PAP-450

MODEL SIMILITUDE - EXTENDED FOR ACTIVE SEDIMENT TRANSPORT

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by

R. A. Dodge

FROM

momentum balance milli...

Fig 6

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Active Sediment Transport Modeling

PRELIMINARY

The information in this memorandum should be considered an addendum to the memorandum "Model Similitude" previously given to you. The material in the previous memorandum was directed toward bank protection studies where bank stability with no sediment transport was the main goal. However, some of the sections, equations, and table examples also apply to the continuous transport cases. These will be identified by title, page, figure and table number where and if needed in the following parts of this memorandum. These are in appendix. RAD

Sufficient field samples, hydraulic data, and hydrologic computations are needed to make adequate friction and transport scaling analyses. The data should include bed profile and cross section elevations, water surface profiles and analyzed historical hydrograph data, suspended sediment samples, bed material samples, and sediment discharge rating curves calculated with sediment transport equations.

Estimates of friction performance in terms of equations (1) and (8) are needed to help ensure that the model produces similar hydraulic shear on the boundary, reproduces secondary flows, bed slopes, and water surface elevations with respect to various discharges. Darcy-Weisbach pipe friction coefficient (f) curves should be used. However, four times the hydraulic radius (R) should be substituted for pipe diameter in the Reynolds number and relative roughness parameters. Thus,

$$N_r = 4RV/\nu \qquad (19)$$

where: R = hydraulic radius
V = velocity
ν = kinematic viscosity

$$\text{Relative roughness} = K_s/4R \qquad (20)$$

where: K_s = rugosity

PRELIMINARY

Using the field data and these two equations, a generated table, like table 1, will estimate the range of discharges for which the model is expected to reproduce boundary shears, velocities, and secondary flow adequately.

Prototype rugosity (K_s) for equations (19) and (20) can be determined by using the field data and computed hydrologic data with equation (1) and friction curves. Doing this provides the best estimate of (K_s). A less reliable way to estimate (K_s) is to use the bed material sample grain analysis. Kamphius [6] ~~found~~ reported that approximately

$$K_s = 2d_{90} \qquad (21)$$

where: d_{90} = the particle diameter at which 90 percent of the grains have smaller diameters

A prior ~~t~~report by Brown and Chu [ ] there is also ~~data~~ data that ~~can be used to~~ verifies this.

For three recent model studies we have found that relative roughness scales if the (d_{90}) scales geometrically. Settling velocity and geometrical scaling (Figure 5) are simultaneously attained for model diameter sizes of 1 mm and greater. However, this should be followed by calculating flow Reynolds numbers and relative roughness and developing values of friction for both the model and prototype. The friction ratios such as shown in table 1 should be determined to investigate the discharges at and below which cause significant flow Reynolds number defect. affect

Model friction performance should be checked during model operation by comparing model river staging with field and hydrologic data. The same kind of checks should be made later in the studies to determine if there are bed form distortions causing friction to change. The larger the prototype particle diameter the less likely there will be unnatural dun~~ing~~e formation.

Transport Function

PRELIMINARY

Shields' entrainment function, figure 6, can be considered a transport function for a special case where transport is very nearly equal to zero. Gessler's [10] modification of Shields' function, figure 7, with parallel curves for different probabilities of moving further suggests that transport is related to the Shields' parameters and approximately parallels Shields' critical curve. Graf and Paiz [9] make an even firmer connection of to transport by developing nearly parallel curves~~, figure 8, to~~ Shields' curve in the transition zone in terms of the number of particles moving. Thus it seems quite logical that Shields' parameters should define higher and very active transport as well as incipient motion of sediment. For modeling, the most useful form is the transport function, figure 9, given by Taylor and discussed by Vanoni [11] which uses a dimensionless sediment discharge (q_{s*}) as a third or nesting parameter along with Shields' parameters in equation (11a). The dimensionless sediment discharge parameter (q_{s*}) is defined as

$$q_{s*} = q_s/U_*d \tag{22}$$

where: q_s = sediment discharge in volume per unit width per second
U_* = shear velocity
d = diameter of particle

Taylor curves are approximately parallel to Shields' curve, including the transition dip, figure 9.

Thus ~~model~~ predicting sediment discharge requires investigating the following functional relationship:

$$\frac{q_s}{U_* d} = q_{*s} = \phi\left(\frac{\tau}{(\gamma_s - \gamma_w)d}, \frac{U_* d}{\nu}\right) = \phi\left(T_*, R_*\right)$$

for ~~sed~~ time scaling

The dimensionless sediment discharge parameter uses shear velocity which can be expressed in the following forms:

$$U_* = \sqrt{\frac{\tau}{\rho}}$$
$$= V\sqrt{\frac{f}{8}}$$
$$= \sqrt{RSg}$$

most useful because f must be account for ~~for~~ secondary flow and friction loss

PRELIMINARY

where: q_s = Vol. sediment per unit width per sec.
τ = tractive shear
γ_s = specific weight of sediment
γ_w = specific weight of water
ν = kinematic viscosity
ρ = density of water
V = velocity
f = Darcy-Weisbach friction factor
R = hydraulic radius
S = slope of energy gradient
g = acceleration of gravity

Despite difficulties of determining (f), the second form for U_* is the most convenient to use close to dams.

