

THE GYARMATI PRINCIPLE AND THE THEORY OF
MINIMUM ENERGY DISSIPATION RATE
(REPORT No. I)

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1. Introduction

Gyarmati presented his Principle (GP) as shown by a functional integral:

$$G = \int_V (\sigma - \psi) dV = \max \quad (1)$$

where σ = local entropy production rate, which can be assumed as the input of energy to the system; ψ = dissipation potential rate, which represents the "ineffective" output from the system, for it refers to that part of energy transferred into heat.

On the relationship of GP to other principles, include the Hamilton Principle (HP), Prigogine Principle (Prigogine, 1963; PP) and the Theory of Minimum Energy Dissipation Rate (TMEDR). Gyarmati himself discussed this problem only briefly (1965, 1971). Furthermore, his description was quite abstract. Indeed, this problem becomes important not only for the study of the TMEDR, but also for criticizing the well-known Clausius prediction about "heat death," that is the entropy in universe should trend to maximum (Kestin, 1976), developed in last century and not satisfactorily answered to date.

Though in our previous papers (Hou, 1987; 1989) some relations between these principles had been analyzed, many problems still need to be clarified, especially those relationships between the Gyarmati Principle and the TMEDR and the Clausius prediction. The main goal of this paper is the analysis on these problems.

2. Prigogine Principle and TMEDR

Before the relationship between the GP and the TMEDR is analyzed, the relationship between the Prigogine Principle (PP) and the TMEDR needs to be analyzed. Basically, the TMEDR can be considered as an alternative presentation of the PP in a special form; their similarities, differences, and connection can be condensed in the following three points:

(1) Both in the Prigogine Principle and the TMEDR, only the dissipation was concerned. They did not concern the global balance of energy (entropy) in the system; that is, they did not need to be concerned with the input and output to and from giving system.

(2) The Prigogine Principle was concerned with the change of entropy, whereas the TMEDR was concerned with the change of energy. However, the water is an incompressible fluid. Its internal energy should remain unchanged. Furthermore, the river should proceed under isothermal condition. In this and only in this special case, the entropy and entropy-variance, $dS/dt = P$, should become linearly proportional to the energy and energy-variance, respectively. The validity of this transformation can be verified easily from the first law of thermodynamics:

$$S \sim E; \text{ and } P = \frac{dS}{dt} \sim \frac{dE}{dt} \quad (2)$$

Thus, the TMEDR can be considered as a special case of the Prigogine Principle.

(3) The TMEDR (Song, Yang, 1982; Yang, Song, 1986) concerned only the criterion for the equilibrium state, which can be expressed as:

$$\frac{dE}{dt} = \min \quad (3)$$

or, for river flow, it takes this special form, known as unit stream power:

$$\gamma u S = \min \quad (4)$$

whereas for the Prigogine Principle, besides the equilibrium state, it concerned the nonequilibrium state. In this case, the rate of entropy production must be less than zero (process damped out with time):

$$\frac{dP}{dt} < 0 \quad (5)$$

Because water flow is isothermal, the entropy production corresponds to the rate of energy:

$$P \sim \frac{dE}{dt}$$

so that the criterion for the change of entropy production with time becomes:

$$\frac{d^2 E}{d t^2} < 0 \quad (6)$$

Because the time process can be transformed into the course process through this operator:

$$\frac{\partial}{\partial t} = u \frac{\partial}{\partial x} \quad (7)$$

instead of equation (5), the criterion of nonequilibrium state, specified for the river course process, can be expressed in this form:

$$u^2 \frac{\partial}{\partial x} (\gamma u S) < 0 \quad (8)$$

This characteristic had not been dealt with before.

3. The Gyarmati Principle and the Prigogine Principle

From equation (1), we can see, if the input of energy (entropy) remains constant, then:

$$G = \int_V (const - \psi) dV = \max$$

This should reduce to a minimum value of ψ :

$$\int_V \psi dV = const - \max = \min \quad (9)$$

This transformation can be shown explicitly by the geometry in figure 1 (Hou, 1987). From this characteristic, Gyarmati (1971) concluded that the Prigogine

Principle was not an independent principle, but only an alternative form of the Gyarmati Principle (Onsager, by Gyarmati) in the stationary case.

However, in our opinion, some concepts need to be distinguished. The Gyarmati Principle, equation (1), considered only the state at some giving moment, though thoroughly, but did not refer to the whole process, whereas the Prigogine Principle referred to the whole irreversible process, though simply. Hence, we can say that the Gyarmati Principle considered only the "cross-sectional" profile of the process, whereas the Prigogine Principle had considered the "longitudinal profile" of the process.

4. Restrictions of the Gyarmati Function

Though some research concerned with the characteristics and the relationship of the Gyarmati Principle to other principles has been carried out since the Gyarmati Principle was established, but little attention was paid to its restrictions (region for its effectiveness). In our opinion, the applicability of the Gyarmati Integral equation (1) has its own range, beyond which its usage become meaningless.

Physically, the input σ and the dissipated part ψ in equation (1) can be bounded in this range. It may be that σ is much greater than ψ , that is,

$$\sigma \gg \psi \quad (10)$$

or, in the giving process, the dissipation part is negligible; that is:

$$\psi \approx 0 \quad (11)$$

In these cases, the Gyarmati Principle should transform into the Hamilton Principle, suitable for the reversible process.

The second limit case is that σ occurs close to the magnitude of ψ , that is,

$$\sigma \approx \psi \quad (12)$$

Another limit case would be impossible, for example, $\sigma < \psi$. Physically, any irreversible process could be maintained only under these conditions: when an energy input was supplied continuously from outside on one hand, and on the other hand, the value of input was sufficiently great to cover entirely the whole dissipation. Otherwise, the giving process could not be sustained.

Any irreversible process could not sustain all the time (and over the course) with such an inequality $\sigma \gg \psi$. In another words, it could not remain in a state far away from the equilibrium state all the time (and over the course). It would evolve spontaneously from a state farther from the equilibrium state to one nearer to the equilibrium state. And finally, it would be closed tangentially to this equilibrium state. In the later case, the energy dissipation rate should take a minimum value, $\psi = \min$. On the other hand, the input of rate of energy would take a minimum value also, if this process should proceed spontaneously and the input of energy should be supplied not from a man-made energy source, but from the process itself.

The input energy of surface water flow is the effective component of the potential energy in a gravitational field. In the upper reach, the input of potential energy would be greater or much greater than that of the dissipated one through the friction, that is, $\sigma > \psi$ or $\sigma \gg \psi$.

However, as the river flowed toward the lower reach, the input of energy (effective component of potential energy) would be dissipated entirely and locally, as for the case of laminar flow. In this limit case, $\sigma \sim \psi$.

Thus, an irreversible process would be established dynamically under two conditions:

$$\psi = \min \quad (13)$$

$$\sigma - \psi = 0 \quad (14)$$

However, if the process should reach that moment, when σ was close to ψ , as shown by equation (12), from equation (1), we would have:

$$G = \int_V (\sigma - \psi) dV = \int_V (-0) dV = 0 \quad (15)$$

Thus, in this case, the function G could not be sustained with a maximum value once more.

Hence, we can conjecture that the Gyarmati Principle would be invalid in the region very close to the equilibrium state. In our case, when the surface water flowed to the lower reach of the river, its flow pattern would follow solely the TMEDR; the input of energy should almost be dissipated directly and wholly. Thus, the mathematical value of G can be changed within this range:

$$0 < G < \infty$$

(16)

Meanwhile, the numerical value of G could be diminished along the time (and course).

5. Natural and Man-Made Irreversible Processes

These two different concepts about the process must be strictly distinguished. Gyarmati (1971) stated that the Prigogine Principle can be considered a special case of the Gyarmati Principle under the stationary state, that is, under the constraint $\sigma = \text{const}$. In this case, the relationship between σ , G , ψ was shown in figure 1. This form of irreversible process can be referred to as the man-made process, for the input of energy could be sustained with a constant value with time only under artificial conditions (as if a constant water head were pumped for maintaining this process). However, such a maneuver could be excluded and the process allowed to proceed spontaneously with time and/or along the course. Most processes in nature belong to this category. An important feature of natural irreversible processes, which should be distinguished from the man-made one, was that for the previous case, not only the energy dissipation rate ψ could be diminished with time (and along the course), but sometimes the input of energy σ could be diminished also. An obvious example is a river in an alluvial bed. As is well known, the surface slope S , and mean velocity u of any river should always diminish along the course. This means that not only should the unit stream power diminish along the course, but the effective component of the potential energy of the river should be diminished along the course simultaneously. This specific feature of the river process is caused by the fact that the water is a dissipative medium, and also a dense fluid. Due to its "dense effect," the elevation of a river should be lowered progressively along the course. Meanwhile, due to its "viscous effect," the energy dissipation rate would be diminished along the course accordingly. How should both of these effects be coupled? This is a very profound problem for which many features remain to be clarified. Nevertheless, this coupling seems to be flexible. It constitutes an unified system, by which not only the mechanical energy of water mass should decrease continuously along the course, but the energy dissipation rate should decrease continuously along the course also.

Figure 1 was presented only for analysis of the relationship between G , ψ , and σ . In the real world, the upward tendency of ψ would be impossible due to the irreversibility of the process; it ought to be diminished monotonically with time. Thus, if the process were a natural one, then instead of figure 1, we would predict the variance of these parameters for man-made processes shown in figure 2. Thus, as with ψ , the value of G would be changed with time. But contrary to ψ , which should attain a maximum value at $t = 0$, the value of G should gain a minimum value at $t = 0$.

The statement above is valid for the case with constant input. For the natural irreversible process, $\sigma \neq \text{const}$, it should change in time also, as for G and ψ . The limitation of σ is that it ought to be equal to the dissipated

part of system, so that the system should sustain not only a stationary, but also an equilibrium condition. Corresponding to this, the value of G should approach zero eventually. Therefore, the natural irreversible process can be expressed by the geometry in figure 3.

In his monograph, Gyarmati (1971) had stated the relationship between ϕ , σ , and G only formally, but did not predict how they should interrelate to each other even for any specific irreversible process. Though the Brussel's school (Glansdorff, 1971) did not predict an explicit relationship between σ , ϕ and G , as was done by Gyarmati, an alternative concept, the so-called "kinetic potential" or "local potential" as a locomotive force of irreversible process, was accepted. However, it is difficult to realize and measure this parameter. In neither the Hungarian nor the Brussel's school was the concrete relationship between input and output of energy in any irreversible process studied.

In our opinion, in any natural spontaneous irreversible process, the input and output could not be isolated from each other, they are mutually interrelated in the evolutionary process of a system. In the limiting case, the input may follow the output, when the process should approach the equilibrium stage. In another words, the energy dissipation within the system should result in the input being diminished along the course (and with time), as with the energy dissipation itself. Any irreversible process attempts to approach and establish its own dynamic equilibrium state. Even this "potential energy" leads the input to be diminished along the course together with the output.

However, in some cases, the output may follow to the input, when the process is far away from the equilibrium stage, as in, for example, the river at an upper reach with a very steep slope and high velocity, or the runoff formed from heavy precipitation on a steep slope of a barren mountain, or the water fall over a weir. In these limiting cases, the friction, inherent in the system, can be neglected and the water process can be approximated to a "reversible" one. They would in turn follow the Hamilton Principle.

A free waterfall can be considered as such a category of surface flow, by which the constraints (friction) have been excluded entirely, so that a surface water flow would flow down with quasi-infinite (vertical!) slope and velocity, even under such a change the water flow could undergo a transformation from the finite rate of energy to the maximum rate.

Those regions, in which the energy dissipation plays a dominant role, can be referred to simply as the "Prigogine region" (or TMEDR region), and that region, where the potential energy plays a dominant role, can be called the Hamilton region. If the giving process should stay in the Prigogine region, then the process should proceed with a decelerated with time (and/or along the course); and if this process should stand in the Hamilton region, then the process becomes an accelerating one. For both of these processes belonged to different category, then intermediate region would exist, in which both principles mentioned should played an equal-dominant role. In another words, there would possibly exist a critical and/or transitional region, beyond which the system ought be followed to HP and PP simultaneously.

This turning point (or turning region) for transformation actually exists in some natural processes, for example, on a certain loess-plateau with serious soil erosion in China, there were not only widely developed rills/gullies, but also, on many slopes some slight foot prints of meanders could be found. This phenomenon can be explained by the following: During intense precipitation, the surface runoff should follow the Hamilton principle; hence the runoff should be accelerated. Due to this effect, an intense soil erosion would follow, and as its byproduct, a rill/gully would be formed. However, as the intensified precipitation diminished gradually, the accelerated surface runoff would be transformed into a decelerated one; it in turn should follow the Prigogine Principle instead of the Hamilton Principle. As its result, the surface runoff would turn into meanders on the slope. To diminish the energy dissipation rate, an outcome would be the formation of a network looking like slight strikes over the whole slope.

6. Evidence of Accelerated and Decelerated Processes

The system of criteria for irreversible processes (processes with friction) had been derived earlier by Prigogine (1963); however, the original formulation of this system was expressed by the entropy production P , for evidence it has been replaced in terms of energy. The system by the Prigogine Principle is shown in table 1:

Table 1

State	Prigogine Principle	Hamilton Principle
Nonequilibrium	$\frac{dE}{dt} > 0$	$\frac{dE}{dt} > 0$
	$\frac{d^2E}{dt^2} > 0$	$\frac{d^2E}{dt^2} > 0$
Equilibrium (Dynamic)	$\frac{dE}{dt} = \min$	$\frac{dE}{dt} = \max$
	$\frac{dE}{dt} = 0$	$\frac{dE}{dt} = \infty$
Equilibrium (Static)		$E = \min$

Accordingly, the system of criteria for reversible processes can be formulated based on equation (1) (Hou, 1989). The last column in table 1 refers to the static equilibrium, which corresponds to the minimum mechanical energy.

The question of the conditions, under which the surface flow becomes an accelerating or decelerating process, remains to be studied. But the question of whether a free-falling body in a gravitational field without consideration of its ambient friction belongs to an accelerating process is addressed by elementary physics:

$$\begin{aligned} V &= V_0 + gt \\ y &= V_0 t + \frac{gt^2}{2} \end{aligned} \quad (17)$$

from which we have:

$$\begin{aligned} E &= \gamma y = \gamma \left(V_0 t + \frac{gt^2}{2} \right) \\ \dot{E} &= \gamma(V_0 + gt) > 0 \\ \ddot{E} &= \gamma g = \rho g^2 > 0 \end{aligned} \quad (18)$$

furthermore, when $t \rightarrow \infty$, then $\dot{E} \rightarrow \infty$.

Whether the irreversible process is a decelerating one for the river process, is quite evident, though the analytical prediction remains to be done.

7. Surface Flow as an Accelerating Flow

If a surface water flows along a long rigid boundary with a small or moderate hydraulic gradient, eventually it should flow uniformly. This is a byproduct of constant rate of dissipated energy, when the dynamic equilibrium has been established, as shown in table 1.

If the surface flow is proceeding along an alluvial bed, then it should form a concave longitudinal profile to follow the Prigogine criteria in the nonequilibrium state (table 1).

Both of these cases belong to the process with friction. Imagine now the friction within a surface flow along a rigid or movable bed can be neglected; the input of energy to the system should be much greater than the output. This case is for a surface flow with a very steep hydraulic gradient. This surface flow should begin to follow the Hamilton Principle. In this case, the surface flow becomes an accelerating one. In the following, an appropriate approach to this flow is presented: At $x = 0$, $V = V_0$; the effective component of gravity should be $g \sin \theta$; then instead of equation (17) for vertical motion, the equation of motion for a single fluid particle along a slope can be written as:

$$\begin{aligned} V &= V_o + g \sin \theta t \\ y &= V_o t + g \sin \theta \frac{t^2}{2} \end{aligned} \quad (19)$$

Let the unit discharge q be constant,

$$q = Vh = V_o h_o = \text{const} \quad (20)$$

where h_o , h are the depths for respective cross sections. In whatever case, both V , y are variable, they are functions of t , or of l ; l = course-length. For:

$$l = Vt, \quad dl = t dV + V dt \quad (21)$$

From equation (19),

$$t = \frac{V - V_o}{g \sin \theta}, \quad dV = g \sin \theta dt$$

then

$$dl = \frac{V - V_o}{g \sin \theta} dV + \frac{V dV}{g \sin \theta} = \frac{2V - V_o}{g \sin \theta} dV \quad (22)$$

if θ , remained a constant value, then:

$$L = \int_0^L dl = \int_{V_o}^V \frac{2V - V_o}{g \sin \theta} dV = \left| \frac{V^2}{g \sin \theta} - \frac{VV_o}{g \sin \theta} \right|_{V_o}^V = \frac{V^2 - VV_o}{g \sin \theta} \quad (23)$$

Substituting equation (20) into equation (23), we get:

$$L = \frac{q^2}{h_o^3 g \sin \theta} \left(\frac{h_o}{h} \right)^2 (h_o - h) = Fr \left(\frac{h_o}{h} \right)^2 (h_o - h) \quad (24)$$

where:

$$Fr = \frac{q^2}{h_o^3 g \sin \theta} \quad (25)$$

But equation (24) should remain valid within this range:

$$h_c < h < h_o \quad (26)$$

where h_c = critical depth, corresponding to the minimum mechanical energy.

