



## CONTRACTION COEFFICIENTS FOR RADIAL AND SLUICE GATES IN WINGATE SOFTWARE

Albert Clemmens<sup>1</sup>  
Tony Wahl<sup>2</sup>  
Gilles Belaud<sup>3</sup>

### ABSTRACT

WinGate is a new interactive computer program that provides discharge calibration of canal check structures containing radial gates and/or vertical slide gates. The software provides a graphical user interface to define dimensions and hydraulic properties of the canal, check structure, and gates, and enables the user to compute discharge for specific gate settings or the gate openings needed to pass a specified discharge. The software utilizes the latest energy-momentum calibration equations that enable calibration for a wide range of upstream and downstream cross sections. Rating tables covering ranges of canal water level conditions can be generated for both free flow and submerged flow cases. The software can also accommodate check structures with non-uniform gate geometries and gate openings. New relationships have been developed for the contraction coefficient, based on analysis that shows that the pressure distribution on the gates is non-hydrostatic. These analyses show that the contraction coefficient is dependent on the relative gate opening. This is important for understanding the transition from free orifice to submerged orifice flow, particularly at low heads where submerged conditions may be difficult to distinguish. As the vena contracta becomes submerged, there is a transition to a new contraction coefficient, but after complete submergence of the gate lip the contraction coefficient remains essentially unchanged with further submergence, except as influenced by the relative opening.

### INTRODUCTION

Flow measurement is a key element of most modernization efforts and is often provided by dedicated flow measurement structures, such as weirs and flumes, or modern instruments, such as acoustic Doppler flow meters. At points of flow control, such as in-line checks and bifurcations, it is often desirable to combine flow measurement and control functions by calibrating existing gates for flow measurement. This can provide a monetary savings as well as eliminate lag between changes in gate setting and changes in measured flow rate. The software described in this paper implements the energy-momentum calibration method for both radial gates and vertical slide gates.

Free flow radial and sluice gate calibrations generally use the energy equation.

---

<sup>1</sup> WEST Consultants, 8950 South 52<sup>nd</sup> Street, Tempe, AZ 85284 [bclommens@westconsultants.com](mailto:bclommens@westconsultants.com)

<sup>2</sup> U.S. Bureau of Reclamation, Hydraulic Investigations and Laboratory Services Group, Denver, CO, [twahl@usbr.gov](mailto:twahl@usbr.gov).

<sup>3</sup> SupAgro, UMR G-eau, BP 5095, 34196 Montpellier cedex 5, France [belaud@supagro.inra.fr](mailto:belaud@supagro.inra.fr).

$$Q = b_c \delta w \sqrt{\frac{2g(H_1 - y_2)}{\alpha_2 + \xi}} = b_c \delta w \sqrt{\frac{2g(H_1 - y_2)}{k}} \quad (1)$$

where  $Q$  is discharge,  $b_c$  is gate width,  $\delta$  is the contraction coefficient,  $w$  is the vertical gate opening,  $g$  is the acceleration due to gravity,  $H_1$  is the upstream energy head, and  $y_2$  is the depth at the vena contracta ( $\delta w$  for free flow). The coefficient,  $k > 1$ , accounts for the effects of upstream velocity distribution ( $\alpha_2$ ) and energy losses at the gate ( $\xi$ ). The gate profile is shown in Figure 1 for a radial gate.

