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Design of Bellmouths for Entrances to Conduits of Circular and Square and Rectangular Cross Sections

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DESIGN OF BELLMOUTHS FOR
ENTRANCES TO CONDUITS OF CIRCULAR
SQUARE AND RECTANGULAR CROSS SECTIONS

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(1) Introduction

In designing a bellmouth for the entrance to a large conduit operating under high head, two opposing considerations are encountered. (1) The entrance width of the bellmouth should be kept to a minimum, in order to promote economy and convenience in closing the entrance by means of an emergency bulkhead. Moreover, keeping the entrance width to a minimum promotes economy of expensive material, in case the bellmouth is made of metal. (2) Excessive sharpness in the entrance curvature of the bellmouth may be a source of cavitation.

In case of a circular conduit, safety against cavitation may be secured by making the shape of the bellmouth identical with that of the nappe of the jet from a sharp-edged circular orifice. The absolute pressure at all points on the surface of such a bellmouth will be exactly the same as on the surface of the cylindrical conduit where it leaves the bellmouth and enters the conduit. This follows conversely because in such a jet discharging intothe air, the (atmospheric) pressure acting on the cylindrical jet is identical with that acting on the converging portion of the nappe. If a bellmouth is thus designed it must therefore be free from cavitation if the conduit itself is free from cavitation at the upper end of its cylindrical portion.

In the practical design of a bellmouth, for a circular conduit, it is necessary to modify the entrance profile slightly from that of the ideal jet from a sharp-edged circular orifice, in order to make the curve touch, rather than be asymptotic to, the straight line representing the crown or invert of the circular conduit.

The corresponding procedure for designing the bellmouth for a square or rectangular conduit is not so clear cut or well established since the cross section of the jet from a square orifice is not square, and the cross section of the jet from a rectangular orifice is not rectangular. The design of such bellmouths forms the principal topic of this paper.

No exact mathematical analysis exists for computing the nappe shapes of jets from sharp-edged circular orifices, square orifices or rectangular orifices of finite length, but Kirchoff in 1869 developed such an analysis for a sharp-edged rectangular orifice of infinite length. This analysis will be discussed in subsequent paragraphs. Since the jet from a circular orifice is a very convenient object for laboratory experimentation, precise profiles of the nappe of such jets have been obtained in various laboratories, one of these curves being presented in a subsequent paragraph. It will be shown that by plotting certain quantities in terms of the "hydraulic radius" of the cross section, of the jet, a close relationship between this curve and Kirchoff's curve may be demonstrated.

(2) Properties of Jet From Sharp-Edged Circular Orifice

(See Fig. 1 for notation and coordinates.)

The first two columns of Table I give coordinates of points on the nappe of a jet 3 inches in diameter, issuing from a sharp-edged circular orifice. These coordinates are scaled from a curve based on experiments made by the U. S. Reclamation Service.

Fig. 2 shows the nappe profiles of this jet as plotted from Columns 6 and 7 of Table I. It is noted that the coefficient of contraction of this jet is:

$$C_c = \left(\frac{d_j}{d_o}\right)^2 = \left(\frac{3.000}{3.892}\right)^2 = 0.594.$$

The "Vena contracta," that is: the section where the perceptible contraction of the jet ceases, is seen to be at a distance of about 2/3 d_j or 3/4 d_j (or 1/2 d_o or 3/5 d_o) from the plane of the orifice. The uncertainty inherent in locating this section is discussed in a subsequent paragraph.

The foregoing profiles were actually obtained on a jet from a 3-inch sharp-edged circular orifice, under three different heads, of approximately 12, 43, and 93 feet. No perceptible variation of the profile with the head was detectable. This agrees by analogy with Kirchoff's analysis, which indicates the profile of the jet of an ideal fluid from a sharp-edged rectangular slot of infinite length to be independent of head. Of course, as published in numerous textbooks, the coefficient of contraction, and therefore the profile of the jet from a circular orifice, varies slightly in the case of jets of actual fluids having viscosity and surface tension and issuing from orifices in metal not infinitely smooth. In general the coefficient of contraction tends to become smaller as the size and velocity of the jet increase, i.e., as the Reynolds number increases. No information is available regarding the coefficient of contraction for enormous free jets corresponding in size to the outlet conduits of large dams, but what evidence is at hand on this general subject points to the conclusion that even the largest Reynolds numbers will not produce coefficients materially lower than 0.592. This coefficient is only slightly less than that for the jet from a 3-inch orifice under a head of 93 feet, referred to in Table I. The profiles of that jet may therefore be considered as applicable with considerable accuracy to the design of bellmouths for circular conduits of the largest size.

TABLE I DATA FOR JET FROM SHARP EDGED CIRCULAR ORIFICE

1	2	3	4	5	6	7	8	9
Scaled from U.S.B.S. Curve		Computed ($r_j = 1/4\ d_j = 0.750''$)						
x	y	$d = (3 + 2y)$	$r = (d/4)$	$\frac{d}{d_j}$	$\frac{x}{d_j}$	$\frac{y}{d_j}$	$\frac{x}{r_j}$	$\frac{u}{u_j} = \left(\frac{d}{d_j}\right)^2$
in.	in.	in.	in.					
0	0.446	3.892	0.973	1.298	0.0000	0.4887	0.0000	1.685
0.05	0.350	3.700	0.925	1.233	0.0167	0.1167	.0667	1.5203
0.10	0.306	3.612	0.903	1.204	0.0333	0.1020	.1333	1.450
0.20	0.235	3.470	0.868	1.157	0.0667	0.0783	.2667	1.339
0.30	0.189	3.378	0.844	1.126	0.1000	0.0630	.4000	1.268
0.40	0.155	3.310	0.828	1.103	0.1333	0.0517	.5333	1.217
0.60	0.103	3.206	0.802	1.069	0.2000	0.0343	.8000	1.143
0.80	0.068	3.136	0.784	1.044	0.2667	0.0227	1.0667	1.090
1.00	0.042	3.084	0.771	1.028	0.3333	0.0140	1.3333	1.057
1.20	0.025	3.050	0.763	1.017	0.4000	0.0083	1.6000	1.034
1.40	0.010	3.020	0.755	1.007	0.4667	0.0033	1.8667	1.014
1.60	0.008	3.016	0.754	1.005	0.5333	0.0027	2.1333	1.010
1.80	0.003	3.006	0.752	1.002	0.6000	0.0010	2.4000	1.004
2.00	0.001	3.002	0.750	1.000	0.6667	0.0003	2.6667	1.000
2.50	0.000	3.000	0.750	1.000	0.8333	0.0000	3.3333	1.000

(3) Properties of Ideal Jet From Infinitely Long Rectangular Orifice

Kirchoff's derivation of the equations for the profile of the jet from a sharp-edged rectangular orifice of infinite length may be found in Lamb's "Hydrodynamics" (Fifth edition, p. 91). These equations in terms of the slope angle θ (see Fig. 3) are:

$$\frac{x}{d_j} = \frac{1}{\pi}\left(\log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2} \right) + \sin\theta \right) \qquad (1)$$

$$\frac{y}{d_j} = \frac{1}{\pi}\left(1-2\cos^2\frac{\theta}{2}\right) = -\frac{1}{\pi}\cos\theta \qquad (2)$$

(For computation with ordinary tables of common logarithms of the trigonometric functions, the first of these equations may be written in the form:

$$\frac{x}{d_j} = \frac{1}{\pi}\left[2.302585 \log_{10}\left(\tan\frac{\pi}{4} - \frac{\theta}{2}\right) + \sin\theta \right] \qquad (3)$$

The values tabulated in Table II are computed from these equations. The profile of the nappe is plotted in Gib. II, from the values in Columns 2 and 3 of Table II. It is to be noted that Kirchoff's analysis establishes a value of $C_d = \frac{d_j}{d_o} = .611$ for the coefficient of contraction of the ideal jet from the sharp-edged rectangular orifice of infinite length. As indicated in Fig. 4, the ideal nappe profile ABC is not tangent to the straight line OF, representing the boundary of an ideal parallel-plane-sided jet of thickness d_j, within a finite distance of the plane of the orifice. Thus the old conception of a "vena contracta," or section where the contraction of the jet is sensibly complete is more or less vague and incapable of exact definition.

In applying Kirchoff's curve to the design of bellmouths for conduits in engineering structures, it is necessary for practical reasons to make the bellmouth of finite length. This can be done most conveniently by replacing the curve BC lying to the right of some point B in Fig. 4, by a straight line BE tangent to the curve at B. The total length L of the bellmouth will then be given by

$$L = x_1 - y_1 \tan \theta \qquad (5')$$

(See figure for notation.)

