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Sediment Transport Similitude for Scaled Physical Hydraulic Modeling

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Abstract
Studies of the hydraulics of sediment transport – a field with extensive associated complexities – have yielded a diverse set of empirically-derived predictive methodologies. Researchers at the Bureau of Reclamation's Hydraulic Investigations and Laboratory Services Group have identified an approach for design of movable bed physical scale models based on a relationship between dimensionless bed shear (Shields parameter) and dimensionless unit sediment transport (Taylor's function). Using a method that includes selection of model particle size and density based on terminal velocity (fall velocity) for particles in both scale model and prototype; model design parameters may be identified to produce model-prototype similitude for aspects of sediment transport including incipient motion and approximate transport capacity. This paper further expands the methods described in ASCE's Manual 110, Appendix C (Pugh, 2008) on "Sediment Transport Scaling for Physical Models." When the model is not large enough to compensate for the scale effects, the sediment density and or slope of the model may need to be adjusted to match the model and prototype Taylor's Function curve for constant dimensionless transport rate.

Introduction
A 1:24 scale (M:P, model : prototype) physical hydraulic model study was conducted in 2004 at Reclamation's Hydraulics Laboratory in Denver CO that examined alternatives for limiting sediment intake at a planned diversion on the Rio Grande River near Albuquerque, NM. This paper follows methodologies used to identify appropriate parameters for model design.

Scale Physical Hydraulic Modeling & General Hydraulic Similitude
Scale model studies of hydraulic systems have proven a cost-effective means of investigating performance of a proposed structure or of proposed system modifications, provided requirements for hydraulic similitude are met. Idealistically, this requires matching the ratio of appropriate pairs of forces in both scaled model and prototype that play significant roles in the physical processes being examined. Often of interest is the ratio of inertial forces to viscous forces (a dimensionless ratio known as the Reynolds Number) as well as the ratio of inertial forces to gravity forces (also dimensionless, known as the Froude Number) in both model and prototype. The stream Reynolds Number (Re) for wide, shallow channels is the product of fluid velocity (V) and flow depth (y) divided by the fluid's kinematic viscosity (ν). The Froude Number (Fr) is the fluid velocity (V) divided by the square root of the product of the gravitational constant (g) and the flow depth (y).

$$\mathrm{Re} = \frac{Vy}{\nu} \qquad\qquad Fr = \frac{V}{\sqrt{gy}}$$

In practical applications, meeting both criteria would require scaling of not only physical dimensions, but scaling of fluid properties (i.e. viscosity, fluid density) – which can almost never be achieved due to the fact that fluids with suitably scaled properties almost never exist. In most physical model studies of water conveyance and control systems, water is both the model and prototype fluid for economic reasons. If turbulent flow conditions exist in both model and prototype for the aspect(s) of a system being examined, viscous force effects are significantly diminished and observations from model performance will relate to prototype performance within a useful degree of accuracy. Hence physical open channel flow hydraulic models are commonly designed to adhere to Froude number scaling and to maintain turbulent flow conditions for the modeled aspects of interest in order to avoid having viscous forces (commonly referred to as "Reynolds effects") impact model performance. A stream Reynolds number of 2000 represents the minimal range for turbulent flow conditions.

Additional Sediment Modeling Considerations

For sediment movement, the hydraulic scale of interest is at the bed sediment particle diameter. Particle movement is a function of shear force – or the drag force – exerted by fluid moving past bed particles that exceeds forces holding the particles in place. Bed shear (τ_o) is calculated as the product of fluid density (ρ) (or more correctly density of the fluid and suspended particle mixture) and the square of the shear velocity (u_*). Shear velocity is calculated as the square root of product of the gravitational constant (g), the channel's hydraulic radius (R) and slope (S). For wide shallow channels like the modeled stream reach of the illustrative example, hydraulic radius is approximated by the depth of flow (y). The magnitude of drag force exerted depends on degree of turbulence present and thus is a function of the Reynolds Number. The form of the Reynolds Number used for consideration at the bed particle scale is known as the "Grain" Reynolds Number, (Re_*), defined as the product of the shear velocity (u_*) and grain size (d_s) divided by the fluid kinematic viscosity (ν) or $Re_* = u_* d_s / \nu$. Hence:

$$\tau_o = \rho\sqrt{gyS} \qquad\qquad \mathrm{Re}_* = \frac{u_* d_s}{\upsilon}$$

Generally, it is not feasible to simply reduce particle size according to geometric model scale. As particle size is reduced, cohesiveness properties may change dramatically, completely altering the sediment transport mechanics between model and prototype. Using a model particle size in excess of the scaled value may necessitate using a lower density bed material in the model, increasing the model slope, or a combination of density and slope adjustments to produce transport mechanics with a useful degree of similarity between model and prototype.

