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Mean Air Concentration of Self-Aerated Flows

By Henry T. Falvey[1]

Introduction

A designer frequently needs to estimate the quantity of air that is transported by open channel flow. A large number of experiments and field measurements have been conducted but none has led to a simple correlation that relates the mean air concentration with the flow quantities.

Bormann (1) identifies three distinct regions in self-aerating flows. These are: (1) A regime of no air entrainment where the turbulent boundary layer has not reached the water surface; (2) a regime of developing air entrainment in which the air concentration profiles are not constant with distance; and (3) a regime of fully developed air entrainment in which the air concentration profiles are constant with distance. Keller, Lai, and Wood (3) divide Bormann's middle regime into two sections. The first is a region where the aeration is developing but the air has not reached the bottom of the chute. The second is a region

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where the air has reached the bottom of the chute but the air concentration profile continues to vary with distance (Fig. 1).

The purpose of this technical note is to develop a correlation between the mean air concentration and the most significant flow parameters in the fully aerated zone. Dimensional analysis is used to evaluate the relative importance of the various parameters. This approach follows somewhat that used by Kobus (5) to examine the motion of bubbles in liquids.

Definition of Mean Air Concentration

The conventional definition of concentration is the quantity (usually measured by volume) of a material, A, either dissolved or suspended in another material, B. Thus, the air concentration would be the volume of air in a given volume of water. However, when the amount of suspended material becomes large,

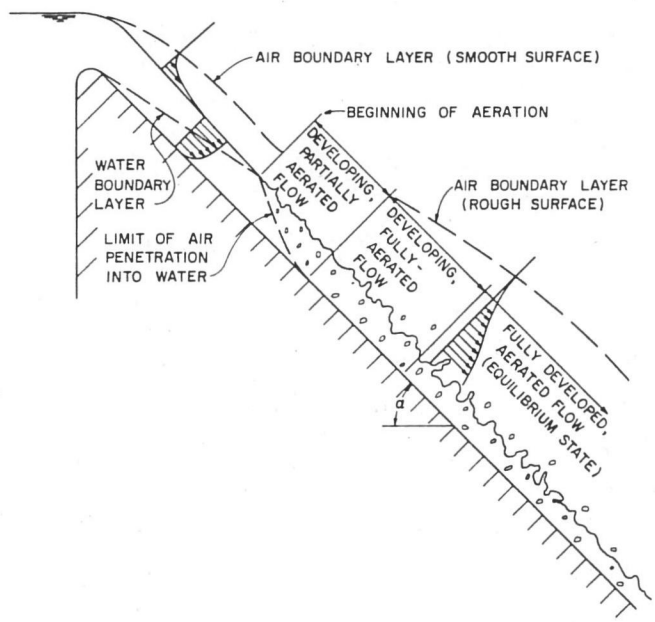

FIG. 1.—Development of Air Entrainment on Wide Steep Chutes

the reference volume is often taken as the volume of material B plus the volume of material A. In this case, the concentration of air is

$$\bar{C} = \frac{I_a}{I_a + I_w} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (1)$$

This latter definition is used throughout this paper.

The early method of measuring air concentrations of air-water flows consisted of a pitot tube-type sampler developed by Viparelli (8). This type of a device gives accurate results in the zone beneath the waves. However, in the zone where waves exist, the sampler records not only the air in the water but also the air between the waves. Therefore, the measurements indicate air concentrations that are too large in the mixing zone.

Almost all of the references to air concentration in the literature include the amount of air between the waves. Therefore, the published data report

values are greater than the true amount of air in the flowing water. As a gross approximation, the maximum mean air concentration based on the actual air concentration distribution can be estimated by the following reasoning. Within the flow, the air concentration is assumed to consist of bubbles that have a spherical shape. It can be shown that the air concentration for the bubbles packed in their most dense configuration is approx 75%. If the entire flow consists of densely packed spherical bubbles, then the maximum mean air concentration is also on the order of 75%.

