Computational Procedures used for Radial Gate Calibration in WinGate

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ABSTRACT

New laboratory experiments were conducted on a radial gate to evaluate the energy-momentum procedure for calibration. Prior work had used an artificial energy correction, E_{corr}, to match observed conditions. In the current work, non-hydrostatic pressures, velocity, and momentum distribution coefficients were used instead. This avoids the quirky behavior associated with E_{corr}. Non-hydrostatic pressure distributions were found in the jet, but they had an insignificant influence on the head-discharge calibration. Velocity-distribution coefficients were added to both the energy and momentum equations. The value of the momentum coefficient was related to the relative submergence; roughly 1.08 for low submergence, increasing to 1.3 at high submergence. For this data set, the energy equation was able to predict free-flow discharge within ± 2%. The energy-momentum equations were able to predict the discharge for submerged flow to ± 5% when the downstream channel width was equal to the gate chamber width, and within ±8% for a wider downstream channel. No improvement was found for estimating the wall force exerted downstream when the tailwater channel is wider than the gate. These new calibrations have been implemented in the new WinGate software.

INTRODUCTION

Henderson (1966) outlined a basic energy-momentum procedure for sluice gates. Clemmens et al. (2003) developed a calibration procedure that used the energy equation on the upstream side of the gate and the momentum equation on the downstream side, the so called E-M method. This method included empirical factors to account for upstream energy loss, velocity-distribution effects, estimation of downstream channel wall forces, and estimation of effects of submerged hydraulic jumps. The method can be used to calibrate gates with any upstream and downstream channel size and shape, and thus has advantages over the strictly energy-based methods. The E-M method presented by Clemmens et al. (2003) required an energy correction term (Ecorr) and an associated modification to the jet velocity. Wahl (2005) developed an alternative correction term. Lozano et al. (2009) applied the method successfully to vertical sluice gates. Castro-Orgaz et al. (2010) suggested the use of energy and momentum coefficients to improve the calibration of vertical slide gates.

Additional experimental data were collected in the hydraulics laboratory of the U.S. Water Conservation Laboratory, before it was closed in 2006. Preliminary analysis of this data uncovered a fundamental flaw in the adjustment procedure of the E-M method presented by Clemmens et al. (2003). The energy correction term is not in effect until submergence starts, while data showed that submergence was actually delayed (occurred at a higher downstream water level). This suggests that the E_{corr} correction was working on the wrong aspect of the process.

In this paper, we present the results from analysis of the new experimental data and propose E-M equations incorporating adjustments in the form of velocity-distribution and pressure-distribution coefficients that account for deviation from hydrostatic pressures and uniform velocity profiles. Our application of these terms varies significantly from that proposed by Castro-Orgaz et al. (2010), as will be demonstrated. These new procedures will be implemented in the WinGate software.

THEORY

We use the Energy-Momentum method suggested by Clemmens et al. (2003), where the energy equation is used upstream from the vena contracta and the momentum equation is used downstream. The equations are redeveloped, considering non-uniform velocity distributions and non-hydrostatic pressure distributions. The general energy and momentum equations for open channel flow are derived in Henderson (1966) based on point forms. Applying those equations for steady flow, for the energy equation we have:

$$H_1 = \lambda_{E1} y_1 + \alpha_1 \frac{v_1^2}{2g} = \lambda_{E2} y_2 + \alpha_2 \frac{v_2^2}{2g} + \xi \frac{v_2^2}{2g} = \lambda_{E2} y_2 + (\alpha_2 + \xi) \frac{v_2^2}{2g} \tag{1}$$

where y is flow depth, v is average flow velocity, g is acceleration due to gravity, α is the velocity-distribution coefficient, and the λ_E coefficients account for the effects of a non-hydrostatic pressure distribution in the energy equation. The pressure-distribution coefficient can be computed from:

$$\lambda_E = \frac{\int_{invert}^{water\ surface} \left(\frac{P(z)}{\gamma} + z\right) dz}{\int_{invert}^{water\ surface} \left(\frac{P(z)_H}{\gamma} + z\right) dz} = \frac{\int_{invert}^{water\ surface} \left(\frac{P(z)}{\gamma} + z\right) dz}{y^2} \tag{2}$$