An approach that has been employed for moveable bed scale model studies at Reclamation's Hydraulics Lab is based on an apparent relationship between dimensionless bed shear known as Shields' Parameter (τ_*), and dimensionless unit sediment transport (q_{*s}). Shields' Parameter is defined as the bed shear (τ_o) divided by the product of buoyant specific weight (γ_s - γ) and particle size (d_s). Dimensionless unit sediment transport, known as Taylor's function, is defined as the unit sediment ($q*_s$) divided by the product of shear velocity ($u*$) and grain sediment size (d_s)

$$\tau_* = \frac{\tau_o}{(\gamma_s - \gamma) d_s} \qquad\qquad q*_s = \frac{q_s}{u* d_s}$$

The threshold condition for either mobilization or deposition of a given particle is commonly referred to as the condition of incipient motion. Using data from laboratory flume studies, Shields was able to show that for a given grain Reynolds number there is a unique dimensionless shear value at which the state of incipient motion exists. Dimensionless shear values representing the incipient motion state are plotted against grain Reynolds number to produce a curve for the condition of incipient motion. Dimensionless shear values that lie on this curve are known as "critical" Shields parameter values. (Vanoni, 1975)

In studies with low sediment discharge rates by Taylor (as discussed in Vanoni, 1975), the amounts of sediment being transported from the flume were measured and results analyzed in terms of dimensionless unit sediment transport. Taylor determined Shields' parameter values and Taylor function values for each data point. He found that when Shields' values for data points of constant Taylor's function value were plotted against grain Reynolds number, curves approximately parallel to the critical Shields parameter were produced. (Vanoni, 1975).

When plotted on the Shields' diagram, Taylor's data appears below the critical Shield's parameter suggesting that the critical Shields' curve represents a constant Taylor's function value of some small value of dimensionless unit sediment transport. Figure 1 is the Shields' diagram showing the apparent parallels Taylor found for Shields' parameter values associated with constant Taylor's function values, and the critical Shield's parameter curve.

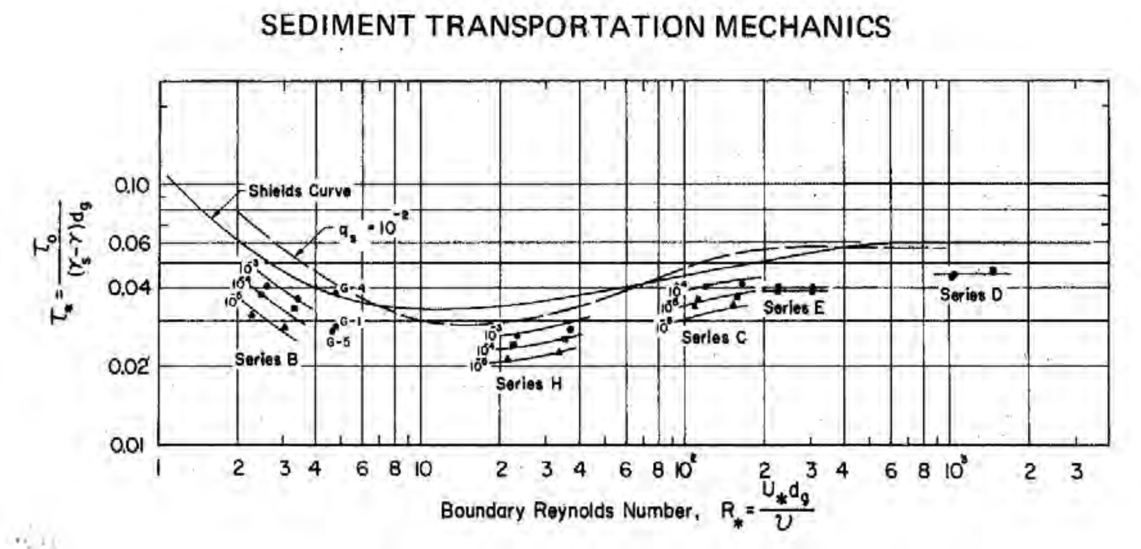

Figure 1. Plot showing the "critical" Shields values along with Shield's values for constant-value segments of Taylor's function data (from Vanoni 1975)

In keeping with theory relating similitude and dimensionless parameters it follows that when Taylor's function values for scale model and prototype are equivalent, similitude would exist in sediment transport. Pugh and Dodge (1991) proposed that the parallel relationship Taylor had shown between Shields' parameter values associated with constant Taylor's function values for small rates of sediment discharge and the critical Shields' values might hold for higher sediment discharge rates. If so, the target Shields' value for the model and the corresponding prototype Shields' value would lie on the same curve paralleling the critical Shields' values curve. By equating the differentials between actual Shields values and critical Shields' values at the respective grain Reynolds numbers for both model and prototype, similitude in sediment transport rates could be achieved.

