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Design of Labyrinth Spillways

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Abstract: The capacity of a labyrinth spillway is a function of the total head, the effective crest length, and the crest coefficient. The crest coefficient depends on the total head, weir height, thickness, crest shape, apex configuration, and the angle of the side legs. Data and a procedure are presented for designing labyrinth weirs for angles between 6° and 35°, and for a range of heads. The design procedure allows the angle of the side legs and the number of cycles to be varied until the desired layout and capacity are achieved. The solution is presented in a spreadsheet format that automatically calculates the dimensions for the labyrinth. Even though the design procedure is quite accurate, it is recommended that the capacity and performance be verified with a model study. The model can evaluate factors not included in the design procedure, like aeration effects at low heads, unusual flow conditions in the approach channel, and flow conditions in the discharge channel.

INTRODUCTION

A labyrinth spillway is an overflow weir folded in plan view to provide a longer total effective length for a given overall spillway width. Fig. 1 shows a typical layout. A labyrinth spillway has advantages compared to the straight overflow weir and the standard ogee crest. The total length of the labyrinth weir is typically three to five times the spillway width. Its capacity varies with head and is typically about twice that of a standard weir or overflow crest of the same width. Labyrinth weirs can be used to increase outlet capacity for a given spillway crest elevation and length or to increase storage by raising the crest while maintaining spillway capacity. The variables that need to be considered in designing a labyrinth include the length and width of the labyrinth, the crest height, the labyrinth angle, the number of cycles, and several other less important variables such as wall thickness, crest shape, and apex configuration.

An extensive investigation dealing with the behavior of labyrinth weirs was performed by Taylor (1968). He presented his results in terms of a magnification ratio of the labyrinth flow to the flow for a sharp-crested linear weir having the same channel width. As a follow-up to that work, Hay and Taylor (1970) published a design procedure for labyrinth weirs, including criteria for estimating the discharge over triangular or trapezoidal labyrinth weirs.

A number of hydraulic models have also been tested in order to learn about the design of labyrinth spillways. Darvas (1971) for example used the experimental results of the model studies of Woronora and Avon weirs in Australia and developed a family of curves for designing labyrinth weirs. Mayer (1980) used a 1:20 scale model to study the effect on discharge of a proposed labyrinth weir spillway to be added to the Bartlett's Ferry project. The conceptual design of the structure was based on the approach of Hay and Taylor (1970) and was found to be inadequate as the structure would not pass the required flow. Lux (1984) assessed the hydraulic performance of labyrinth weirs using data obtained from flume studies and site-specific models. He developed an equation for discharge over labyrinth weirs.

The U.S. Bureau of Reclamation tested models for the labyrinth spillways of the Ute Dam and Hyrum Dam (Houston 1982, 1983; Hinchliff and Houston 1984). Testing of the originally designed 10-cycle model of the Ute Dam labyrinth spillway, which was based on Hay and Taylor (1970) design curves, showed that the design discharge could not be passed by the spillway at the maximum reservoir elevation. They found that the discrepancy between their results and those of Hay and Taylor was partly due to the difference in head definition. Houston (1982, 1983) and Lux (1984) used the total head instead of the piezometric head. Use of piezometric head does not allow for differences in the approach velocity and can introduce significant errors in the predictions.

More recently the Bureau of Reclamation completed a model of the Ritschard Dam labyrinth spillway (Vermeyen 1991). The model results of that study were used to design a labyrinth for Standley Lake (Tullis 1993).

Researchers have been involved in developing data and a procedure to improve and simplify the design of labyrinth weirs. Several experimental programs were completed at the Utah Water