where z is vertical elevation above the invert, $P(z)$ is the actual pressure at location z, $P(z)_H/\gamma$ is the hydrostatic pressure head at location z (= y-z), y is the water depth, and γ is the unit weight of water. The velocity-distribution coefficient α is:

$$\alpha = \frac{\int v^3 dA}{v_m^3 A} \tag{3}$$

(Henderson 1966 pg 19 and Chow 1959 pg 29) where v is flow velocity, v_m is average flow velocity, A is cross-sectional flow area, and ξ is an energy loss coefficient. Subscripts refer to section locations, where section 1 is in the channel upstream from the gate, and section 2 is at the vena contracta. (Figure 1).

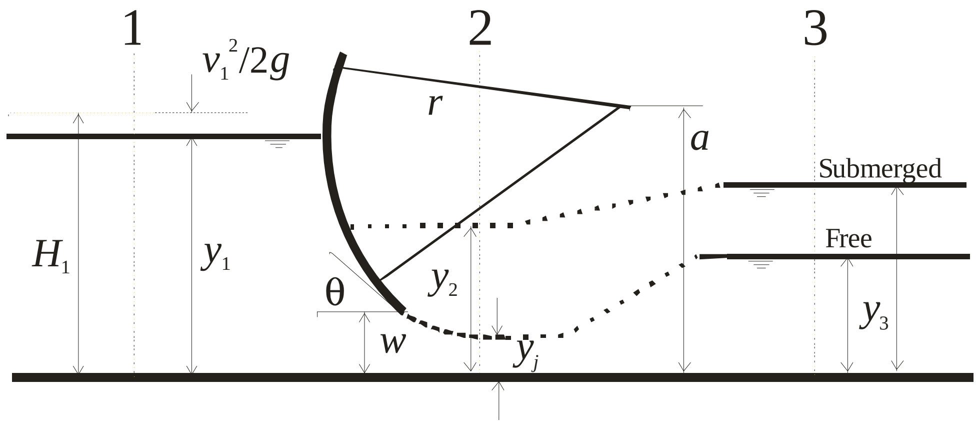


Figure 1. Definition sketch for radial gate.

For a rectangular gate, the vena contracta depth, y_j, is usually computed from an empirically determined contraction coefficient, δ,

$$y_j = \delta w \tag{4}$$

where w is the gate opening. If the velocity is replaced by $Q/b_2 y_j$ in Eq. 1, and then solved for discharge with $\lambda_{E1} = 1$ (i.e., hydrostatic pressure upstream), we get

$$Q = b_c \delta w \sqrt{\frac{2g(H_1 - \lambda_{E2} y_2)}{\alpha_2 + \xi}} \tag{5}$$

Note that under free flow conditions, $y_2 = y_j$. Under submerged conditions, the jet thickness (δw) remains the same as under free flow, as described by Eq. 5, but the depth y_2 increases.

For a rectangular gate, the momentum equation also follows from Henderson(1966):

$$\beta_2 Q v_e + \lambda_{M2} b_2 g \frac{y_2^2}{2} + \frac{F_{wall}}{\rho} = \beta_3 Q v_3 + \frac{F_3}{\rho} + \frac{F_{drag}}{\rho} \tag{6}$$

where Q is discharge, b is channel width, v_e is the vena contracta velocity, F_{wall} is the force on the structure, F_3 is the pressure force in the downstream channel, F_{drag} is the drag force on the floor and walls of the downstream channel due to friction, ρ is the density of water and β is the momentum coefficient (Henderson 1966 pg 19 and Chow 1959 pg 29):

$$\beta = \frac{\int v^2 dA}{v_m^2 A} \tag{7}$$

The coefficient on the pressure term of the momentum equation to account for non-hydrostatic pressure distributions, λ_M, is

$$\lambda_M = \frac{\int_{invert}^{water\ surface} P(z) dz}{\int_{invert}^{water\ surface} P(z)_H dz} \tag{8}$$

