Presented at the USSD International Symposium on the
Mechanics of Internal Erosion for Dams and Levees
Salt Lake City, UT – August 8-10, 2016

THE SUBMERGED JET EROSION TEST: PAST-PRESENT-FUTURE

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SUMMARY

Major advances in the modeling of dam breach processes have occurred as a result of improved methods for quantifying erosion resistance of embankment materials through field and laboratory testing. The submerged jet erosion test has become one of the most widely applied methods for evaluating erodibility of cohesive soils due to its adaptability to lab and field situations and its robustness across a wide range of materials. This presentation reviews the development of the jet test methods and equipment and highlights significant results obtained from its application. The presentation also offers a critical look at recent developments in jet test equipment, data analysis methods, and associated erosion modeling laws (equations). Standardization of the jet test method is discussed, including the recent retirement of the original standard and the status of efforts to create a new ASTM jet test standard.

INTRODUCTION

Erosion of cohesive soils affects a host of engineering problems including embankment erosion and breach, stream bank erosion, bridge scour, spillway headcut erosion, and rill erosion of agricultural soils. Three classes of erosion test have become prominent in recent years for making quantitative estimates of erodibility parameters: jet erosion tests, internal erosion tests, and flume-type erosion tests. Jet erosion tests utilize a hydraulic jet impinging normally on an exposed soil surface, internal erosion tests utilize pressurized flow through a pre-formed slot or hole in a soil specimen, and flume-type tests utilize flow parallel to a soil surface. In all three types of tests, rates of erosion are observed or inferred from other measurements, applied stresses are estimated, and soil erodibility parameters are determined that correlate applied stress to observed erosion according to selected erosion modeling equations. Specific examples of devices and procedures implementing these approaches are the submerged jet erosion test (Hanson and Cook 2004; ASTM D5852), the Hole Erosion Test (Wan and Fell 2004), and the Erosion Function Apparatus (EFA) (Briaud 2001).

JET TEST DEVELOPMENT

Efforts to develop devices for evaluating the erodibility of cohesive soils date back to at least 1959 (Hanson 1990). To describe soil erodibility numerically, a mathematical model or erosion law is needed, along with methods for estimating the parameters of the model. A commonly adopted mathematical model is the linear excess stress equation:

$$\varepsilon_r = k_d(\tau - \tau_c)^a \qquad (1)$$

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where ε_r is the volume of material removed per unit surface area per unit time (units of velocity), τ is the applied shear stress, τ_c is the critical shear stress needed to initiate sediment detachment, and k_d is a detachment rate coefficient (units of length per time per stress). The exponent a is typically assumed to have a value of 1; when it is allowed to have another value this becomes the nonlinear excess stress model.

Many of the earliest erosion tests were developed only to assess the critical shear stress. An early forerunner of today's submerged jet test and a device focused on erosion rate was the relatively large (more than 1.5-ft diameter) in situ jet test apparatus developed at the USDA Agricultural Research Service Hydraulic Engineering Lab, Stillwater, Oklahoma (Hanson 1990). A pin profiler was used to map a cross section of the scour hole produced by a ½-inch diameter hydraulic jet. Analysis of the data yielded a detachment rate coefficient (i.e., k_d) that was compared to erosion rate tests carried out in large open channel flumes. Subsequently, the analysis method was modified to determine a dimensionless jet index parameter (Hanson 1991) that could be related back to k_d. In both of these methods the critical stress was assumed to be negligible. An ASTM standard for the device (D5852) was first published in 1995, and the standard described how to determine the jet index parameter and the detachment rate coefficient.

THE BLAISDELL ANALYSIS METHOD

Hanson and Cook (1997) examined methods for determining both the critical shear stress and detachment rate coefficient. Three methods were considered:

- A nonlinear curve fitting routine to simultaneously estimate both parameters,
- A method that first estimates τ_c by fitting logarithms of dimensionless scour and jet velocity-time parameters to a hyperbolic function (Blaisdell et al. 1981) found to fit scour progression data from plunge pools below cantilevered spillway outlet and culvert pipes. Once τ_c is determined, k_d is estimated by fitting dimensionless scour depths and times to a model predicting the evolution of the scour depth during the testing period.
- A method that first estimated τ_c based on particle size via Shield's diagram and then determined k_d similarly to the second method.

