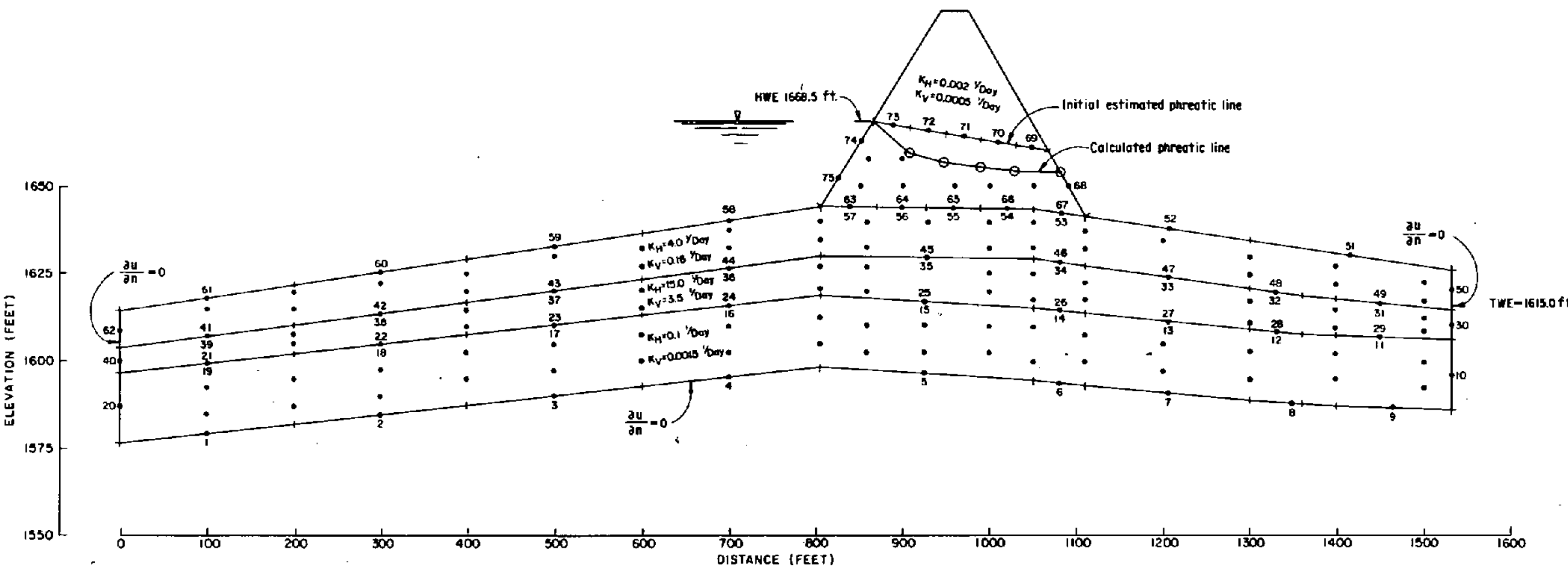



Figure 12.—Sample problem No. 6 — Steady-state seepage through the dam and its foundation zones.

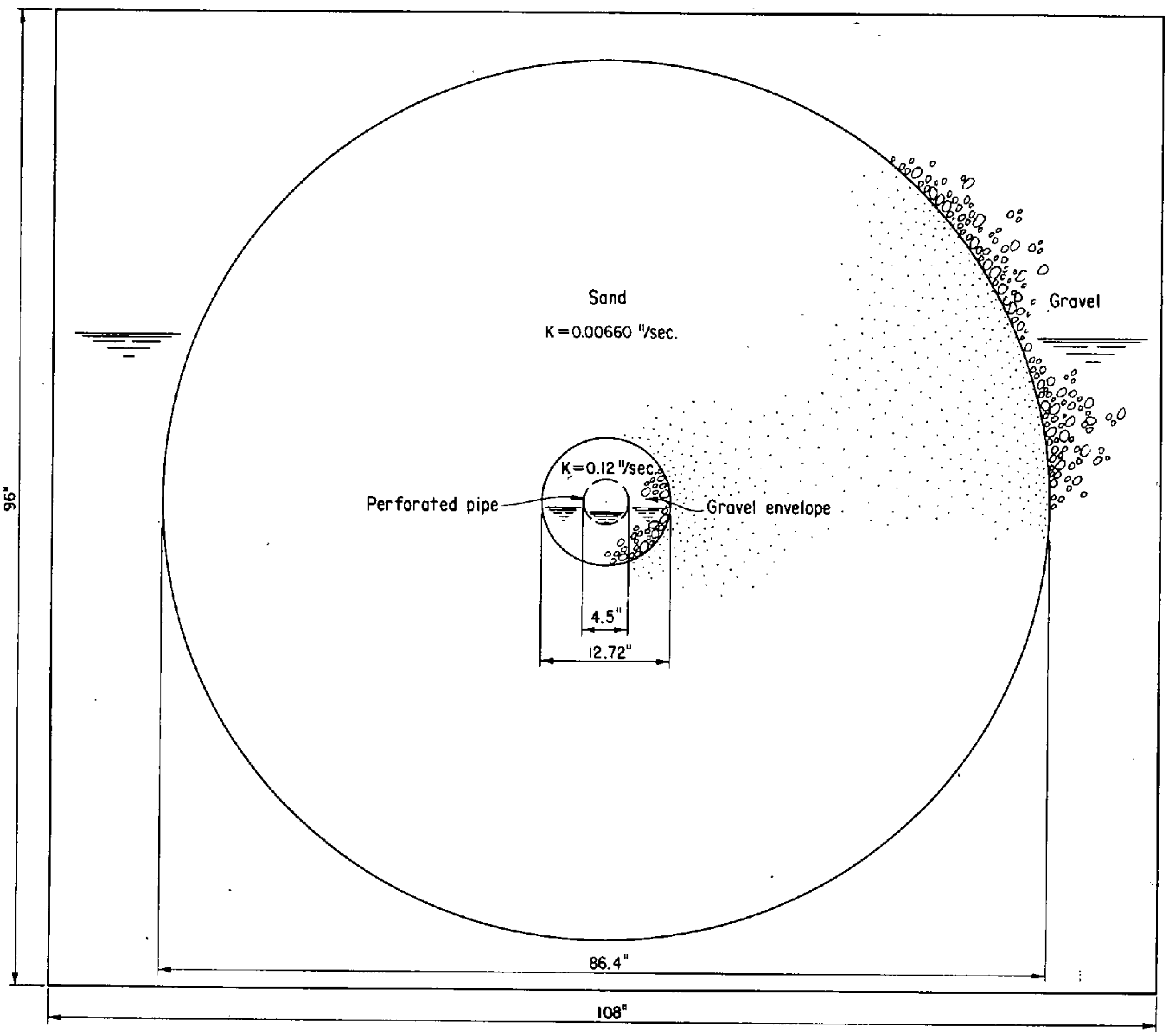



Figure 13.—Sample problem No. 7 — Laboratory model of a drain with gravel envelope.