The equation of line BE is as follows

$$y = y_1 + (x - x_1) \operatorname{ctn} \theta$$

The following table shows the relation between $\frac{L}{d_j}$ and θ,

θ,	x_1/d_j	y_1/d_j	y_1/d_j	$\tan \theta$,	L/d_j
88°	.970	.0111	.3181		1.286
89°	1.191	.0056	.3185		1.509
89°30'	1.412	.0028	0.3186		1.730

In a bellmouth designed as indicated in Fig. 4, the pressures in the vicinity of B will be slightly lower than the constant pressure for which the remainder of the Bellmouth surface is designed. The particular value of θ, to be used in establishing a given bellmouth will obviously depend on the size of the conduit, the velocity of flow and the pressure to be maintained in the conduit where it leaves the bellmouth. For large conduits under high heads, and where the existence of low-pressure in the conduit makes it necessary to guard carefully against the occurrence of cavitation, it is suggested that $-\Theta_1$ be taken as 89°. As noted in the foregoing table, in the case of a conduit whose cross section is a rectangle whose width is very large in comparison to its depth, this makes the length of bellmouth slightly greater than 1.5 d_j. For this value of Θ, it is noted that the point of tangency B, the thickness of the jet exceeds d_j by slightly more than 1/2 of 1%. The application of Kirchoff's curve to the design of bellmouths for circular, square and rectangular conduits will be discussed in subsequent paragraphs.

Certain engineers have suggested or used ellipses or other empirical curves for the entrance profiles in bellmouth design. In general, none of these curves can be expected to fit the nappe profiles as closely as Kirchoff's curve, or certain curves derived therefrom. Since the computation of coordinates of points of Kirchoff's curve is simple and easy, there seems to be no good reason for using other types of curves in bellmouth design.

TABLE II DATA FOR KIRCHOFF'S PROFILE OF IDEAL JET FROM INFINITELY LONG RECTANGULAR ORIFICE

1	2	3	4	5
θ_1	$\frac{x}{d_j}$	$\frac{y}{d_j}$	$B_k = \frac{x}{r_j} = \frac{2x}{d_j}$	$A_k = \frac{a}{a_j} = 1 + \frac{2y}{d_j}$
0	0	0.3183	0	1.6366
-15°	0.0019	0.3075	.0038	1.6149
-30°	0.0157	0.2757	.0314	1.5513
-45°	0.0555	0.2251	.1110	1.4502
-60°	0.1435	0.1592	.2871	1.3183
-75°	0.3379	0.0824	.6759	1.1648
-85°	0.6796	0.0278	1.3593	1.0555
-88°	0.9704	0.0111	1.9409	1.0222
-89°	1.1910	0.0056	2.3819	1.0111
-89°30'	1.412	0.0028	2.8231	1.0056
-90°		0.000		1.0000

(4) Relation between Jet from Circular Orifice and Jet From Infinitely Long Rectangular Orifice

If the cross section of a circle is considered as built up of elementary triangles, as indicated in sketch A of Fig. 5, and if the area is developed as shown in sketch B, the height of the rectangle of equal area is seen to be $d/4$, this being the "hydraulic radius" of the cross section of the circle. This suggests the use of the hydraulic radius "r" as a parameter in comparing the contraction of area in the circular jet with that in the jet from the infinitely long rectangular orifice. (In the case of the latter it is evident that $r = \frac{d}{2}$ and $\frac{a}{a_j} = \frac{d}{d_j} = \frac{r}{r_j}$).

In Fig. 6 the area ratio a/a_j is plotted against $\frac{x}{r_j}$ for both the jet from the circular orifice and from the infinitely long rectangular orifice. The close similarity in the rates of area contraction for the two jets is brought out clearly by this method of plotting. The curve for the circular orifice lies slightly above that for the infinitely long rectangular slot, because the coefficient of contraction (0.594) for the former jet, is somewhat less than that (.611) for the latter. For the region to the right of that shown in this figure, that is for values of x/r_j greater than 2.0, the two curves are practically identical when plotted to this scale.

In Fig. 6 let A denote the area ration a/a_j. The ordinate of any point on the lower curve (Kirchoff's curve) may then be denoted by A_k while that of any point on the upper curve may be denoted by A_c. The ordinate A'_c of any point on the dotted curve is computed by

$$A'_c = \frac{.685}{.636}(A_k - 1) + 1 = 1.077\,A_k - 0.077$$

It is noted that the dotted curve thus computed from Kirchoff's curve furnishes a close approximation to the experimental curve for the circular orifice. The process of obtaining the dotted curve may be explained in more general terms as follows: If C_c is the coefficient of contraction for a circular orifice, then

$$A_c = \left(\frac{\pi}{2}\right)\left(\frac{1}{C_c} - 1\right)(A_k - 1) + 1$$

(5) Properties of jets from Square and Rectangular Orifices

In order to investigate the shape of jets from rectangular orifices, the apparatus shown in Fig. 7 was installed in the Hydraulic Laboratory at Carnegie Institute of Technology in 1945. The apparatus consists of a cylindrical steel tank 30" in diameter by 54" long, with its axis in a vertical position. The jet b issues vertically downward through an orifice cut in a galvanized steel plate c. A set of such plates was prepared,having rectangular orifices of the following sizes.

3.5" x 3.5" , l/d = 1
2.5" x 5" , l/d = 2
2" x 6" , l/d = 3
1.75" x 7" , l/d = 4
1.5 x 8.0" , l/d = 5.33

Water was supplied through an 8-inch pipe d. A perforated stilling plate e was provided. Measurements in three mutually perpendicular directions to determine the contour of the nappe of the jet were obtained by a pointer on the end of a sliding arm attached to a sliding frame h resting on a platform f. In designing this apparatus, the vertical position of the jet was chosen, in order to eliminate difficulties in measurement due to the parabolic trajectory which would exist if the jet were directed horizontally. A piezometer p was provided for measuring the head, but was not used in these tests, because, in agreement with theory, no perceptible change could be detected in the shape of the jet above the vena contracta as the head was varied.

The square and rectangular jets issuing from this apparatus, exhibited the familiar phenomenon known as "inversion of the jet."

This is described in numerous textbooks on hydraulics. The cross section of the jet from a square orifice is not a square. The corners of the jet are beveled off, so that, in departing from the plane of the orifice the cross section of the jet first assumes an octagonal form and then the form of an equilateral cross. In a large jet where surface tension is small in comparison with other forces, the jet usually breaks into spray before or after attaining the cross-shaped section.

The reason why the corners of the jet from a square or rectangular orifice become beveled off may be understood from inspection of Fig. 8, which represents the water inside the tank, in the plane of the orifice, approaching a square orifice. As indicated, the two velocity vectors v at each corner combine into a single velocity vector $v\sqrt{2}$ which is directed toward the center, and thus causes the cross section of the jet to become beveled as indicated by the dotted line.

In the apparatus here described, the tank was not sufficiently large to provide as perfect stilling as might be desired, and therefore the surfaces of the jet fluctuated slightly. This was not objectionable from the standpoint of visual observation, but prevented the attainment of the highest precision in measurements to determine the contour of the nappe.

Contours of a jet from a sharp-edged square orifice 3.5" by 3.5", obtained in the apparatus here described, are shown in Fig. 9. The lowest contour in this figure represents the cross section of the jet at a distance of 4 inches below the plane of the orifice.

At this location the cross section of the jet is just starting to attain the cross-shaped form* which becomes much more pronounced at a lower elevation. Although the measurements used for plotting these contours were taken as carefully as the apparatus would permit, difficulties due to fluctuations of the nappe surfaces prevented attainment of high precision, particularly at the lower levels. For this reason, the reader is cautioned against attempting to draw important conclusions based on planimeter measurements of the areas enclosed by these contours.

As indicated in Fig. 9 the contour taken one inch below the plane of the orifice shows greater contraction at the side of the jet than any other contour in this figure. The actual vena contracta, as determined by visual observation, was slightly below this level. The jet-diameter of 2.80 inches, measured across this contour, normally to the sides of the orifice, is thus slightly greater than the actual minimum jet diameter. If the jet were square at the vena contracta and the coefficient of discharge (under the head of about 4') were 0.604 the diameter at the vena contracta would be 3.5 x .604 = 2.72 inches.

Fig. 10 shows contours of the jet from a sharp-edged rectangular orifice 2.5 inches wide by 5 inches long. It is noted that the beveling off of the corners of the jet progresses so extensively as to reduce the cross section approximately to a six-sided polygon, at a distance of 3 inches below the plane of the orifice. Visual observation indicated the maximum contraction at the sides of this jet to

*Gibson p. 133

occur at a distance of about 1.5 inches below the plane of the orifice. Thus the contraction of the sides of the jet, as indicated by the contour taken one inch below the plane of the orifice, is slightly less than the maximum contraction. This contour shows a jet width of 1.70 inches and a length 4.43 inches, measured on the center lines of the cross section. If the coefficient of discharge (under the head of about 4 feet) were .605 and the cross section at the vena contracta were a rectangle with sides in the ratio of 1.70 to 4.43 the area of that rectangle would be .605 (2.5) 5 = 7.56 sq. in. and its width and length respectively would therefore also be 1.70 and 4.43 inches. Because of small but unavoidable errors in the measurements from which these contours are plotted, the reader is again cautioned against drawing far-reaching conclusions based on areas planimetered from the contours.