Equation (24) represents the waterfall curve along a steep slope when the friction can be neglected. Thus, the depth h should decrease along the course. The mechanical energy should increase along the course also, but this in the form of an analytical expression is quite cumbersome.

Similar to equation (18) for the case of free-falling body, for the present case, we have:

$$\begin{aligned} E &= \gamma y = \gamma \left(V_o t + \frac{g \sin \theta t^2}{2} \right) \\ \dot{E} &= \gamma (V_o + g \sin \theta t) > 0, \text{ when } t > 0 \\ \ddot{E} &= \gamma g \sin \theta > 0 \end{aligned} \quad (27)$$

Thus, the time rate of energy should increase linearly with time, similar to the case of equation (18) for the case of a body falling.

8. Turning Point of Surface Flow

When the surface flow is proceeding along a channel with moderate slope, it would tend to follow the Prigogine Principle. In another words, the rate of energy in the system should be diminished with time (and along the course), as shown by the family of curves "1" in figure 5. However, when this flow proceeds along a channel with a very steep slope, then the rate of energy tends to increase with time (and along the course), as shown by the family of straight lines "2." Obviously, when the Froude number Fr increases in an alluvial reach, then the curve "1" tends upwards as was shown by arrow P in

figure 5. On the other hand, when Fr decreases on a steep slope, then the straight line goes downwards as shown by arrow H in figure 5. Both of these families would meet each other at some point T on the axis $t = 0$. This intersection T can be named as the "turning point" due to its neutrally stable character.

Formally, for T is a function of E when $t = 0$,

$$T = T(E|_{t=0}) \quad (28)$$

and from equation (27) we have:

$$E|_{t=0} = \gamma V_0 \quad (29)$$

then

$$T = \gamma V_0 \quad (30)$$

The turning point exists not only in surface water flow, but also in outer flow. But contrar to the previous case, a body would be statically established at low or zero speed. Their comparison can be shown in table 2:

Table 2

Motion	Speed of motion (rate of energy)	
	High	Low
Surface flow	H	P
Motion of body	P	H

where P,H = Prigogine and Hamilton Principles, respectively.

9. Practical Significance of a Turning Point Study

In the area of soil conservation, this study seems of basic importance: The soil erosion on a slope in a period of heavy precipitation is caused directly by the concentrated surface runoff, and intensive erosion could be caused by these combined conditions:

(1) Barren, long, and steep slopes with easily erodible material for a surface mantle;

(2) Temporary heavy precipitation.

Under these conditions, the runoff at some moment could exceed the value corresponding to the turning point, so that intensified soil erosion would be caused.

An effective solution for soil conservation is a thick sublayer of low vegetation. The main function of a thick vegetation layer on the slope would be that this layer would distribute, disperse and delay the surface runoff so that the flow pattern would drop into the region of the Prigogine Principle. Once the surface runoff fell into this region, the flow would spontaneously diminish its energy dissipation rate, so that the goal of soil conservation were achieved.

The turning point is a critical point. The real flow regime could be in either the subcritical region, or the supercritical region. If the flow pattern should be within the subcritical region, then the flow would be established automatically in time and course. This is even the main goal of soil conservation. However, if the surface flow is within the supercritical region, then the flow would accelerate with time and along the course automatically. In this case, the quantity of released water energy would be increased with time and along the course. This released water energy, increasing in time and space, becomes the energy source of soil erosion on the slope.

Besides the sub or supercritical regions, another factor, which influences the rate and intensity of soil erosion, is the concentration process of surface runoff. Basically, the deeper the surface flow depth, the lower the center of gravity of the water mass, so that the surface flow with shallow depth should trend to join the deeper one. After this concentration, the unit energy of surface runoff would be doubled. Hence, the concentration of surface flow should cause a phenomenon of "stress concentration" on the slope. It leads the surface erosion to become a localized one (rill-or-gully), and to intensify this erosional process along the course.

10. Maximum Entropy and Minimum Dissipation

As mentioned before, in the nineteenth century Clausius predicted that the entropy of the universe would approach a maximum, whereas in recent years, the Prigogine school predicted that the irreversible process would spontaneously minimize its energy dissipation rate with time. Do these concepts conflict with each other? Furthermore, are Gyarmati's maximum and Clausius' maximum the same or different? These problems are of basic importance and need thorough analysis.

In above paragraphs those processes with constant input were being referred as manmade processes. In fact, if the radiation from the sun to our Earth surface can be considered constant, then the compound dissipation process on Earth can be considered an irreversible process under constant input.

The Clausius' maximum was concerned with dissipated energy (heat) from the Earth back to the universe attaining a maximum value. The accumulating process of "useless heat" into the universe ought to be of actual, it could not be avoided. But the Clausius' maximum is maintained in "approaching," whenever the attended onto the Earth solar-radiation has not been changed. Therefore, the Clausius' concept applied only with this unavoidable phenomenon.

The "minimum energy dissipation rate" is concerned with the overall tendency of any individual process; in most of these processes, the state with minimum dissipation rate really exists.

The concept of "minimum energy dissipation rate" ought not be confused with that of "maximum entropy." The former is concerned with the tendencies of individual processes, whereas the latter is concerned with the "accumulation" of these though minimum (greater than zero) but infinitely added waste energy. An infinite accumulation of minimum is equal to maximum!

11. Gyarmati Principle and TMEDR

Figures 2, 3, and 5 are summed up in figure 6 as (a), (b), (c), in which (a) represents the case when the surface flow dropped into the region of the Hamilton Principle. In this case, the rate of input energy should increase with time and/or along the course. Case (c) represents that case when the surface flow dropped into the region guided by the Prigogine Principle. In this case, not only should the output gradually decrease in time (and the course), but also the input would be forced to decrease. Case (b) is the case of constant input.

From figure 6, we can see that for any case, the function G does not represent an independent variable, but one related to the dissipated energy. Furthermore, G would approach a maximum only for the '(a)' and '(b)' cases, when the process has reached the equilibrium state. There are some distinguishing features: For case (a), the value of G still would increase with time, even though the dissipation rate of energy has attained a constant value already. For case (b), the value of G should approach not only a maximum, but a constant value also. For case (c), the Gyarmati Principle could not remain valid, for in this case, G should approach zero, but not a maximum.

Basically, any irreversible process would approach the state with minimum energy dissipation rate, irrespective of the case, whether the input should either increase, decrease or remain a constant with time. But the GP should remain valid only with constant input.

But the most important difference between the Gyarmati Principle to the TMEDR is that the Gyarmati Principle was concerned only with the "lateral profile" of the giving process, that is, the relationship between relevant parameters, when the equilibrium state of the open or closed system has been established already; whereas the TMEDR was concerned with its "longitudinal" profile. In another words, the TMEDR deals with the transition from the nonequilibrium state to the equilibrium state.

12. Further Relationship of the TMEDR to the Prigogine Principle

Both of these principles are basically the same, but with some differences:

(1) The criteria of the PP have been expressed only in general form, whereas for the TMEDR, the criteria for open channel flow have been expressed in a concrete form. (2) Prigogine predicted that when the system should stay in a nonequilibrium state but very close to an equilibrium state, then it would approach this state with decreasing entropy production, whereas the TMEDR referred to the system under an equilibrium condition only, though Yang (1986) had indicated: "If a system is not at dynamic equilibrium, its rate of energy dissipation is not at its minimum value. However, the system will adjust itself in such a manner that the rate of energy dissipation can be reduced to a minimum value and regain equilibrium.", but this statement was qualitative only.

In the common case, when the system was in a nonequilibrium condition, the input of energy needs not equal to the output; the input would be greater than the output. But if the system of a natural process had evolved spontaneously into the equilibrium state, then the input and output would approach the output. At this moment, the input and output should coincide with each other, similar to the laminar flow (for laminar flow, the input of energy should be dissipated directly and wholly into heat through the effect of viscosity to the velocity distribution).

13. Concluding Remarks

Water is a viscous medium, so that water flow should unavoidably be accompanied by energy dissipation, which should become heat and transfer back into the surroundings. Hence, the water flow belongs to a thermal system, and it can be analyzed on background of thermodynamics.

However, classic thermodynamics is concerned with isolated systems with "static" equilibrium only, so that it could not be used for explaining water flow phenomena.

Though the nonequilibrium thermodynamics have been concerned with closed and open systems before, so that they became an useful tool for studying the water process. However, the main attention in this field before had been paid to the dissipative structure of the system (Prigogine, 1983; Glansdorff, 1971), that is, to its external form of expression, its mathematical description, etc., and not to the input and output of the energy of the system. Perhaps Gyarmati and his school have primarily attacked this important problem, though many problems remain to be studied, especially in the area of surface water flow.

In this paper, only a qualitative analysis for surface flow is presented. Some of predictions remain for further experimental verification.

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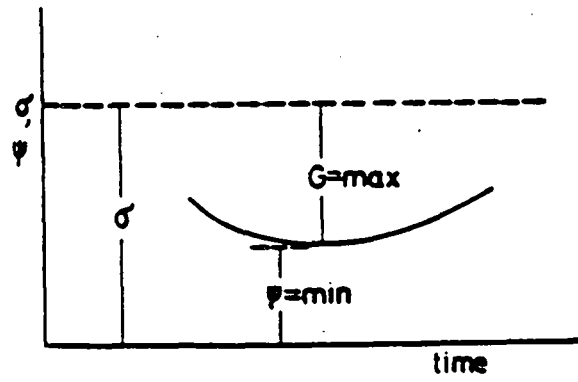


Figure 1. - Relationship between G , σ , and ψ .

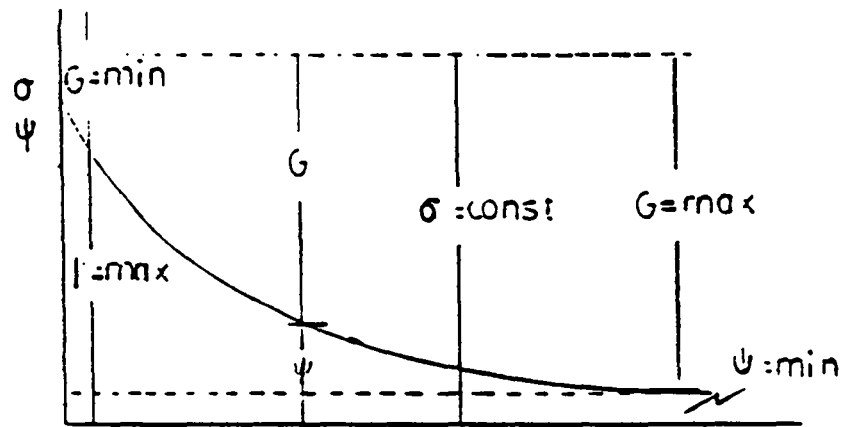


Figure 2. - Process under constant input ($\sigma = \text{const}$).

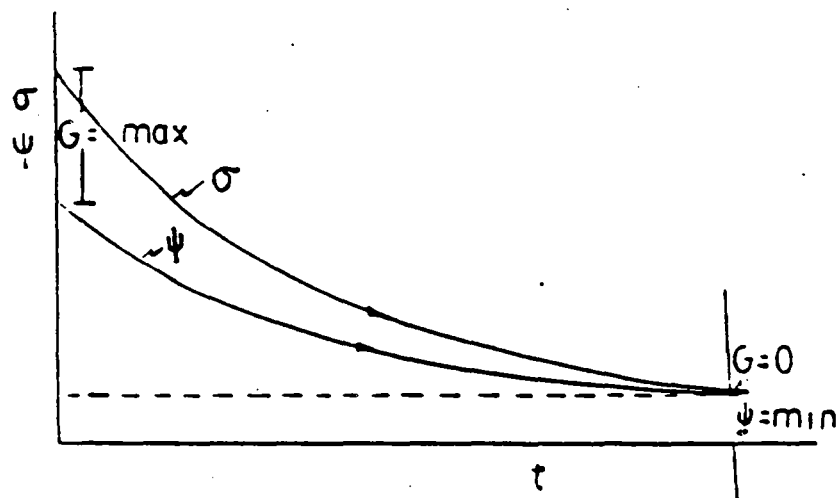


Figure 3. - Natural process under nonconstant input ($\sigma \neq \text{const}$)

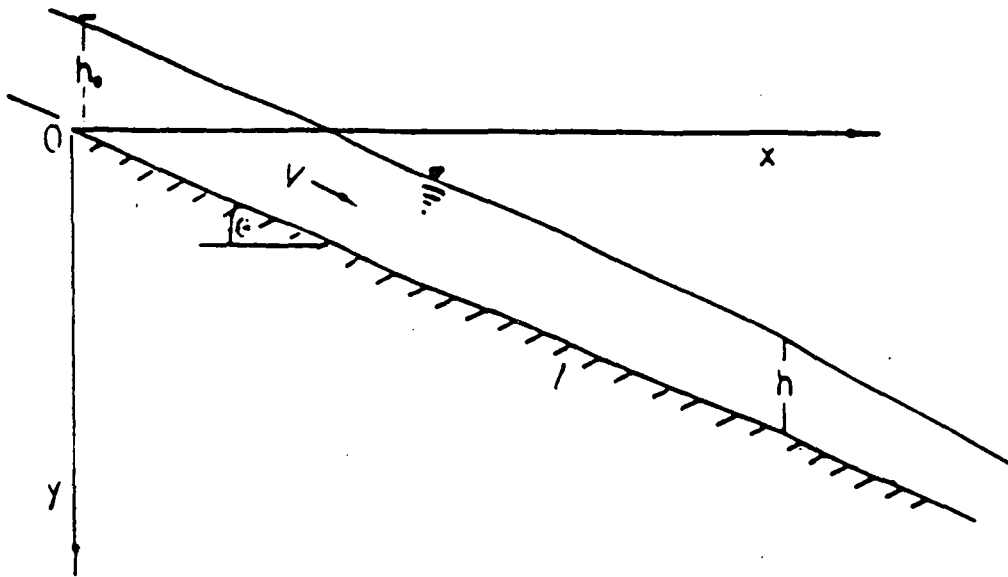


Figure 4. - Dropping-surface curve on a steep slope.

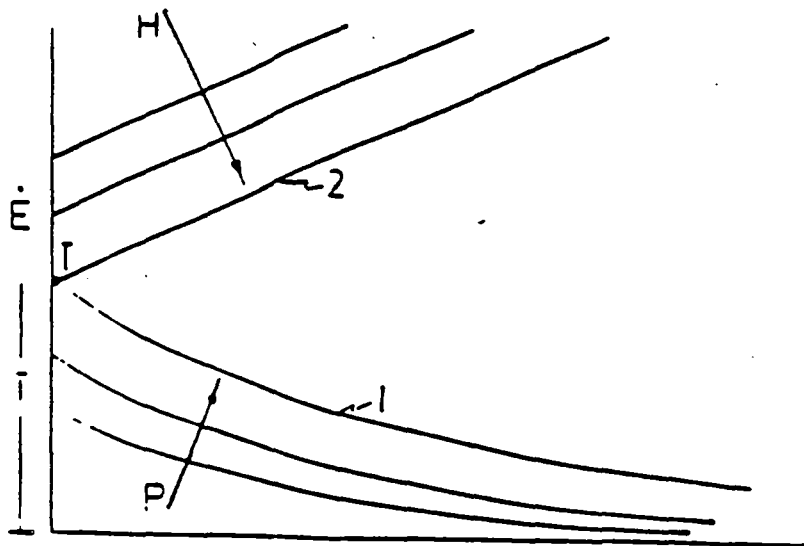


Figure 5. - Determination of turning point; '1' curves follow PP; '2' straight lines follow HP.

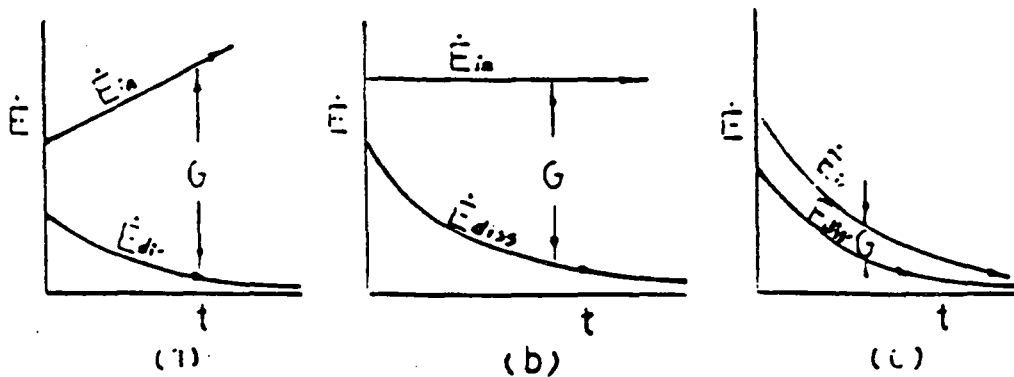


Figure 6. - Different conditions of input for water flow: (a) Flow follows the Hamilton Principle (steep slope); (b) Flow with constant input; (c) Flow follows the Prigogine Principle (alluvial bed).

