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PAP-1156

FRICTION RESISTANCE IN OPEN AND CLOSED CONDUITS

Technical Update Lecture
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April 1987

HISTORICAL DEVELOPMENT

The prediction of the flow resistance in open and closed conduits began with empirical relationships which were derived from physical measurements. Since open channel flows are easier to observe and measure, most of the early relationships dealt with the prediction of losses in canals. Some of the early investigators included Chezy (1718-1798), Hagen (1797-1884), Darcy (1803-1858), Weisbach (1806-1871), Manning (1816-1897), Ganguillet (1818-1894), Kutter (1818-1888), and Bazin (1829-1917), [1].

An empirical relationship is developed by first identifying the significant parameters which affect the flow. Then, the parameters are grouped into some type of a functional form with unknown coefficients. Finally, the coefficients are determined by measuring the parameters in the laboratory or in the field. The investigators listed above realized that the parameters which influenced the velocity in a pipe or in a canal were the slope of the water surface or the difference in elevation between the ends of a pipe, some characteristic dimension, and the surface texture or roughness. Increasing the slope or the characteristic dimension increased the velocity. Whereas, increases in the surface roughness, decreased the velocity.

Since the relationships were empirical, a large number of correlations existed for specific canal shapes, roughness, and slopes. Gauckler tried to find a universal equation, but discovered that two equations were necessary to fit the available data. These are

$$V = C_1 R^{4/3} S \quad \text{for } S < 0.0007 \quad (1)$$

and $V = C_2 R^{2/3} S^{1/2} \quad \text{for } S > 0.0007. \quad (2)$

where C_1 = constant
 C_2 = constant
 R = hydraulic radius
 S = friction slope
 V = average flow velocity

The second of these equations was also developed by an Irishman named Manning. However, Mannings comment on the relationship was

"if modern formulae are empirical, with scarcely an exception, and are not homogeneous, or even dimen-

sional, then it is obvious that the truth of any such equation must altogether depend on that of the observations themselves, and it cannot in strictness be applied to a single case outside them".

In other words, he realized that empirical correlations which have dimensional coefficients can only be used to interpolate between observed points. The correlations cannot be relied upon for extrapolations outside the range of the observed data. Therefore, the Manning equations was limited by the slope and by the range of the observed data.

Because of the nondimensional coefficient in equation 2, Manning discarded the equation in favor of a homogenous relationship, which now only has historical interest.

Chezy was the one who developed the concept of the hydraulic radius in an attempt to compare the flows in different streams. He also tried to apply the correlation to pipes and discovered that the hydraulic radius was equal to one-fourth of the pipe diameter.

In 1923, a Swiss engineer, Strickler, published his experiments with flow in beds composed of cobble stones or small boulders. He used the form of equation 2, but proposed that the value of the coefficient could be given as

$$C_2 = 25.6/D^{1/6} \quad (3)$$

where D = average bed grain size in feet.

In the English speaking world, equation 2 is referred to as the Manning Equation. In the French speaking world it is normally referred to as either the Strickler or the Gauckler-Strickler Equation.

In 1902, Prandtl (1875-1953) proposed the idea of boundary layer theory which revolutionized the way in which frictional resistance was considered. In 1932, he developed an expression for the frictional resistance of smooth pipes. A year later, Nikuradse (1894-) developed the concept for flow in rough pipes. He showed that the frictional resistance curves have three characteristic zones; smooth, fully rough, and a transitional zone. The smooth and the fully rough zones were associated with specific resistance laws, figure 1.

The smooth zone now included the effect of another significant parameter which had not been considered by the 19th Century experimenters; the viscosity of the fluid. The significant parameters for the fully rough zone included the relative height of the roughness and a characteristic dimension. Since the Manning equation does not contain a viscous term, it was now apparent that its' range of applicability must be limited not only by slope and the range of observed data but also by the type of flow.

Although the relationships developed by Nikuradse were still empirical, they had the advantage that they were

- . based on a better understanding of the mechanics of the flow and
- . they were nondimensional.

Therefore, they could be applied not only outside of the range of the observed data, but also to fluids other than water.

