DESIGN OF TRASHRACKS FOR
HYDRAULIC STRUCTURES

BY

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Dimensionamento de grades de protecção de circuitos hidráulicos

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Abstract

This technical paper summarizes present knowledge about the hydrodynamic actions on trashracks placed in hydraulic structures and about methods for forecasting hydraulic operating conditions that may induce instabilities on the individual elements of such racks. Emphasis is given to the presentation of simplified analytical methods to calculate flow-induced forces on the individual elements and hydraulic operating conditions that may lead to trashrack failure. Some criteria that may be adopted, in the design phase, to reduce the amplitude of vortex-induced vibrations are suggested; some solutions to reduce dangerous levels of vibration that may exist during the operation are also presented. Some cases of trashrack failures occurring as a result of flow-induced vibrations are presented in an appendix.

Symbols

Only the principal indices and symbols are indicated. Other symbols and indices are indicated in the text.

- $A$ - area of the transverse section of water intake in the plane of the trashrack and without the trashrack
- $A_8$ - area of the elements of the trashrack perpendicular to flow
- $C_D$, $C_D_0$ - drag coefficient of the trashrack
- $C_d$ - drag coefficient of the elements of the trashrack
- $C_k$ - effective lift coefficient
- $C_L$ - lift coefficient
- $C_r$ - coefficient of head loss
- $C_y$ - coefficient of transverse force
- $D$ - transverse dimension of the trashrack
- $E$ - modulus of elasticity
- $F_D$ - drag force in the trashrack
\( F_d \) - drag force in the elements of the trashrack
\( F_k \) - lift force
\( F_y \) - force in the direction perpendicular to flow
\( F(t) \) - generalized force
\( F_0 \) - amplitude of the force
\( H \) - length of the bar (distance between supports)
\( I \) - momentum of inertia
\( L \) - dimension of the trashrack in the direction perpendicular to \( H \)
\( M \) - momentum
\( Re \) - Reynolds number
\( St \) - Strouhal number
\( T \) - transmissibility
\( U \) - transverse stress
\( V \) - velocity
\( <V> \) - average velocity in the post section
\( V^*, V^{**} \) - reduced velocities
\( \Omega \) - generalized mass
\( a_y \) - amplitude of vibration in the direction perpendicular to flow
\( c \) - structural damping
\( d \) - transverse dimension of the bars
\( f \) - frequency of detachment of the vortices (Hz)
\( f_{res} \) - resonance frequency (Hz)
\( f_0 \) - frequency of the structure itself (Hz)
\( k \) - rigidity of the structure
\( l \) - dimension of the bars perpendicular to the plane of the trashrack
\( m \) - mass of the structure per unit of length
\( m_a \) - hydrodynamic or additional mass per unit of length
\( p \) - pressure
\( q \) - generalized displacement
\( r \) - index characteristic of the mode of vibration
\( t \) - time
\( x, y \) - coordinates
\( y \) - displacement due to vibration perpendicular to flow
\( \dot{y} = dy/dt \)
\( \ddot{y} = d^2y/dt^2 \)
α - coefficient which depends on the conditions of support and the mode of vibration
β - angle of attack of the flow
γ - dimensionless mass parameter
δ - hydrodynamic mass coefficient
ζ - damping coefficient
ρ - volume mass of the water
ρ_8 - volume mass of the material in the elements of the trashrack
ν - kinematic viscosity of the water
ω - excitation frequency (rad/s)
ω_r - resonant frequency (rad/s)
DESIGN OF TRASHRACKS FOR HYDRAULIC STRUCTURES

1. INTRODUCTION

In addition to the problems of economics, operation and maintenance, the design of trashracks for hydraulic structures must take into account four important factors:

a) Static stress on the trashrack;
b) Spacing between the bars;
c) Head loss;
d) Vibrations.

Stress on the trashrack refers to the maximum value of stress to be considered in its design, which results from the pressure differential between the planes immediately upstream and immediately downstream from the trashrack. It is obvious that this difference is dependent on the head loss that occurs when the flow crosses this element. In defining this stress, it is important to take into account the existing cleaning system, since the value of the stress naturally increases with the quantity and type of materials which are transported by the flow and retained by the trashrack. Carelessness in cleaning of the reservoir is one of the principal causes of trashrack obstruction.

As a rule, the spacing between bars is defined by the recommendations of the equipment suppliers, and it is imposed by the maximum dimension of the materials that can be included in the hydraulic conduit without damaging the equipment that is to be protected. Although it is difficult to enumerate completely the factors that influence this spacing, in almost all hydraulic structures it depends, to some degree, on the height of the drop, the discharge, the equipment to be protected, the characteristics of the flow, the type of reservoir, and the cleaning system used.
The head loss resulting from the installation of trashracks is, in itself, a very important factor in the design and planning of the trashracks, since the adoption of inappropriate measures will result in a sizeable energy loss throughout the life of the structure. The bars are not to be considered alone in determining these head losses; all elements of the trashracks must be considered together.

To some extent, these three factors are related to each other, since the head losses will depend on the size and shape of the structural elements; the orientation of the bars in relation to the direction of flow; their spacing and, consequently, the relation between the area of the passage section where the trashrack is lodged and the area occupied by the bars, support beams, reinforcement plates and braces; the the additional obstruction caused by the debris that are deposited on the trashrack itself.

The stability of the trashrack in relation to the vibrations must be taken into account because the structural elements may be subjected to various dynamic stresses induced by the flow. The necessity for taking this stability into account is attested by failures that have occurred in this type of structure in some hydraulic applications, described in the appendix.

One of the factors contributing to the increasing importance accorded the vibrations is the installation of larger hydraulic machinery in existing structures, particularly turbines and pumps. In many cases, these replacements have resulted in high velocities approximating those of relatively small trashracks.