VELOCITY DISTRIBUTION AND MINIMUM ENERGY
DISSIPATION RATE
(Report No. II)

by

H. C. Hou

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HISTORICAL ACCOUNT

Velocity distribution for both pipe and open channel water flow is a classic problem in fluid mechanics, for which many experimental results have been accumulated (Nikuradse, 1932; 1933; Keulegan, 1938; Vanoni, 1946). Comparison of those experiments to the classic theories showed that the Prandtl-Karman's logarithm law seemed to fit the experiments very well in the core region, but some deviation still occurred at the boundary. Van Driest (1956) revised this classic model to include the viscous effect at the boundary, but the basic feature of the "mixing-length theory" was not changed.

The study of velocity distribution has gone on for a long time, but concerning its physical modeling, great achievements have not been attained. Perhaps, prior to the 1960's, the problems of velocity distribution still had not been related to the energy and energy dissipation rate (Malkus, 1956; Song, Yang, 1979).

In recent years, even though the concept that velocity distribution must relate to energy dissipation and entropy has been commonly accepted (Song, Yang, 1979, 1986; Hou, 1987, Chiu, 1989, 1990), many problems remain to be studied.

This paper is devoted to analysis of the following problems:

- (1) Relation between energy dissipation and velocity distribution
- (2) How the energy dissipation rate should distribute along the cross section
- (3) The difference between the energy dissipation for laminar velocity distribution and for turbulent flow
- (4) Regions governed by different laws (i.e., the region for laminar flow, the region for semilog turbulent flow, and the region for accelerating high speed flow)
- (5) How the velocity distribution should relate to the longitudinal profile of surface water flow
- (6) Experimental evidence and analysis
- (7) Practical application

LOG VELOCITY DISTRIBUTION AND ENERGY DISSIPATION

Though in recent times velocity distribution has been related to the energy-dissipation rate, many aspects still need to be clarified.

Although turbulent velocity distribution is referred to as the fluctuation of a fluid mass, and the fluctuation of individual particles seems to be quite random, as predicted by Prandtl/Karman's model, in our opinion, the overall

tendency of fluctuation could not be "random" and "spontaneous". It is guided by mechanical and thermodynamic laws.

Water is a dense fluid on one hand; on the other hand, it is also a viscous fluid. Because it is dense, water flow must follow the Hamilton principle (HP); because it is viscous, it ought to follow the Prigogine principle (PP), or the theory of minimum energy dissipation rate (TMEDR). In some limited cases, the water flow may follow only one of these principles. For example, when water flows on a steep slope, the viscous effect becomes second order, and the water flow is guided basically by the HP. But when the surface flow is proceeding along a flume with a very small hydraulic gradient, then its dense effect can be neglected, and the water flow follows solely the PP or TMEDR. But in the common case, for the surface flow in the intermediate hydraulic condition, both of these principles ought to guide the water flow simultaneously; the dense effect and the viscous effect ought to be coupled. The coupling phenomenon leads the problem of water flow to become quite cumbersome. But first of all, water flow guided by the PP or HP ought to be strictly distinguished: the laws for them occur quite different.

If the surface water flow is located in the region governed by the PP, it eventually establishes the state with minimum energy dissipation rate (irrespective of whether the boundary is rigid or erodible). Once the water flow has attained this state, the rate could not be further lowered (for it could not be less than "min"), nor increased (because of the irreversibility of the process). The only possible outcome ought to be that this minimum rate of dissipated energy should remain the same value along the course. In another words, if the surface water flow should drop into the region governed by the PP, then the energy dissipation rate would remain at a constant value along the course. In this case, the water flow should stand in the equilibrium state. The constant energy dissipation rate should become uniform flow pattern, such that:

- (1) The hydraulic gradient should be unchanged along the course and parallel to the flume boundary slope.

- (2) The hydraulic elements, mean depth, and velocity should be unchanged along the course.

- (3) Accordingly, the form of velocity distribution should be unchanged along the course.

Thus, the semilog velocity distribution should satisfy the requirement for minimizing the energy dissipation rate.

For the Prandtl/Karman's semilog law for the core, together with the linear law for the laminar sublayer occurred fit well with the real distribution, then in following a thorough analysis of the dissipated energy along the cross section will be analyzed upon this type of velocity distribution.

ENERGY DISSIPATION IN UNIFORM TURBULENT FLOW

As was mentioned, the whole cross section can be divided into two regions, that is, the core and the laminar sublayer. Corresponding to this, the dissipation can also be divided into two parts.

It needs to be emphasized that the energy dissipation picture for turbulent flow and that for laminar flow are quite different. For laminar flow, the energy should be dissipated wholly and directly through its velocity distribution; whereas, for turbulent flow, only a part of energy should be dissipated directly through its time-mean velocity distribution (see Hinze, 1975). In any case, the form of velocity distribution still can be used for conjecturing the physical picture concerned with the energy dissipation.

(1) Dissipation in Laminar Sublayer. - Because in this layer the energy is dissipated entirely through the Newtonian viscous effect, the rate of dissipated energy can be decomposed into two parts:

$$\dot{E}_{diss} = \int_0^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \int_0^\delta \mu \left(\frac{\partial u}{\partial y} \right)^2 dy + \int_\delta^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \dot{E}_{diss,1} + \dot{E}_{diss,2} \quad (1)$$

In the sublayer region, the velocity distribution follows to the linear law:

$$u^+ = y^+, \quad u = u_*^2 y / \nu$$

then

$$\frac{\partial u}{\partial y} = \frac{u_*^2}{\nu}, \quad \left(\frac{\partial u}{\partial y} \right)^2 = \frac{u_*^4}{\nu^2} \quad (2)$$

Substituting (2) into (1), the first term on the right-hand side, we get:

$$\int_0^\delta \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \int_0^\delta \mu \frac{u_*^4}{\nu^2} dy = \frac{\mu u_*^4 \delta}{\nu^2} = \frac{\mu u_*^3}{\nu} \frac{u_* \delta}{\nu} = \rho \delta \cdot u_*^3 \quad (3)$$

where u^* = shear velocity and $\gamma = \mu/\rho$

$$\delta^+ = \frac{u_* \delta}{v} = 11.6 \quad (4)$$

then

$$\dot{E}_{diss,1} = \int_0^\delta \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = 11.6 \rho u_*^3 \quad (5)$$

(2) Dissipation in the Core. - Energy is dissipated partly through the Newtonian viscous effect. The velocity distribution follows the semilog law, that is,

$$u^+ = A \ln y^+ + B, \quad u = u_* \left(A \ln \frac{y u_*}{v} + B \right) \quad (6)$$

where A,B remained to be determined by experiments, but for a first approximation, let $A = 2.5$, $B = 5.5$. For:

$$y^+ = \frac{u_* y}{v}, \quad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial y^+} \frac{\partial y^+}{\partial y}, \quad \frac{\partial u}{\partial y^+} = \frac{A u_*}{y^+}, \quad \frac{\partial y^+}{\partial y} = \frac{u_*}{v},$$

then

$$\frac{\partial u}{\partial y} = \frac{A u_*^2}{v y^+}, \quad \left(\frac{\partial u}{\partial y} \right)^2 = \frac{A^2 u_*^4}{v^2 y^{+2}} \quad (7)$$

Substituting equation (7) into the second term of right-hand side of equation (1), we get:

$$\begin{aligned}\dot{E}_{diss,2} &= \int_{\delta}^h \mu \left(\frac{A^2 u_*^4}{v^2 y^{*2}} \right) dy = \frac{v}{u_*} \int_{\delta}^h \mu \left(\frac{A^2 u_*^4}{y^{*2} v^2} \right) dy^* = \rho A^2 u_*^3 \left(-\frac{1}{y^*} \right) \Big|_{\delta^*}^{h^*} \\ &= \rho A^2 u_*^3 \left(\frac{1}{\delta^*} - \frac{1}{h^*} \right)\end{aligned}\quad (8)$$

If the depth h is much greater than δ , then $h^* \gg \delta^*$, and equation (8) can be approximated as (if $A = 2.5$, $= 11.6$):

$$\dot{E}_{diss,2} = \rho A^2 u_*^3 \left(\frac{1}{\delta^*} \right) = 0.54 \rho u_*^3 \quad (9)$$

Comparing equation (5) to equation (9) we can see that the rate of energy, dissipated at the boundary, occurs much higher than that in the core:

$$\frac{\dot{E}_{diss,1}}{\dot{E}_{diss,2}} = \frac{11.6 \rho u_*^3}{0.54 \rho u_*^3} = 21 \quad (10)$$

Therefore, most of the energy of the surface water flow was dissipated in the region near the boundary.

The numerical result of equation (1) is:

$$\dot{E}_{diss} = \dot{E}_{diss,1} + \dot{E}_{diss,2} = 11.6 \rho u_*^3 + 0.54 \rho u_*^3 = 12.14 \rho u_*^3 \quad (11)$$

For surface water flow, the shear-velocity is measured by the depth h and the hydraulic gradient S :

$$u_* = \sqrt{ghS} = \sqrt{\tau_o/\rho} , \quad \tau_o = \gamma hS \quad (12)$$

where τ_o is the shear stress at the boundary, then:

$$\dot{E}_{diss} = 12.14 \rho (ghS)^{3/2} \quad (13)$$

Therefore, if h, S are given, then the rate of dissipated energy can be calculated from equation (13).

Because the core constitutes most of the depth, and the amount of dissipated energy in it should constitute only a small part of the total dissipated energy, it can be conjectured that the semilog distribution is the most energy-saving distribution. In another words, the semilog distribution is a distribution for minimum energy dissipation rate (because most of the energy is dissipated in a thin layer at the channel bed, not in the core). This does not mean that the rate of dissipated energy in the laminar sublayer should exhibit a "maximum" value. This is only the comparison of the rate between the core and the laminar sublayer. If we consider solely the energy dissipation in the laminar sublayer, then the dissipation rate for the linear velocity distribution is still a distribution with minimum dissipation rate comparative to any other form of velocity distribution (Hou, 1987). Therefore, the dissipation of energy in the laminar sublayer is still a minimum rate of dissipation but under its own constraint condition.

Thus, we can conjecture that the total energy dissipation rate of the core plus the laminar sublayer exhibits a minimum value.

A traditional concept about open channel flow was: At the boundary, the viscous effect must be included, and far from the boundary, the viscous effect could be neglected, as if in the core, the flow became ideal fluid flow in whatever case. But the above analysis indicated that the formation of a semilog velocity distribution did not mean that the flow in the core was ideal, but of quasiminimum (greater than zero!) energy dissipation rate.

(2) Calculation of input. - The above conclusion should remain valid only for energy dissipation, that is, the output of surface flow. It would be incorrect to predict that if the input can be divided into the laminar sublayer part and the core part, then the laminar sublayer part should constitute the majority of the input. On the contrary, the input attributed to the core should constitute a majority in whatever case. This conjecture can be verified in following: The input (called "unit stream power" by Yang, 1976) can be divided into two parts:

$$\dot{E}_{in} = \dot{E}_{in,1} + \dot{E}_{in,2} = \int_0^{\delta} (\gamma u S) dy + \int_{\delta}^z (\gamma u S) dy \quad (14)$$

For

$$u^* = y^* , u = u_* y^* = \frac{u_*^2 y}{v} \quad (15)$$

Substituting (15) into (14), the first term becomes:

$$\begin{aligned} \dot{E}_{in,1} &= \gamma S \int_0^{\delta} \frac{u_*^2 y}{v} dy = \frac{\gamma S u_*^2}{v} \left(\frac{y^2}{2} \right) \Big|_0^{\delta} = \frac{\gamma S u_*^2 \delta^2}{2v} = \frac{\gamma v S}{2} \left(\frac{\delta u_*}{v} \right)^2 = \\ &= \frac{S \gamma v}{2} \delta^{*2} = 67.28 \gamma v S \end{aligned} \quad (16)$$

In the core,

$$u^* = A \ln y^* + B, u = u_* (A \ln y^* + B) \quad (17)$$

Substituting (17) into the second term of equation (14), we get:

$$\begin{aligned} \dot{E}_{in,2} &= \int_{\delta}^h \gamma u_* (A \ln y^* + B) S dy = \gamma S \int_{\delta}^h u_* (A \ln y^* + B) d \frac{u_* y}{v} \frac{v}{u_*} = \\ &= \gamma v S \int_{\delta}^h (A \ln y^* + B) dy^* = \gamma v S [A(y^* \ln y^* - y^*) + B y^*] \Big|_{\delta}^h = \\ &= \gamma v S A(h^* \ln h^* - h^* - \delta^* \ln \delta^* + \delta^*) + B(h^* - \delta^*) \end{aligned} \quad (18)$$

Because h^* should be much greater than δ^* the value in brackets would be much greater than '67.28' in equation (16).

ENERGY DISSIPATION IN LAMINAR FLOW

Proceeding now to the question: Whether the dissipation rate for the laminar regime would be greater than that for the turbulent one under the same hydraulic conditions? The criterion

$$\gamma u S = \min \quad (19)$$

itself remains valid both for laminar and turbulent flow, though for different flow patterns its own value of 'min' would be different one from another. Furthermore, the turbulence can be considered as a fluctuation of the laminar flow. It ought to further diminish the energy dissipation rate. Thus, we have:

$$(\gamma u S)_l > (\gamma u S)_t \quad (20)$$

which, under the same hydraulic condition, that is, $S, q (=uh)$, should remain unchanged for both cases.

For laminar flow in an open channel, as is well known, the velocity distribution should follow to the parabolic law:

$$u = \frac{gS}{\nu} \left(yh - \frac{y^2}{2} \right) \quad (21)$$

then

$$\frac{\partial u}{\partial y} = \frac{gS}{\nu} (h - y), \quad \left(\frac{\partial u}{\partial y} \right)^2 = \frac{g^2 S^2}{\nu^2} (h - y)^2 \quad (22)$$

The dissipated energy over the whole depth can be expressed by:

$$(\dot{E}_{diss})_l = \int_0^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \int_0^h \mu \frac{g^2 S^2}{\nu^2} (h - y)^2 dy = \rho h^3 u_*^3 / 3 \quad (23)$$

In the common case, $h^+ \gg \delta^+ = 11.6$. For example, if $h = 50$ cm, and $\delta \sim 1$ mm, then $h^+ = 500$, $\delta^+ \approx 5,000$, and the energy is occurs much higher than $(\dot{E}_{diss})_t$, presented by equation (11). The conclusion is: The surface flow ought to be transformed from the laminar regime to the turbulent one under the same hydraulic conditions with the target to diminish its energy dissipation rate!

ENERGY BALANCE OF SURFACE WATER FLOW

(1) Laminar flow. - The general energy-balance equation for laminar or turbulent flow can be derived from the Navier-Stokes Equation or Reynolds Equation. It has been thoroughly stated in many monographs on fluid mechanics

(Hinze, 1975). However, this general statement (without concerning the concrete velocity distribution and concrete boundary condition) is quite insufficient for real surface flow. Furthermore, we have predicted that for laminar flow, the input of energy should be dissipated entirely and directly through the velocity field, but for turbulent flow, the input should be dissipated only partly and directly through the time-mean velocity field. Proceeding now to the first problem. The input of the rate of energy to open-channel laminar flow for a unit width is the effective part of the potential energy:

$$\dot{E}_{in, \epsilon} = \int_0^h \gamma u S \, dy = \gamma S \int_0^h u \, dy \quad (24)$$

Substituting (21) into (24), we get:

$$\dot{E}_{in} = \gamma S \int_0^h \frac{gS}{\nu} \left(yh - \frac{y^2}{2} \right) dy = \frac{\rho^2 g^2 h^3 S^2}{3\mu} \quad (25)$$

On the other hand, the rate of dissipated energy can be calculated by this expression:

$$\dot{E}_{diss} = \int_0^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy \quad (26)$$

Substituting (22) into (26), we get the same result as equation (25):

$$\int_0^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \frac{\gamma^2 S^2 h^3}{3\mu} \quad (27)$$

This is an alternative expression of equation (23). Therefore, for the laminar flow we have:

$$\dot{E}_{in} = \dot{E}_{diss} \quad (28)$$

Besides this, parabolic distribution of laminar flow is flow at the minimum energy dissipation rate. This outcome has already been obtained by Song, Yang (1979).

(2) Turbulent flow. - The input and output (dissipated) in the core and laminar sublayer has been calculated as equations (5), (8), (16), and (18), as shown in the following table:

Region	Input	Output (dissipated)
Core	$[2.5h^*(\ln h^*-1)+5.5h^*]\gamma\nu S$	$0.54 \rho u_*^3$
Laminar Sublayer	$67.28 \gamma\nu S$	$11.6 \rho u_*^3$
Sum		$12.14 \rho u_*^3$

Without giving h^* , u_* a quantitative estimate of the input/output seems impossible. However, the energy dissipated mostly at the boundary region should be compensated by the energy taken from the core. In any case, the input and output for turbulent flow shown in the table could not equal each other; the input should always be greater than the output, because some part of the energy is transferred into turbulence generation, dissipated in sediment transport, etc.