The first method was found to be unstable, with results varying unpredictably based on initial guesses of the parameter values. The second and third methods both provided consistent, useful results and the Blaisdell method (the name ascribed to the second method in recent literature) became widely adopted. Hanson and Cook (2004) described a reduced-scale jet test device with a 1-ft diameter submergence tank and ¼-inch diameter jet nozzle, and an accompanying spreadsheet that implemented the Blaisdell solution procedure. Notably, the pin profiler was absent from this device as it had been found that reliable results could be obtained from measurements of the scour depth along just the jet axis. Hanson et al. (2002) described a similar device adaptable to use on sloped soil surfaces, also utilizing a ¼-inch diameter jet (Figure 1). Although the Blaisdell analysis method was never incorporated into the ASTM standard, it has been the predominant method of jet data analysis since the late 1990s.

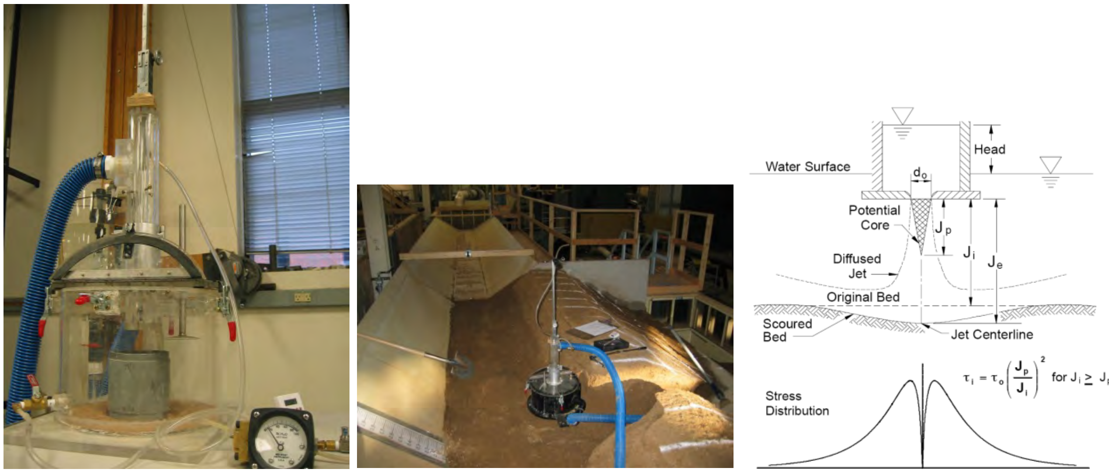

Figure 1. — Submerged jet erosion test device for laboratory and in situ use, and a schematic diagram including the stress profile applied to the soil boundary (Hanson and Cook 2004).

LESSONS LEARNED USING THE SUBMERGED JET TEST

The ease of use of the submerged jet test prompted its application as a research tool in studies aimed at understanding relationships between soil erodibility and basic soil properties.

Hanson (1996) investigated relationships between soil erodibility (as indicated by jet index values) and soil strength and stress-strain indicators, including stress-strain curves obtained from unconfined compression tests. An interesting finding was that soil strength indices were less reliable indicators of erodibility than were the total strain at failure and the volume beneath the stress-strain curve (failure energy per unit soil volume). A simple physical interpretation of this finding is that the total strength of a soil has less correlation with erosion rate than the ability of the soil to absorb energy and deform prior to failure. Although there is still great interest in correlating erodibility with other basic soil properties, this author is not aware of subsequent studies that have exploited this finding.

Hanson and Hunt (2007) showed how soil compaction conditions could dramatically affect the erodibility of soils. Jet test results were used to demonstrate that erosion resistance was maximized when soils were compacted near optimum water content. The effects of varying compaction effort and wet- and dry-of-optimum compaction were also demonstrated. The potential for dry compaction to produce highly erodible soils was dramatically shown. Hanson et al. (2010) developed tables that could be used to estimate τ_c and k_d values as a function of soil clay content, compaction energy, and compaction water content. These tables are useful when jet test results are not available, but they do not take the place of actual jet testing to obtain refined estimates.