For some time there was a problem in defining what the relationship should be used in the transition zone. The experiments of the flow with uniformly rough surfaces, such as those formed by coating a pipe with sand grains, experienced a peculiar dip in the resistance curve. Whereas, the resistance curves for a nonuniform roughness did not experience the dip. Experiments with commercially available pipes showed a gradual transition from the smooth to the rough flow zones. Therefore, for engineering purposes, the curves without the dip seemed to be the most practical.

In 1937, Colebrook and White developed an equation for commercial pipes which connects the smooth and the fully rough zones. Actually, their equation is valid for all three zones. Their equation (known as the Colebrook-White equation) is

$$\frac{1}{(f)^{1/2}} = - C_2 \log \left[\frac{k_s}{C_5 R} + \frac{C_4}{R_{\infty} (f)^{1/2}} \right] \quad (4)$$

where C_2 = constant
 C_4 = constant
 C_5 = constant
 f = Darcy-Weisbach friction factor
 k_s = equivalent sand grain roughness
 R = Hydraulic radius
 R_{∞} = Reynolds number.

The Darcy-Weisbach friction factor is defined in the following expression for the slope of the energy gradient

$$S = \frac{f}{4R} \frac{V^2}{2g} = \frac{f V^2}{4(R)^2} \quad (5)$$

where g = acceleration of gravity.

The Reynolds number is defined as

$$R_{\infty} = \frac{4RV}{v}$$

A. Closed Conduit Developments

In the period 1920 to 1939, an engineer with the Department

of Agriculture, Mr. F.C. Scoby, made extensive measurements of friction losses in both open and closed conduits throughout the western United States [6], [7], [8]. He developed equations which matched the observed data rather closely. However, his correlations were of the empirical nature eschewed by Manning. Nevertheless, these equations found their way into the design process of the Bureau of Reclamation and are still used in some segments of the organization today. Their weakness lies in their range of limited applicability and in the difficulty in obtaining the original references.

Building on the boundary layer theory and the correlations of Colebrook and White, Bradley and Thompson, of the Bureau of Reclamation, published a monograph of friction factors for large conduits flowing full. They used observed data on losses in closed conduits which were available either within the Bureau or from the technical literature. The data base was enlarged by Thomas and Dexter and the curves were revised by Schuster in 1962, [5]. Using the Colebrook-White formulation, they developed recommended ranges of equivalent values of the sand grain roughness for typical surfaces found in the field.

In 1965, a Task Committee of the American Society of Civil Engineers summarized the factors influencing the flow in large conduits, [9]. They pointed out the weaknesses of both the Manning formulation and of the Colebrook-White equation. In essence, the Manning formula has a limited range of applicability. For instance, they showed that the value of the Manning n varies by more than 10-percent when approximating the smooth flow zone, figure 2. This zone is obviously outside the range of applicability of Mannings equation. The majority of the cases examined in the Engineering Monograph of the Bureau show that the flow in pipelines is typically in the transition zone. Therefore, Mannings equation is generally not within its valid range for closed conduit flow computations. With respect to the Colebrook-White equation, the Task Committee concluded that more research was necessary to investigate the effects of roughness spacing, etc. The Committee did not recommend one method over another.

In 1974, the Bureau of Reclamation published a design manual on small dams which developed the head loss relationships through closed conduits based upon the concept of the Darcy-Weisbach friction factor, [10]. However, it recommended that the losses be computed with the empirical formulation of Manning! The rationale behind this step backward in technology is difficult to understand.

In 1980, the Corps of Engineers [20] recommended the Darcy-Weisbach formula, and specifically the Colebrook-White equation, for all of the closed conduit shapes normally used in the Corps' outlet works structures which includes, rectangular, horseshoe, oblong, sloping-side horseshoe, and circular. It is worth noting that the only reference to Mannings equation in this manual is on a drawing from the Bureau which