This problem becomes more serious then the trashracks are installed in hydraulic conduits with a non-uniform velocity distribution. There are conduits in which, for some operating conditions, significant variations in ve-
Locality are registered [1, 2]. It is easy to understand this phenomenon if we keep in mind the fact that the water reservoirs function, during pumping, as diffusers (sometimes capriciously), and can cause separation and reversal of the flow. On the other hand, the flow upstream from the plane of the trashrack may show characteristics of vibration and surges resulting from the machine's functioning (rotation velocity and number of vanes) and may therefore be a source of significant dynamic stresses.

Evaluating the dynamic effects involves determining the elastic characteristics of the trashrack and its elements and a knowledge of the mechanisms of hydrodynamic stress.

2. HEAD LOSS. STATIC STRESS

Coefficient of head loss, $C_p$, is the name given to the coefficient defined by the relation

$$C_p = \frac{\Delta p}{\frac{1}{2} \rho <v>^2}$$

(2.1)

in which $\Delta p$ is the difference between the pressures upstream and downstream from the trashrack; $\rho$ is the volume mass of the water; and $<v>$ is the average velocity in the upstream section. In expression (2.1), $\frac{1}{2} \rho <v>^2$ is the average dynamic pressure of the flow in the trashrack.

In the case of trashracks and similar elements, it is customary to express the loss coefficient on the basis of the drag coefficient $C_D$, since the drag force $F_D$ depends on the friction stresses, the distribution of pressures on the surface, and on the areas affected.

This coefficient, defined by the expression

$$C_D = \frac{F_D}{\frac{1}{2} \rho <v>^2 A_i}$$

(2.2)
in which \( V \) is the velocity of approximation and \( A_c \) is a characteristic area, commonly considered to be equal to \( A_0 \), depends on the following parameters:

a) Geometry (shape and size) and orientation of the bars (Figure 2);

b) Spacing between the elements, expressed by the coefficient of projected area \( \phi \), defined by the quotient between the solid area, perpendicular to the flow, occupied by the structural elements, \( A_0 \), and the total area in the plane of the trashrack and without it, \( A \) (Figures 1 and 3);

c) Slenderness of the trashrack \( \lambda \), defined by the quotient between the length \( L \) and the width \( D \) (Figure 1);

d) The presence upstream of another structure of the same type located at a relative distance \( s \), defined by the quotient between the distance between the planes of the trashracks \( S \) and the length \( L \) (Figure 4);

e) Angle of attack of the flow \( \beta \) in relation to the plane of the trashrack (Figure 2).

Figure 1. Trashrack in a hydraulic structure. Diagram.

Therefore, for a given trashrack, the following is valid:

\[
C_p = \lambda \phi \Sigma C_o.
\]

in which
\( \Lambda \) - is the parameter that includes slenderness;

\( \Pi \) - is the parameter that expresses the influence of the angle of attack;

\( \Sigma \) - is the parameter that includes the influence of the upstream trashrack;

\( C_{D_0} \) - is the drag coefficient for the trashrack, considering it to be of infinite dimension, isolated, and installed perpendicular to the flow, which depends on the geometrical characteristics of its elements (bars, support beams, reinforcement plates, braces, etc.) and on \( \phi \)

\[
C_{D_0} = C_{D_0}' \text{(geom. char. } \Phi \text{)}
\]  

(2.4)

\[
\begin{array}{c|cc|ccc|ccc|ccc}
\beta & C_{D_x} & C_{D_y} & C_{D_x} & C_{D_y} & C_{D_x} & C_{D_y} & C_{D_x} & C_{D_y} \\
\hline
0^\circ & +2.05 & 0 & +2.0 & 0 & +2.1 & 0 & +2.0 & 0 \\
45^\circ & +1.95 & +0.6 & +1.5 & +1.5 & +1.8 & +0.1 & +1.4 & +0.7 & +1.55 & +1.55 \\
90^\circ & +0.5 & +0.8 & +0 & +1.0 & 0 & +0.1 & 0 & +0.75 & 0 & +2.0 \\
\end{array}
\]

\( C_{D_x} \) - Component of \( C_D \) in direction of \( X \)

\( C_{D_y} \) - Component of \( C_D \) in direction of \( Y \)

Figure 2. Drag coefficients of profiles with different geometric shapes. Two-dimensional flow [3].

\[
\text{Figure 3. Variation in drag coefficient with the coefficient of projected area [4].}
\]

The literature [4-10] shows values for these parameters, for the drag co-
efficient, and for the relation between this coefficient and the loss coefficient.

If \( \phi < 0.15 \), the elements making up the trashracks can be considered isolated, and when the flow is normal to the trashrack, the head loss coefficient of this element can be expressed with close approximation as [4]:

\[
C_r = \left( \frac{0.5 + \phi}{1 - \phi} \right)^2
\]

(2.5)

if the elements have sharp edges, and as

\[
C_r = \left( \frac{\phi}{1 - \phi} \right)^2
\]

(2.6)

if they have rounded edges.

We should emphasize the fact that the total resistance of the trashrack results from the contribution of all its elements (bars, support beams, reinforcement plates, braces, etc.) and from the mutual interference of all these elements.

The different values of \( C_r \), for elements with sharp or rounded edges is due to the fact that the flow along these elements and downstream from them has different characteristics. In fact, in the elements with sharp edges, separation of the boundary layers occurs in these edges, while in the elements with
a rounded shape these separations occur irregularly and their behavior may even be different depending on the characteristics of the flow, defined in general by the Reynolds number (Figure 5) and Froude number, the characteristics of the structural elements (geometry and roughness, Figure 6) and the turbulence of the flow.

Table 1 shows the drag coefficients $C_d$ relative to the various geometric shapes of the cross sections of the structural elements most commonly used in trashracks.

Figure 7 shows the $C_d$ values of different-sized bars with rectangular cross section. We should point out that the $C_d$ values presented in Figure 7 are higher than those shown in Table 1. In designing trashracks, the higher values are recommended.