PRELIMINARY

To model time rates for sediment transport, (q_{*s}) model and (q_{*s}) prototype must be made to fall on common (q_{*s}) curves parallel to Taylor's curves at different discharges. This can be done through adjustments of model (τ_{*s}) and (R_*) by commensurate changes of selected model grain diameters and/or specific weight. Thus any selected model sediment diameter and/or weight can be considered arbitrary until they have been checked on a Taylor's plot. Even settling velocity scaling for diameters can be considered arbitrary but settling velocity frequently shifts model (q_{*s}) values toward the prototype parallel (q_{*s}) values. Thus we often start Taylor transport analyses using assumed settling velocity scaling for sediment diameters. Then change diameters to attain closer Taylor function conformance avoiding changing specific weight of sediment unless it is the only way to attain conformance. The larger the model, the larger the flow Reynolds number, and the larger the sediment diameters, the more likely assumed settling velocity diameters will scale near the (q_{*s}) curve.

The general process of using the Taylor's function is to calculate (τ_{*p}) and (R_{*p}) prototype for some diameter ($d_{\%}$) percentage size of particular interest on the grain analysis curve. Plot this parameter pair of prototype values along with a plot of Taylor's base curve, figure 10, for constant (q_{s*}). Then find the vertical linear offset of the prototype data point from the base curve. Next plot a parallel curve to the base curve passing through the prototype parameter pair. Next, compute (τ_{*m}) and (R_{*m}) for a trial model diameter selected hopefully to represent the prototype selected percentage and particle size. If the plotted model pair (τ_{*m}) and (R_{*m}) does not fall on the parallel (q_{s*}) curve passing through the prototype point then adjust model diameter and/or model specific weight of model sediment to make (τ_{*m}) and (R_{*m}) shift and to plot on the parallel curve. For the impractical small sediment sizes, model diameters can be increased to obtain a workable size and then specific weight can be reduced to make the model transport sediment at proper rates.

For a proposed dam, it is recommended that the model investigations start by finding the model sediment that best simulates the river reach in question compared to hydrologic field data without the dam. This should be checked in the river model without the dam or with the dam in a model with sufficient approach length outside of the pooling effects. The reach outside of the pooling action requires sufficient length to accurately measure water surface and bed slopes developed at computed transport rates. Poor performance of the river part of the model could be an indication of insufficient sediment bed sampling, insufficient hydrologic data, poor model sediment design, distorted dun~~ing~~e formation, or a model that is too small ~~making~~ resulting in model flow Reynolds numbers ~~that are~~ OK too low, ~~in the model~~. Prototype verification of model slopes, depths, velocities, transport and general performance of a river part of the model can provide considerable confidence in a model. After the river verification, the reservoir sediment deposits should be formed by model operation with sediment transport. One possible way would be by overfilling the model with sediment. Then operate the model without model-fed transport until scour transport is very low. Finish deposit formation by operating the model with the proper transport rate associated with the discharge. The bed form, observations, and updated calculations of (τ_*) and (R_*) may indicate that the model sediment needs to be redesigned to study certain locations of pertinent interest near the dam.

We consider the information that we now have concerning your Dam as insufficient to study the river transport modeling case. Therefore, examples were based on the sluiceway discharge-head calibration of 570 m^3/s assuming a prototype flow approach width of about 6 m, approach depth equal to calibration head, a (K_s) of 144 mm and settling velocity scaling of diameters. Three cases are for sizes where 20, 50, and 80 percent are finer.

The plotted prototype values for the 80, 50, and 20 percent diameter sizes of (τ_{*p}) and (R_{*p}) are the open circle data points at the right end of the curves. The subscript p denotes prototype. The curves were drawn through the prototype points parallel to the lower base (q_{s*}) curve taken from Vanoni and Taylor. The model diameter size (d_{50_m}) that was selected by settling velocity to represent (d_{50_p}) was used to calculate (τ_{*m}) and (R_{*m}) where m denotes model were plotted as the open circle data points on the left.

The prototype and model particle d_{50} have about a 0.30 Gessler probability of moving. The plotted point of the Shields' parameters for the model particle (d_{50m}) is too low and is to the left of the transition dip. Any attempt to correct by diameter size change results in diagonal shifting parallel to the left sloping part of the Taylor function. Only a reduction of the model specific weight will shift the data point vertically upward toward the common (q_{s*}) curve.

(τ_{*m}) versus (R_{*m}) for the 80 percent size plotted high over the middle of the transition dip. This data point can be shifted vertically downward by increasing the specific gravity of the sediment particle, but is much easier and more practical to shift diagonally toward the (q_{s*}) curve by increasing the diameter and using natural sediment. The solid data point on the red (q_{s*}) curve was attained by doubling the diameter determined by settling velocity and is considered an adequate representation. The prototype and model particle (d_{80}) should have a Gessler probability of moving less than 0.05.

The model (τ_{*m}) versus (R_{*m}) for the 20 percent size plotted considerably lower than the prototype (q_{s*}) curve and it cannot be shifted by diameter changes

PRELIMINARY

because the shift is approximately parallel to the (q_{s_*}) curve and to the left of the transition dip. However, it could be shifted vertically upward by decreasing the specific gravity of the sediment particle. It might be necessary to change the model scale to match the specific gravity of some known material. The prototype and model particle (d_{20}) d_{50} have a probability of moving greater than 0.85. It should be noted these results are for one river approach discharge (570 m^3/s) and depth. Similar plots should be made at other discharges to determine the range or part of the hydrograph that can be programmed at different model discharges. Mixing model sediment specific weights could extend the range of the diameter analysis represented and range of the discharge that could be operated in the model.