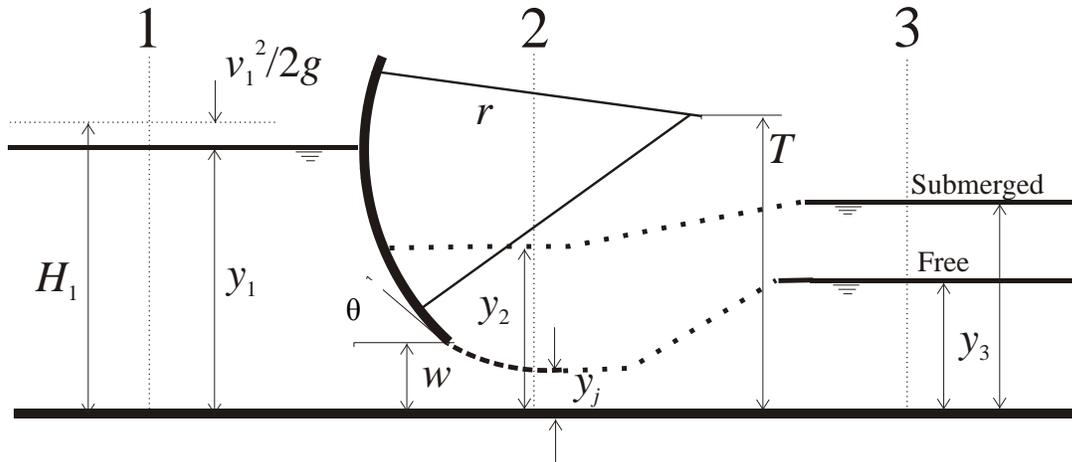


Figure 1. Definition sketch for radial gate.

Clemmens et al. (2003) developed a procedure for determining submerged radial gate discharge based on the energy equation applied between sections 1 and 2 and the momentum equation applied between sections 2 and 3, the Energy-Momentum or E-M method. The key to the approach was an energy correction term added to the energy equation for submerged flow. The E-M method has seen further development since its introduction. Wahl (2005) developed an improved energy correction term utilizing the large data set of Buyalski (1983). Lozano et al. (2009) tested it for vertical slides gates and found the method to be sound. However, providing reliable values for the energy correction term over a wide range of conditions has been elusive. A key obstacle to progress has been the assumption that the contraction coefficient,  $\delta$ , is solely a function of the gate lip angle,  $\theta$ .

Direct observation of contraction coefficients in submerged flow is not possible, so the general approach has been to use the same contraction coefficient in submerged flow and free flow. Laboratory and theoretical studies have provided estimates for  $\delta$  in free flow (see Montes 1997). These studies show that there are very slight variations in  $\delta$  as a function of relative gate opening,  $w/H_1$ , but the effect is small enough that it has been neglected. Belaud et al. (2009) evaluated the contraction coefficient as a function of both relative gate opening and relative submergence. They used potential-flow theory on a vertical sluice gate to determine the contraction. Their solution was similar to prior results, but they were able to show that the contraction coefficient in free flow varied

with the value of  $k$ . Their approach also allowed them to determine the contraction coefficient under submerged flow conditions, where they showed that the contraction coefficient varies significantly, approaching a value of 1.0 as the relative gate opening approaches  $w/H_1 = 1$ . This is theoretically sound, since one would not expect a contraction in flow downstream when the gate lip is barely in the water.

Free orifice flow can only exist when the gate opening is less than two-thirds of the upstream head. For larger gate openings, the flow drops below the gate lip and passes through the opening as weir flow if there is insufficient tailwater to prevent critical flow through the gate opening. However, submergence can keep the gate in the water at a large relative gate opening. Previous work assumed that the contraction coefficient for these cases was the same as that computed for free flow, even though there is no possibility of free flow at the same upstream head and gate opening. The work by Belaud et al. (2009) showed that this is a flawed assumption; the previous attempts to apply energy correction terms were an indirect means for dealing with the problem. Using their improved contraction coefficients for submerged flow, Belaud et al. (2009) obtained good results applying the E-M method to vertical sluice gates without the use of an energy correction. This paper presents the application of the Belaud approach directly to radial gates, utilizing laboratory data to develop and test the concept.

## **LABORATORY EXPERIMENT**

Laboratory experiments on a model radial gate were conducted in 2004 and 2005 at the U.S. Water Conservation Laboratory before it was closed down in 2006. Experiments were conducted in two configurations: 1) a downstream channel with the same width as the gate; and 2) a wider downstream channel.

The 1.24-m (4-foot) wide, 0.61-m (2-foot) high, 15.2 m (50-ft) long glass-sided flume at the (former) U.S. Water Conservation Laboratory was used to perform these tests. Water was supplied from a constant head tank and discharges were weighed in a large weigh-tank and scale system. The radial gate was 0.457 m wide and had a radius of 0.457 m, with a sharp metal edge gate lip. The gate was set between two 17.5-mm-thick plexiglass side walls that were 1.219 m long. The trunnion pin height was 0.366 m, and was located 0.091 m upstream from the downstream end of the side walls, which, if scaled, is typical of installations at the Salt River Project, from whom the experimental gate was obtained. Downstream water depths were measured 4.9 m downstream from the trunnion pin. Additional details of the hydraulics lab facility can be found in Clemmens et al. (2003).