In view of the above discussion, and keeping in mind the static stresses to which the trashracks are subjected and the resulting head losses, when the trashracks are designed, all their elements and the characteristics of the

![Flow systems diagram]

Figure 5. Influence of Reynolds number on characteristics of flow around cylinders [11, 12].
cleaning system must be taken into account. This system can be specified as a function of the value of the maximum admissible static stress.

![Influence of geometry and roughness on the value of the drag coefficient](image)

**Figure 6.** Influence of geometry and roughness on the value of the drag coefficient [13, 14].

<table>
<thead>
<tr>
<th>Shape of cross section</th>
<th>$C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square with side perpendicular to flow</td>
<td>2.0 (1.2)(1)</td>
</tr>
<tr>
<td>Square with corner perpendicular to flow</td>
<td>1.55 (1.5)</td>
</tr>
<tr>
<td>Rectangle with small side perpendicular to flow</td>
<td>1.5 (0.6)</td>
</tr>
<tr>
<td>Rectangle with large side perpendicular to flow</td>
<td>2.0 (1.6)</td>
</tr>
<tr>
<td>Equilateral triangle, corner in the direction of flow</td>
<td>1.2 (1.1)</td>
</tr>
<tr>
<td>Right triangle, corner in the direction of flow</td>
<td>1.55</td>
</tr>
<tr>
<td>Equilateral triangle, base perpendicular to flow</td>
<td>2.0 (1.3)</td>
</tr>
<tr>
<td>Circle</td>
<td>1.2 (0.7)</td>
</tr>
</tbody>
</table>

(1) Numbers in parentheses refer to minimum values

![Variation in Strouhal number and in the drag coefficient](image)

**Figure 7.** Variation in Strouhal number and in the drag coefficient with $z/d$ [16].
3. VIBRATORY SYSTEMS. RESONANT FREQUENCIES AND MODES OF VIBRATION. 
TRANSMISSIONIBILITY

3.1. General remarks

When mechanical systems are exposed to impact or to the action of forces that vary over time, they are subjected to vibratory phenomena which may result in undesirable disturbances capable of reducing the useful life of the system because of excessive stresses or, more frequently, because of fatigue.

These phenomena of vibration appear especially when the actions operating in the system are periodic. For certain frequencies (called resonant frequencies) the amplitudes of vibration become high, even when the excitation is low. The configuration assumed by the system when it vibrates in one of its resonant frequencies is called the modal configuration, or, more simply, the mode.

The general equation of movement in a mechanical system is:

\[ \ddot{q}_r + 2\zeta\omega_n \dot{q}_r + \omega_n^2 q_r = \frac{F_r(t)}{\Omega_r} \]  

(3.1)

where

- \( q \) - the generalized displacement;
- \( \omega \) - the resonant frequency;
- \( \zeta \) - the damping factor;
- \( F_r(t) \) - the generalized force;
- \( \Omega \) - the generalized mass;

and in which \( r \) is the index characterizing the order of the mode in question.

Finding the integral of this equation, with boundary conditions appropriate to the end connections of the system, makes it possible to obtain the law of movement of the body and, in particular, to determine the resonant frequencies \( \omega_r \).
For simple systems, such as the case of the bars forming the trashracks which have a uniform section and a continuous and uniform mass distribution, the resonant frequencies in the absence of damping are given by:

\[ \omega_n = \alpha \sqrt{\frac{EI}{mH^2}} \]  

(3.2)

where 

- \( E \) - the material's modulus of elasticity;
- \( I \) - momentum of inertia of the cross section in relation to the axis which is perpendicular to the direction of vibration;
- \( m \) - mass per unit of length;
- \( H \) - length of the bar (distance between two supports);
- \( \alpha \) - coefficient which depends on the type of connection and on the mode of vibration.

Section 3.2 shows the values of \( \alpha \) for the most common cases. If there is damping, we find:

\[ \omega' = \omega \sqrt{1 - \xi^2} \]  

(3.3)

in which \( \omega' \) is the apparent resonant frequency.

If the driving force is periodic and of the following type:

\[ F(t) = F_0 \cos(\omega t) \]  

(3.4)

in which \( \omega \) is the frequency of excitation and \( F_0 \) is its amplitude, the general law of movement is expressed by:

\[ y = Y_0 + Y_1 \]  

(3.5)

in which \( Y_0 \) is the integral of equation (3.1) without a driving force (free vibration) and \( Y_1 \) is a particular integral of the driving force [forced vibration (a)].

(a) The term "forced vibration" is used to describe vibration which is sustained by a periodic force which acts independently of the vibration.
The presence of damping guarantees the more or less rapid decrease of the free movement of the system, which persists only in forced vibration, expressed by:

\[ y \approx y_i = \frac{F_0/k}{\sqrt{1 - \left(\frac{\omega}{\omega_i}\right)^2 + \left(\frac{2\zeta \omega}{\omega_i}\right)^2}} \cdot \cos(\omega t - \theta) \]  

(3.6)
in which \( \theta \) is the phase shift between the stress and the response, and \( k \) is the rigidity of the system.

It is possible to use this expression as a starting point to obtain the relation between the applied force and that transmitted by the system to its connections, which is called transmissibility, \( T \), being:

\[ T = \frac{1 + \left(\frac{2\zeta \omega}{\omega_i}\right)^2}{\sqrt{1 - \left(\frac{\omega}{\omega_i}\right)^2 + \left(\frac{2\zeta \omega}{\omega_i}\right)^2}} \]  

(3.7)

which is represented in graph form in Figure 8.

When \( \frac{\omega}{\omega_i} \rightarrow 1 \), or when the frequency of excitation is close to a resonant frequency of the system, this system is said to be in resonance. As shown by the analysis of equation (3.7), if the damping is low, under conditions of resonance, the transmissibility becomes very high.

