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WATER HAMMER STUDIES IN A PRESSURE CONDUIT RUPTURING SUDDENLY

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WATER HAMMER STUDIES IN A PRESSURE CONDUIT RUPTURING SUDDENLY

"Etude des Coups de Belier dans une Conduite Sous Pression

Eclatant Brusquement"

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Translated from the French by Marie L. Murphy

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WATER HAMMER STUDIES IN A PRESSURE CONDUIT RUPTURING SUDDENLY

1 - THEORY

1.1. Definition of Water Hammer

The term water hammer is given to pressure increases or decreases which propagate in water-filled conduits and are produced by flow variations occurring at the ends of the conduits.

1.2. Analytical Method

Only the case of a circular-section conduit of constant diameter and wall thickness will be discussed here.

A velocity variation occurring at a point in the conduit is accompanied by a corresponding pressure variation and the resulting instantaneous perturbation quickly becomes a plane wave. Therefore, the velocity and pressure variations in a section are identical at all points of the section.

The chosen x axis is coincident with the axis of the conduit, letting p = pressure and u = velocity of the liquid.

If E designates the modulus of elasticity of the conduit wall, D its diameter, e its thickness, ε the coefficient of elasticity of the liquid, the equation of continuity is written:

$$- \frac{\partial u}{\partial x} = \left(\frac{1}{E} \frac{D}{e} + \frac{1}{\varepsilon} \right) \frac{\partial p}{\partial t} \qquad (1)$$

ρ being the specific mass of the liquid, the general equations of motion and fluid are reduced to an equation which is expressed as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} \qquad (2)$$

$u \dfrac{\partial u}{\partial x}$ being negligible relative to $\dfrac{\partial u}{\partial t}$ the result is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \qquad (3)$$

Presenting

$$\frac{1}{\rho a^2} = \frac{1}{E} \frac{D}{e} + \frac{1}{\varepsilon}$$

it is possible to write $\dfrac{\partial u}{\partial x} = -\dfrac{1}{\rho a^2} \dfrac{\partial p}{\partial t}$ (4)

The equations (3) and (4) make it possible to solve the problem analytically.

Taking derivatives from the two terms of equation (3) with respect to x, and from the two terms of equation (4) with respect to t and by successively eliminating the terms, we obtain

$$\frac{\partial^2 p}{\partial t^2} = a^2 \frac{\partial^2 p}{\partial x^2} \qquad (5)$$

The general integral of this equation is expressed by

$$p = p_0 + F(x - at) + f(x + at) \qquad (6)$$

which indicates that the increased pressure $p - p_0$ may be considered always as the sum of a pressure increase F moving in the direction of the positive x's and of a pressure increase f moving in the direction of the negative x's at velocity a.

$$\frac{\partial p}{\partial t} = -a\frac{\partial F}{\partial t} + a\frac{\partial f}{\partial t}$$

$$\frac{\partial^2 p}{\partial t^2} = a\frac{\partial^2 F}{\partial t^2} + a\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 f}{\partial x^2}$$

1.3. The Louis Bergeron Graphical Method

Since the determination of the functions F and f is very complicated, we have devised a method which makes it possible to solve the problem graphically.

This method is based on the following law:

An observer moves at celerity "a" along a constant characteristic in a conduit. An arbitrarily chosen point of the regime $P-P_o$ is found on a line which passes through the origin of the characteristic in time and space. The angular coefficient is $\pm \frac{a}{gS}$ depending upon whether the wave moves in the opposite direction (+) or in the same direction (-) of the discharge q_o initially chosen as the positive direction.

Therefore, in order to obtain the various regimes encountered by the observer, it is necessary to construct the diagram of pressure variations as a function of the discharge which will be formed by the sloping characteristic $\pm \frac{a}{gS}$. This will be possible if we know the regime at the time and location of the starting point of the observer, as well as the pressures and flows imposed by the conduit.