Figure 12 shows contours of the nappe of a jet from a non-rectangular orifice, based on a rectangle 2.5 inches by 5 inches in plan, but having deep V-shaped notches cut in the corners of the rectangle. The cutting of these notches was done by trial-and-error process in which an endeavor was made to eliminate the beveling of the corners of the jet, so as to produce an approximately rectangular jet at a distance of about 2 inches below the plane of the orifice. It was thought that an orifice which was capable of producing a rectangular jet might be especially suitable for use in the design of bell-mouths for rectangular conduits.

As indicated in the figure, only a small amount of success was attained in the attempt to obtain an orifice shaped so as to make the

2-inch contour approximate a rectangle more closely than in the case of a rectangular orifice. A bellmouth of shape based on the nappe of this special orifice with notched corners would have two disadvantages as compared with one based on the nappe from a rectangular orifice: (1) greater cost, due to the more complicated shape, and (2) larger over-all dimensions, unfavorable to the placing of an emergency bulkhead over the entrance. For these reasons, the attempt to develop an orifice delivering a rectangular jet was abandoned.

Figure 11 shows contours of a jet from a rectangular orifice 1.5 inches by 8.0 inches.

The present tests did not include a determination of the coefficients of contraction or discharge for the orifices tested. Tables of the coefficients of discharge for circular, square or rectangular orifices are published in numerous textbooks on Hydraulics. Tables of such coefficients indicate that at large Reynolds numbers, i.e. large values of the product of the velocity and diameter, the coefficient of velocity for sharp-edged orifices with smooth approach surfaces becomes very nearly unity, while the coefficient of discharge, and therefore the coefficient of contraction, approaches a minimum value which is constant for an orifice of given shape.

For the purposes of the remainder of this paper, the coefficients of contraction for jets of the large size and high velocity encountered in the conduits of dams, where the design of cavitation-free bellmouths would be an important problem, may be taken as follows.

Type of Orifice	Coef. of Contraction	Reference
Circular	0.592	Hamilton Smith's "Hydraulics"
Square	0.598	" " "
Rectangular	$.611 - .15 \frac{b_o}{d_o}$ 0.013 ?	Fanning's treatise on Hydraulic and Water-Supply Engineering. (Empirical formula fitting Fanning's data)*

(6) Bellmouths for Square and Rectangular Conduits

In the following paragraphs, the term "rectangular" is taken to include "square" as a special case.

Since the jet from a rectangular orifice is not rectangular in cross section, and since it is not possible to find any orifice shape which will produce a rectangular jet, it is obvious that cavitation-free bellmouths for rectangular conduits cannot be designed by the simple and satisfactory process applicable to circular conduits, i.e., giving the bellmouth the same shape as the nappe of a free jet. In general, for the sake of simplicity in design and construction, engineers will prefer to use a rectangular entrance for the bellmouth of the rectangular conduit. This design also has the advantage of minimizing the entrance width, thereby facilitating the use of temporary bulkheads for closure of the entrance. Since a bellmouth does not simulate the beveling off of the corners of the jet from a rectangular orifice, it is evident that a region of slightly subnormal pressure will exist in the corners of the cross section in the region where the bellmouth merges into the conduit. No satisfactory method

*$C_c = 0.611 - 0.013 \frac{b_j}{d_j} \pm 0.0012 \left(\frac{b_j}{d_j} - 1\right)$; if $\frac{b_j}{d_j} < 1$, use minus sign; if $\frac{b_j}{d_j} > 1$, use plus sign; when $\frac{b_j}{d_j} = 0$, $C_c = 0.611$.

of computing this pressure lowering by theory or by experiments on open air jets has yet been devised or discovered.

Thus the only feasible method of obtaining a precise determination of the cavitation--potentialities of a given bellmouth designed for a rectangular conduit, is to construct and test a model of that particular bellmouth. Two methods of testing are available: (1) use of cavitation apparatus wherein the whole liquid system is subject to sub-atmospheric pressures, causing cavitation pockets to open at places corresponding to their occurrence in the prototype, and (2) use of ordinary open-air model with numerous piezometer holes to investigate pressures on the inside surface of the bellmouth and adjacent portions of the conduit. If desired, the latter method may be modified by replacing the ordinary piezometers by the device developed at Vicksburg for measuring instantaneous pressure fluctuations. A program of such tests on a series of bellmouths of standardized design would form a desirable research project, whose results would be of substantial value to the engineering profession.

The tests on open-air jets from sharp-edged rectangular orifices, described in this report, have emphasized the conclusion that such tests do not supply a complete and adequate basis for the design of cavitation-free bellmouths for rectangular conduits. The most that can be expected from such tests is to furnish suggestions for the empirical design of bellmouths which will presumably have low potentialities for cavitation. However, such empirical designs should be subjected to actual model tests before being used in important structures where general pressure conditions are such that an improperly streamlined bellmouth might cause cavitation.

The following empirical design process is suggested. Since the coefficient of contraction for any rectangular orifice lies between that for the circular orifice and the infinitely wide rectangular slot, it is reasonable to assume that the proper rate of taper of cross sectional area in the bellmouth for a rectangular conduit can be derived from Kirchoff's curve by the same equation which expresses the rate of taper of the cross sectional area of the jets from both the circular orifice and the wide rectangular slot, namely

$$\frac{bd}{b_j\,d_j} = \frac{\pi}{2}\left(\frac{1}{\frac{\pi}{\pi+2} - .013 \overset{32}{\times} \frac{b_o}{d_o}} - 1\right)(A_k - 1) + 1 \qquad (6)$$

for A_k see pp 10 Fig. 6

where b_j and d_j are the breadth and depth of the rectangular cross section of the conduit, and b and d are the breadth and depth of the rectangular cross section of the bellmouth at a distance of e feet from the plane of the entrance. (See Equation Art. 4 and the table at the end of Art. 5).

Let b_o and d_o be the breadth and depth of the rectangular entrance to the bellmouth. Then at the entrance where, $b = b_o$, $d = d_o$ and $A_k = \frac{2 + \pi}{\pi}$, equation 6 reduces to

$$\frac{b_o d_o}{b_j\,d_j} = \frac{1}{\frac{\pi}{\pi+2} - .013 \frac{b_o}{d_o}} = \frac{1}{.611 - .013 \frac{b_o}{d_o}}$$

Referring to Figures 11 and 12 it is seen that, in general, the width of the contraction at the ends of a jet from a rectangular orifice is slightly less than at the sides of the jet. However, in designing a bellmouth having a rectangular cross section at all stations, it is desirable to relieve the sharpness of the curvature at the ends of the rectangular section in order to minimize the tendency for abnormally low pressures to occur in the corners of the cross section, where beveling would normally occur if the jet were free to contract as in the open air. For this reason, it is suggested that in bellmouths of the type here under discussion for rectangular conduits, the width of the end contraction be made equal to that of the side contraction, at all stations. That is

$$b_o - b = d_o - d \tag{8}$$

At the entrance section this reduces to

$$b_o - b_j = d_o - d_j \tag{9}$$

By eliminating b_o between Eqs. (7) and (9) the following equation is obtained

$$\frac{d_o(d_o + b_j - d_j)}{b_j\, d_j} = \frac{1}{0.611 - 0.015 \dfrac{d_o + b_j - d_j}{d_o}} \tag{10}$$

Thus if b_j and d_j are given, the value of d_o may be found by solving equation 10 by trial. If desired, this solution can be facilitated by preparing a curve of $\frac{d_o}{d_j}$ versus $\frac{b_j}{d_j}$, the equation for this purpose being written in the form.

$$\frac{\frac{d_o}{d_j}\left(\frac{d_o}{d_j} + \frac{b_j}{d_j} - 1\right)}{\frac{b_j}{d_j}} = \frac{1}{0.611 - 0.013\left(\frac{\frac{d_o}{d_j} + \frac{b_j}{d_j} - 1}{\frac{d_o}{d_j}}\right)} \qquad (11)$$

After finding d_o, the value of b_o is readily found by Eq. 9.

In order to find the values of b and d at a cross section of the bellmouth e feet from the plane of the orifice, the following procedure may be used:

(1) In Kirchoff's formulas (Eqs. 1 and 2) assume a value of θ, and compute the corresponding values of $B_k = \frac{2x}{d_j}$ and $A_k = 1 + \frac{2y}{d_j}$ as in Table II*.

(2) Using Eq. 6 compute the value of bd.

(3) By combining this result with Eq. 8 find the values of b and d.

