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SPECTRAL ANALYSIS AND CHARACTERIZATION OF PRESSURE FLUCTUATION IN FLUID FLOW

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Abstract

This paper treats the basic concepts and calculation methods of spectral analysis for pressure fluctuation. The criteria for choosing the basic parameters are considered and a method for synthesizing multi-point pressure fluctuation and calculating the mean pressure is given. Formulae for evaluating spectral densities are given and the spectral densities for some typical flow patterns are briefly discussed.

I. Introduction

Pressure fluctuation in water flow is one of the important problems in hydraulic engineering and hydraulic research. The extensive construction of high dams and light dams has revealed a number of high-speed flow problems. Investigators in hydraulics are most concerned with the dynamic effects of the water flow. The vibration problems of hydraulic structures such as sluice gates, inlet towers, overflow dams, arch dam overflow, overflow-type building structures and steel conduits under the dynamic effects of water flow are important research topics of today and have attracted broad attention here and abroad. Am important aspect of the vibrational research is the study of the dynamic characteristics of water flow or the vibration source under various flow conditions. In some cases pressure fluctuation is also a consideration in the cavitation erosion of hydraulic structures.

The research of pressure fluctuation in water flow can be traced back to the 1930's [1]. Extensive data have been accumulated on the pressure fluctuation load of different types of water outlet structures in prototype and model experiments. As the measurement technique progressed from non-electrical methods to electrical transduction, the study of pressure fluctuation advanced a major step and the new method provided the possibility of recording the temporal history of the pressure fluctuation $x(t)$. The oscillogram not only clearly shows the changing characteristics, but also affords further analysis. In China, the research on pressure fluctuation began to receive attention in the late 1950's. Beijing Hydraulics and Hydroelectric Power Institute, Qinghua University, Dalian Engineering College, Tianjin University, and Nanjing Hydraulics Research Institute have all studied engineering problems using prototype observation and model experiments, and have used electrical measurement techniques extensively.

Over the years a number of problems remained regarding the analysis of pressure fluctuations $x(t)$. As is well known, the pressure fluctuation $x(t)$ is generally random. Even though attempts have been made to find representative frequency and pulse amplitude on the random $x(t)$ curve, a set of artificial "criteria" was often used in the selection of the waveforms. For example, N. A. Puleabulazhinskii *et al.* [A/] only looked at the maximum amplitude for a given time interval. N. P. Zrelov *et al.* first determined the mean pressure line and then calculated the maximum amplitude above or below the mean pressure line, and N. P. Loznov took the average of the two largest amplitudes. In China some use the average of several of the largest pulse amplitudes and others use the pulse amplitude corresponding to the principal frequency in the amplitude frequency distribution curve. A probability distribution repre-

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A/ Translator's note: This and the next two Russian names have been transliterated from the Chinese, so they are purely educated guesses on our part.

sentation using statistical methods has also been used. The fluctuation frequency has been treated using interval average and probability analysis. These analysis methods share the following shortcomings:

1. Artificial "criteria" are used in the selection of waveforms in both the frequency analysis and the amplitude analysis of the fluctuation curve.

2. Representative values of the amplitude and frequency must be found.

As a result, the methods above lacked a comprehensive scientific basis and neglected the random nature of the pressure fluctuation and cannot accurately represent the objective laws in pressure fluctuation.

The principle of stochastic functions is a new branch of mathematics developed in the 1950's. It provided a theoretical basis for stochastic data analysis, and quickly found extensive applications in space, aviation, navigation, physics, earthquake engineering, electronics, control, hydrometeorology, geology, petroleum exploration, medicine, biology, economics and industrial management. Large high-speed computers have provided the necessary prerequisites for the general application of stochastic data analysis and led to the rapid development of this new branch of mathematics.

In recent years stochastic functions have begun to be used in the analysis of pressure fluctuation here and abroad, but many problems still remain unsolved. Based on the work done in previous years and the fundamental concepts of random functions, we shall discuss the calculation methods in this paper and analyze the spectral characteristics of some typical flow patterns.

II. Basic Concepts

The mathematical meaning of the basic concepts discussed here often can be found in probability theory or random function courses, but rarely in the context of hydraulics. It is therefore necessary to introduce some fundamental concepts before a discussion on spectral analysis is given.

Figure 1 shows actually measured waveforms $x_k(t)$ of pressure fluctuation. They are all random functions of time and the value for a given time is the random variable. $x(t)$ may be the pressure at a given point or the load acting on a given surface and it depends on the choice of transducer. In the space below we briefly discuss the methods of treating such waveforms and the techniques of finding the representative data and curve.

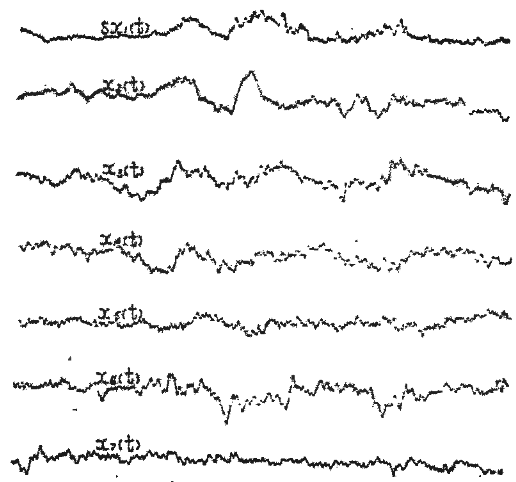

Figure 1. Typical pressure fluctuations at a point.

(1) Random variable

As is well known in probability theory, a random variable ξ is an irregular and unpredictable quantity for a given test condition. Random variables can be divided into continuous random variables and discrete random variables. For a given real number x, there is a corresponding probability. If $\{x_j\}$ represents all the possible values of the discrete random variable ξ and $P_j(j = 1, 2, \ldots, n)$ represents the probability that ξ has a value x_j, then

$$P\{\xi = x_j\} = p_j \qquad (j = 1,\ 2,\ \cdots\cdots,\ n) \qquad (2\text{-}1)$$

For a continuous random variable in an arbitrarily small interval, the corresponding probability is

$$F(x) = P(\xi < x) \tag{2-2}$$

where $F(x)$ is known as the distribution function. Equations (2-1) and (2-2) are the distribution laws of the random variable. If the distribution law of a random variable is known, then the entire set of characteristics can be understood.