| Location Identification | Potential in Feet of Head Above Datum of Elevation 3.939 Feet | |
|---|---|---|
| | Calculated | Measured |
| C1-R2* | .04658 | .032 |
| R3 | .0465 | .033 |
| R4 | .047 | .038 |
| R5 | .04883 | .046 |
| R6 | .05517 | .058 |
| R7 | .11242 | .178 |
| R8 | .2375 | .268 |
| C2-R2 | .21625 | .222 |
| R3 | .2177 | .213 |
| R4 | .2202 | .214 |
| R5 | .2268 | .215 |
| R6 | .2418 | .238 |
| R7 | .2787 | .317 |
| R8 | .3244 | .377 |
| C3-R2 | .3883 | .433 |
| R3 | .3903 | .420 |
| R4 | .3942 | .409 |
| R5 | .4012 | .414 |
| R6 | .4141 | .414 |
| R7 | .4338 | .466 |
| R8 | .4495 | .500 |
| C4-R2 | .5535 | .594 |
| R3 | .5561 | .579 |
| R4 | .5597 | .579 |
| R5 | .5656 | .578 |
| R6 | .5743 | .591 |
| R7 | .5846 | .609 |
| R8 | .5952 | .614 |
| C5-R2 | .7268 | .735 |
| R3 | .7291 | .736 |
| R4 | .7315 | .728 |
| R5 | .7352 | .743 |
| R6 | .7397 | .751 |
| R7 | .7432 | .753 |
| R8 | .7432 | .745 |
| C6-R2 | .8963 | .880 |
| R3 | .8978 | .883 |
| R4 | .8996 | .892 |
| R5 | .9014 | .897 |
| R6 | .9024 | .913 |
| R7 | .9013 | .914 |
| R8 | .8947 | .902 |
| C7-R2 | 1.0639 | not measured |
| R3 | 1.0648 | not measured |
| R4 | 1.0657 | 1.058 |
| R5 | 1.0663 | 1.055 |
| R6 | 1.0657 | 1.074 |
| R7 | 1.0618 | 1.077 |
| R8 | 1.0514 | 1.072 |
| C8-R2 | 1.232 | 1.248 |
| R3 | 1.2323 | 1.249 |
| R4 | 1.2327 | not measured |
| R5 | 1.2328 | not measured |
| R6 | 1.2318 | not measured |
| R7 | 1.2283 | not measured |
| R8 | 1.218 | not measured |
| C9-R2 | 1.3994 | 1.409 |
| R3 | 1.3995 | 1.410 |
| R4 | 1.3994 | not measured |
| R5 | 1.3995 | not measured |
| R6 | 1.3993 | 1.417 |
| R7 | 1.3988 | not measured |
| R8 | 1.3966 | 1.423 |

* $C_r$-$R_\theta$ is the intersection of circumferential path r and radial line θ. Radial lines are spaced at 15° intervals. R1 extends vertically downward from the drain centerline. Circumferential paths are located at radial distances of 0.54, 0.68, 0.86, 1.08, 1.37, 1.73, 2.18, 2.75, and 3.47 feet and are identified as C1, C2, C3, . . . C9 respectively.

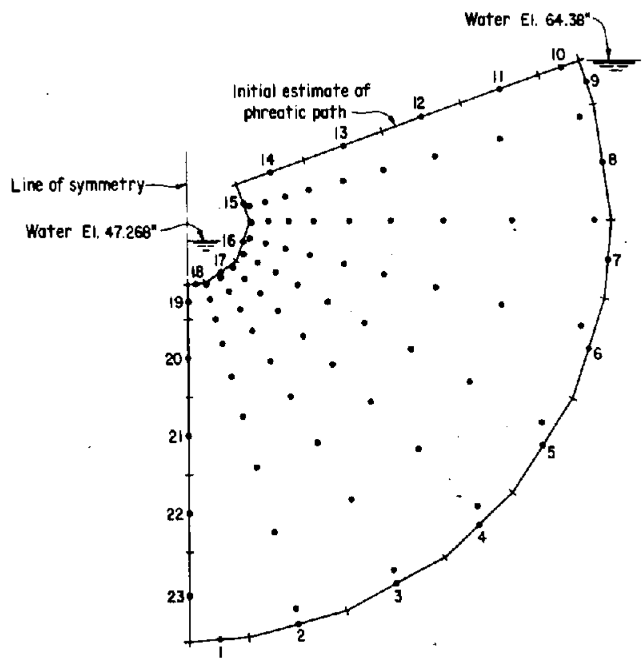



Figure 14.—Pressure head distribution for sample problem No. 7 (fig. 13).

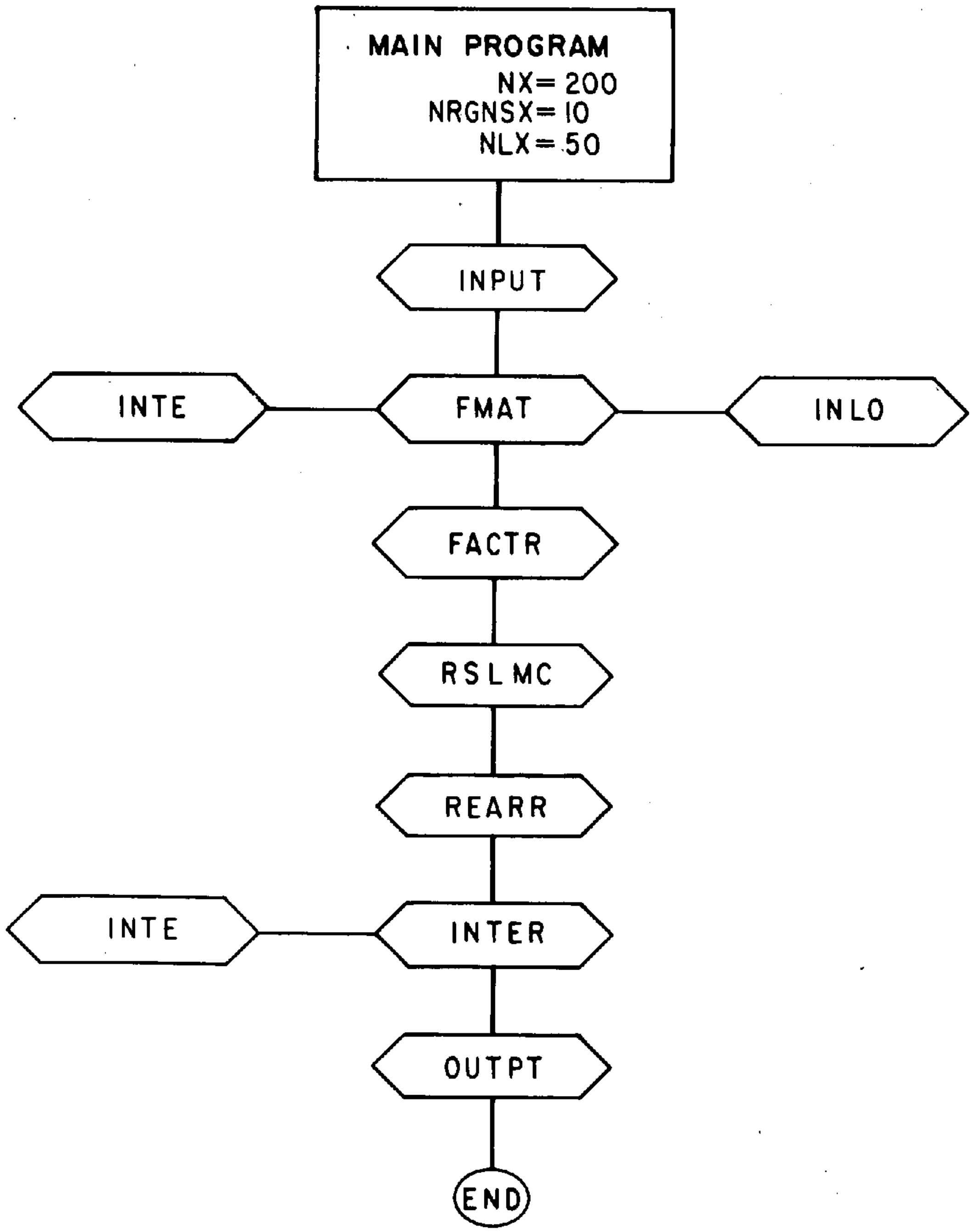



Figure 15.—Macroflow diagram.