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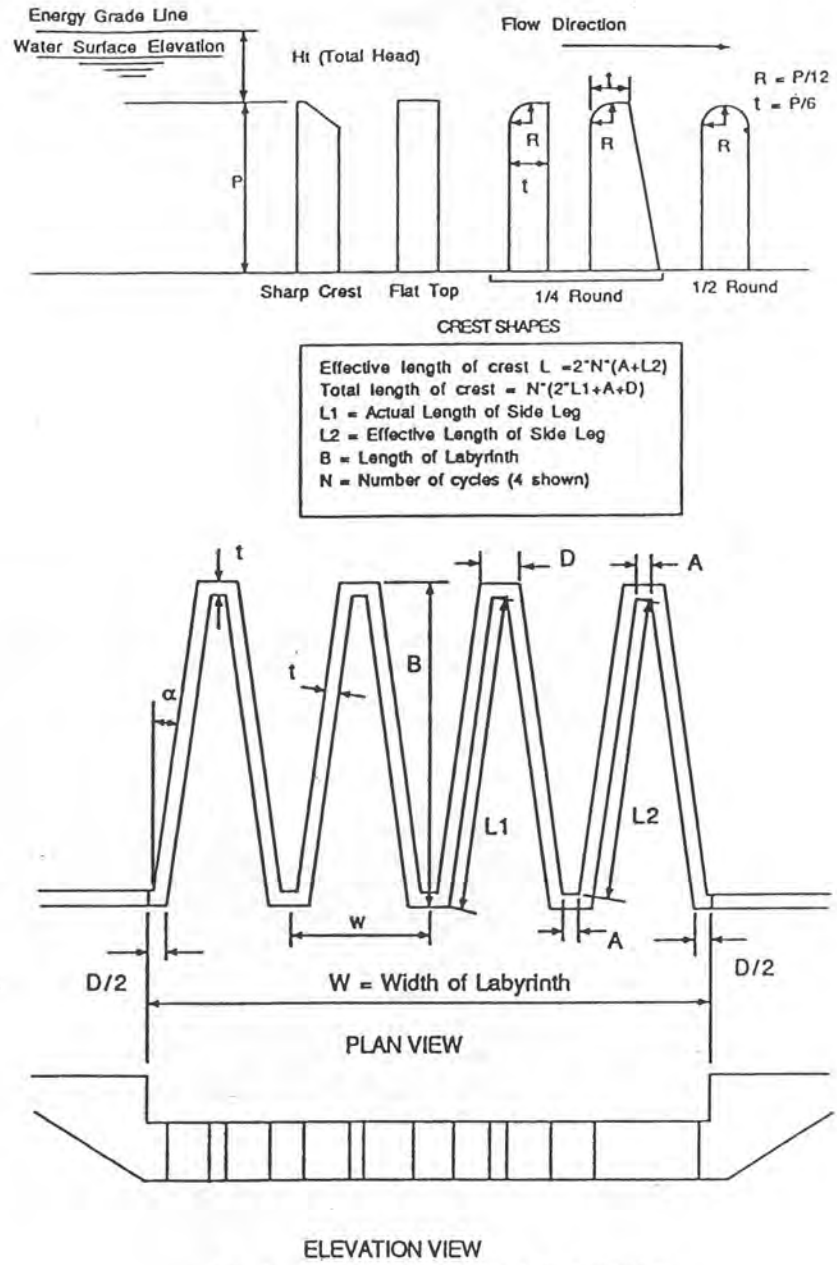

FIG. 1. Layout and Details of Labyrinth Weir

Research Laboratory (UWRL) to evaluate the crest coefficient for labyrinth weirs (Aman 1987; Baasiri et al., unpublished report, 1992; and Waldron 1994). Each researcher used same (3-ft) 1.0-m-wide flume. Linear weirs with flat, quarter-round and half-round crest sha were tested over a range of H_t/P from about 0.05 to about 1.0. The model weirs were 152- 229-mm high. Labyrinth weirs were tested at angles from 6° and 35°. Most of the weirs w 152-mm high and 25.4-mm thick with quarter-round and half-round crest shapes. The weirs w carefully aligned, leveled, sealed, and measured to determine the net length (defined in I 1). A point gauge, readable to 0.3 mm, located 1 m upstream from the weir was used to meas water depth. The point gauge was accurately referenced to the crest elevation and chec periodically during the testing. Flow was measured by weigh tanks and volumetric tanks to accuracy of 0.25%. Details of the experimental studies are contained in the original works the three researchers (Amanian 1987; Baasiri et al., unpublished report, 1992; Waldron 19

The end results of these experimental studies was the development of a database and des procedure. It is based on a specific crest geometry. The input-system data required for the I procedure is the design flow and head. The procedure allows complete flexibility in selec the number of cycles and the angle of the side legs. Limitations are placed on some of the des variables, such as the height of the weir and the width to length ratio of the labyrinth. With input data selected, the spreadsheet automatically solves for the corresponding labyrinth

mensions. Each of the input variables can be varied to determine its influence on the design. There will be many layouts that provide the design flow at design head. The final choice should be based on which design fits best into the overall layout of the project, is cost-effective, and produces an acceptable outflow hydrograph.

WEIR EQUATION

The proposed method for designing a labyrinth weir using the basic equation developed for linear weirs is

$$Q = \frac{2}{3} C_d L \sqrt{2g} H_t^{1.5} \tag{1}$$

where C_d = a dimensionless crest coefficient; g = acceleration of gravity; L = effective length of the weir; and H_t = total head on the crest. The total head is normally determined a short distance upstream from the weir and is equal to the measured depth of water above the crest plus the velocity head of the approach flow at the point of measurement. For a weir with a short approach where inlet losses are negligible, H_t is the elevation difference between the reservoir water level and the elevation of the weir crest.