APPLICATION TO A VARIETY OF EROSION-RELATED PROBLEMS

The jet test was first developed to investigate soil erodibility in the context of headcut erosion in earthen spillways and embankment breach, and the linear excess stress equation using jet-determined parameters is an integral component of the SITES and

WinDAM models developed by USDA. The submerged jet erosion test has also been adopted for other purposes, including erosion of agricultural lands and erosion processes associated with stream channel migration. Hanson and Simon (2001) used the jet test to evaluate erodibility of cohesive stream bed soils, establishing in the process that there was an inverse relation between τ_c and k_d. Several subsequent investigators have proposed variations of this relation. The same study also established a five-tier descriptive erodibility classification scheme based on τ_c-k_d zones, and these descriptive terms have since become widely adopted; their use in the dam safety field is one example (Bureau of Reclamation and U.S. Army Corps of Engineers 2015).

Development of the non-vertical jet test device was spurred by interest in applying the jet test to problems of stream bank erosion and stream channel migration. The Bank Stability and Toe Erosion Model (BSTEM) was developed by the USDA-ARS National Sedimentation Laboratory. It utilizes the linear excess stress equation to predict channel bank migration rates. Simon et al. (2010), Daly et al. (2016) and others have applied jet testing to this field.

COMPARING TO OTHER TESTS

The availability of several methods for determining soil erodibility parameters has prompted studies to compare the jet method and other tests. Wahl et al. (2008) compared the jet and Hole Erosion Test (HET) methods by applying them to paired samples of remolded soils. Both tests ranked erodibility of different soils in a similar order, but the magnitudes of τ_c and k_d values were dramatically different, with the HET indicating greater erosion resistance (larger τ_c, smaller k_d), often by an order of magnitude in k_d. HET data have generally been applied to develop empirical correlations to probabilities and rates of development of internal erosion, whereas jet test data have been used more for quantitative modeling, correlated to lab tests of erosion rates in flumes, spillways, and embankments; despite the differences in parameter values, within each of these realms, the two tests both adequately serve the intended purposes. However, one must be aware of the differences when attempting to use results from either test in an application area where the other test has traditionally been used. The comparison by Wahl et al. (2008) also demonstrated that the jet test could be more successfully applied over a wide range of soil erodibilities; the jet test could span a range of more than 5 orders of magnitude of k_d, whereas the HET could span only about 3 orders of magnitude.

RECENT DEVELOPMENTS

NEW DATA REDUCTION METHODS

Although the Blaisdell method has been the prevalent approach to analyzing jet test data since the late 1990s, researchers have continued efforts to improve data analysis methods. Several motivations probably drive this, including:

- The inherent variability of soil erosion makes it always challenging to obtain consistent results, and outlying results are not uncommon.

- Conducting tests in one stress range and then extrapolating to larger or smaller stresses during modeling efforts sometimes leads to poor outcomes that prompts questioning of erosion models and parameter values.

The greatest criticism of the Blaisdell method has been its tendency to yield large estimates of the equilibrium scour depth and hence low estimates of the critical shear stress, τ_c. In theory, the equilibrium scour depth is the scour that would occur if a jet test could be continued for infinite time (and thus often seems quite large), and the critical shear stress is the stress that would be applied by the jet when that scour depth is achieved. As a result, the critical shear stress determined by the Blaisdell method often seems quite small. Indeed, Blaisdell et al. (1981) cautioned that the equilibrium scour depth predicted by his "hyperbolic logarithmic velocity-of-scour" analysis was not comparable to a *practical equilibrium* (Blaisdell's emphasis), but that the advantage of his method was that it was objective and avoided the varying personal judgments and interpretations that were so often affecting estimates of equilibrium scour depth at the time of his work.

Table 1. — Jet data analysis methods.