The upstream and downstream water levels were measured with point gauges in stilling wells outside the glass flume. The water levels at Section 2 were measured with the static side of a 5 mm-diameter Prandtl tube which was placed in the middle of the stream. The depth of water at Section 2 under free-flow conditions was also measured with a point gauge. Velocity distributions at Section 2 were measured with a 2 mm-diameter Prandtl tube. All water levels and pressures were registered to the channel invert elevation immediately under the gate.

Tel (2000) used this same facility to conduct a limited number of submerged tests, at one gate opening and with the upstream and downstream channel set to the full 1.24 m width. The tests reported here were conducted with a wider range of submergence conditions and with the downstream channel both at the full width (1.24 m) and at essentially the same width as the gate (0.457 m). To accomplish this, plywood side walls were constructed and anchored such that they could maintain their width. Even so, the plywood and supports did swell, making this channel only 0.445 m wide. When the side walls were not in place, a pressure tap (6-mm copper pipe) was added to the downstream plywood end wall on one side. Tests with the side walls in place are referred to as “narrow” and tests without the side walls in place are referred to as “wide.”

Table 1 shows the conditions that we attempted to establish for each run starting with free flow. These were chosen to give a wide range for  $w/H_1$  and for relative submergence. Before each test, standing water was used to register point gauges to one another and to the bottom of the channel. Machined blocks of the desired thickness were used to set the gate position, and the gate was clamped in place. The weir at the downstream end of the glass-sided flume was lowered to eliminate tailwater on the gate. The flow was turned on, and the supply valve was opened to provide the desired upstream depth under free gate-controlled flow. Flow was stabilized for at least 20 minutes. The depth at the vena contracta was measured at 5 locations across the width of the channel. Several weight-tank measurements of flow were made while collecting these data.

After the free-flow tests were run, the downstream tailwater gate was raised to increase the downstream water depth and cause submergence of the model radial gate. The initial submergence was determined by observing the increase in the upstream water depth. Once the appropriate downstream water depth was obtained, the flow was stabilized for at least 20 minutes. Readings included the upstream depth ( $y_1$ ), depth at the vena contracta location ( $y_2$ ), downstream depth ( $y_3$ ), and depth at the end wall ( $y_w$ ). Further increases in the downstream tailwater level were used to set new submerged conditions. All tests for a given gate position and initial flow were run sequentially, all on the same day. Tests were run first with the narrow downstream channel. Then the walls were removed and tests with the wide downstream channel were run.

Different testing procedures were needed for cases where the flow was too low to allow gate-controlled free flow – or in other words, submergence was required for the gate to remain in the water. The gate position was set as described above. We then adjusted the flow rate to the smallest value that would still allow orifice flow. At this flow rate, all the standard measurements were taken. Then the tailwater level was raised and measurements were taken at 4 to 5 tailwater levels. Next, the flow was reduced by 20%. Then, the tailwater level was raised until gate submergence just occurred, at which point additional measurements were taken. Again 4 to 5 tailwater levels were measured. The same measurements were taken at 100%, 80%, 60%, 40% and 20% of the original inflow. No measurements were taken for weir-flow conditions.

Table 1. Experimental tests, starting at free-flow conditions.

Free flow $y_1$ (m)	Gate Opening $w$ (m)	Expected $y_2$ for free flow (m)	Target values of submerged $y_2$ (m)*						
			0.055	0.077					
0.45	0.05	0.0375	0.055	0.077					
0.48	0.10	0.0709	0.085	0.11	0.145				
0.48	0.15	0.1013	0.11	0.13	0.155	0.17			
0.40	0.05	0.0375	0.05	0.065	0.1	0.15	0.18	0.189	
0.40	0.20	0.1299	0.145	0.17	0.2	0.23	0.27		
0.33	0.05	0.0375	0.07	0.09	0.12	0.145	0.17	0.24	
0.33	0.10	0.0709	0.09	0.11	0.14	0.2	0.24	0.28	
0.33	0.20	0.1299	0.135	0.15	0.2	0.25	0.31		
0.30	0.15	0.1013	0.11	0.14	0.2	0.3	0.327		
0.30	0.20	0.1299	0.15	0.19	0.27	0.35			
0.25	0.05	0.0375	0.048	0.07	0.1	0.14	0.18	0.24	0.31
0.15	0.10	0.0709	0.085	0.12	0.2	0.28	0.39		
0.13	0.05	0.0375	0.044	0.065	0.08	0.15	0.27	0.4	