Section 3 is in the channel a sufficient distance downstream from the structure that flow is relatively stable (Figure 1). If the downstream channel is rectangular the downstream pressure is:

$$\frac{F_3}{\rho} = \lambda_3 b_3 g \frac{y_3^2}{2} \tag{9}$$

The wall force occurs when the downstream channel cross-section is different from the gate cross section (usually larger). For a rectangular gate and downstream channel, the wall pressure is found from

$$\frac{F_{wall}}{\rho} = (b_3 - b_{2)}\, g\, \frac{y_w^2}{2} \tag{10}$$

where we estimate y_w as a weighted average of the depths at sections 2 and 3

$$y_w = p_w y_3 + (1 - p_w) y_2 \tag{11}$$

The drag force can be estimated from a boundary-layer drag calculation, where

$$\frac{F_{drag}}{\rho} = \frac{C_F}{2} \int_{section\ 2}^{section\ 3} w_p v^2 dx \approx \frac{C_F}{2} \left([1 - \phi] w_{p2} v_2^2 + \phi w_{p3} v_3^2 \right) \tag{12}$$

where C_F is the drag coefficient, w_p is wetted perimeter and ϕ is a weighting coefficient to model the relative contribution of drag at sections 2 and 3.

LABORATORY EXPERIMENT

Unfortunately, the shortness of this paper does not allow us to fully describe the laboratory experiments that were conducted in 2004 and 2005. Experiments were conducted with a downstream channel with the same width as the gate and with a wider downstream channel.

DISCHARGE CALCULATIONS

Free Flow. Free flow discharge for a given upstream water level, y_1, gate width, b_c, and gate position, w, can be found from iterative solution of Eq. 5, where H_1 is a function of y_1 and found from the left part of Eq. 1. The solution is iterative since H_1 is a function of velocity, which is a function of discharge (velocity = discharge divided by area). WinGate currently uses this procedure to determine free flow.

Unknowns from these equations are δ, α_1, $\alpha_2+\xi$, and λ_{E2}. The contraction coefficient, δ, was empirically determined from prior laboratory studies (Tel 2000), and confirmed by the current study (2004-2005).

$$\delta = 1.0016 - 0.2349\theta + 0.1843\theta^2 + 0.1133\theta^3 \tag{13}$$

The approach velocity coefficient, α_1, can be assumed based on a well-developed profile. Clemmens et al (2001) suggest a value of 1.04.

Clemmens et al. (2003) present a relationship for $\alpha_2+\xi$, however that relationship was in error due to an error in the assumed temperature, which resulted in the wrong values for Reynolds Numbers. A new relationship found here is

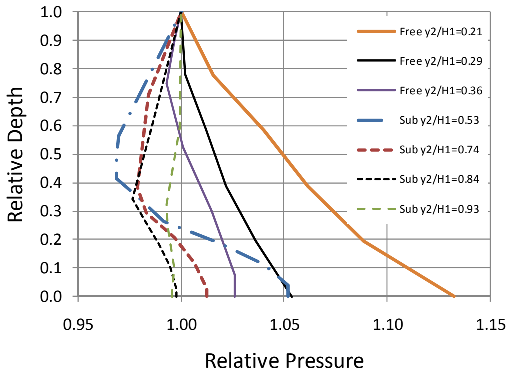
$$\alpha_2 + \xi = 1 - 0.2e^{-Re/174000} \tag{14}$$

Where Re is the Reynolds number based on the velocity and wetted perimeter at the entrance to the gate.

The 2004-2005 experiments found non-hydrostatic pressure distributions in the vena contracta, as shown in Figure 2. The relationship found here (Figure 3) was

$$\lambda_{E2} = 1.06 - 0.21\,\frac{y_2}{H_1} + 0.15\left(\frac{y_2}{H_1}\right)^2 \tag{15}$$

However, very little change in discharge results from use of Eq. 15 rather than λ_{E2} = 1.0 in Eq. 5.