Development of Dimensionless Parameters

The two factors that govern the air concentration at any point in the fluid mass are the buoyancy of the bubbles and the turbulent diffusion characteristics of the flow. The buoyant forces are governed primarily by the bubble size. Hinze (2) indicates that bubbles tend to be broken up by viscous shear forces and by turbulent shear forces. This tendency is resisted by interfacial tensile forces. For high enough degrees of turbulence, it is reasonable to expect that the viscous shear forces are insignificant with respect to the turbulent shear forces. For nonisotropic wall turbulence, the significant parameter defining the turbulent shear forces is the wall shear stress, τ_o.

From these considerations, it is reasonable to expect that the air flow rate, Q_a, can be expressed as a function of the turbulent shear stress, the interfacial forces, the two-phase flow fluid properties, and the given flow conditions. The expression is

$$Q_a = f(\tau_o, \sigma_f, \rho_a, \mu_a, \rho_w, \mu_w, V_w, d, b, g) \quad (2)$$

in which $f(\ )$ denotes "a function of"; and the subscripts a and w refer to air and water, respectively.

Using V_w, d, and ρ_w as the repeating variables results in the following dimensionless parameters:

$$\frac{Q_a}{V_w d^2} = f\left(\frac{\tau_o}{V_w^2 \rho}, \frac{\sigma_f}{V_w^2 d \rho}, \frac{\rho_a}{\rho_w}, \frac{\mu_a}{\mu_w}, \frac{\mu_w}{V_w d \rho}, \frac{b}{d}, \frac{gd}{V_w^2}\right) \quad (3)$$

By examining these dimensionless terms and transforming some of them, it is possible to develop parameters that could be used for correlating available model and prototype data. It can be shown that one possible combination of parameters is

$$\bar{C} = f\left(\frac{(\sin\alpha)^{1/2}\,\mathrm{W}}{\mathrm{F}}, \mathrm{W}, \frac{\rho_a}{\rho_w}, \frac{\mu_a}{\mu_w}, \mathrm{R}, \mathrm{F}\right) \quad (4)$$

In this equation all variables are written in terms of the hydraulic radius. The first term inside the parentheses can be interpreted as the ratio of the turbulent shear forces to the interfacial tensile forces. The second term is the ratio of the interfacial tensile forces to the inertial forces.

The normal range of temperatures encountered in hydraulics works and in models is usually between about 0° C and 30° C. Over this range, the Weber number, the density ratio, and the viscosity ratios vary only moderately. Therefore, these terms can be considered as unimportant when compared with

the variations encountered in the remaining terms.

For flows that are turbulent enough to entrain air naturally, the dynamic pressure forces determine the size of the largest air bubbles. These dynamic forces are a result of changes in velocity over distances that are about the same scale as the diameter of the bubble. Since the dynamic forces predominate over the viscous forces, the dimensionless term involving viscosity, the Reynolds number, can be considered to be small with respect to the magnitude of the other terms.

Thus, dimensional analysis leads to the conclusion that for fully developed flow, the mean air concentration, $\bar{C}$, is primarily a function of the Froude number, F, and the turbulent-interfacial tension force ratio, or

$$\bar{C} = f\left(\frac{(\sin\alpha)^{1/2}\,\mathrm{W}}{\mathrm{F}}, \mathrm{F}\right) \quad (5)$$

Correlations Using Measured Data

Three types of data are available for the correlations. These include tests with the inception of aeration, developing aeration, and fully aerated flow. Portions of the model tests by Straub and Anderson (6) could be identified as being fully aerated flow. These experimental values were selected for the correlations. Field tests by Michels and Lovely (4) and Thorsky, Tilp, and Haggman (7) contained data in which the air content at the equilibrium state was small. Thus, these data represent the limiting case of aerated flow. All data that appeared to be in the developing flow regime were rejected.

Eq. 5 indicates the significant parameters that must be correlated. However, the functional form of the correlation cannot be inferred from the equation. In some cases theoretical considerations indicate how the terms are related. However, for aeration, the theory is inadequate and the relationships must be discovered from experimental data.