For a discrete random variable, the distribution function may be written as

$$F(x) = \sum_{x_j < x} P(x = x_j) \tag{2-3}$$

In reality it is often difficult to determine the analytic form of the distribution function and sometimes a knowledge of some average characteristics may suffice in the description of the changing characteristics. Origin moments and center moments are often used in the description. The first order origin moment is also known as the mathematical expectation, generally written as

$$E[x] = m_x = \frac{1}{n} \sum_{j=1}^{n} x_j \tag{2-4}$$

It is an important feature of the random variable, but it only represents the static characteristics of a random variable. In terms of pressure fluctuation in water flow, this is the mean pressure. The difference between the random variable x and its mathematical expectation is called the centralized random variable and it may be used to express the fluctuating part of the pressure, which is the quantity of interest in our study. The k-th order central moment is defined as

$$\mu_k = E[(x_j - m_x)^k] \tag{2-5}$$

and it is the k-th order mathematical expectation of the fluctuating part of the pressure. We now shall describe the special physical significance when k is equal to 1, 2, 3 and 4.

The first order central moment is the mathematical expectation value of the pressure fluctuation above and below the mean pressure and this average value is identically zero:

$$\mu_1 = E[(x_j - m_x)] = 0 \qquad (j = 1, 2, \cdots\cdots, n) \tag{2-6}$$

The second order central moment is the square deviation of the fluctuation and is an important indicator in evaluating the intensity of the pressure fluctuation:

$$\mu_2 = E[(x_j - m_x)^2] = \frac{1}{n} \sum_{j=1} (x_j - m_x)^2 \tag{2-7}$$

The third order moment is given by

$$\mu_3 = E[(x_j - m_x)^3] = \frac{1}{n} \sum_{j=1}^{n} (x_j - m_x)^3 \tag{2-8}$$

and the fourth order moment is given by

$$\mu_4 = E[(x_j - m_x)^4] = \frac{1}{n} \sum_{j=1}^{n} (x_j - m_x)^4 \tag{2-9}$$

μ_3 and μ_4 are important physical quantities related to the distribution characteristics of the pressure fluctuation, but the following parameters are more widely used in practice:

$$\sigma = \sqrt{\mu_2} \tag{2-10}$$

$$s = \frac{\mu_3}{\sigma^3} \tag{2-11}$$

$$E = \frac{\mu_4}{\sigma^4} - 3 \tag{2-12}$$

where σ is known as the mean square deviation or standard deviation and represents the spread of the random variable. It is a representative indicator of the intensity of the pressure fluctuation. S is called the skew or the asymmetry factor of the distribution and $S = 0$ when the distribution curve is symmetric. E , known as the elevation, is the sharpness of the distribution preak and $E=0$ for a normal distribution. Knowing S and E, one can roughly estimate the distribution characteristics of the pressure fluctuation. The digital characteristics of these random variables are important parameters in the description of the pressure fluctuation.

(2) **Random Process**

The pressure fluctuation in a water flow usually has a waveform like that shown in Figure 2. This is an actual record for a given point under a specific set of conditions. The abscissa t is the time and the ordinate $x(t)$ is the pressure. Obviously, the value of $x(t)$ in the figure changes with time in an arbitrary manner and one cannot predict the value of $x(t+\Delta t)$ at $t+\Delta t$ from the value of $x(t)$ at time t. Another way of looking at this property is that for the same flow conditions the n waveforms measured at the same point but at different times $x_k(t)$, $k=1, 2, \ldots, n$ are all different. Such a function is called a random function of the independent variable t and the process is called a random process. Pressure fluctuation is a random process.

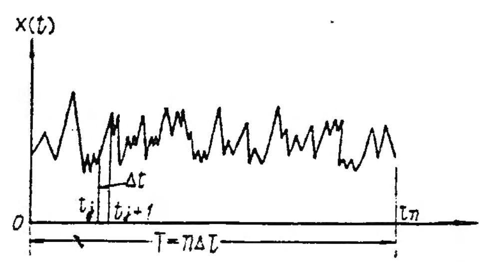

Figure 2. Diagram depicting the calculation method.

(3) **Steady random process and state history**

If the statistical characteristics of a random process $x(t)$ do not change with time, that is, $x(t-\tau)$ and $x(t)$ have the same statistical properties with respect to a time translation τ, then $x(t)$ is called a steady random process. For such a process the distribution function is invariant under a time translation τ:

$$F_{t+\tau}(x)=F_t(x) \tag{2-13}$$

Generally speaking, if the main conditions producing the random phenomenon are invariant in time, the process will be a steady random process. In the study of the water flow, if the controlling parameters such as the structure, boundary conditions, upstream and downstream water levels, gate position and the aerating condition of the pipe remain unchanged, then the statistical characteristics of the flow will also be time independent. Therefore, for given boundary conditions and flow conditions, the statistical properties of pressure fluctuation in a steady flow are invariant in time and the process may be regarded as a steady random process. For a steady random process the statistical characteristics are independent of the measurement time.

Under identical flow conditions, n measurements of the pressure fluctuation at the same location $x_k(t)$, $k = 1, 2, \ldots, n$ constitute an ensemble. An ensemble is made up of n samples all looking different but sharing the same statistical characteristics. Strictly speaking, the statistical characteristics of the pressure fluctuation should be found from the ensemble $x_k(t)$, $k = 1, 2, \ldots, n$, but for a steady random process measured over a long enough period of time the ensemble average may be replaced by the time average. This will greatly simplify the analysis and allow us to obtain the statistical characteristics of physical quantities on which a large number of sampling is not possible.

It should be pointed out that rigorous mathematical proof of the state history is often difficult, but experience has shown that almost all meaningful steady random processes possess the state history property. Many application problems are therefore examined by assuming the state history property. For steady flows with fixed flow conditions, the pressure fluctuations are analyzed as a steady random process with state history. The ensuing analysis is based on this assumption.

