FLOW CONDITIONS IN UNIFORM FREE-SURFACE FLOW IN CLOSED CONDUITS

by

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United States Department of the Interior
Bureau of Reclamation
Engineering and Research Center

Translation No.:
Team No.: General
Book No.: 12,688
Paper No.:

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Fliesszustand bei Freispiegel-Normalabfluss in geschlossenen Kanalquerschnitten

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Vladimir Šifalda

Translated from the German

Source:
Wasserwirtschaft, Vol. 76, No. 5, pp. 215-16 (1986)

Translated by:
Ben Teague

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Division of Foreign Activities
Foreign Training and Coordination Branch
Code 2000

USBR Translations
Denver, Colorado
April 1988

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FLOW CONDITIONS IN UNIFORM FREE-SURFACE FLOW IN CLOSED CONDUITS

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SUMMARY

Many design problems call for an assessment of whether flow is subcritical or supercritical. The amount of computer time required will depend on the decision criterion selected. This article shows that the use of the "critical slope" criterion in an appropriate graphical representation offers a quick way to characterize the state of flow.

1. Basic Equations and Definitions

The expression for the critical slope results from the definition of the critical state of flow and from a formula for the hydraulic resistance. Both the initial equations are referred to the relative depth of part-full conduit flow. The solution is demonstrated for the example of a circular cross section.

From the general equation defining critical flow

$$Fr^2 = 1, \text{ where } Fr^2 = \alpha(Q^2B/gA^3), \tag{1}$$

if it is assumed that $\alpha \cong 1.0$, it follows that

$$v_{crit}^2 = g(A/B) = gh_m. \quad (2)$$

Here Fr is the Froude number [1];

α is the Coriolis number [1];

g is the acceleration of free fall = 9.81 m/s^2;

Q is the flow rate, m^3/s;

v_{crit} is the critical velocity, m/s;

B is the width of the water surface, m;

A is the cross-sectional area of the flow, m^2; and

$h_m = A/B$ is the mean depth (or, in Anglo-American terminology, the hydraulic depth), m.

Fig. 1 illustrates the definition of the mean depth.

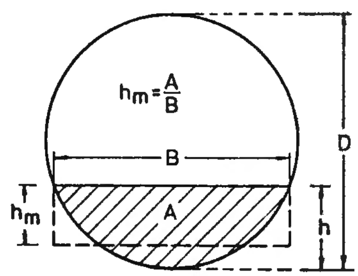

Fig. 1. Definition of "mean [hydraulic] depth" h_m.

Introducing the relative quantity $h_{mr} = h_m/D$, where D is the diameter or cross-sectional height, we put Eq. (2) in the form

$$v_{crit}^2 = gDh_{mr}. \quad (3)$$

If the cross-sectional shape is given, the geometry (Fig. 2) gives the relative mean depth h_{mr} as a function of the relative depth $h_r = h/D$.

For practical reasons, the Manning-Strickler formula is used for the velocity:

$$v = (1/n)R^{2/3}J^{1/2}. \quad (4)$$

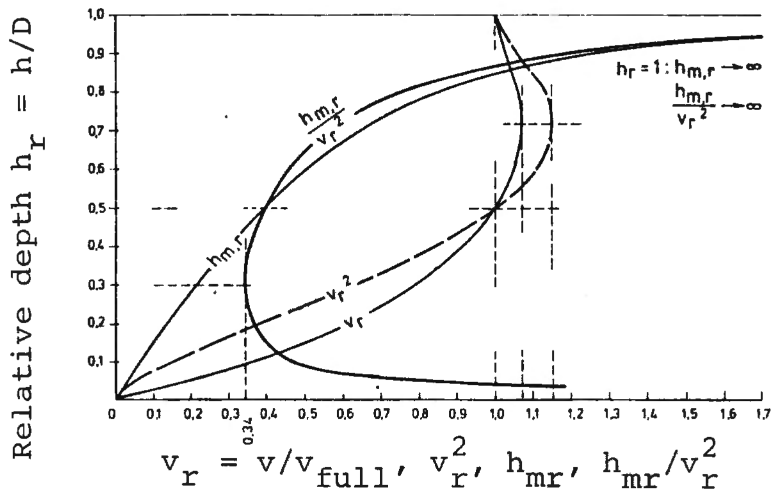

Fig. 2. Dimensionless groups used to determine the critical state of flow in a circular conduit.

The Manning roughness coefficient n is the reciprocal of the Strickler coefficient k_{st}. For the condition of full-cross-section flow in the circular cross section (R = D/4), this formula becomes

$$v_{full} = (1/n)(D/4)^{2/3}J^{1/2}. \tag{4a}$$

For the calculations to be equivalent to the Prandtl-Colebrook calculations with the most commonly selected roughness coefficient k_b = 1.5 mm, the Manning roughness coefficient should be taken in the range of n = 0.013-0.014 (valid for D = 0.25-2.5 m).

In common practice, the velocity for part-full flow is determined not directly from Eq. (4) but from a "part-full velocity curve":

$$v = v_{full}v_r, \tag{4b}$$

where v_r is the relative velocity of part-full flow in the conduit, that is, the abscissa of the part-full velocity curve.

Combining Eqs. (3), (4a) and (4b) yields an expression for the critical slope in the form

$$J_{crit} = 62.3n^2(1/D^{1/3})(h_{mr}/v_r{}^2). \tag{5}$$

Fig. 2 is a plot of the dimensionless group h_{mr}/v_r^2 and the intermediate quantities v_r, v_r^2 and h_{mr}. The part-full curve (v_r) is shown in the usual version with the "Thormann correction."