For a linear weir without side contractions and with normal approach flow, the effective length L is the actual measured length of the weir. The crest coefficient is dependent on H_t/P, the wall thickness t, crest configuration, and nappe aeration. Fig. 2 shows the variation of the crest coefficient with H_t/P for an aerated linear weir with t/P = 1/6 and the crest rounded on the upstream corner at a radius of $P/12$. Three sets of data obtained by three different researchers at the UWRL (Amanian 1987; Baasiri et al., unpublished report, 1992; and Waldron 1994) are plotted in Fig. 2 for the linear weir with an aerated nappe. Establishing reliable data for the linear weir is important in the analysis of the crest coefficients for labyrinth weirs because it represents the upper boundary of the C_d values.

For a labyrinth, the effective length to be used in (1) is defined in Fig. 1. The crest coefficient is dependent on the same variables influencing a linear weir plus the configuration of the labyrinth at its apex, and the angle of the labyrinth.

VARIABLES AFFECTING CREST COEFFICIENT

The height, thickness, and shape of the crest have a significant influence on the crest coefficient (Amanian 1987). There are four basic options for the shape of the crest (shown in Fig. 1): sharp-crested, flat, quarter-round on the upstream side and half-round.

Wall thickness is determined from structural analysis and is dependent on height of the crest, hydraulic forces, ice loading, and specific site conditions. For economy and strength, it may be preferable to have the downstream side of the wall tapered. This will not influence the crest coefficient. To make the coefficients in the present paper applicable, it is necessary to make the radius of curvature $R = P/12$, as shown in Fig. 1.

Sharp-crest and flat-crest weirs are generally not preferred because their crest coefficients are measurably less than those for rounded crested weirs. The most efficient and practical shape

FIG. 2. Crest Coefficient for Linear Weir

appears to be the quarter round (Amanian 1987). Even though the quarter-round crest has smaller C_d at low heads ($H_t/P < 0.5$) compared to a full half-round crest, it has a large C_d high heads and is easier to construct. Therefore, the proposed design procedure assumes quarter-round crest with the top wall thickness $t = P/6$ or a tapered wall with $R = P/12$. If tl wall is made thicker for structural reasons, it has little effect on the crest coefficient. Howeve a significant decrease in the wall thickness and the corresponding reduction in the radius curvature causes separation and reduces the crest coefficient.

The weir height P is the difference between the crest elevation and the elevation of tl upstream apron. The weir height influences losses in the approach channel and spillway capacit For a linear weir, C_d reaches a maximum and becomes constant at large values of H_t/P (s Fig. 2). For a labyrinth weir, as the head increases, C_d continues to decrease and the spillw capacity eventually approaches that of a linear weir having a length equal to the apron widt It is therefore necessary to limit the H_t/P to maintain the effectiveness of the labyrinth. Tl writers recommend that a maximum flow H_t/P be less than about 0.9. The labyrinth still functio at higher heads but the advantage of the labyrinth design continues to diminish as the he continues to increase. The final decision will probably be based on economics.

The width A of the apex (defined in Fig. 1) influences spillway capacity. It reduces the n length of the labyrinth weir and decreases spillway capacity. Consequently, A should be as sm as possible. Typically, the inside apex is one or two times the wall thickness.

The design is based on a labyrinth sitting on a horizontal apron with the upstream a downstream portion at the same elevation. The flow downstream from the labyrinth should super critical to avoid submergence effects. To use the design data herein, the downstrea channel must have a supercritical slope or at least a short section at a steep slope to preve submergence of the crest.

Depending on the configuration of the inlet channel and the placement of the spillway, t angle of the approach flow β may not be perpendicular to the axis of the labyrinth. Data a available for $\beta = 15°$, 30°, and 45° that identify the influence of the approach angle β on capac (Amanian 1987). The percent reduction of flow for these three approach angles is only 1' 4%, and 6%, respectively. Since it would be highly unlikely that approach conditions wo result in an angle over 15°, the effect of β is generally negligible. If the approach conditions a very unusual, a physical-model study would be recommended. All data included in the pres paper are for $\beta = 0\%$.