2 - APPLICATION OF THE LOUIS BERGERON GRAPHICAL METHOD TO A PRESSURE CONDUIT RUPTURING SUDDENLY

2.1. Hypotheses chosen

For a better understanding of the diagrams, the phenomena have been simplified by placing air bubbles at points which facilitated drawing the figures and we have supposed that:

-- the head losses are zero;

-- the water flow velocity in the conduit is very low in relation to the velocity of the wave caused by the flow change;

-- when an air pocket totally occupies a point in the conduit, the flow is slowed because this air pocket moves slowly.

The flow at the end of the conduit is directed toward the rupture point and cannot become negative.

At the rupture point, the pressure is equal to the atmospheric pressure.

2.2. The conduit does not contain any air pockets (Figure 1)

This is a horizontal conduit A B, of length L, with a constant section and wall thickness.

We take as the unit of time $\frac{L}{a}$, which is the time necessary for a wave to travel from A to B; therefore, a perturbation produced at B at time 0 will arrive at A at time 1.

The direction chosen for the flow goes from A toward B.

Supposing that at time 0 the conduit ruptures at B, the observer leaves A (or A_1 the given point) at time 1 when the regime is still normal, the pressure is equal to the test pressure P_o, the flow is zero. For the observer going from A toward B, the regime will be indicated on the diagram as a sloping characteristic $-\frac{a}{gS}$ (- because it moves in the positive direction of the flow); when the observer arrives at B at time 2, the conduit is ruptured and the pressure is now equal to the atmospheric pressure.

The intersection of the given characteristic and of the characteristic p = 1 atm. gives the arbitrary point B_2 of the regime at B at time 2. Arriving at B, the observer takes off again toward A on the sloping characteristic $+\frac{a}{gS}$ (traveling in a negative sense to the discharge). At A, the pressure cannot exceed the water vapor pressure of the test temperature; hence, the arbitrary point A_3 of the regime at time 3.

Moving onward again toward B, the regime goes in the direction of the flow on a sloping characteristic $-\frac{a}{gS}$ or point B_4 at a pressure equal to that of B_2.

By following this line of reasoning, we obtain points A_5, B_6, etc.

The diagram indicates no pressure increase; however, there is cavitation at A, that is, a partial void.

If the asbestos-cement conduit has been designed to withstand outside pressures higher than 1 kg/cm^2, the conduit cannot be damaged.

2.3. <u>The conduit contains one air pocket at its center</u> (Figure 2)

This is a horizontal conduit A B, of length L, with an air pocket located at its center. The unit of time used is the time it takes for a wave to travel from B to C.

Assuming that at time 0 the conduit ruptures at B, the observer arriving at A at time 2, finds the test pressure discharge to be zero, hence point A_2. It then moves on the characteristic $-\frac{a}{gS}$, passing C at time 3 where it encounters a certain amount of pressure due to the expansion of the air pocket, and arrives at time [1] at B_4 where the pressure is equal to one atmosphere. Moving on toward A on a characteristic $+\frac{a}{gS}$, it encounters some pressure at C at time 5 and continues toward A where it finds a pressure equal to the vapor pressure, or point A_6. It is assumed that upon arriving at C at time 7, the air pocket occupies the entire conduit; therefore, the flow (point C_7) is very low if the observer returns toward A (it follows the liquid and does not cross the air pocket), it then arrives at the point of zero flow at A_8, drops again at C_9 - A_{10}, etc.

Based on the diagram, it can be seen that the increased pressures are lower than the operating pressure. The only dangerous phenomenon is cavitation (due to voids) observed at the time of rupture.

2.4. <u>The conduit contains one air pocket at its center and one at its end</u> (Figure 3)

The same units are used as in the preceding case.

If the conduit ruptures at B at time 0, the observer arriving at A finds air pressure and zero flow, or point A_2. It moves on the characteristic $-\frac{a}{gS}$, passing to time 3 at C_3 where it finds a certain pressure due to the air pocket. It arrives at B where the pressure is equal to the atmospheric pressure, or B_4; it takes off again toward A on a sloping characteristic $+\frac{a}{gS}$, it finds at C some pressure due to the expansion of the air bubble, or C_5, and a slightly higher pressure at A, or A_6.

[1] Translator's Note: No time given in original text.

Assuming that at this moment the air pocket occupies the entire conduit at C, the observer is traveling toward C, the regime moves on a sloping characteristic $-\frac{a}{gS}$ up to point C_7, where the flow is very low because of the presence of the air pocket.