## APPENDIX 1. – SOURCE LISTING OF THE COMPUTER PROGRAM BIE2DCP

### Introduction

The macroflow diagram for the boundary element computer program is shown on figure 15. The computer program BIE2DCP consists of one main program and nine subroutines. The main program defines the maximum dimensions of the system of equations (or boundary nodes) to 200, maximum number of zones to 10, and maximum number of interior points in any one zone to 50. It calls seven of the following nine subroutines.

INPUT: Reads the program input.

FMAT: Forms the two matrices H and G and rearranges them according to the boundary conditions to form the matrix A of equation (35).

INTE: Computes the values of the off-diagonal elements of the H and G matrices by means of numerical integration along the boundary elements.

INLO: Computes the values of the diagonal elements of the G matrix.

FACTR: Factors the matrix A into a product of a lower triangular matrix and an upper triangular matrix. The lower triangular matrix has unit diagonal which is not stored.

RSLMC: Solves the system of linear equations AX = B when the coefficient matrix A has been factored into a product of two triangular matrices by the subroutine FACTR.

REARR: Reorders the computed solution to correspond to the boundary parameters of each zone.

INTER: Computes the values of the potential at the selected internal points.

OUTPT: Outputs the results.

Subroutines INTE and INLO are called by the subroutine FMAT. Also, subroutine INTER calls the subroutine INTE.

PROGRAM BIE2DCP

```
C***********************************************************************BIE2DCP
C***********************************************************************BIE2DCP
C**********  2.D.POTENTIAL PROGRAM (CONSTANT ELEMENTS) B.E.M.  *********BIE2DCP
C**********  FOR A ZONED MEDIUM WITH PHREATIC LINE SEARCH      *********BIE2DCP
C***********************************************************************BIE2DCP
C***********************************************************************BIE2DCP
C                                                                       BIE2DCP
C  PROGRAM FOR SOLUTION OF TWO DIMENSIONAL POTENTIAL PROBLEMS           BIE2DCP
C  BY THE B.I.E. METHOD WITH CONSTANT ELEMENTS                          BIE2DCP
C                                                                       BIE2DCP
      PROGRAM BIE2DCP (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)           BIE2DCP
      COMMON N                                                          BIE2DCP
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)BIE2DCP
     1             ,NEQNS,NL(10),EFFP(10)                               BIE2DCP
      COMMON /ADD2/ XM(200),YM(200)                                     BIE2DCP
      COMMON /ADD3/ PERMX(10),PERMY(10),XT(200),YT(200),CXT(10,050),    BIE2DCP
     1              CYT(10,050),XMT(200),YMT(200)                       BIE2DCP
      COMMON /ADD4/ NPHREL,IDPHR(25),MALIK,ACC,ITRMAX                   BIE2DCP
      COMMON /BIE1/ X(200),Y(200)                                       BIE2DCP
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE                            BIE2DCP
      DIMENSION G(200,200),FIP(200)                                     BIE2DCP
      DIMENSION KODE(200),CX(10,050),CY(10,050),SOL(10,050),H(200,200)  BIE2DCP
      DIMENSION WA(200),FID(500),DFI(500)                               BIE2DCP
      DIMENSION G2(200,200),PER(200),F(200)                             BIE2DCP
      EQUIVALENCE (G2(1,1),H(1,1)),(WA(1),FID(1))                       BIE2DCP
C                                                                       BIE2DCP
C  INITIALIZATION OF PROGRAM PARAMETERS                                 BIE2DCP
C  NX= MAXIMUM DIMENSION OF THE SYSTEM OF EQUATIONS.                    BIE2DCP
C  NRGNSX= MAXIMUM DIMENSION FOR THE NUMBER OF REGIONS.                 BIE2DCP
C  NLX= MAXIMUM DIMENSION FOR THE NUMBER OF INTERIOR POINTS IN EACH REGIBIE2DCP
C                                                                       BIE2DCP
      NX=200                                                            BIE2DCP
      NRGNSX=10                                                         BIE2DCP
      NLX=050                                                           BIE2DCP
      ITER=0                                                            BIE2DCP
      ITRMAX=10                                                         BIE2DCP
      DO 6 I=1,NX                                                       BIE2DCP
      FIP(I)=0.                                                         BIE2DCP
    6 CONTINUE                                                          BIE2DCP
C                                                                       BIE2DCP
C  INPUT                                                                BIE2DCP
C                                                                       BIE2DCP
   10 CALL INPUT(CX,CY,KODE)                                            BIE2DCP
C                                                                       BIE2DCP
C  FORM SYSTEM OF EQUATIONS                                             BIE2DCP
C                                                                       BIE2DCP
      CALL FMAT(XT,YT,XMT,YMT,G,H,DFI,KODE,NX,N)                        BIE2DCP
C                                                                       BIE2DCP
C  SOLUTION OF THE SYSTEM OF EQUATIONS                                  BIE2DCP
C                                                                       BIE2DCP
      DO 15 I=1,NEQNS                                                   BIE2DCP
      DO 15 J=1,NEQNS                                                   BIE2DCP
      G2(I,J)=G(I,J)                                                    BIE2DCP
   15 CONTINUE                                                          BIE2DCP
```

```
      CALL FACTR(G,PER,NEQNS,NX,IER1,NX*NEQNS)                          BIE2DCP  55
      EPSI=1.E-06                                                       BIE2DCP  56
      CALL RSLMC(G2,G,DFI,F,NEQNS,EPSI,IER2,NX,WA,PER,NX*NEQNS)         BIE2DCP  57
      DO 16 I=1,NEQNS                                                   BIE2DCP  58
      DFI(I)=F(I)                                                       BIE2DCP  59
   16 CONTINUE                                                          BIE2DCP  60
      WRITE (6,21) IER1,IER2                                            BIE2DCP  61
   21 FORMAT (1H0,'ERROR INDICATORS FROM THE EQUATION SOLVER = ',       BIE2DCP  62
     1I5,5X,I5)                                                         BIE2DCP  63
C                                                                       BIE2DCP  64
C     REARRANGE THE SOLUTION VECTOR                                     BIE2DCP  65
C                                                                       BIE2DCP  66
      CALL REARR(DFI,FID,KODE)                                          BIE2DCP  67
C                                                                       BIE2DCP  68
C  COMPUTE THE POTENTIAL VALUES FOR INTERNAL POINTS                     BIE2DCP  69
C                                                                       BIE2DCP  70
      CALL INTER(DFI,KODE,CXT,CYT,XT,YT,SOL)                            BIE2DCP  71
C                                                                       BIE2DCP  72
C  OUTPUT                                                               BIE2DCP  73
C                                                                       BIE2DCP  74
      CALL OUTPT(XM,YM,DFI,CX,CY,SOL)                                   BIE2DCP  75
    1 FORMAT (1H0,70H***************************************************BIE2DCP  76
     1*******************//)                                            BIE2DCP  77
    2 FORMAT (1H0,49HPHREATIC LINE ELEMENTS GEOMETRY IS BEING ADJUSTED, BIE2DCP  78
     15X,13HITERATION NO.,I4)                                           BIE2DCP  79
    3 FORMAT (1H0,45HREADJUSTED PHREATIC LINE ELEMENT COORDINATES //)   BIE2DCP  80
    4 FORMAT (1H0,22H           X             Y//)                      BIE2DCP  81
    5 FORMAT (1H0,2F11.3)                                               BIE2DCP  82
      MALIK=0                                                           BIE2DCP  83
      IF (NPHREL.LE.0) GO TO 200                                        BIE2DCP  84
      DO 100 I=1,NPHREL                                                 BIE2DCP  85
      J=IDPHR(I)                                                        BIE2DCP  86
      DIFF=ABS(FI(J)-FIP(J))                                            BIE2DCP  87
      IF (DIFF.LE.ACC) GO TO 100                                        BIE2DCP  88
      MALIK=1                                                           BIE2DCP  89
  100 CONTINUE                                                          BIE2DCP  90
      ITER=ITER+1                                                       BIE2DCP  91
      IF (MALIK.LE.0.OR.ITER.GT.ITRMAX) GO TO 200                       BIE2DCP  92
      MALIK=0                                                           BIE2DCP  93
      WRITE (6,1)                                                       BIE2DCP  94
      WRITE (6,2) ITER                                                  BIE2DCP  95
      WRITE (6,1)                                                       BIE2DCP  96
      WRITE (6,3)                                                       BIE2DCP  97
      WRITE (6,4)                                                       BIE2DCP  98
      SIGN=Y(IDPHR(1))-Y(IDPHR(NPHREL))                                 BIE2DCP  99
C                                                                       BIE2DCP 100
C     IF SIGN.LT.0.; Y(IDPHR(1)) NEEDS TO BE ADJUSTED                   BIE2DCP 101
C     IF SIGN.GT.0.; Y(IDPHR(NPHREL)) NEEDS TO BE ADJUSTED              BIE2DCP 102
C                                                                       BIE2DCP 103
      IF (SIGN.LT.0.) GO TO 160                                         BIE2DCP 104
      DO 150 I=1,NPHREL                                                 BIE2DCP 105
      J=IDPHR(I)                                                        BIE2DCP 106
      K=J+1                                                             BIE2DCP 107
      DIFF=ABS(FI(J)-FIP(J))                                            BIE2DCP 108
```