(4) Compute the distance e of the given section from the plane of the orifice by

$$e = B_k \frac{b_j\, d_j}{2\,(b_j + d_j)} \qquad (12)$$

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*As pointed out in connection with the discussion of Table II, for values of $-\theta_1$, only slightly less than 90° Kirchoff's curve must be replaced by straight line (see Eq. 3) in order that the curved surfaces of the bellmouth may touch rather than be asymptotic to the conduit walls.

Recapitulation of Assumptions Underlying the Proposed Rules for Bellmouth Design

The proposed rules for designing bellmouths for rectangular conduits are based on the following assumptions:

(1) The cross section of the bellmouth is rectangular at all stations.

(2) The width of the top and bottom tapers are made equal to those of the side tapers at all stations.

(3) The rate of taper, so that the change in cross sectional area with respect to the change in hydraulic radius of the cross section based on the corresponding rate of taper for Kirchoff's ideal jet, slightly modified (according to Eq. 6, P 18*) is such as to make the coefficient of contraction agree with Fanning's data on coefficients of contraction for rectangular orifices (See bottom of p. 16).

(4) The final transition from the curved bellmouth surfaces to the plane walls of the conduit is made by inclined plane surfaces tangent to the curve (as indicated in Eq. 3" Page 7).

*Approximates a Tractrix.

Recapitulation of Empirical Rectangular Bellmouth Design Procedure

Method I

Suggested by Professor Harold A. Thomas

1. Given the conduit dimensions, b_j, and d_j.
2. Solve Eq. 11 (page 20) by trial for d_o, the larger dimension of the bellmouth at face of Dam.
3. Subtract: $d_o - d_j = 2y$, the total maximum flare of the bellmouth.
4. Assume that the flare for the shorter dimension equals flare for larger dimension, then $b_o = b_j + 2y$.
5. Knowing b_j, d_j, b_o, and d_o, solve Eq. 6 (page 18) for bd. Eq. 6 reduces to: $bd = \text{const}\ (A_k - 1) + b_j d_j$.
6. Take values of θ_t, and corresponding values of A_k from Table II (page 9), and substitute in equation in step 5 to determine numerical values of bd.
7. Combine values of bd thus found, with Eq. 8 (page 19), solving simultaneously, to determine b and d.
8. Determine e, the x distance of points on bellmouth curve from face of Dam by Eq. 12, page 20.
9. From point on bellmouth curve where $\theta_t = 89°$, extend the curve by a tangent line to its intersection with the conduit wall. The total length of the bellmouth in the x direction will be: $L = e\ (\text{for } \theta_t = 89°) + y\ (\text{for } \theta_t = 89°) \tan \theta_t$.
10. Tabulate results and plot the bellmouth.

Recapitulation of Empirical Rectangular Bellmouth Design Procedure

Method II

Suggested by Dr. Clarence E. Bardsley

1. Given the Conduit dimensions, b_j and d_j.

2. Determine the Fanning Coefficient of contraction:

$$C_c = 0.611 - 0.015 - \frac{b_j}{d_j} + 0.0012 \left(\frac{b_j}{d_j} - 1 \right) \text{ (See footnote page 16)}$$

3. Solve for $A_c' = \left[\dfrac{\dfrac{1}{C_c} - 1}{\dfrac{1}{0.611} - 1} \right] \left(A_k - 1 \right) + 1$ (page 10)

4. Take values of $\theta_{,,}$ and corresponding values of A_k from Table II (page 9) (between $\theta_{,,} = 0°$ and $\theta_, = 89°$), and substitute in equation in step 3 to determine numerical values of A_c'.

5. Determine area of bellmouth as follows:

   $a = A_c' \, a_j = A_c' \, b_j \, d_j$.

6. Solve for bellmouth flare, 2 y, by the equation of area:

   $(b_j + 2y)(d_j + 2y) = a$

7. Determine the dimensions of bellmouth, b and d:

   $b = b_j + 2y$, and $d = d_j + 2y$.

8. Determine e, the x distance of points on bellmouth curve from face of Dam by Equation 12, page 20.

9. From point on bellmouth curve where $\theta'_, = 89°$, extend from the curve a straight line whose length is $\frac{b_o - b_j}{2}$, one end being tangent to the curve and the other end intersecting the conduit

wall. Since θ is nearly a right angle, the total length of the bellmouth in the x direction will be:

$$L = e \text{ (for } \theta' = 89°) + \frac{b_o - b_j}{2}$$

10. Tabulate results and plot the bellmouth.

<u>Notation</u>

d_j = longer cross sectional dimension of rectangular conduit.

b_j = shorter cross sectional dimension of rectangular conduit.

d_o = longer cross sectional dimension at mouth of rectangular bellmouth.

b_o = shorter cross sectional dimension at mouth of rectangular bellmouth.

d = longer cross sectional dimension of rectangular bellmouth at any station on curve.

b = shorter cross sectional dimension of rectangular bellmouth at any station on curve.

e = x = longitudinal dimension of point on curve measured from mouth of bellmouth.

y = ordinate of curve from conduit wall (Same for all walls).

$$C_c = 0.611 - 0.015 \frac{b_o}{d_o} = 0.611 - 0.015 \frac{b_j}{d_j} + 0.0012 \left(\frac{b_j}{d_j} - 1 \right)$$

the coefficient of contraction. (See footnote, page 16).

θ = the angle the tangent to curve makes with horizontal line = slope of curve.

θ' = complimentary angle to θ.

$$A_x = \frac{a}{a_j} = 1 + \frac{2y}{d_j}$$

$$B_k = \frac{x}{r_j} = \frac{2x}{d_j}\ ; \quad e = B_k \frac{b_j d_j}{2\,(b_j + d_j)}$$

a = Area of bellmouth at any station on curve

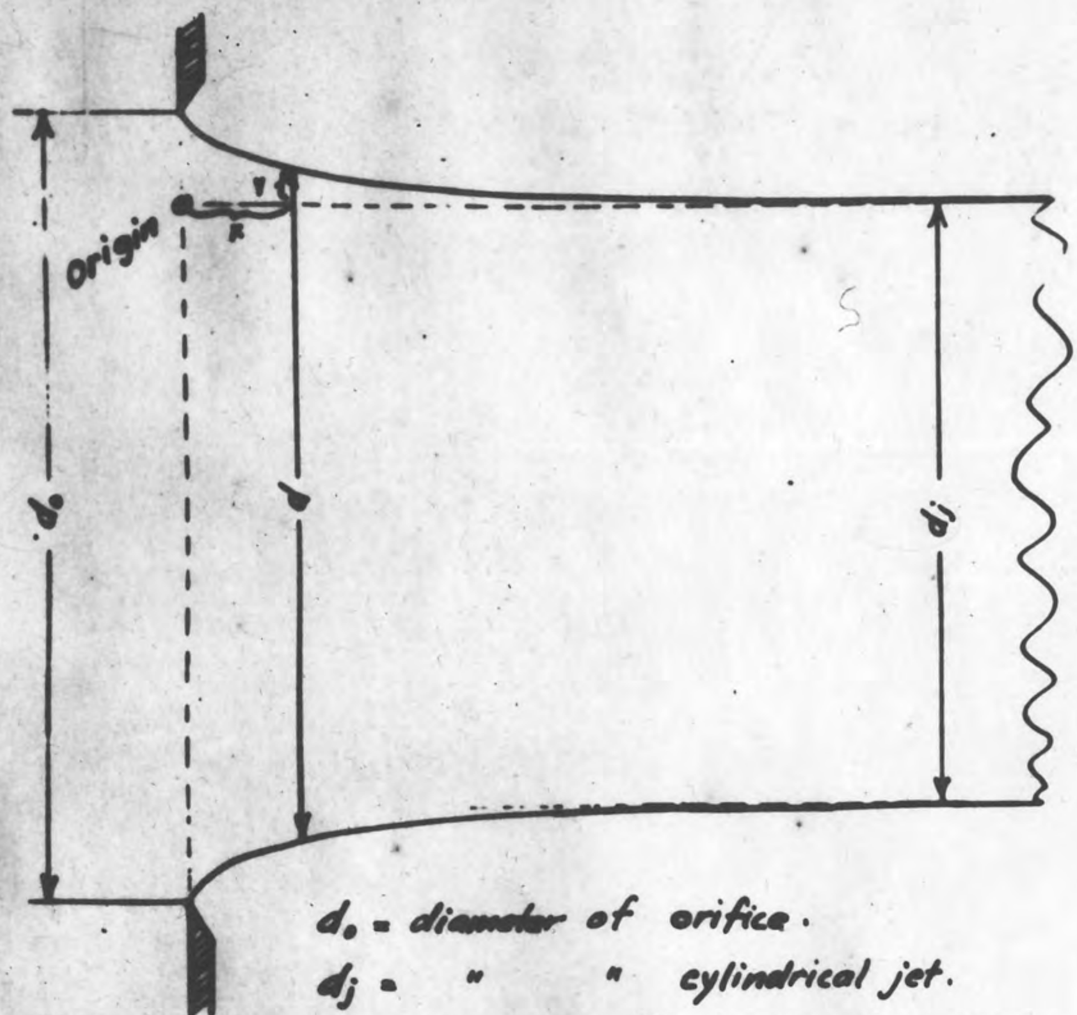
a_j = Area of rectangular conduit.

a_o = Area of bellmouth at mouth

$$A'_c = \frac{\frac{1}{C_c} - 1}{\frac{1}{0.611} - 1} \quad (A_k - 1) + 1;$$ the area ratio

$$L = X_{89°} + Y_{89°} \tan 89° = X_{89°} + \frac{b_o b_j}{2}$$ the total longitudinal length of bellmouth.