In the case of metal structures, the absorption is normally low (\( \zeta < 0.02 \)), which means that it will be \( \omega_i = \omega' \), and that when a condition of resonance is approached (\( \frac{\omega}{\omega_i} \rightarrow 1 \)) the transmissibility increases rapidly, reaching values higher than 25, as shown by the analysis of Figure 8.
3.2. The coefficient α

The classic solution of the derivation of the equation of motion uses the equation of elasticity, which is:

\[ EI \frac{d^2y}{dx^2} = M \]  

(3.8)

Figure 9 shows schematically the state of deformation of one element of a continuous bar which is made of a Hookean material.

The balance of the vertical forces in this element, including the forces of inertia, is:

\[ U + \frac{\partial U}{\partial x} - U = \frac{\partial U}{\partial x} - dx = m \dot{y} \]  

(3.9)

Figure 9. Diagram of deformation of an element of a continuous bar.

and given that

\[ U = \frac{\partial M}{\partial x} \]

the result is

\[ \frac{\partial}{\partial x} \left( \frac{\partial^2 y}{\partial x^2} \right) = m \ddot{y} \]

(3.9) [sic]

In the cases when \( EI \) is constant, the solution is of the following type:

\[ y = Y(x) \cos (\omega t + \theta) \]  

(3.10)

Considering

\[ \eta = \frac{\omega m}{EI} \]  

(3.11)

after some transformations, which include the derivation of equation (3.10)
and its previous substitution in (3.9), the result is

\[ y'' = \frac{d^2y}{dx^2} = \eta^4 y \]  

(3.12)

a differential equation which, with the appropriate boundary conditions, makes it possible to obtain a description of the movement.

The general solution of this equation is of the following type:

\[ y = A_1 \text{sen}(\eta x) + A_2 \cos(\eta x) + A_3 \text{senh}(\eta x) + A_4 \cosh(\eta x) \]  

(3.13)

with the following boundary conditions:

- the deflection is proportional to \( y \), being negligible in a rigid support;
- the rotation is proportional to \( y' \), being negligible in an end constraint;
- the momentum is proportional to \( y'' \), being negligible in free ends or in supports with gratings;
- the transverse stress is proportional to \( y''' \), being negligible in free ends.

Table 2 presents a summary of the values of \( \alpha \) under various boundary conditions.

Table 3 shows the values of the coefficient \( \alpha \) and the three first modes of vibration for the situations indicated in Table 2.

To include the cases in which the support conditions are other than the conditions presented here, which correspond to ideal conditions, it is customary to define \( \alpha \) as a function of the degree of constraint \( \eta \). Figure 10 shows the values of \( \alpha \) in the first mode for bars with a constant section subjected to a uniformly distributed load and with support conditions corresponding to the various degrees of constraint.
Table 2. Values of $\alpha$ for various boundary conditions

<table>
<thead>
<tr>
<th>Constraint conditions</th>
<th>Boundary conditions</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple support</td>
<td>$x=0$ { $Y=0$ } { $Y'=0$ }</td>
<td>$r$</td>
</tr>
<tr>
<td>Simple support</td>
<td>$x=H$ { $Y=0$ } { $Y'=0$ }</td>
<td>$r$</td>
</tr>
<tr>
<td>Fixed ends</td>
<td>$x=0$ { $Y=0$ } { $Y'=0$ }</td>
<td>$2r+1$</td>
</tr>
<tr>
<td>Fixed ends</td>
<td>$x=H$ { $Y=0$ } { $Y'=0$ }</td>
<td>$r$</td>
</tr>
<tr>
<td>Fixed ends</td>
<td>$x=0$ { $Y=0$ } { $Y'=0$ }</td>
<td>$4r+1$</td>
</tr>
<tr>
<td>Simple support</td>
<td>$x=H$ { $Y=0$ } { $Y''=0$ }</td>
<td>$4r$</td>
</tr>
</tbody>
</table>

Table 3. Values of $\alpha$ in bars with a constant section with a uniformly distributed load

<table>
<thead>
<tr>
<th>Constraint conditions</th>
<th>$r$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Simple support</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Fixed ends</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Simple support &amp; fixed ends</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Laboratory tests described in [18] indicate that the values of $\alpha$, in the first mode, for structural elements of the trashracks that are normally used in hydraulic structures, in which the ends are welded to the reinforcement beams, are between 16 and 20.

Tests were conducted at the LNEG [2] with a panel made of rectangular metal bars, welded on the ends, with $d = 12$ mm, $L = 120$ mm.
\( b = 150 \text{ mm} \) and \( H = 765 \text{ mm} \), in which \( d \) and \( l \) respectively represent the transverse dimensions in the direction of the flow and perpendicular to it, and \( b \) is the distance between the bars. These tests indicated a value of about 20 for \( \alpha \).

3.3. Influence of the fluid on the equations of movement

3.3.1. Hydrodynamic or additional mass

When a body vibrates in a fluid, even when this fluid is at rest, a flow is set up which is non-stationary, although it may be periodic. Consequently, additional forces appear which, according to the second principle of mechanics, should continuously compensate for the variation in flow of the quantity of movement induced by the vibration.

Additional forces, which are in phase with the displacement of the structure, give rise to a hydrodynamic mass, commonly known as additional mass and, in special cases, to a hydrodynamic rigidity. Forces which are not in phase with the displacement but which are in phase with the velocity of vibration, normally give rise to energy damping (hydrodynamic or additional damping) or possibly to energy production (negative hydrodynamic damping).

Thus these additional forces have components proportional to \( \ddot{y}, \dot{y}, \) and \( y \). In this way, the existence of the fluid affects the mass \( Hm \) or absorption \( c \), and in some cases the rigidity of the structure \( k \). This effect may be expressed by additional values of \( m, c, \) and \( k \).