For your study

The Gessler probabilities associated with the (q_{*s}) curves are low. This is expected because the pooling caused by a dam decreases the (τ_*) and (R_*) values considerably. It is doubtful that doubling the natural river slope (your Dam) prior to sluicing tests represents any natural condition that may occur. It is expected the reservoir will be filled by an advancing steep fronted sediment deposit as shown on figure 17 of the "Blanco" report [3]. This is especially likely for steep mountain streams with coarse sediment. Eventually this deposit will toe out in the dam, figure 18. If the previous sediment scaling analyses indicate that hydrographs can be operated in your model, it would be better to deposit the bed naturally by simulating a series of flood hydrograph-sediment rates. If not, some significant or dominate discharge should be determined and operated at its appropriate bed sediment transport rate until the model bed attains its required slope, bed shape, and the transport rate over the entire model reach are the same. This is when the amount of sediment entering the model is the same as that leaving the model and matches the prototype sediment discharge curves.

PRELIMINARY

If the river has steep sides - like in a canyon with nearly vertical walls - the river width cannot change much with elevation and the river will eventually tend to revert to the same slopes, flow depths, and velocities as existed before building the dam, approximating river data determined from field measurements and hydrologic studies before the project was built. This provides a way of estimating the bed shape for testing sluicing. We expect that the proper shape would be like figure 18, Blanco report [3]. It would be better to let the model bed form during flow with the proper transport with respect to discharge.

Our experience, Blanco report [3], indicates that there is an optimum discharge for sluicing. This discharge is generally low and cleaning is by cut bank action sweeping out by meandering with edge sloughing of sediment. Once this optimum discharge is approximately found by the model, transport sediment model design should be rechecked using the optimum discharge. Different sediment might be required to model bed material at the low optimum sluicing discharge.

Falling head sluicing tests should be done after checking to find the minimum discharge or range that will scale in terms of friction analysis and in terms of transport parameter analysis of selected model grain sizes and distribution.

This discussion may not solve all problems associated with sediment modeling; however, it provides a way to ~~determine~~ gain insight as to where and when a model is adequate. This approach also shows the modeler what might be adjusted or distorted to help make model transport conform to the prototype. The examples indicate the same adjustment in magnitude and direction can improve model approximation for one case and decrease it for another. The same problem can apply for one

PRELIMINARY

model at different discharges or for different percent grain diameters ($d_{\%}$). A close look at (q_{s*}) modeling is at present time the best way known to keep from unintentionally making arbitrary modeling adjustments in wrong directions and finding out how well a model really approximates the prototype at different discharges and diameter sizes.

Shields curve indicated minor effect of ρ_s

However, it can be argued that sediment density effects have still not been fully accounted for. For example, there may be added mass effect due to flow acceleration and oscillation. However, it seems that q_* function show when density corrections are req'd @ low R_* values yet

Deriving Shields' function by dimensional analysis can give some indication of what other parameters might need to be included in modeling; however, dimensional analysis does not resolve which parameters are redundant or when they need to be included. These issues can only be resolved by sufficient data aquisition taken and analysed in a manner that doesn't produce spurious correlation.

Simons and Senturk showed by dimensional analysis that noncohesive material has an entrainment function restated as:

$$\frac{\tau_c}{(\gamma_s - \gamma_w)d_g} = \phi\left(\frac{U_{*c}\, d_g}{\nu}, \frac{d_g}{R_c}, \frac{\gamma_s}{\gamma_w}\right)$$

$$U_{*c} = \sqrt{\frac{\tau_c}{\rho_w}} = V_c\sqrt{\frac{f}{8}} = \sqrt{RSg}$$

where symbols not previously defined are:

τ = Tractive shear
γ = Specific weight
ν = kinematic viscosity
c = subscript denoting critical
s = subscript denoting sediment
w = subscript denoting water
d_g = Particle diameter
U_* = Shear velocity
V = Velocity

ρ = density
f = Friction factor
R = Hydraulic Radius
S = Slope (energy gradient)

It should be noted this relationship includes all variables to define lift or drag. This function, except for relative roughness ~~ratio~~ (d/R_c) and relative specific weight $[\gamma_s/\gamma_w]$, is the same as Shields' function derived on the basis of drag and logarithmic velocity distribution theory. Relative roughness is commonly dropped from the relationship on the basis of relatively deep flow and/or fine grain sediment bed material. The relative specific weight term is commonly dropped on the basis of being concerned with water and natural sediment only or using sand in the model to represent prototype sediment. Thus equation (9) is commonly reduced to just the Shields' function parameters.

$$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = \tau_* = \phi\left(\frac{U_* d}{\nu}\right) = \phi\left(R_*\right) \quad \ldots (7a)$$

However, the data on figure 6 strongly indicate that relative specific weight has little influence.

In summary, the effects of dropping (d/R) and (γ_s/γ_w) still need to be investigated. Also, there is still a need to modify this approach for cohesive materials. Until these investigations are made, we will continue to approach modeling using Taylor's transport function.