Fig. 1



d_o = diameter of orifice.

d_j = " " cylindrical jet.

d = diameter of jet at section having abscissa x.

$a = \frac{1}{4}\pi d^2$ = cross-sectional area at this section.

$p = \pi d$ = air perimeter " " "

$r = a/p = \frac{1}{4}d$ = hydraulic radius " " "

$r_j = \frac{1}{4}d_j$ = hydraulic radius of jet.

FIG. 1. Notation for Jet from Circular Orifice.

Fig. 2

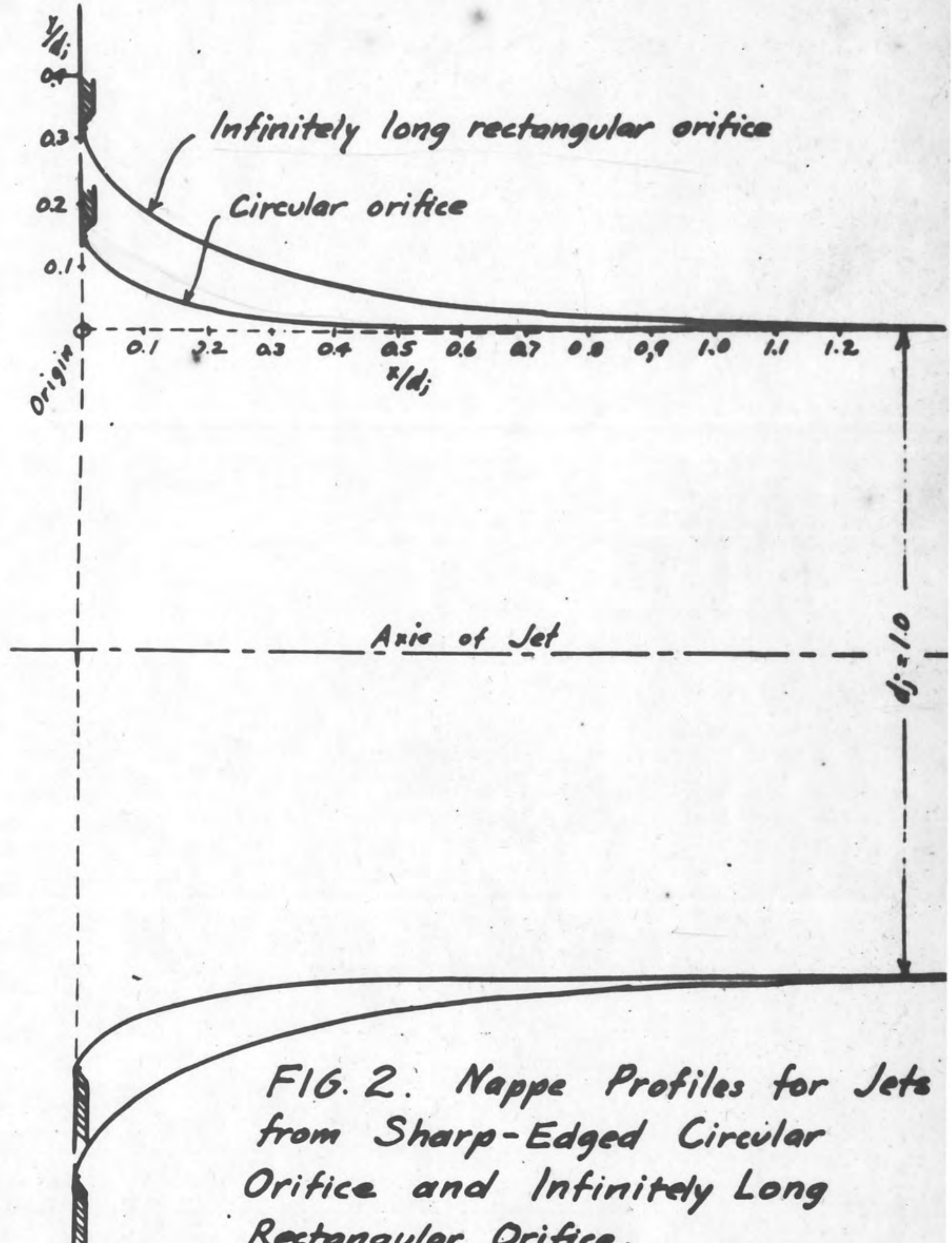


FIG. 2. Nappe Profiles for Jets from Sharp-Edged Circular Orifice and Infinitely Long Rectangular Orifice.

$$x = \frac{d_j}{\pi}\left[\log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + \sin\theta\right] \ldots\ldots (1)^*$$

$$y = \frac{d_j}{\pi}\left(1 - 2\cos^2\frac{\theta}{2}\right) = -\frac{d_j}{\pi}\cos\theta \ldots\ldots (2)$$

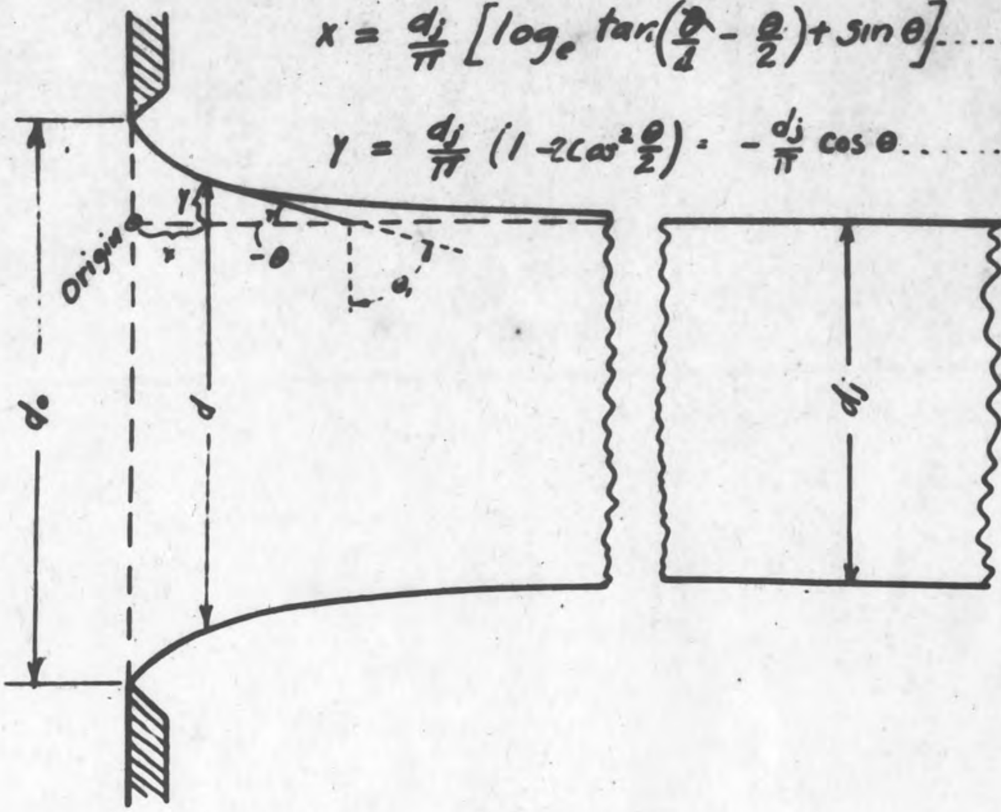


FIG. 3. Notation for Kirchoff's Profile of Jet from Infinitely Long Rectangular Orifice.

$$x = \frac{d_j}{\pi}\left[2.302585 \log_{10} \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + \sin\theta\right] \ldots\ldots\ldots (3)$$

* $x = \frac{d_j}{\pi}(fgd\, x + \sin\theta)$
where $gd = \left[\log_e \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$ = Gudermannian Function