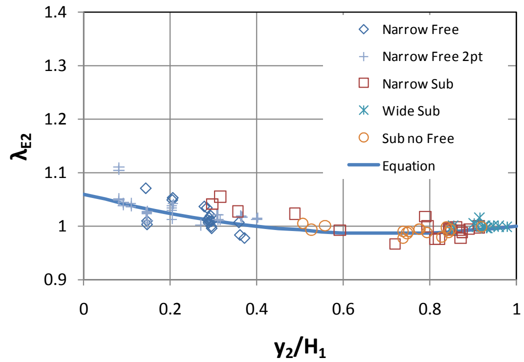


Figure 2. Static pressures measured at the vena contracta of experimental radial gate.

Figure 3. Pressure-distribution coefficients at the vena contracta of radial gate.

Free-Flow Discharge Prediction: Eq. 5 was used to determine discharge based on the measured upstream water level and gate position. A value for λ_{E2} was computed from Eq. 15, δ was computed from Eq. 13, and $\alpha_2+\xi$ from Eq. 14. For free flow, $y_2 = \delta w$. The results are shown in Figure 4. The average error for these data was -0.05% and the standard deviation was 0.9%. Two standard deviations gives an uncertainty better than ±2%. A comparison between discharge error and error in vena-contracta depth showed a positive correlation. Since these errors are reasonably small, the effect of error in the vena contra depth was not investigated further.

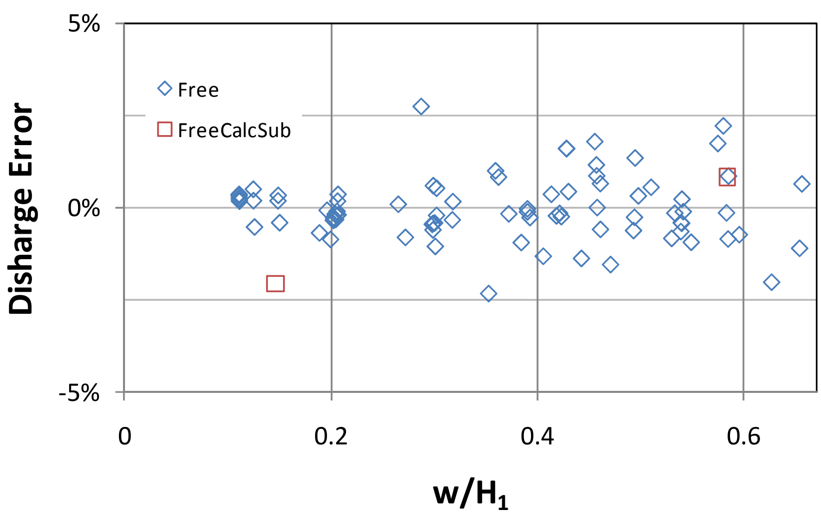


Figure 4. Free flow discharge predictions from energy equation.

Submerged Flow. Calculation of submerged flow requires simultaneous solution of both the energy equation (Eq. 5) and the momentum equation (Eq. 6). Known conditions are upstream depth, y_1, gate width, b_c, gate opening, w, and downstream depth, y_3. For these two equations, discharge, Q, and depth at the vena contracta, y_2, are unknown. Additional parameters determined experimentally are: for the energy equation δ, α_1, $\alpha_2+\xi$, and λ_{E2}; and for the momentum equation (only needed for submerged flow) β_2, λ_{M2}, β_3, p_w from Eq. 11 and C_F, and ϕ from Eq. 12.

For this analysis, we assume that the cross-sectional area of the vena contracta is the same under free and submerged flow. Thus the effective velocity in the jet is $v_e = Q/b_2y_j$, while the average velocity in the section $v_2 = Q/b_2y_2$. We disagree with Castro-Orgaz et al. (2010) on the interpretation of β. Since the momentum is only applied by the vena contracta jet, the velocity-distribution coefficient should only apply to that term, not the entire depth, y_2. The intent of α and β are to show minor deviations from a uniform velocity field, not to represent flow-regime changes (i.e., values should be slightly above, but close to unity).

