ATMOSPHERIC SIMULATION USING A STRATIFIED LIQUID MODEL

BY

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ABSTRACT

Analytic and physical model studies were made to demonstrate the feasibility of using stratified liquid to simulate mesoscale (2 to 20 km) atmospheric wind fields over mountainous terrain. The similitude law for the atmospheric properties is based upon the concept of potential density. Topography in the model is produced using plastic relief maps with a 2:1 vertical distortion. The maps are placed in a circular tank having a diameter of about 6 meters. Wind velocities are induced through the rotation of a movable lid which floats on the liquid in the tank. Wind direction is varied by changing the orientation of the maps within the tank. The types of atmospheric gradients which can be simulated include inversions, pre-storm, during-storm, and post-storm conditions. Streamlines in the model are traced with dyes injected into the liquid. Local velocities are measured with a laser-doppler anemometer.

Field studies in the Sierra mountains have shown that the model accurately simulates the atmospheric wind field for a wide variety of atmospheric conditions. The model can be used in cloud seeding studies, pollution tracking from point sources, and wind farm prospecting.

NOMENCLATURE

\( a \) = radius of earth
\( b \) = pressure force
\( c_p \) = specific heat at constant pressure
\( c_v \) = specific heat at constant volume
\( f \) = frictional force
\( F \) = Coriolis parameter
\( g \) = gravitational force
\( g_0 \) = gravitational constant
\( H \) = height of atmosphere from sea level
\( I \) = 1 km
\( K \) = horizontal curvature of trajectory
\( k_o \) = minimum curvature
\( L \) = maximum horizontal length in domain of interest
\( n \) = coordinate axis in plane of trajectory and normal to streamline
\( p \) = atmospheric pressure
\( p_0 \) = standard atmospheric pressure at sea level
\( q \) = angular velocity of model lid
\( R \) = rotational Reynolds number
\( s \) = coordinate axis in plane of trajectory and tangent to streamline
\( t \) = time
\( T \) = temperature
\( u \) = component of velocity vector \( V \) in the easterly direction
\( U \) = maximum wind velocity
\( V \) = instantaneous velocity vector relative to the observer
\( V_r \) = velocity of the surface of the earth relative to a fixed inertial system
\( z \) = coordinate axis perpendicular to the plane of the trajectory
\( \theta \) = angle between velocity vector and isobar
\( \nu \) = kinematic viscosity
\( \rho \) = density of atmosphere
\( \rho_d \) = density of dry air at sea level
\( \rho_0 \) = 1.225 kg/m at 15 C
\( \phi \) = latitude of coordinate center
\( \Omega \) = angular velocity of Earth
\( \Omega \) = 2\( \pi \) radians per 86,400 seconds

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A bar over a parameter signifies a dimensionless number which varies between 0 and 1.

The subscript n refers to values in nature. The subscript m refers to model values.

An arrow over an expression signifies a vector quantity.

**INTRODUCTION**

Models have been used to simulate atmospheric phenomena for almost half a century. One of the earliest investigations was by Abe (1), who modeled the air currents and cloud formations around Mount Fujiyama in Japan.

In general, two types of atmospheric models are used. The first, and most common type uses air as the working fluid. With this type of model, flow patterns, eddies, and diffusion characteristics can be studied. Air models are extremely useful in simulating flows around buildings and diffusion from point sources. Because of the difficulties in establishing density gradients, this type of simulation is usually limited to studies in the lowest layer of the atmosphere.

The second type of model uses a liquid as the working fluid. Liquid models have their greatest range of applicability where gravity effects are significant. Studies in the past have included the progress of a density flow (such as a dust cloud or a cold front), wave motion and mixing across an interface of a density flow, and the effect of mountain ranges upon motion in the upper atmosphere (2).

The model described in this paper uses a stratified liquid to simulate the atmosphere. In contrast to previous studies of simplified topographies, this investigation uses the complicated three-dimensional land form actually found in nature. A derivation of the model laws is presented followed by a description of the most important features of the physical model. Model predictions are compared to actual field measurements.

**SIMILITUDE LAWS**

For a model to accurately simulate conditions in nature, the model must have geometric, kinematic, and dynamic similarity. This is another way of stating that length, velocity, and force ratios are similar at corresponding locations in the model and in nature.

For geometric similitude, the ratio of the length dimension must be the same in all coordinate directions. The similitude requirement applies not only to the topography, but also to the curvature of the geostrophic streamlines. If the topography is vertically distorted, verification and/or calibration studies are necessary.

Verification studies compare model predictions with observed values in nature to determine the validity of the predictions. Calibration studies show how a model must be adjusted to reproduce an event observed in nature.

