EFFECTS OF SALINITY GRADIENTS ON DEFLECTION OF ACOUSTIC WAVE IN THE SACRAMENTO-SAN JOAQUIN DELTA

February 1980

HENRY T. FALVEY, TECHNICAL SPECIALIST

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INFORMATIONAL ROUTING

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403.

Mr. Stuart Hoffard, Associate Chief
Menlo Park Subdistrict
Geological Survey
Water Resources Division
855 Oak Grove Avenue
Menlo Park CA 94025

Dear Bud:

Enclosed is a brief report of my studies on the effects of salinity gradients on the acoustic velocity meter.

I have tried to think of a way to measure the deflection of the ray path in the field but have not come up with a good method. What is needed is a set of perhaps 5 to 10 hydrophones suspended at equal depth increments at some station, say midchannel. The difference in the arrival times of the rays at the various hydrophones would show where the minimum path lay. Obviously, this technique is not practical.

Should the card decks be returned to Carl Winkler? Considering the effort required to produce them, it would be a shame to dispose of the decks. However, unless someone would like some additional analysis, I see no need for me to retain them.

Due to the increased restrictions on travel funds, I am allotted only one more trip to California on this problem this year. Therefore, I would like to postpone our scheduled meeting until some of the results of the field tests are available and some experience has been gained in the use of the acoustic velocity meter. That way, we can maximize the benefits of our joint meeting. By postponing the meeting, I could also do some more analysis if any of the team members see the need for it after reviewing the enclosed report.

I would appreciate your distributing the enclosed report to the other team members.

Sincerely yours,

HENRY T. FALVEY

Henry T. Falvey, Dr. - Ing.

Enclosure

Blind to: Regional Director, Sacramento, Attention: Mr. Don Hebert, MP-710
D-1530 ~~D~~-1530A (2)

HTFalvey:jdm-V6

☆ GPO 1979 678-520

UNITED STATES GOVERNMENT

Memorandum

Memorandum
TO :Files

Denver, Colorado
DATE: February 20, 1980

FROM :Henry T. Falvey, Technical Specialist

SUBJECT: Effects of Salinity Gradients on Deflection of Acoustic Wave in the Sacramento-San Joaquin Delta

BACKGROUND

Sound and light obey the same laws of physics when traveling through a nonuniform density medium. The deflection of a sound or light path from a straight line can be predicted from Snell's law of refraction. Based on Snell's law it can be shown that the sound path travels a circular arc in the presence of a linear density gradient 1/, 2/. The radius of curvature of the circular arc, R (m), is given by:

$$R = -1450\left(3.63 \frac{dT}{dy} + 1.13 \frac{dS}{dy}\right)^{-1} \qquad (1)$$

Where T = Temperature (°C)
y = Depth measured downward from the water surface (M)
S = Salinity (g/kg)

This equation shows that the deflection of the ray path is affected more by temperature than by salinity gradients.

Stable salinity gradients always result in positive values for the gradient. These gradients produce upward curvatures of the ray paths. On the other hand, a stable temperature gradient produces a downward curvature.

1/ Urick, R. J., Principles of Underwater Sound for Engineers, McGraw Hill Book Company, 1967.
2/ Cole, J. A., The Deflection of an Acoustic Beam by Temperature and Salinity Gradients, Support Paper, Water Research Centre, Henley Road, Medmenham, PO Box 16, Marlow Bucks SL7 2HD, England, March 1979.

The bending of the sound path results in an increase in the travel distance for the sound ray to pass between two points a distance L apart. The percent increase in the travel length due to the curved path is given by:

$$\frac{X-L}{L} = \left[\frac{2R}{L} \sin^{-1}\left(\frac{L}{2R}\right) -1\right] 100 \qquad (2)$$

Where X = True path length
L = Chord length between points
R = Radius of curvature of sound path

As well as traveling a longer distance, density gradients also cause the ray to be deflected from a straight line path. The amount of deflection, D, is given by:

$$D = R\left(1 - \sqrt{1 - \frac{1}{4}\left(\frac{L}{R}\right)^2}\right) \qquad (3)$$

A longer sonic path and the deflection from a straight path can potentially affect the accuracy of an acoustic velocity meter. This memorandum investigates the magnitude of these effects at the Chipps Island Channel in the Sacramento-San Joaquin Delta.