Method	Erosion Model	Details
Blaisdell method (Hanson and Cook 2004)	Linear excess stress	1. Predicts τ_c based on estimate of equilibrium scour at $t=\infty$. (Asymptote of hyperbolic scour-time curve) 2. Adjusts k_d with Excel Solver to minimize sum of squared errors in **predicted times** to reach measured scour depths. Data-fitting uses dimensional times, although data are plotted nondimensionally.
Iterative method (Simon 2010)		Uses Blaisdell solution as starting point. Constrains τ_c to not exceed stress applied at end of test. Adjusts k_d and τ_c simultaneously with same objective as Blaisdell method.
Scour depth method (Daly et al. 2013)		Adjusts k_d and τ_c simultaneously with objective of minimizing sum of squared errors in **predicted scour depths** (dimensional) at specific times.
Khanal et al. (2016)	Nonlinear excess stress	Similar to Daly et al. (2013), but allows value of exponent *a* to vary
Al-Madhhachi et al. 2013	Wilson model	Adjusts b_0 and b_1 simultaneously to minimize sum of squared errors in **predicted erosion rates**. Optimizing to minimize errors in predicted **scour depths** was also tested and has been adopted for more recent work (personal communication with Al-Madhhachi).

Table 1 summarizes several alternative methods of jet data analysis. Innovations have been considered in several areas. Some methods (discussed in the next section) attempt to fit data to nonlinear erosion models, while others continue to rely upon the linear excess stress equation. Among the latter, two new methods use goal-seeking tools that simultaneously determine τ_c and k_d (similar to the Hanson and Cook [1997] first method which was considered unstable), but with different numerical fitting objectives. Specifically, some methods minimize the error in predicted times required to reach observed scour depths, while others minimize the error in predicted scour depth at specific times. The former approach tends to adjust parameters to better fit the latest observations of scour (at the end of a test), while the latter approach emphasizes fitting the earlier observations (from the beginning of a test).

NONLINEAR EROSION MODELS

Recently, researchers have begun to fit jet erosion data to nonlinear models. The most popular has been the model developed by Wilson (1990a; 1990b).

$$\varepsilon_r = \frac{b_0\sqrt{\tau}}{\rho_d}\left(1 - e^{-e^{3-\frac{b_1}{\tau}}}\right) \quad (1)$$

in which ρ_d is the soil dry density and b_0 and b_1 are soil erodibility parameters. This model developed from mechanistic principles predicts that erosion rate vs. shear stress curves will exhibit three regions, an initial region with a low but exponentially increasing erosion rate as stress is increased, a linear region, and a final region in which erosion rate is proportional to the square root of applied shear stress (Figure 2). Like the linear excess stress equation, it is a two-parameter model, with b_0 related to erosion rate and b_1 related to the threshold for initiating erosion. Al-Madhhachi et al. (2013) demonstrated the use of the model for jet data analysis. Khanal et al. (2016) applied the Wilson model to JET, HET, and rill erosion data sets and showed that the Wilson model seemed to fit very well to some nonlinear erosion behavior observed in HET data sets collected by Wahl et al. (2008). Although not directly related to jet erosion testing, Criswell et al. (2016) measured erosion rates of clean gravel beds in a laboratory flume, determined parameters of the linear excess stress model and the Wilson model, and related them to the particle grain size. This provides a first step toward predicting erosion rates of cohesionless soils using models similar to those used for cohesive soils.