Kinematic and dynamic similarity can be achieved by properly reproducing Newton's Second Law in the model. In addition, since the fluid properties are not constant in the atmosphere, an equation of state must also be properly represented in the model.

Newton's Second Law must be written relative to an inertial coordinate system. If the motion is viewed from a moving coordinate system, the equations must account for this movement. To simulate the atmospheric motions on a mesoscale, the relationship of the earth to the sun can be considered as an inertial coordinate system. The rotation of the earth is then considered as a moving coordinate system. The velocity of an element of the atmosphere can be described relative to an observer traveling with the moving coordinate system.

In this case, the absolute velocity is the vector sum of the movement of the earth and the velocity of the particle as observed from the earth, or

\[
\mathbf{V}_a = \mathbf{V}_e + \mathbf{V}_p
\]  

(1)

The acceleration of the particle in vector notation is given by

\[
\frac{d\mathbf{V}}{dt} = \mathbf{\Omega} \times \mathbf{V} + \mathbf{f}
\]  

(2)

Substitution of equation (1) into (2) results in

\[
\frac{d\mathbf{V}}{dt} = \mathbf{\Omega} \times \mathbf{V}_e + \mathbf{f} + \mathbf{\Omega} \times (\mathbf{V}_e + \mathbf{V}_p) - \mathbf{\Omega} \times \mathbf{V}_p
\]  

(3)

Newton's Second Law states the sum of the forces is equal to the mass times the acceleration. In terms of force per unit mass the equation is expressed as

\[
\frac{d\mathbf{V}}{dt} = b + g + f
\]  

(4)

With respect to the observed velocities the equation becomes

\[
\frac{d\mathbf{V}}{dt} = b + g + f + \mathbf{\Omega} \times \mathbf{V}_e - \mathbf{\Omega} \times \mathbf{V}_p
\]  

(5)

The vector equation (5) does not depend upon the coordinate system used as a reference frame. One method of describing the atmospheric motion relative to the point of observation on the surface of the earth is to use a natural coordinate system. A natural coordinate system is defined in
terms of a trajectory of a particle. At the coordinate origin, the velocity vector lies tangent to the trajectory. The tangent is called the s axis. This axis forms one of three mutually perpendicular axes. The other two are the n and the z axes where n is in the plane of the motion and z is directed perpendicular to the plane. This coordinate system was used in the model with the coordinate origin taken at the center of the model.

The equation of horizontal motion in the three natural coordinate directions (neglecting friction) are

\[
\frac{dV}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial s} - fV 
\]

\[
\frac{V^2}{a} = \frac{1}{\rho} \frac{\partial p}{\partial n} - g - 2u\Omega \cos \phi 
\]

The term horizontal in the above equation refers to surfaces which are parallel with mean sea level.

For the most general case, the tangent to the particle trajectory does not coincide with the tangent to the isobars in nature. The system of equations can be transformed for use with isobaric maps by defining n as the coordinate direction of the horizontal pressure gradient. The angle that the wind makes with the isobars is defined as \( \theta \). If the wind is deflected toward lower pressures, \( \theta \) is positive. For deflections toward higher pressure, \( \theta \) is negative.

Equations (6), (7), and (8) can be normalized in such a fashion that all of the variables, both independent and dependent, vary between zero and unity over the domain of concern. This is accomplished by defining the following dimensionless variables:

\[
\bar{p} = \frac{p}{p_0} \quad \bar{n} = \frac{n}{L} \quad \bar{z} = \frac{z}{H} \quad \bar{u} = \frac{U}{U_0} \quad \bar{\Omega} = \frac{\Omega}{\Omega_0} \quad \bar{\rho} = \frac{\rho}{\rho_0} 
\]

Realizing that the terms \( V^2/\alpha \) and \( 2u\Omega \cos \theta \) are small, it can be shown that the equation of motion in terms of the dimensionless variables is given by

\[
\frac{d\bar{V}}{dt} = \left( \frac{\bar{p}}{\bar{\Omega} \bar{\rho} \bar{U}} \right) \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{n}} \sin \theta 
\]

\[
\frac{\bar{V}^2}{\bar{a}} = \left( \frac{\bar{p}}{\bar{\Omega} \bar{\rho} \bar{U}} \right) \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{n}} - 2 \sin \theta \left( \frac{\bar{\rho}}{\bar{\rho}_0} \right)^2 \frac{\bar{\rho}}{\bar{\rho}_0} - \bar{g} 
\]

The dimensionless parameters are generally defined as:

\[
\frac{\bar{p}}{\bar{\rho} \bar{U}} = \text{pressure parameter} \\
\bar{\Omega} \bar{\rho} \bar{U} = \text{Rossby number} \\
\frac{\bar{U}^2}{\bar{g} \bar{H}} = \text{Froude number} \\
\frac{\bar{\rho}}{\bar{\rho}_0} = \text{density ratio}
\]

Equations (9), (10), and (11) form the basis for designing the model. For instance, if the simulations are all of steady state conditions, the wind vector will be coincident with the isobars. From equation (9), the angle is equal to zero. It should be noted that all terms containing the Rossby number drop out. Therefore it is not necessary to be concerned with the Coriolis forces. This limits the model to the simulation of "snap shots" of the dynamic process that occurs during the development of a storm. One can observe the flow lines before the storm, during the storm, or after the storm, but not the continuous unsteady process of the storm development.