(4) **Correlation function and spectral density**

1. Correlation function

We use the correlation function to represent the relationship of the pressure fluctuation $x(t)$ between t and $t+\tau$. It is defined as

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t+\tau)\, x(t)\, dt \tag{2-14}$$

and has the following properties:

① $$R_x(\tau) = R_x(-\tau) \tag{2-15}$$

② $$R_x(0) = \sigma^2 \tag{2-16}$$

③ $$|R_x(\tau)| \leqslant R_x(0) \tag{2-17}$$

2. Spectral density

We wish to know the relationship between the pressure fluctuation at a certain frequency and the corresponding energy density, that is, the characteristics of the pressure fluctuation in the frequency domain. Since the correlation function and the spectral density are related by a Fourier transform, a time domain function may be transformed into a frequency domain function. That is,

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau)\, e^{-i\omega\tau}\, d\tau \tag{2-18}$$

and

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega)\, e^{i\omega\tau}\, d\omega \tag{2-19}$$

Once the correlation function $R_X(\tau)$ is found, the spectral density $S_X(\omega)$ may be calculated using equation (2-18). The general relationships are:

$$S_x(\omega) = \frac{1}{\pi} \int_0^{\infty} R_x(\tau) \cos \omega\tau \, d\tau \tag{2-20}$$

$$R_x(\tau) = 2\int_0^{\infty} S_x(\omega) \cos \omega\tau \, d\omega \tag{2-21}$$

and the special cases are:

$$S_x(0) = \frac{1}{\pi} \int_0^{\infty} R_x(\tau) \, d\tau \tag{2-22}$$

$$R_x(0) = 2\int_0^{\infty} S_x(\omega) d\omega \tag{2-23}$$

The spectral density function $S_x(\omega)$ shows the distribution of the fluctuation energy, as a function of frequency ω, of the fluctuation load $x(t)$. It is a desomposition of the pressure fluctuation in terms of frequency. Our goal is to analyze the vibration of the structure by finding the region of large fluctuation energy in this decomposition. In other words, we wish to arrive at the output spectrum $S_y(\omega)$ by multiplying the input load spectrum $S_x(\omega)$ with the transfer function $H(i\omega)$ of the system:

$$S_y(\omega) = |H(i\omega)|^2 S_x(\omega) \tag{2-24}$$

The correlation function $R_y(\tau)$ can then be obtained by an inverse Fourier transform. This application in random vibration falls outside the scope of this work and will not be discussed here. The readers are referred to [6] for details.

III. Calculation Methods

(1) Selection of basic parameters [9]

In the analysis of the random waveforms of the pressure fluctuation some elementary parameters must first be selected. Since causal relationships exist between the parameters, the selection is not arbitrary and the selection closely affects the analysis results and errors. The selected parameters are the step length Δt, the number of points N, the record length T and the delay m. These parameters are not only related but they also affect the maximum frequency f_N, the equivalent resolution bandwidth B_e of the power spectrum and the standard error ϵ_r. The following simple relations exist among the parameters:

$$\Delta t = \frac{1}{2f_N} \tag{3-1}$$

$$T = N\Delta t \tag{3-2}$$

$$m = \frac{1}{B_e \Delta t} \tag{3-3}$$

$$\epsilon_r^2 = \frac{m}{N} \tag{3-4}$$

Naturally, the 7 parameters should be considered simultaneously because if three of them are chosen the remaining four can be calculated using equations (3-1)-(3-4).

1. Choosing Δt

From equation (3-1), once Δt is chosen, f_N is fixed and the analysis range $0 \sim f_N$ is also fixed. f_N is sometimes called the Nyquist cut-off frequency. The highest possible frequency in an actual flow should fall within this frequency range.

2. Choosing the record length T

In an actual computation it is neither possible nor necessary to choose T as infinity. In general, the larger the T, the closer it is to reality and the smaller the cut-off error. But a large T means more computation, which sometimes is limited by the capacity of the computer. From equation (3-2) the number of points N is fixed once T is chosen.

3. Choosing m

After Δt and N are chosen, the choice of m has a large effect on B_e and ϵ_r. Equation (3-3) shows that the resolution bandwidth B_e is fixed after m is chosen.

We have tested different record lengths and some of the typical results are shown in Table 1. The spectral density curves for $\Delta t = 0.004$ s are shown in Figure 3. Evidently, when we choose $m/N = 0.1$, increasing the record

Table 1. Relationship between elementary parameters

Δt (s)	N	T (s)	f_N (Hz)	m	ϵ_r	B_e (Hz)
0.01	400	4	50	40	0.316	2.5
〃	1600	16	〃	160	〃	0.625
0.004	400	1.6	125	40	〃	6.25
〃	1600	6.4	〃	160	〃	1.56

length cannot reduce the error, but merely improves the frequency resolution. A comprehensive consideration should therefore be made for all the parameters before performing the calculation. For example, when certain requirements are expected of B_e, ϵ_r and f_N, the proper values of Δt, T and m may be chosen.

(2) **Calculation Procedure**

1. First the mathematical expectation value m_x is calculated, and the data obtained are then centralized by substituting $x_j - m_x$ with x_j, and then the central moments are calculated.

2. Calculate the correlation function: equation (2-14) is changed into the following useful form:

$$R_x(\tau) = \frac{1}{n-m} \sum_{j=1}^{n-m} x(t_j + m)\, x(t_j) \tag{3-5}$$

3. Initial estimate of spectral density: Integrate equation (2-18) using the trapezoid formula,

$$S'_{x_{Kf}} = 2\Delta t\left[R_x(0) + 2\sum_{r=1}^{m-1} R_x(r)\cos\frac{\pi kr}{m} + R_x(m)\cos\frac{\pi k}{m}\right] \qquad (k=0,\ 1,\ \cdots\cdots,\ m) \tag{3-6}$$

4. Smoothing the spectrum: Because of the errors produced by a finite T and a discrete x_j, we must smooth $S'x_{kf}$. We used

$$S_{x_K} = 0.23\, S'_{x_{K-1}} + 0.54\, S'_{x_K} + 0.23\, S'_{x_{K+1}} \qquad (k=1,\ 2,\ \cdots\cdots,\ m-1) \tag{3-7}$$

for the two end-points

$$\begin{cases} S_{x_0} = 0.54\, S'_{x_0} + 0.46\, S'_{x_1} \\ S_{x_m} = 0.46\, S'_{x_{m-1}} + 0.54\, S'_{x_m} \end{cases} \tag{3-7'}$$

The spectral curves in Figure 3 are after smoothing.