Past research on linear weirs has documented that nappe aeration influences the crest co ficient and, therefore, the spillway capacity. When the weir is aerated, the cavity beneath t nappe is near atmospheric pressure and the crest coefficient has a minimum value. A lin weir is easy to aerate at heads below about $H_t/P \approx 0.7$. The primary purpose for venting lin weirs is to reduce vibrations caused by pressure variations under the nappe. The data for t linear weir in Fig. 2 are for a fully aerated nappe. It is recommended that these values be us for design unless it is certain that the nappe is not aerated. When the nappe is not aerated, t weir passes more flow than predicted by the C_d values shown in Fig. 2 (Amanian 1987; Waldr 1994). The crest coefficient values for nonaerated flows can be slightly above $C_d = 0.75$ small H_t/P values.

Assuming a constant crest shape and similar aerated nappe conditions, the C_d values fo labyrinth cannot be higher than those for a linear weir as shown in Fig. 2. Several sets published crest coefficient values show C_d values at low heads exceeding 0.75. These values above the curve for the linear weir in Fig. 2. Such values can be obtained when the crest operating nonaerated with a negative pressure below the nappe. A labyrinth tends to oper with a slight negative pressure at H_t/P between about 0.1 and 0.2. Experimental data poi that exceeded those for the linear weir were ignored. The reason was to provide conservativ low C_d values for designing the labyrinth. When it is important to accurately know the rat curve at low reservoir elevations, a physical-model study should be conducted.

The most important part of the research at the UWRL was to determine the value of crest coefficient C_d for the full range of variables studied. The crest coefficients for a labyri weir are shown in Fig. 3 for labyrinth angles between 6° and 35° (Amanian 1987; Baasiri et unpublished report, 1992; Tullis 1993; Waldron 1994). To facilitate using a spreadsheet to des a labyrinth, regression equations [(2)–(9)] were determined for variation of C_d with H_t/P. The equations are valid for apex width $t \le A \le 2t$; $H_t/P < 0.9$; $t \approx P/6$; crest shape is a quar round (on upstream side); and the radius of crest curvature $R = P/12$

$$C_d = 0.49 - 0.24(H_t/P) - 1.20(H_t/P)^2 + 2.17(H_t/P)^3 - 1.03(H_t/P)^4; \quad \text{for } \alpha = 6°$$

$$C_d = 0.49 + 1.08(H_t/P) - 5.27(H_t/P)^2 + 6.79(H_t/P)^3 - 2.83(H_t/P)^4; \quad \text{for } \alpha = 8°$$

$$C_d = 0.49 + 1.06(H_t/P) - 4.43(H_t/P)^2 + 5.18(H_t/P)^3 - 1.97(H_t/P)^4; \quad \text{for } \alpha = 12°$$

$$C_d = 0.49 + 1.00(H_t/P) - 3.57(H_t/P)^2 + 3.82(H_t/P)^3 - 1.38(H_t/P)^4; \quad \text{for } \alpha = 15°$$

$$C_d = 0.49 + 1.32(H_t/P) - 4.13(H_t/P)^2 + 4.24(H_t/P)^3 - 1.50(H_t/P)^4; \quad \text{for } \alpha = 18°$$