ANALYSIS

Data on salinity as a function of depth were obtained from Mr. Stuard Hoffard, Geological Survey, Menlo Park Subdistrict Office, California, to analyze. These data were measured during the period September 11-27, 1954, in the Chipps Island Channel. A computer program was written to determine a histogram of the radii of curvature of the data, appendix I. By varying the parameters in the program, it was possible to obtain histograms for both the flood and slack tide periods. In addition, histograms as a function of depth were also determined. These latter histograms represent 10-foot-deep segments of the water prism.

The program output was written so that the numerical results appear on the left side of the page and a rough plot of the histogram appears on the right side, figures 1-9.

The results are biased somewhat toward the larger values of radii of curvature because the distance between the last depth and the river bottom was not known. In the computations, this interval was assumed to be equal to 10 feet. Actually, this distance is less than 10 feet. The equation used to determine the radius of curvature is:

$$R(m) = -1450 \left(\frac{1.13\ \Delta S}{10 \times 0.305}\right)^{-1} = \frac{-3911}{\Delta S} \quad (4)$$

Where ΔS = Difference between salinity readings at adjacent 10-foot depths

A positive value for R indicates downward curvature of the acoustic ray, whereas, negative values indicate upward curvature of the rays.

The maximum acoustic path length at Chipps Island is about 1130 m. Since a radius of curvature of the acoustic path of 1×10^6 m represents only a 15-cm deflection, equation 3, all radii equal to or greater than 1×10^6 m were assumed to be the same as a direct path.

RESULTS

All Data

The histogram indicated a concentration of the radii of curvature in the range 1×10^4 to 5×10^4 m. These acoustic rays have an upward curvature. Only 10 percent of the rays could be considered as being on the direct path.

0- to 10-foot Depth

In the top 10 feet, over 30 percent of the radii curvature lie in the 1×10^4-to 5×10^4-m range, figure 2. Another 30 percent of the data have a radius of curvature less than 1×10^4 m.

10- to 20-foot Depth

Nearly 50 percent of the radii curvature lie in the 1×10^4-to 5×10^4-m range, figure 3. Only 5 percent of the rays follow a direct path.

20- to 30-foot Depth

At this depth 40 percent of the radii of curvature lie in the 1×10^4-to 5×10^4-m range, figure 4. The histogram is somewhat biased toward larger radii of curvature.

30- to 40-foot Depth

Although over 30 percent of the radii of curvature are in the 1×10^4 to 5×10^4 range, the histogram is strongly biased toward larger values of the radii of curvature, figure 5. Over 40 percent of the data have a radius of curvature greater than 1×10^4 m.

40- to 50-foot Depth

At this depth range, the radii of curvature are centered around the 1×10^5 to 5×10^5 range. Over 20 percent of the rays follow the direct path, figure 6.

50- to 60-foot Depth

Over 40 percent of the rays follow a direct path. It should be noted that 25 percent of the rays have a downward curvature, figure 7.

Ebb and Flood Tides

These two tides were analyzed by using only that data which were greater than a 3-foot or less than a 4-foot elevation. These data show a clustering around the 1×10^4-to 5×10^4-m radii of curvature range, figure 8.

High and Low Slack Water

These two conditions were analyzed by using only that data which were greater than 5-foot or less than a 2-foot elevation. These data show a clustering around the 1×10^4 to 5×10^4 radii with a slight bias toward the larger values, figure 9.

EFFECT OF RAY PATH CURVATURE

Deviation from Horizontal Path

Using a horizontal path length, 1127 m, and a radius of curvature of 5×10^4 m results in a deflection from the horizontal of 3.18 m, figure 10a. This path is the shortest one connecting the transducers on opposite ends of the measuring section. The next shortest ray is one which reflects off the water surface.

With a radius of curvature of 1×10^4 m, the shortest ray is one which reflects off the water surface one time, figure 10b. For a hydrophone set located 5.70 m below the water surface, the ray passes about 1.5 m below the horizontal before rising to reflect off the water surface. In this case, the total deflection is about 7.2 m vertically. The next shorter path has two reflections off the water surface. This ray traverses about 8.0 m vertically as it passes from one hydrophone to the other at the 5.7-m depth.

