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SHIELDS PARAMETER IN LOW SUBMERGENCE OR STEEP FLOWS

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INTRODUCTION

Shields parameter is an integral part of riprap design. No other single parameter matches Shields parameter in describing the mechanics of incipient motion, or in riprap terminology, failure. This paper presents three methods for deriving Shields parameter and one method for deriving the boundary or particle Reynolds number. This paper is not a continuation of the spurious correlation argument that seems to arise when discussing Shields parameter. The purpose of this paper is to investigate the behavior of Shields parameter in flow regimes not considered by Shields.

Some investigators assert that Shields parameter is not constant above boundary Reynolds number 10^4. Characteristics of this flow regime are low relative submergence ($d/D_{50} < 5$) and bulked or aerated water. This paper is an attempt to show that Shields parameter is indeed constant in this flow regime.

Background

Shields published "Anwendung der Aehnlichkeitsmechanik und der turbulenzforschung auf die geschiebebewegung: Mitteilung der Preussischen Versuchsanstalt fuer Wasserbau und Schiffbau" in 1936, in the German language. Most papers cite the translated title "Applications of similarity principles and turbulence research to bed-load movements". Quoting from a translation [6], Shields main proposition is:

> "The ratio of the active force of the water parallel to the bed, to the resistance of a grain on the bed is a universal function of the ratio of the grain size to the thickness of the laminar sublayer."

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From Gessler [3] the following relationship puts Shields words into mathematical form.

$$T = \frac{\tau_c}{(\gamma_s - \gamma)k} = f_1\left(\frac{k}{\delta_1}\right) = f_2\left(\frac{u_* k}{\nu}\right) \tag{1}$$

T .... Shields Parameter
τ_c .... Critical shear stress
γ_s .... Specific weight of the grain
γ .... Specific weight of the fluid
δ_1 .... Thickness of the laminar sublayer
ν .... Kinematic viscosity
u_* .... Shear velocity, $u_* = \sqrt{\tau/\rho}$
ρ .... Density of fluid
τ .... Bottom shear stress
k .... Grain size

Shields determined the functions f_1 and f_2 experimentally. Gessler [3] describes the difficulty of defining incipient motion.

> "During the course of the experiments the difficult question of the definition of incipient motion arose; due to various reasons the uniform size grains did not all begin to move at the same time. Shields defined the beginning of motion in the following way: he found in experiments with different bottom shear stresses, that were just above critical, small bedload transport rates per unit time. He extrapolated the function for bedload movement dependent on bottom shear stress obtained in this way in the direction of decreasing bottom shear stress to the point where the bedload transport is zero. He called the corresponding shear stress of this point the "critical shear stress"."

Therefore, in the simplest terms, Shields parameter is a delineator of particle motion and particle stability.

Stevens and Simons [9] analyzed the forces and moments acting on riprap. By balancing the turning forces or moments with the righting forces or moments, they created a stability factor for riprap mixtures based upon a characteristic particle size that was a function of the riprap gradation. A stability factor of 1.0 defines incipient motion. Values less than 1.0 indicate instability and values greater than 1.0 indicate stability. The stability factor functions very much like the safety factors prevalent in engineering design practice. By definition, Stevens and Simons correlate a stability factor of 1.0 to a Shields parameter of 0.047. This correlation was a convenient method of estimating the forces by water parallel to the bed. Therefore, in more complex terms, Shields parameter is a safety factor.

Shields and others developed data in flows with large relative submergence. The definition of relative submergence, λ_{sub}", is the ratio of water depth to particle size.

$$\lambda_{sub} = \frac{d}{k} \tag{2}$$

d .... Fluid depth
k .... Grain size

There is a question whether the stability factor method applies to flow that is steep, rough, and has relative submergence much less than tested by Shields. Wittler and Abt [10] show that the stability factor method of Stevens and Simons becomes increasingly conservative as slope increases. Shen [7] [8] proposes that the value of Shields parameter is roughly five times greater at boundary Reynolds numbers greater than 10^4 than the accepted value of 0.047 at boundary Reynolds number $170 < \mathrm{Re}_* < 10^4$. This raises the question of the basis of the stability factor, in steep flow conditions, the basis being the value of Shields parameter.

