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ABSTRACT

Recent physical model studies at the Bureau of Reclamation's hydraulic laboratory requiring scaling of sediment transport rates were designed with a method based on the Shields function as modified by Taylor. Taylor developed a dimensionless sediment discharge parameter, which provides nesting curves that parallel Shields' incipient motion curve at constant values of dimensionless sediment discharge. Using this method clearly demonstrates when lightweight materials are required or when the sand particle sizes can be adjusted in the model to represent the sediment transport rates. A method to adjust the sand particle sizes based on settling velocity was developed. This method appears to make the correct adjustment to fit the transport curves on the Shields diagram. The method was used for modeling the lateral scour of fuse plugs and compared well with prototype experience. The method also illustrates the limitations and applicability of sediment models. The sediment transport rates may be properly scaled in one area of a model and improperly scaled in a different area where the Reynolds number is different.

INTRODUCTION

Background

Reclamation experience with sediment models started in the early 1950's. Several predesign models were conducted for the Missouri River Basin and Middle Rio Grande diversions. The sediment used for all these studies was a fine uniform sand with a mean diameter of 0.2 mm. Settling velocity is very important in determining when a particle will remain at rest or how far it will travel once lifted into the flow. The 50-percent model sizes were scaled by settling velocity according to the Froude law, that is by the square root of the length ratio.

These models were force fed sediment to develop bed slopes sufficient to move sediment at rates estimated by sediment bedload equations. Bed slopes that were developed this way were generally exaggerated because of friction differences between model and prototype. However at diversions, flow splits occur within a short reach in the direction of flow, and the structure does not need to be distorted.

Sediment concentration was measured in all the component flows of the models. Concentration ratios of the measured delivery rate to the river concentration were used to compare the relative amount of sediment in the delivery for different trial diversion arrangements.

The scale relationships for open channel flow based on Froude number are as follows:

Length = L_r

Area = L_r^2

Volume = L_r^3

Time = $L_r^{1/2}$

Force = L_r^3

Shear = L_r

Velocity = $L_r^{1/2}$

Discharge = $L_r^{5/2}$

Similitude Deficiency of Froude Scaling

Models involving erosion of noncohesive bed material must simulate tractive shear (τ), because the tractive shear causes the drag force which overcomes the forces holding the particle in place.

The tractive shear on a particle fluctuates because of the turbulence. The drag force and the turbulence are a function of Reynolds number. Therefore, a model based on Froude law does not necessarily simulate the tractive forces and the erosion accurately. In some models, the sediment sizes must be adjusted to compensate for Reynolds number defect. Reynolds number defect (R_{ed}) of a Froude number model can be determined by the Froude scaled variables being substituted in the Reynolds number resulting in:

$$R_{ed} = L_r^{3/2} \tag{1}$$

If erosion occurs by chunking action such as cutbank scour or in a controlled manner, such as in an impervious core in a fuse plug, a model needs to include effects of elastic forces as well as gravity. Taking the ratio of gravity and elastic forces results in the structural integrity number **M** expressed as:

$$\mathbf{M} = \gamma L/E \tag{2}$$

where E is the modulus of elasticity of the chunking material or the core in a fuse plug. **M** must be made the same in the model and prototype.

Dimensionless Unit Sediment Discharge

Vanoni [1975] used Taylor's data [1971] to show that dimensionless unit sediment discharge at low transport rates falls close to Shields curve for threshold of movement (fig. 1). To properly simulate sediment discharge, the dimensionless unit sediment discharge rate (q_s*) must be the same in the model and prototype.

For a model with a grain Reynolds number (**R***) between 5 and 100, the dimensionless sediment unit discharge will be too high if the sand grain diameters are scaled geometrically. Therefore, model sediment sizes or specific gravity should be adjusted in this range to properly simulate sediment transport.