Following this line of reasoning, we arrive at points A_8, C_9.

Therefore, it can be seen that the presence of air pockets at both the center and at the ends of the conduit can cause excessive pressures to develop between the air pocket and the end of the conduit; however, this occurs only in the liquid and not in the air of the previously mentioned air pocket.

On the other hand, the construction of the diagram shows that the maximum pressure increase is achieved when the pressure at A_6 is equal to that at A_2, i.e., equal to p_o. The path on the diagram shows this maximum pressure to be equal to $3p_o$.

Physically, the phenomenon of excessive pressure illustrated by the diagram occurs in the following manner: the air pocket under pressure at A imposes on the water in the conduit a certain velocity which may be relatively great until the formation of an air bubble at C suddenly brakes the water motion by provoking the formation of a water hammer wave.

2.5. <u>The conduit contains two air pockets</u> (Figure 4)

This is a horizontal conduit A B, of length L, with air pockets located in the first third and second third of the conduit, i.e., at C and D.

The diagram is drawn on the basis of the preceding reasoning and using as a unit of time, the time taken to travel along one third of the length of the conduit. Two different occurrences may take place, as indicated below.

If the conduit is totally filled by air at C at time 11, but not at all at D, the observer therefore encounters the pressures and flows indicated by the points A_3, D_4, C_5, B_6, C_7, D_8, A_9, D_{10}, C_{11}, D_{12}, A_{13}, D_{14}, C_{15}. There is a pressure drop at A at time 9. A reversal of the flow is noted at D at time 12.

If the conduit is totally filled with air at C at time 11 and at time 12 at D, the observer encounters the pressures and flows indicated by the points A_3, D_4, C_5, B_6, C_7, D_8, A_9, D_{10}, C_{11}, D_{12}; C_{13}. The pressures at C are always lesser than the initial pressure; this is also the case at D.

2.6. <u>The conduit contains three air pockets of which one is at its end</u> (Fig. 5)

This is a horizontal conduit A B, of length L [2/], with three air pockets located at A and C, and at D (its first and second thirds).

This diagram shows the formation of excessive pressure at C when the air pocket at this point brakes the motion of the water. The amplitude of this phenomenon depends solely on the pressure imposed at A by the air pocket and not by the presence of an air pocket at D.

It is obvious from the diagram that the excess pressure cannot exceed $3\,p_o$ as in the case of Item 2.4.

2/ The French text shows "length C "

3 - CONCLUSION

As indicated in the diagrams, the rupture of a conduit causes the appearance of water hammer having a pressure that can exceed three times the test pressure when there is an air pocket at the end of the test pipe. Water hammer can occur only if the air pocket close to the rupture point is sufficiently large to occupy the entire conduit at the moment of the expansion of the air.

These pressure increases, which occur only in the water column, develop and move between the air pocket and the end of the conduit. Water being a slightly compressible fluid, the perturbation in the conduit does not cause any visible rupture, which only appears the next time the conduit is placed under pressure.

There is no formation of excessive pressure in the air pockets.