Kinematic similitude is achieved by proper scaling of the Froude number. This requires that

\[
\bar{U}_m = \bar{U}_n \sqrt{\bar{H}_n / \bar{H}_m} 
\]

Therefore, if there is vertical distortion, the velocity is scaled by the vertical and not the horizontal scale ratio.

The equation of state for the atmosphere can be obtained by assuming that the atmosphere behaves like a polytropic process of a perfect gas. The expression is

\[
\rho \bar{p}^n = \bar{\rho}_0 \bar{p}_0^n = \text{constant} 
\]

The value of the exponent \( n \) can be considered to be constant for discrete layers of the atmosphere. The absolute value of the exponent varies with the atmospheric conditions. For specific processes, \( n \) assumes certain values. These are as follows:

- **Constant pressure** \( n = 0 \)
- **Isothermal** \( n = 1 \)
- **Isentropic** \( n = k \)
- **Constant volume** \( n = \infty \), \( C_p/C_V > 1 \)

For dry air, \( k \) is equal to 1.4.
Equation (13) can be solved for the reference density giving
\[
\rho_p = \rho_0 \left( \frac{p}{\rho} \right)^{1/K}
\]
(14)
This expression is known as the potential density. For a compressible fluid, the potential density is defined as the resulting density when a parcel of air is brought isentropically to a reference elevation. Potential density is analogous to potential temperature, which is commonly used in meteorology to describe atmospheric conditions. The advantage of using potential density becomes apparent if the potential density at any elevation is divided by the potential density at sea level. This results in an expression known as the relative density.

Relative Density = \frac{\rho}{\rho_0} = \left( \frac{p}{\rho_0} \right)^{1/K}
(15)

In terms of temperature and pressure, the relative density is given by

Relative Density = \left( \frac{T}{T_0} \right)^{1/(K-1)}
(16)

The temperature in these equations must be expressed on the Kelvin scale.

With an incompressible fluid, the relative density is defined as the ratio of the density at any elevation to the density at some fixed elevation. Therefore, the atmosphere can be properly simulated when the potential density distribution in the model matches that in the atmosphere.

THE MODEL

In order to reduce costs in reproducing the topography, commercially available three dimensional plastic relief maps are used. These maps have a 1:250 000 horizontal and a 1:125 000 vertical scale. A lightweight concrete backing is used to maintain the vertical alignment. Several maps are placed together and the boarders are cut to form a disk about 2 m in diameter. This disk is placed in a 5.75 m diameter tank which is 150 mm deep. The stratified liquid which simulates the atmosphere enters the tank through four ports in the tank floor. Wind motion over the model is generated by a rotating disk which floats on the liquid. The disk is driven by a variable speed motor which is connected to a vertical spindle located in the center of the tank. With this configuration the curvature of the isobars can be closely approximated, figure 1. In addition, the density gradient remains essentially constant during the course of an experiment.

To obtain complex density gradients the flow from two containers is combined and their resultant product is introduced into the tank. One container holds freshwater and the other contains a saline solution at the maximum required density. By properly varying inflow rates from the two containers, it is possible to simulate any density gradient.

Tests have shown that the flow lines tend to remain at constant density elevations and flow around obstacles rather than pass over them. This effect is so strong that the model essentially truncates the approaching density profile, figure 2. This characteristic can be used to accurately produce a desired density gradient over the topography.

The wind velocities induced by the rotating disk have some very interesting characteristics. Schlichting (3), who studied the flow induced by a rotating disk, found that the flow remains laminar up to a rotational Reynolds number of 100 000 with homogeneous fluids. The rotational Reynolds number is defined as

\[
R = \frac{R \cdot \omega}{V}
\]
(17)

The shear induced rotational flow can be divided into three distinct zones: a boundary layer at the upper plate, an intermediate core, and a boundary layer at the lower plate. In this model, the two boundary layers are each about 10 mm thick. To avoid the boundary layer effect, the model is placed about 20 mm above the tank floor. The center core rotates at half the velocity of the floating disk.
If the disk is rotated at too large a rotational velocity, the upper boundary layer tends to grow rapidly. This growth tends to destratify the density gradient. The upper limit for the rotational Reynolds number with the stratified liquid has been found to be about 200 000. This condition puts an upper limit on the magnitude of the free stream velocities which can be simulated.