\*Each target value of  $y_2$  is a separate run.

### FREE-FLOW CONTRACTION COEFFICIENT

Each run of the laboratory data was analyzed to determine the pressure on the gate relative to hydrostatic, based on the momentum equation. This was similar to the analysis performed by Belaud et al. (2009) for vertical sluice gates. The average pressure was roughly 80% of the hydrostatic pressure, but the scatter in the data was too wide to allow it to be used to develop useful contraction coefficients. Since this analysis was not considered when the experiments were originally run, insufficient data were collected to make this approach effective. As an alternative, since the influence of  $w/H_1$  on  $\delta$  under free flow is relatively minor, the results of Belaud et al. (2009) were used directly, even though they had been developed for vertical gates. The resulting relationship was

$$\delta_{\text{free}} = \delta_0 - 0.06a + 0.026a^2 + 0.026a^3 \quad (2)$$

where  $a=w/H_1$ . For a vertical sluice gate,  $\delta_0 = 0.617$ . For radial gates,  $\delta_0$  varies with the gate lip angle. A polynomial equation for  $\delta_0$  was fitted to the measured radial gate contraction coefficients in free flow

$$\delta_{0,\theta} = 0.98 - 0.2515\theta - 0.0489\theta^2 + 0.0393\theta^3 \quad (3)$$

where  $\theta$  is the gate angle in radians. For a vertical sluice gate,  $\theta=\pi/2$  and the computed value is 0.617, which is in agreement with the value for  $\delta_0$  used in Eq. 2. The resulting equation for a radial gate contraction coefficient in free flow is

$$\delta_{\text{free}} = (0.98 - 0.2515\theta - 0.0489\theta^2 + 0.0393\theta^3) - 0.06a + 0.026a^2 + 0.026a^3 \quad (4)$$

Once the contraction coefficient is known, it is then possible to use the free flow data from the laboratory tests to solve Eq. 1 for  $k$ . Analysis of these data led to a predictive relation for  $k$ :

$$k = 1 + 0.16[1 - a^{3.8}]e^{-Re/335,000} \quad (5)$$

where  $Re$  is the Reynolds number for the flow approaching the gate (see Clemmens et al. 2003 for details). Prior analysis had not included the influence of  $a$  on  $k$ . The flow approaching the gate is assumed to have fully developed flow, which would give a velocity distribution coefficient of  $\alpha_1 = 1.04$  (as recommended by Clemmens et al 2001). Here we limit  $k$  to  $\leq 1.04$  so that conditions in the vena contracta will match the approach channel at high submergence.

### FREE-FLOW DISCHARGE PREDICTION

Once relationships for  $\delta$  and  $k$  are known, the discharge can be computed from Eq. 1. For the laboratory data, the confidence interval for the error was 1.9%. The results are shown in Figure 2. The results suggest that the contraction coefficient for free flow is reasonable.