RELATION OF VELOCITY DISTRIBUTION TO UNIT STREAM POWER

The unit stream power $\gamma u S$, or simply $u S$, as an indicator for the stability of an alluvial river, was introduced by Yang (1976). Here u was the cross-sectional mean velocity. Now we can see from equations (16) and (18), that with different velocities, the concrete analytical expression for the unit stream power would be different.

An important question faced is: If the unit stream power is referred to the input of surface flow, then, whether the minimization is referred to the input or output? Strictly speaking, the minimization must be related to the dissipated energy, that is, the output. However, for alluvial river flow (or uniform open channel flow with a rigid boundary), where the equilibrium state was established, the flow pattern should approach a uniform one. The input of a river ought to be proportional to the output (though not equal to each other for turbulent flow). In this case, the idea that the minimization was directly related to the input, ought to be approximately correct (see fig. 3 of paper: Hou, H. C., "Gyarmati Principle and Theory of Minimum Energy Dissipation Rate").

EXPERIMENTS ON VELOCITY DISTRIBUTION IN ACCELERATING FLOW

So far, only the velocity distribution under equilibrium conditions has been considered. For open channel flow, this type of velocity distribution (semilog) would exist in a flume with a moderate or small hydraulic gradient. In another words, semilog velocity distribution for open channel flow would exist only in the region where the water flow is guided mainly by the Prigogine principle.

A question immediately arises: Where should the boundary of this region be, within which the surface flow would be guided by the PP, and beyond which the PP has no force? Unfortunately, there were no experimental results on this. Obviously, the following three variables and their combinations should influence this transformation:

- (1) Hydraulic gradient (i.e., channel bed slope)
- (2) Discharge
- (3) Roughness of bed

A surface flow along a steep slope with a smooth boundary and a large discharge would be out of the region for the PP. But with a very rough boundary and a small discharge, flow would remain in the region of the PP. Recent experiments on a very steep slope had revealed some characteristics of this accelerating flow. Figures 1-3 show the velocity distribution on stepped spillways of Beaver Run Dam. The slope of spillways is 1:2; the height of each step is 2 ft. From these figures some important characteristics of accelerating flow can be revealed in following:

(1) In the core, the velocities turn to uniform distribute; velocity-gradient disappeared entirely. This means that in this region there was no energy dissipation in this case. It indicates that as the Froude number increases sharply, $Fr > 1$, the "minimum energy dissipation rate" turns to "zero energy dissipation rate". "Zero" is minimum among all the minimums!

Because there was no (or very little) energy dissipation in the core, the entire input should have been dissipated in the laminar or turbulent boundary layer.

(2) According to the requirement of the acceleration along the course, the depth should decrease along the course. Meanwhile, the surface velocity should increase along the course.

In general, the generation of accelerating flow was such that the input of energy rate was so large that it could not be dissipated entirely locally by conventional measures, resulting in energy accumulating. If this input of energy could be dissipated entirely and locally, so that energy could not accumulate, the surface flow would reach the equilibrium state, so that a uniform flow pattern would be sustained. In the above experiments, this transformation was observed under the conditions of a small discharge and a shallow water depth.

APPROXIMATE MATHEMATICAL MODELING AND VERIFICATION

(1) Ideal fluid case. - In our previous paper (Gyarmati Principle and Theory of Minimum Energy Dissipation Rate) a model of accelerating flow for ideal fluid (i.e., where the friction can be neglected), by which the local velocity can be represented in this form:

$$L = \frac{U^2 - UU_o}{g \sin\theta}, \text{ or } U^2 - UU_o - Lg\sin\theta = 0 \quad (29)$$

where U_o = velocity at $L=0$. The distance is measured from $L=0$ downwards parallel to the flow distance along the steep slope. From equation (29), we get:

$$U = [U_o \pm \sqrt{U_o^2 + 4Lg\sin\theta}]/2 \quad (30)$$

in which the root with negative sign has no physical meaning (The negative sign represents decelerating flow but not accelerating one.). Therefore, we have finally:

$$U = [U_o + \sqrt{U_o^2 + 4Lg\sin\theta}]/2$$

or in nondimensional form:

$$\bar{L} = [1 + \sqrt{1 + 4\bar{L}\sin\theta}]/2 \quad (31)$$

in which:

$$\bar{U} = U/U_o, \quad \bar{L} = Lg/U_o^2 \quad (32)$$

Data from figures 1-3 are used for verification. Since in these experiments $\sin\theta = 1/\sqrt{5}$, (31) can be simplified to:

$$\bar{U} = [1 + \sqrt{1 + 1.788\bar{L}}]/2 \quad (33)$$

The numerical results of equation (33) are shown as a solid curve in figure 4, in which some experimental results of surface flow on a steep slope from figures 1-3 are also shown as points.

(2) Verification of velocity distribution. - From figures 1-3, velocity distribution for accelerating flow can be divided into two parts, that is, the core part and the sublayer part. The core part is nondissipative flow. It is characterized by constant vertical velocity:

$$u = \text{const, or } u^* = \text{const} \quad (34)$$

Assume that in the sublayer region the velocity distribution conforms to linear law as for the equilibrium case. Thus, when $0 < y < \delta$, we have:

$$u = \alpha y, \text{ or } u^* = y^* \quad (35)$$

But in this case, the thickness of the laminar/turbulent sublayer could not remain constant; it would increase with the increase of velocity in the core, as shown in figures 1-3. The position of the edge of this layer ought to be constrained by the unit discharge q . By the definition of mean velocity:

$$u = \frac{1}{h} \int_0^h u dy \quad (36)$$

Since the unit discharge must be a constant value:

$$q = \bar{u}h = \text{const} \quad (37)$$

Therefore, the area bounded by the velocity distribution curve and the depth axis ought to maintain a constant value:

$$\int_0^h u \, dy = \text{const} \quad (38)$$

or

$$\int_0^\delta u \, dy + \int_\delta^h u \, dy = q = \text{const} \quad (39)$$

Since in the core the velocity is uniformly distributed:

$$\int_\delta^h u \, dy = u_c \int_\delta^h dy = u_c (h - \delta) \quad (40)$$

where u_c = velocity in the core; both h, δ are functions of distance.

$$\int_0^\delta u \, dy = \frac{y^2 \alpha}{2} = \frac{\alpha \delta^2}{2} \quad (41)$$

Substituting (40), (41) into (39), we get:

$$\frac{\alpha \delta^2}{2} + u_c (h - \delta) = q$$

or

$$\alpha \delta^2 - 2\delta u_c + (hu_c - q) = 0 \quad (42)$$

from which we have:

$$\delta = u_c \pm \sqrt{u_c^2 - \alpha (hu_c - q)} \quad (43)$$

Equation (42) or (43) can be considered as the equation of continuity for the present case.

Another equation is the equation of motion. From the Hamilton principle (see the table) of our previous paper:

$$\frac{d^2 E}{dt^2} = u^2 \frac{d^2 E}{dx^2} = u^2 \frac{d}{dx} \left(\frac{dE}{dx} \right) > 0, \quad \frac{dE}{dt} = u \frac{dE}{dx} > 0 \quad (44)$$

For nonuniform surface flow, if its friction must be included, then the alternative forms for equation (44) can be written in this form:

$$u \frac{d}{dx} \left(\gamma u S - \tau_b \frac{du}{dy} \right) > 0, \quad \gamma u S - \tau_b \frac{du}{dy} > 0 \quad (45)$$

Assuming this inequality can be replaced by a linear relationship:

$$u \frac{d}{dx} \left(\gamma u S - \tau_b \frac{du}{dy} \right) = A \left(\gamma u S - \tau_b \frac{du}{dy} \right) \quad (46)$$

where A is a constant. For turbulent boundary layer flow, let:

$$\tau_b = e \frac{du}{dy} \quad (47)$$

ϵ is the eddy viscosity coefficient at the boundary region:

$$u \frac{d}{dx} \left[\gamma u S - e \left(\frac{du}{dy} \right)^2 \right] + A \left(\gamma u S - \tau_b \frac{du}{dy} \right) \quad (48)$$

but from equation (35),

$$\left(\frac{du}{dy} \right) \sim \alpha, \quad \left(\frac{du}{dy} \right)^2 \sim \alpha^2$$

Furthermore, for surface flow with a rigid boundary, u and S are not independent; they are related through the Chezy formula:

$$u = \sqrt{\frac{f}{2} g} \sqrt{hS} \quad (49)$$

If the change of h can be neglected, then:

$$S = ku^2 \quad (50)$$

$k = \text{constant}$. Thus, equation (48) can be replaced by:

$$\frac{d}{dx} (\gamma ku^4 - e\alpha^2 u) = A (\gamma ku^3 - \alpha^2)$$

resulting in

$$\int \frac{(4\gamma ku^3 - e\alpha^2)}{\gamma ku^3 - \alpha^2} du = Ax + \text{const.}$$

ON A RATIONAL PRINCIPLE FOR SPILLWAY DESIGN

The classic concept concerned with the hydraulic design of spillways was restricted to its streamlined body with the objective of nonseparation to prevent cavitation and increase discharge capacity. In the common case, spillway flow is an accelerating flow, tending to increase overflow. The accelerating process would be strengthened simultaneously, producing an energy-accumulating process. This phenomena would make dissipation of energy at the downstream end of a hydraulic structure difficult, especially for a structure with a high head. Therefore, some conflict exists in design, and this classic approach could not entirely satisfy the engineering requirements.

Perhaps, an optional design for this type of structure would simultaneously address these constraints:

- (1) To maximally increase the discharge capacity

(2) To restrict or to obstruct the accelerating process, i.e., the energy-accumulating process.

To satisfy these requirements, the energy of overflow needs to be dissipated not entirely downstream, but partly, along the route to the downstream end before the energy has a change to accumulate. In recent years, stepped spillways have been used widely (Essery, 1971; Rajaratnam, 1990). These types of spillways could satisfy the requirements, although further studies are still needed.

CONCLUSION AND DISCUSSION

(1) From the standpoint of energy dissipation, surface flow can be divided into two categories, uniform surface flow and accelerating nonuniform flow. The formation of uniform flow means that the flow is following the Prigogine principle, and that it has attained the equilibrium state.

(2) Because the energy dissipation rate of surface laminar flow is greater than that of surface turbulent flow under the same hydraulic conditions, according to the TMEDR, the surface laminar flow should certainly be transformed into surface turbulent flow.

(3) Semilog velocity distribution in the core not only is the outcome of the mixing process in a turbulent mass, as predicted by the classic models, but it also represents a distribution by which the surface flow should gain a very low energy dissipation rate.

(4) Uniform surface flow represents flow at an equilibrium state. Not all the surface water flow could attain uniform flow, only the flow in the region guided by the PP. Because the surface water flow in this region should maintain its (minimum) energy dissipation rate along the course, a uniform flow pattern would be formed in this reach.

(5) Surface accelerating flow is that flow guided by the HP and for which the energy dissipation rate can be neglected. Energy accumulates for this flow so that it needs to be seriously considered in design of hydraulic structures.

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RELATION BETWEEN COEFFICIENTS α , β , AND VELOCITY DISTRIBUTION

(Report No. III)

by

H. C. Hou

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Acknowledgements

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1. Introduction

Classic hydraulics studied water flow problems usually by using a one-dimensional approach to the equation of motion. However, due to the viscous effect, the velocity is often nonuniformly distributed; at the boundary it approaches zero. Due to this nonuniform effect two coefficients α , β were introduced for correction early in last century. Coriolis introduced the energy coefficient α which was defined in this mathematical form:

$$\alpha = \frac{\int u^3 dA}{u_m^3 A} \quad (1)$$

where A is cross-sectional area; u_m is mean velocity; u is local velocity; α represents "the effect" of nonuniform velocity distribution at a channel cross-section on the kinetic energy of the flow" (Watts, 1967). Eq. (1) is valid for three-dimensional flow. For two-dimensional flow (flow with very great width), Eq.(1) can be simplified to the problem for unit width as:

$$\alpha = \frac{\int u^3 dy}{u_m^3 h} \quad (1a)$$

Another coefficient β was introduced by Bousinesq. The latter coefficient can be expressed in this form:

$$\beta = \frac{\int u^2 dy}{u_m^2 h} \quad (2)$$

β represents "the effect of the nonuniform velocity distribution at a cross section on the momentum flux of the flow".

Similar to (1a), for two-dimensional flow, Eq.(2) is simplified to:

$$\beta = \frac{\int u^2 dy}{u_m^2 h} \quad (2a)$$

For a long time, α , β could be determined numerically only by experiments; they fluctuate within a large range.

Chow (1959) firstly attempted solve this problem analytically. He obtained the expressions α , β in these forms:

$$\alpha = 1 + 3e^2 - 2e^3 \quad (3)$$

$$\beta = 1 + e^2 \quad (4)$$

where,

$$e = \frac{u_*}{u_m} = \frac{u_{\max}}{u_m} - 1 \quad (5)$$

Experimental methods for determining the numerical values of α , β seems certain. Many factors influence their precise determination.

The main goal of this paper is three-fold:

(1) To derive a radical expression for α and β . For simplicity, the surface flow is restricted to two dimensions (the distribution along the vertical, without regarding side slope influence).

(2) To compare the change of α , β with different idealized velocity distributions.

(3) To analyze Chow's assumption: should α , β be determined by only the velocity ratio u_*/u_m ?

2. Expression for α , β with semi-log distribution

2.1 Mean velocity distribution

Although in Chow's derivation the log distribution for a rough surface was studied, but the sublayer and the free constant in the semilog profile were not included. Strictly speaking, the semi-log profile could not be attain the boundary surface in whatever case. Therefore, we would prefer to use the log distribution, conjugated with a laminar sublayer.

Because in the core the law of semilog distribution is effective,

$$u^* = A \ln y^* + B \quad (6)$$

where A, B are constants; $A = 2.5$; $B = 5.5$; u^* , y^* are nondimensional velocity and distance, respectively; they are nondimensionalized by shear velocity u_* :

$$u^* = \frac{u}{u_*}, \quad y^* = \frac{yu_*}{v} \quad (7)$$

In the laminar sublayer, the velocity distribution should follow to the laminar law, that is,

$$u^* = y^* \quad (8)$$

The junction of the semilog line (6) and the linear line (8) should determine the nondimensional sublayer thickness δ^* , that is, from (6),

$$u_\delta^* = A \ln \delta^* + B \quad (9)$$

$$u_\delta^* = \delta^* \quad (10)$$

where u_e^+ is the velocity at the edge of this layer. From (9), (10) we have

$$\delta^+ = A \ln \delta^+ + B \quad (11)$$

By interpolation δ^+ can be obtained from (11) as

$$\delta^+ = 11.6 \quad (12)$$

Since the velocity distribution both in the core and the laminar sublayer was different, the mean velocity for the core and the laminar sublayer, respectively, first needs to be calculated. Mean velocity for the laminar sublayer u can be expressed:

$$u_{m,1} = \frac{1}{\delta} \int_0^\delta u \, dy \quad (13)$$

Substituting (8) into (13), we get

$$u_{m,1} = \frac{1}{\delta} \int_0^\delta y^+ u \, dy \quad (14)$$

Let

$$\frac{u_{m,1}}{u_*} = u_{m,1}^+ \quad (15)$$

substituting (15), (7) into (14), we get the integral in nondimensional form

$$u_{m,1}^* = \frac{1}{\delta^*} \int_0^{\delta^*} y^* dy^* = \frac{\delta^*}{2} = \frac{11.6}{2} = 5.8 \quad (16)$$

The mean velocity in the core u is expressed by

$$u_{m,2} = \frac{1}{h-\delta} \int_{\delta}^h u dy \quad (17)$$

Substituting (6) into (17), we get:

$$u = \frac{1}{h-\delta} \int_{\delta}^h u_* (A \ln y^* + B) dy \quad (18)$$

or in a nondimensional form:

$$u_{m,2}^* = \frac{1}{h^* - \delta^*} \int_{\delta^*}^{h^*} (A \ln y^* + B) dy^* \quad (19)$$

This results in:

$$u_{m,2}^* = \frac{1}{h^* - 11.6} [2.5 (h^* \ln h^* - h^* - 16.83) + 5.5 (h^* - 11.6)] \quad (20)$$

For calculating the mean velocity over the whole depth, it needs to divide the mean velocity of different layers by respective different thickness: in general, the expression for the global mean velocity can be written as:

$$u_m = \frac{\sum u_{m,i} h_i}{\sum h_i} \quad (21)$$

where $u_{m,i}$ = mean velocity in layer "i"; h_i = thickness of layer "i". In our case, (21) can be expressed in this explicit form:

$$u_m = \frac{u_{m,1} \delta + u_{m,2} (h - \delta)}{h} \quad (22)$$

or in nondimensional form:

$$u_m^* = \frac{u_{m,1}^* \delta^* + u_{m,2}^* (h^* - \delta^*)}{h^*} \quad (23)$$

Some numerical results of (23) are shown in Figure 1.