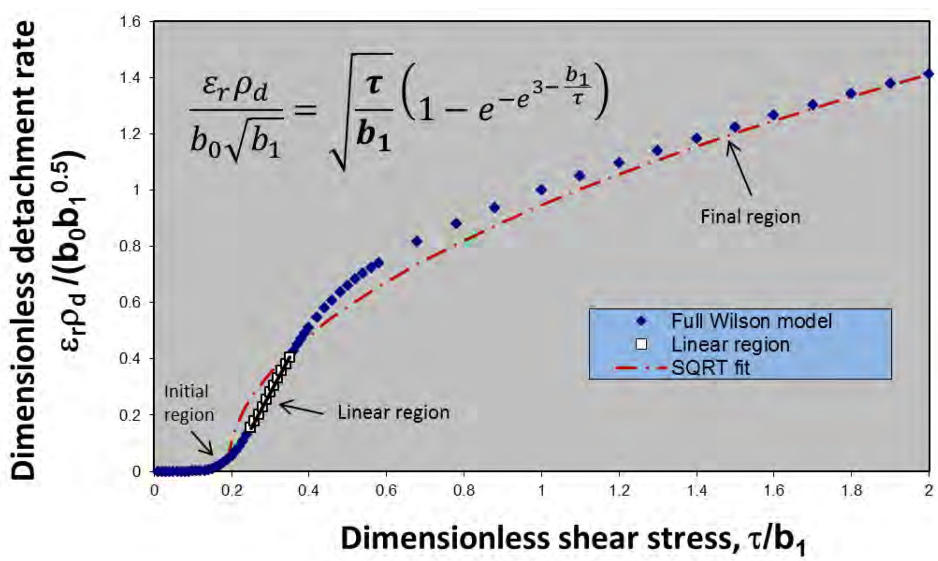

Figure 2. — Dimensionless erosion rates vs. shear stress predicted by the Wilson model, exhibiting three stages of erosion behavior.

A preliminary study recently undertaken by the author examined the ability of several of the newly proposed solution methods and models to fit various jet data sets. One example that illustrates some of the pitfalls is shown in Figures 3-5. The soil tested here was a sandy lean clay s(CL) with 19% clay and a plasticity index (PI) of 9, compacted near optimum water content with standard Proctor compaction effort. Figure 3 shows the observed erosion rates versus applied stresses. The data points follow the general shape trend of the Wilson model and the model can be optimized to fit the scour rate data very well (solid black line). However, when those parameters are used to predict scour depths vs. time (black dashed line in Figure 4) they produce poor predictions of the scour depth in the later stages of the test. In contrast, when the model is optimized to fit scour depths (solid black line in Figure 4), the fit to the scour rate data is poor (Figure 3, black dashed line). The Blaisdell method fits the erosion rate data very poorly at the start of the test and very well near the end of the test period (on Figure 3 the test proceeds from high stress to low stress, left-to-right). The Blaisdell method predicts scour depths adequately, but underpredicts early scour (Figure 4). The scour-depth method (Daly et al. 2013) provides a mediocre fit to the nonlinear erosion rate data but predicts the scour depths well (Figure 4). This jet test lasted about 2 hours, but Figure 5 extends the prediction time of each model out past 5 hours to so that we can evaluate the ability of each solution to predict longer-term scour depths. The Wilson model and scour-depth solutions (Daly et al. 2013) predict almost no continuing scour, but the Blaisdell model predicts scour that appears to follow the trend of the observations in the latter part of the test.

This one example illustrates that the "best" model depends greatly on the perspective used to analyze the data and that visual examination of the data and how the models are reacting to the data is important during analysis. The author has found example jet data sets that exhibit a range of characteristics that add difficulty and the need for judgment to the analysis process. I plan to show some of these additional examples during my presentation.

Some conclusions reached from the author's preliminary investigation of the various solution methods and models include:

- The Wilson model appears to describe real soil behavior in some jet tests.
- Nonuniformity of erosion resistance within a tested specimen may confound attempts to define parameters of the Wilson model. Example data sets illustrate that many specimens exhibit a high degree of random variation of erosion rates, and the Wilson model may fit these data no better than a linear or simple square-root model. Also, fluctuating erosion rates may lead to misidentification of the initial and final regions of the Wilson model erosion curve.
- Defining all three regions of the Wilson model in a single JET test is a challenge. If a soil's erodibility parameters are not already known, multiple tests may be needed to find a starting stress that will allow all three erosion zones to be experienced in a single test. Multiple tests at different starting stresses may be needed to define all three regions.