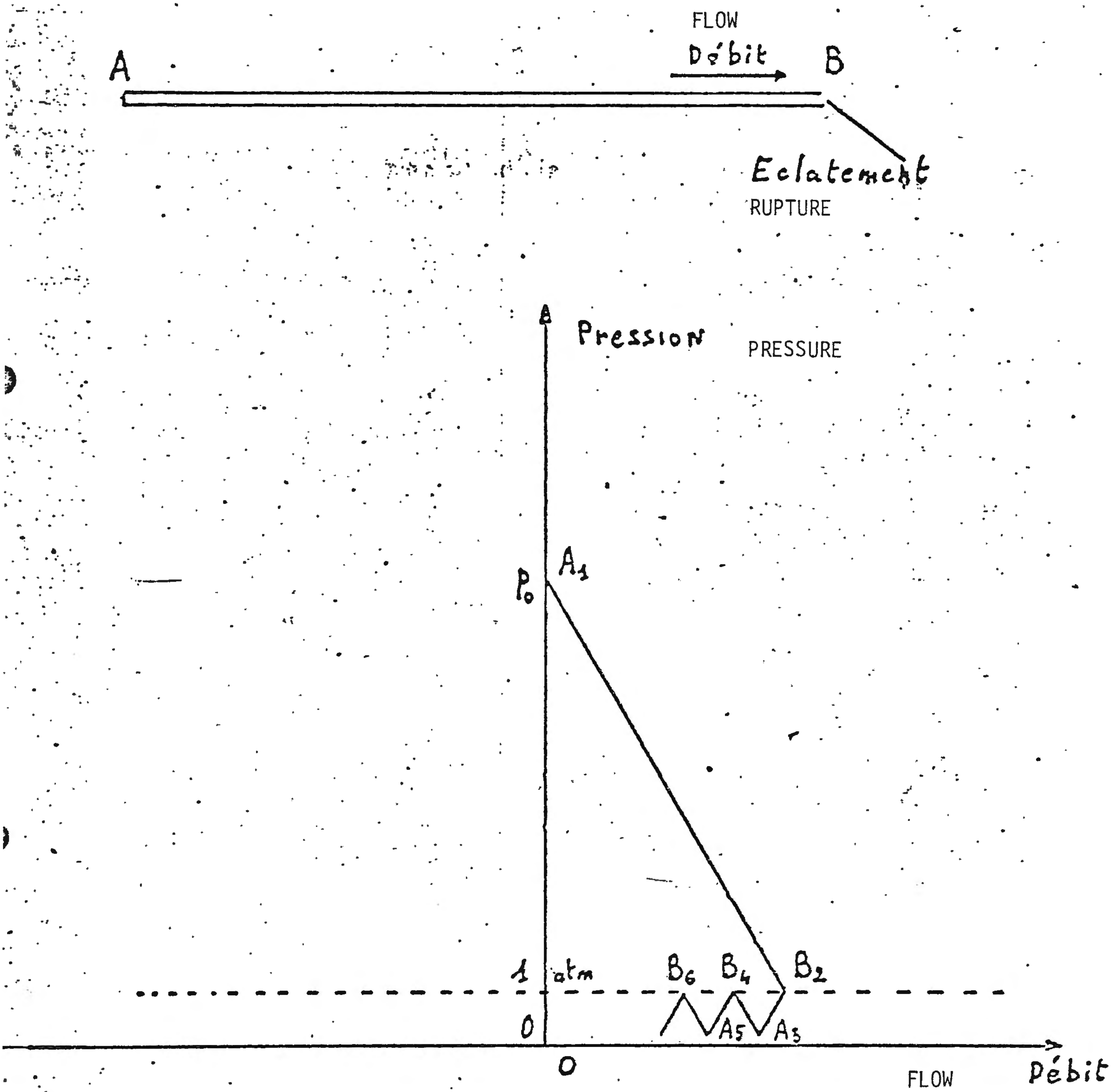
Two remedies are possible to keep the pressure increases caused by the rupture from having a higher value than the test pressure.

Avoiding the formation of air pockets at the ends of the sections of conduit being tested.

Avoiding the testing of conduit sections which have high points where large air pockets can form and which may, with air expansion, occupy the entire conduit.

In cases where it becomes absolutely necessary for the water in the conduit to be under pressure, such as when testing for imperviousness, and if there is an air pocket in the conduit, it is preferable to arrange for this air pocket to be in the center of the conduit and not at its ends.