PROGRAM BIE2DCP

```
      YDIFF=Y(K)-(FI(J)+TWE)                                            BIE2DCP 109
      SLOPA=(Y(K)-Y(K+1))/(X(K)-X(K+1))                                 BIE2DCP 110
      IF (DIFF.GT.ACC) MALIK=1                                          BIE2DCP 111
      Y(K)=(0.5*(FI(J)+FI(K)))+TWE                                      BIE2DCP 112
      IF (I.LT.NPHREL) GO TO 150                                        BIE2DCP 113
      Y(K)=FI(J)+TWE                                                    BIE2DCP 114
      X(K)=X(K)-(YDIFF/SLOPA)                                           BIE2DCP 115
      FIS(K)=0.5*(Y(K)+Y(K+1))-TWE                                      BIE2DCP 116
  150 CONTINUE                                                          BIE2DCP 117
      GO TO 175                                                         BIE2DCP 118
  160 DO 170 I=1,NPHREL                                                 BIE2DCP 119
      II=NPHREL-I+1                                                     BIE2DCP 120
      J=IDPHR(II)                                                       BIE2DCP 121
      K=J-1                                                             BIE2DCP 122
      DIFF=ABS(FI(J)-FIP(J))                                            BIE2DCP 123
      YDIFF=Y(J)-(FI(J)+TWE)                                            BIE2DCP 124
      SLOPA=(Y(J)-Y(K))/(X(K)-X(J))                                     BIE2DCP 125
      IF (DIFF.GT.ACC) MALIK=1                                          BIE2DCP 126
      Y(J)=(0.5*(FI(J)+FI(K)))+TWE                                      BIE2DCP 127
      IF (I.LT.NPHREL) GO TO 170                                        BIE2DCP 128
      Y(J)=FI(J)+TWE                                                    BIE2DCP 129
      X(J)=X(J)+(YDIFF/SLOPA)                                           BIE2DCP 130
      FIS(K)=0.5*(Y(K)+Y(J))-TWE                                        BIE2DCP 131
  170 CONTINUE                                                          BIE2DCP 132
  175 DO 178 I=1,NPHREL                                                 BIE2DCP 133
      J=IDPHR(I)                                                        BIE2DCP 134
      K=J                                                               BIE2DCP 135
      IF (SIGN.GT.0.) K=J+1                                             BIE2DCP 136
      WRITE (6,5) X(K),Y(K)                                             BIE2DCP 137
  178 CONTINUE                                                          BIE2DCP 138
      DO 180 I=1,NEQNS                                                  BIE2DCP 139
      FIP(I)=FI(I)                                                      BIE2DCP 140
      FI(I)=0.                                                          BIE2DCP 141
      DFI(I)=0.                                                         BIE2DCP 142
  180 CONTINUE                                                          BIE2DCP 143
      DO 190 I=1,N                                                      BIE2DCP 144
      FI(I)=FIS(I)                                                      BIE2DCP 145
  190 CONTINUE                                                          BIE2DCP 146
      IF (MALIK.EQ.0) GO TO 200                                         BIE2DCP 147
      GO TO 10                                                          BIE2DCP 148
  200 STOP                                                              BIE2DCP 149
      END                                                               BIE2DCP 150
```

SUBROUTINE INPUT

```
      SUBROUTINE INPUT(CX,CY,KODE)                                      INPUT     1
C                                                                       INPUT     2
C  THIS SUBROUTINE READS THE INPUT DATA                                 INPUT     3
C                                                                       INPUT     4
C  N= NUMBER OF BOUNDARY ELEMENTS                                       INPUT     5
C  L= NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED        INPUT     6
```