Using the curves of Smith 3/, the value of mean velocity correlation coefficient C will vary between 0.94 to 0.96 for a 1×10^4 radius of curvature and 1.02 to 1.04 for a 5×10^4 radius of curvature. The mean velocity correlation coefficient is defined as:

$$C = \frac{\overline{V}}{V_p} \qquad (5)$$

Where $\overline{V}$ = Q/A for the entire cross section
V_p = Line velocity over the selected acoustic path
Q = Total discharge
A = Total cross sectional flow area

Therefore, if the salinity and temperature gradients are not measured simultaneously with the acoustic velocity measurements, errors of ±5 percent in the total discharge may result as the shortest sound path is deflected either above or below the horizontal path.

Change in Path Length

The sound from a hydrophone occurs as a short burst of pulses at a specific frequency. One effect of rays following two separate paths is noticeable in the modulation of the received signal.

With a radius of curvature of 5×10^4 m, the two rays traveling on the shortest paths reach the receiving hydrophone with a 27-μ s delay, figure 10a. This delay would tend to eliminate the second pulse of a 20 kHz (50-μ s period) signal since the second pulse would be about 180° out of phase with the initial signal.

With a radius of curvature of 1×10^4 m, the two rays traveling on the shortest paths reach the receiving hydrophone with a 108-μ s delay. This delay would tend to amplify the second pulse since the two rays are approximately in phase.

The rays traveling on the shortest paths for the two radii of curvature experience a significant delay relative to the horizontal path. However, Smith 3/ has shown that these delays do not cause errors if the delay is the same in both travel directions.

3/ Smith, W., Feasibility Study of the Use of the Acoustic Velocity Meter for Measurement of Net Outflow From the Sacramento-San Joaquin Delta in California, Geological Survey Water-Supply Paper 1877, 1969, 54 pp.(see p. 25).

Reflection off Water Surface

At a low enough frequency and a sufficiently small grazing angle, the water surface can act as a nearly perfect reflector of sound. Urick 1/ states that the surface is essentially smooth if:

$$\lambda > 8\ H \sin\theta \qquad (6)$$

Where λ = Wave length of acoustic signal
H = Wave height roughness
θ = Angle with which acoustic wave meets water surface

For both the 1×10^4 and the 5×10^4 radii of curvature ray paths, the grazing angle is approximately 1°. The wave length for a 20-kHz signal is 0.07 m with a 1487-m/s acoustic velocity. Substitution of these values into equation 6 shows that the water surface can be considered a smooth surface for wave roughness values less than 0.5 m.

CONCLUSIONS

1. For the majority of the measurements, salinity gradients will cause the acoustic rays to have a significant deflection.

2. The radius of curvature of the ray path will most probably lie between 1×10^4 and 5×10^4 m.

3. With a 1×10^4-m radius of curvature, the shortest ray path will lie almost entirely above the horizontal path. The ray will reflect off the water surface.

4. With a 5×10^4-m radius of curvature, the shortest ray path will lie entirely below the horizontal path.

5. Unless salinity and temperature gradients are measured coincidently with the acoustic velocity measurements, deflections of the acoustic path can introduce an error of ± 5 percent in the determination of the mean discharge.

6. Increases in the acoustic path length due to deflection of the ray will not introduce errors in the discharge determination.

Figure 1

ALL DATA

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	.00	
RANGE -5EXP5 TO -1EXP6	.00	
RANGE -1EXP5 TO -5EXP5	.15	
RANGE -5EXP4 TO -1EXP5	.12	
RANGE -1EXP4 TO -5EXP4	.37	
RANGE -5EXP3 TO -1EXP4	.11	
RANGE -1EXP3 TO -5EXP3	.06	
RANGE 1EXP3 TO -1EXP3	.00	

DOWNWARD CURVATURE OF ACOUSTIC RAY	
RANGE 5EXP3 TO 1EXP3	.00
RANGE 1EXP4 TO 5EXP3	.00
RANGE 5EXP4 TO 1EXP4	.02
RANGE 1EXP5 TO 5EXP4	.02
RANGE 5EXP5 TO 1EXP5	.06
RANGE 1EXP6 TO 5EXP5	0.00