IV. Synthesis of Pressure Fluctuation and Spectral Density of the Mean Pressure Fluctuation

Up to this point we have only discussed the analysis of a single random process or the method for treating the pressure fluctuation data at one point. In the design of hydraulic structures, consideration of the dynamic effects often requires a knowledge of the pressure fluctuation on a certain interface or the two-dimensional distribution of the pressure fluctuation.

In the past the simulation of an area pressure fluctuation in a model was often achieved by direct measurement with an area pressure transducer and the relationship between the point pressure fluctuation and the area pressure fluctuation was studied. This type of study remains in the exploratory stage because of the complexity of the apparatus and difficulties in the analysis.

The measurement of a plane pressure fluctuation for a prototype would require even more elaborate measurement and will be very difficult to implement. As a result, a study of the synthesis of pressure fluctuation and computation method of the spectral density of the mean pressure fluctuation are urgently needed. Some preliminary attempts were made in [6] which we shall now describe.

The pressure fluctuation X on a plane A is a random function of the

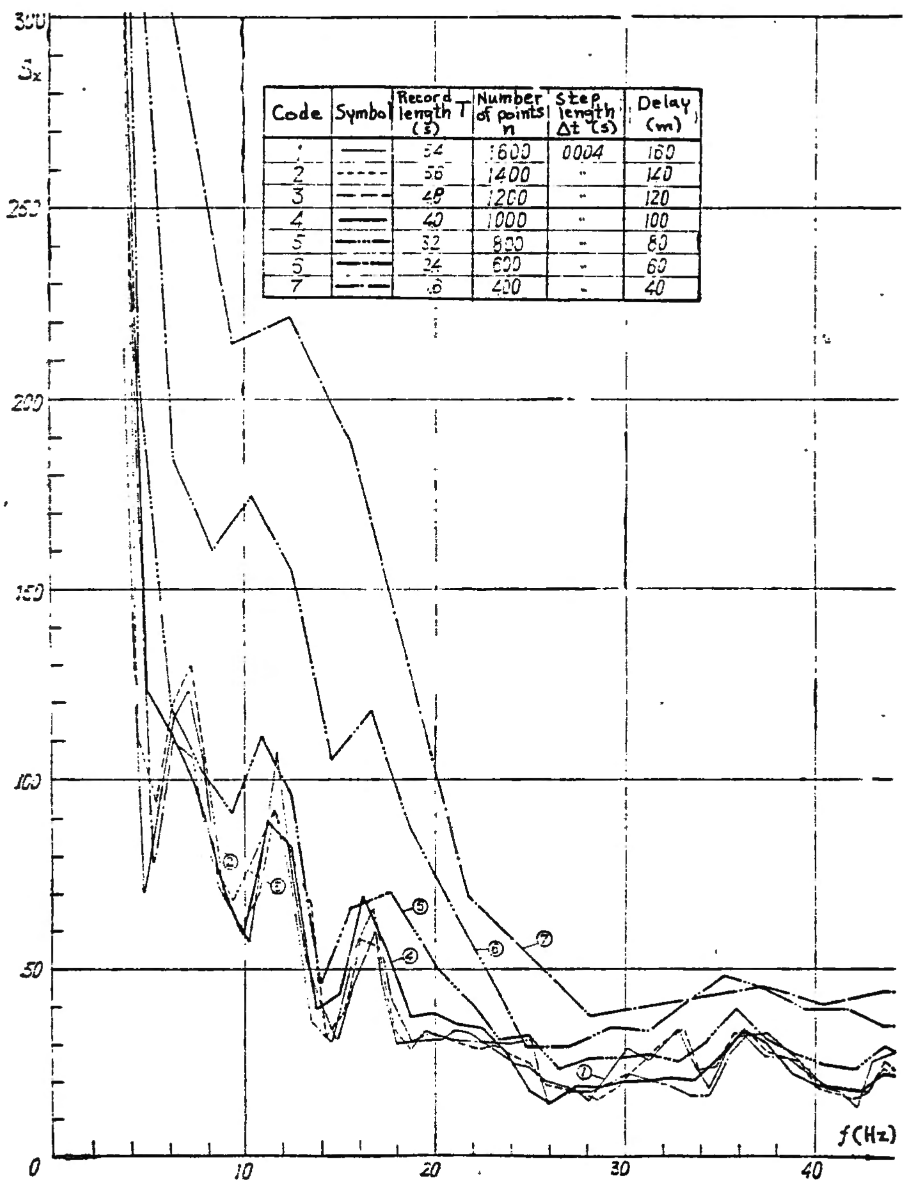

Figure 3. Relationship between spectral curves and record length.

plane coordinates (ξ, η) and time t. In other words, $X(\xi, \eta, t)$ is a three-dimensional random field. Assuming $X(\xi, \eta, t)$ is a uniform and steady three-dimensional random function, then the correlation function and the spectral density may be defined as follows:

$$R_x(\xi', \eta', \tau) = \lim_{\substack{T\to\infty \\ \Xi\to\infty \\ H\to\infty}} \frac{1}{2T} \frac{1}{2\Xi} \frac{1}{2H}$$

$$\cdot \int_{-T}^{T} \int_{-\Xi}^{\Xi} \int_{-H}^{H} x(\xi, \eta, t)\, x(\xi+\xi', \eta+\eta', t+\tau)\, d\xi d\eta dt \tag{4-1}$$

$$S_x(\omega_1, \omega_2, \omega_3) = \left(\frac{1}{2\pi}\right)^3 \iiint_{-\infty}^{\infty} R_x(\xi', \eta', \tau)\, e^{-2\pi i(\xi'\lambda_1+\eta'\lambda_2+\tau f)} d\xi' d\eta' d\tau \tag{4-2}$$

While

$$R_x(\xi', \eta', \tau) = \iiint_{-\infty}^{\infty} S_x(\omega_1, \omega_2, \omega_3)\, e^{i2\pi(\xi'\lambda_1+\eta'\lambda_2+\tau f)}\, d\omega_1\, d\omega_2\, d\omega_3 \tag{4-3}$$

where ξ and η are respectively the coordinates on plane A along the flow direction and perpendicular to the flow direction;

ξ' and η' are the increments of ξ and η, respectively;

$\omega_1 = 2\pi\lambda_1$, $\omega_2 = 2\pi\lambda_2$, $\omega_3 = 2\pi f$;

λ_1 and λ_2 are the wavenumbers in the ξ and η directions, respectively, and f is the frequency.