Fig. 4-5

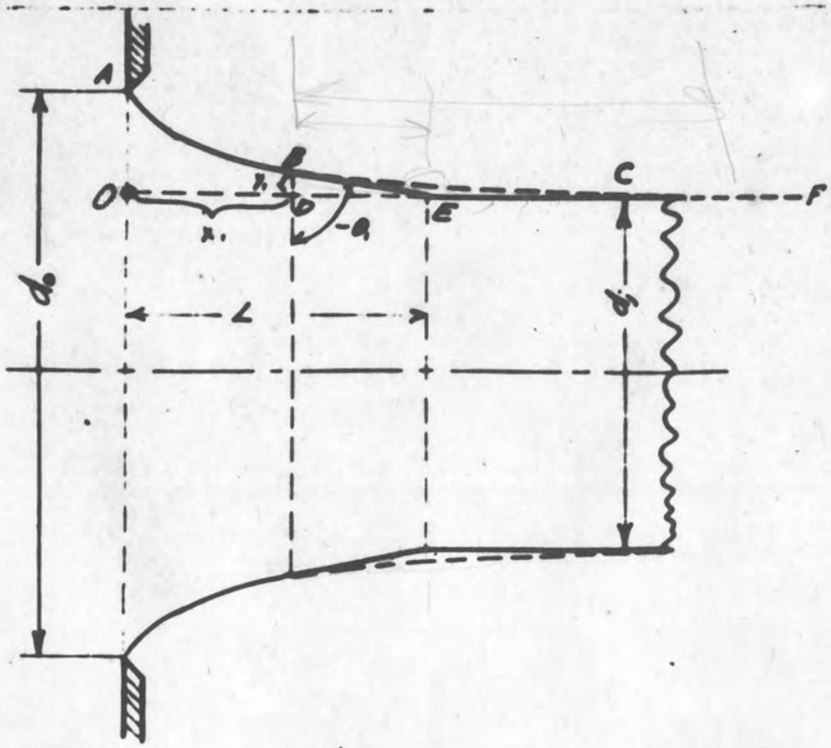


FIG. 4 BELLMOUTH BASED ON MODIFICATION OF KIRCHOFF'S JET PROFILE

Area of circle $= a = \frac{\pi d^2}{4}$

Perimeter " " $= p = \pi d$

Hydraulic radius " " $= r = \frac{a}{p} = \frac{d}{4}$

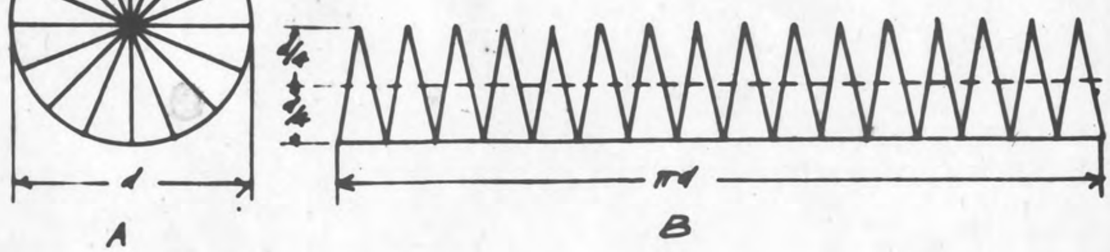


FIG. 5. DEVELOPMENT OF CIRCULAR AREA

Fig. 6

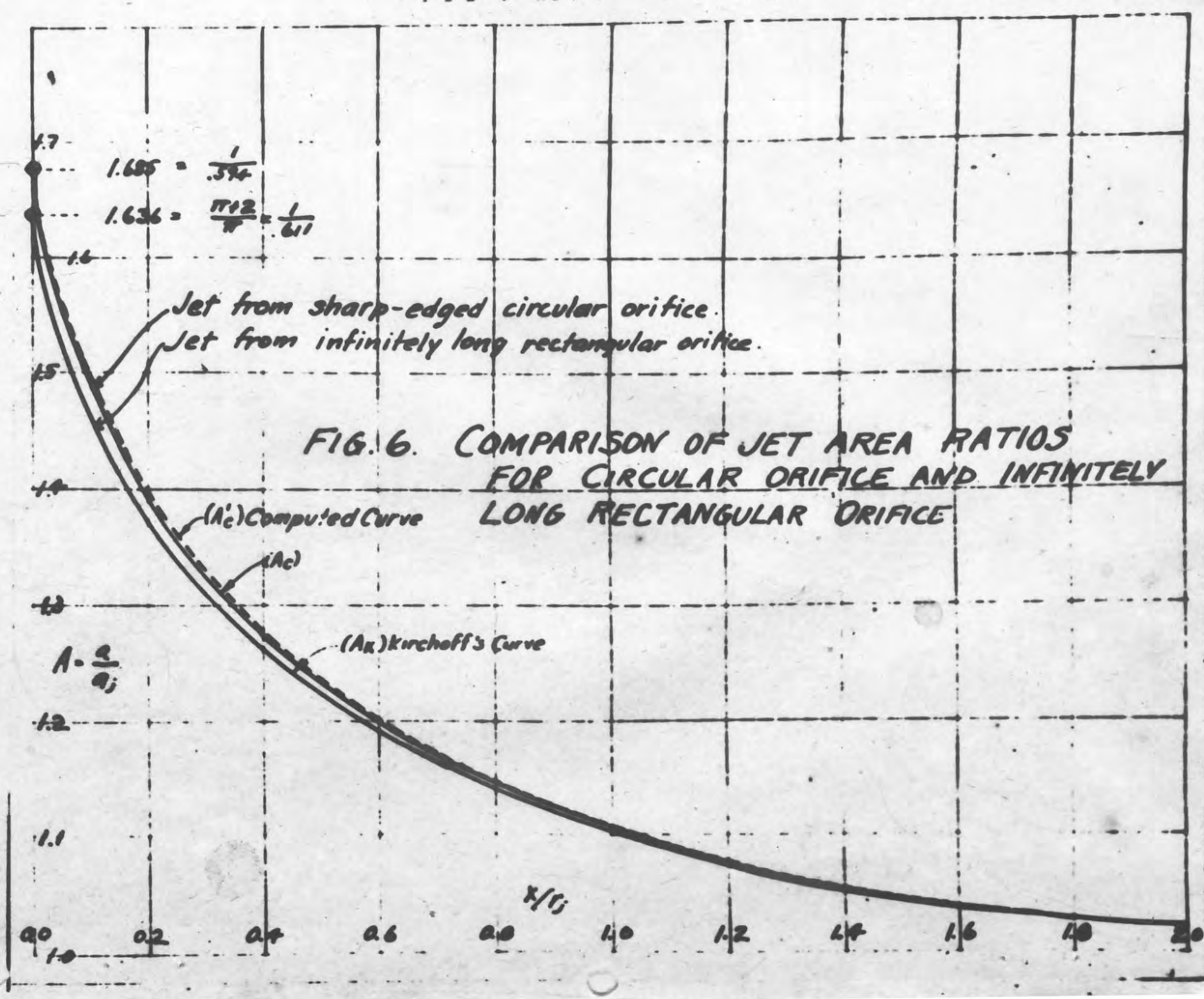


FIG. 6. COMPARISON OF JET AREA RATIOS FOR CIRCULAR ORIFICE AND INFINITELY LONG RECTANGULAR ORIFICE

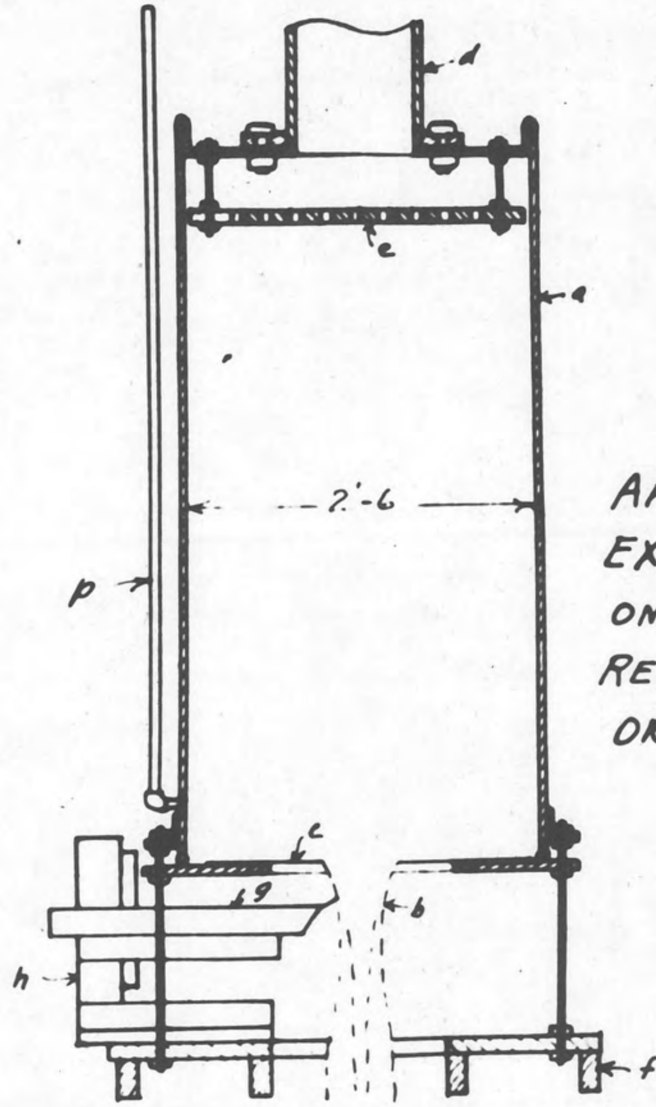


FIG. 7.
APPARATUS FOR EXPERIMENTS ON JETS FROM RECTANGULAR ORIFICES.