FIELD VERIFICATION

Previous weather modification studies have indicated that the proper placement of cloud seeders is extremely important for the successful operation of the program. One weather modification project of the Bureau is the Sierra Cooperative Pilot Project. The initial part of the investigations was to study the airflow patterns within and near the American River Basin with the stratified liquid model, (4). The model studies were then followed by field studies to determine the trajectory and diffusion characteristics during storm periods and in clear air situations, (5).

The topography of the model was oriented in the tank to simulate cyclonic curvature of the wind fields as they encounter the Sierra Nevada barrier, figure 3. The model was 250 km in diameter. It was located within the tank to simulate a 355 km radius of curvature of the isobars over the middle of the American River basin.

Figure 3. Comparison of Streamline Curvature in Model with Isobars

Figure 4. Atmospheric Density Gradients Tested in the Model

The free stream velocity was determined from a statistical sample of storms observed during the Central Sierra Research Experiment (CENSARE Project). The median wind velocity was 26 m/s at the 50 kPa level. The maximum wind velocity that could be achieved over the center of the model and not create turbulence in the core was 15.5 m/s. Thus, the simulations were limited by the rotational Reynolds number of the model. To reduce this effect, all wind velocities were normalized with respect to the wind velocity at the 50 kPa level.

An analysis of many atmospheric conditions from the CENSARE Project revealed that a storm history could be simulated with three representative density gradients, figure 3. These profiles consisted of a prestorm deep inversion, a during storm, and a poststorm condition, figure 4.

The direction of the approaching wind was varied in the model by rotating the model topography about the center of the target area. The center of the target area was assumed to be located at 725 000 m east and 4 322 500 m north, Universal Transverse Mercator Grid, Zone 10. The approaching wind directions varied in azimuth from 185 to 365 degrees. The simulations were conducted at 15 degree intervals between these two extremes.
The streamlines were made visible by injection of dye at various locations in the model. The motion near the ground was made visible by dye injection tubes which passed through the map. The open ends of these tubes were flush with the map surface. Another set of tubes were placed at various elevations at nine locations upwind of the topography. Three of the tubes were at 500 m, three at 1500 m and the other three at 2500 m above the sea level elevation. The crest of the mountain barrier is at an elevation of about 2600 m.

After the model investigations were completed, the Cloud Physics Group of the University of Washington conducted tracer and diffusion studies in the American River Basin (5). Although no major storms passed through the study area, a good set of transport and diffusion measurements were obtained in postfrontal showers.

Thirteen of the plumes tracked during the field studies could be compared with the model results. The field atmospheric conditions were similar to the three characteristic atmospheric profiles and four of the wind directions studied in the model. Both ground and aerial release points were used in the field studies.

The predicted location of the 13 plume axes was within 5 km of the observed axes for distances up to 40 km. In general, the predicted axis occurred within the estimated boundaries of the field plume. No significant differences were apparent in the accuracy of the prediction between aerial and ground sources.

A typical comparison can be seen for the case of ground seeder trajectories during a storm with a 245 degree wind direction, figure 5. Some of the difference between the predicted and the observed plume is due to the fact that the field wind direction was actually coming from 230 degrees. This case of ground seeding represents the worst possible combination of vertical distortion and boundary layer development effects.

Figure 5. Comparison of Model Predictions with Field Measurements.
Ground Seeders, 245 Degree Wind, During Storm
The excellent agreement between the model and field studies essentially verifies the technique. Calibration of the model to obtain better results does not seem to be necessary. This conclusion applies to the location of the plume trajectories. The field data were insufficient to determine if an equally good correlation exists for the velocity predictions. This latter question is presently being investigated in a study of a wind farm in Medicine Bow, Wyoming.

APPLICATIONS

The application of the model to investigate both ground and airborne seeder locations for a wide variety of atmospheric conditions has been demonstrated in this paper.

The model also has potential for use in the field of investigating the energy production by the wind. All existing wind surveys are based primarily upon data from airports. Because airports are normally sited in low wind areas, the surveys tend to underestimate the true potential. If the wind velocity studies currently being conducted yield good correlations, the model could be used to identify prospective sites for wind farms.

Satellite photographs have been useful in defining the path of airborne pollution from point sources, such as coal-fired powerplants. The model could be used in a similar manner to trace pollution trajectories over distances up to 200 km. The model would be especially advantageous for simulations during storm periods when the trajectories are not visible to the satellites.

REFERENCES