For cases where prototype grain Reynolds' number is greater than 100, but where geometrically scaled particle size produce grain Reynolds numbers below 100, Pugh and Dodge reported success achieving target differentials between model dimensionless shear and critical Shield's values by increasing model particle size. A guideline used was to attempt to equate model and prototype particle fall velocity (ω_o). Fall velocity is terminal velocity of a particle of given shape, density and size falling through a fluid of given properties. Fall velocities are commonly approximated using empirically derived relationships based on laboratory observations. An empirical relationship for fall velocity developed at the Reclamation Hydraulics Laboratory (Dodge, 1983) based on settling chamber data for sand particles in clear water is:

For d_s >0.3 mm, $\omega_o = 11 * \frac{d_s^{0.5}}{100}$ For d_s < 0.3 mm, $\omega_o = 80 * \frac{d_s^{2}}{100}$

The parallel relationship between grain shear for constant value Taylor's functions and critical Shields' values provides a design guideline for similitude in sediment transport capacity. Actual transport rates are a function of both transport capacity and sediment availability. A scaled sediment load relationship may be derived by applying an empirical bed load transport equation to both model and prototype over a range of discharges. A mathematical relationship can then be identified between corresponding predicted loads. A formulation of the Meyer-Peter & Mueller (M-P&M) equation as presented in Vanoni, (1975) was utilized for this purpose in the illustrative model study. M-P&M was developed for sand grain sediments and can be applied to sediments of varying density. The derived relationship was utilized to compare measured sediment feeding rate in the model with prototype bed load field data.

Design of the Illustrative Physical Model

Selection of Scale

Selecting an appropriate scale was a function of space availability, including as many of the prototype channel features as possible, and maintaining a large enough scale to limit the effect of viscous forces. The available model box was approximately 8.86 m wide and 20.5 m long. A 1:24 scale factor was identified as near the upper limit for scale that would enable construction of the entire diversion structure and bank-full model of the upstream channel within the 8.86 m box width. Even at this scale, the Reynolds number in the diversion bays assuming 1.84 m^3/s diversion per bay and 0.984 m depth (prototype) is approximately 1600. This falls below a minimum value of 2000 for turbulent flow conditions. Thus model flows in the diversion bays are subject to some degree of Reynolds' effects. Due to higher velocities, flows through gate openings on the diversion structure should be in the turbulent range and be less impacted by Reynolds effects.

Examination of Sediment Transport Similarity

For this study model adjustments were identified following an iterative sediment transport model scaling methodology described by Pugh (Pugh 2008). The initial step was to look at Shields' values for particles of prototype density and of geometrically scaled size with equivalent channel slope in both model and prototype. Analysis of field samples of prototype sediments indicated a prototype grain size of 51mm. A geometrically scaled model grain size would be 0.021 mm. Corresponding Shields values calculated for these model and prototype grain sizes were determined for flow depths of 0.248 m, 0.328 m, 0.656 m, 0.984 m, 1.640 m, and 2.297 m. Formulas were entered into spreadsheet cells to calculate dimensionless shear and grain Reynolds number for model and prototype at each of the selected flow depths. These Shields values are shown in Figure 2.