DIRECT PATH

GREATER THAN 1EXP6	.10

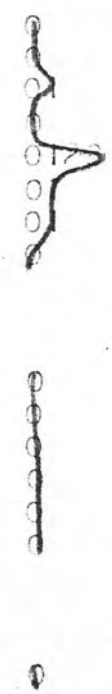
TOTAL NUMBER OF INCREMENTS = 13248

Figure 2

0 TO 10-FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	0.00	
RANGE -5EXP5 TO -1EXP6	0.00	
RANGE -1EXP5 TO -5EXP5	.12	
RANGE -5EXP4 TO -1EXP5	.09	
RANGE -1EXP4 TO -5EXP4	.33	
RANGE -5EXP3 TO -1EXP4	.16	
RANGE -1EXP3 TO -5EXP3	.14	
RANGE 1EXP3 TO -1EXP3	.00	
DOWNWARD CURVATURE OF ACOUSTIC RAY		
RANGE 5EXP3 TO 1EXP3	.00	
RANGE 1EXP4 TO 5EXP3	.00	
RANGE 5EXP4 TO 1EXP4	.02	
RANGE 1EXP5 TO 5EXP4	.02	
RANGE 5EXP5 TO 1EXP5	.05	
RANGE 1EXP6 TO 5EXP5	0.00	
DIRECT PATH		
GREATER THAN 1EXP6	.07	



TOTAL NUMBER OF INCREMENTS = 3380

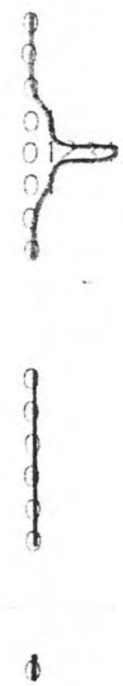
Figure 3

10- TO 20-FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY	
LESS THAN -1EXP6	0.00
RANGE -5EXP5 TO -1EXP6	.00
RANGE -1EXP5 TO -5EXP5	.10
RANGE -5EXP4 TO -1EXP5	.10
RANGE -1EXP4 TO -5EXP4	.47
RANGE -5EXP3 TO -1EXP4	.16
RANGE -1EXP3 TO -5EXP3	.07
RANGE 1EXP3 TO -1EXP3	0.00
DOWNWARD CURVATURE OF ACOUSTIC RAY	
RANGE 5EXP3 TO 1EXP3	.00
RANGE 1EXP4 TO 5EXP3	.00
RANGE 5EXP4 TO 1EXP4	.01
RANGE 1EXP5 TO 5EXP4	.01
RANGE 5EXP5 TO 1EXP5	.03
RANGE 1EXP6 TO 5EXP5	0.00
DIRECT PATH	
GREATER THAN 1EXP6	.05

HISTOGRAM



TOTAL NUMBER OF INCREMENTS = 3198

Figure 4

20- TO 30- FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY

LESS THAN -1EXP6	0.00
RANGE -5EXP5 TO -1EXP6	0.00
RANGE -1EXP5 TO -5EXP5	.16
RANGE -5EXP4 TO -1EXP5	.13
RANGE -1EXP4 TO -5EXP4	.41
RANGE -5EXP3 TO -1EXP4	.09
RANGE -1EXP3 TO -5EXP3	.02
RANGE 1EXP3 TO -1EXP3	0.00

DOWNWARD CURVATURE OF ACOUSTIC RAY

RANGE 5EXP3 TO 1EXP3	.00
RANGE 1EXP4 TO 5EXP3	.00
RANGE 5EXP4 TO 1EXP4	.02
RANGE 1EXP5 TO 5EXP4	.02
RANGE 5EXP5 TO 1EXP5	.05
RANGE 1EXP6 TO 5EXP5	0.00

DIRECT PATH

GREATER THAN 1EXP6	.08

TOTAL NUMBER OF INCREMENTS = 3081

HISTOGRAM

Figure 5

30- TO 40-FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	.00	
RANGE -5EXP5 TO -1EXP6	0.00	
RANGE -1EXP5 TO -5EXP5	.20	
RANGE -5EXP4 TO -1EXP5	.14	
RANGE -1EXP4 TO -5EXP4	.32	
RANGE -5EXP3 TO -1EXP4	.04	
RANGE -1EXP3 TO -5EXP3	.01	
RANGE 1EXP3 TO -1EXP3	0.00	