The spectral density for a different location is expressed as

$$S_x(\xi, \eta, \omega_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\xi, \eta, \tau)\, e^{-i2\pi f\tau} d\tau \tag{4-4}$$

and the wavenumber spectra in the ξ and η directions are, respectively,

$$S_x(\omega_1, \eta, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\xi', \eta, \tau)\, e^{-i2\pi\xi'\lambda_1} d\xi' \tag{4-5}$$

and

$$S_x(\xi, \omega_2, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\xi, \eta', \tau)\, e^{-i2\pi\eta'\lambda_2} d\eta' \tag{4-6}$$

Equation (4-2) is the wavenumber-frequency spectrum of the pressure fluctuation $x(\xi, \eta, t)$ and equation (4-3) is the time-space correlation function.

The pressure fluctuation in a two-dimensional flow may be simplified to a two-dimensional random process and there exists the following basic relationships:

$$R_x(\xi', \tau) = \lim_{\substack{T\to\infty \\ \Xi\to\infty}} \frac{1}{2T} \frac{1}{2\Xi} \int_{-T}^{T}\int_{-\Xi}^{\Xi} x(\xi, t)\, x(\xi+\xi', t+\tau)\, d\xi\, dt \tag{4-7}$$

$$S_x(\omega_1, \omega_3) = \left(\frac{1}{2\pi}\right)^2 \iint_{-\infty}^{\infty} R_x(\xi', \tau)\, e^{-i2\pi(\xi'\lambda_1+\tau f)} d\xi'\, d\tau \tag{4-8}$$

and

$$R_x(\xi', \tau) = \iint_{-\infty}^{\infty} S_x(\omega_1, \omega_3)\, e^{i2\pi(\xi'\lambda_1+\tau f)} d\omega_1\, d\omega_3 \tag{4-9}$$

The frequency spectrum and the wavenumber spectrum for different locations are given by

$$S_x(\xi, \omega_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\xi, \tau)\, e^{-i2\pi f\tau} d\tau \tag{4-10}$$

$$S_x(\omega_1, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\xi', \tau)\, e^{-i2\pi\xi'\lambda_1} d\xi' \tag{4-11}$$

For n measurement points equally spaced along the flow direction on the bottom surface A under a two-dimensional flow, the pressure fluctuations are

$$x_k(t) \qquad (k = 1, 2, \cdots\cdots, n)$$

or

$$x(\xi_k, t) \qquad (k = 1, 2, \cdots\cdots, n)$$

Suppose $\xi_{k+1} - \xi_k = \Delta\xi_k$ is a constant and the width of the observation plane is b ; then,

$$A = \sum_k b\,\Delta\xi_k = \sum_k \Delta A_k \tag{4-12}$$

The total fluctuation load acting on surface A may be defined as

$$P(t) = \int_A x_k(t)\,dA \doteq \sum_k x_k(t)\,\Delta A_k \tag{4-13}$$

and the spatial mean pressure fluctuation will be

$$x_m(t) = \frac{P(t)}{A} = \frac{1}{n}\sum_k x_k(t) \tag{4-14}$$

Based on the discussion above, the mean pressure fluctuation $x_m(t)$ on a surface is not always less than the pressure fluctuation at a point, that is, the condition

$$x_m(t) < x_k(t) \tag{4-15}$$

does not always hold.

Based on these definitions, one can easily derive the correlation function and spectral density of the mean pressure fluctuation and the total load fluctuation:

$$R_{x_m}(\tau) = \frac{1}{n^2}\sum_k\sum_j R_{x_{kj}}(\tau) \tag{4-16}$$

$$R_p(\tau) = \frac{A^2}{n^2}\sum_k\sum_j R_{x_{kj}}(\tau) \tag{4-17}$$

$$S_{x_m}(\omega) = \frac{1}{2\pi}\int_{-T}^{T} R_{x_m}(\tau)\,e^{-i\omega\tau}\,d\tau = \frac{1}{n^2}\sum_k\sum_j S_{x_{kj}}(\omega) \tag{4-18}$$

$$S_{x_p}(\omega) = \frac{1}{2\pi}\int_{-T}^{T} R_p(\tau)\,e^{-i\omega\tau}\,d\tau = \frac{A^2}{n^2}\sum_k\sum_j S_{x_{kj}}(\omega) \tag{4-19}$$

The correlation function $R_{x_{kj}}(\tau)$ and the spectral density $S_{x_{kj}}(\omega)$ each have n^2 terms and may be represented in matrix form:

$$\begin{pmatrix} R_{11} & R_{12} & \cdots\cdots & R_{1n} \\ R_{21} & R_{22} & \cdots\cdots & R_{2n} \\ & \cdots\cdots\cdots\cdots & & \\ R_{n1} & R_{n2} & \cdots\cdots & R_{nn} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} S_{11} & S_{12} & \cdots\cdots & S_{1n} \\ S_{21} & S_{22} & \cdots\cdots & S_{2n} \\ & \cdots\cdots\cdots\cdots & & \\ S_{n1} & S_{n2} & \cdots\cdots & S_{nn} \end{pmatrix}$$

where the diagonal terms are the autocorrelation function and its spectral density and the off-diagonal terms are the cross-correlation function and the corresponding spectral density. Furthermore, when $j = k$, we have

$$S_{x_{kk}}(\omega) > 0 \qquad (4\text{-}20)$$

and for the general case $j \neq k$, we have

$$S_{x_{kj}}(\omega) \neq S_{x_{jk}} \neq 0 \qquad (4\text{-}21)$$

In other words,

$$S_{x_{kj}}(\omega) < 0 \qquad \text{or} \qquad S_{x_{kj}}(\omega) > 0 \qquad (4\text{-}22)$$

$\therefore$ condition $\quad S_{x_m}(\omega) < S_{x_{kk}}(\omega) \qquad k = 1,\ 2,\ \cdots\cdots,\ n \qquad (4\text{-}23)$

does not always hold. $S_{x_{kk}}(\omega)$ is the spectral density of the pressure fluctuation at a given point and $S_{x_m}(\omega)$ is the spectral density of the mean pressure fluctuation on an observation plane. The calculation results are shown in Figure 4.