PRELIMINARY

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PRELIMINARY

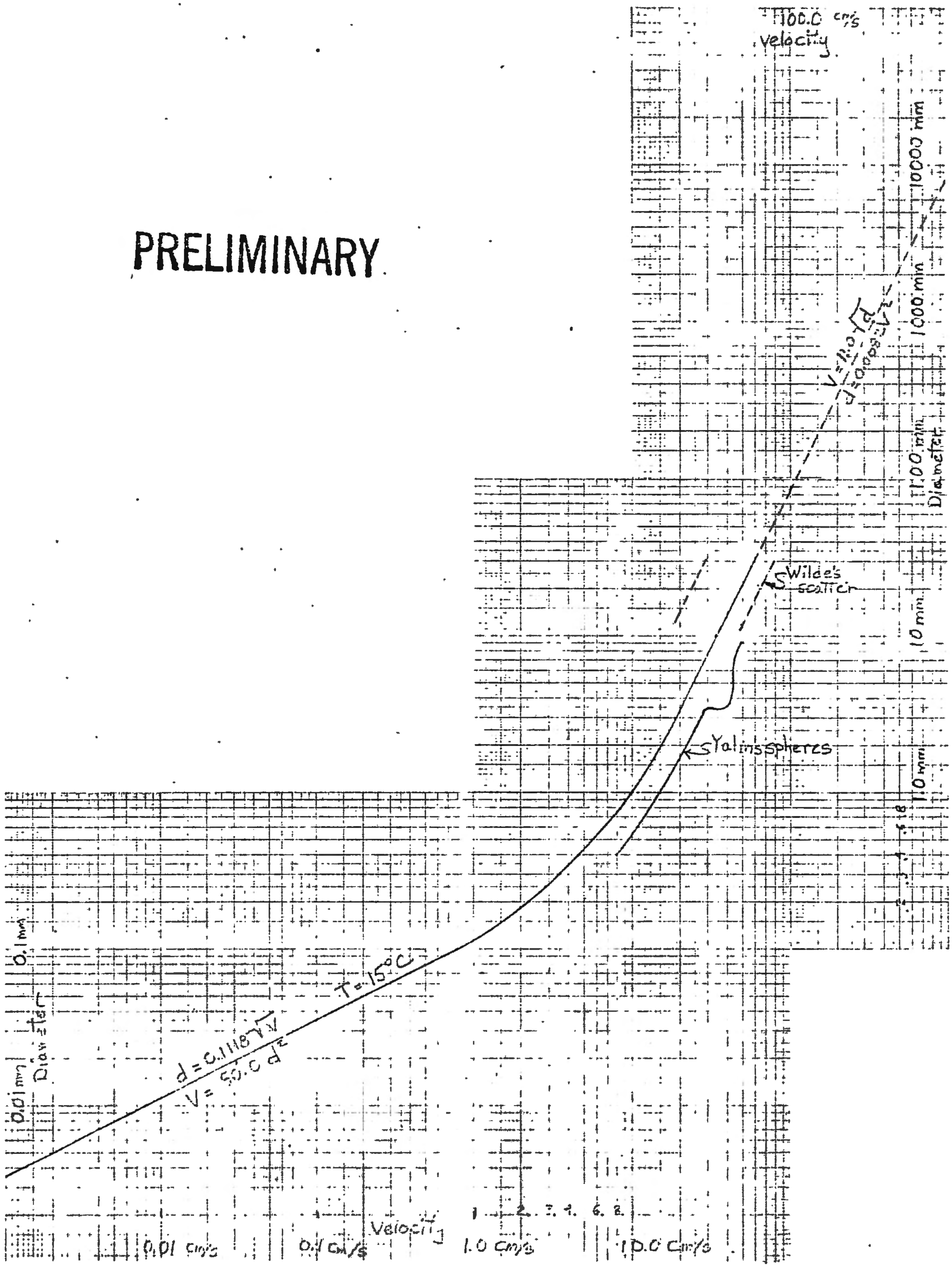
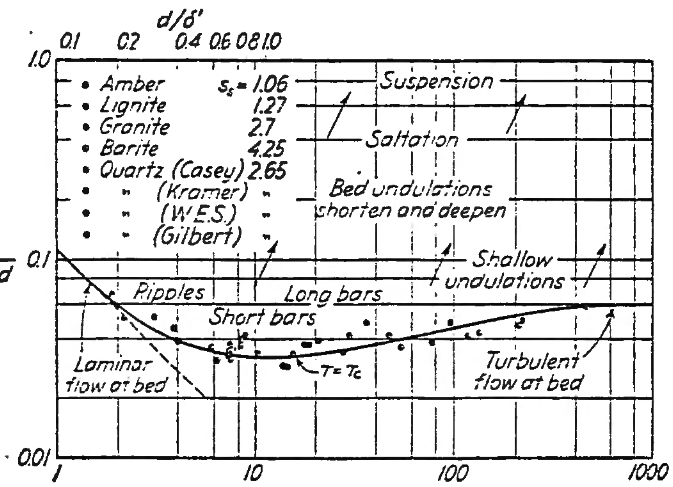