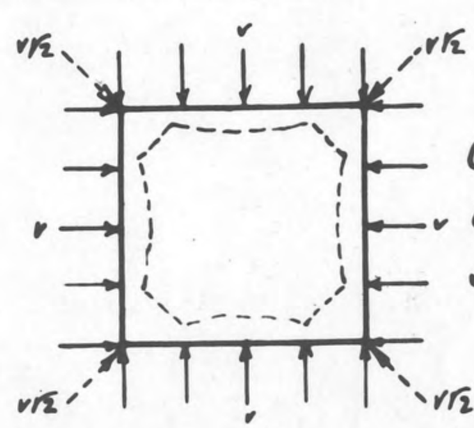


FIG. 8.
CONTRACTION OF SQUARE JET.

Fig 9

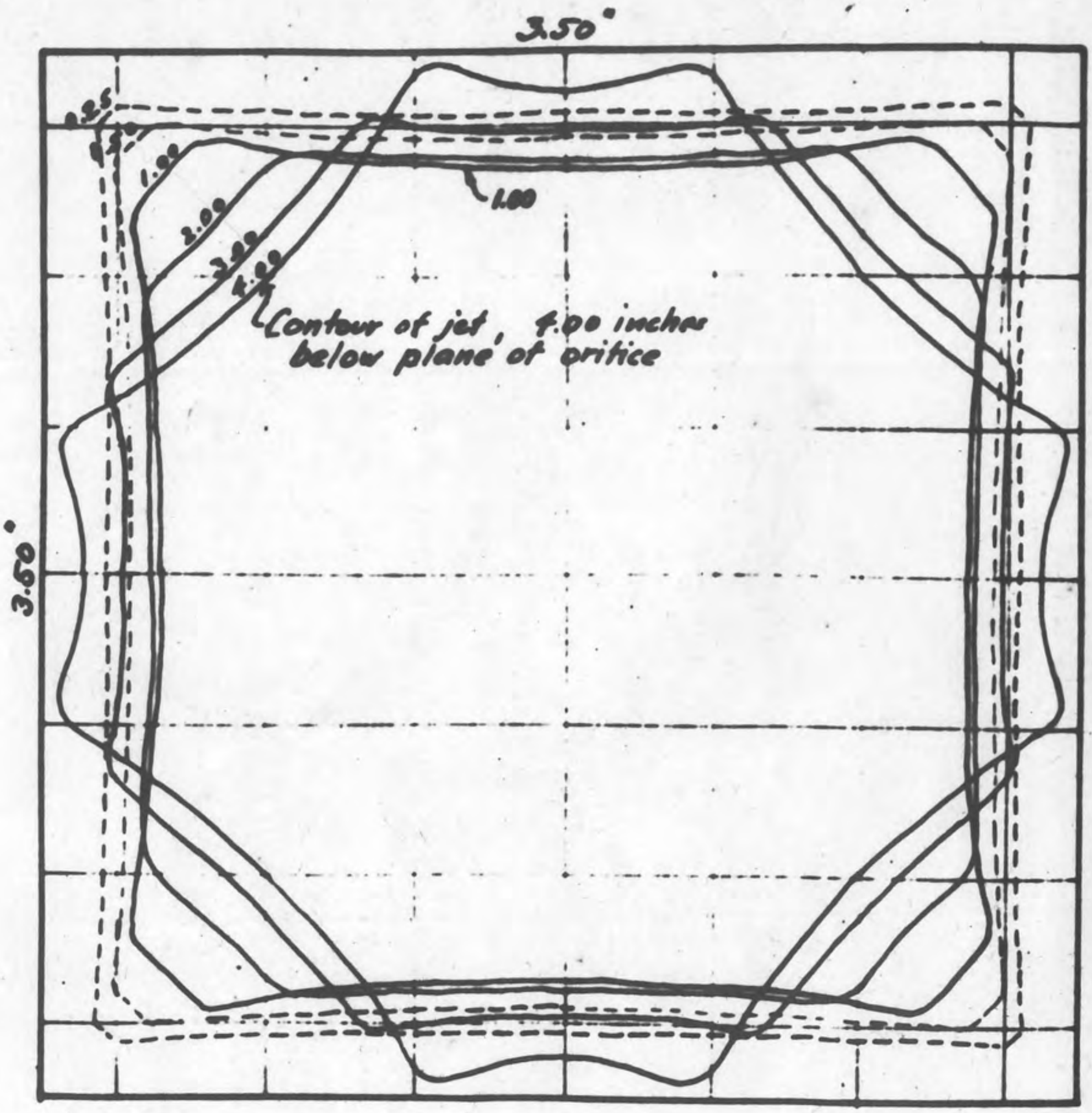


FIG. 9. CONTOURS OF JET FROM SQUARE SHARP-EDGED ORIFICE $3\frac{1}{2}" \times 3\frac{1}{2}"$.

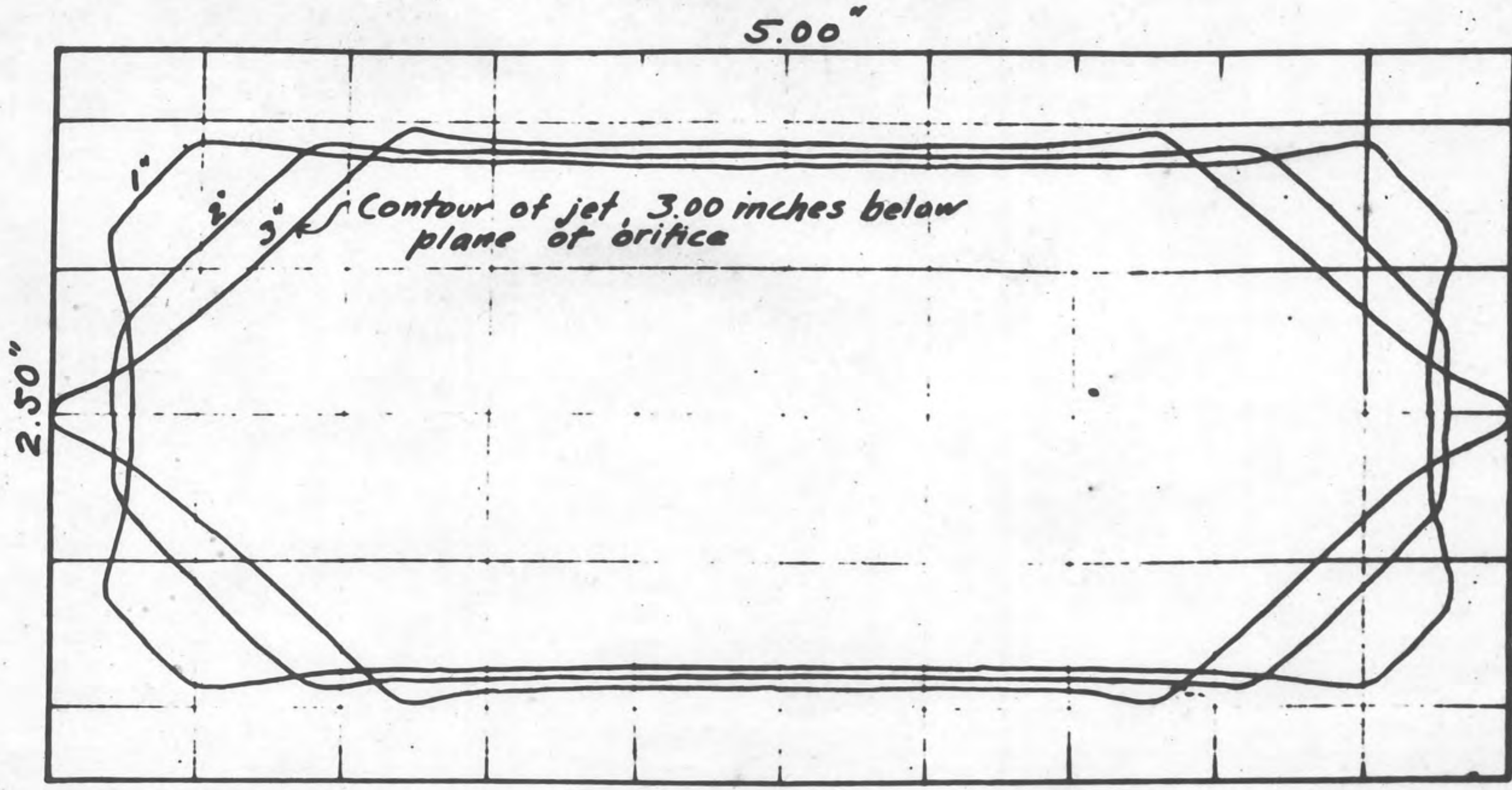


FIG. 10. CONTOURS OF JET FROM SHARP-EDGED RECTANGULAR ORIFICE $2\frac{1}{2}$" by 5".