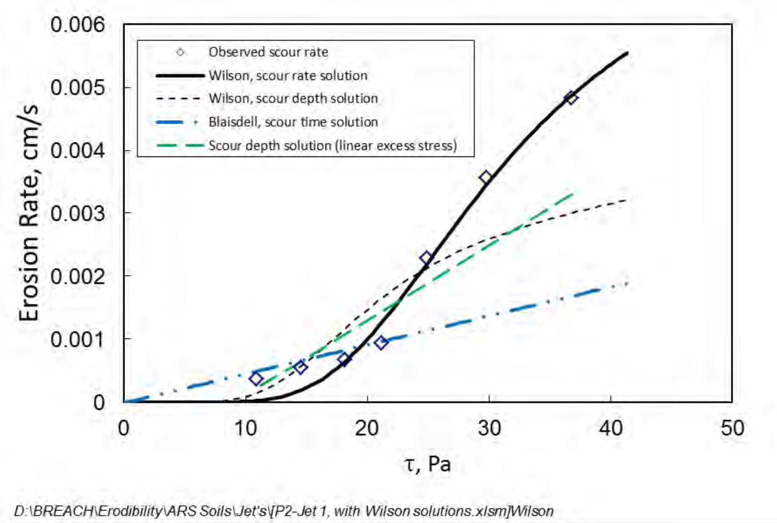

Figure 3. — Jet erosion rate observations analyzed with several predictive models.

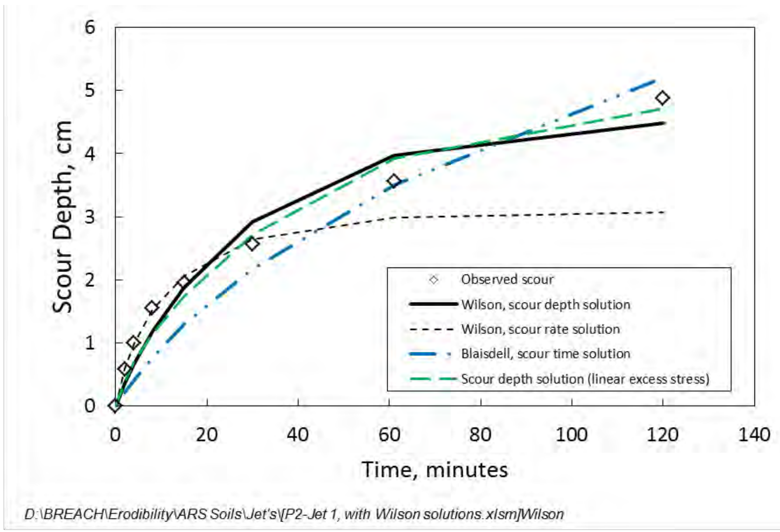

Figure 4. — Jet test scour depth observations analyzed with several predictive models.

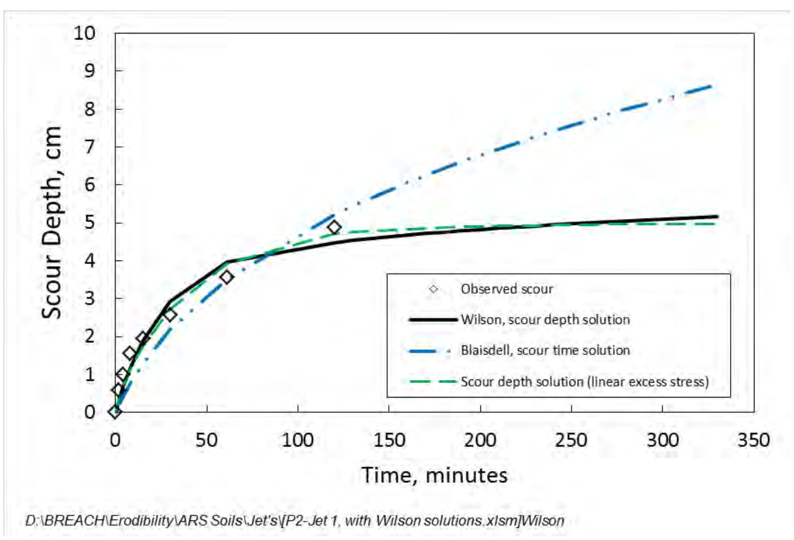

Figure 5. — Jet test scour depth predictions by several predictive models at extended times.