Figure 5

PRELIMINARY



$$T_* = \frac{\tau}{\gamma(s_s-1)d}$$

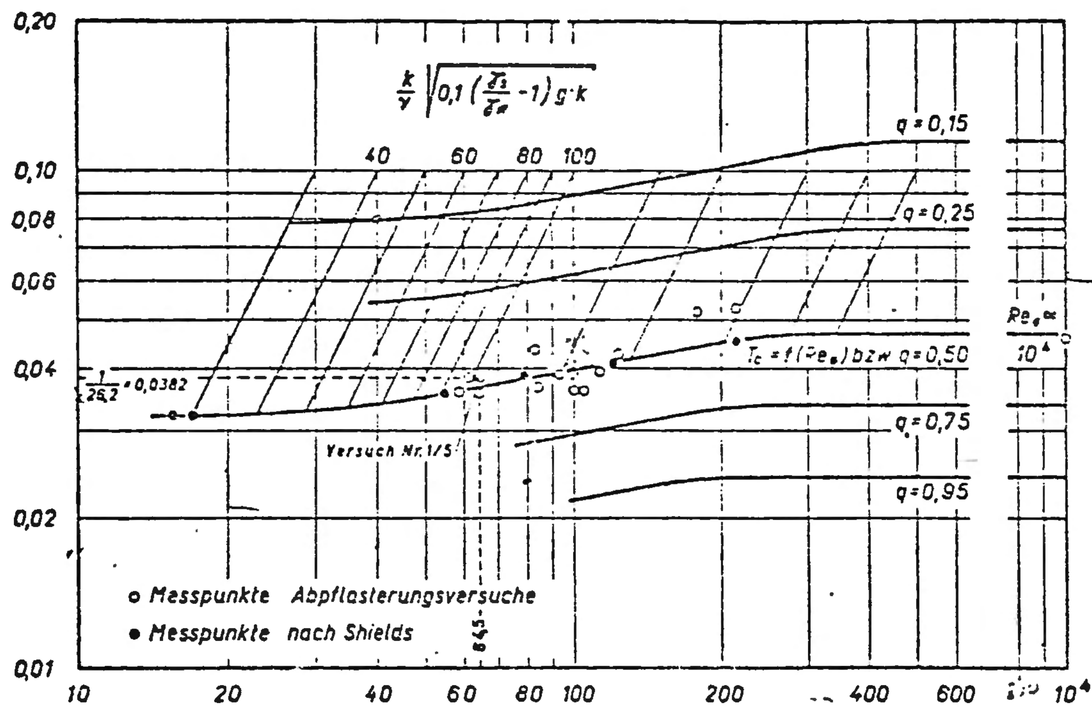
$$\frac{d\sqrt{\tau/\rho}}{\nu} = \frac{d\sqrt{gyS}}{\nu} = R_*$$