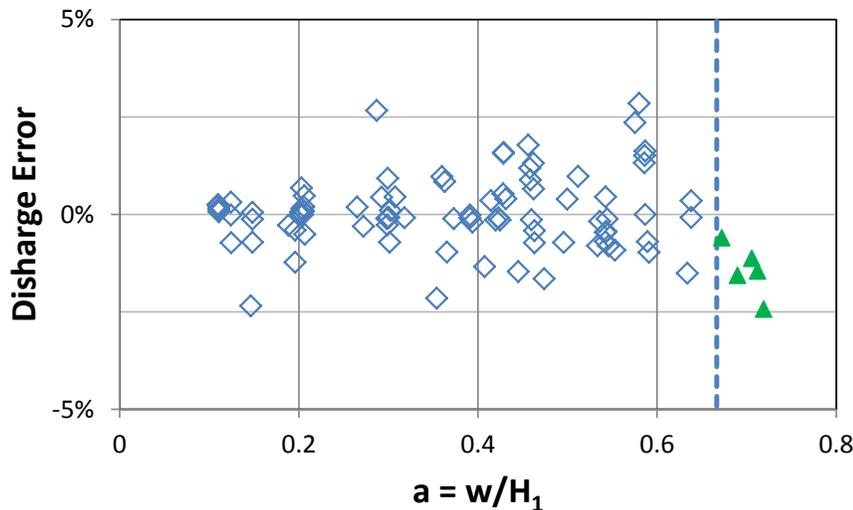


Figure 2. Errors in predicted and measured discharge for free-flow laboratory data. The vertical line at  $a = 0.667$  in Figure 2 represents the theoretical limit for free orifice flow. The points to the right of that line represent submerged conditions. Visual observation suggested that free flow conditions existed. The lower discharges observed there show that the flow was partially submerged. This demonstrated the difficulty of determining flow conditions under low relative head (or high relative gate opening). (Note: the values to the right were correctly computed by WinGate to be submerged flow).

### SUBMERGED-FLOW CONTRACTION COEFFICIENT

For vertical sluice gates ( $\theta=\pi/2$ ), Belaud et al. (2009) computed values for the contraction coefficient under fully submerged flow,  $\delta_{subm}$ . They considered fully submerged flow to occur when the depth  $y_2$  was greater than the gate opening. Values of  $\delta_{subm}$  increased with relative gate opening, starting from the free-flow contraction at zero opening,  $\delta_0=0.617$ . Belaud et al. (2009) developed a relatively complex relation for predicting  $\delta_{subm}$ . We fitted the same data to a polynomial equation:

$$\delta_{\text{subm},v} = 0.617 - 0.092a + 1.515a^2 - 7.86a^3 + 20.03a^4 - 22.98a^5 + 9.77a^6 \quad (6)$$

The subscript  $v$  indicates that this relation applies to vertical sluice gates. The polynomial terms containing  $a$  describe how the fully submerged contraction coefficient increases from  $\delta_0$  to 1.0 as the relative gate opening increases to 1.0.

Belaud et al. (2009) developed the relation above through a momentum analysis between sections 1 and 2 that accounted for nonhydrostatic pressure distribution against the gate. Since the pressures on a radial gate have not been determined experimentally or analytically, a similar methodology cannot be used to determine the influence of  $a$  on  $\delta_{\text{subm}}$  for a radial gate. Also, the value of  $\delta_0$  varies with gate lip angle for a radial gate, so a different amount of correction is needed for each gate lip angle. Thus, the polynomial  $a$  terms in Eq. 6 cannot be applied directly to radial gates.

To address this situation, we recognize that the contraction for a relative gate opening of 1 should still be equal to 1 for radial gates. Thus, we assume that the variation observed for vertical sluice gates can be applied proportionally to a radial gate to produce variation of  $\delta_{\text{subm}}$  from  $\delta_{0,\theta}$  to 1.0 for any gate lip angle. The resulting equation for radial gates is

$$\delta_{\text{subm}} = 1 - (1 - \delta_{\text{subm},v}) \frac{(1 - \delta_{0,\theta})}{(1 - \delta_0)} \quad (7)$$

In this equation  $\delta_{\text{subm},v}$  comes from Eq. 6 and includes variations in  $a$ ,  $\delta_{0,\theta}$  comes from Eq. 3 and indicates radial gate contraction at relative gate opening  $a=0$ , and  $\delta_0=0.617$  is the contraction coefficient for a vertical sluice gate at  $a=0$ . Alternately, since there would be no contraction (and no flow!) associated with a real gate opening of zero, the contraction coefficients at  $a=0$  should be understood to represent the contraction that occurs for a finite gate opening under an infinite head. To illustrate the behavior of Eq. 7, Figure 3 shows the data from Belaud et al. (2009) and the polynomial curve fit (Eq. 6), and an example of the variation of  $\delta_{\text{subm}}$  versus  $a$  for a radial gate with a non-vertical gate lip angle.