SHIELDS PARAMETER

The following section presents several methods for deriving Shields parameter. Two methods of fractional analysis show that Shields parameter is physically derivable.

Pi

The process for creating dimensionless variable groups is straight forward. The following matrix identifies the pertinent variables.

Variable	Mass	Length	Time
k		1	
g		1	-2
ρ	1	3	
ρ_s	1	3	
u		1	-1

The rank of the matrix is one indicating a single dimensionless variable group.

$$T = \mathrm{f}\left(\frac{\rho_w u^2}{\rho_s g k}\right) \Leftrightarrow \frac{\frac{m}{l^3}\frac{l^2}{t^2}}{\frac{m}{l^3}\frac{l}{t^2}l} \tag{3}$$

If u is proportional to u_*, then

$$T = \mathrm{f}\left(\frac{\tau}{\gamma k}\right) \tag{4}$$

a form of Shields parameter.

Similitude

Similitude is an imaginative comparison of forces, moments, or energy acting on a particle in a flow field. Force similitude is the simplest case, considering magnitude and direction of forces. Moment similitude adds another dimension to force similitude by considering the moment arm through which forces act. Energy supersedes force magnitude and direction by considering potential and mechanical energy.

Forces

There are two primary forces acting on a particle in flowing water: The gravitational force associated with the particle, and the inertial force of the flowing water. The gravitational force is a function of the mass and volume of the particle. The inertial force is a function of the form of the particle, the projected area perpendicular to the direction of flow, and the mass density of the fluid.

$$\text{Gravity:}\quad \begin{aligned} F_g &= mg \\ m &\propto (\rho_s - \rho\cos\theta)k^3 \\ F_g &\propto g(\rho_s - \rho\cos\theta)k^3 \end{aligned} \tag{5}$$

$$\text{Inertia:}\quad \begin{aligned} F_i &= C_d\frac{u^2}{2}A\rho \\ A &\propto k^2 \\ F_i &\propto \rho k^2 u^2 \end{aligned} \tag{6}$$

The velocity term represents the mean local velocity about the particle. The shear velocity, u_*, is proportional to u, so that

$$\begin{aligned} \frac{F_g}{F_i} &\propto \frac{g(\rho_s - \rho\cos\theta)k^3}{\rho k^2 u^2} \\ u \propto u_* &= \sqrt{\tau/\rho} \\ \frac{F_g}{F_i} &\propto \frac{(\gamma_s - \gamma\cos\theta)k}{\tau} = constant \end{aligned} \tag{7}$$

The ratio of gravitational and inertial forces is the inverse of Shields parameter. In engineering sense, a safety factor is a ratio of positive to negative. In terms of force, the safety factor is a ratio of the stabilizing or resistive forces to the destabilizing or motivating forces. As shown in Equation 7, Shields parameter is the inverse of a safety factor, if gravitational forces are stabilizing and fluid inertial forces are destabilizing.

Moments

The similitude of moments acting on a particle is very similar to the force similitude. The additional factor in the moment analysis is a moment arm. The line of action of both gravitational and inertial forces is difficult to define. The direction of the gravitational force is well known, but the other component, buoyancy, is not. The line of action of each force, with respect to the pivot point, is a factor of the particle diameter, *k*. The line of action of the inertial components, lift and drag, is even less detectable. However, the line of action must pass within some factor of the particle diameter regardless of the direction of either component.

To write the following equations take the moment about the downstream contact point, and assume that the upstream contact points are zero force at incipient motion.

$$\begin{aligned} &M_g = mgl \\ &l \propto k \\ &m \propto (\rho_s - \rho\cos\theta)k^3 \\ &M_g \propto g(\rho_s - \rho\cos\theta)k^4 \end{aligned} \tag{8}$$

$$\begin{aligned} &M_i = C_d \frac{u^2}{2} A \rho l \\ &l \propto k \\ &A \propto k^2 \\ &F_i \propto \rho k^3 u^2 \end{aligned} \tag{9}$$

$$\begin{aligned} &\frac{M_g}{M_i} \propto \frac{g(\rho_s - \rho\cos\theta)k^4}{\rho k^3 u^2} \\ &u \propto u_* = \sqrt{\tau/\rho} \\ &\frac{M_g}{M_i} \propto \frac{(\gamma_s - \gamma\cos\theta)k}{\tau} = constant \end{aligned} \tag{10}$$