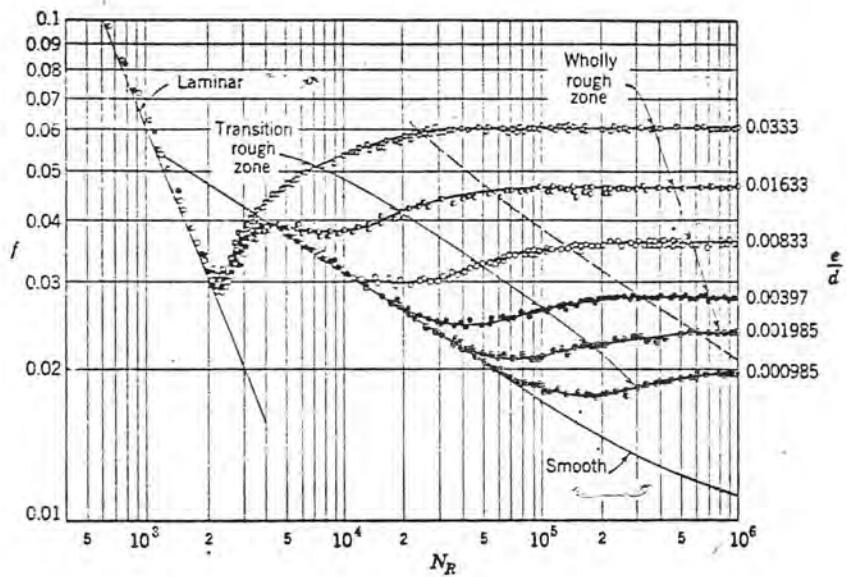


Figure 1. Frictional Resistance Relationships in a Pipe

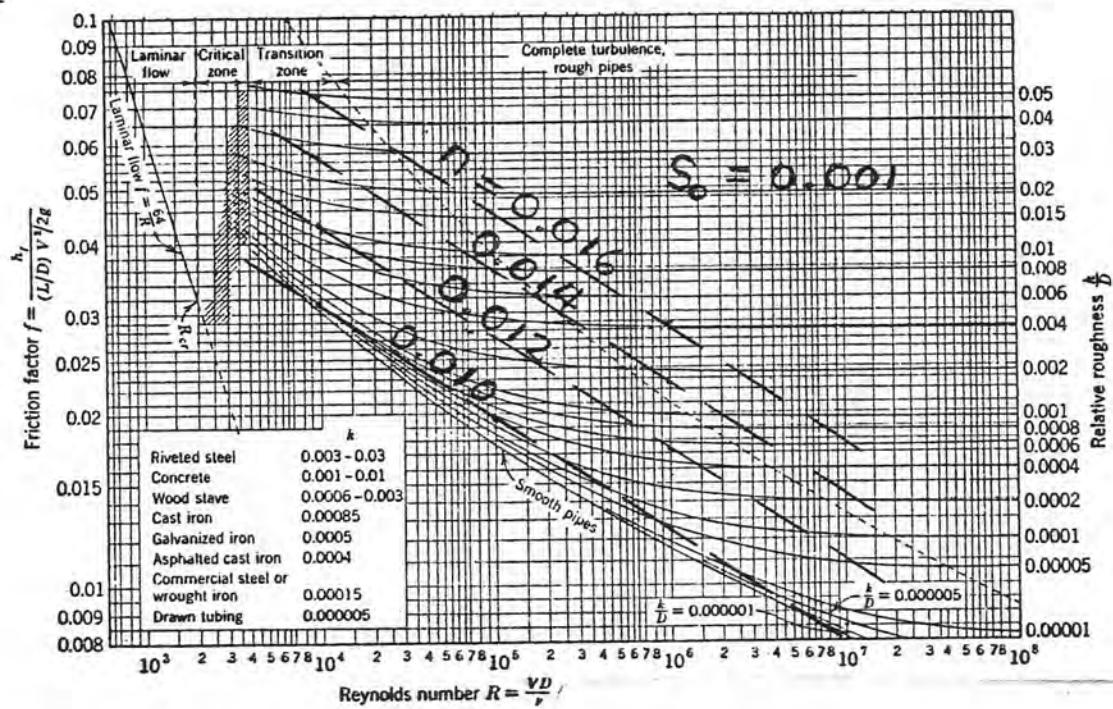


Figure 2. Comparison of Manning Equation with Frictional Resistance Relationships

gives the physical properties of the Bureaus' horseshoe shape.