Fig 1



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PRESSURE
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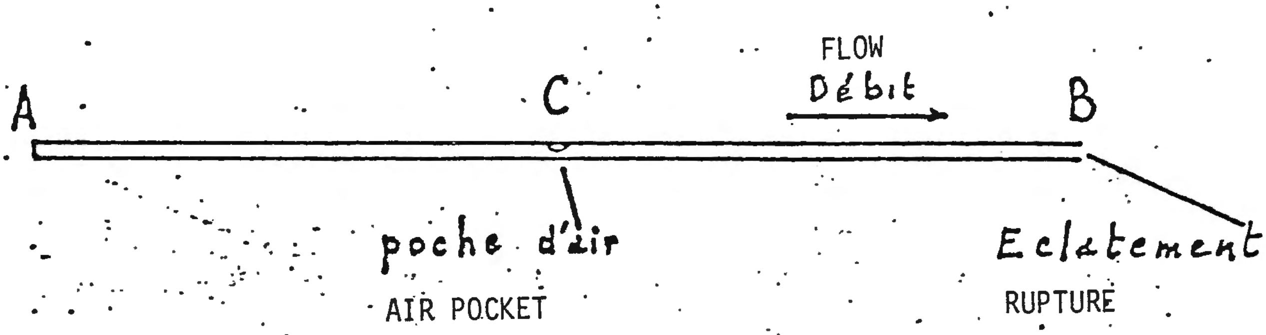
Fig 2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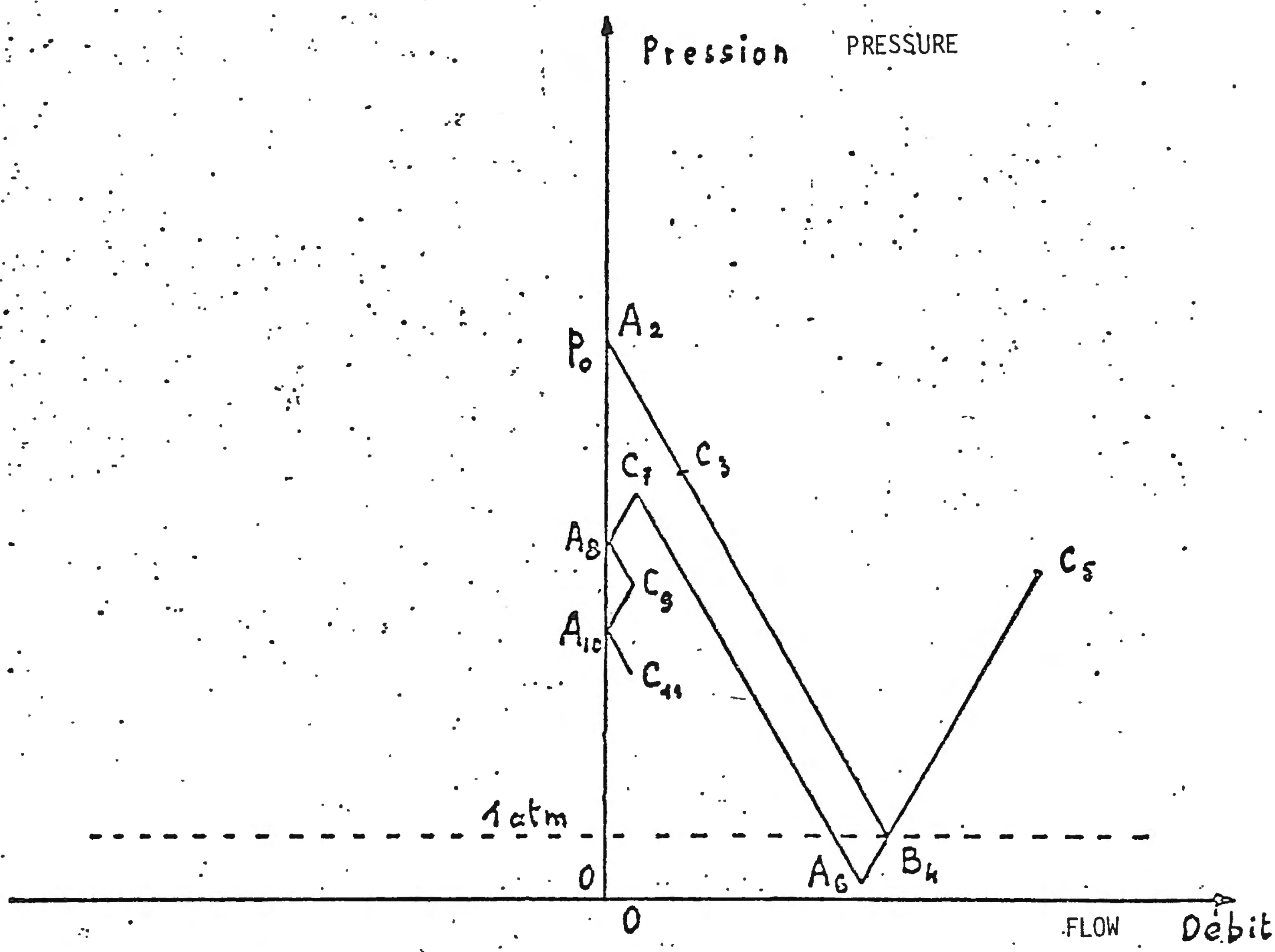
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Fig 3