The ratio of the moments causes the extra length dimension to divide out. The same proportionality, which is Shields parameter, therefore exists whether considering ratios of forces or moments. With the uncertainty in direction of forces and lines of action, it helps to explore another facet of the physics of flowing water, a facet that is a scalar, energy.

Energy

The energy required to lift a particle from a position of rest is equal to the potential energy of the particle one particle diameter above it's initial position.

$$E_L = mgk \tag{11}$$

The mass of the particle is equal to the volume times the buoyant mass density of the particle. The volume of the particle is proportional to the diameter cubed. The inertial energy is a function of mass and the local velocity squared.

$$E_L \propto gk(\rho_s - \rho\cos\theta)k^3 = gk^4(\rho_s - \rho\cos\theta) \tag{12}$$

$$\begin{aligned} E_I &= \tfrac{1}{2}mu^2 \\ E_I &\propto (\rho_s - \rho\cos\theta)k^3u_*^2 \end{aligned} \tag{13}$$

Again the assumption is that the shear velocity is proportional to the local particle velocity. The ratio of the inertial energy to the potential energy is Shields parameter.

$$\frac{E_I}{E_L} \propto \frac{\rho k^3 u_*^2}{(\rho_s - \rho\cos\theta)gk^4} = \frac{\tau}{(\gamma_s - \gamma\cos\theta)k} \tag{14}$$

This section shows three methods of deriving Shields parameter. Gessler [2] presents an exhaustive treatise on the derivation of Shields parameter. In following sections the assumptions made in the derivations will help explain the behavior of Shields parameter. The next section derives the boundary Reynolds number.

BOUNDARY REYNOLDS NUMBER

Like Shields parameter, the boundary or particle Reynolds number is the ratio of two forces. In this case the two forces are the inertial and viscous forces.

$$\begin{aligned} F_I &\propto \rho k^2 u^2 \\ F_V &\propto ku^2\mu\frac{du}{dy} \end{aligned} \tag{15}$$

Again the assumption is that the shear velocity approximates the local velocity. Also, the differential $\frac{du}{dy}$ is proportional to $\frac{u_*}{k}$. Following these two assumptions, the boundary Reynolds number, Re_* results.

$$
\begin{aligned}
&\frac{F_I}{F_V} \propto \frac{\rho k^2 u^2}{k^2 \mu \dfrac{du}{dy}} = \frac{u^2}{\mu \dfrac{du}{dy}} \\
&\frac{F_I}{F_V} \propto \frac{u_* k}{\nu} \\
&\therefore \mathrm{Re}_* = \frac{u_* k}{\nu}
\end{aligned}
\tag{16}
$$

This completes the background of Shields parameter, T, and the boundary Reynolds number, Re_*. Shields presents the functional relationship between Shields parameter and the boundary Reynolds number in graphical form, Shields diagram.

SHIELDS DIAGRAM

Shields published a diagram [6] showing a functional relationship between what he described as a dimensionless shear stress and the particle Reynolds number. Rouse [5] later added the curve. The original curve plots a constant Shields parameter, T_c, of 0.06 at $\mathrm{Re}_* > 10^4$. The basis of this value is Shields definition of incipient motion, that is the dimensionless shear at zero bedload. Gessler proposes that the correct value of T_c is 0.047. The basis of this assertion is that Shields neglected to account for shear stress due to form drag from ripples and bed forms, thus over estimating the value of the parameter. Therefore, the basis of the correction by Gessler is a different definition of incipient motion than Shields. Gessler [3] defines incipient motion as the state where a particle has a fifty percent probability of motion. Figure 1 shows Shields diagram as corrected by Gessler. The basis of the Stability Factors riprap design procedure [9] is the corrected value of T_c.