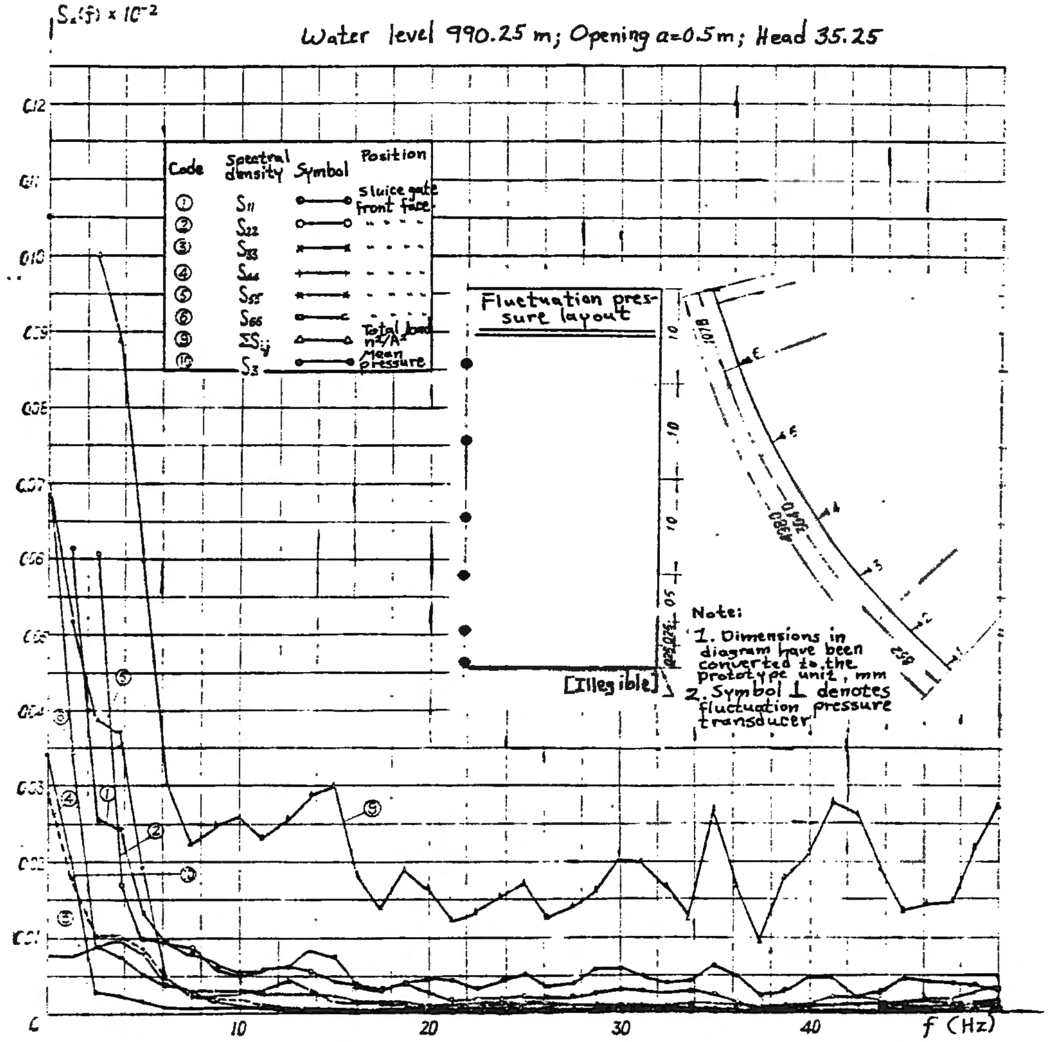

Figure 4. Synthesis of pressure fluctuation and spectral density of the mean pressure fluctuation.

V. Spectral Characteristics of Pressure Fluctuation

In 1883 the famous experiments of Reynolds revealed two types of flow — laminar and turbulent. A dimensionless number — the Reynolds number — has since been used to distinguish the two types of flow. The criterion is an empirical one. Today, even though the theory on the mechanism leading to turbulence is still in the development stage, a connection has been made between turbulent flow and vortex. At the boundary of the flow, the occurrence and development of a boundary layer depend on the flow velocity and local variations of pressure. Under certain circumstances vortex currents of a certain size will occur.

L. Prandtl [5] referred to turbulent flow as irregular vortex motion. Such irregular vortex movement manifests pressure fluctuation at the boundary. One might say that different size vortex currents in the flow lead to velocity fluctuations and pressure fluctuations of different frequency and intensity. In the study of hydraulic structure vibration, pressure fluctuation is of interest. The spectral density function, a Fourier integral representation of the pressure fluctuation, expressed the above physical concept in mathematical form and expresses the energy relationship of different frequency vortices.

Thus, the pressure fluctuation at a given point or a given cross-section is closely related to the flow state, the flow velocity and the upstream flow conditions. The spectral characteristics of the pressure fluctuation for different hydraulic structures under different flow conditions will be different. In a certain sense, one of the tasks of high-speed flow research is to find the spectral curves of the pressure fluctuation for different flow conditions. They are indispensable in the study of hydraulic structure vibrations. In the remaining space we shall describe some representative results.