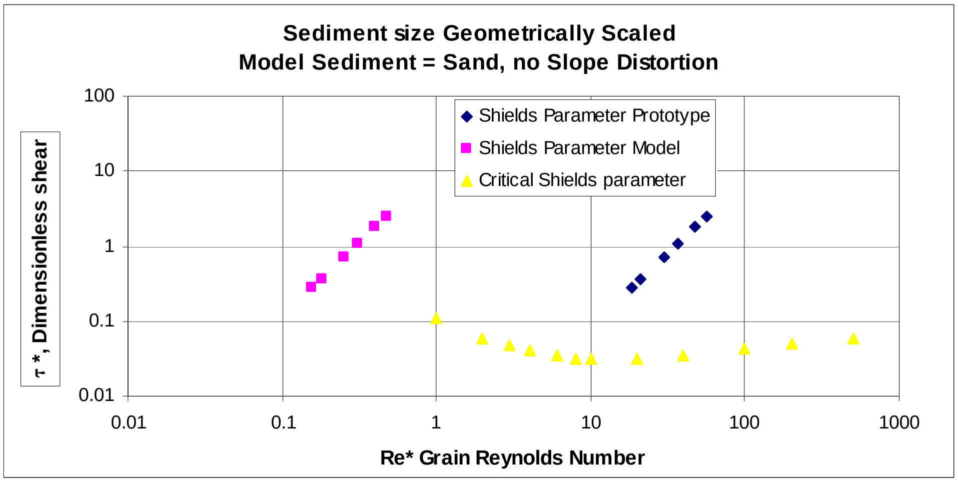

Figure 2. Shields diagram showing dimensionless shear for prototype And geometrically scaled grain model grain sizes

It is readily apparent from Figure 2 that corresponding model and prototype dimensionless shear values come nowhere near lying on curves parallel to the critical dimensionless shear. For the next adjustment, model grain size is increased to equate fall velocity between model and prototype. For the 1:24 scale model, a resulting model grain size of 0.142 mm was derived. Figure 3 shows the model and prototype Shields values for fall velocity adjusted model particle size.

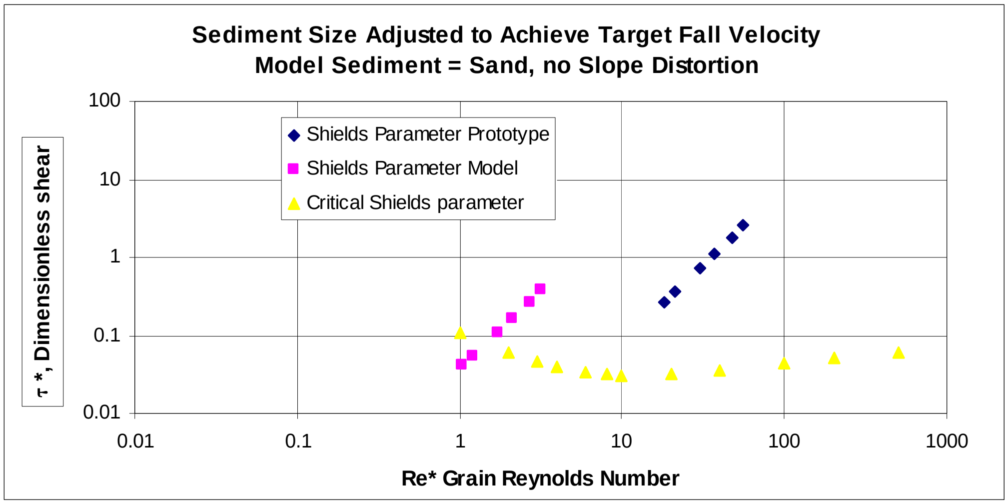

Figure 3. Shields diagram showing dimensionless shear for prototype and model grain size adjusted for fall velocity

As seen in figure 3, the model particle adjustment to equate model and prototype fall velocities resulted in a shift of model dimensionless shear values for the selected stream depths, but does not bring these values near enough to lying on parallel curves with corresponding prototype dimensionless shear values. For the next adjustment, model sediment of reduced density was utilized. An available stock of crushed coal was found to have a grain size of 0.88 mm and a specific gravity of 1.27. Figure 4 shows the dimensionless shear values, assuming the available crushed coal stock is used as the model sediment, in comparison with prototype dimensionless shear for corresponding stream depths.