2.2 Calculation of the integrals in (1a) and (2a)

To split the integrals into two parts, and calculate their α , β respectively:

$$\int_0^h u^2 dy = \int_0^\delta u^2 dy + \int_\delta^h u^2 dy \quad (24)$$

and

$$\int_0^h u^3 dy = \int_0^\delta u^3 dy + \int_\delta^h u^3 dy \quad (25)$$

Substituting (10) into the first integral of (24), we get:

$$\int_0^\delta u^2 dy = \int_0^\delta u_*^2 y^{+2} dy = \int_0^{\delta^+} v u_* y^{+2} dy = \frac{v u_* \delta^{+3}}{3} \quad (26)$$

$$= 520.30 v u_*$$

Similarly, the first integral in (25) becomes:

$$\int_0^\delta u^3 dy = \int_0^\delta u_*^3 y^{+3} dy = \int_0^{\delta^+} v u_*^3 y^{+3} dy = \quad (27)$$

$$= \frac{v u_*^3 \delta^{+4}}{4} = 4526.60 v u_*^2$$

Calculating α , β for the laminar sublayer:

$$\alpha_1 = \frac{\int u^3 dy}{u_{m,1}^3 \delta} = \frac{4526.60 v u_*^2}{(5.8 u_*)^3 \delta} = \frac{4526.60}{5.8^3 \times 11.6} = 2.0 \quad (28)$$

$$\beta_1 = \frac{\int u^2 dy}{u_{m,1}^2 \delta} = \frac{520.30 v u_*}{(5.8 u_*)^2 \delta} = \frac{520.30}{5.8^2 \times 11.6} = 1.33 \quad (29)$$

2.3 Expressions for α , β

α , β for the core can be expressed by:

$$\alpha_2 = \frac{\int_\delta^h u^3 dy}{u_m^3 h}$$

Similarly to mean velocity (Eq.22), the expression of global mean value of α can be written in this form:

$$\alpha = \frac{\alpha_1 \delta + \alpha_2 (h - \delta)}{h} \quad (30)$$

This procedure is quite complicated. On the other hand, whether the residue method could give a precise result, was uncertain. Therefore, we would prefer to solve this problem with approximation in following paragraphs.

3. Approximate method for determining α , β

3.1 Calculation of nondimensional mean velocity

Since the laminar sublayer constitutes only a small part of the depth, from the standpoint of mechanical energy, it can be neglected. According to this assumption, the expression for α or β for the whole depth can be simplified to:

$$\alpha = \frac{\int_0^h u^3 dy}{u_m^3 h} \quad (31)$$

Eq.(31) can be replaced by this form:

$$\alpha = \frac{\frac{1}{h} \int_0^h u^3 dy}{u_m^3} \quad (31a)$$

where

$$u_m = \frac{1}{h} \int_0^h u dy \quad (32)$$

Substituting (6) into (32), we get:

$$u_m = \frac{1}{h} \int_{\delta}^h u_* (A \ln y^* + B) dy = \frac{v}{h} \int_{\delta^*}^{h^*} (A \ln y^* + B) dy =$$

$$= \frac{v}{h} (A[h^* \ln h^* - h^* - (\delta^* \ln \delta^* - \delta^*) + B(h^* - \delta^*)]$$

or

$$\frac{u_m}{u_*} = u_m^+ = \frac{1}{h^*} \{2.5(h^* \ln h^* - h^*) - 42.08 + 5.5 h^* - 63.8\} =$$

$$= \frac{1}{h^*} (2.5 h^* \ln h^* + 3 h^* - 105.88) \quad (33)$$

Some numerical results of (33) are shown in figure 2, showing that the relation of $(u_m/u_*)^2$ to h^* conforms to the semilog law.

Chow stated that if the velocity distribution followed the semilog law, then the coefficient α or β would be determined solely by u_m/u_* , as was shown in Eqs.(3)-(5). However, as we can see from (33), this ratio should change with respect to the increase of h^* .

3.2 Calculation of α , β

From (6),

$$(A \ln y^* + B)^3 = A^3 (\ln y^*)^3 + 3A^2 B (\ln y^*)^2 + 3AB^2 \ln y^* + B^3$$

then

$$\int_{\delta}^h u^3 dy = \int_{\delta}^h [A^3 (\ln y^*)^3 + 3A^2 B (\ln y^*)^2 + 3AB^2 \ln y^* + B^3] dy$$

in which

$$\begin{aligned} \frac{1}{h} \int_{\delta}^h B^3 dy &= \frac{1}{h} B^3 (h - \delta) = \frac{v}{u_{*h}} 166.38 (h^* - 11.6) = \\ &= \frac{1}{h^*} 116.38 (h^* - 11.6) = 166.38 (1 - 11.6/h^*) \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{1}{h} \int_{\delta}^h 3AB^2 \ln y^* dy &= \frac{1}{h^*} \int_{\delta^*}^{h^*} 3AB^2 \ln y^* dy^* = \\ &= \frac{1}{h^*} 3AB^2 (y^* \ln y^* - y^*) \Big|_{\delta^*}^{h^*} = \\ &= \frac{3AB^2}{h^*} (h^* \ln h^* - h^* - 16.83) \\ &= \frac{226.87}{h^*} (h^* \ln h^* - h^* - 16.83) \end{aligned} \quad (35)$$

Since

$$d[y^*(\ln y^*)^2] = [(\ln y^*)^2 + 2(\ln y^*)] dy^*$$

then

$$\begin{aligned} \int_{\delta^*}^{h^*} (\ln y^*)^2 dy^* &= [y^*(\ln y^*)^2] \Big|_{\delta^*}^{h^*} - \int_{\delta^*}^{h^*} 2 \ln y^* dy^* = \\ &= h^* \ln h^* - \delta^* \ln \delta^* - 2 [h^* \ln h^* - h^* - (\delta^* \ln \delta^* - \delta^*)] = \\ &= 5.23 - h^* \ln h^* + 2h^* \end{aligned}$$

and since

$$d[y^*(\ln y^*)^3] = (\ln y^*)^3 dy^3 + 3(\ln y^*)^2 dy^*$$

then

$$\begin{aligned}
\int_{\delta^*}^{h^*} (\ln y^*)^3 dy^* &= y^* (\ln y^*)^3 \Big|_{\delta^*}^{h^*} - 3 \int_{\delta^*}^{h^*} (\ln y^*)^2 dy^* \\
&= h^* (\ln h^*)^3 - \delta^* (\ln \delta^*) - 3 \times (5.23 - h^* \ln h^* + 2h^*) = \\
&= h^* (\ln h^*) + 3h^* \ln h^* - 6h^* - 30.41
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{1}{h} \int_{\delta}^h 3A^2 B (\ln y^*)^2 dy^* &= \frac{1}{h^*} \int_{\delta^*}^{h^*} 3A^2 B (\ln y^*)^2 dy^* = \\
&= \frac{103}{h^*} (5.23 - h^* \ln h^* + 2h^*) = \\
&= \frac{538.69}{h^*} - 103 \ln h^* + 206h^*
\end{aligned} \tag{36}$$

$$\begin{aligned}
\frac{1}{h} \int_{\delta}^h A^3 (\ln y^*)^3 dy^* &= 15.62 [(\ln h^*)^3 + 3 \ln h^* - 6h^* - 30.41] = \\
&= 1562 (\ln h^*)^3 - 46.86 \ln h^* - 93.72h^* - 475
\end{aligned} \tag{37}$$

Although these results are quite cumbersome, they are algebraic equations, which are easy to solve by simple programming.

4. Comparison with Chow's model

Chow's result (1959) about the influence of coefficients α , β on the velocity distribution was shown by Eqs.(3)-(5). As evidence, some numerical results, for these equations are shown in figure 2. Thus, according to Chow's model, although his derivation was based on the velocity for a rough boundary, the final expression was related to the nondimensional velocity u_*/u_m only. In other words, Chow considered the ratio u_*/u_m as an "independent" variable for determining the value of α and β . However, as we see in this paper, the ratio u_*/u_m is determined by the nondimensional depth h^* .

To precisely determine the value of α , β was quite difficult before (Watts, 1967). This may be partly due to the influence of different values of h^* .

Furthermore, the relation of values α , β to u_m/u_* predicted by Chow, was too simple. It approximated a nonlinear combination of ϵ . In our numerical results, the relation is quite

cumbersome. They could be solved only by either a program, or with the help of subsidiary graphs.

According to Chow's model, α and β are not independent of each other. Their relation can be obtained by co-solution of Eqs. (3)-(4):

$$\alpha = 1 + 3 (\beta - 1) - 2 (\beta - 1)^{3/2}$$

This numerical result is shown by the dotted line in figure 4, in which the analytical results of this paper are shown as a solid line (see below) for comparison. We can see from this figure that in the region of small α (probably $\alpha < 1.3$) our results seem close to Chow's, but as α increases beyond this region, the predicted relationships of α to β diverge.

5. Influence of velocity distribution on the values of α , β

5.1 Case of ideal fluid

In general, α and β represent a gauge of velocity distribution. In the following are some limiting cases from the standpoint both of type of velocity distribution and of different flow pattern.

The surface flow along a steep slope is close to the ideal case, because in the core, the velocity was almost uniformly distributed.

Once the surface flow is uniformly distributed, then

(1) the velocity should independent from y ; and

(2) the velocity at any point should represent the mean velocity. In this case, Eqs. (1) and (2) become

$$\alpha = \frac{u^3 \int dA}{u_m^3 A} = \frac{u_m^3 \int dA}{u_m^3 A} = 1 \quad (38)$$

$$\beta = \frac{u^2 \int dA}{u_m^2 A} = 1 \quad (39)$$

2

This is one of the limiting cases for α , β .

5.2 Case of linear velocity distribution

This type of flow and velocity distribution would be realized when the water surface suffered a shear stress. In the laminar sublayer of open channel flow, the velocity distribution was close to a linear distribution also. Although the velocity should be distributed along the vertical, there is no curvature. This is of another type of limiting case.

Like Eqs. (26), (27), the integral in (1a) and (2a) can be easily performed as:

$$\int_0^h u^2 dy = \int_0^h u_*^2 y^{+2} dy = \frac{v}{u_*} \int_0^h u_*^2 y^{+2} dy^+ = v u_* \frac{h^{+3}}{3} \quad (40)$$

$$\int_0^h u^3 dy = \int_0^h u_*^3 y^{+3} dy = \frac{v}{u_*} \int_0^h u_*^3 y^{+3} dy^+ = v u_*^2 \frac{h^{+4}}{4} \quad (41)$$

and the mean velocity can also be easily calculated:

$$u_m = \frac{1}{h} \int_0^h u dy = \frac{1}{h} \int_0^h u_* y^+ dy = \frac{v}{h} \int_0^h y^+ dy^+ = \frac{v}{h} \frac{h^{+2}}{2} = \frac{u_* h^+}{2} \quad (42)$$

Substituting (41), (42) into (1a), we get:

$$\alpha = \frac{\int u^3 dy}{u_m^3 h} = \frac{v u_*^2 h^{+4}/4}{u_*^3 h^{+3} h/8} = 2 \quad (43)$$

and

$$\beta = \frac{\int u^2 dy}{u_m^2 h} = \frac{v u_* h^{+3}/3}{u_*^2 h^{+2} h/4} = 4/3 = 1.3333 \quad (44)$$

5.3 Case of parabolic velocity distribution

This type of velocity distribution belongs to laminar surface flow, that is:

$$u = \frac{Sg}{v} \left(yh - \frac{y^2}{2} \right) \quad (45)$$

Substituting (44) into (42), we get the mean velocity as:

$$u = \frac{gS}{vh} \int_0^h \left(yh - \frac{y^2}{2} \right) dy = \frac{gSh^2}{3v} \quad (46)$$

then

$$u_m^2 = \frac{g^2 S^2 h^4}{9v^2}, \quad u_m^3 = \frac{g^3 S^3 h^6}{27v^3} \quad (47)$$

The integrals in (1a) and (2a) become

$$\begin{aligned} \int_0^h u^3 dy &= \frac{S^3 g^3}{v^3} \int_0^h \left(y^3 h^3 - \frac{3y^4 h^2}{2} + \frac{3y^5 h}{4} - \frac{y^6}{8} \right) dy = \\ &= \frac{S^2 g^3}{v^3} \left(\frac{h^7}{4} - \frac{3h^7}{10} + \frac{h^7}{8} - \frac{h^7}{56} \right) = \frac{2 g^3 S^3 h^7}{35} \end{aligned}$$

Then

$$\alpha = \frac{\int_0^h u^3 dy}{u_m^3 h} = \frac{2/35}{1/27} = 1.5428 \quad (48)$$

Furthermore,

$$\begin{aligned} \int_0^h u^2 dy &= \int_0^h \frac{g^2 S^2}{v^2} (y^2 h^2 - y^3 h + \frac{y^4}{4}) dy = \frac{g^2 S^2}{v^2} \left(\frac{h^5}{3} - \frac{h^5}{4} + \frac{h^5}{20} \right) = \\ &= \frac{2 g^2 S^2 h^5}{15 v^2} \end{aligned}$$

Then

$$\beta = \frac{\int_0^h u^2 dy}{u_m^2 h} = \frac{2 g^2 S^2 h^5 / 15 v^2}{g^2 S^2 h^4 h / 9 v^2} = 1.20 \quad (49)$$

6. Coefficients α , β and energy dissipation

6.1 Case of ideal fluid

This is the problem of whether the coefficients α , β should relate to the energy dissipation. Solving this problem requires calculating the rate of energy dissipation separately for the individual profiles mentioned above.

Since in an ideal fluid flow field velocity distribution is uniform, (i.e., $u = \text{const}$),

$$\frac{\partial u}{\partial y} = 0$$

so that it can be conjectured that in this field the energy dissipation rate can be neglected.

6.2 Case of linear velocity distribution

Since

$$u^* = y^*, \text{ or } \frac{u}{u_*} = \frac{yu_*}{v}$$

then

$$u = \frac{yu_*^2}{v}, \quad \frac{\partial u}{\partial y} = \frac{u_*^2}{v}, \quad \left(\frac{\partial u}{\partial y}\right)^2 = \frac{u_*^4}{v^2}$$

Energy dissipation rate along the whole depth \dot{E} can be written as:

$$\dot{E} = \int_0^h \mu \left(\frac{\partial u}{\partial y}\right)^2 dy = \frac{\mu u_*^4}{v^2} \int_0^h dy = \frac{\mu u_*}{v^2} h = \rho h^* u_*^3 \quad (50)$$

6.3 Case of parabolic velocity distribution

Since

$$u = \frac{gS}{v} \left(gh - \frac{y^2}{2}\right), \quad \frac{\partial u}{\partial y} = \frac{gS}{v} (h - y), \quad \left(\frac{\partial u}{\partial y}\right)^2 = \frac{g^2 S^2}{v^2} (h - y)^2$$

then

$$\begin{aligned}
\dot{E} &= \int_0^h \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \frac{\mu g^2 S^2}{v^2} \int_0^h (h^2 - 2hy + y^2) dy = \\
&= \frac{\mu g^2 S^2}{v^2} \left(h^3 - h^3 + \frac{h^3}{3} \right) = \\
&= \mu g^2 S^2 h^3 / 3v^2 = \rho h^* u_*^3 / 3
\end{aligned} \tag{51}$$

in which

$$\sqrt{ghS} = u_*, \quad v = \mu/\rho, \quad h^* = u_* h / v.$$

Comparing (50) and (51) we can see that the formation of the curvature of the distribution curve decreases the dissipated energy.

All these numerical values by analytical approaches are listed in the following table:

Flow pattern	Velocity profile	α	β	Energy dissipation	
				Dimensional	Nondimensional
Couette flow	Linear	2	1.33	$\rho h^* u_*^3$	h^*
Laminar sublayer	linear	2	1.33	$\rho \delta^* u_*^3$	δ^*
Laminar	Parabolic	1.5428	1.20	$\rho h^* u_*^3 / 3$	$h^* / 3$
Turbulent	Semilog	1.03- 1.06		$A^2 \rho u_*^3 / \delta^*$	A^2 / δ^*
Ideal fluid	Uniform	1	1	0	0

This relationship between α and β is shown as a solid curve in figure 4, and can be considered an analytical relationship.

7. Concluding remarks

The coefficients α , β were not deduced directly from the consideration of energy dissipation, but they indeed indirectly represent one of the indicators of dissipated energy. However, since they are related to velocity distribution, they could give only relative values, not its absolute quantity. For example, for laminar sublayer flow and laminar flow over the whole depth, the values of α , β are the same, but the amounts of dissipated energy would be different. Whatever the value of α , or β they

can be considered indicators of stability of flow: If α , β were high, the flow pattern would be relatively unstable situation; when α is very close to 1.0 (representing no energy dissipation), then the flow ought to be stand relatively stable.

Coefficient α , or β does not represent a purely empirical coefficient, but can be obtained by an analytical approach.