More recent comparisons (1981) of the various flow resistance equations for closed conduit flow reveal that most are relatively accurate if they are applied to situations from which they were derived. However, only the Colebrook-White equation yields reliable results when applied to unusually high or low flow situations, [14].

B. Open Channel Flow Developments

The development of the Colebrook-White equation for open channel flow tended to parallel its development for closed conduit flows. However, its implementation within the Bureau has been slower than that for closed conduit flows.

In 1938, both Keulegan, in the United States, and Zegzhda, in Russia, independently developed equations based on equation 4 to predict the frictional resistance of open channel flow, [2].

A period of intense investigations followed these developments since there is a great benefit in having one equation which is valid for the prediction of frictional resistance in both open and closed conduit flows. One of the main points of discussion centered around the influence of the shape of the canal.

One of the beneficial characteristics of using the Colebrook-White equation is also one of the reasons that its experimental verification causes so many discussions. That characteristic is that large variations in estimating the magnitude of the equivalent sand grain roughness produce only minor variations in the resulting friction losses. However, it can be seen that if the friction losses are measured, then large variations can be expected in computing the magnitude of the sand grain roughness. To reduce the variations in the computed values, extremely carefully conducted experiments are necessary. In open channel flow, the losses are usually small, therefore the evaluation of the equivalent sand grain roughness from observations is much more difficult in canals than it is in pipes.

In 1958, Ackers [3] published the results of a study he performed of the frictional resistance laws. He concluded that the Colebrook White equation form of the resistance law was the best available. The characteristic length to be used in the relationship is the hydraulic radius. He recommended that the equivalent sand grain roughness be increased by 20-percent over that used for closed conduit flows.

In 1963, a special task group of the American Society of Civil Engineers also recommended the used of the Colebrook-White equation [2]. They stated "It is believed

that, for other than fully rough flow, the Colebrook-White formula is more trustworthy than the Manning formula with constant n ; that is, for any given channel, k_s is more likely to be constant than n ".

The following year, Tilp and Scrivner of the Bureau of Reclamation, published a report on their studies of friction factors in large concrete-lined canals, [4]. Their recommended design procedure used Mannings formula. However, the value of the Manning n value is determined from a curve on a Reynolds number versus friction factor plot. In other words, they were essentially using a type of the Colebrook-White equation to determine the frictional losses.

Frederiksen and DeVries found that use of the Manning equation resulted in canals which were under designed when applying observed n values from small canals to large canal systems, [11]. Therefore, they compared a large number of friction equations and finally adopted the Colebrook-White equation for the calculation of the friction loss in large canals.

Due to the problems encountered with the range of applicability of Manning equation, more and more organizations are using the Colebrook-White equation to predict the friction losses in open channels. For example, in 1985, the Committee on Channel Stabilization of the Corps of Engineers, US Army, recommended that the Los Angeles District use a Colebrook-White type of formulation to calculate the losses in the Arizona Diversion Channel, [12]. Even in their 1968 Hydraulic Design Criteria [19], they recommended the use of the Chezy equation. The Chezy equation is defined as

$$V = C (R S)^{1/2} \quad (7)$$

where $C =$ the Chezy coefficient
 $= (8g/f)^{1/2}$.

The friction equation they recommended for the determination of the Chezy coefficient is

$$C = - 32.6 \log (C/5.2 Re + ks/12.2 R) \quad (8)$$

This can be shown to be equivalent to the Colebrook-White equation.

The displacement of the Manning formula by the Colebrook-White equation in the computation of open channel flow resistance is also taking place in Europe, [13].

EVALUATION OF THE COLEBROOK-WHITE EQUATION

A. Determination of Coefficients

The primary discussion in the use of the Colebrook-White Equation is concerned with the determination of the

appropriate values of the constants C_3 , C_4 , and C_5 .

With closed conduit flow, the constants are as follows;

$$\begin{aligned}C_3 &= 2. \\C_4 &= 2.51 \\C_5 &= 14.8\end{aligned}$$

With open channel flow, the magnitude of the constants can be expected to be different than those for closed conduit flow because of the presence of the free water surface. However, the range of variation in the values is not very large. In fact, some investigators use the same values that are used for closed conduit flow, [2], [3].