The relative importance of the various additional forces depends on the type of structural elements immersed in water, the most important component in the equation of movement is the component due to the hydrodynamic mass.

If we consider only this component, equation (3.2) takes on the following
Finally, everything occurs as though the "fictitious" mass of water $m_a$ were closely associated, during movement, with the mass of the body, which is expressed by a reduction of the natural frequency of the system. It is customary to designate the value of $m_a$ by the hydrodynamic or additional mass of the vibrating system.

The value of the hydrodynamic mass generally depends on the geometrical characteristics of the body in movement, the characteristics of this movement (frequency, amplitudes, and modes of vibration), the degree of submergence, the viscosity of the water, and the characteristics of the flow. Other factors, such as the compressibility of the water and free surface radiation are negligible in this case.

For bodies vibrating at a low amplitude in a liquid at rest, the value of the hydrodynamic mass may be obtained by applying the theory of potential flow.

Figure 11 shows the values of the coefficients of hydrodynamic mass $\delta$.
for cross sections of bidimensional structures \((H >> d)\), vibrating in a fluid at rest with infinite dimensions. In this type of structure the coefficient of hydrodynamic mass \(\delta\) is defined by

\[
\delta = \frac{\Delta m}{\rho A}
\]

in which \(m_\alpha\) is the hydrodynamic mass per unit of length.

Figure 11 also shows the coefficients of hydrodynamic mass of plates vibrating perpendicularly to the direction of the largest dimension of the cross section. In this case, the hydrodynamic mass \(m_\alpha\) results from the fact that part of the kinetic energy transferred from the body to the fluid dissipates in the form of surface waves. Contrary to the effect of the free surface, the proximity of a rigid boundary increases the value of the hydrodynamic mass. Another report [21] presents values of hydrodynamic mass resulting from the movement of plates vibrating perpendicularly to the direction of the largest dimension of the cross section as a function of the distance to the rigid boundary.

For structures vibrating in a flowing liquid, the hydrodynamic mass is independent of the amplitude and the frequency of vibration, provided that the flow velocities are not too high. In fact, considering the flow velocity \(V\) to be substantially lower than the displacement velocity of the body \(\omega_A\),

\[
\frac{V}{\omega_A} << 1
\]

it is easy to observe that the structure behaves in the same way as in a liquid at rest [21].

For values of \(\frac{V}{\omega_A} >> 1\), the field of pressures induced by the movement depends on the characteristics of the flow. According to the experimental results presented in [22] on cylinders, under these conditions the hydrodynamic mass is lower than that obtained in a liquid at rest.
However, as of the present we have no information available which would allow us to quantify the effects of flow on the value of the hydrodynamic mass.

![Diagram of section types]

Figure 12. Influence of submergence on the value of the coefficient of hydrodynamic mass [20]

3.3.2. Resonant frequency of the bars with a rectangular cross section

Generally speaking, the cross sections of bars which make up the panels of trashracks installed in hydraulic structures are rectangular. For this reason, the equation proposed by Levin [9] can be used to calculate the resonant frequency of this type of bar, vibrating perpendicularly to the largest dimension

\[ \omega_r = \frac{r}{H} \sqrt{\frac{E}{b \rho_x + \frac{d}{2} \rho}} \]  

(3.17)

in which:

- \( r \) - the radius of rotation of the upright section of the bars in relation to the axis which is perpendicular to the direction of vibration;
- \( b \) - the distance between bars;
- \( d \) - the thickness of the bars;
- \( \rho_x \) - the volume mass of the material from which the bars are made;

and the remaining nomenclature has already been defined.
In this equation the term of hydrodynamic mass per unit of length $\frac{b}{d} \rho$ resulted from experiments that took into account the proximity of the nearby bars.

According to Levin, expression (3.17) is valid provided that $b \leq 0.7l$. If $b > 0.7l$, he suggests using the value $b = 0.7l$.

It is clear that considering the value $b = 0.7l$ amounts to adopting a coefficient of hydrodynamic mass with a value essentially equal to 0.9, which is lower than the value defined in Figure 11. After some simple transformations, obtaining this value results in substituting equation (3.16) for (3.15).

According to Sell [18], the value of $b$ to be adopted in expression (3.17) also depends on the value of the quotient $l/d$ and the degree of constraint of the bars. For bars with a degree of constraint such that $\alpha > 16$ (in the fundamental mode), this author proposes that the value of $b$ adopted be limited to $0.55l$ for relations of $l/d = 10$ and $1.0l$ for relations of $l/d = 5$.

4. FLOW-INDUCED VIBRATIONS

4.1. Preliminary remarks

Flow-induced vibrations may result from the action of various mechanisms, and are generally designated according to the mechanism from which they originate.

They can be divided into two major groups:

a) vibrations that occur especially in flows of low turbulence, although they may exist and develop in very turbulent flows;

b) vibrations that originate in the flow's turbulence itself or in its pulsations.

Belonging to the second group are the vibrations induced by the fluctuation of the longitudinal component of the flow velocity; the results of the function-
ing of the turbomachines (pump-turbine) during pumping merit special attention. The frequencies associated with this stress will naturally be related with the number of vanes, the speed of the pump-turbine and, of course, the shape of the flow passages. The periodic characteristics transmitted to the flow by the machine are generally altered by the shape of the flow passages.

In the first group, we distinguish the two phenomena that will be analyzed below: stress due to separation of vortices and gallop.

4.2. Separation of vortices

When a structure with a certain geometric shape is immersed in a flow, there is formation, development, and then separation of vortices on a large scale. This separation operates alternatively on each side of the structure (Figure 5). The frequency of separation of these vortices increases linearly with the velocity of flow. However, when resonance is reached, it is the structure which controls the frequency of separation of vortices, which is maintained constant for small variations in average velocity (Figure 13).