Acknowledgements

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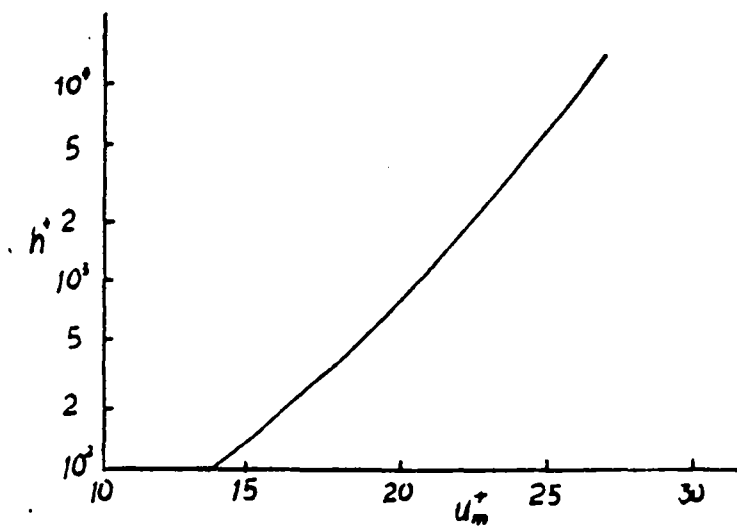


Figure 1

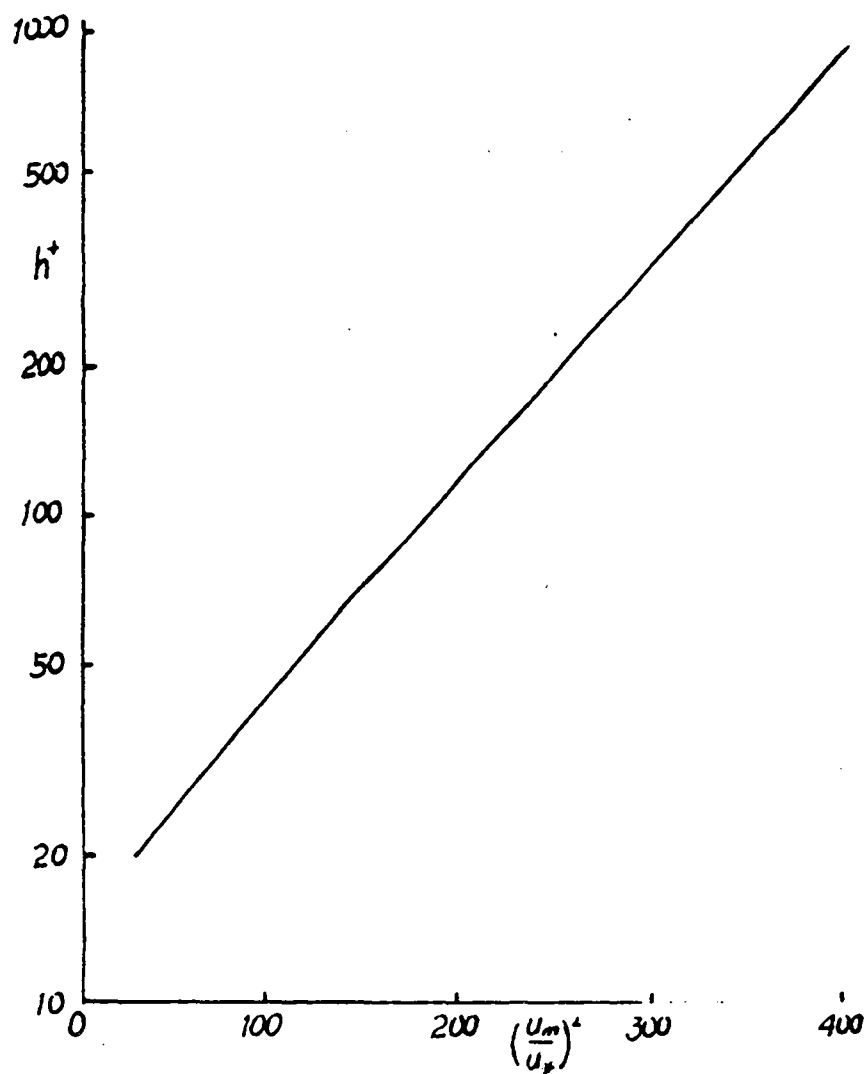


Figure 2

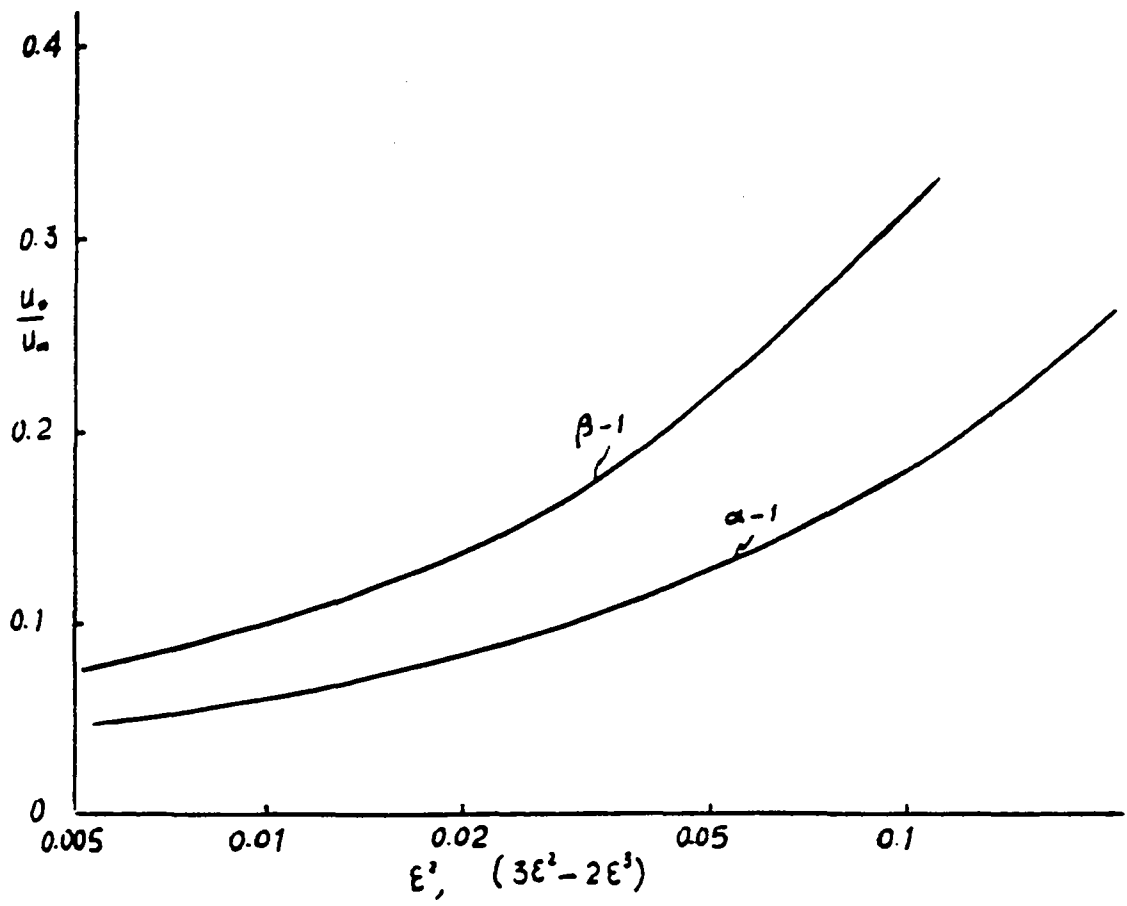


Figure 3

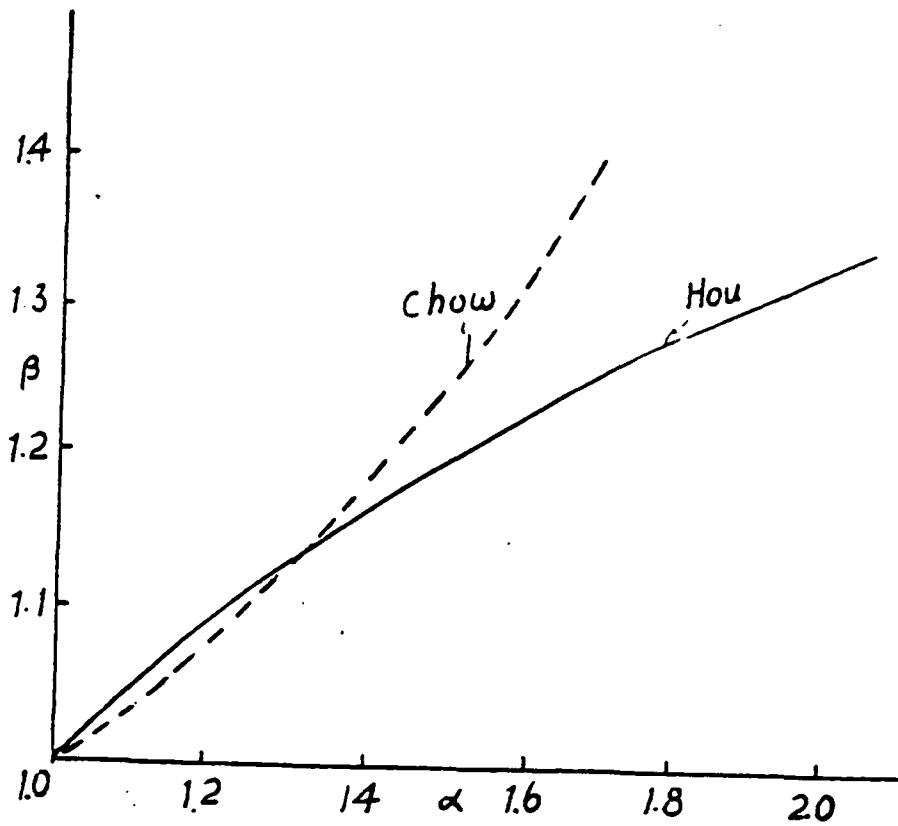


Figure 4

HYDRAULICS AND THE THEORY OF MINIMUM ENERGY DISSIPATION RATE

(Report No. IV)

by

H. C. Hou

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References

1. Introduction

Hydraulics is a longstanding scientific branch of classical mechanics. From the standpoint of engineering applications, hydraulics was first constituted mainly of two components. The main component (Bakhmeteff, 1932; Chow, 1959; Henderson, 1966) was concerned with the 'macroscopic' (long-range) forms of water surfaces due to man made obstructions (dams, hydraulic structures, sharp changes of channel slopes, etc.). The second component was concerned with local 'microscopic' considerations of velocity/energy fields around hydraulic structures, including the determination of different coefficients, different measures for dissipating energy in tailwater, etc.

Although most of the principles, laws, and methods of calculation for different problems in the above two components of hydraulics were established long ago, and the problem of energy dissipation has been studied extensively in engineering hydraulics, some problems still seem to need to be refined. Especially important problems are whether the theory of minimum energy dissipation rate (TMEDR) conflicts or conforms with classic hydraulics and how to explain different hydraulic phenomena from the standpoint of energy dissipation and TMEDR.

This paper is devoted to the analysis on these problems.

2. Basic Concepts of Theory of Minimum Mechanical Energy (TMME) and the Theory of Minimum Energy Dissipation Rate (TMEDR)

2.1 Basic Concepts of TMME

Although some of the following phenomena are common sense, they need to be reconsidered against the background of energy and energy dissipation:

Because water is a dense fluid, when a water mass is still within some container, it is stabilized with respect to the lowest position of its center of gravity, as with a rigid body. But why does a quiet water mass form a horizontal straight line surface instead of another form? The answer is: The center of gravity for a water mass with a horizontal surface is at the lowest position compared to the position with whatever form of free surface, such as a sloping free surface (Yang & Song, 1986). The lowest position of the center of gravity corresponds to the minimum potential energy for a given water mass, and a still water mass should exhibit a tendency to establish its own state with minimum potential energy.

Why does water flowing freely over a weir form a 'critical' depth on the crest, but not any other depth? The answer is: The magnitude of mechanical energy of a given water mass, corresponding to the formation of critical depth, attains the smallest value of

mechanical energy compared with the magnitude for whatever different depth. In other words, moving water exhibits a tendency to establish its own state with minimum mechanical (kinetic plus potential) energy. Even on the crest of a weir (a free uncontrolled fall) water could change in both its depth and velocity 'freely', so that minimum of mechanical energy would be maintained.

Obviously, the minimization of potential energy is a special case of minimization of mechanical energy.

This characteristic of a minimization-approaching tendency can be named the THEORY OF MINIMUM MECHANICAL ENERGY (TMME).

This mechanical characteristic is only one of the basic characteristics of water flow. Besides its dense character, water is also a viscous fluid. The viscosity should point to another mechanical/thermodynamic characteristic of water flow, which will be analyzed in the following paragraph.

2.2 Basic Concepts of TMEDR

The viscosity of water flow should correspond to its energy dissipation. Flowing water should always dissipate some part of its energy, and this energy should be transformed into heat and transferred to the external environment. Thus, if no energy were supplied constantly from outside (man made or natural) to compensate this dissipated energy of surface water flow, then the existence of any form of water flow would be impossible. Flowing water would stop suddenly. This has been a quite well understood phenomenon.

However, the fact that water flow is unavoidably accompanied by energy dissipation is only one of the basic characteristics of water flow, as viscous flow. Other more important basic characteristics are that (1) the energy dissipation rate falls as much as possible under given constraints, and (2) along with this, the time rate (or course rate) of energy dissipation for a given water mass diminishes with time (and along the course). If the boundary of flowing water were a deformable surface such as an alluvial bed, then this characteristic would be expressed quite obviously: The energy dissipation rate (i.e., the unit stream power) should be diminished along the course on one hand; on the other hand, the rate should fall to a value for a given bed material in the lower reach, whereby the representative particles on the riverbed remain in a statistically incipient/equilibrium condition.

For most problems in engineering hydraulics, the boundaries of flowing water are rigid, so that the energy dissipation rate can be diminished neither along the course, nor with time by bed deformation. It is as if this characteristic cannot be expressed

in engineering hydraulics. However, it is expressed indirectly in the velocity distribution.

As mentioned before (Hou, II), the formation of semilog velocity distribution in an open channel (or a pipe) is not only the result of random fluctuating fluid particles (as described by the classical Prandtl's model), but also minimization of dissipated energy: Comparison between the dissipated energy for the core (by semilog) and for the laminar sublayer showed that, though the core constitutes most of the depth, and the thickness of the laminar sublayer is negligible, dissipation in the core energy was still much lower than that in the laminar sublayer. Therefore, the semilog distribution is the most 'energy-saving' distribution for water mass transport. This fact indirectly indicated that the water flow would spontaneously maneuver its own velocity distribution toward the minimum energy dissipation rate.

2.3 Coupling of TMME and TMEDR

TMME is concerned with macromechanical processes without considering energy dissipation, whereas TMEDR is concerned with thermodynamic processes with energy dissipation. These principles belong to two different areas, representing two different laws. However, due to the dual mechanical-thermodynamic characteristic of surface water flow, the TMME and the TMEDR ought to be coupled. Thus, the question becomes whether TMME and TMEDR conflict or conform with each other in an united system. In general, both TMME and TMEDR are in conflict. Only in some restricted cases do they conform.

In this paper, we will see that if water processes proceed without man made constraints, that is, is proceeding spontaneously forward, then they evolve toward a situation, whereby TMME and TMEDR conform with each other. However, if the water process is accompanied by some man made constraint, then they conflict with each other.

3. Engineering Hydraulics and TMEDR

3.1 Expression of Energy Dissipation in Hydraulics

'Energy-dissipation' is not a new concept in hydraulics, for most hydraulic structure projects are concerned with energy dissipation in the tailwater downstream. But this is a global form of energy dissipation, and it is constrained mainly by the man made obstruction. However, energy dissipation should exist within the whole system of water flow.

Irrespective of the different forms of surface water flow, any energy dissipation can be decomposed into these different forms:

(1) Dissipation through the action of Newtonian forces (i.e., through the viscous effect of the velocity distribution). - This physical picture and its numerical method was dealt with thoroughly in our previous paper (Hou, II).

(2) Dissipation through separation. - There are two types of separations:

a. Microscopic separation. - When water flows along a streambed, although the streambed should coincide with the boundary, from the microscopic point of view, the streambed could touch the boundary only locally and discontinuously; the streambed would be separated at rough spots, so that in these 'shaded' regions beneath the rough boundary, innumerable small vortices would be forced to form. These vortices should dissipate additional energy, so that the coefficient of friction for rough pipe is greater than that for smooth pipe.

b. Macroscopic separation. - If the boundary did not coincide with the streambed, then a large separation would be generated. The vacuum formed could create a great amount of dissipated energy in the case of high-speed flow, and it would be the source of cavitation and damage to the hydraulic structure.

(3) Dissipation through the impact to the boundary. - In this case, the kinetic energy of a fluid mass is dissipated directly into heat through the random fluctuation of fluid particles.