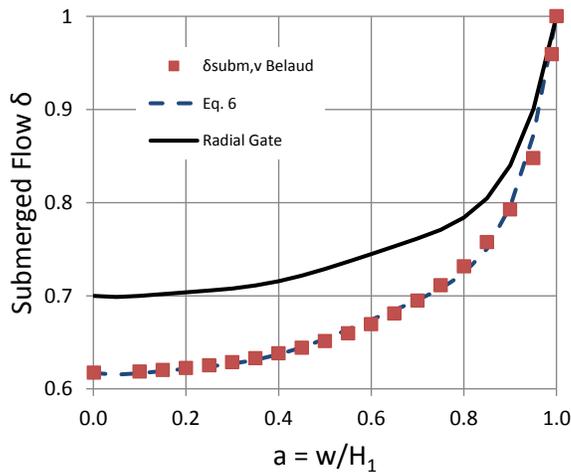


Figure 3. Submerged contraction coefficient as a function of relative opening,  $a$ .

Eq. 7 applies only to fully submerged flow, when the  $y_2$  depth is above the gate lip. If  $y_2$  is greater than the vena contracta depth but less than the gate opening, the flow is partially submerged, which we will call the transition zone. In this range, for a given value of relative gate opening,  $a$ , there is a transition from the free flow value,  $\delta_{\text{free}}$ , to the submerged value,  $\delta_{\text{subm}}$ . This transition occurs as the downstream depth at the vena contracta varies from the free flow value to a value equal to the gate opening. Once  $y_2$  exceeds the gate opening, the contraction coefficient remains essentially constant at  $\delta_{\text{subm}}$ . The data from Belaud et al. (2009) was fit to produce a simple equation for computing the contraction coefficient during the transition,  $\delta_{\text{trans}}$ , where

$$\delta_{\text{trans}} = \delta_{\text{free}} + (\delta_{\text{subm}} - \delta_{\text{free}})(1.63x - 0.5x^2) \quad (8)$$

where  $\delta_{\text{trans}} < \delta_{\text{subm}}$ ,  $0 > x > 1$ ,  $x = (s - a\delta_{\text{free}})/(a - a\delta_{\text{free}})$ ,  $s = y_2/H_1$ , and  $a > s > a\delta_{\text{free}}$ . This relationship is shown in Figure 4. The parameter  $x$  is the fraction of relative submergence with 0 representing  $y_2$  equal to the vena contracta depth and 1 representing  $y_2$  at the gate lip. The data in Figure 4 show that the fully submerged contraction coefficient is realized when the relative submergence exceeds 82 percent; the polynomial of  $x$  in Eq. 8 is limited to a value of 1.0.

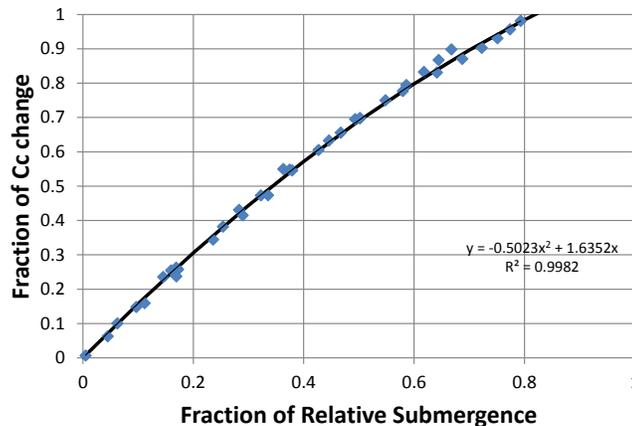


Figure 4. Adjustment to contraction coefficient in transition from free to submerged flow.

### SUBMERGED FLOW DISCHARGE PREDICTION

Several additional details have to be specified to apply the momentum equation between sections 2 and 3. First, there is drag on the flow. Here we use boundary layer drag to compute the force on the stream from boundary resistance. We used a drag coefficient of 0.00235, as suggested by Clemmens et al. (2001). The distance from the vena contracta to the location of the downstream depth measurement was divided into 20 increments. The water depth was assumed to vary linearly between these locations. Then the drag was computed based on boundary layer drag, as described in Clemmens et al. (2003). Also, for the wide channel, there is pressure on the wall resulting from the channel expansion. Here we use the procedure from Clemmens et al. (2003), namely 0.64 times the downstream depth plus 0.36 times the depth at the vena contracta.