SUBROUTINE INPUT

```
C
      COMMON N
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)
     1              ,NEQNS,NL(10),EFFP(10)
      COMMON /ADD2/ XM(200),YM(200)
      COMMON /ADD3/ PERMX(10),PERMY(10),XT(200),YT(200),CXT(10,050),
     1              CYT(10,050),XMT(200),YMT(200)
      COMMON /ADD4/ NPHREL,IDPHR(25),MALIK,ACC,ITRMAX
      COMMON /BIE1/ X(200),Y(200)
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE
      DIMENSION CX(10,050),CY(10,050),KODE(1),TITLE(18)
      IF (MALIK.EQ.1) GO TO 1400
      WRITE(6,100)
  100 FORMAT(' ',120('*'))
C
C  READ NAME OF THE JOB
C
      READ(5,150) TITLE
  150 FORMAT(18A4)
      WRITE(6,250) TITLE
  250 FORMAT(25X,18A4)
C
C  READ BASIC PARAMETERS
C
      READ(5,200)N,NRGNS
  200 FORMAT(2I5)
      WRITE(6,300)N,NRGNS
  300 FORMAT(//' DATA'//2X,'NUMBER OF BOUNDARY ELEMENTS=',I3/2X,'NUMBER
     1OF REGIONS = ',I3)
C
C  READ INTERNAL POINTS COORDINATES
C
      DO 2 INDU=1,NRGNS
      READ (5,200) NL(INDU)
      L=NL(INDU)
      DO 1 I=1,L
    1 READ(5,400) CX(INDU,I),CY(INDU,I)
    2 CONTINUE
  400 FORMAT(2F10.4)
C
C  READ COORDINATES OF EXTREME POINTS OF THE BOUNDARY ELEMENTS
C   IN ARRAY X AND Y
C
      WRITE(6,500)
  500 FORMAT(//2X,'COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELE
     1MENTS',//4X,'POINT',10X,'X',18X,'Y')
      DO 10 I=1,N
      READ(5,600) X(I),Y(I)
  600 FORMAT(2F10.4)
   10 WRITE(6,700)I,X(I),Y(I)
  700 FORMAT(5X,I3,2(5X,E14.7))
C
C  READ BOUNDARY CONDITIONS
C  FI(I)= VALUE OF THE POTENTIAL IN THE NODE I IF KODE=0,
```

```
C   VALUE OF THE POTETIAL DERIVATIVE IF KODE=1.                         INPUT  61
C   KODE(J)=2 IMPLIES THAT THE NODE IS ON THE INTERFACE AND NEITHER     INPUT  62
C   THE POTENTIAL NOR ITS DERIVATIVE IS KNOWN. FI(I) IN THIS CASE MUST BEINPUT  63
C                                                                       INPUT  64
      WRITE(6,800)                                                      INPUT  65
  800 FORMAT(//2X,'BOUNDARY CONDITIONS'//5X,'NODE',6X,'CODE',5X,'PRESCRIINPUT  66
     1BED VALUE')                                                       INPUT  67
      DO 20 I=1,N                                                       INPUT  68
      READ(5,900) KODE(I),FI(I)                                         INPUT  69
  900 FORMAT(I5,F10.4)                                                  INPUT  70
   20 WRITE(6,950)I,KODE(I),FI(I)                                       INPUT  71
  950 FORMAT(5X,I3,8X,I1,8X,E14.7)                                      INPUT  72
      NEQNS=N                                                           INPUT  73
 1000 FORMAT (8I10)                                                     INPUT  74
 1050 FORMAT (2F10.2)                                                   INPUT  75
      DO 1100 I=1,NRGNS                                                 INPUT  76
      READ (5,1050) PERMX(I),PERMY(I)                                   INPUT  77
      READ (5,1000) ISTART(I),IEND(I)                                   INPUT  78
      DO 1060 J=1,NRGNS                                                 INPUT  79
      IF (J.EQ.I) GO TO 1060                                            INPUT  80
      READ (5,1000) NINTF(I,J)                                          INPUT  81
      NINT=NINTF(I,J)                                                   INPUT  82
      IF (NINT.LE.0) GO TO 1060                                         INPUT  83
      READ (5,1000) (ID(I,J,K),K=1,NINT)                                INPUT  84
 1060 CONTINUE                                                          INPUT  85
 1100 CONTINUE                                                          INPUT  86
      WRITE (6,1200)                                                    INPUT  87
 1200 FORMAT (1H0,10HREGION NO.,5X,17HSTARTING ELEM NO.,5X,15HENDING ELEINPUT  88
     1M NO.,5X,8HPERM-^X^,5X,8HPERM-^Y^//)                              INPUT  89
      DO 1250 I=1,NRGNS                                                 INPUT  90
      WRITE (6,1300) I,ISTART(I),IEND(I),PERMX(I),PERMY(I)              INPUT  91
 1250 CONTINUE                                                          INPUT  92
 1300 FORMAT (1H0,3X,I3,16X,I3,16X,I3,9X,E10.4,3X,E10.4,9(I3,2X))       INPUT  93
 1304 FORMAT (1H0,64HZONE NO.     INTERFACED ZONE NO.           INTERFACEINPUT  94
     1 ELEMENT NOS.//)                                                  INPUT  95
 1305 FORMAT (1H0,I5,12X,I5,15X,16I5/37X,16I5)                          INPUT  96
 1306 FORMAT (1H0,41HPHREATIC LINE ELEMENTS IDENTIFICATION NO.//)       INPUT  97
 1307 FORMAT (1H0,15I5)                                                 INPUT  98
 1308 FORMAT (2I10,8F10.0)                                              INPUT  99
 1309 FORMAT (1H0,95HNO. OF BOUNDARY EL. ON PHREATIC LINE     MAX. ITERAINPUT 100
     1TIONS         ACCURACY                                     //)    INPUT 101
 1310 FORMAT (1H0,10X,I2,36X,I2,16X,F5.2,4(3X,F8.2))                    INPUT 102
 1311 FORMAT (1H0,28H           HWE           TWE  //)                  INPUT 103
 1312 FORMAT (1H0,05X,8(F8.2,5X))                                       INPUT 104
      IF (NRGNS.LT.2) GO TO 1280                                        INPUT 105
      WRITE (6,1304)                                                    INPUT 106
      DO 1270 I=1,NRGNS                                                 INPUT 107
      DO 1260 J=1,NRGNS                                                 INPUT 108
      NINT=NINTF(I,J)                                                   INPUT 109
      IF (NINT.LE.0) GO TO 1260                                         INPUT 110
      WRITE (6,1305) I,J,(ID(I,J,K),K=1,NINT)                           INPUT 111
 1260 CONTINUE                                                          INPUT 112
 1270 CONTINUE                                                          INPUT 113
 1280 READ (5,1308) NPHREL,ITRMAX,ACC,HWE,TWE                           INPUT 114
```

SUBROUTINE INPUT

```
      WRITE (6,1311)
      WRITE (6,1312) HWE,TWE
      IF (NPHREL.LE.0) GO TO 1400
      READ (5,1000) (IDPHR(I),I=1,NPHREL)
      WRITE (6,1309)
      WRITE (6,1310) NPHREL,ITRMAX,ACC
      WRITE (6,1306)
      WRITE (6,1307) (IDPHR(I),I=1,NPHREL)
 1400 DO 1500 INDU=1,NRGNS
      L=NL(INDU)
      RATIOP=PERMY(INDU)/PERMX(INDU)
      RATIOX=SQRT(RATIOP)
      RATIOY=1.
      PXY=PERMX(INDU)*PERMY(INDU)
      EFFP(INDU)=SQRT(PXY)
      DO 1480 I=1,L
      CXT(INDU,I)=CX(INDU,I)*RATIOX
      CYT(INDU,I)=CY(INDU,I)*RATIOY
 1480 CONTINUE
C
C  SINCE RATIOY=1., THE TAIL WATER ELEVATION IS NOT BEING
C  TRANSFORMED
C
      IS=ISTART(INDU)
      IE=IEND(INDU)
      DO 1490 I=IS,IE
      XT(I)=X(I)*RATIOX
      YT(I)=Y(I)*RATIOY
      IF (I.EQ.IE) GO TO 1485
      XM(I)=(X(I)+X(I+1))/2.
      YM(I)=(Y(I)+Y(I+1))/2.
      GO TO 1490
 1485 XM(I)=(X(I)+X(IS))/2.
      YM(I)=(Y(I)+Y(IS))/2.
 1490 CONTINUE
 1500 CONTINUE
      IF (MALIK.EQ.1) RETURN
      DO 1550 I=1,N
      IF (KODE(I).EQ.0) FI(I)=FI(I)-TWE
      FIS(I)=FI(I)
 1550 CONTINUE
 1660 FORMAT (1H0,'TOTAL NUMBER OF EQUATIONS = ',I5)
      WRITE (6,1660) NEQNS
      RETURN
      END
```