The momentum equation and associated drag are only relevant once submergence has occurred. Also, the drag mainly occurs downstream from the structure, once the channel width has widened. We tried a variety of values for C_F (0.002 to 0.005) and ϕ (0 to 1). They had a relatively minor influence on the solution. We settled on C_F=0.00235, which represents a fully developed profile, and ϕ =0.7.

The analysis of this data did not suggest an improved method to determine the water depth, and thus wall force, on the downstream side of the structure. We used an average value for p_w = 0.64.

Chow (1959, pg. 32, pg. 50) develops equations for the pressure-distribution coefficients that allow one to show that

$$\frac{\lambda_M - 1}{\lambda_E - 1} = 2 \tag{16}$$

Calculating individual values of the pressure-distribution coefficients from the raw pressure data, this relationship was confirmed exactly for every example tested. Solving Eq. 16 for λ_M gives $\lambda_{M2} = 2\,\lambda_{E2} - 1$.

For the downstream channel, the momentum equation requires a value for β rather than α. The literature does not give recommendations for selecting values of β, but previous studies suggest that α and β values are related to one another: Henderson suggests that the ratio $(1-\alpha)/(1-\beta)$ is typically in the range of 2.7 to 2.8, and Strelkoff (1969) suggests a value of roughly 3. For the downstream channel, the flow conditions are generally a little less stable than upstream. Here, the value for α might be expected to be higher, perhaps even above 1.1. If we use α_3 = 1.11, we get $\beta_3 \approx$ 1.04. Since the velocity in the downstream channel is relatively low, β_3 does not have a large influence on the predicted discharge. (i.e., we also used $\beta_3 \approx$ 1 and 1.05 in the analysis that follows with very little difference in results.)

Momentum Coefficient (β_2), narrow channel: The Energy-Momentum (E-M) method for submerged flow solves Eqs. 5 and 6 for the unknowns y_2 and Q. For this analysis, we used the coefficient values defined above for λ_{E2}, δ, $\alpha_2+\xi$, λ_{M2}, β_3, p_w, C_F and ϕ. With y_1 (and thus H_1 through iteration with Q), y_3 and w measured, the solution of Eq. 6 (with substitutions from Eqs. 9 to 11) also requires a value for β_2. For the narrow downstream channel, p_w does not have an influence. Starting with an initial value of β_2, we can solve Eqs. 5 and 6 for y_2 and Q. If the value of Q does not match the measured value, we can adjust β_2 until the calculated Q matches the observed Q. In this way, β_2 is essentially determined from the observed data, assuming the equations above. The results for β_2 from the laboratory data with a narrow downstream channel are shown in Figure 5 as a function of y_2/H_1. Several other variables were tried, but this had the strongest relationship. The fitted equation is

$$\beta_2 = 1.07 - 0.05\left(\frac{y_2}{H_1}\right)^2 + 0.28\left(\frac{y_2}{H_1}\right)^3 \tag{17}$$

This relationship is somewhat dependent on the relationships used for the drag coefficient. As discussed above, different values of ϕ in Eq. 12 were used. Computing the drag based on only the conditions at section 3 produced slightly lower values of β_2, particularly at lower y_2/H_1. Averaging the conditions (ϕ=0.5) gave higher values at small y_2/H_1 and a drop to lower values before rising again. A value of ϕ=0.7, gave a relatively constant value at low y_2/H_1, and a gradually rising value at higher values. We were not able to find a combination of conditions that produced a constant value of β_2.

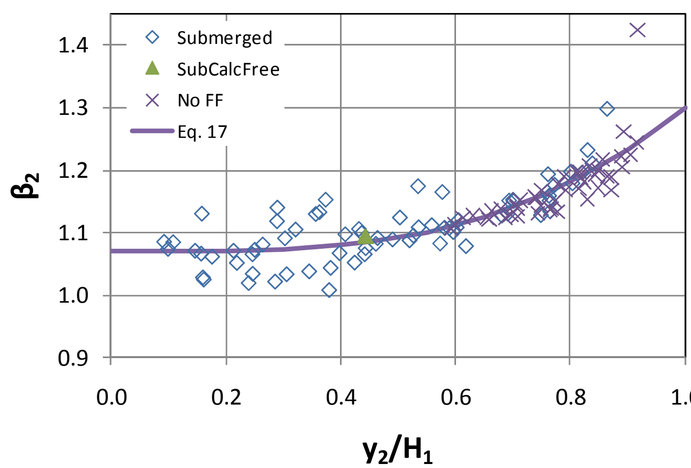


Figure 5. Values of momentum coefficient in vena contracta with narrow downstream channel.