DOWNWARD CURVATURE OF ACOUSTIC RAY

RANGE 5EXP3 TO 1EXP3	.00
RANGE 1EXP4 TO 5EXP3	.00
RANGE 5EXP4 TO 1EXP4	.03
RANGE 1EXP5 TO 5EXP4	.03
RANGE 5EXP5 TO 1EXP5	.08
RANGE 1EXP6 TO 5EXP5	0.00

DIRECT PATH

GREATER THAN 1EXP6	.14

TOTAL NUMBER OF INCREMENTS = 2254

Figure 6

40- TO 50-FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	0.00	0
RANGE -5EXP5 TO -1EXP6	0.00	0
RANGE -1EXP5 TO -5EXP5	.23	012
RANGE -5EXP4 TO -1EXP5	.13	0
RANGE -1EXP4 TO -5EXP4	.17	0
RANGE -5EXP3 TO -1EXP4	.01	0
RANGE -1EXP3 TO -5EXP3	.00	0
RANGE 1EXP3 TO -1EXP3	.00	0
DOWNWARD CURVATURE OF ACOUSTIC RAY		
RANGE 5EXP3 TO 1EXP3	0.00	0
RANGE 1EXP4 TO 5EXP3	0.00	0
RANGE 5EXP4 TO 1EXP4	.06	0
RANGE 1EXP5 TO 5EXP4	.04	0
RANGE 5EXP5 TO 1EXP5	.13	0
RANGE 1EXP6 TO 5EXP5	0.00	0
DIRECT PATH		
GREATER THAN 1EXP6	.23	012

TOTAL NUMBER OF INCREMENTS = 1227

Figure 7

50- TO 60-FT DEPTH SEGMENT

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	0.00	
RANGE -5EXP5 TO -1EXP6	0.00	
RANGE -1EXP5 TO -5EXP5	.16	
RANGE -5EXP4 TO -1EXP5	.09	
RANGE -1EXP4 TO -5EXP4	.09	
RANGE -5EXP3 TO -1EXP4	0.00	
RANGE -1EXP3 TO -5EXP3	0.00	
RANGE 1EXP3 TO -1EXP3	0.00	
DOWNWARD CURVATURE OF ACOUSTIC RAY		
RANGE 5EXP3 TO 1EXP3	.02	
RANGE 1EXP4 TO 5EXP3	.01	
RANGE 5EXP4 TO 1EXP4	.01	
RANGE 1EXP5 TO 5EXP4	.02	
RANGE 5EXP5 TO 1EXP5	.19	
RANGE 1EXP6 TO 5EXP5	0.00	
DIRECT PATH		
GREATER THAN 1EXP6	.42	

TOTAL NUMBER OF INCREMENTS = 108

Figure 8

EBB & FLOOD TIDES

THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY

LESS THAN -1EXP6	.00
RANGE -5EXP5 TO -1EXP6	.00
RANGE -1EXP5 TO -5EXP5	.13
RANGE -5EXP4 TO -1EXP5	.10
RANGE -1EXP4 TO -5EXP4	.40
RANGE -5EXP3 TO -1EXP4	.16
RANGE -1EXP3 TO -5EXP3	.07
RANGE 1EXP3 TO -1EXP3	0.00

DOWNWARD CURVATURE OF ACOUSTIC RAY

RANGE 5EXP3 TO 1EXP3	.00
RANGE 1EXP4 TO 5EXP3	.00
RANGE 5EXP4 TO 1EXP4	.01
RANGE 1EXP5 TO 5EXP4	.01
RANGE 5EXP5 TO 1EXP5	.05
RANGE 1EXP6 TO 5EXP5	0.00