SUBROUTINE FMAT

```
      SUBROUTINE FMAT(X,Y,XM,YM,G,H,DFI,KODE,NX,N)
C
C  THIS SUBROUTINE COMPUTES G AND H MATRICES AND FORM THE
```

```
C  SISTEM  A X = F
C
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)
     1              ,NEQNS,NL(10),EFFP(10)
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE
      DIMENSION X(1),Y(1),XM(1),YM(1),G(NX,NX),H(NX,NX),KODE(1)
      DIMENSION DFI(1)
C
C  COMPUTE THE MID-POINT COORDINATES AND STORE IN ARRAY XM AND YM
C
      DO 4 I=1,NEQNS
      DO 3 J=1,NEQNS
      G(I,J)=0.
      H(I,J)=0.
    3 CONTINUE
    4 CONTINUE
      DO 35 INDU=1,NRGNS
      IS=ISTART(INDU)
      IE=IEND(INDU)
      DO 10 I=IS,IE
      IF (I.EQ.IE) GO TO 5
      XM(I)=(X(I)+X(I+1))/2.
      YM(I)=(Y(I)+Y(I+1))/2.
      GO TO 10
    5 XM(I)=(X(I)+X(IS))/2.
      YM(I)=(Y(I)+Y(IS))/2.
   10 CONTINUE
C
C  COMPUTE G AND H MATRICES
C
      DO 32 I=IS,IE
      JC=IS-1
      DO 30 J=IS,IE
      JC=JC+1
      X1=X(J)
      Y1=Y(J)
      X2=X(IS)
      Y2=Y(IS)
      IF (J.LT.IE) X2=X(J+1)
      IF (J.LT.IE) Y2=Y(J+1)
      IF(I-J)20,25,20
   20 CALL INTE(XM(I),YM(I),X1,Y1,X2,Y2,HE,GE)
      GO TO 1000
   25 CALL INLO(X1,Y1,X2,Y2,GE)
      HE=3.1415926
C
C  CONSTRUCT THE G AND H MATRICES ENFORCING THE INTERFACE
C  CONSTRAINT CONDITIONS
C
 1000 IF (INDU.EQ.1.OR.KODE(J).NE.2) GO TO 2000
C
C  LOCATE THE INTERFACE ON WHICH J LIES
C
      DO 1200 IND=1,NRGNS
```

```
      IF (IND.EQ.INDU) GO TO 1200
      NINT=NINTF(INDU,IND)
      IF (NINT.EQ.0) GO TO 1200
      DO 1150 INDC=1,NINT
      IF (J.EQ.ID(INDU,IND,INDC)) GO TO 1160
 1150 CONTINUE
      GO TO 1200
 1160 JINT=ID(IND,INDU,INDC)
      SIGN=-1.
      IF (IND.GT.INDU) JINT=ID(INDU,IND,INDC)
      IF (IND.GT.INDU) SIGN=+1.
      H(I,JINT)=H(I,JINT)+HE
      G(I,JINT)=G(I,JINT)+(GE*EFFP(IND)/EFFP(INDU))*SIGN
      GO TO 30
 1200 CONTINUE
 2000 H(I,JC)=HE
      G(I,JC)=GE
   30 CONTINUE
   32 CONTINUE
   35 CONTINUE
C
C  ARRANGE THE SYSTEM OF EQUATIONS READY TO BE SOLVED
C
      DO 51 J=1,N
      IF (KODE(J).EQ.2) GO TO 51
      IF(KODE(J))51,51,40
   40 DO 50 I=1,N
      CH=G(I,J)
      G(I,J)=-H(I,J)
      H(I,J)=-CH
   50 CONTINUE
   51 CONTINUE
      IF (NRGNS.LE.1) GO TO 500
C
C  ELIMINATE THE ZERO COLUMN VECTORS
C
      JR=0
      DO 3200 J=1,N
      IC=0
      DO 3000 I=1,N
      IF (G(I,J).EQ.0) IC=IC+1
 3000 CONTINUE
      IF (IC.EQ.N) GO TO 3200
      JR=JR+1
      DO 3100 I=1,N
      G(I,JR)=G(I,J)
 3100 CONTINUE
 3200 CONTINUE
      JC=JR
      DO 100 I=1,NRGNS
      DO 90 J=1,NRGNS
      IF (I.GE.J) GO TO 90
      NINT=NINTF(I,J)
      IF (NINT.EQ.0) GO TO 90
```

SUBROUTINE FMAT

```
      DO 70 K=1,NINT
      JINT=ID(I,J,K)
      JC=JC+1
      DO 65 L=1,N
      G(L,JC)=-H(L,JINT)
   65 CONTINUE
   70 CONTINUE
   90 CONTINUE
  100 CONTINUE
C
C  DFI ORIGINALLY CONTAINS THE INDEPENDENT COEFFICIENTS,
C  AFTER SOLUTION IT WILL CONTAIN THE VALUES OF THE SYSTEM UNKNOWNS
C
  500 DO 60 I=1,N
      DFI(I)=0.
      DO 60 J=1,N
      DFI(I)=DFI(I)+H(I,J)*FI(J)
   60 CONTINUE
      RETURN
      END
```

SUBROUTINE INTE

```
      SUBROUTINE INTE(DXP,DYP,DX1,DY1,DX2,DY2,H,G)
C
C  THIS SUBROUTINE COMPUTES THE VALUES OF THE H AND G MATRIX
C  OFF DIAGONAL ELEMENTS BY MEANS OF NUMERICAL INTEGRATION
C  ALONG THE BOUNDARY ELEMENTS
C
C  DIST=DISTANCE FROM THE POINT UNDER CONSIDERATION TO THE
C  BOUNDARY ELEMENTS
C  RA=DISTANCE FROM THE POINT UNDER CONSIDERATION TO THE
C  INTEGRATION POINTS IN THE BOUNDARY ELEMENTS
C
      DIMENSION XCO(10),YCO(10),GI(10),OME(10)
      DOUBLE PRECISION X1,X2,Y1,Y2,XP,YP,HD,TA,XCO,YCO,GI,OME
      DOUBLE PRECISION AX,BX,AY,BY,DIST,GD,RA
      DOUBLE PRECISION DABS,DSQRT,DLOG
      X1=DX1
      Y1=DY1
      X2=DX2
      Y2=DY2
      XP=DXP
      YP=DYP
      GI(1)=0.973906528517172D0
      GI(2)=-GI(1)
      GI(3)=0.865063366688985D0
      GI(4)=-GI(3)
      GI(5)=0.679409568299024D0
      GI(6)=-GI(5)
      GI(7)=0.433395394129247D0
```

SUBROUTINE INTE

```
      GI(8)=-GI(7)
      GI(9)=0.14887433898163 1D0
      GI(10)=-GI(9)
      OME(1)=0.066671344308688D0
      OME(2)=OME(1)
      OME(3)=0.149451349150581D0
      OME(4)=OME(3)
      OME(5)=0.219086362515982D0
      OME(6)=OME(5)
      OME(7)=0.269266719309996D0
      OME(8)=OME(7)
      OME(9)=0.295524224714753D0
      OME(10)=OME(9)
      AX=(X2-X1)/2.D0
      BX=(X2+X1)/2.D0
      AY=(Y2-Y1)/2.D0
      BY=(Y2+Y1)/2.D0
      IF(AX)10,20,10
   10 TA=(Y2-Y1)/(X2-X1)
      DIST=DABS((TA*XP-YP+Y1-TA*X1)/DSQRT(TA**2+1.D0))
      GO TO 30
   20 DIST=DABS(XP-X1)
   30 SIG=(X1-XP)*(Y2-YP)-(X2-XP)*(Y1-YP)
      IF(SIG)31,32,32
   31 DIST=-DIST
   32 GD=0.D0
      HD=0.D0
      DO 40 I=1,10
      XCO(I)=AX*GI(I)+BX
      YCO(I)=AY*GI(I)+BY
      RA=DSQRT((XP-XCO(I))**2+(YP-YCO(I))**2)
      GD=GD+DLOG(1.D0/RA)*OME(I)*DSQRT(AX**2+AY**2)
   40 HD=HD-(DIST*OME(I)*DSQRT(AX**2+AY**2)/RA**2)
      H=HD
      G=GD
      RETURN
      END
```