Figure 6.—Analysis by Shields of the entrainment function.

PRELIMINARY

Gessler modified and extended Shields' function to incorporate probability of particles to remain on a bed of mixed sizes. This figure below also indicates that transport is a function of $[\tau_*]$ & $[R_*]$.

τ_*



R_*

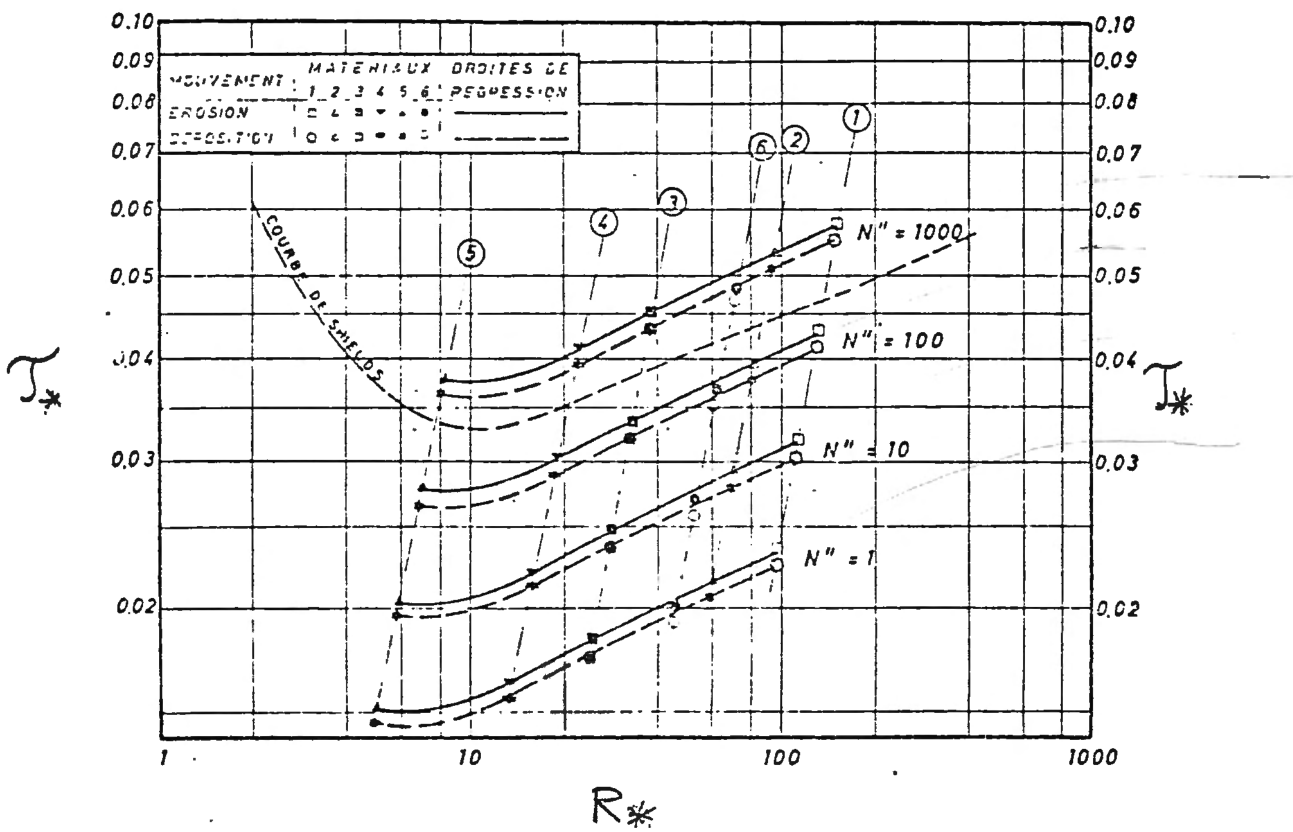
q = Probability of not moving

$\tau_* =$

Figure 7

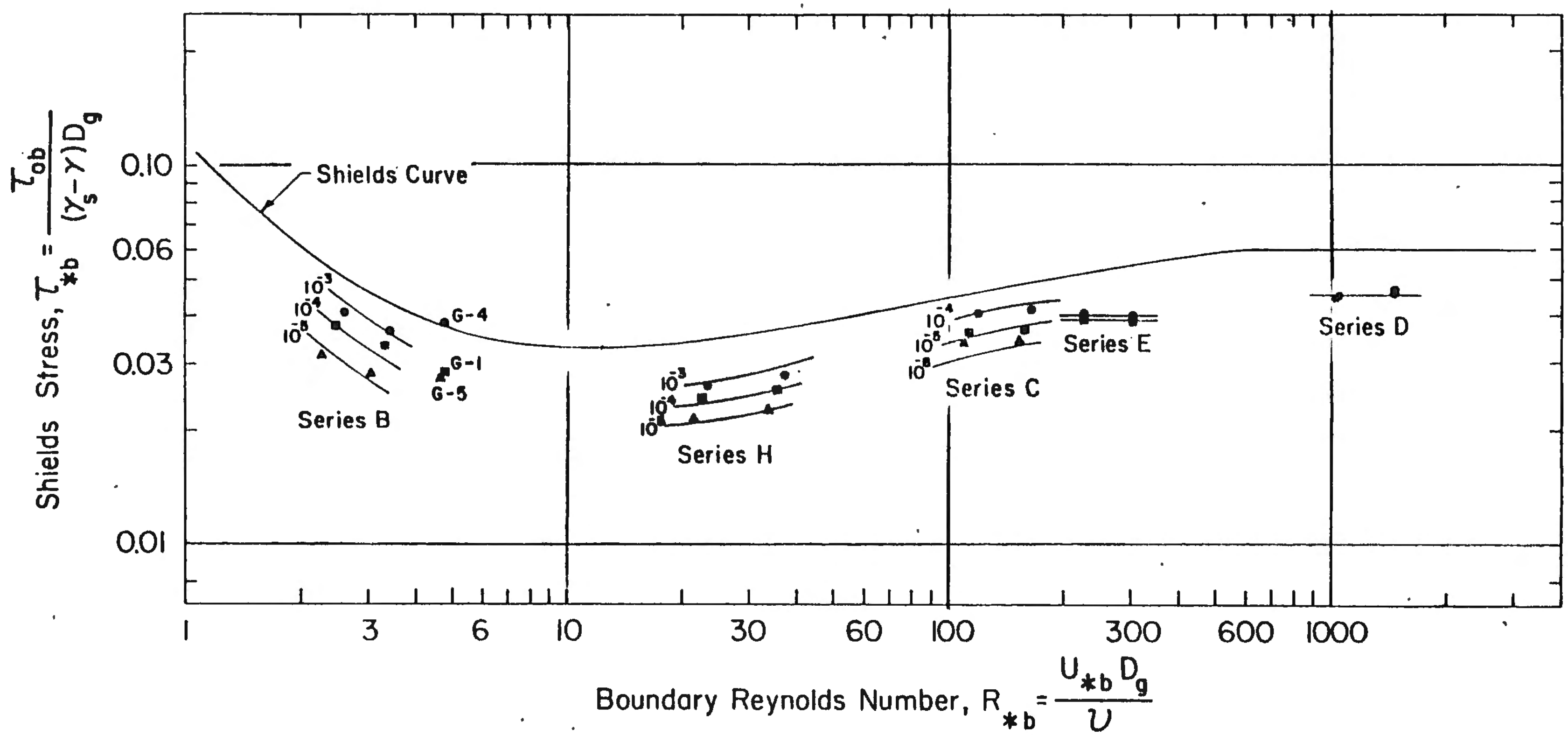
PRELIMINARY

Graf's curves below indicates that transport is a function of $[\tau_*]$ & $[R_*]$.