![Figure 13. Resonance in a cylinder with separation of vortices [23].](image)

This alternating separation of vortices gives rise to a force that can be analyzed as the result of the average and fluctuating drag components (in the direction of flow) and a fluctuating lift component (perpendicular to the direction of flow).

The fluctuating lift component has a frequency \( f \) which can be expressed by the relation

\[
s = \frac{f}{V}
\]

(4.1)

in which \( f \) is the frequency of separation of the vortices; \( V \) is the velocity of
the flow; and \( d \) is a characteristic length, normally the transverse dimension. The fluctuating drag component has a frequency double that of the lift.

In the case of an isolated structure, the dimensionless parameter \( S' \), commonly called the Strouhal number, is a function of the shape of the cross section, the angle of attack of the flow \( \beta \), and the action of the fluid's viscosity, generally expressed by the Reynolds number. In the case of structures that are parallel or in a series, the Strouhal number also depends on their distance from each other. Figure 14 shows an example of this influence.

Figures 7 and 15 show the values of the Strouhal numbers of the structures with the most common cross sections. These values refer to flow situations in which the effects of viscosity are negligible.

When the value of \( S' \) is known, figuring the value of \( f \) will depend on the value of \( V \) adopted.

For this parameter, most authors consider the value of the average velocity.

In the case of the flow passages of reversible units, flow situations may occur with which there is a corresponding field of non-uniform velocities near the trashrack. The variations registered in the flow passages discussed in [1, 18] are significant.

Therefore, the average velocity may be very different from the velocity at one point. For this reason, under these circumstances, it is advisable to adopt the maximum foreseeable value immediately upstream from the bar in question. In many cases, determination of this value requires scale-model studies.
Another factor to be taken into account in the application of expression (4.1) is connected with the orientation of the velocity vector and with the possible existence of fluctuating components of the velocity in relation to the average value.

Under conditions of resonance, the existence of vortices separation creates a force perpendicular to the flow.

This force depends on the damping of the structure, and its maximum value is expressed, if the process can be considered a "narrow band" process, by

$$F_k = \frac{1}{2} C_k \varepsilon \left( \frac{1}{2} \rho V^2 \right)$$

in which $F_k$ represents the force per unit of length, and $C_k$ is the lift coefficient (the other symbols have already been defined). The vortices separation in the metal bars of trashracks, because of their small cross section, can be considered as a "narrow band" process.

Table 4 shows the values of $C_k$ in relation to the various shapes of the cross sections of the structures. Figure 16 shows the evolution of $C_k$ with a Reynolds number for the case of a structure with a circular section.

Finally, there is also a force in the direction of the flow due to this phenomenon. In almost all cases, its value is much lower than the maximum force due to the average stress in the direction of flow.

As already indicated, when the frequency of vortices separation is resonant
Table 4. Values of $C_k$ determined in a wind tunnel

<table>
<thead>
<tr>
<th>Section</th>
<th>$C_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>0.22</td>
</tr>
<tr>
<td>Square</td>
<td>0.50</td>
</tr>
<tr>
<td>Dodecagonal</td>
<td>0.40</td>
</tr>
</tbody>
</table>

with that of the bars, significant movements and forces may occur in them, especially if the vortices separate simultaneously along each of them. For reasons depending on the characteristics of the flow (for example, turbulence) or the elements (for example, small variations in transverse dimension), the vortices only separate, with a high correlation, in more or less limited areas.

However, this situation may be radically altered in resonance. Specifically, when the elements move, the frequency of vortices separation may be maintained, even though the flow velocity varies, which is due to the fact that the structure begins to control this separation. Under these conditions, the correlation of vortices separation is practically equal to one along the bars, and the conditions of significant stress development are established. "Lock in" is the name given to this phenomenon. Figure 13 shows "lock in" for a cylinder. The "lock in" effect may also occur if the exact frequency is equal to a multiple or a sub-multiple of the frequency of vortices separation [27].

4.3. Gallop

This phenomenon consists of the occurrence of significant movements in the direction transverse to the flow. It usually develops for velocities higher than those for which the vortices separation, resonating with the structure, occurs. It is a phenomenon of auto-excitation, which originates in the variation of the lift coefficient $C_L$ with the angle of attack $\beta$. 
Let us consider the model shown in Figure 17. The coefficient of total force in the transverse direction \( C_y \) can be expressed by

\[
C_y = -(C_x \cos \beta + C_x \sin \beta)
\]  

(4.3)

with

\[
\beta = \arctan \frac{\dot{y}}{\dot{x}}
\]  

(4.4)

For the variation of force with the angle \( \beta \), this results in the following expression:

\[
\frac{\partial C_y}{\partial \beta} = \sin \beta \left( -C_x + \frac{\partial C_x}{\partial \beta} \right) + \cos \beta \left( C_x + \frac{\partial C_x}{\partial \beta} \right)
\]  

(4.5)

which, for low values of \( \beta \), leads to

\[
\frac{\partial C_y}{\partial \beta} = \frac{\partial C_x}{\partial \beta} + C_x
\]  

(4.6)

If \( \frac{\partial C_y}{\partial \beta} > 0 \), this will mean that there is a continuous energy flow towards the structure, since the forces act in the direction of the separations (Figure 18). The situation is analogous to that which corresponds to a negative damping induced by the flow. In fact, the general equation of movement will be of the following type:

\[
H(m + m_d)\ddot{y} + c\dot{y} + k_y = C_x \frac{1}{2} \rho V^2_{in} \, d_H
\]  

(4.7)

in which

\[
V_{in}^2 = V^2 + \dot{y}^2
\]

and in which the second member represents the value of force \( F_y \).

If, on the other hand, \( \frac{\partial C_y}{\partial \beta} < 0 \), the forces that develop take a direction contrary to the movement (Figure 18), and thus function as positive damping to dissipate energy.

The condition of instability will be defined by
This condition is also known as Den Hartog's condition [29].