SUBROUTINE INLO

```
      SUBROUTINE INLO(X1,Y1,X2,Y2,G)
C
C  THIS SUBROUTINE COMPUTES THE VALUES OF THE DIAGONAL
C  ELEMENTS OF THE G MATRIX
C
      AX=(X2-X1)/2
      AY=(Y2-Y1)/2
      SR=SQRT(AX**2+AY**2)
      G=2*SR*(ALOG(1/SR)+1)
      RETURN
      END
```

SUBROUTINE REARR

```
      SUBROUTINE REARR (DFI,FII,KODE)
C
C  THIS SUBROUTINE REARRANGES THE COMPUTED SOLUTION VECTOR DFI
C  TO THE EXPANDED FORM AND REORDERS DFI AND FI VECTORS TO
C  PUT ALL THE VALUES OF THE POTENTIALS IN FI AND ALL THE VALUES
C  OF DERIVATIVES IN DFI
C
      COMMON N
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)
     1,NEQNS,NL(10),EFFP(10)
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE
      DIMENSION DFI(1),FII(1),KODE(1)
      NB=NEQNS
      NP=NEQNS
      DO 3000 I=1,NRGNS
      DO 2900 J=1,NRGNS
      IF (J.LE.I.OR.NINTF(I,J).LE.0) GO TO 2900
      NB=NB-NINTF(I,J)
 2900 CONTINUE
 3000 CONTINUE
      IR=0
      IO=0
      DO 2000 INDU=1,NRGNS
      IS=ISTART(INDU)
      IE=IEND(INDU)
      DO 1000 I=IS,IE
      IR=IR+1
      IO=IO+1
      IF (KODE(I).NE.2) FII(IR)=DFI(IO)
      IF (KODE(I).NE.2) GO TO 1000
      DO 500 IND=1,NRGNS
      IF (IND.LE.INDU.OR.NINTF(INDU,IND).EQ.0) GO TO 500
      NINT=NINTF(INDU,IND)
      DO 400 INDC=1,NINT
      IF (I.NE.ID(INDU,IND,INDC)) GO TO 400
      FII(IR)=DFI(IO)
      NP=NP+1
      NB=NB+1
      FII(NP)=DFI(NB)
      GO TO 1000
  400 CONTINUE
  500 CONTINUE
      DO 700 IND=1,NRGNS
      IF (IND.GE.INDU.OR.NINTF(INDU,IND).EQ.0) GO TO 700
      NINT=NINTF(INDU,IND)
      DO 600 INDC=1,NINT
      IF (I.NE.ID(INDU,IND,INDC)) GO TO 600
      FII(IR)=-FII(ID(IND,INDU,INDC))*EFFP(IND)/EFFP(INDU)
      IO=IO-1
      GO TO 1000
  600 CONTINUE
  700 CONTINUE
 1000 CONTINUE
 2000 CONTINUE
```

SUBROUTINE REARR

```
      DO 2500 IND=1,NP
      DFI(IND)=FII(IND)
 2500 CONTINUE
C
C  REORDER FI AND DFI ARRAY TO PUT ALL THE VALUES OF THE POTENTIAL
C  IN FI AND ALL THE VALUES OF THE DERIVATIVE IN DFI
C
      DO 20 I=1,N
      IF (KODE(I).EQ.2) GO TO 20
      IF(KODE(I)) 20,20,10
   10 CH=FI(I)
      FI(I)=DFI(I)
      DFI(I)=CH
   20 CONTINUE
      IF (NRGNS.LE.1) RETURN
C
      N1=N
      DO 200 I=1,NRGNS
      IS=ISTART(I)
      IE=IEND(I)
      DO 150 J=IS,IE
      IF (KODE(J).LT.2) GO TO 150
      DO 140 K=1,NRGNS
      IF (I.EQ.K.OR.NINTF(I,K).EQ.0) GO TO 140
      IF (K.GT.I) GO TO 100
      NINT=NINTF(I,K)
      DO 80 INDC=1,NINT
      IF (J.NE.ID(I,K,INDC)) GO TO 80
      FI(J)=FI(ID(K,I,INDC))
      GO TO 150
   80 CONTINUE
      GO TO 140
  100 NINT=NINTF(I,K)
      DO 110 INDC=1,NINT
      IF (J.NE.ID(I,K,INDC)) GO TO 110
      N1=N1+1
      FI(J)=DFI(N1)
      GO TO 150
  110 CONTINUE
  140 CONTINUE
  150 CONTINUE
  200 CONTINUE
      RETURN
      END
```

SUBROUTINE INTER

```
      SUBROUTINE INTER(DFI,KODE,CX,CY,X,Y,SOL)
C
C  THIS SUBROUTINE COMPUTES THE POTENTIAL VALUE FOR INTERNAL POINTS.
C
```

SUBROUTINE INTER

```
      COMMON N
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)
     1             ,NEQNS,NL(10),EFFP(10)
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE
      DIMENSION DFI(1),KODE(1),CX(10,050),CY(10,050)
     1,X(1),Y(1),SOL(10,050)
C
      DO 50 I=1,NRGNS
      L=NL(I)
      IF (L.EQ.0) GO TO 50
C
C  COMPUTE THE POTENTIAL VALUES FOR INTERNAL POINTS
C
      IS=ISTART(I)
      IE=IEND(I)
      DO 40 K=1,L
      SOL(I,K)=0.
      DO 30 J=IS,IE
      X1=X(J)
      Y1=Y(J)
      X2=X(IS)
      Y2=Y(IS)
      IF (J.LT.IE) X2=X(J+1)
      IF (J.LT.IE) Y2=Y(J+1)
      CALL INTE(CX(I,K),CY(I,K),X1,Y1,X2,Y2,A,B)
   30 SOL(I,K)=SOL(I,K)+DFI(J)*B-FI(J)*A
   40 SOL(I,K)=SOL(I,K)/(2*3.1415926)
   50 CONTINUE
      RETURN
      END
```



SUBROUTINE OUTPT

```
      SUBROUTINE OUTPT(XM,YM,DFI,CX,CY,SOL)
C
C  THIS SUBROUTINE PRINTS THE RESULTS.
C
      COMMON N
      COMMON /ADD1/ NRGNS,ISTART(10),IEND(10),NINTF(10,10),ID(10,10,050)
     1             ,NEQNS,NL(10),EFFP(10)
      COMMON /BIE2/ FI(200),FIS(200),HWE,TWE
      DIMENSION XM(1),YM(1),DFI(1),CX(10,050),CY(10,050),SOL(10,050)
      WRITE(6,100)
  100 FORMAT(' ',120('*')//1X,'RESULTS'//2X,'BOUNDARY NODES'//
     16X,'NODE',12X,'X',19X,'Y',15X,'POTENTIAL',6X,'POTENTIAL DERIVATIVE
     2',3X,'PIEZOMETRIC RISE',4X,'NODE'/)
  600 FORMAT (1H0,'REGION NO = ',I3)
      DO 10 I=1,N
      TP=FI(I)+TWE
      PP=TP-YM(I)
   10 WRITE(6,200) I,XM(I),YM(I).TP,DFI(I),PP,I
```