As examples of what different experimenters recommended, Keulegan found the following values for very wide, smooth channels;

$$\begin{aligned}C_3 &= 2. \\C_4 &= 2.98 \\C_5 &= 12.6\end{aligned}$$

The Corps of Engineers Design Criteria [19] uses the following values for rectangular and triangular channels;

$$\begin{aligned}C_3 &= 2.03 \\C_4 &= 3.08 \\C_5 &= 12.2\end{aligned}$$

Henderson [15], proposed the following values for all channels;

$$\begin{aligned}C_3 &= 2. \\C_4 &= 2.5 \\C_5 &= 12.\end{aligned}$$

Ackers [3] used the same values of the coefficients in open channel flow as are used in closed conduit flow. However, he used higher values of the equivalent sand grain roughness than are commonly accepted.

B. Solution of the Equation

Evaluation of the Colebrook-White equation numerically appears to be difficult because of the presence of the dependent variable, f , on both the left and the right hand sides of the equation. An attempt to circumvent this problem has been made by Haaland [16], who developed a simple explicit approximation to the Colebrook-White equation. However, this is not necessary if one has access to a programmable calculator.

The convergence of numerical approximations to the correct solution is very rapid using a Newton iteration scheme. The equations to use are

$$G = -F - 2 \cdot \log \left[\frac{k_s}{12R} + \frac{2.5 F}{R_s} \right] \quad (9)$$

and

$$F = F_o - \frac{G}{dG/dF} \quad (10)$$

where F = revised approximation of $(1/f)^{1/2}$
 F_o = initial and updated approximations of F
 G = a function whose value is equal to zero
when the correct value of f has been found

However, the derivative can be approximated by

$$\frac{dG}{dF} = -1 \quad (11)$$

Thus,

$$F = F_o + G \quad (12)$$

The procedure to determine the value of the friction factor F is as follows. First a value for the friction factor F is assumed. This is then substituted into equation 9 along with the known values of sand grain roughness, hydraulic radius, and Reynolds number. The function G is calculated and the value of F is revised if G is not very close to zero using equation 12. Usually only about three iterations are needed to determine f to within 1-percent when starting with an initial value of F equal to 10.

C. Estimation of Sand Grain Roughness

An essential element in using the Colebrook-White equation is the ability to estimate the value of the equivalent sand grain roughness. If the coefficients of Henderson [15] are used, then values of the sand grain roughness or rugosity can be estimated from the Bureau's Engineering Monograph No.7, [5]. These values can be used for both open channel and closed conduit flows. If the closed conduit coefficients are used, then the values from Engineering Monograph 7 should be increased by about 20-percent. In lieu of this, values from Ackers paper [3] should be used. It should be stressed that errors in estimating the sand grain roughness do not produce large errors in the estimation of the head loss.

The equivalent sand grain roughness can also be determined from physical measurements in the field. This technique would be useful in estimating the surface roughness of unlined tunnels excavated by blasting. Details of the method are outlined by Bergmann [17]. Basically, the idea is to obtain a statistical measure of the surface roughness from some reference height using a profilometer or by measuring the

vertical distance to the surface with a point gage mounted on a proving ring. From these measurements the mean, the root-mean-square, and the 90-percentile of the deviations can be determined. Brown and Chu [18], using a similar technique to that of Bergmann, showed that the equivalent sand grain roughness of the surface is approximately 2.5 times the 90-percentile size of the surface deviations.

D. Limitations

The Colebrook-White equation has limitations which must be observed. One of the more important limitations is that the ratio of the flow depth to roughness height cannot be less than about 10:1. When the roughness height exceeds 10-percent of the flow depth, the flow is no longer two dimensional and the concept of a two dimensional flow does not apply.

Although it is not truly a limitation of the Colebrook-White equation, effects such as algae growth, bends, waviness in the canal lining, and bridge piers are not included in the resistance equation. These effects must be considered separately. Previous design methods have accounted for many of these effects by merely increasing the value of Mannings n [4]. This procedure is not recommended.