N'' = number of moving particles per unit bed area

Figure 8

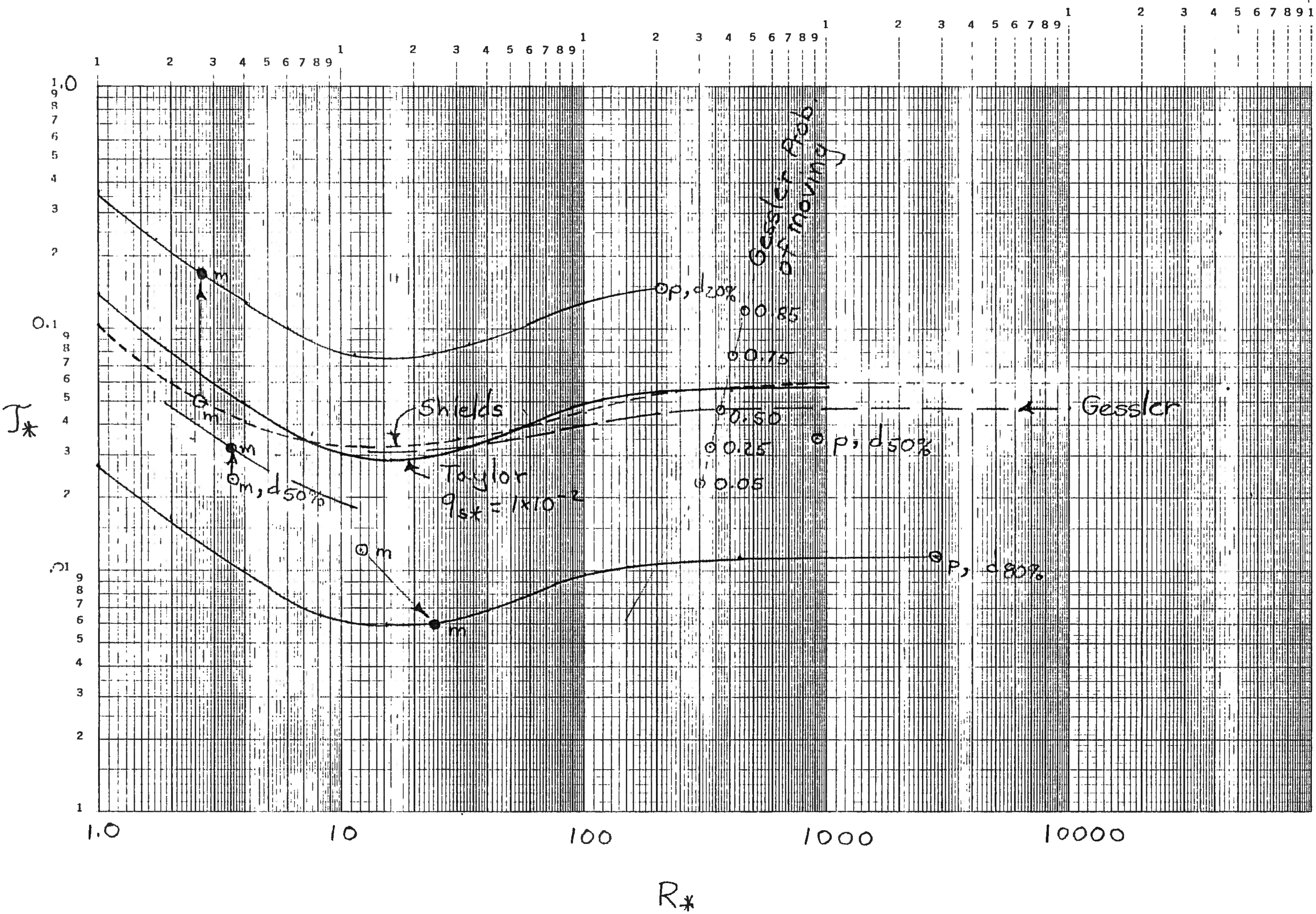


Contours of dimensionless sediment discharge (q_{*b} = constant) versus τ_{*b} and R_{*b} for low-transport, flat-bed experiments. Numbers near the ends of the contours are values of q_{*b}. In each series the data points with the higher values of R_{*b} are for warm water; and constant velocity and depth experiment-pairs are indicated by the same data symbol.

$$q_{*b} = \frac{q_s}{U_* D_g}$$

Figure 9

PRELIMINARY



PRELIMINARY

Figure 10

Appendix

Table 1. - Values of ratio of friction factor model to prototype versus prototype discharge

Prototype discharge (ft^3/s)	Prototype discharge (m^3/s)	Ratio of friction factor model to prototype
* 1 000 000	28 300	1.00
** 405 000	11 500	1.00
160 000	4 530	1.02
80 000	2 270	1.06

* Maximum design flood for original dam design.
** Maximum for total powerplant capacity including possible second extension.

Actual Model Friction Performance

The model was compared with computed water surfaces based on field-verified Manning's "n" values. A sample of 35 model water surface elevations, measured at different river stations and discharges ranging from 90 000 to 600 000 ft^3/s (2 550 to 17 000 m^3/s), agreed with the prototype computed values to within three-fourths percent of cross section hydraulic radius on the average and all values were within 1-1/3 percent. Thus, the roughness of the model and prototype were considered sufficiently verified.

Since friction was reproduced in the model, point velocities, velocity profiles, and secondary flows were expected to scale provided there ~~are~~ was no ~~major~~ significant defects in model geometry ~~similitude~~. Because of the adequate friction scaling, flow shear on the boundary determined from velocity profiles was expected to scale within the precision of measured velocity and differences of elevation between the flow boundary and the velocity measuring device. However, friction performance checks should be made during the course of the study to check for possible changes friction caused by duning or armoring.

Some uniform flow friction equations often used in model studies are:

$$S = \frac{f}{4R} \cdot \frac{V^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1) = \frac{f}{8} \cdot N_F^2$$

$$\tau = \gamma R S \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\tau = \rho \cdot \frac{f}{4} \cdot \frac{V^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\tau = \rho \left[\frac{V_2 - V_1}{2.5 \log_e (Y_1 - Y_2)} \right]^2 \quad \ldots\ldots\ldots (4)$$

The variables in these equations are:

S = slope of the energy gradient

T = tractive shear

γ = specific weight

R = hydraulic radius

f = Darcy-Weisbach friction coefficient

ρ = density

g = acceleration of gravity

V = velocity

Y = distance from bed

All these equations are for uniform flow only and do not incorporate the effects of gradual acceleration and deceleration on average tractive shear at a river flow section or structure. These effects can be accounted for similar to [1]* by using the terminology in figure (1), differential form, and equation (1) for ~~the friction loss~~ shear in opposition to flow and writing the energy equation, ~~as:~~ results in:

$$\frac{V^2}{2g} + D + dh = \frac{(V + dV)^2}{2g} + (D + dD) + \frac{\tau \, dX}{\gamma D} \quad \ldots\ldots\ldots\ldots (5)$$

* Numbers in brackets refer to references.