(1) Pressure fluctuation at the front face of a deepwater arch [6]

In the analysis of sluice gate vibration under the dynamic effect of water flow, one of the indispensable steps is calculation of the load spectrum. Using the measured load in a model study, the smoothed spectral curves are shown in Figures 4 and 5. For an opening $a < 1.0$ m, the curve is a decay curve. For an opening $a > 1.0$ m, pronounced peaks were observed in the frequency range studied. For an arbitrary opening, $S_x(f)$ increases with increasing water

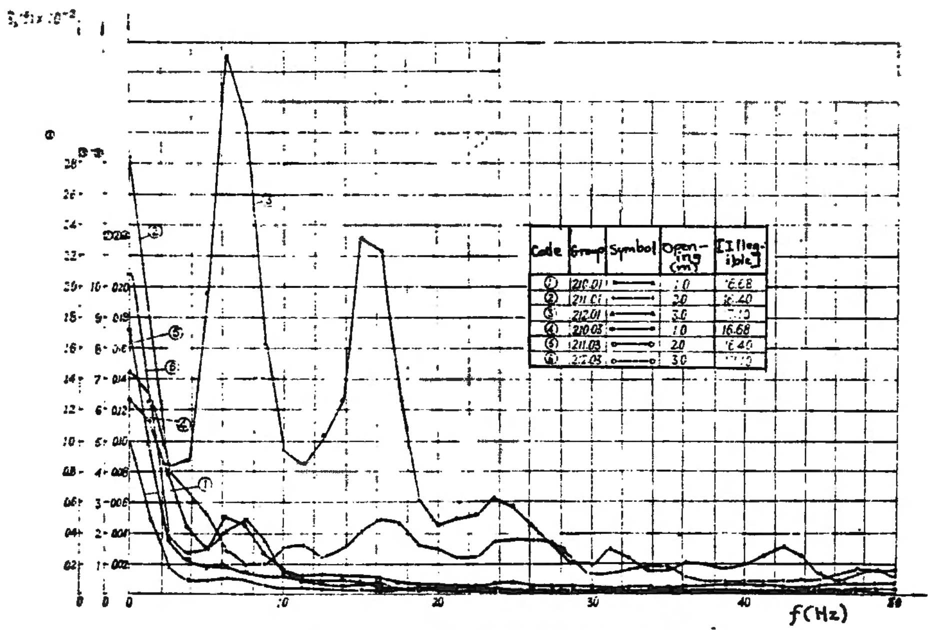

Figure 5. Pressure fluctuation spectral curves on the front face of a deepwater arch.

level and the peaks are more pronounced at high water levels, but the spectral characteristics remain basically the same. At a small opening, the fluctuation is large at the bottom edge and decreases gradually upward. This is not true for large openings.

Figure 6 shows the relationship between the relative fluctuation intensity σ_p/H at the bottom edge of the gate and the upstream Froude number F_{up}. This dependence may be expressed with the following empirical equation

$$(\sigma_p/H) \times 10^2 = 1.25\, F_{up}^{1.5} \tag{5-1}$$

where σ_p is the mean square deviation of the pressure fluctuation;

H is the water head;

F_{up} is the upstream Froude number.

The pressure fluctuation depends on the upstream flow field. The pressure fluctuation consists of two parts: one part is due to the turbulent motion in the flow itself and the small opening spectrum shows this characteristic.

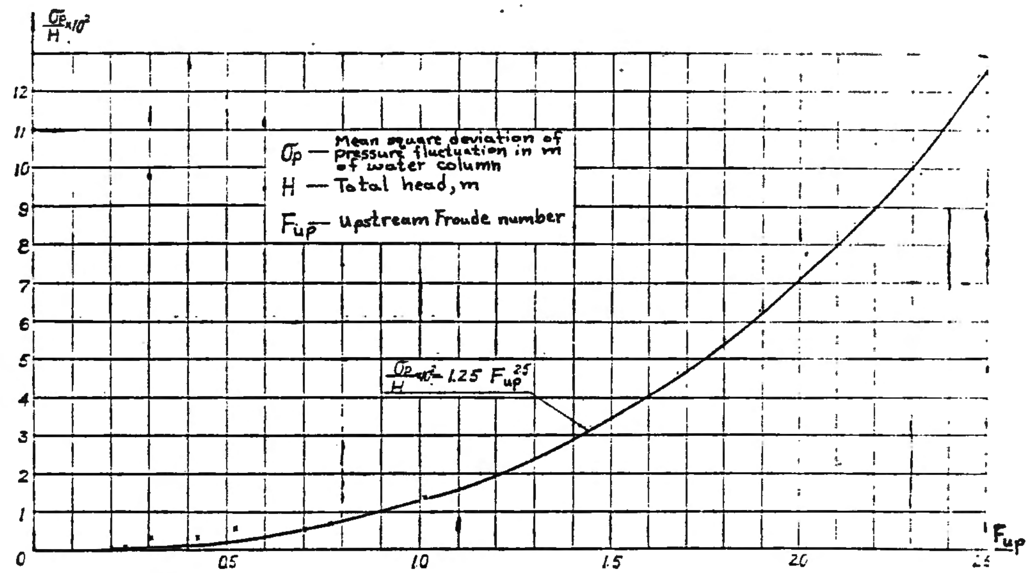

Figure 6. σ_p/H as a function of F_{up}.

The other part is due to vortices caused by non-streamline boundaries in the upstream and shows pronounced peaks. The latter contains much more energy than the former.

(2) **Pressure fluctuation characteristics of a tower inlet structure** [7]

In a study of the vibration of an overflow tower under the influence of moving water, the load fluctuation was obtained as a function of the tower height, and, using spectral analysis, the distribution of the fluctuating load was obtained for different flow conditions and the total load fluctuation for the bottom section was also obtained. When the flow rate $Q < 5\ m^3/s$, the load fluctuation has more energy in the 20 to 60 Hz frequency range. When $Q > 12\ m^3/s$, the fluctuation has more energy in the 2 to 15 Hz range. The amplitude of the pressure fluctuation decreases with increasing tower height and increases with increasing flow rate Q. The fluctuation is a maximum when $Q = 12$ to $13\ m^3/s$.