BEHAVIOR OF SHIELDS PARAMETER AT $\mathrm{Re}_* > 10^4$

Background

There are two phases of Shields diagram. The characteristic of the first phase is the non-constant value of Shields parameter that corresponds to the laminar boundary layer regime where the boundary Reynolds number is less than roughly 200. The second phase is the range characterized by boundary Reynolds number greater than 200. In this range, the value of Shields parameter is constant. Shen and others propose that a third phase exists. In this range of boundary Reynolds number greater than 10^4 Shen proposes that the constant value

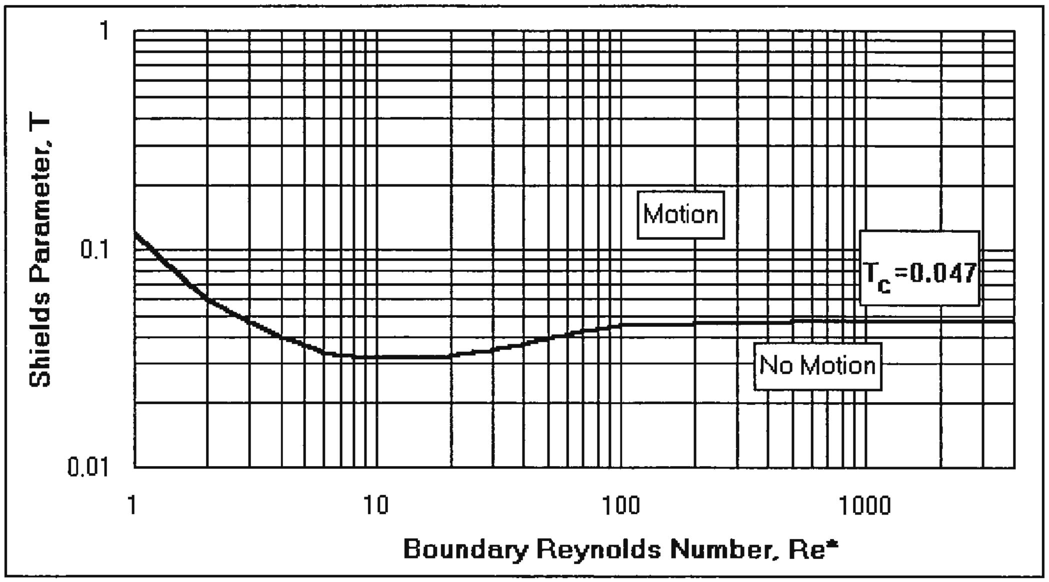

Figure 1. Shields diagram, corrected by Gessler.

increases from 0.06 (corrected to 0.047 by Gessler) to 0.25, and then remains constant. Application of riprap design procedures such as Stability Factors in a steep, low submergence flow regime, requires knowledge of the behavior of Shields parameter. Two researchers have reported values of Shields parameter much greater than 0.047 in the region where $Re_* > 10^4$.

Shen

Shen and Wang [7] [8] propose that Shields parameter achieves a constant value of 0.25 at $Re_* > 10^4$. They base this proposition on data collected in China in flood channels. The mean relative submergence of the tests by Shen and Wang is 7.9 and the average energy slope is .047, or 4.7 percent. Shen and Wang explain the increase in Shields parameter as a drag reduction effect. They base their proposition on the drag reduction that occurs at Reynolds number of 10^5 to 10^6. Schlicting [4] reports the drag reduction phenomenon. The median stone size is the basis for k in calculations of Shields parameter. Depth and slope measurement techniques are undocumented.

Abt et al

Abt et al [1] achieved similar results for a wide variety of median riprap sizes and slopes. They report maximum Shields parameter values of 0.12. The median stone size is the basis for k in calculations of Shields parameter. Median rock sizes include 1, 2, 3, 4, and 6 inches. Bed slopes tested include 1%, 2%, 5%, 10%, 15%, and 20%. Relative submergence ranges from 0.48 to 2.01. Abt reports highly aerated flow, periods of instability followed by periods of stability, followed by ultimate failure of the riprap blanket. Depth is piezometric head measured at the filter-riprap interface corrected to the virtual riprap surface.