Until applications incorporate the Wilson model or other nonlinear models, there will still be a need to use realistic linear models. Models like WinDAM and SITES presently offer only the linear excess stress equation. These points are important to keep in mind when using a linear model:

- It is important to consider the stress ranges used in the jet test vs. those experienced in the application environment. Whenever possible, erosion tests should be performed in the stress range that will be experienced in the field.
- By virtue of its method for estimating the equilibrium scour at $t = \infty$ and thus τ_c, the Blaisdell method may more accurately represent the fact that erosion rates decrease gradually to zero (the initial region of the Wilson model), and may also be less prone to overestimating erosion rates at high stresses.
- Methods that simultaneously optimize τ_c and k_d (e.g., Daly et al. 2013; Simon et al. 2010) tend to predict either $\tau_c = 0$ or a τ_c value close to the final stress applied during any given test, depending on whether the data suggest a positive or negative x-intercept on the erosion rate vs. applied stress chart.
- If soils behave as described by the Wilson model, linear models fit to data primarily collected in the linear region will underestimate erosion rates at low stresses and overestimate erosion rates at high stresses.

STATUS OF ASTM STANDARD

ASTM Standard D5852 "Standard Test Method for Erodibility Determination of Soil in the Field or in the Laboratory by the Jet Index Method" was withdrawn in 2016 with no direct replacement immediately provided. This standard had not reflected current practice for many years, since it was based on the original large apparatus (1.5-ft diameter soil sample, ½-inch nozzle, pin profiler to measure scour hole depth and shape) and the jet index analysis method that did not provide an estimate of the critical shear stress. Most practitioners since the late 1990s or early 2000s have utilized the Blaisdell analysis method and devices similar to those described in Hanson et al. (2002) or Hanson and Cook (2004).

Al-Madhhachi et al. (2013) described the "mini-JET" device which utilizes a 1/8-inch diameter jet nozzle and a 4-inch diameter submergence tank. This apparatus has quickly become popular for field use due to its small size, light weight, and small water volume requirement. Initial testing (Al-Madhhachi et al. 2013) showed that k_d values obtained from the mini-JET were consistent with those obtained from the earlier, larger devices, but τ_c values were consistently smaller, so an adjustment was recommended to bring τ_c values into agreement. The need for this adjustment has made some users reluctant to adopt the mini-JET. The small size of the 1/8-inch diameter jet also creates concerns about applying the device to soils with a coarse texture or structure.

A new ASTM standard focused on the mini JET has been proposed. At this time the proposed standard presents all three of the linear excess stress model solutions listed in Table 1. The proposed standard does not discuss any solutions based on nonlinear erosion models.

LIMITATIONS OF THE JET TEST

The submerged jet test was originally developed for use on cohesive soils. Application to soils containing significant quantities of fine and medium sand generally works well, especially when silt and/or clay are also present. Pure sands are often so rapidly eroded that it can be challenging to collect sufficient data, but if time intervals between scour depth measurements are kept very short, meaningful results can still be obtained. Soils that contain large quantities of coarse sand or significant amounts of gravel become problematic; as the scour hole deepens, the jet is not able to clear these materials from the scour hole after they become detached. The scour hole quickly becomes stable, limited either by armoring of the scour hole surface or transport capability of the jet. One work-around for this problem is to perform tests with the soil specimen inclined so that detached coarse particles can be assisted out of the scour hole by gravity. Wahl (2014) conducted a study using this approach and proposed that the erodibility of mixed gravelly soils might be evaluated by measuring the erodibility of just the portion of the sample passing the #4 sieve. This would depend upon the sample containing sufficient finer material to prevent interparticle contact of gravel pieces that would limit compaction of the finer fraction. There is presently interest in the dam safety community in creating larger-scale jet test devices that could evaluate erodibility of soils that have previously been difficult to test with small-scale jet devices.

CONCLUSIONS

The submerged jet erosion test has been one of the most successful devices used to quantify erodibility of embankment soils in dam engineering history and has also had a significant impact on other engineering problems related to soil erosion. Although the equipment and methods were standardized for a time, a new wave of development work has produced an era of non-standardization again. While the new developments offer potential for better understanding of soil erosion processes, there is also a great need to fully understand the limitations of the equipment and methods and to continue to exercise care in the analysis of data related to the highly variable processes of soil erosion.

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