By dimensional analysis, it has been shown that noncohesive material has a functional relationship which can be expressed as a dimensionless shear stress,

$$\tau^* = \tau/(\gamma_s - \gamma_w)d_s = \Phi(U^* d_s/\nu,\ d_s/R,\ \gamma_s/\gamma_w) \tag{3}$$

where the grain Reynolds number is,

$$\mathbf{R}^* = U^* d_s/\nu \tag{4}$$

The shear velocity can be expressed as:

$$U^* = (\tau/\rho)^{1/2} \tag{5}$$

Where d_s = sand grain diameter
ν = kinematic viscosity of water
τ = tractive shear
ρ = density of water
R = hydraulics radius
γ_s = specific weight of sand
γ_w = specific weight of water

This function, except for the roughness ratio [the ratio of the sand grain diameter to the hydraulic radius (d_s/R)] and the relative specific weight [the ratio of the specific weight of sand to the specific weight of water (γ_s/γ_w)], is the same as the Shields function [1936]. The relative weight term is usually dropped on the basis of the same materials, water and natural sand, being used in the model as in the prototype. The relative diameter is dropped on the basis of relatively deep flow or fine grain sediment bed material. Thus, the equation reduces to Shields function. In addition, Shields' data for a large spread in particle specific weight fall within the data scatter for sand alone.

Shields developed a dimensionless diagram relating dimensionless shear to a boundary or grain Reynolds number. Shields used this diagram to define threshold of movement. This concept has been expanded by others to include transport parameters, such as number of particles moving, probability of particles moving, and dimensionless sediment discharge.

Investigators since Shields have verified that Shields' entrainment curve is a special transport case, representing nearly zero transport. Gessler [1968] shows that the probability of particles moving, used as a nesting parameter produces curves that parallel the Shields curve. Graf and Pazis [1977], using the number of particles moving per square foot of bed as a third parameter, produced curves that also parallel Shields' curve. Taylor [1971] developed (by dimensional analysis) a dimensionless unit sediment discharge parameter:

$$q_s^* = q_s/U^* d_s \tag{6}$$

where q_s is the sediment discharge in volume per transverse flow width and U^* is the shear velocity.

Taylor's analysis, discussed by Vanoni [1975], indicates that using q_s* as a third parameter with Shields' parameters produces curves of equal q_s* that nearly parallel Shields' incipient motion curve. Thus,

$$q_s* = \Phi(U* d_s/\nu,\ \tau/(\gamma_s - \gamma_w)d_s) \tag{7}$$

These functions can be used by physical modelers to predict transport scaling, transport volume and time model defect, possible correction factors and possible model adjustments. To use Taylor's function, the data point with coordinates $[(R*)_p, (\tau*)_p]$ corresponding to the unit dimensionless discharge, q_s*, is located on a plot of Shields-Taylor function. A curve is drawn through this point parallel to the Shields-Taylor curves. Then a trial model particle size and specific gravity combination is found that causes $(q_s*)_m$ to lie on the $(q_s*)_p$ curve. An increase in trial diameter size causes a shift of the point downward and to the right toward the target $(q_s*)_p$ curve. A change in trial specific gravity causes the trial point to move vertically toward the target $(q_s*)_p$ curve. If the geometrically scaled model grain diameter is adjusted in size according to the settling velocity as explained below, the correct adjustment is made to cause the value of $(q_s*)_m$ to approximate the target value of $(q_s*)_p$. Examples of sediment scaling for the fuse plug model study are shown in figure 1. In this study, 12.5-ft- and 25-ft-high prototype fuse plug embankments were simulated at scales of 1:25 (tests 1-6) and 1:10 (tests 7 and 8). The prototype and model data points and model adjustments are shown in the figure.