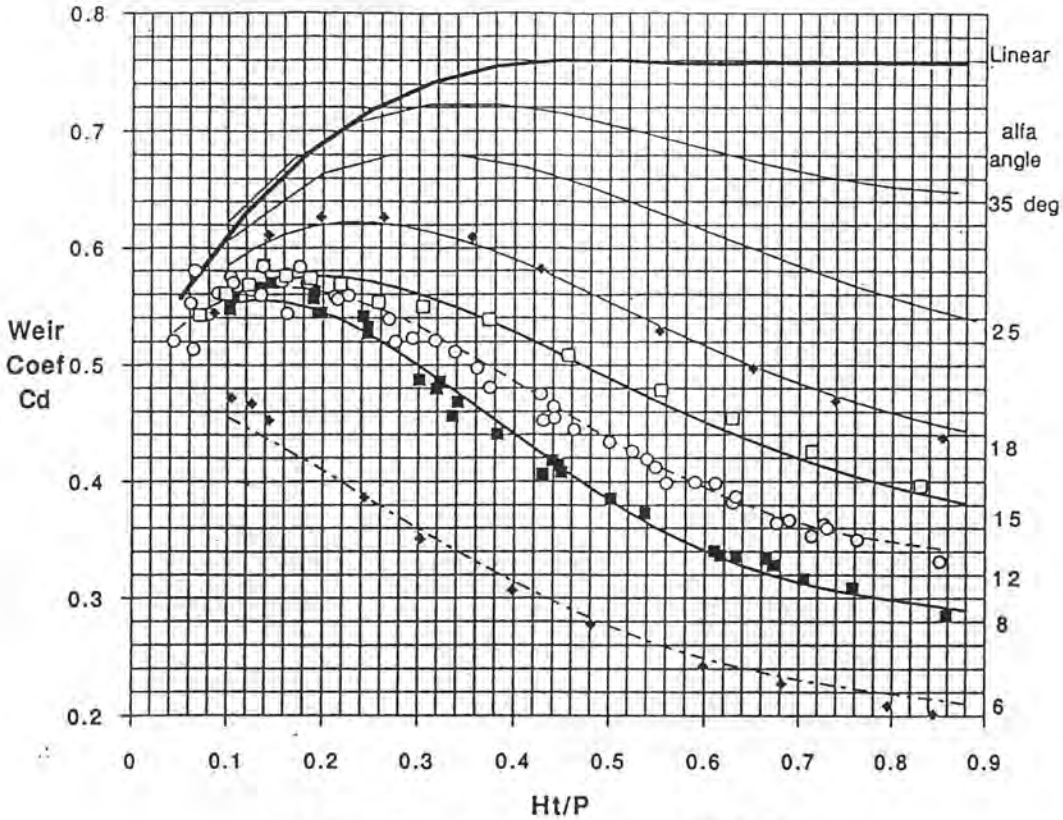

FIG. 3. Crest Coefficient for Labyrinth Spillways

$$C_d = 0.49 + 1.51(H_t/P) - 3.83(H_t/P)^2 + 3.40(H_t/P)^3 - 1.05(H_t/P)^4; \quad \text{for } \alpha = 25° \tag{7}$$

$$C_d = 0.49 + 1.69(H_t/P) - 4.05(H_t/P)^2 + 3.62(H_t/P)^3 - 1.10(H_t/P)^4; \quad \text{for } \alpha = 35° \tag{8}$$

$$C_d = 0.49 + 1.46(H_t/P) - 2.56(H_t/P)^2 + 1.44(H_t/P)^3; \quad \text{for } H_t/P < 0.7 \text{ and } \alpha = 90° \tag{9}$$

Eqs. (2)–(9) and some of the experimental data are plotted in Fig. 3.

The value of C_d does not vary significantly with a small change of α. Therefore, each of the equations can be used for angles close to those listed. For angles different by more than about $\pm 1°$ from the values listed for (2)–(9), a new regression equation should be developed or the data interpolated from Fig. 3.

An analysis was made to determine the accuracy of the regression equations compared to the experimental data from which they were developed. The standard deviation between the measured and calculated data for angles between 6° and 18° was less than $\pm 3\%$ with the maximum difference about $\pm 6\%$. Therefore, (2)–(6) provide sufficient accuracy for $0.1 \leq H_t/P \leq 0.9$. No data were used above $H_t/P = 0.9$ because the labyrinth becomes increasingly inefficient with increased head. Below $H_t/P = 0.1$, the value of C_d is very difficult to evaluate experimentally. For a reduced-scale model operating at $H_t/P < 0.1$, a slight error in measuring the water-surface elevation or the crest elevation can cause a significant change in the crest coefficient. The model crests tested by the writers were 15- and 30-cm high.

The data for 25° and 35° are interpolated values and are accurate only to about $\pm 10\%$. This is because the experimental data were for a different crest shape. Therefore, the data points for those angles are not plotted in Fig. 3.

DESIGN PROCEDURE

The recommended design procedure is demonstrated using an example based on data from a model study conducted at the UWRL of the Standley Lake labyrinth spillway, located near Denver (Tullis 1993). The original design of the spillway for Standley Lake, which is used in Table 1 to demonstrate the design procedure, was based on a model study of the Ritschard Dam labyrinth (Vermeyen 1991). Table 1 provides the design calculations setup in a spreadsheet format.

The upper block lists typical input data that would come from a hydrologic analysis of the system. This includes the maximum required spillway flow (Q_{max} = 1,539 m^3/s for this example), the corresponding maximum reservoir elevation (*res* = 1680.91 m), and the normal pool ele-