Next, we need to apply momentum (Boussinesq) coefficients (Henderson 1966) to the vena contracta ( $\beta_2$ ) and downstream section ( $\beta_3$ ). For the downstream channel, the momentum equation requires a value for  $\beta$  rather than  $\alpha$ . The literature does not give recommendations for selecting values of  $\beta$ , but there is guidance that  $\alpha$  and  $\beta$  values are related to one another. Henderson suggests that the ratio  $(\alpha-1)/(\beta-1)$  is typically in the range of 2.7 to 2.8, and Strelkoff (1969) suggests a value of roughly 3. We use  $\alpha_1 = 1.04$  for the approach channel which is assumed to be fully developed flow, following Clemmens et al (2001). If we use this same assumption for the tailwater channel, this above relationship for  $\alpha$  and  $\beta$  would give  $\beta_3 = 1.014$ . With this value of  $\beta_3$ , and the methods assumed above for computing drag and wall force, we determined the value of  $\beta_2$  that would balance the momentum for the measured discharge. For the approach section,  $k$  from Eq. 5 is our best estimate of  $\alpha$ . The best fit relationship between  $k$  and  $\beta$  was determined, namely

$$\frac{k - 1}{\beta - 1} = 2.9 \quad (9)$$

With the above values, the E-M method was applied to predict discharge. The results for the narrow downstream channel are shown in Figure 5. In this figure,  $y_2$  is the computed depth at the vena contracta and  $y_j$  is the free flow depth there.

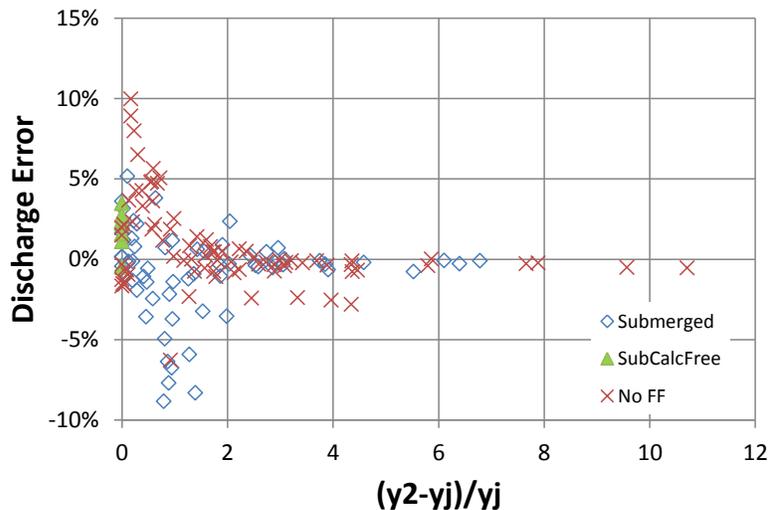


Figure 5. Submerged discharge prediction as a function of relative submergence.

The standard deviation of the error was 2.8%, with a 95% confidence interval of 5.5%, and with essentially no bias. This is a significant improvement over previous implementations of the E-M method that used energy corrections (Clemmens et al. 2003; Wahl 2005). There were some cases where the E-M method predicted free flow, while the observer considered flow to be in the transition. Even for these conditions, the error in discharge prediction is acceptable. Although not shown here, the errors that remain appear to have some relation to the value of  $a = w/H_1$ . It is likely that the assumed linear variation in depth when computing drag is inappropriate. One might expect the nature of changes in depth to vary with  $a$ .

The calculations for the data runs with the wide channel showed larger errors. The standard deviation was 7.6% (CI 15%), with significant bias (2.5%). There are clearly significant errors associated with the wall force. At times, inappropriate values cause calculation of free flow conditions when the flow is clearly submerged. Ongoing efforts are being made to improve modeling of the wall force and the depth variations used for computing drag.