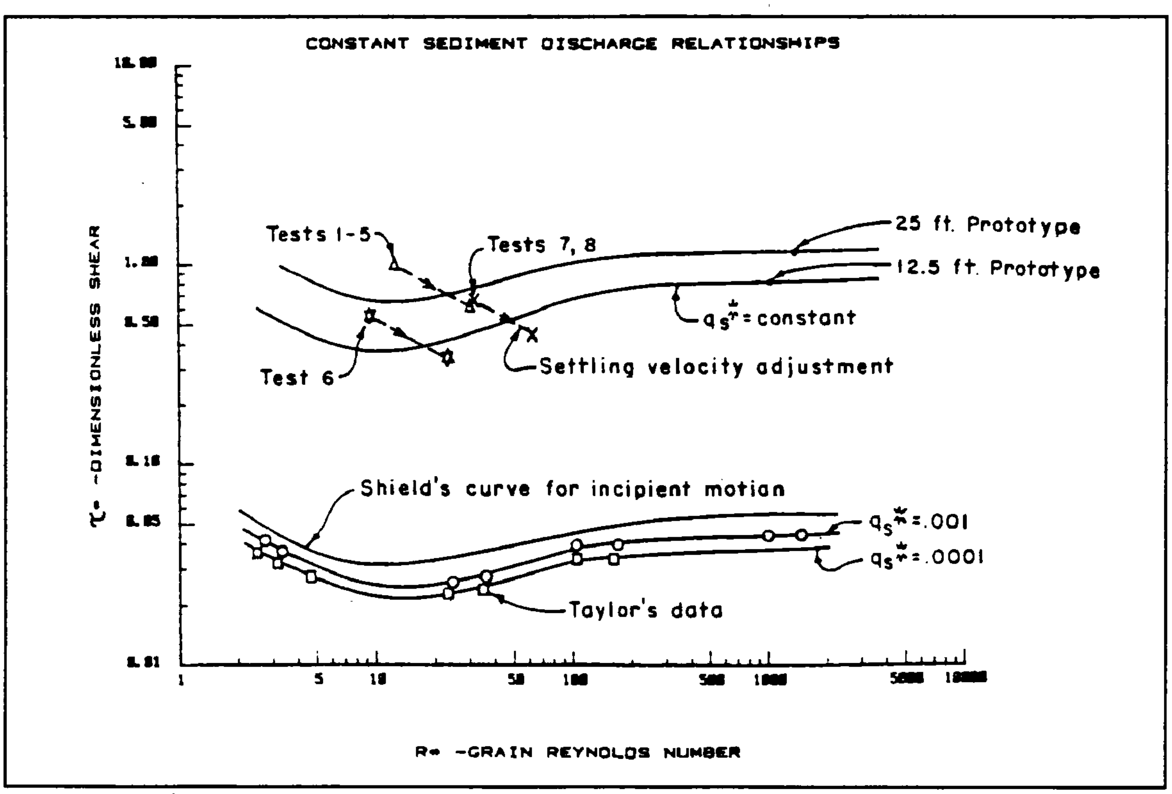

Figure 1. - Constant sediment discharge relationships.

Friction Scaling

It is sometimes convenient to use other equivalent relations for U* that are easier to work with in terms of field and the model data. These forms are

$$U^* = (gRS)^{1/2} = (\tau/\rho)^{1/2} = V(f/8)^{1/2} \quad (8)$$

To determine the Darcy-Weisbach friction factor f in an open channel, the Reynolds number R_e is computed as $4RV/\nu$ where R is the hydraulic radius and V is the average velocity at the area in question. To determine f, relative roughness is defined as $K_s/4R$ where K_s is the rugosity. Kamphius [1974] shows that K_s is equal to $2d_{90}$ where d_{90} is the particle size at which 90 percent of the grains are smaller in diameter.

The following relationship in terms of f can be used to determine τ^*:

$$\tau^* = (V^2 f/8gd)(\gamma_w/\gamma_s-\gamma_w) \quad (9)$$

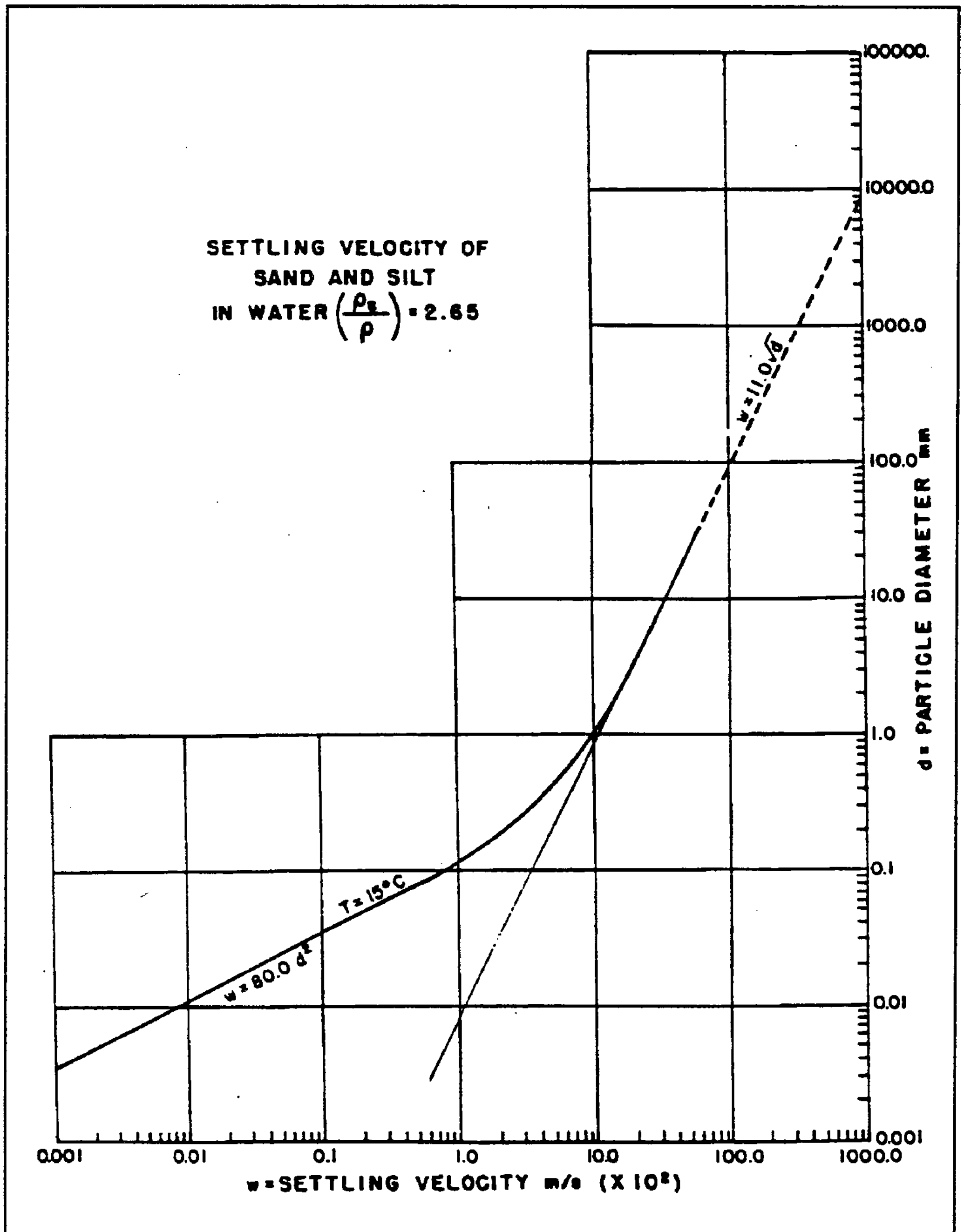

Figure 2. - Settling velocity of sand and silt in water.

Settling Velocity

Settling velocity is an important parameter that determines when a particle will remain at rest or how far it will travel once lifted into the flow. A diagram of settling velocity (w) of sand and silt particles is given in figure 2. For particles larger than 1 mm, settling velocity is a function of the particle diameter (d) raised to the 1/2 power. Also, for particles larger than 1 mm, the grain Reynolds number is generally larger than 100, and no adjustment in grain diameter is necessary to achieve the prototype value of q_s*. This is consistent with Froude scaling for velocity as discussed earlier. However, there is a defect in Froude velocity scaling for particles less than 1 mm because of fluid shear and Reynolds number defect.