DIRECT PATH

GREATER THAN 1EXP6	.08

TOTAL NUMBER OF INCREMENTS = 3491

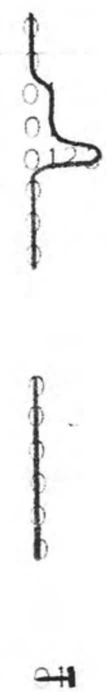
HISTOGRAM

Figure 9

HIGH & LOW SLACK WATER

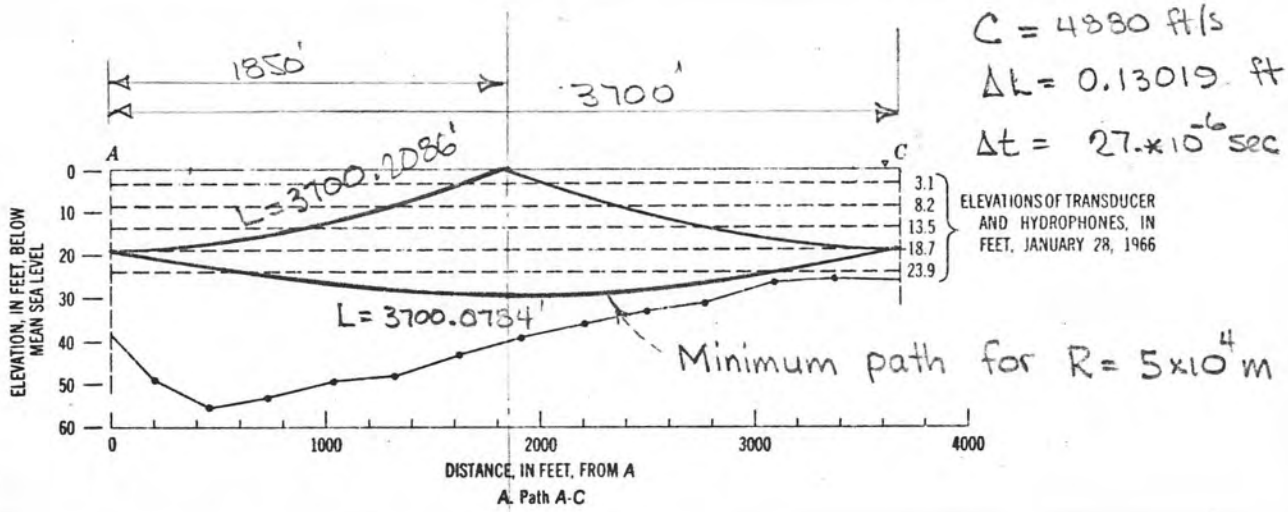
THE NUMBER OF SECTIONS = 4324

UPWARD CURVATURE OF ACOUSTIC RAY		HISTOGRAM
LESS THAN -1EXP6	0.00	
RANGE -5EXP5 TO -1EXP6	0.00	
RANGE -1EXP5 TO -5EXP5	.16	
RANGE -5EXP4 TO -1EXP5	.13	
RANGE -1EXP4 TO -5EXP4	.33	
RANGE -5EXP3 TO -1EXP4	.07	
RANGE -1EXP3 TO -5EXP3	.07	
RANGE 1EXP3 TO -1EXP3	.00	
DOWNWARD CURVATURE OF ACOUSTIC RAY		
RANGE 5EXP3 TO 1EXP3	.00	
RANGE 1EXP4 TO 5EXP3	.00	
RANGE 5EXP4 TO 1EXP4	.03	
RANGE 1EXP5 TO 5EXP4	.03	
RANGE 5EXP5 TO 1EXP5	.07	
RANGE 1EXP6 TO 5EXP5	0.00	
DIRECT PATH		
GREATER THAN 1EXP6	.11	