Rationale for a Two-phase Shields Parameter

The following sections address reasoning supporting a two-phase Shields parameter. The basis for analyzing Shields parameter in the large boundary Reynolds number range are the assumptions made in the derivation of Shields parameter and making hypothetical modifications to the force, moment, or energy ratios.

Aeration-Bulking

Aeration or bulking is the process of ingesting air into flowing water. Air is ingested by a mechanism related to the momentum and viscosity of the air-water interface. The practical effect of aeration is an inflated depth of flow and a deflated fluid density. Equation 7 facilitates a hypothetical demonstration of the effects of aeration upon Shields parameter.

$$\begin{aligned} &\frac{F_g}{F_i} \propto \frac{g(\rho_s - \rho\cos\theta)k^3}{\rho k^2 u^2} \\ &u \propto u_* = \sqrt{\tau/\rho} \\ &\frac{F_g}{F_i} \propto \frac{(\gamma_s - \gamma\cos\theta)k}{\tau} = constant \end{aligned} \qquad (7)$$

The gravitational force has four components: gravitational acceleration, g, particle mass density, ρ_S, fluid mass density, ρ, and particle size, k. Gravitational acceleration is uninfluenced by aeration as is the particle size and particle mass density. Therefore, in the numerator of the force ratio, aeration influences only the mass density of the fluid.

The inertial force has three components: fluid mass density, ρ, particle size, k, and the local velocity, u. The particle size is uninfluenced by aeration. It is unclear whether aeration increases or decreases the local velocity. Therefore aeration influences only fluid mass density in the inertial force.

A simplified form of equation 7 shows the modified factors.

$$\begin{aligned} &T_a \propto \frac{(\rho_s - \rho_a)}{\rho_a} \\ &\rho_a = \text{aerated fluid mass density} \\ &T_a = \text{aerated Shields parameter} \end{aligned} \qquad (7.a.)$$

Let's assume that the air water mixture is 50% air and 50% water.

$$\frac{T_a}{T}=\frac{\dfrac{(\rho_s-\rho)}{\rho}}{\dfrac{\rho_s-\rho_a}{\rho_a}} \tag{17}$$

$$\frac{T_a}{T}=\frac{\dfrac{(2.65-1)}{1}}{\dfrac{2.65-.5}{.5}} \tag{18}$$

$$\frac{T_a}{T}=0.384$$

The result of equation 18 shows that aeration reduces the value of Shields parameter. Clearly, aeration is not a rationale for asserting that Shields parameter increases in any range of boundary Reynolds number.

One effect that aeration has on Shields parameter is the determination of measured factors in laboratory or field measurements. The final form of Shields parameter demonstrates the errors that can creep into field or laboratory measurements.

$$T=\frac{\tau}{(\gamma_s-\gamma)k}=\frac{\gamma RS}{(\gamma_s-\gamma)k} \tag{19}$$

Again, assuming a 50%-50% air-water mixture, let's examine the effect upon measuring Shields parameter.

In aerated flow, fluid unit weight, γ, is overestimated. It is common to assume that γ has a value of 62.4 pcf, when in actuality, in a 50% air-water solution, it is 31.2 pcf, an error factor of two. The hydraulic radius is similarly overestimated by a factor of two. Meanwhile, the buoyant weight of the particle is underestimated by a factor of 1.3. Therefore, the overall error results in overestimating Shields parameter by a factor of 5.21.

$$T = \frac{\tau}{(\gamma_s - \gamma)k}$$
$$\tau = \gamma RS$$
$$\frac{T}{T_a} = \frac{\dfrac{\gamma RS}{(\gamma_s - \gamma)k}}{\dfrac{\gamma_a R_a S}{(\gamma_s - \gamma_a)k}}$$
$$\frac{T}{T_a} = \frac{\dfrac{62.4(2R)S}{62.4(2.65-1)k}}{\dfrac{31.2RS}{62.4(2.65-0.5)k}} = 5.21 \tag{20}$$

Recall that Shen reported a four-fold increase in the apparent value of Shields parameter, while Abt reported a two-fold increase using piezometric depth, negating the illusory hydraulic radius factor.