PRELIMINARY

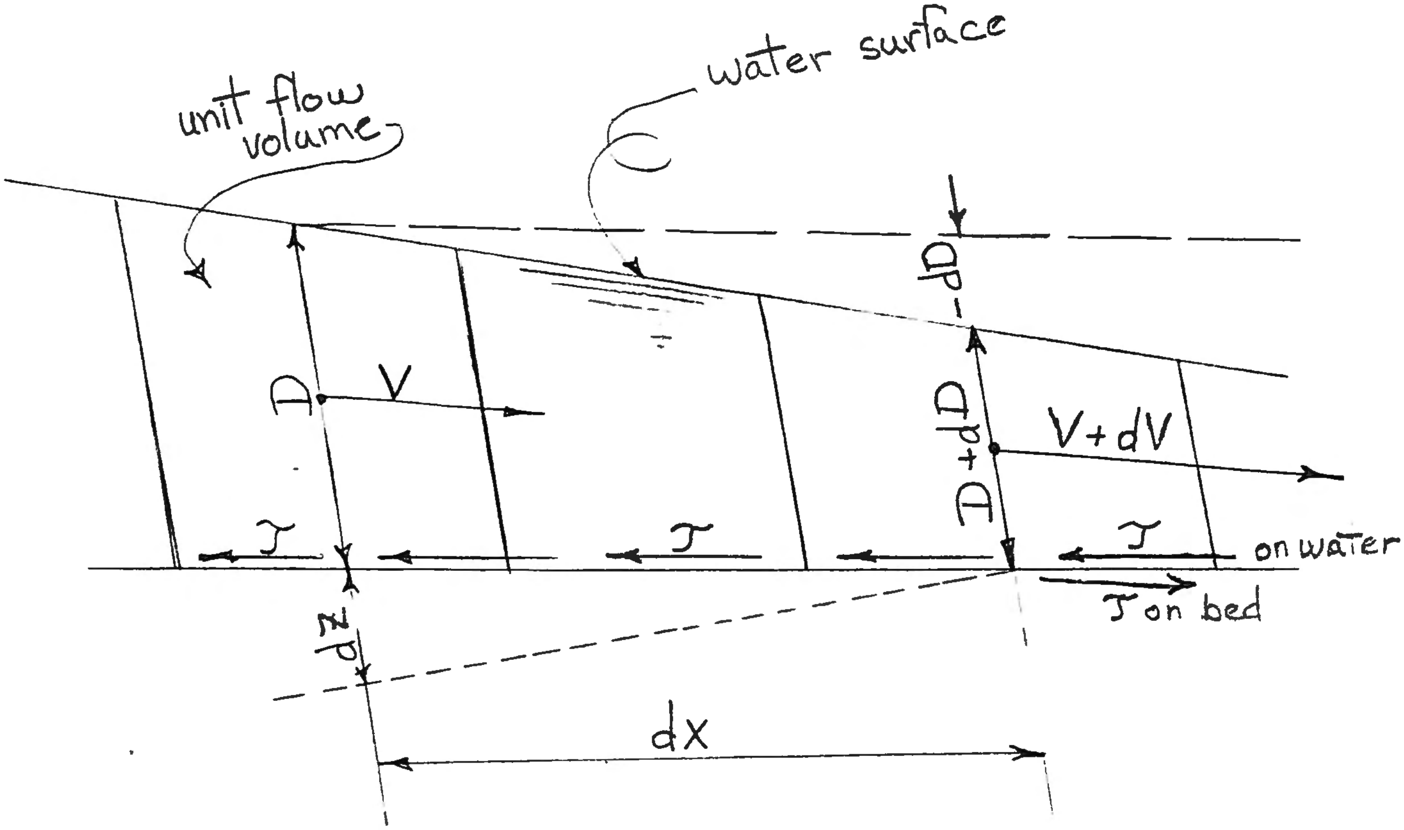


Figure I - Definition Diagram for gradually varying flow.

The term $(dV)^2$ is small relative to $(2VdV)$ and solving for (τ) results in:

$$\tau = D\gamma\left(-\frac{VdV}{gdx} - \frac{dD}{dx} + \frac{dh}{dx}\right) \cdots\cdots (6)$$

This equation does include the effects of gradually accelerating or decelerating flow.

The Darcy-Weisbach friction loss equation was used to define dimensionless tractive shear. This and the other dimensionless variables were defined as:

$$\tau_* = 8\tau / f\rho V_c^2 \qquad h_* = h/X_c$$

$$V_* = V/V_c \qquad dV_* = dV/V_c$$

$$X_* = X/X_c \qquad dX_* = dX/X_c$$

$$D_* = D/X_c \qquad dD_x = dD/X_c$$

$$dh_x = dh/X_c$$

Where an asterisk denotes dimensionless variables, c denotes characteristic values, and (f) is the Weisbach friction coefficient. Solving for the dimensionless variables, substituting them into equation (6), grouping characteristic variables with constants into terms enclosed in parentheses, and letting density (ρ) equal specific weight (γ) divided by acceleration of gravity (g) result in:

$$\frac{\tau_* dX_*}{D_*}\left(\frac{V_c^2 f X_c}{8g}\right) = -V_* dV_*\left(\frac{V_c^2}{g}\right) - dD_*(X_c) - dh_*(X_c) \cdots (7)$$

Dividing this equation by any one group of variables such as (X_c) results in ,for instance:

$$\frac{J_* dX_*}{D_*}\left[\frac{V_c^2}{gX_c}\cdot\frac{f}{8}\right] = V_* dY_*\left[\frac{V_c^2}{gX_c}\right] - dD_* - dh_* \quad \ldots(8)$$

where (V_c^2/gX_c) is the Froude number squared and ($f/8$) is a function of Reynolds number [$V_c R_c/\nu$] and relative roughness [$K/4R_c$].

Equation (8) is dimensionless and the terms in brackets are dimensionless parameters or pi terms. To exactly apply results from a model or from any laboratory test clay erosion test facility, all these pi terms should be the same for the ~~facility~~ model or facility as well as for the river or canal in question. Satisfying this requirement would assure that forces, turbulence, and secondary flows are similar for an actual channel and the erosion test device or a hydraulic model. In practice, complete compliance with this requirement usually cannot be accomplished. However, efforts should be made to determine the degree of compliance and the effects of noncompliance in the interpretation and use of data obtained with any model or erosion test facility.

To apply equation (8) to both the model and prototype, its dimensionless parameters must be carefully considered for design, assessing modeling potential, determining model limitations, and determining model verification needs, ~~for a model.~~ By tradition, the model Froude number [V_c^2/gX_c] is made equal to the prototype for a free surface model making necessary use of analysis, field data, ~~or~~ and best hydrologic estimates to check friction, time, sediment entrainment, and transport scaling.