By an increase of the size of a model sediment grain, the settling velocity can be corrected to the proper value for Froude scaling. For example, 1:10 scaling of a 2-mm prototype sand particle would be 0.2 mm. However, from figure 2, the settling velocity would then be 0.02 m/s when it should be 0.049 m/s according to Froude scaling. If the particle size is increased from 0.2 to 0.4 mm, the settling velocity is corrected to 0.049 m/s, the proper value for Froude scaling. This adjustment moves the location of the point plotted on the Shields-Taylor curves (fig. 1) close to the target value of the prototype unit dimensionless sediment discharge rate.

Prototype Comparison

A field test was performed in 1959 (Albrook [1959]) on a 1:2 scale model of the 8.2-m-high fuse plug used for the Oxbow Project on the Snake River in Idaho. The gradation curve for the 4.1-m-high test embankment was very close to the prototype gradation simulated in the fuse plug model study conducted at the Bureau of Reclamation's hydraulic laboratory in 1985.

The geometrically scaled sand grain diameters in the model were adjusted in size with the settling velocity adjustment correcting for the Reynolds number defect (Pugh [1985]).

The lateral erosion rate predicted by Reclamation's 1:10 scale model for the Oxbow field test was 1.66 m/min as compared to 1.71 m/min measured during the Oxbow test. The difference of 2.5 percent is well within experimental accuracies and seems to confirm the scaling technique.

CONCLUSIONS

The model design method described in this paper to simulate sediment transport in physical models compensates for the fact that the Reynolds number is generally too low to properly simulate sediment transport in a Froude scale model. The method uses an adaptation of the Taylor dimensionless unit sediment discharge curves to determine the appropriate sediment diameters or adjustment in model sediment specific gravity. Adjusting the model sediment size according to settling velocity to correct to the proper value for Froude scaling confirms the adjustment shown by the Taylor-Shields curves. Comparisons of sediment erosion rates predicted by a fuse plug model using this technique and erosion rates measured during a field test on the Oxbow Project support the validity of the technique.

Carlson [1970] summarized these early studies and concluded that physical modeling of proposed diversion schemes in close liaison with design engineers results in much better sediment control than when studies are not conducted.

Some of the more recent Reclamation experience has been with steep and fast flowing mountain streams where sediment is courser. Segregation and armoring can be important in regard to river and diversion performance. Therefore, as much of the range of sediment diameters as possible was represented by settling velocity scaling. During these later studies and during a fuse plug model study, Reclamation model design methods were reviewed and a more rational model design method was developed. The fuse plug model study results (Pugh [1985]) compared favorably with the Oxbow field data [1959].

MODELING CONSIDERATIONS

Similitude

Hydraulic models are used because of the large number of variables involved and because of complicated boundary conditions. A physical model is designed in a manner such that the flow characteristics of the model simulate in a known manner the physical behavior of the prototype.

A model and prototype must be similar geometrically, kinematically and dynamically. Geometric similarity exists when the ratios of all homologous dimensions between model and prototype are the same. The geometric scale ratio, or length ratio, is denoted by $\mathbf{L_r}$ which is the ratio $\mathbf{L_m/L_p}$, where the subscripts m and p refer to the model and prototype, respectively. Kinematic similarity, or similarity of motion, implies that the ratios of velocities and accelerations between model and prototype are equal. Dynamic similarity requires that the ratios of homologous forces between the model and prototype be the same. Possible hydraulic forces are caused by gravity, viscosity, pressure, surface tension and elasticity.

For hydraulic modeling, the more important dimensionless parameters are dimensionless ratios formed with respect to inertia:

Froude number (inertia/gravity) $\mathbf{F_r} = V/(Lg)^{1/2}$

Reynolds number (inertia/viscosity) $\mathbf{R_e} = VL/\nu$

Euler number (inertia/pressure) $\mathbf{E_u} = \rho V^2/\Delta p$

Where ν = kinematic viscosity
ρ = density
g = acceleration due to gravity
V = the average velocity
L = a characteristic length
Δp = differential pressure.

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