3.2 Energy-Related Meaning of Hydraulic Coefficients

In engineering hydraulics, there are many coefficients such as:

(1) The coefficient of friction, f , along a pipeline or open channel (or expressed in the form of Manning's coefficient).

(2) The coefficient of water head loss due to a local sharp change in the configuration of a conduit.

(3) The coefficients of discharge for different forms of weirs, spillways, etc.

(4) The coefficients of hydromachines (pumps, hydroturbines, etc.). As if all these coefficients were of empirical constant only. Of course, most of their numerical values could be determined only by experiments. But an important fact is that, from the viewpoint of energy dissipation, these coefficients in fact represent a relative measure of dissipated energy.

For example, the pipe flow the Darcy formula can be written as:

$$\Delta h = f \frac{L}{d} \frac{v^2}{2g} \quad (1)$$

where d = diameter of pipe; v = mean velocity of pipe flow; Δh = water head loss along a distance, L . Equation (1) can be replaced by:

$$f = \frac{\gamma \frac{\Delta h}{L} d}{\rho v^2 / 2} \quad (2)$$

where

$$\frac{\Delta h}{L} = S = \text{hydraulic gradient}$$

The numerator of (2) can be considered a dissipated energy, represented in the form of potential energy loss, divided by the diameter, d . The denominator of (2) is the kinetic energy per unit volume of water mass, and it can be considered to be the input. Hence, the coefficient 'f' can be considered as a measure of dissipated energy relative to the input in the form of kinetic energy. Therefore, we have:

$$f = \frac{\text{dissipated energy in the form of potential energy}}{\text{input in the form of kinetic energy}} \quad (3)$$

Another example is the free fall of surface flow over a weir. It can be written as:

$$Q = C B \sqrt{2g} H^{3/2} \quad (4)$$

where Q = discharge; B = width of channel; H = water-head above the weir; C = discharge-coefficient. Equation (4) can be written as:

$$Q^2 = C^2 B^2 2 \frac{\gamma}{\rho} H^3 \quad (5)$$

for

$$V = Q/BH$$

where v = mean velocity. Then Equation (5) can be replaced by:

$$C^2 = \frac{\frac{\rho v^2}{2}}{\gamma H} \quad (8)$$

in which the numerator is the kinetic energy of the water mass, or effluent from the weir. It can be referred to as the output of dissipated energy. The denominator is the potential energy in the form of the water head of the weir. Therefore, we have:

$$C = \sqrt{\frac{\text{Dissipated energy in the form of kinetic energy}}{\text{Input in the form of potential energy}}} \quad (7)$$

3.3 Different methods for diminishing dissipated energy

In engineering hydraulics, an important measure for diminishing the energy dissipation rate is to exclude the separation region. In general, in whatever case, for the streambed being a continuous smooth curve, then accordingly, the boundary contour of all the conduits must be designed such so that it contacts the adjacent streambed as much as possible. Otherwise, the energy dissipation rate could be aroused, and the coefficient of discharge (or velocity) would be increased accordingly.

In figure 1 are serial experimental results on an elliptical contour entrance in submerged square tubes. From these results some general insights can be obtained.

(1) Irrespective of different lengths of tubes, for a given length of tube, the more closely the entrance of tube comes to an

elliptical contour, the higher the coefficient of velocity, that is, the lower the energy dissipation rate.

(2) The influence of length on the coefficient of velocity seems twofold: If the entrance was only partially elliptical or not elliptical at all, then the longer the tube length, the higher the value of the velocity coefficient. If the entrance was elliptical, an inverse physical picture would appear: the longer the tube length, the lower the value of the velocity coefficient.

(3) Although both the length of the tube and shape of the entrance could increase the value of the coefficient of velocity, an elliptical entrance with the shortest tube length would be the most effective one. Thus, a short tube with an entrance with a wholly elliptical surface would be the outlet with the quasi-minimum energy dissipation rate.

From these experiments, we can see that the influence of an elliptical entrance on the coefficient of discharge occurs monotonical, whereas the influence of the tube length is of twofold. It would channel water in a straight line, suppressing separation downstream and decreasing the energy dissipation rate on one hand. On the other hand, a long tube would increase the friction loss, increasing the energy dissipation rate. This twofold character of tube length is expressed in figure 2, where the abscissa is the tube length, L , normalized by the tube height, D , and C is the coefficient of velocity, that is,

$$V = C_v \sqrt{2gH} , C_v = \frac{V}{\sqrt{2gH}}$$

H is water head.

These experiments are concerned with the case of low water heads. If the water head increased, separation would intensify. A sharp-edged entrance would not only generate vortexes behind it, but sometimes the cross section of water flow would be compressed, leading the discharge to decrease sharply. Figure 3 (Bureau of Reclamation, 1977) shows this comparison.

3.4 Velocity-Distribution and TMEDR, TMME

In our previous paper (Hou, II), the relation between the velocity distribution and the TMEDR was analyzed, and some numerical results were derived. Most of the energy was dissipated not in the core, but in the laminar sublayer region. The numerical results showed that the core, which occupied most of the depth, represented an energy-saving distribution.

Therefore, the main part of the dissipated energy was concentrated into a narrow region at the boundary. Dissipated energy corresponds to velocity gradient. Therefore, if we could decrease the velocity gradient at the boundary, we would achieve the goal of diminishing the energy dissipation rate for surface water flow. However, hydraulic structures (weirs, conduits, etc.) have rigid boundaries, where the velocity ought to equal zero, so that the velocity gradient at the boundary would be unchangeable. In this case, the question is whether the velocity gradient could be diminished in hydraulic engineering. There may be different measures available for this change:

- (1) Movable bed of fine particles
- (2) Ejection beneath the boundary
- (3) Flexible boundary, etc.

The effectiveness of each measure used can only be verified by experiments.

3.5 Local Scour Downstream and TMEDR

Energy dissipation downstream of a high-speed water flow issuing from a spillway gate under a high head is an important and difficult problem in engineering hydraulics, especially for structures founded on fluvial beds with fine particles, because aroused local scour for this case would be serious and sometimes dangerous for structure foundations without proper engineering treatment. Of course, treatments for different hydrological/hydraulic, geological, and engineering conditions would differ from each other. This paper does not cover this skill in design, and does not conclude with some successful experience or failure in design. This paper covers the general principles for treating this problem.

Design work for avoidance of serious local scour focuses on hydraulic jumps that are separate from the structures. Of course, hydraulic jumps extensively mix the water mass, and dissipate a large amount of kinetic energy, so that eases the problem to some degree. However, hydraulic jumps are a general concept; their structure would be different for different cases.

In recent years, many authors have studied the turbulent diffusional process in water flowing from structures. Local scour has indeed been found to be closely related to the evolutionary diffusional process of velocity distribution downstream. Local scour and its effects are the 'byproducts' or 'footprints' of the diffusional process of velocity distribution downstream. Scour is caused by turbulence fluctuation generated by a velocity gradient, which penetrates deeply through the turbulent-diffusional process,

on one hand, and on the other hand, a deep clockwise circulation (fig. 4a) intensifies the deep transport of sediment. The relation between the velocity distribution along the course, the formation of circulation and the corresponding formed local scour is shown in figures 4a and 4b.

Difficulties in solving the local scour problem are that the outflow velocity from the structure is high and quasi-uniformly distributed along the vertical, and energy is concentrated within a small region(depth).

Because the energy dissipation process of water flow (including the turbulent decaying process) is continuous and gradual from the viewpoint of TMEDR, the principle for compensating for this manmade concentrated high energy must be that the energy must be dissipated continuously and gradually without significant differences along the course. According to this requirement, protection (by either reinforced or flexible material) must be designed so that the diffusional process of water flow follows this principle.

4. Open-Channel Hydraulics and TMME, TMEDR

4.1 Horizontal Surface and TMME, TMEDR

Different classifications of surface profiles and their corresponding methods of calculation are dealt with thoroughly in many monographs on open channel hydraulics (Chow, 1959), but this paper will reconsider their characteristics from the standpoint of minimum energy and energy dissipation rate, without regard to their numerical method of calculation. In this respect, the free-surface profile of a quiet water mass, as the simplest form and limit case of surface 'flow', ought to be included in the category of free surface flow. In this case, both the hydraulic gradient, S , and velocity, u , should equal zero.

A horizontal free surface has two meanings: (1) The potential energy of this water mass should be in the minimum state, and (2) The energy dissipation rate along the course should equal zero (i.e., minimum among the minima!). The first statement refers to TMME, the second to TMEDR. In another words, if a surface water mass is flowing 'tangentially' (with very small hydraulic gradient!) to a water mass with a horizontal free surface, then TMME and TMEDR conform to each other. All the spontaneously evolved rivers in nature flow along this way! Therefore, conformation between TMME and TMEDR is conditional.

4.2 Uniform Flow and TMME, TMEDR

It must be emphasized that not all surface water flow could form a uniform pattern. As mentioned by Hou (II), formation of uniform patterns in open channel flow is conditional. When the

flow is in the region governed basically by TMEDR (in the case of not a very great channel bed slope), then the surface flow would ultimately establish its uniform pattern spontaneously. Roughly speaking, when the input of surface flow, that is, the effective component of the rate of potential energy should equal the energy rate dissipated directly through the velocity field plus that part of the energy dissipated at the rough boundary, then a uniform flow pattern would be formed. If the effective input were much greater than the directly dissipated part, then the flow pattern would be transformed to an accelerating one.

Different flow regions and their corresponding flow patterns are listed in table 1:

Table 1.

Slope	Horizontal	Small hydr.	Medium	Steep	Very steep
Theory	TMME	TMEDR & TMME	TMEDR	TMME & TMEDR	TMME
Flow	Quiet	Uniform	Uniform	Quasi-Uniform	Nonuniform Accelerat.

Table 1 is only a qualitative classification; there are not yet distinct specific value, by which each case could be quantitatively defined.

From table 1, we can see that TMME should occupy two limit regions. It should be effective in the region with very high speed and slope. Meanwhile, it should be effective in the region with zero gradient and velocity. The regions, guided by TMEDR would be in some intermediate region. The remaining regions would be mixed, where TMME and TMEDR would be coupled.

4.3 Criteria for Flow Pattern Transfer

Our previous paper (Hou, II) predicted that a turning point exists for surface water flow, beyond which the uniform pattern should transform to the accelerating one, and vice versa.

The classification of flow pattern in table 1 is only a rough one. In fact, as indicated in (Hou, II), the factors that influence flow pattern, are not restricted to the bed slope, S , but include the discharge, Q (or velocity), and the roughness (n or f). For example, on some steep slope with a smooth boundary under a given discharge, the flow pattern may be accelerating, but with a rough surface under the same hydraulic conditions, it would become uniform. Or, on a steep slope, the flow pattern would be uniform

under the condition of small discharge, but it would become an accelerating pattern with a high discharge.

Considering these factors separately: Let an increase of N represent the flow pattern transfer from uniform toward accelerating. Then N should be directly proportional to the discharge; that is, the higher the value of Q , the greater the possibility of this transfer. N should also be directly proportional to S but inversely proportional to the roughness n . A possible form of this criterion would be:

$$N = \frac{Q^x S^y}{n^z} \quad (8)$$

here the indexes x, y, z can be determined only by experiment.

Another approach is the input-to-output approach. Let the magnitude of N be constituted by the input to the output. Here output must include that part dissipated through the velocity field plus that dissipated by the rough boundary. This criterion can be obtained by an analytical procedure:

From (Hou, II), the input of surface flow was obtained as:

$$\dot{E}_{in} = 67.28 \gamma v S + \gamma v S A (h^* \ln h^* - h^* - \delta^* \ln \delta^* + \delta^*) + B (h^* - \delta^*) \quad (9)$$

Let $\delta^* = 11.6$; $A = 2.5$; $B = 5.5$, then equation (9) can be simplified to:

$$\dot{E}_{in} = (2.5 h^* \ln h^* + 3 h^* - 42.6) \gamma v S = C \gamma v S \quad (10)$$

where

$$C = (2.5 h^* \ln h^* + 3 h^* - 42.6) = f(h^*)$$

The output is:

$$\dot{E}_{diss} = 12.14 \rho u_*^3 = 12.14 \rho (ghS)^{3/2} \quad (11)$$

Defining the N as:

$$N = \frac{\dot{E}_{in}}{\dot{E}_{diss}} \quad (12)$$

Substituting (10), (11) into (12), we get:

$$N = \frac{C_{\gamma} v S}{12.14 \rho u_*^3} = 0.88 v g \frac{S}{u_*^3} = k \frac{S}{u_*^3} \quad (13)$$

This can be used as an indicator for pattern transition also. But the concrete value of coefficient 'k' remains unknown, it could be determined only by the experiments.

4.4 Gradually Varied Flow and TMME, TMEDR

The classification of different surface profiles for gradually varied flow is thoroughly analyzed in textbooks on open channel hydraulics. A widely accepted classification is shown in figure 5 (Chow, 1959). However, some cases in figure 5 could be realized only in an artificial surface flow. For example, neither the normal depth nor the critical depth of surface flow could be realized in an open channel with zero or adverse bed slope. Besides this, in an open channel with a steep slope, surface flow does not necessarily need to approach a uniform pattern. In another words, the existence of 'S2' and 'S3' in figure 5 must remain to be verified by experiments.

The profiles M1, M2 in figure 5 seem to be most interesting and profound among all the profiles from the standpoint of TMEDR and TMME.

Backwater curve M1 in a river is formed by a high elevation preceding a dam (or the sea). The specific characteristic of profile M1 is: It coincides tangentially with the normal free surface upstream on one hand, and on the other hand, it coincides tangentially with the horizontal surface downstream. The method for calculating the form of this profile is well known in hydraulic textbooks. We are interested only in its energetic essence here: Backwater curves represent river processes, by which the surface

curve is initiated from a uniform flow, where the state has reached equilibrium, and it is terminated downstream by the 'zero energy dissipation rate', that is, the horizontal water surface. Hence, the backwater curves, as a river process, conform to TMEDR: Thus, at the junction of the backwater curve upstream, we have:

$$\frac{dE}{dt} = u \frac{dE}{dx} = \min = \text{const}, (\min, \text{const} > 0) \quad (14)$$

At the junction downstream we have:

$$\frac{dE}{dt} = u \frac{dE}{dx} = 0 \quad (15)$$

However, a horizontal surface means that the backwater curve should start to follow to the TMME, that is, the backwater curve is terminated by:

$$E = \min \quad (16)$$

Hence, in this river process the TMME and TMEDR conform.

The profile M2 in figure 4 represents a process by which the state is initiated from the uniform pattern upstream, that is, according to equation (14), but terminated with maximal release of mechanical energy downstream, that is, the profile M2 is terminated by:

$$\frac{dE}{dt} = u \frac{dE}{dx} = \max \quad (17)$$

according to TMME, at the point of a junction (crest of weir), the mechanical energy should approach a minimum value, as shown by the equation (16).

5. Conclusion and Discussion

Classic hydraulics is entirely related to TMME and TMEDR:

(1) Different coefficients in engineering hydraulics relate to the different expression of relative dissipated energy.

(2) Because dissipated energy is concentrated at the boundary, if the 'boundary' conditions of water flow could be changed, then the value of dissipated energy would decrease accordingly.

(3) Some surface profiles in open channels are river processes, terminated either by TMME or by TMEDR downstream and upstream. Many problems remain for further study on whether these constraint conditions could be used for determining the surface profile.

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Appendix

Backwater curve was studied thoroughly in any textbook on open channel hydraulics. Its method of calculation was founded on the balance equation of water head (Chow, 1959).

From the standpoint of energy dissipation, the backwater curve is such a surface water-process, by which the energy dissipation rate should be diminished along the course to that limit value, where the energy dissipation rate would be approached to zero! (for the hydraulic gradient at the junction to the reservoir should equal to zero).

Specified to the surface water flow, the energy dissipation rate is expressed in form of ' γuS ', or shortly, in ' uS ', for in common case, the specific weight was remained a constant value. At the upper junction A-A the value of energy dissipation rate ought be given. But whatsoever its value should be, it ought be grater than zero:

$$(\gamma uS)_A > 0 \quad (A1)$$

otherwise any surface flow could not exist. At the lower junction B-B its value should approach to zero:

$$(\gamma uS)_B = 0 \quad (A2)$$

And in the range the change of unit stream power ought be less than zero, i.e.,

$$\frac{d(\gamma uS)}{dt} < 0, \text{ or } u \frac{d(\gamma uS)}{dx} < 0 \quad (A3)$$

Irrespective to that, whatever the form of backwater curve should be, the value of (γuS) within the range of back water curve ought be satisfied to this inequality:

$$0 < (\gamma uS) < (\gamma uS)_A \quad (A4)$$

Furthermore, the change of ($\gamma u S$) along the course ought be remained of continuous.