For a given roughness, the critical slope as a function of the parameters D and h_r is found with Fig. 3. The plot in Fig. 3 is based on the assumption n = 0.014.

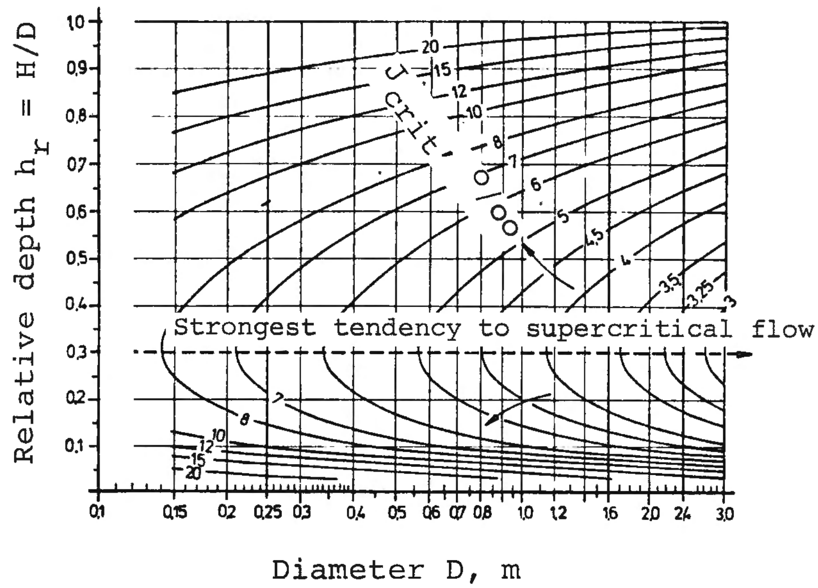

Fig. 3. Critical slopes for circular conduits.

Example of the use of the plot: For diameter D = 1.0 m and bottom slope[1] $J_s = J_{crit}$ = 4.5 °/oo, supercritical flow (shooting) is found in the range of h_r = 0.20 to 0.43. Below and above this range, flow is subcritical (streaming).

2. The J_{crit} Curves

The shape of the J_{crit} curves is strongly influenced by the last factor in Eq. (5), the expression h_{mr}/v_r^2.

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[1]Translator's Note: The symbol °/oo denotes "per thousand"; 4.5°/oo = 0.45%.

For $h_r = 0$ and $h_r = 1$, this expression tends to infinity; it has a minimum at $h_r = 0.30$.

The flow in a closed cross section is always subcritical at very shallow depths and in the transition to full-pipe flow. The tendency to go over to supercritical flow in circular cross sections is strongest at a depth/diameter ratio of 30%.

Therefore, to assess whether supercritical flow can occur at all, it is sufficient to examine the state of flow for $h_r = 0.30$.

For the sizing of cross sections, on the other hand, what is of interest is the condition when the conduit is running nearly full. In such cases, the J_{crit} values corresponding to h_r = approx. 0.70 can be regarded as representative.

The shape of the J_{crit} curves in Fig. 3 holds for flows that do not involve changes in air pressure.

3. Effect of Compression of Air Enclosed in a Duct

The definition of critical flow in Eq. (1) holds provided the pressure conditions over the water surface remain constant during the passage of a flow wave. In closed conduits, this condition does not always hold. If we write p_L for the pressure difference relative to atmospheric pressure and express this pressure difference as the pressure head (p_L/γ_w) in meters water column, the general expression for the specific energy head becomes

$$h_E = h + \alpha \cdot \frac{v^2}{2g} + \frac{p_L}{\gamma_w}$$

and from the critical flow condition

$$\frac{dh_E}{dh} = 1 + \frac{\alpha}{2g} \cdot \frac{d(v^2)}{dh} + \frac{d(p_L/\gamma_w)}{dh} = 0$$

it follows that

$$v^2_{crit} = \frac{1}{\alpha} \cdot g \cdot h_m \cdot (1 + \frac{d(p_L/\gamma_w)}{dh}) \qquad (2a)$$

that is, the previously assumed Eq. (2) is multiplied by $1 + d(p_L/\gamma_w)/dh$. Eq. (5) also takes on a corresponding form:

$$J_{crit} = 62.3 \; n^2 \cdot \frac{1}{D^{1/3}} \cdot \frac{h_{m,r}}{v_r^2} \cdot (1 + \frac{d(p_L/\gamma_w)}{dh}) \qquad (5a)$$

If the vents are blocked, a higher pressure can occur in the leading region of a flow wave and a lower pressure can occur behind the wave. When the pressure is higher, the tendency to supercritical flow is inhibited; when the pressure is lower, this tendency is promoted. As the peak discharge passes in large conduits, the air pressure gradient can exert a perceptible effect. According to my estimate, gradient values of $d(p_L/\gamma_w)/dh$ = 0.1 to 0.5 (i.e., 1 to 5 mbar per 0.1 m rise in water level) can occur in such cases.

4. Conclusion

This discussion of the "critical slope" characteristic was limited to the example of a circular conduit. The method of development can, however, be extended to other cross-sectional shapes provided the geometry of the cross section is stated unambiguously and the velocity curve for part-full flow is known.

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