Fig. 10

Max. end contraction occurs about 1" below plane of orifice
" side " " " 1½" " " " "

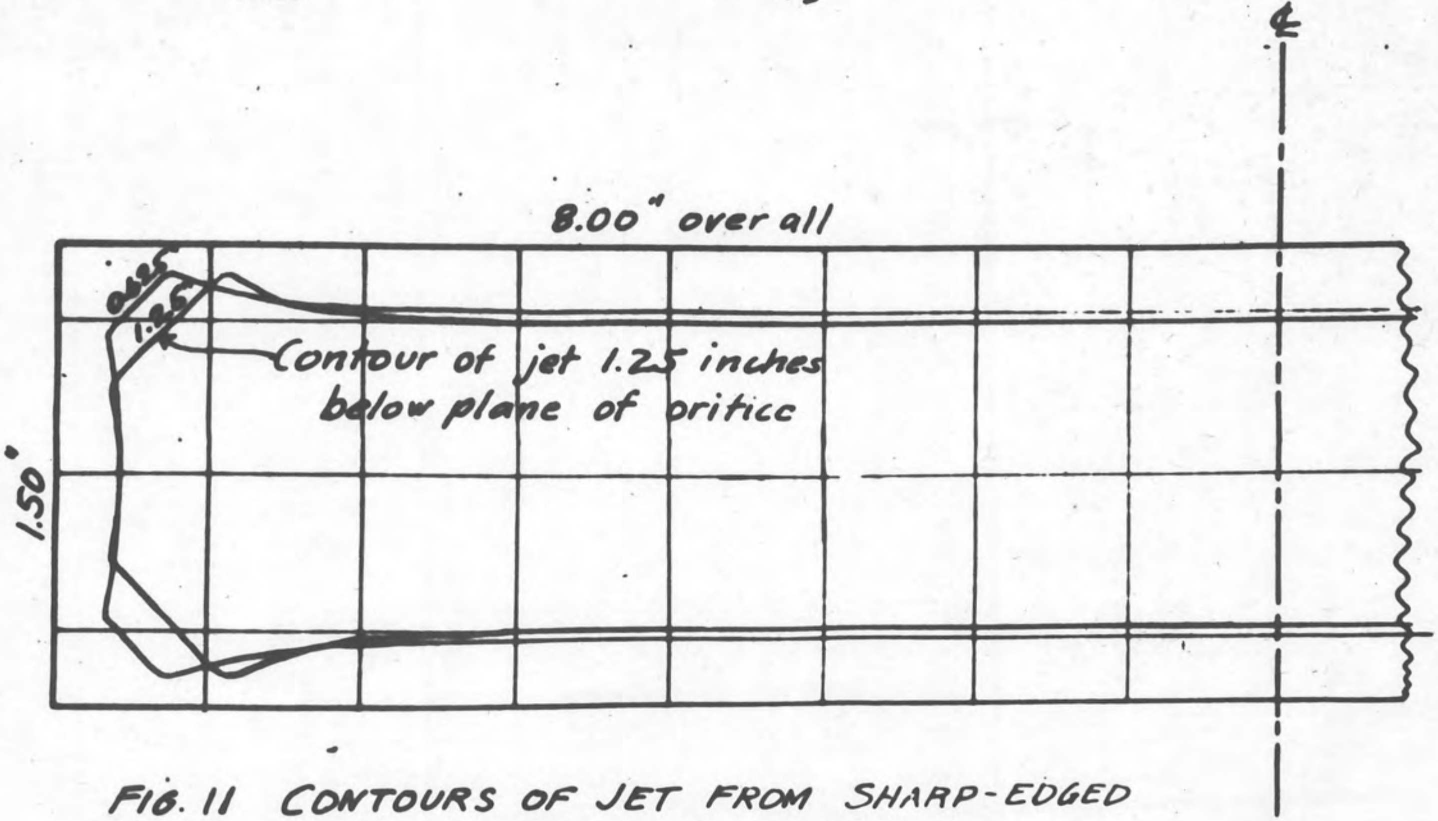


FIG. 11 CONTOURS OF JET FROM SHARP-EDGED RECTANGULAR ORIFICE 1½" by 8"

Scale: Twice full size

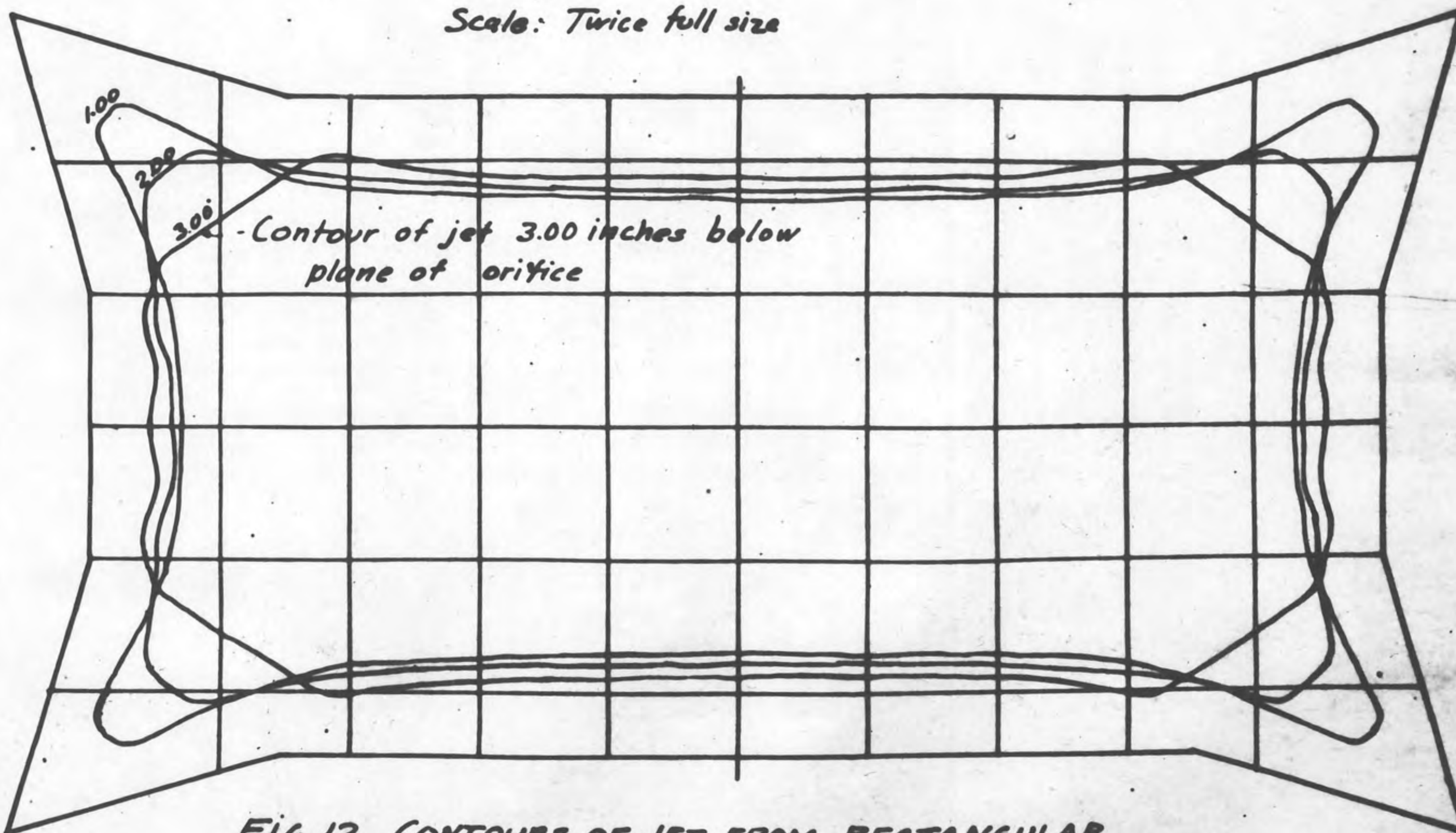


FIG. 12. CONTOURS OF JET FROM RECTANGULAR ORIFICE $2\frac{1}{2}$" by 5", WITH CORNERS MODIFIED.

Fig 12