TABLE 1. Spreadsheet for Designing Labyrinth Weirs

Parameter (1)	Symbol (2)	Value (3)	Units (4)	Source/equation/notes (5)
		(*a*) Given input-system data		
Maximum flow	Q_{max}	1,538	(m^3/s)	Input
Maximum reservoir elevation	*res*	1,680.91	(m)	Input
Approach channel elevation	—	1,675.75	(m)	Input
Crest elevation	el	1,678.80	(m)	Input
Total head	H_t	1.975	(m)	H_t = res-crest-loss
		(*b*) Assumed data		
Estimated inlet loss at Q_{max}	Loss	0.13	(m)	Estimated
Number of cycles	N	13	—	Select to keep $w/P \sim 3$ to 4
Crest height	P	3.05	(m)	Set $P \sim 1.4\ H_t$
Angle of side legs	α	8.0	(deg)	Normally 8°–16°
		(*c*) Calculated data		
Thickness of wall	t	0.51	(m)	$t = P/6$
Inside width at apex	A	0.95	(m)	Select between t and $2t$
Outside width of apex	D	1.83	(m)	$D = A + 2t \cdot \tan(45 - \alpha/2)$
Total head/crest height	H_t/P	0.648	—	—
Crest coefficient	C_d	0.3255	—	Eq. (3)
Effective crest length	L	576.6	(m)	$1.5Q_{max}/[(C_d \cdot H_t^{1.5}) \cdot (2g)^{0.5}]$
Length of apron (parallel to flow)	B	22.72	(m)	$B = [L/(2N) + t \cdot \tan(45\text{-}\alpha/2)]\cos(\alpha) + t$
Actual length of side leg	L_1	22.45	(m)	$L_1 = (B - t)/\cos(\alpha)$
Effective length of side leg	L_2	22.02	(m)	$L_2 = L_1 - t \cdot \tan(45 - \alpha/2)$
Total length of walls	L_3	620	(m)	$L_3 = N(2L_1 + D + A)$
Distance between cycles	w	9.42	(m)	$w = 2L_1 \cdot \sin(\alpha) + A + D$
Width of labyrinth (normal to flow)	W	123	(m)	$W = N \cdot w$
Length of linear weir for same flow	—	249	(m)	$1.5 \cdot Q_{max}/[(C_d \cdot H_t^{1.5}) \cdot (2g)^{0.5}]$; ($C_d$ for linear w
Distance between cycles/crest height	w/P	3.09	—	Normally between 3 and 4
		(*d*) Concrete volume		
Wall concrete volume	—	293	(m^3)	vol. = $L_3 \cdot P \cdot t$
Apron concrete volume[a]	—	431	(m^3)	vol. = $W \cdot B \cdot t$
Total	—	655	(m^3)	—

[a]For apron concrete volume it is assumed that apron thickness is the same as wall thickness.

TABLE 2. Influence of Labyrinth Angle on Spillway Width

Angle (α) (1)	Width (W) (m) (2)	Length (B) (m) (3)	Number of cycles (4)	C_d (5)	Effective leng (L) (m) (6)
6°	139	20.2	20.0	0.232	811
7°	126	20.1	16.5	0.280	670
8°	122	20.2	14.0	0.327	572
9°	120	19.9	13.5	0.346	543
10°	127	20.6	12.5	0.359	523
12°	133	20.1	12.0	0.381	493
16°	136	20.3	10.0	0.445	421
32°	176	20.2	6.5	0.604	311
49°	213	20.1	4.5	0.682	275
Linear	249	—[a]	—[a]	0.755	249

[a]Not applicable.

vation that would generally correspond to the weir crest elevation of the labyrinth (el = 1678. m).

The second block contains assumed data. The inlet loss at maximum flow can either be s to zero or estimated from preliminary calculations. For this example, a loss of 13 cm is assum at maximum flow. The number of cycles has a significant effect on the overall layout of t labyrinth. The value of N is varied to determine the most appropriate number of cycles th gives the least cost and a hydraulically effective layout. The example uses N = 13 and an ang of 8.0°.

The third block of data contains the detailed calculations identifying the geometry of t labyrinth and equations used for each calculation. Such calculations are most efficiently do using a spreadsheet. Table 1 also contains two guides regarding the acceptable ranges of variable $H_t/P < 0.9$, and $3 \leq w/P \leq 4$. It is the opinion of the writers that these limitations help ke the design in an economical and hydraulically efficient range.

The bottom set of data in Table 1 provides a rudimentary economic analysis based on concre

volume only. The volume of the concrete in the walls is easily calculated from the data in the table. The volume of concrete in the apron assumes that the apron thickness is equal to the wall thickness. The total volume is listed as a sum of the two. One can do a preliminary economic analysis by varying the number of cycles and/or the angle of the side legs and comparing the total concrete volume.

The angle α of the side legs significantly affects both the capacity and the layout of the labyrinth. Table 2 shows calculated spillway width based on different angles for the following prototype conditions: Q_{max} = 1,538 m³/s, P = 3.05 m, H_t = 1.975 m, and the recommended weir configuration and apex width [see limitations for (2)–(9)]. The number of cycles was varied for each angle to maintain the length of the apron B essentially constant.

The angle has an impact on both the economics and the performance. The data in Table 2 show that the optimal range of angle, based on minimizing W (the width of the labyrinth) for a given discharge and B, is between 7° and 16°. Below 7° and above 16°, the width increases. The total length of the weir wall is also a factor in the economics. As the angle increases, the length of the weir decreases so the least cost may not be obtained by minimizing the width.