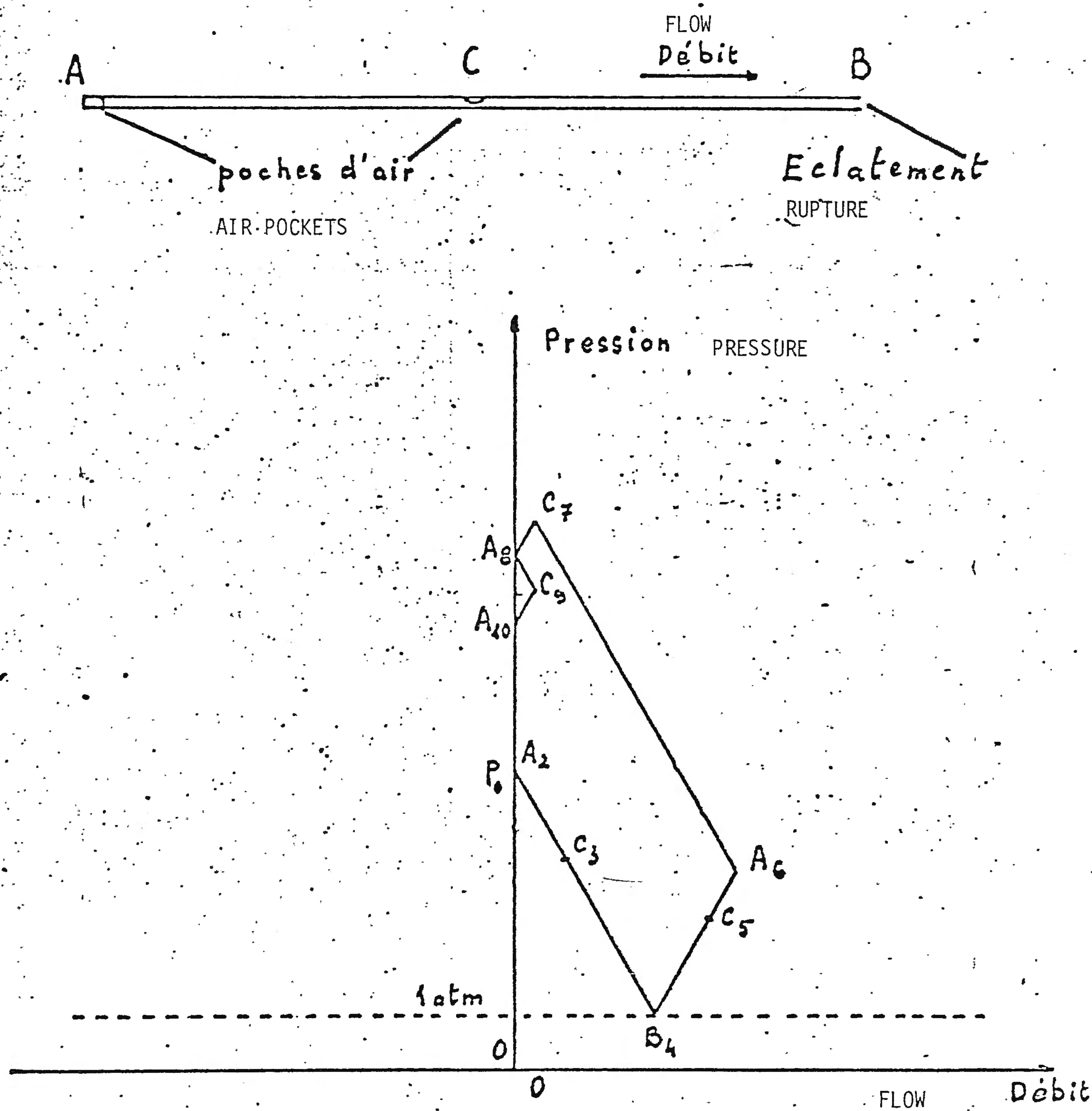
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Fig 4