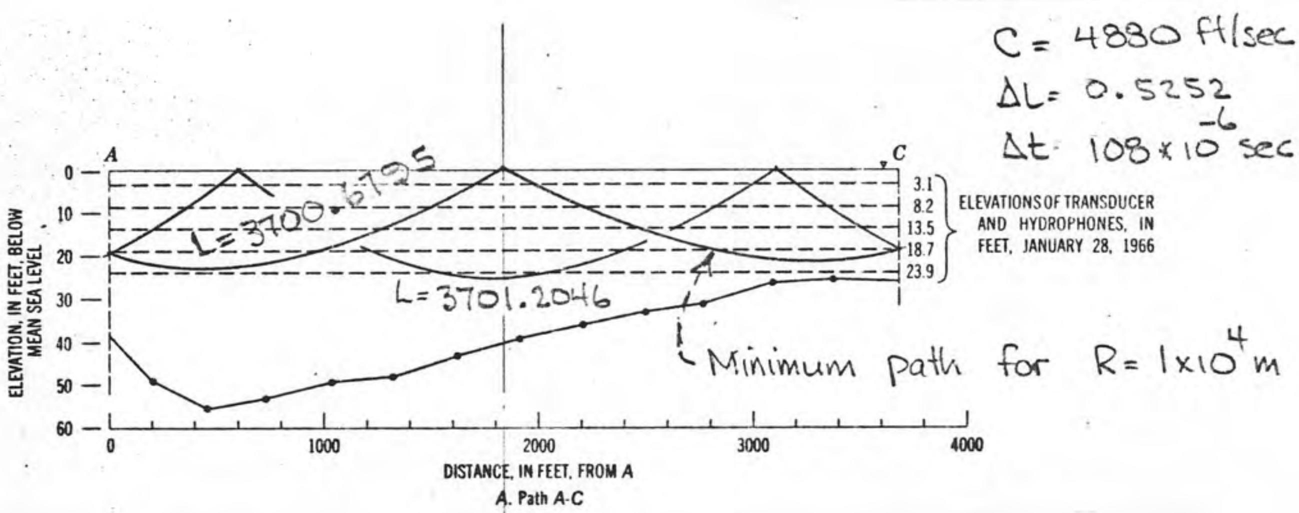


TOTAL NUMBER OF INCREMENTS = 4543

Figure 10



a. $R = 5 \times 10^{4}$ m



b. $R = 1 \times 10^{4}$ m

APPENDIX I

Computer Program to Determine the Radius of Curvature of an Acoustic Ray Due to Salinity Gradients

```
      PROGRAM HFSAC(HFSFD1,HFSFD,HFSFD3,OUTPUT,TAPE3=HFSFD,
     +TAPE1=HFSFD1,TAPE2=HFSFD3,TAPE5=OUTPUT)
C
C     PROGRAM TO DETERMINE A HISTOGRAM OF THE ACOUSTIC RADIUS OF
C     CURVATURE FOR DATA FROM THE SACRAMENTO-SAN JOAQINE DELTA
C     STUDY.
C
C
      DIMENSION N(15),S(7),RN(15)
      DATA N/15*0/
      NUM= 10H0123456789
      NR= 1
C
C     DO LOOP TO READ IN DATA
C
      DO 12 I=1,5000
        GO TO (1,3,5)NR
    1   READ(1,6)D,(S(J),J=1,7)
        IF(EOF(1))2,7
    2   NR= 2
    3   READ(2,6)D,(S(J),J=1,7)
        IF(EOF(2))4,7
    4   NR= 3
    5   READ(3,6)D,(S(J),J=1,7)
    6   FORMAT(12X,F4.1,4X,7F8.0)
        IF(EOF(3))13,7
C
C       DO LOOP TO REDUCE READINGS TO G/KG AND TO ELIMINATE
C       THE ZEROS BETWEEN THE LAST DATA POINT AND THE BED
C
C   7   IF(D.LT.(5.).AND.D.GT.(2.))GO TO 12
C   7   IF(D.GT.(4.).OR.D.LT.(3.))GO TO 12
    7   DO 8 K=1,7
          S(K)= S(K)/1000.
          IF(ABS(S(K)).LE.0.001)GO TO 9
    8   CONTINUE
        GO TO 10
    9   IF(S(1).LE.0.001)GO TO 12
        SSAV= S(7)/1000.
        IF(S(6).LE.0.001)S(7)= 0.
        S(K)= SSAV
C
C       DO LOOP TO COMPUTE THE RADIUS OF CURVATURE OF THE ACOUSTIC
C       PATH.  TEMPERATURE GRADIENTS AND DEPTH GRADIENT EFFECTS ARE
C       NEGLECTED.  THE DISTANCE BETWEEN THE LAST DATA POINT AND
C       THE BED IS ASSUMED TO BE 10 FEET.  THE EQUATION USED HERE
C       HAS BEEN CONVERTED TO METRIC UNITS.
C
   10   DO 11 J=2,K
          IF(S(J).LE.0.001)GO TO 12
          DELTS= S(J)-S(J-1)
          IF(ABS(DELTS).LE.1.E-10)DELTS= -1.E-05
          R= -3911./DELTS
```

```
C
C         COUNTING OF DATA IN APPROPRIATE RANGES
C
          IF(R.LE.(-1.E+06))N(1)= N(1)+1
          IF(R.LE.(-5.E+05).AND.R.GT.(-1.E+06))N(2)= N(2)+1
          IF(R.LE.(-1.E+05).AND.R.GT.(-5.E+05))N(3)= N(3)+1
          IF(R.LE.(-5.E+04).AND.R.GT.(-1.E+05))N(4)= N(4)+1
          IF(R.LE.(-1.E+04).AND.R.GT.(-5.E+04))N(5)= N(5)+1