Several runs were observed to be submerged, but the calculations identified free flow conditions, depending on the value of β_2 used. These are labeled “SubCalcFree” for “submerged but calculated as free.” There were also several runs where the discharge was below what would be possible for a free flow gate (i.e., below flow at H_1/w =1.5, when weir flow starts). All data seemed to follow the same trend. Values of β_2 of 1.1 are not too surprising, since the flow is rapidly accelerating. These results assume that the above equations are all correct, in particular that the submerged jet retains the same cross sectional area.

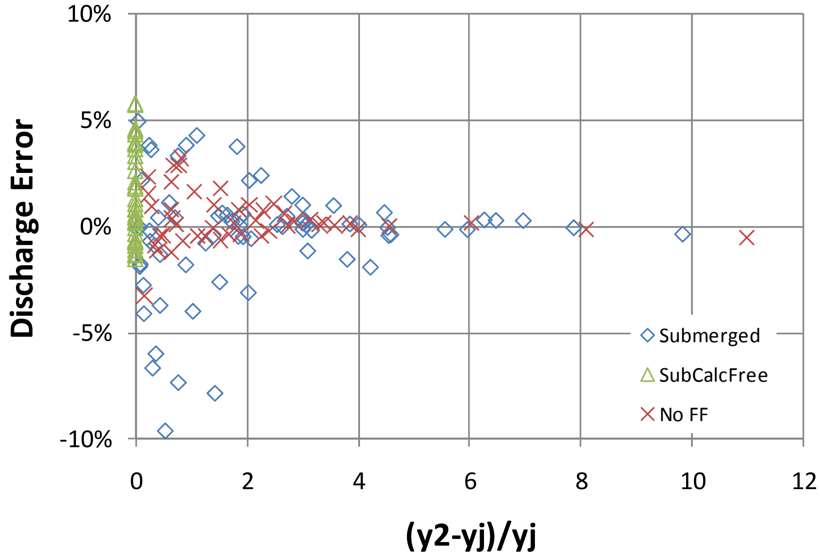


Figure 6. Discharge error for submerged radial gate with narrow downstream channel.

Submerged-Flow Discharge Prediction, narrow channel: With Eq. 17 for β_2, Eqs. 5 and 6 were solved for discharge. The results are shown in Figure 6. Note that there were quite a few cases where the calculations suggested free flow conditions. The discharge errors are reasonable, suggesting that these were just at the transition to submerged flow. The cases where there is no free flow possible also showed reasonable discharge predictions (i.e., flow is less than that computed with H_1/w = 1.5). The average discharge error was -0.1% and the standard deviation was 2.4%. The expanded uncertainty based on two standard deviations is roughly ±5%. This is slightly worse than for free flow, but still acceptable.

Momentum Coefficient (β_2), wide channel: If p_w and Q are known, then the E-M method (Eqs. 5 and 6) can be used to predict β_2 for the conditions where the downstream channel is wider than the gate. Results are shown in Figure 7. The general trend of the data follows Eq. 17, but the scatter in the data is much wider than in Figure 5. There are also β_2 values below unity, which is not theoretically possible. Despite these problems, comparing Figure 5 and Figure 7 shows that the relations between β_2 and y_2/H_1 are similar for the wide-channel and narrow-channel data. Since β_2 reflects conditions in the gate chamber, we would not expect it to be influenced by conditions in the downstream channel. Thus, we used the relationship developed in the narrow channel and applied it to the wider downstream channel.