The equations (A1)-(A3) can be derived in following:

According to the Theory of Minimum Energy Dissipation Rate (or named as the Prigogine Principle), the water flow, as an irreversible process, its energy dissipation rate must be of a positive value; it would be deminished along the time; and finally, it should approach to a minimum value (or, in limit case, to zero). These statements can be expressed by the mathematical formulation in following:

$$\frac{dE}{dt} > 0, \quad \frac{d^2 E}{dt^2} < 0, \quad \text{and} \quad \frac{dE}{dt} = \min \quad (A5)$$

But we are interesting how the energy should change along the course, but not along the time. In this case,

$$\frac{d}{dt} = u \frac{d}{dx}$$

Furthermore, the energy, which should sustain the surface water to flow, is only the potential energy of this water mass, then:

$$E \sim \gamma y$$

so that:

$$\frac{dE}{dt} = u \frac{d}{dx} \gamma y = \gamma u \frac{dy}{dx} > 0$$

For

$$\frac{dy}{dx} = S.$$

then

$$\frac{dE}{dt} = \gamma u S > 0$$

and

$$\frac{d^2 E}{dt^2} = \frac{d}{dt} \frac{dE}{dt} < 0$$

or

$$\frac{d}{dt} (\gamma u S) < 0$$

or

$$u \frac{d}{dx} (\gamma u S) < 0$$

Principally, if the law of change rate of $(\gamma u S)$ was known, then the backwater curve could be derived. Unfortunately, this problem is remained to be studied. We assume temporarily that the change of $(\gamma u S)$ is linearly proportional to the local value of $(\gamma u S)$, i.e.,

$$\frac{d (\gamma u S)}{dt} = -A (\gamma u S) \quad (A6)$$

where A is a constant of proportionality. From (A6) we have:

$$\frac{d (\gamma u S)}{(\gamma u S)} = -A dt = -A \frac{dx}{u}$$

or

$$\ln (\gamma u S) = -A \int \frac{dx}{u} + \text{const} \quad (\text{A7})$$

The boundary condition is: when

$$x = 0, (\gamma u S) = (\gamma u_o S_o) \quad (\text{A8})$$

where S_o , h_o , u_o are the values for uniform flow at upstream beyond the backwater curve. Substituting (A8) to (A7), we get:

$$\text{const} = \ln (\gamma u_o S_o) \quad (\text{A9})$$

then

$$\ln \frac{(\gamma u S)}{(\gamma u_o S_o)} = A \int \frac{dx}{u}$$

or

$$\frac{\gamma u S}{\gamma u_o S_o} = \exp \left(A \int \frac{dx}{u} \right)$$

for

$$q = u_o h_o = u h, \text{ or } u_o/u = h/h_o$$

q is unit discharge, then

$$\frac{hS_o}{h_o S} = \exp \left(A \int \frac{h dx}{q} \right)$$

Furthermore, if the Chezy formula is remained valid for the nonuniform flow, then:

$$q = C\sqrt{hS}$$

and

$$\frac{C^2 S_o}{h_o q^2} = \exp \left(A \int \frac{h dx}{q} \right) \quad (A10)$$

the equation (A10) is remained for further experimental verification.

CASE	ENTRANCE CONDITIONS	SERIES 1	SERIES 2	SERIES 3	SERIES 4	SERIES 5	SERIES 6	SERIES 7
A		 $K_e = 1.60$ $C = 0.62$	 $K_e = 1.44$ $C = 0.64$	 $K_e = 1.37$ $C = 0.65$	 $K_e = 0.93$ $C = 0.72$	 $K_e = 0.69$ $C = 0.77$	 $K_e = 0.56$ $C = 0.80$	 $K_e = 0.52$ $C = 0.81$
B		 $K_e = 1.44$ $C = 0.64$			 $K_e = 1.04$ $C = 0.70$	 $K_e = 0.64$ $C = 0.78$		 $K_e = 0.49$ $C = 0.82$
C		 $K_e = 1.16$ $C = 0.68$	<p>NOTES All tubes 4'-0" x 4'-0". Where elliptical entrance is not indicated corners are square, cut in wood. Values of C given are averages for the formula $V = C\sqrt{2gh}$ Loss coefficient $K_e = \left(\frac{1}{C^2} - 1\right)$</p>			 $K_e = 0.93$ $C = 0.72$	 $K_e = 0.52$ $C = 0.81$	 $K_e = 0.45$ $C = 0.83$
D		 $K_e = 0.64$ $C = 0.78$			 $K_e = 0.88$ $C = 0.73$	 $K_e = 0.38$ $C = 0.85$		 $K_e = 0.38$ $C = 0.85$
								 $K_e = 0.35$ $C = 0.86$ Wall
E		 $K_e = 0.08$ $C = 0.96$			 $K_e = 0.18$ $C = 0.92$	 $K_e = 0.16$ $C = 0.93$	 $K_e = 0.23$ $C = 0.90$	 $K_e = 0.29$ $C = 0.88$

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Fig. 1

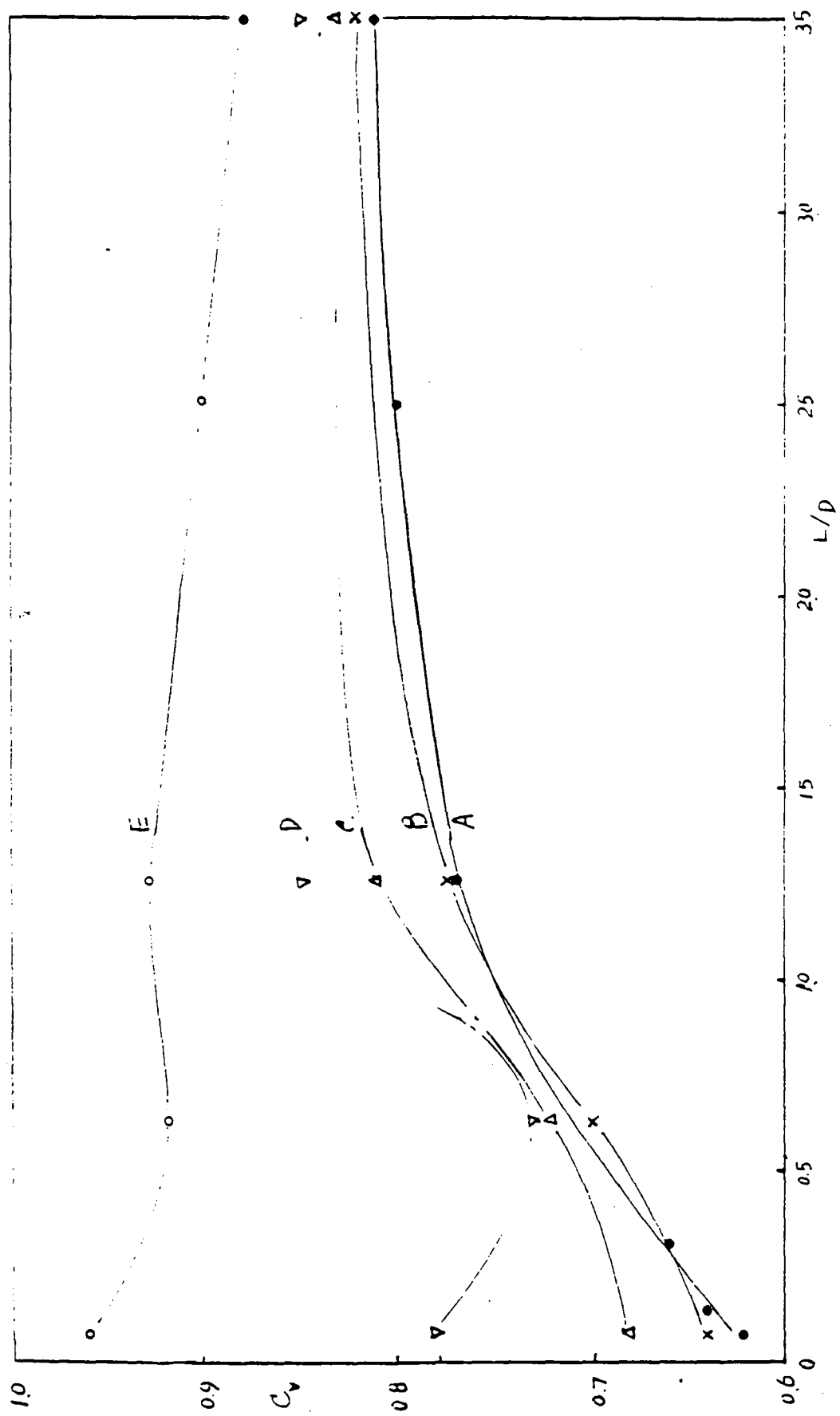


Fig. 2

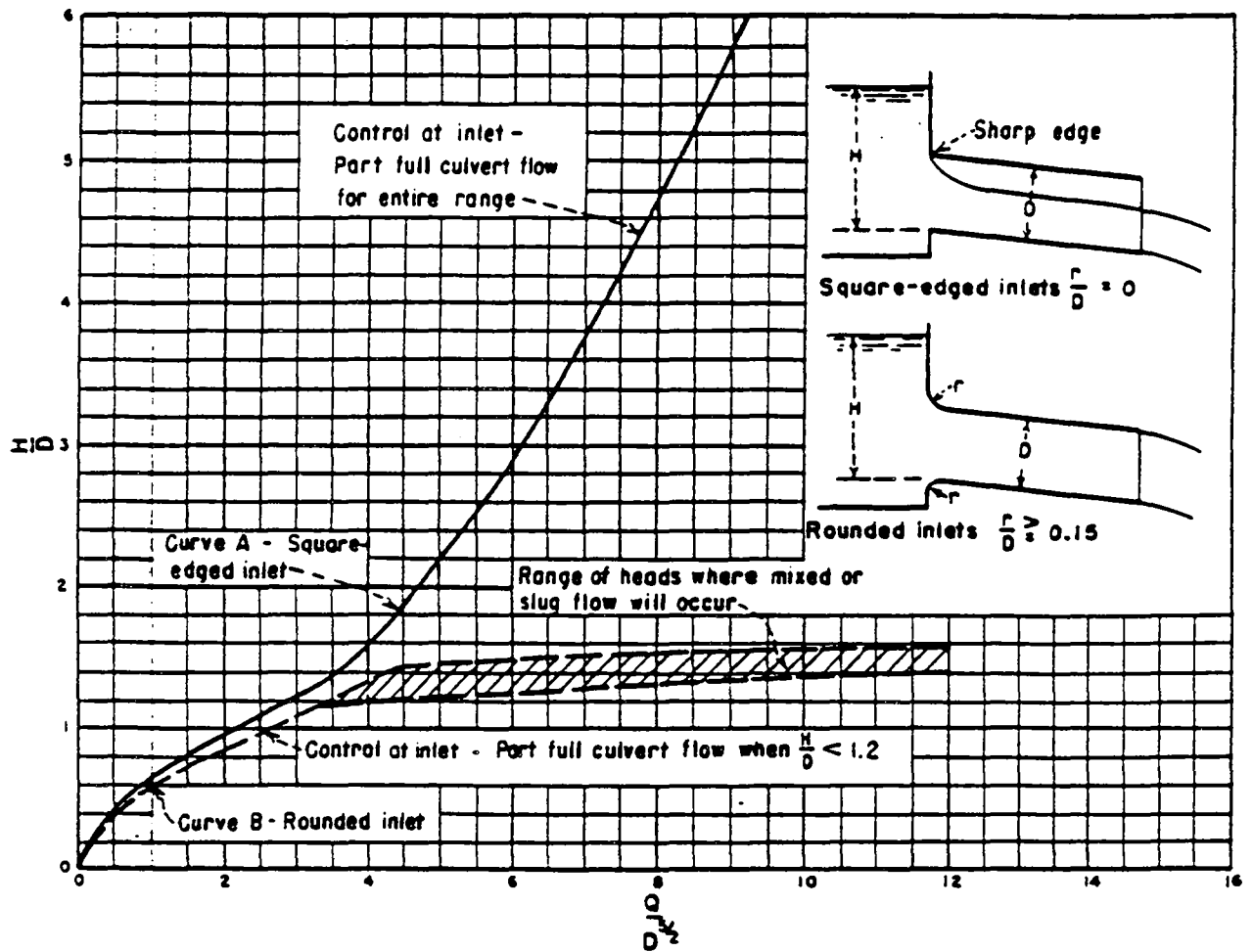


Figure 3 Head-discharge curves for square-edged and rounded inlets for circular culverts on steep slopes.

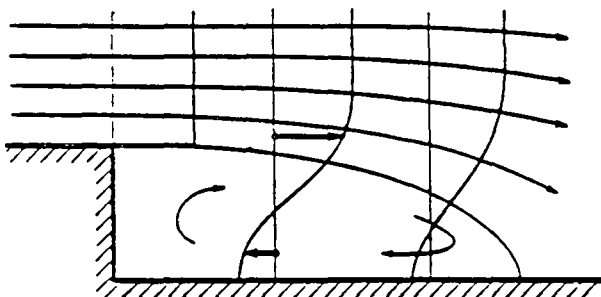


FIG. 4a. Pattern of flow at a limited expansion.

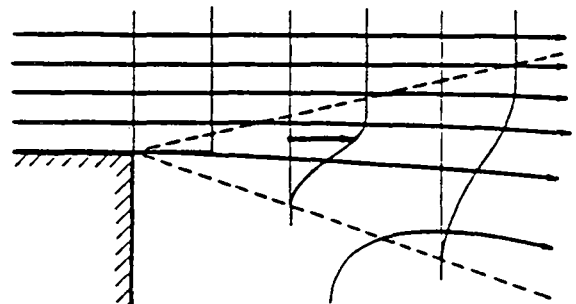


FIG. 4b. Pattern of flow at an unlimited expansion.

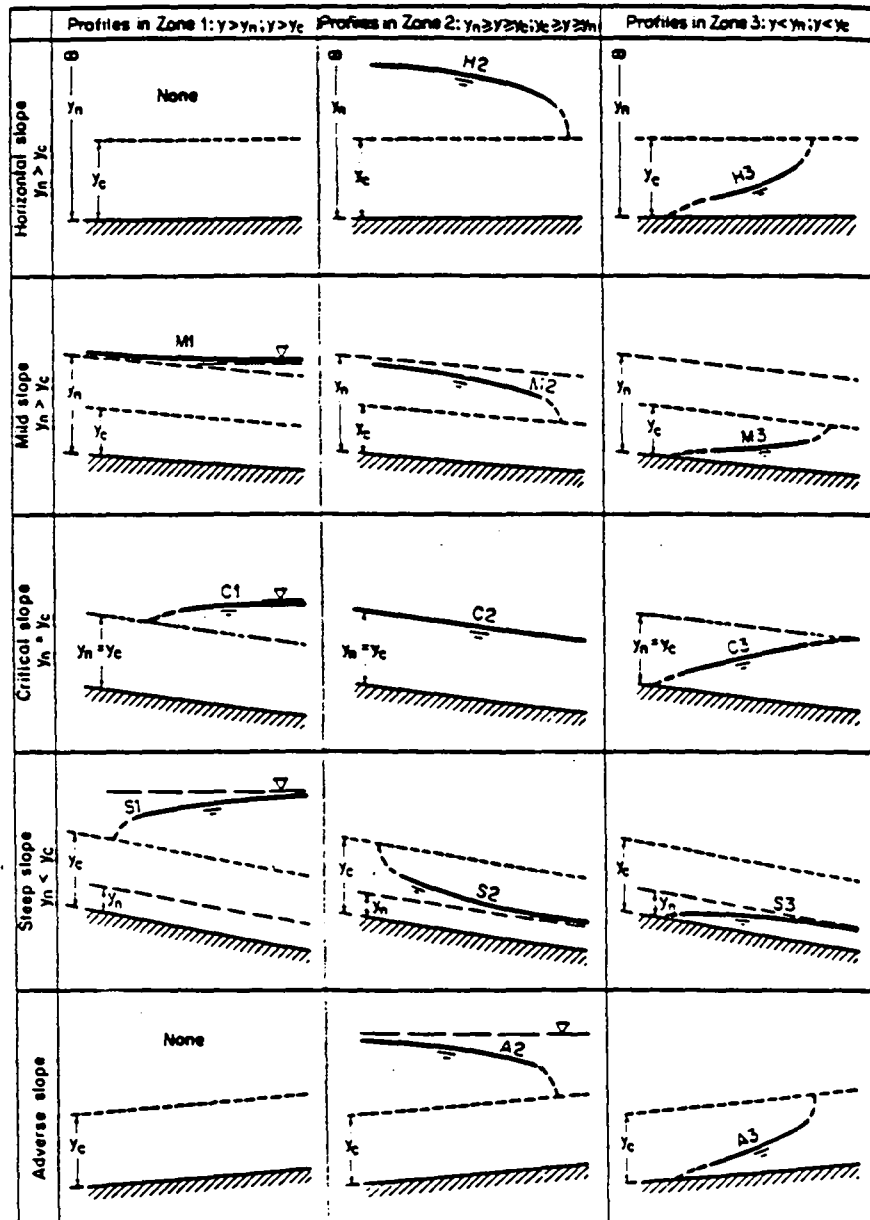


FIG. 5 Classification of flow profiles of gradually varied flow.