Instability is observed when the following relation is valid [27]:

$$v^* = \frac{v}{2\xi d} > \frac{1}{\gamma C_1 A_1}$$

in which \(\gamma\) is a dimensionless parameter of mass defined by \(\gamma = \frac{p_d^2}{4(m + m_a)}\); \(C_1\) is a factor of participation that depends on the form of the mode and the profile of average velocities along the element; and \(A_1 = dC_y/\partial B\).

It is common to analyze the stability to gallop, representing the response, in amplitude, as a function of velocity. Specifically, this representation, based on dimensionless parameters, leads to diagrams of universal stability of the type shown schematically in Figure 19.

The values of \(a^*\) and \(V^{**}\) correspond to

$$a^* = \gamma \frac{a_x}{\xi d}$$

and

$$V^{**} = \frac{v}{\zeta 2\xi f_d}$$
Figure 19. Response curves of rectangular prisms \((d/l = 0.5)\) [30].

Figure 20 shows the variation in \(C_y\) with \(\beta\) for two types of bars currently used in trashracks and in which this phenomenon may occur. Figure 21 shows the variation in \(\partial C_y / \partial \beta\) with \(l/d\) for prismatic bars with a rectangular section, in which the flow is perpendicular to dimension \(d\).

![Image](image.png)

Figure 20. Variation in the coefficient of transverse force with the angle of attack [28].

These figures refer to the results obtained in low-turbulence flows. The presence of turbulence alters these relations. Figure 22 shows an example of this influence.

We also note that Den Hartog's condition is sufficient, but not necessary, since gallop may occur with \(\partial C_y / \partial \beta < 0\). In this case the problem is much more complex and requires analysis of the stability of the oscillations and a precise knowledge of how \(C_y\) varies with \(\beta\).

Figure 23 shows the response, in transverse direction, obtained experimentally, of a rectangular prism in a low-turbulence flow. This response is expressed by the variation of the quotient between the amplitude of vibration \(a_y\)
and the transverse dimension \( d \) with \( V/fd \).

Figure 21. Variation in \( \partial C_y/\partial \beta \) with \( l/d \) when the flow is perpendicular to the side with dimension \( d \) [19,27].

The two phenomena described can be observed: separation of the vortices and gallop, which occurs at a higher flow velocity. The effect of damping is still seen in the response, and its influence is seen in the separation of the vortices and in the development of gallop.

5. CONCLUDING REMARKS

When the designer evaluates the stability of trashracks in hydraulic structures, he must keep in mind not only the aspects related to static stress, but also the possibility of dynamic phenomena which are induced by flow and are capable of causing failure of the elements.

For this reason, trashracks must be designed to prevent the occurrence of these phenomena, particularly since the levels of internal damping of the struc-
tural elements of trashracks (usually made of steel) are low.

To control the dynamic stresses acting on the elements of the trashracks, particularly in the bars, the following principles should be observed:

- Definition of the geometry of the hydraulic structure to create the most regular possible conditions of flow upstream from the trashracks (field of uniform velocities, and with orientation perpendicular to the plane of the trashrack).

- Definition of the geometry of the bars to minimize hydrodynamic stresses. In situations with multidirectional flow, it is appropriate to orient the bars so that their largest transverse dimension is oriented according to the flow.

- Design of the bars and their braces so that the resonant frequency, in the fundamental mode, will be sufficiently high so that resonance phenomena will not occur

\[ f_{n} = \frac{\omega_{1}}{2\pi} > 1.4 f_{re} \]

- When this condition cannot be observed, every effort should be made, for all functioning situations, to keep their resonant frequency, in the fundamental mode, sufficiently low. On the other hand, the resonant frequency in the second mode should be high enough so that resonance phenomena do not occur

\[ \frac{\omega_{1}}{2\pi} = f_{n1} < 0.6 f_{re} \]

\[ \frac{\omega_{2}}{2\pi} = f_{n2} > 1.4 f_{re} \]

- When this technique is used, the structure should be analyzed, and the possibility of gallop should be considered.

If instability is observed in the elements of the trashrack after it is installed in the hydraulic structure, some of the following measures can be
taken:

- Increasing the resonant frequency of these elements, for example, by reducing the distance between braces.

- Introducing helical coils along the bars to interrupt the regularity of the vortices separation. These coils may be made of wire with a diameter about one-tenth the thickness of the bar. The disadvantage of this solution is that it increases the drag forces and, consequently, the head losses. However, we know of no instances in which this technique, which is common in structural aerodynamics, has been used with trashracks.

- Introducing deflecting plates. In the case of "lock in," the purpose of these plates is to interrupt the regularity of the vortices separation. It presents the same disadvantage as the coils.

- Installing devices to introduce damping. This damping may be created, for instance, by neoprene plates.

- Redesigning the trashrack.
APPENDIX

Failures of trashracks in hydraulic structures

This Appendix shows some cases of failures of metal trashracks in hydraulic structures. These cases were chosen to illustrate the mechanisms of excitation discussed throughout the text.

These cases were presented at the IAHR/IUTAM Symposium held in Karlsruhe in 1979, entitled "Experience with flow-induced vibrations."
I. Fracture of support beams

Problem:
The accumulation of materials upstream causes an increase in the velocity through the trashrack. The support beams (Figure I.1) broke. Examination of the fractures indicated that the failure was caused by fatigue.

Geometry:
See Figures I.1 and I.2. The area for passage of flow is 81 m² (5 panels). The maximum length between support beams is 1565 mm.

Boundary conditions:
The beams are steel structures, and are welded and supported as shown in Figure I.2.

Flow conditions:
The average velocity through the trashrack, with the accumulation of materials shown in Figure I.1, is 2.1 m/s.

Dynamic characteristics of the beams:
Their resonant frequency, in the fundamental mode, is 24 Hz.