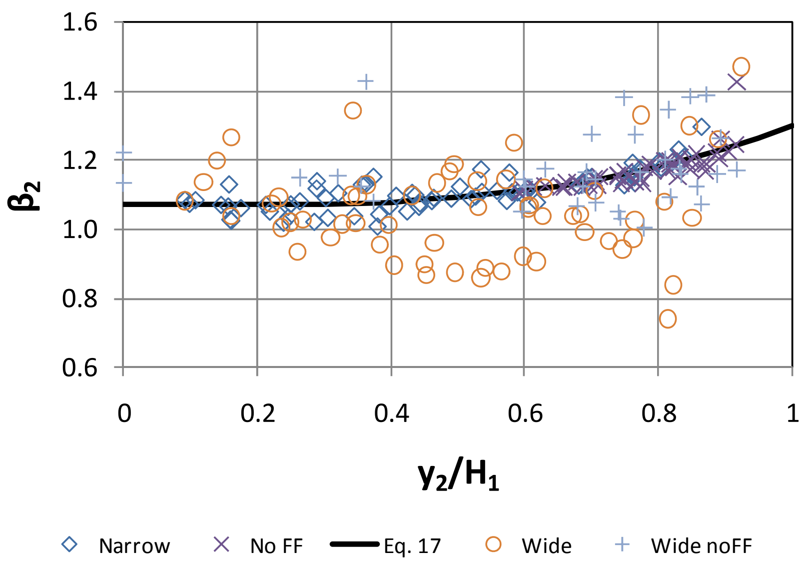


Figure 7. Comparing values of the momentum coefficient in the vena contracta for wide and narrow downstream channels.

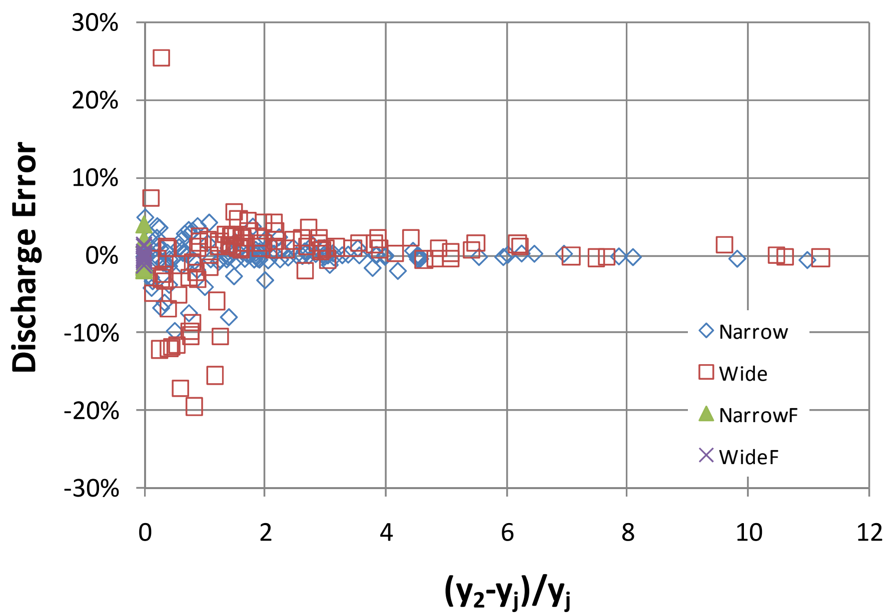


Figure 8. Discharge error for submerged radial gate, with constant wall force coefficient.

Submerged-Flow Discharge Prediction, wide channel: We used the relationship for β_2 described by Eq. 17 and computed the discharge by solving Eqs. 5 and 6 for Q and y_2. The results are shown in Figure 8. The average error was -0.2% and the standard deviation was 3.8% (expected uncertainty 7.6%). While these values are reasonable, the concern is that there are a few cases at low relative submergence where the errors are greater than these statistics might suggest. Comparisons were made between the discharge error and the difference between the values of β_2 from the data analysis and Eq. 17 (different between points and line in Figure 7). The discharge error was slightly positive for values of β_2 less than unity. The large negative discharge errors resulted from predicted values of β_2 that were above the value predicted by Eq. 17. These large errors in β_2 suggest that further refinement may be needed for the momentum equation (Eq. 6) and for some of these coefficient values, particularly δ, β_2, and p_w.

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