A small α produces a high spillway capacity at lower reservoir elevations. The spillway capacity increases as α is reduced because of the increase in effective length of the spillway. With the values of C_d defined by (2)–(9) or obtained from Fig. 3, a rating curve for the complete range of operation can be constructed. The results show that at low reservoir elevations, the capacity is noticeably greater for smaller α. If the reservoir elevation is to be limited at flows less than the design flow, the design with a smaller angle is preferred. On the other hand, if the outflow must be restricted at low reservoir elevations, then a larger angle is better.

A large angle may be chosen in the case of a spillway replacement where there is an existing apron width available and it is desirable to use the full width to minimize changes to the upstream and downstream channels. For such an installation, efficiency may not be the controlling criteria. The largest labyrinth angle is selected that provides the required flow and the best layout for the given width of apron.

Another important variable that influences the general layout and economy of a labyrinth is the number of cycles, N. Past test results (Waldron 1994) have shown that the crest coefficient is not influenced by the number of apexes. This simplifies the design process and allows (2)–(9) to be used without concern for the influence of N. As the number of cycles N is reduced, the length of the apron B gets large and can equal or exceed the width of the apron W. This increases concrete volume and cost. The data in Table 3 were generated using the spreadsheet in Table 2 keeping everything constant but N. The data show that increasing N has little effect on the actual weir length but it decreases B, increases W, and reduces the concrete volume. Selecting either too few or too many cycles produces a layout that may not be hydraulically efficient or cost-effective. Following the criteria $3 \leq w/P \leq 4$ keeps the length and width in proper proportion.

The cost of the approach and discharge channels is a significant cost factor not included in the simplified analysis. Even though the concrete volume decreases with increased N and α, the cost of the channels increases as W increases. There will be a number of designs that pass the required flow. The final design must be based on a complete economic analysis and an evaluation of any site conditions that may limit the maximum width of the structure.

Another factor related to selection of the preferred labyrinth design is the unit discharge in the approach and discharge channels. If the head is high and α and N are selected to give the minimum spillway width, the result is large depths and high velocities. This increases scour potential and may increase the difficulty of designing energy dissipating structures. These factors

TABLE 3. Influence of Number of Cycles on Spillway Layout

Number of cycles[a] N (1)	Length of apron B (m) (2)	Width of apron W (m) (3)	Weir length L_3 (m) (4)	Concrete volume (m³) (5)	w/P (6)
7	40.5	103	1,209	3,034	4.83
8	35.4	106	1,124	2,821	4.34
9	31.4	108	1,058	2,655	3.95
10	28.3	111	1,006	2,525	3.64
11	25.6	114	963	2,417	3.39
12	23.5	116	928	2,329	3.18
13	21.6	119	899	2,255	3.00
14	20.1	122	874	2,192	2.85
15	18.9	124	852	2,138	2.71
16	17.7	127	833	2,092	2.60

[a]N = 10 – 13 is the preferred range; α = 8°.

must be compared to the increased excavation and construction cost for the approach a discharge channels if the spillway is made wider.

FLOOD ROUTING

As part of the spillway design procedure, the maximum flood should be routed through t reservoir with the design layout. A preliminary assumption about spillway capacity as a functi of reservoir elevation must be made to generate the preliminary outflow hydrograph. If the fi spillway design is significantly different from the one originally assumed, the outflow hydrogra will need to be recalculated because the outflow hydrograph and the corresponding des spillway flow will change.

Assume that the original flood routing was done using a spillway with a large labyrinth an and the preferred design has a small angle. The spillway with a small angle has significan more capacity at low reservoir elevations. With the increased spillway capacity, more of flood is passed through the reservoir, which reduces the maximum reservoir elevation. This n allow the spillway length to be reduced, saving construction costs.

The other side of the problem is matching the outflow to downstream flow limitations. example would be where previous water rights limit releases from the reservoir at floods bel the hundred-year flood. If the labyrinth is to be added to an existing reservoir where downstream requirements limit the flows at low water-surface elevations, a labyrinth wit small angle may provide more capacity than can be tolerated. For such an installation, a lar angle labyrinth may better fit the outflow requirements. If the low flow requirement is extrem small, a short section of weir at a lower elevation could be used to pass small flows. The labyrinth would not activate until the flood exceeds some predetermined level, such as hundred-year flood.