CONCLUSIONS

The first frictional resistance equations to be developed were very simple, but their range of applicability was limited by the range of flow conditions which were observed. Because of this, separate equations were developed for the flow in canals and pipes. Many of the early investigators tried to develop a universal resistance equation which could be applied to both open-and closed conduit flow. However, this effort was not successful because the mechanics of the flow were not well understood. As the understanding of fluid mechanics grew, it finally became possible to write an equation which was applicable to both flow conditions. This equation has become known as the Colebrook-White equation.

The Colebrook-White equation is extremely useful because it allows the computation of frictional resistance for conditions far outside the range of the test data. The only disadvantage of the equation is its complexity. However, with the advent of computers, it is possible to write programs which eliminate this disadvantage.

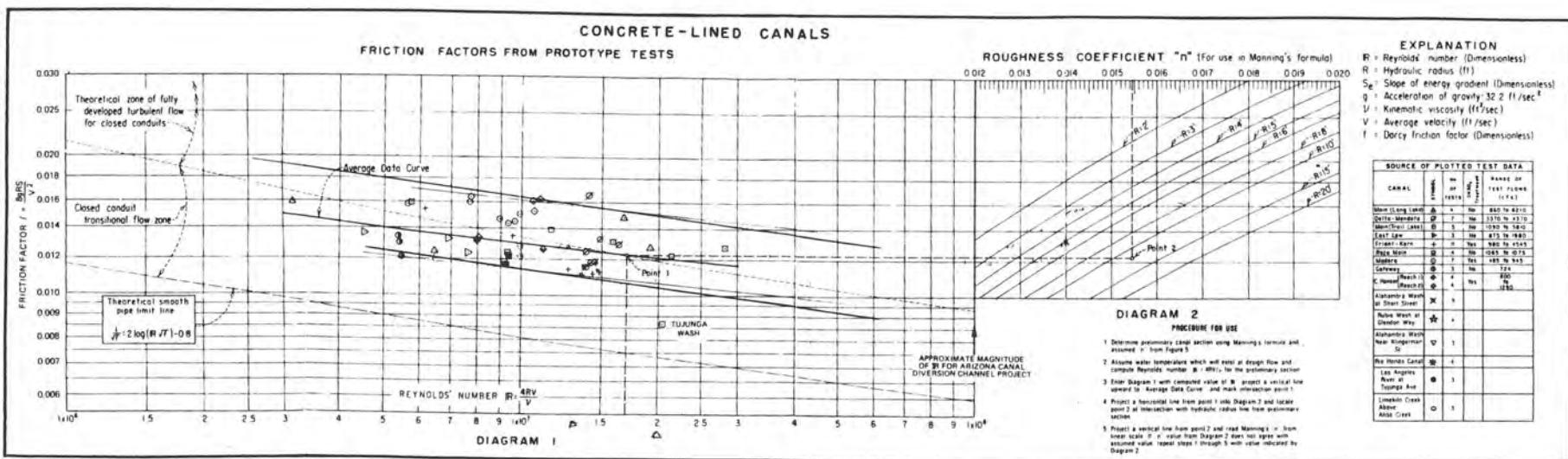
Most of the major engineering organizations in the world are now using the Colebrook-White equation to determine the frictional resistance of open and closed conduit flows. The universal use of the Colebrook-White equation is recommended. In this manner, verification of designs and the interchange of data among disciplines would be easier.