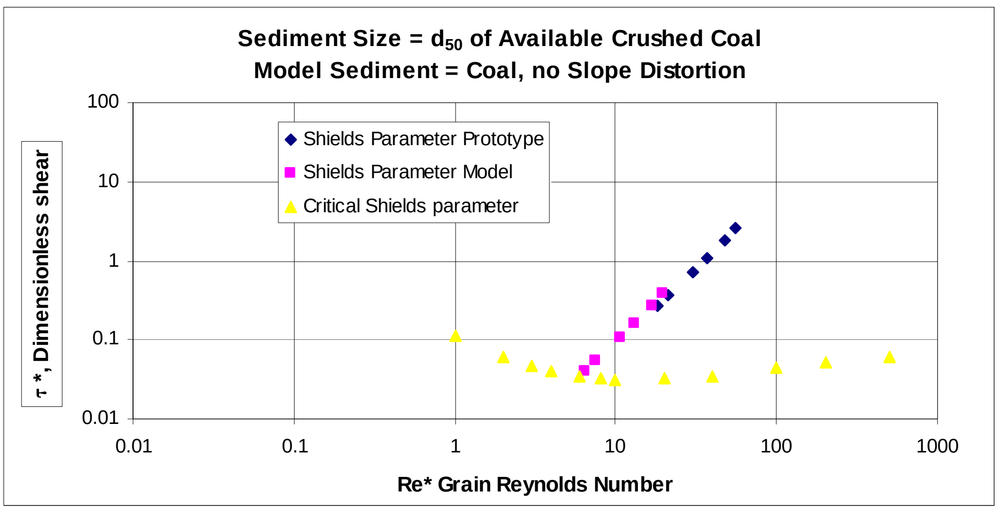

Figure 4. Dimensionless shear for crushed coal as model sediment in comparison with prototype dimensionless shear and critical dimensionless shear values.

Figure 4 shows that consideration of crushed coal as the model sediment produced a significant shift in model dimensionless shear values, but corresponding model and prototype are still below the prototype curves parallel to the critical dimensionless shear plot. The remaining parameter that could be adjusted for the iterative design method would involve exaggerating the model slope. Different slope distortions were examined until the relationship plotted in Figure 5 was identified.

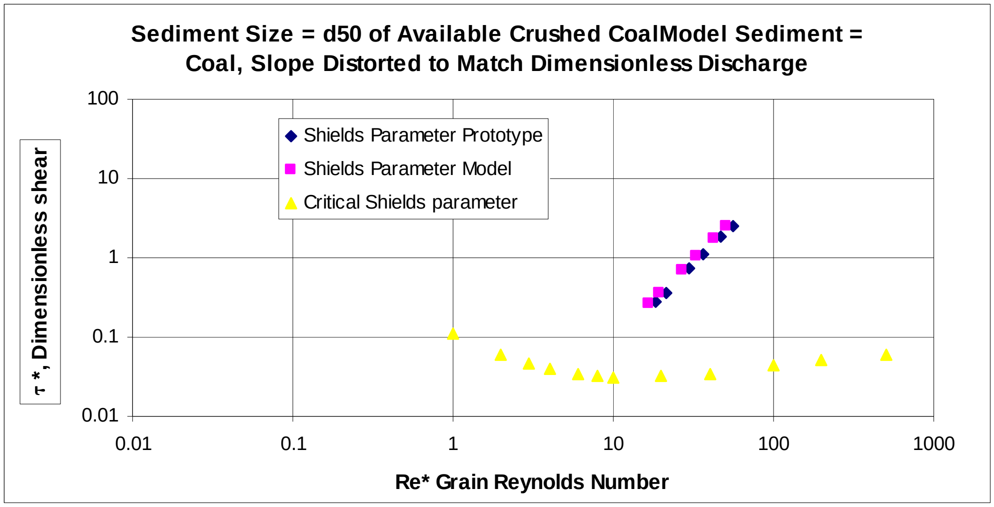

Figure 5. Dimensionless shear using crushed coal as the model sediment with a 6.5:1 slope exaggeration (model: prototype)

From the appearance of Figure 5, the combination of a 6.5:1 slope exaggeration coupled with use of lighter weight model sediment (the available crushed coal) as the model sediment appears to have created appropriate adjustments to enable the model and prototype to have sediment transport similitude. At this point, model parameters of 1:24 (M: P) geometric scale, available crushed coal for model sediment, and 6.5:1 (M:P) bed slope exaggeration were settled upon for model testing.

Sediment Discharge Scaling Methodology for Model Operation

An approximation of the relationship between the sediment discharge rates of model and prototype for the selected model parameters was needed to determine an approximate rate for feeding sediment into the model. This rate was determined by applying the Meyer-Peter and Muller (M-P&M) bed load transport equation to each case for corresponding Froude-scaled stream discharges. The Meyer-Peter and Mueller equation was developed from studies of sand-sized bed particles with particles of varying densities and provides unit sediment discharge (g_s) in metric tons/meter/second. The formulation of this equation presented by Vanoni (Vanoni, 1975) as follows:

$$\left(\frac{k_r}{k'_r}\right)^{\frac{3}{2}} r_b S = 0.047(\gamma_s - \gamma)d_m + 0.25\left(\frac{\gamma}{g}\right)^{\frac{1}{3}}\left(\frac{\gamma_s - \gamma}{\gamma_s}\right)^{\frac{2}{3}} g_s^{\frac{2}{3}}$$