DESIGN VERIFICATION

Before recommending the design procedure proposed herein, its accuracy was verified comparing it to the flows estimated for nine other labyrinth designs. Table 4 shows the c parisons. The comparisons shown are only at design flow. The difference between the estima flow and the flow calculated with the recommended design procedure varies by less than 1 In making the comparisons, slight modifications to the design procedure were needed to acco for differences in crest configuration and head. For Ute Dam, the radius of curvature for c is $P/30$, which is significantly smaller than the $P/12$ specified for our design procedure. This cause more separation and make the crest coefficient and the flow for the Ute model sma than the values predicted by the equations in the present paper. It was assumed that the c coefficient would fall between that for the quarter-round and a flat crest weir. The coeffic listed in Table 4 is the average of these two values. With this assumed coefficient, the flow the Ute Dam data is within 3.3% of the calculated flow.

For Bartletts Ferry, the head given was the measured piezometric head. The approach velo head was estimated and the total head of 2.44 m was used for the calculations.

The total crest length was reduced using the procedure identified in Fig. 1 to obtain effective length. The difference is basically the short outside lengths at the apex where the essentially no flow.

Comparisons were also made by calculating the flow over the full range of H_t/P. Above $P = 0.3$, the calculated values were within 10% of the reported values. At small heads, calculated flows were as much as 20% smaller than the reported values. This is because crest coefficients calculated by (2)–(9) are for aerated nappes and will be too low if the proto nappe is not aerated. This is one reason why it is advisable to generate the final rating c with a model study.

TABLE 4. Evaluation of Recommended Design Procedure

Location (1)	Reference (2)	Angle of side legs (α) (3)	Weir height (P) (m) (4)	Total head (H_t) (m) (5)	Total crest length (m) (6)	Effective crest length (m) (7)	Maximum design discharge (m^3/s) (8)	Calculated flow (m^3/s) (9)	Perc differe (10
Bartletts ferry	Meeks (1983)	14.5°	3.43	2.44	1,441	1,412	6,796	6,740	−0.
Ute Dam	Houston (1982)	12.15°	9.14	5.80	1,024	1,020	15,574	15,065	−3.
Avon Dam	Hinchliff (1984)	27.5°	3.05	2.16	265	252	1,416	1,417	+0.
Boardman	Cassidy (1985)	19.44°	3.51	1.77	107	104	387	399	+3.
Woronora	Hinchliff (1984)	25.4°	2.23	1.36	344	344	1,019	991	−6.
Navet	Hinchliff (1984)	23.58°	3.05	1.52	137	137	481	477	−3.
Rollins Dam	Tullis (1986)	9.23°	3.35	2.74	472	457	1,841	1,890	+2.
Ritchsard Dam	Vermeyen (1991)	8.13°	3.05	2.74	411	399	1,555	1,549	−0.

CONCLUSIONS

The capacity of a labyrinth spillway is a function of the total head H_t, the effective crest length L and the crest coefficient C_d. C_d depends on weir height P, total head H_t, weir wall thickness t, crest shape, apex configuration, and the angle of the side legs α. The design procedure is based on a weir with a of height $H_t/P \approx 0.9$ (at maximum flow), and with $t = P/6$, which is rounded on the upstream corner at a radius of $P/12$. The crest coefficients should be valid for a battered wall as long as the radius of the crest is still $P/12$. With the crest geometry fixed, C_d is only a function of α and head. Values of the crest coefficient for α between 6° and 35°, for the recommended weir configuration, can be determined from Fig. 3 or from polynomial equations [(2)–(9)] for eight different labyrinth angles and a linear weir. The choice of α and the number of cycles N significantly influences the width, length, and other details of the labyrinth. With complete freedom to vary α and N, numerous layouts can be generated. The most appropriate design is determined after considering site-specific limitations, completing an economic analysis in parallel with the hydraulic analysis, and routing the flood through the reservoir using the final spillway design.

Even though the design procedure gives an accurate analysis of the labyrinth's capacity, it is still advisable to verify the performance of the spillway with a model study. The model accounts for site-specific factors outside the scope of the spillway design, such as flow conditions in the approach and discharge channels, inlet losses, scour, submergence, and energy dissipation. If the flow in the discharge channel is supercritical, the model can also provide valuable information on wave heights and superelevation caused by channel convergence or bends.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = inside apex width;
B = length of labyrinth apron;
C_d = crest coefficient;
el = crest elevation;
H_t = total head on crest;
L = effective length;
N = number of cycles;
P = weir height;
Q = flow;
Q_{max} = design flow;
R = radius of crest curvature;
res = maximum reservoir elevation;
t = wall thickness at crest;
W = total width of labyrinth;
w = width of one cycle;
α = labyrinth angle; and
β = angle of approach flow.