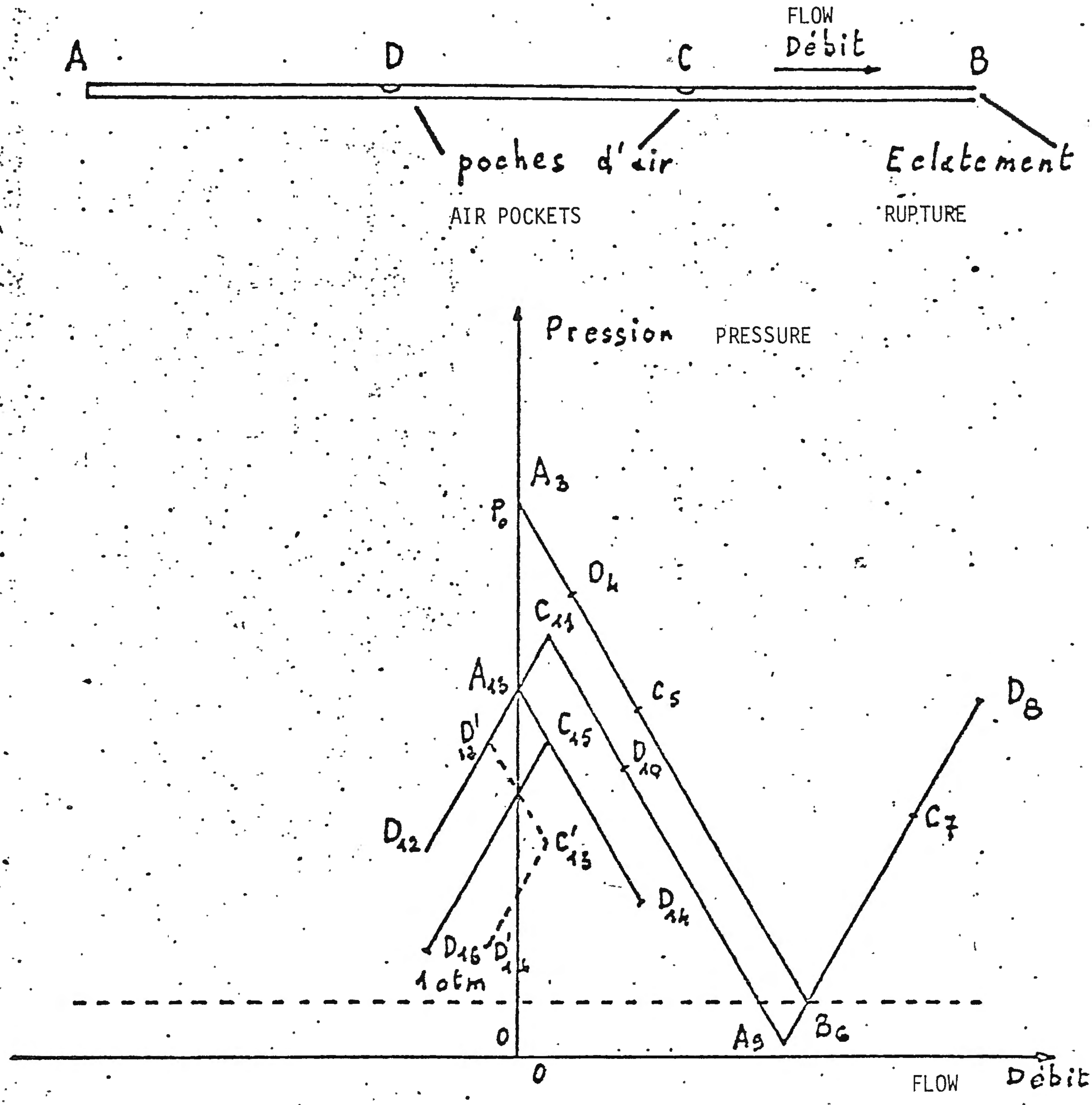
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