Coefficient of Drag

Recall that Shields parameter is the inverse of the ratio of the gravitational and inertial forces. More simply, Shields parameter is the ratio of the inertial and gravitational forces.

$$\frac{F_g}{F_i} \propto \frac{g(\rho_s - \rho\cos\theta)k^3}{\rho k^2 u^2}$$
$$u \propto u_* = \sqrt{\tau/\rho} \tag{7}$$
$$\frac{F_g}{F_i} \propto \frac{(\gamma_s - \gamma\cos\theta)k}{\tau} = constant = 1/T$$

The numerator of Shields parameter then is an expression of the inertial force, that before any assumptions is the product of the square of velocity, area, fluid density, and the coefficient of drag, C_d. Shen bases the increase in Shields parameter upon a corresponding decrease in C_d, as previously noted by Schlicting. If C_d decreases, then the numerator of Shields parameter decreases, and therefore, Shields parameter decreases.

$$F_i = C_d \frac{u^2}{2} A \rho \tag{6a}$$

Therefore, drag reduction is not a rationale for asserting that Shields parameter increases in the range of boundary Reynolds number greater than 10^4.

Steep Slope: S or $\sin\theta$?

Engineers often assume the bottom tractive force, or shear stress, τ, to be equal to the unit weight of water, γ, times the hydraulic radius, R, or depth, d, times the slope, S.

$$\tau = \gamma RS \tag{21}$$

The basis of equation 21 rests upon the following assumptions: Normal depth of flow, wide channel such that the hydraulic radius is roughly equal to the depth, and a shallow slope such that S is roughly equal to $\sin\theta$. The tractive force is the resolved weight of a control volume of water. The volume has an area in contact with the wetted perimeter, p, with dimension pl, where l is a unit distance in the stream wise direction while p is perpendicular to the flow. The thickness of the volume is the depth, that is perpendicular to the bed, d. The bed slopes at an angle θ. The flow is uniform so that the hydrostatic forces at the upstream and downstream ends of the control volume negate each other.

$$\begin{aligned} &w = \text{control volume width} \\ &A = wd \\ &\tau = \frac{\gamma A l}{pl}\sin\theta = \gamma R \sin\theta \end{aligned} \tag{22}$$

The basis of τ is the assumption that $\sin\theta$ is roughly equal to S. This assumption is accurate to less than one percent until the slope exceeds roughly 0.15 or fifteen percent. On embankments where slopes approach 33% the error accumulates quickly.

Relative Submergence

The assumption that the local velocity, u, is proportional to the shear velocity, u_*, bears further investigation. Gessler [2] states "Some of the discussion about effect of relative depth on incipient motion probably reflects the fact that introducing mean velocity instead of shear velocity necessarily leads to such an apparent effect." Shear velocity, u_*, and mean velocity, u, are related by the log-velocity law.

$$\frac{\overline{u}_y}{u_*} = 5.75\log_{10}\left(30.2\frac{yx}{k}\right) \tag{23}$$

The logarithmic term contains a relative submergence term, y/k. The effects of relative submergence are not clear. However, in an intuitive sense, the radically differing regimes of flow between Shields and steep, air-entrained, low submergence flows, suggests that some effect may exist. Shields tested millimeter sized material in flow with a slope less than one-hundredth of one percent and greater than a meter in depth. Shen and Abt tested flow with slopes three orders of magnitude greater, material size two and one-half orders of magnitude greater, and relative submergence three orders of magnitude greater than Shields. Even though the basis of Shields parameter is a ratio of forces, the assumptions that allow simple quantification of those forces may not apply to these vastly different regimes of flow.

CONCLUSIONS

The purpose of this paper is to investigate the behavior of Shields parameter at boundary Reynolds number greater than 10^4. The hypothesis of this paper is that Shields parameter is constant in this flow regime. Investigation of the background and derivation of Shields parameter show that there is no reason to expect Shields parameter to increase, and indeed it is more likely that Shields parameter decreases in this regime. The apparent and reported increase in Shields parameter is the result of measuring flow properties for the calculation of Shields parameter.

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