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Figure 1



$f \approx 0.012$
 $n \approx 0.012$

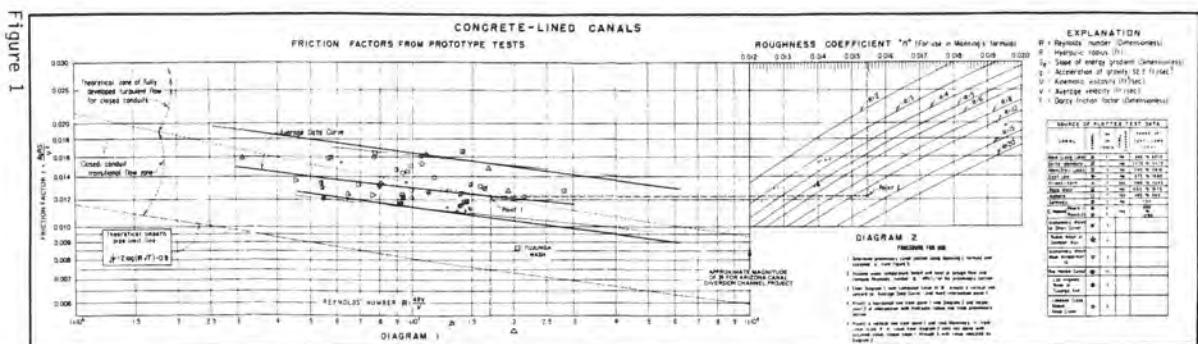


Figure 1

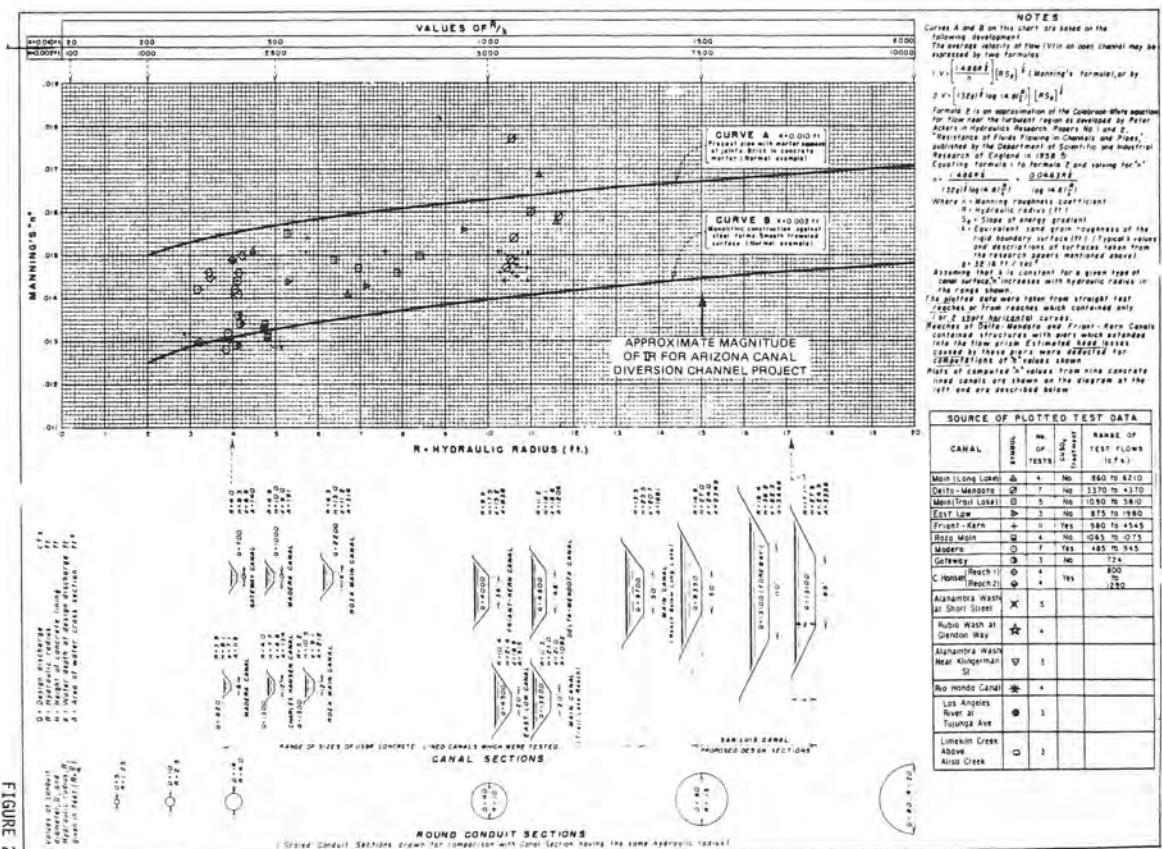


FIGURE 2