SUBROUTINE OUTPT

```
   11 CONTINUE
  200 FORMAT(6X,I4,5(6X,E14.7),6X,I4)
      WRITE(6,300)
  300 FORMAT(//,2X,'INTERNAL POINTS',//11X,'X',18X,'Y',14X,'POTENTIAL',
     114X,'PIEZOMETRIC RISE'/)
      DO 30 INDU=1,NRGNS
      WRITE (6,600) INDU
      L=NL(INDU)
      DO 20 K=1,L
      TP=SOL(INDU,K)+TWE
      PP=TP-CY(INDU,K)
   20 WRITE(6,400)CX(INDU,K),CY(INDU,K),TP,PP
   30 CONTINUE
  400 FORMAT(4(5X,E14.7))
      WRITE(6,500)
  500 FORMAT(' ',120('*'))
      RETURN
      END
```

## SUBROUTINE FACTR

```
      SUBROUTINE FACTR (A,PER,N,IA,IER,IAN)
C
C  THIS SUBROUTINE FACTORS THE MATRIX A INTO A PRODUCT OF A LOWER
C  TRIANGULAR MATRIX L AND AN UPPER TRIANGULAR MATRIX U. L HAS UNIT
C  DIAGONAL WHICH IS NOT STORED.
C
      DIMENSION A(IAN),PER(N)
       DOUBLE PRECISION DP,DABS
C  COMPUTATION OF WEIGHTS FOR EQUILIBRATION
      DO 30 I=1,N
      X=0.
      IJ=I
      DO 20 J=1,N
      IF (ABS(A(IJ))-X)20,20,10
   10 X=ABS(A(IJ))
   20 IJ=IJ+IA
      IF (X) 180,180,30
   30 PER(I)=1./X
      I0=0
      DO 170 I=1,N
      IM1=I-1
      IP1=I+1
      IPIVOT=I
      X=0.
C  COMPUTATION OF THE ITH COLUMN OF L
      DO 80 K=I,N
      KI=I0+K
      DP=A(KI)
      IF (I-1) 180,60,40
   40 KJ=K
      DO 50 J=1,IM1
      IJ=I0+J
      DP=DP-1.D0*A(KJ)*A(IJ)
   50 KJ=KJ+IA
      A(KI)=DP
C  SEARCH FOR EQUILIBRATED PIVOT
   60 IF (X-DABS(DP)*PER(K))70,80,80
   70 IPIVOT=K
      X=DABS(DP)*PER(K)
   80 CONTINUE
      IF (X)180,180,90
C  PERMUTATION OF ROWS IF REQUIRED
   90 IF (IPIVOT-I) 180,120,100
  100 KI=IPIVOT
      IJ=I
      DO 110 J=1,N
      X=A(IJ)
      A(IJ)=A(KI)
      A(KI)=X
      KI=KI+IA
  110 IJ=IJ+IA
      PER(IPIVOT)=PER(I)
  120 PER(I)=IPIVOT
      IF (I-N) 130,170,170
```

SUBROUTINE FACTR

```
  130 IJ=I0+I
      X=A(IJ)
C  COMPUTATION OF THE ITH ROW OF U
      K0=I0+IA
      DO 160 K=IP1,N
      KI=I0+K
      A(KI)=A(KI)/X
      IF (I-1)180,160,140
  140 IJ=I
      KI=K0+I
      DP=A(KI)
      DO 150 J=1,IM1
      KJ=K0+J
      DP=DP-1.D0*A(IJ)*A(KJ)
  150 IJ=IJ+IA
      A(KI)=DP
  160 K0=K0+IA
  170 I0=I0+IA
      IER=0
      RETURN
  180 IER=3
      RETURN
      END
```



SUBROUTINE RSLMC

```
      SUBROUTINE RSLMC (A,AF,B,X,N,EPSI,IER,IA,V,PER,IAN)
C
C  THIS SUBROUTINE SOLVES A SYSTEM OF LINEAR EQUATIONS AX=B
C
      DIMENSION A(IAN),AF(IAN),B(N),X(N),V(N),PER(N)
      DOUBLE PRECISION DP
C  INITIALIZATION
      D0=0.
      IER=0
      ITE=0
      DO 10 I=1,N
      V(I)=B(I)
   10 X(I)=0.
   20 ITE=ITE+1
C  THE PERMUTATIONS OF ROWS OF A ARE APPLIED TO V
      DO 40 I=1,N
      K=PER(I)
      IF (K-I)30,40,30
   30 D1=V(K)
      V(K)=V(I)
      V(I)=D1
   40 CONTINUE
C  SOLUTION OF THE LOWER TRIANGULAR SYSTEM
      DO 60 I=2,N
```

```
      IM1=I-1
      DP=V(I)
      IK=I
      DO 50 K=1,IM1
      DP=DP-1.D0*AF(IK)*V(K)
   50 IK=IK+IA
   60 V(I)=DP
C  SOLUTION OF THE UPPER TRIANGULAR SYSTEM
      IF(AF(IK)) 80,70,80
   70 IER=4
      GO TO 130
   80 V(N)=DP/AF(IK)
      DO 100 I=2,N
      IM1=N-I+1
      INF=IM1+1
      DP=V(IM1)
      IK=(IM1-1)*IA+IM1
      D1=AF(IK)
      DO 90 K=INF,N
      IK=IK+IA
   90 DP=DP-1.D0*AF(IK)*V(K)
  100 V(IM1)=DP/D1
C  TEST OF PRECISION
      D1=0.
      D2=0.
      KLE=0
      DO 120 I=1,N
      D1=D1+ABS(V(I))
      D2=D2+ABS(X(I))
      IF (ABS(V(I))-EPSI*ABS(X(I))) 120,120,110
  110 KLE=1
  120 CONTINUE
      IF (KLE)240,130,140
  130 RETURN
  140 IF (ITE-1)240,160,150
C  ITERATIONS ARE STOPPED WHEN THE NORM OF THE CORRECTION IS MORE
C  THAN HALF OF THE ONE OF THE FORMER
  150 IF (D0-2.*D1)200,160,160
  160 DO 170 I=1,N
  170 X(I)=X(I)+V(I)
      DO 190 I=1,N
      DP=B(I)
      IK=I
      DO 180 K=1,N
      DP=DP-1.D0*A(IK)*X(K)
  180 IK=IK+IA
  190 V(I)=DP
      D0=D1
      GO TO 20
  200 IF(ITE-2)240,240,210
  210 IF (D1-EPSI*D2)220,220,230
  220 IER=1
      RETURN
```

```
      230 IER=2
          EPSI=D1/D2
          RETURN
      240 IER=3
          RETURN
          END
```

## Mission of the Bureau of Reclamation

*The Bureau of Reclamation of the U.S. Department of the Interior is responsible for the development and conservation of the Nation's water resources in the Western United States.*

*The Bureau's original purpose "to provide for the reclamation of arid and semiarid lands in the West" today covers a wide range of interrelated functions. These include providing municipal and industrial water supplies; hydroelectric power generation; irrigation water for agriculture; water quality improvement; flood control; river navigation; river regulation and control; fish and wildlife enhancement; outdoor recreation; and research on water-related design, construction, materials, atmospheric management, and wind and solar power.*

*Bureau programs most frequently are the result of close cooperation with the U.S. Congress, other Federal agencies, States, local governments, academic institutions, water-user organizations, and other concerned groups.*

A free pamphlet is available from the Bureau entitled "Publications for Sale." It describes some of the technical publications currently available, their cost, and how to order them. The pamphlet can be obtained upon request from the Bureau of Reclamation, Attn D-922, P O Box 25007, Denver Federal Center, Denver CO 80225-0007.

CHIEF HYDRAULICS BRANCH V
CODE D-1530