(3) **Observation of a hydraulic powerplant sluice and dewatering tunnel** [8]

In a prototype study of the Nanwan Hydroelectric Powerplant sluice and power production, careful observation of the pressure fluctuation was made and statistical and spectral analyses were also performed. Using the least squares method, the following correlation function was obtained:

$$R(\tau) = \sigma^2 e^{-\alpha\tau} \cos \beta\tau \qquad (5\text{-}2)$$

and the expression of the spectral curve was found to be:

$$S_x(f) = \frac{1}{\pi}\int_0^\infty R(\tau) \cos \omega\tau \, d\tau = \frac{\sigma^2\alpha}{2\pi}\left[\frac{1}{\alpha^2+(\beta+2\pi f)^2} + \frac{1}{\alpha^2+(\beta-2\pi f)^2}\right] \qquad (5\text{-}3)$$

where σ^2 is the square deviation of the fluctuating pressure and the parameters α and β depend on the flow condition. For a given set of flow conditions, α and β are constants. The spectral curve shows a pronounced peak at 15.1 to 16.5 Hz. This peak is due to the vortex currents produced at

Table 2. Statistical characteristics of pressure fluctuation at a hydraulic jump

Froude number in section before jump	Mean pressure (relative)	Standard deviation (relative)	3rd-order central moment	4th-order central moment	Fluctuation range	
F_1	$\frac{\bar{P}}{\frac{1}{2}\rho v_1^2}$	$\frac{\sigma}{\frac{1}{2}\rho v_1^2}$	$\frac{\mu_3}{\sigma^3}$	$\frac{\mu_4}{\sigma^4}$	$\frac{P_{min}}{\frac{1}{2}\rho v_1^2}$	$\frac{P_{max}}{\frac{1}{2}\rho v_1^2}$
4.7	0.307	0.080	0.41	4.71	0.148	0.498
5.9	0.274	0.082	0.38	4.41	0.075	0.520
6.6	0.241	0.085	"	4.58	0.083	0.495

an upstream fork. The vortex-induced pressure fluctuation propagates downstream at a rate of 160 to 200 m/s.

Table 3. Peak frequencies of pressure fluctuation at a hydraulic jump

Froude number F_1	4.7	5.9	6.6
Peak frequency f_{peak} (Hz)	1.47~5.13	2.40~8.40	3.61~12.20

(4) Pressure fluctuation at a hydraulic jump [4]

M. H. Abdul Kharder of India has studied the pressure fluctuation at the bottom of a stilling basin downstream from an overflow dam. He made a detailed calculation of the statistical characteristics, correlation function and spectral density function of the pressure fluctuation. The statistical properties under different flow conditions are shown in Table 2, where ρ is the mass density of water, v_1 is the cross-sectional flow velocity before the jump, $\overline{P}$ is the average pressure, P_{min} is the minimum pressure, P_{max} is maximum pressure, and the other symbols were defined above. Abdul Kharder expressed the correlation function and the spectral density function in terms of dimensionless parameters. It is noteworthy that the correlation function $R(\tau v_1/y_1)$ curves for different Froude numbers basically coincide. The peak frequencies of the spectral curves at different F_1 are shown in Table 3.

VI. Conclusions

Today the principles of random function are widely used in the analysis of pressure fluctuation data here and abroad. In all cases the pressure fluctuations are assumed to be steady random processes and possess the state history property. Although the theories are becoming increasingly complete, many problems remain to be studied in the actual application.

The spectral curves of pressure fluctuation show that it is realistic to link the flow turbulence with the idea of vortex current. The objective of

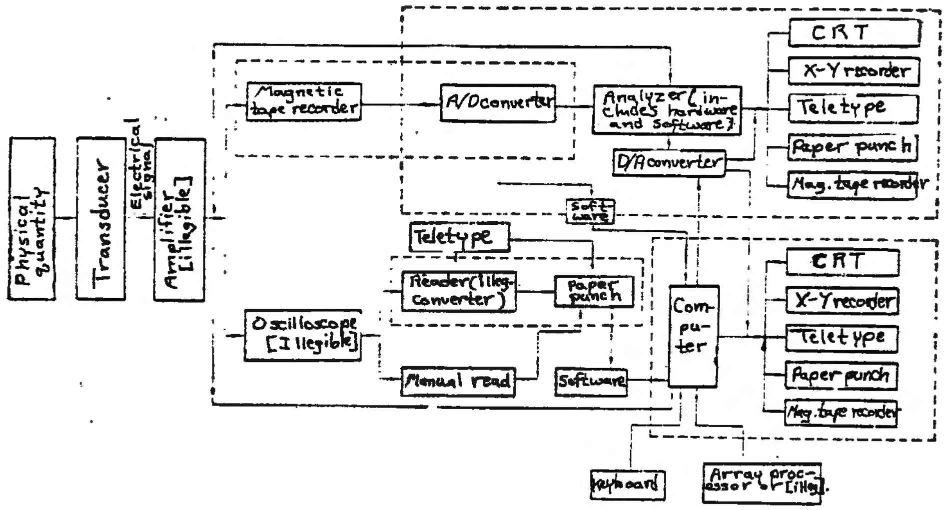

Figure 7. Block diagram of the data processing system.

the spectral analysis is to obtain the energy associate with vortices of different frequency and amplitude. The work in [8] clearly shows that the vortices produced by a branching flow are extremely important for the pressure fluctuation downstream. The spectrum for a large opening in [6] has the same feature. Several sets of representative data show that even though the spectral characteristics of the pressure fluctuation are different under different flow conditions, most of the energies are generally in the ultralow frequency range (< 20 Hz).

The synthesis of pressure fluctuation and the spectral analysis of the mean fluctuation, first proposed in [6], provided a convenient method for calculating the total load and the mean spectral density from multi-point measurement results.

It should be pointed out that the commonly used method today is to calculate the spectral density using the correlation function and the Fourier transform. This direct method was used in all the works cited so far. In other fields, the fast Fourier transform (FFT) is often used to reduce computer time and to improve computation accuracy. In foreign countries, the magnetic tape recorder is often tied directly to the analyzer or the computer and the system is very convenient to use. The statistical properties of interest, such as the correlation function, spectral density, probability density and coherence function, can all be displayed on the fluorescent screen instantly and plotted on the x-y recorder. The ideal recording and processing system [9] is shown as a block diagram in the upper part of Figure 7.

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