Causes:
The frequency of the vortices separation is 19 Hz. The proximity of frequencies \( f \) and \( f_{s1} \) suggests the possibility of resonance.

Solution adopted:
Since it was impossible to prevent the accumulation
of materials, the beams were replaced by others that were more rigid (Figure I.3). The resonant frequency of these beams, in the fundamental mode, is 40 Hz.

Reference: [31].

II. Fracture of bars with rectangular section

Problem:
Fracture of trashrack bars shown in Figure I.1.

Geometry:
See Figures I.1, II.1 and II.2. The dimensions of the bars are $H=1320 \, \text{mm}; \, d=19 \, \text{mm}; \, t=76 \, \text{mm}$.

Boundary conditions:
The bars are welded, and the support conditions are those shown in Figures II.1 and II.2.

Flow conditions:
The average velocity through the trashrack is 1.57 m/s. With accumulation of materials, this value change to 2.51 m/s.

Dynamic characteristics of the bars:
The resonant frequency of the bars, in the fundamental mode, is essentially equal to 36 Hz.

Causes:
The frequency of vortices separation is equal to 22 Hz or 35 Hz, without accumulation of materials. These values suggest the possibility of resonance when there is accumulation of materials.
Solution adopted:

Welded an anchoring bar, Figure II.2, to reduce by half the distance between supports. The resonant frequency of the bars, in the fundamental mode, was raised to 147 Hz.

Reference: [31].

III. Fracture of bars with circular section

Problem:

Flow-induced vibrations led to rapid fracture of the bars of the trashrack in the water passages of the reversible units of the "Raccoon Mountain Pumped Storage Project."

Geometry:

As shown in Figure III.1.

Boundary conditions:

The trashrack is fixed in the groove at 6 points (3 on each side). The vertical bars have a circular cross section with diameter of 25.4 mm and are welded to four horizontal beams with rectangular section 25.4 mm thick and 254 mm long, and to the top and bottom beams.

Flow conditions:

In accordance with calculated operating conditions, the average velocity is 2.4 m/s. The possibility that the trashracks could operate at velocities of 1.2 to 4.9 m/s was considered.
Operating conditions:
The frequency of vortices separation varies from 20 Hz to 81 Hz, corresponding to the velocities of 1.2 m/s and 4.9 m/s (Figure III.4). These values were obtained by considering $S_t = 0.21$.

"Lock in" was observed after studies on a model, for $f = 112 \text{ Hz}$ and velocities of 1.7 to 2.6 m/s, for $f = 120 \text{ Hz}$ and velocities of 2.6 to 3.0 m/s, and for $f = 185 \text{ Hz}$ and velocities of 3.0 to 4.3 m/s. Fractures occurred in 17 of the bars of the tested panel, after a testing time of less than 30 minutes and for $v = 4.3 \text{ m/s}$.

Dynamic characteristics of the bars:
The resonant frequencies of the bars in air are:

- In the plane of the trashrack (Figure III.2)
  
  \begin{align*}
  (a) & f_1 = 28 \text{ Hz} \quad \left( \frac{1}{2\pi} = 83 \right) \\
  (b) & f_2 = 63 \text{ Hz} \quad \left( \frac{1}{2\pi} = 39 \right)
  \end{align*}

- In the perpendicular plane (Figure III.3)
  
  \begin{align*}
  (a) & f_3 = 50 \text{ Hz} \quad \left( \frac{1}{2\pi} = 21 \right) \\
  (b) & f_4 = 87 \text{ Hz} \quad \left( \frac{1}{2\pi} = 250 \right) \\
  (c) & f_5 = 113 \text{ Hz} \quad \left( \frac{1}{2\pi} = 940 \right) \\
  (d) & f_6 = 122 \text{ Hz} \quad \left( \frac{1}{2\pi} = 635 \right) \\
  (e) & f_7 = 193 \text{ Hz} \quad \left( \frac{1}{2\pi} = 1115 \right)
  \end{align*}

Vibrations:
Test results are shown in Figure III.4.
Solution adopted:
Redesign of the trashrack; the circular bars were replaced by rectangular bars.

References: [32, 33].

IV. Fracture of bars with rectangular section

Problem:
Reduction or elimination of vibrations registered with the circular bars described in III.

Geometry:
Described in III (Figure IV).

Operating conditions:
Approximate velocity varying between 0 and 5.5 m/s; "lock in" was observed after the studies on the model, for \( f = 30 \) Hz and velocities between 1.5 and 1.9 m/s; for \( f = 116 \) Hz and velocities of 3.1 to 3.7 m/s; for \( f = 267 \) Hz and velocities of 4.6 to 5.5 m/s.

Dynamic characteristics of the bars:
The frequencies of the bars in air are:
- In the plane of the trashrack (Figure IV.2)
  \[ f_a = 80 \text{ Hz} \left( \frac{1}{24} = 20 \right) \]
- In the perpendicular plane (Figure IV.3)
  \[ \begin{align*}
  (a) f_1 &= 41 \text{ Hz} \left( \frac{1}{24} = 19 \right) \\
  (b) f_2 &= 56 \text{ Hz} \left( \frac{1}{24} = 24 \right) \\
  (c) f_3 &= 119 \text{ Hz} \left( \frac{1}{24} = 411 \right) \\
  (d) f_a &= 275 \text{ Hz} \left( \frac{1}{24} = 1042 \right) \\
  (e) f_a &= 502 \text{ Hz} \left( \frac{1}{24} = 1240 \right) \\
  (f) f_a &= 770 \text{ Hz} \left( \frac{1}{24} = 1500 \right)
\]

Figure IV.
Vibrations:
The test results are shown in Figure IV.4.

Solution adopted:
Neoprene supports improved damping. Deflecting plates (15 cm x 30 cm) mounted in the center of the panel eliminated "lock in." The recommended solution included the neoprene supports and the deflecting plates.

References: [32, 33].
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