Where:

k_r = roughness coeff. (= 1/n where n = Manning's roughness coeff.)
$k'_r = 26/d_{90}^{1/6}$ (d_{90} in meters)
γ = specific weight of water (metric tons/cubic meter)
γ_s = specific weight of sediment (metric tons/cubic meter)
r_b = hydraulic radius (~ depth for wide channel)
S = channel slope
d_m = effective sediment diameter (= $\Sigma_i\, p_i d_{si}$ where p_i = % by wt. of size d_{si})
g = gravitational constant
g_s = bed load (metric tons/meter/second)

This equation was manipulated to calculate unit bed load (g_s) for both model and prototype, then multiplied by respective channel widths, as a basis for scaling transport rate, given differing model/prototype sediment densities as well as model slope distortions. A numerical approximation of the relationship between M-P&M predicted transport for prototype and model was obtained as linear fit of calculated values. This relationship is shown in Figure 8. The relationship shown in Figure 8 was subsequently used as basis for sediment feed rates in the 2004 illustrative model study.

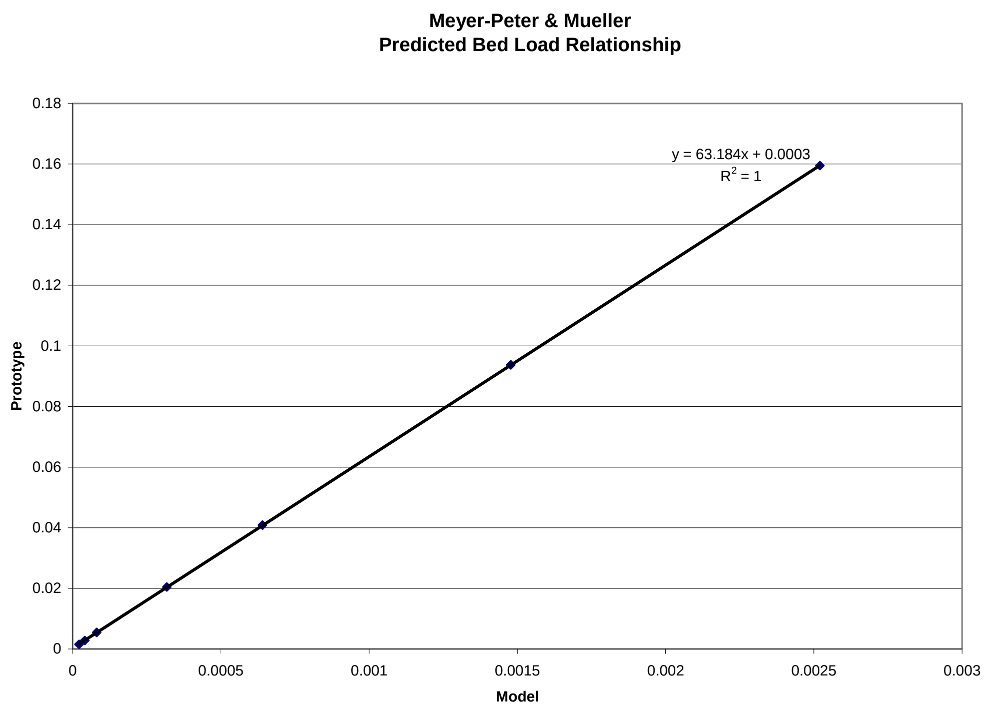

Figure 8. Plot of the M-PM model/prototype bed sediment transport comparison (in metric tons/second) for the identified model parameters at selected flow depth.

Summary

Franco (1978) described alluvial channel engineering as ". . . a matter of experience and general judgment . . ." Despite the widespread availability of vastly enhanced computational tools since this statement was made, sediment transport engineering tools can offer limited precision at best. When attempting to account for the impacts of scaling, the degree of imprecision is magnified for physical scale modeling of sediment transport. Numerous assumptions have been a part of development of the iterative model parameter identification tool described in this paper. For multiple physical scale model sediment transport studies conducted at Reclamation's Hydraulic Laboratory, it has enabled researchers to examine prototype transport process with a useful degree of similitude. The results of applying these adjustments to sediment management at the proposed Rio Grande Diversion in the example model study in this paper have produced reasonable results and allowed evaluation of changes to the proposed diversion structure to exclude sediment from the intake. All of the model distortions available were used in this study, making it a good illustration of how to use the procedures described in ASCE's Sedimentation Engineering Manual-No.110.

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