The variation of the mean concentration as a function of the Froude number was determined from the model data of Straub and Anderson (6) that included the effect of the air between the waves. Therefore, the values are larger than the true air concentration. The upper limit for the measured mean concentration was about 84%. To account for the air between the waves, all reported concentrations were multiplied by 0.9. In the Straub and Anderson tests, the sets of data were taken with essentially constant values of the turbulent-interfacial tension force ratio. With mean air concentrations less than 0.6, the data appeared to fit an equation of the form

$$\bar{C} = a + m\mathrm{F} \quad (6)$$

in which a is a function of the turbulent-interfacial tension force ratio. The values of m for the turbulent-interfacial tension force ratios of 0.116, 0.114, and 0.085 are 0.047, 0.049, and 0.056, respectively. Although the value of m apparently increases as the turbulent-interfacial tension force ratio decreases, the data are insufficient to support the conclusion. Therefore, m was assumed constant and equal to the mean of the values or 0.050.

The function, a, was determined from both the model data and the tests on prototype chutes and spillways. The values of a were determined from

$$a = \bar{C} - 0.05\,F \quad \ldots \quad (7)$$

The curve $$a = -\frac{(\sin\alpha)^{1/2}\,W}{63\,F} \quad \ldots \quad (8)$$

approximately fits the data (Fig. 2).

Therefore, the mean air concentration correlation is given approximately by

$$\bar{C} = 0.05\,F - \frac{(\sin\alpha)^{1/2}\,W}{63\,F} \quad \ldots \quad (9)$$

for $0 \leq \bar{C} \leq 0.6$. For $\bar{C} > 0.6$, the data deviated erratically from Eq. 9. The

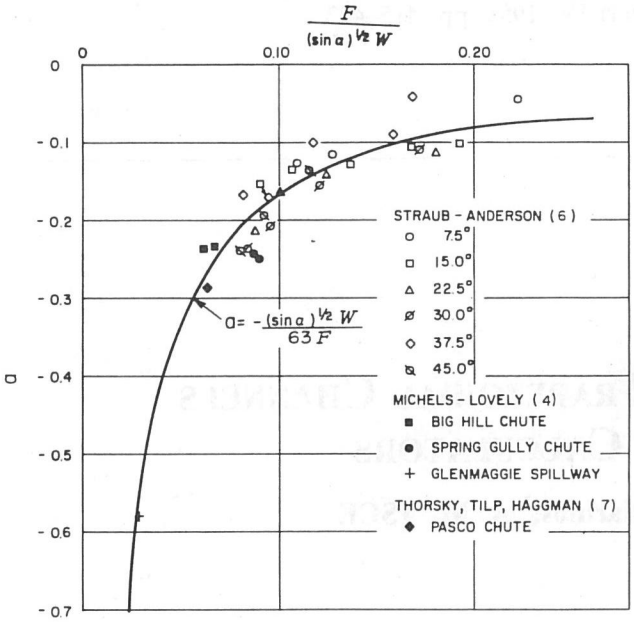

FIG. 2.—Correlation of Model and Prototype Data

physical significance of this deviation is not known. When using Eq. 9 in design, the flow depths and velocities are to be determined as if the flow is not aerated.

Conclusions

Through dimensional analysis, an expression was developed to approximate the value of the mean air concentration in fully aerated flow. The expression correlates both field and laboratory observations relatively well. Although the expression was developed for fully aerated flow, it could possibly be used to estimate air concentrations in developing flow.

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Flow Profiles in Trapezoidal Channels by Pocket Calculators

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Introduction

Determining backwater-curves in trapezoidal channels, constitutes a typical problem in open channel hydraulics which requires for some extreme cases of bulk calculations the employment of a computer, but most commonly, the use of a desk top one. Excluding the case of a time-sharing system which may not exist in some engineer's' offices, the second alternative has the advantage of displaying the results almost immediately after the machine has been fed with the design parameters of the problem. However the present day costs of desk top computers are still rather high, while on the other hand the price-performance ratio of programmable pocket calculators, decreases more drastically compared to the corresponding ratio for desk top computers.

For the last reason it seems more or less worth while to present a framework for organizing the calculations needed for an old problem in hydraulic engineering, which still would maintain the possibility of obtaining the results in an efficient way and at the same time reducing the cost of electronic equipment to a minimum.

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