          IF(R.LE.(-5.E+03).AND.R.GT.(-1.E+04))N(6)= N(6)+1
          IF(R.LE.(-1.E+03).AND.R.GT.(-5.E+03))N(7)= N(7)+1
          IF(R.LE.(+1.E+03).AND.R.GT.(-1.E+03))N(8)= N(8)+1
          IF(R.LE.(+5.E+03).AND.R.GT.(+1.E+03))N(9)= N(9)+1
          IF(R.LE.(+1.E+04).AND.R.GT.(+5.E+03))N(10)= N(10)+1
          IF(R.LE.(+5.E+04).AND.R.GT.(+1.E+04))N(11)= N(11)+1
          IF(R.LE.(+1.E+05).AND.R.GT.(+5.E+04))N(12)= N(12)+1
          IF(R.LE.(+5.E+05).AND.R.GT.(+1.E+05))N(13)= N(13)+1
          IF(R.LE.(+1.E+06).AND.R.GT.(+5.E+05))N(14)= N(14)+1
          IF(R.GT.(+1.E+06))N(15)= N(15)+1
   11     CONTINUE
   12 CONTINUE
C
C     DO LOOP TO MAKE HISTOGRAM DIMENSIONLESS
C
   13 NSUM= N(1)+N(2)+N(3)+N(4)+N(5)+N(6)+N(7)+N(8)+N(9)+
     +N(10)+N(11)+N(12)+N(13)+N(14)+N(15)
      IF(NSUM.EQ.0)NSUM= 1
      DO 14 M=1,15
        RN(M)= FLOAT(N(M))/FLOAT(NSUM)
        N(M)= IFIX(10.*RN(M))+1
   14 CONTINUE
C
C     OUTPUT OF DATA
C
      WRITE(5,15)I,(RN(M),N(M),NUM,M=1,15),NSUM
   15 FORMAT(1H1,10X,25HTHE NUMBER OF SECTIONS = ,I4//
     +          9X,32HUPWARD CURVATURE OF ACOUSTIC RAY ,7X,9HHISTOGRAM //
     +          10X,16HLESS THAN -1EXP6 ,6X,F8.2,T50,A= /
     +          10X,22HRANGE -5EXP5 TO -1EXP6 ,F8.2,T50,A= /
     +          10X,22HRANGE -1EXP5 TO -5EXP5 ,F8.2,T50,A= /
     +          10X,22HRANGE -5EXP4 TO -1EXP5 ,F8.2,T50,A= /
     +          10X,22HRANGE -1EXP4 TO -5EXP4 ,F8.2,T50,A= /
     +          10X,22HRANGE -5EXP3 TO -1EXP4 ,F8.2,T50,A= /
     +          10X,22HRANGE -1EXP3 TO -5EXP3 ,F8.2,T50,A= /
     +          10X,22HRANGE  1EXP3 TO -1EXP3 ,F8.2,T50,A= //
     +          8X,34HDOWNWARD CURVATURE OF ACOUSTIC RAY //
     +          10X,22HRANGE  5EXP3 TO  1EXP3 ,F8.2,T50,A= /
     +          10X,22HRANGE  1EXP4 TO  5EXP3 ,F8.2,T50,A= /
     +          10X,22HRANGE  5EXP4 TO  1EXP4 ,F8.2,T50,A= /
     +          10X,22HRANGE  1EXP5 TO  5EXP4 ,F8.2,T50,A= /
     +          10X,22HRANGE  5EXP5 TO  1EXP5 ,F8.2,T50,A= /
     +          10X,22HRANGE  1EXP6 TO  5EXP5 ,F8.2,T50,A= //
     +          19X,11HDIRECT PATH//
     +          10X,18HGREATER THAN 1EXP6 ,4X,F8.2,T50,A=//
     +          11X,29HTOTAL NUMBER OF INCREMENTS = , I5///)
      CALL EXIT
      END
```