## **WINGATE SOFTWARE**

WinGate is a stand-alone application written in Microsoft Visual Basic.NET. The user interface provides a graphical environment for entering check structure and gate dimensions and other properties. Once a structure has been defined in the software, it can be saved in a commented text file format for later reuse.

Internally, the software uses an object-oriented architecture. Check structure objects are defined by an upstream section, downstream section, and a collection of gates, among other properties. Each of the components of a check structure is itself an object, and the definition of each object is given in a class module. Thus, there are class modules for gates, the gate collection, channel sections, and complete check structures, which can also include some gate bays that are occupied by weirs. The bulk of the program code for calculations resides at the lowest level, within subroutines of the gate class. When the flow rate through an individual gate is needed at a higher level in the program, the higher level object simply asks the gate class to return the flow rate property of the lower level object. This initiates an iterative solution of the energy and momentum equations for that gate. Another level of iteration for the structure as a whole is often needed to fine-tune estimates of energy and velocity head for the upstream and downstream channels, which cannot be fully determined until the total flow is known. With this architecture, the resulting high level subroutines are relatively simple in form.

The program operates in two basic modes, a single-flow solution which provides detailed information about each gate, and a rating table mode that gives results for a range of upstream and downstream water levels. In each mode, the program can solve for the flow through the check structure at given gate settings or the gate setting needed to produce a given flow rate. The single-flow solution can be especially useful for operations. A base data file defining the check structure could be quickly loaded and actual gate positions could be adjusted using the graphical user interface. The flow rate through each gate and the check structure as a whole could then be computed, or target flow rates through each gate could be entered and the software could determine the appropriate opening for each gate. The solution process is very fast, making it feasible to use the program to support real-time operational decisions.

Rating tables are available for both submerged flow situations and gates that operate solely in free flow. The rating table mode can only be applied to check structures in which all gates are similar; the flow is computed assuming that all gates are set to the same position, or a single gate setting is computed that could be applied uniformly to all

gates to produce a desired total flow. For a gate that experiences submerged flow, multiple tables are required to provide information covering the range of upstream and downstream water levels, as well as a varying gate setting or discharge. For a gate that always operates in free flow, the program offers a free-flow rating table that condenses the entire range of operations (varying upstream water level and varying gate setting or discharge) into a single table.

## STATUS

The WinGate software is under continuing development at this time. A beta version of the software has been made available that uses the original E-M method, with energy corrections. Work is now underway to incorporate the new methods described in this paper for both radial gates and vertical sluice gates. Research continues on application of the E-M method for situations where the downstream channel is wider than the gate.

## REFERENCES

- Belaud, G., Cassan, L., and Baume, J-P. 2009. Calculation of contraction coefficient under sluice gates and application to discharge measurement. *Journal of Hydraulic Engineering*, 135(12), 1086-1091.
- Buyalski, C.P. 1983. Discharge Algorithms for Canal Radial Gates. U.S. Department of the Interior, Bureau of Reclamation, Research Report REC-ERC-83-9, Denver, CO.
- Clemmens, A.J., Wahl, T.L., Bos, M.G., and Replogle, J.A. 2001. Water Measurement with Flumes and Weirs. Publication #58, International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands. 382 pp.
- Clemmens, A.J., Strelkoff, T.S., and Replogle, J.A. 2003. Calibration of submerged radial gates. *Journal of Hydraulic Engineering*, 129(9), 680-687.
- Henderson, F.M. 1966. Open Channel Flow. MacMillan Publishing Co., Inc., New York.
- Lozano, D., Mateos, L., Merkley, G.P., and Clemmens, A.J. 2009. Field calibration of submerged sluice gates in an irrigation canal. *Journal of Irrigation and Drainage Engineering*, 136(6), 763-772.
- Montes, J.S. 1997. Irrotational flow and real fluid effects under planar sluice gates. *Journal of Hydraulic Engineering*, 123(3), 219-232.
- Tel, J. 2000. Discharge Relations for Radial Gates. MSc Thesis, Delft Technical University, Delft, The Netherlands, 86 pp. plus Appendices.
- Wahl, T.L. 2005. Refined energy correction for calibration of submerged radial gates. *Journal of Hydraulic Engineering*, 131(6), 457-466.