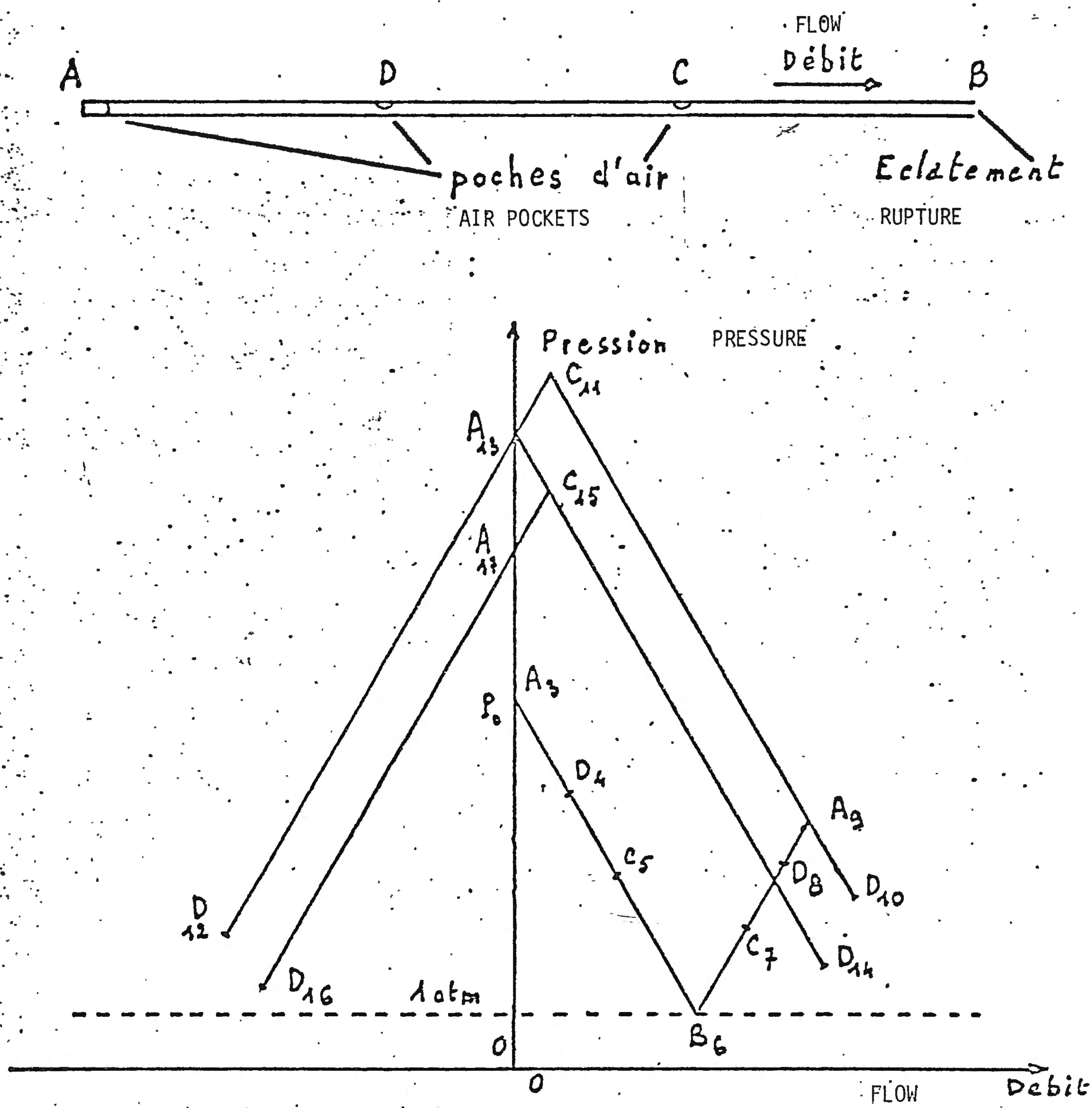
Fig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