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Memorandum

April 12, 1978

HYDRAULICS BRANCH
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Henry T. Falvey, Technical Specialist

Summary Report on Pleasant Oak Main Surge Problem

Chief, Mechanical Branch
Chief, Water Conveyance Branch

Chief, Hydraulics Branch
Chief, Division of Research
Chief, Division of Design

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This memorandum transmits the results of my studies relative to the surging of the Pleasant Oak Main. These studies have included the following: participation in field tests, development of a computer program to determine resonant periods of the pipeline, derivations of electrical equivalents of hydraulic elements, and development of a device to modify the operation of the pilot valve.

Surging in the pipeline can be excited either externally (waves on the regulating reservoirs or surges in the weir structure at station 5) or through self-excitation. The most probable explanation for the surging exhibited in the Pleasant Oak Main is that the system is susceptible to self-excited or auto-oscillations. If an outside excitation were the cause, the surging would continue with the valves blocked. However, the field tests demonstrated that the surging disappeared when the valves were made inoperable. Therefore, it is reasonable to conclude that the self-excitation is provided by the pressure-reducing valves.

Two theories explain the auto-oscillation characteristics of the valves. In the first, the valve is assumed to have a response characteristic which is close to the response characteristic of some part of the pipeline. The valve and the line interact to produce a sustained operation at a frequency which is close to one of the resonant frequencies of the two elements. This type of interaction can be stopped by detuning the system through changing the resonant characteristics of either the line or the valve.

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The second theory was suggested by Dr. E. B. Wylie (see my memorandum dated January 12, 1978). This theory states that the control concept, upon which the operation of each valve is based, is inherently unstable. Basically, the control concept produces a negative slope on a head versus discharge curve. The tendency for the valve to go into a limit cycle can easily be shown for a simple system. However, for a complex system, such as the Pleasant Oak Main, it is much more difficult to demonstrate the unstable characteristic. The definition of a limit cycle, as used here, is a cyclical operation in which nonlinearities in the system limit the amplification (pressure-regulating valve closes, pressure relief valve opens, etc.). The frequency of the limit cycle may or may not occur at any critical period of any element in the system. It should be noted that this instability is predicated upon a rapid rate of valve motion relative to the wave travel time in the system. This type of instability can be controlled by the introduction of proper elements in the feedback loop on the valve.

The field tests performed the week of November 14, 1977, demonstrated conditions under which the pressure-reducing valves can excite the pipeline to resonance. For instance, with only the 6-inch valves in operation, the observed periods of pressure fluctuations upstream of PRV-3 were 2-1/2 and 12 seconds per cycle. The critical periods from the impedance computations, figure 3, appendix A, are identical with the observed periods. Therefore, either theory can explain the occurrence of surging with only the 6-inch valves operating.

With only the 12-inch valves in operation, the observed periods upstream of station 3 were on the order of 20 seconds. When the 16-inch valve at station 4 operated, the observed period at station 3 increased to 26 seconds per cycle. The impedance diagram, figure 2, appendix A, clearly indicates that the observed periods do not correspond with critical periods of any segment of the pipeline. The occurrence of the noncritical periods is not predicted by the first theory.

From the evidence it can be concluded that the most probable explanation for the surging is that suggested by Wylie. As a consequence, attempts to modify the surging by detuning the valves or the line will not be fruitful. Instead, either the control concept must be modified or devices must be placed on the pipeline itself to keep the amplitude of the surge within acceptable limits.

Two approaches are possible in modifying the control concept. One would be to replace each valve with a set of orifices which are either fully open or fully closed. The flow through a particular orifice would be determined by pressure-limit sensors located downstream from the station.

The other approach is to design appropriate elements into the feedback loop of the valve using control system theory concepts. This approach has been successful in controlling open channel flow which also has a negative head versus discharge characteristic 1/. The studies being

1/ Falvey, Henry T., Frequency-Response Analysis of Automated Canals, REC-ERC-77-3, July 1977.

done by Maytum in the Mechanical Branch should lead to a mathematical model of the feedback loop on the main valve which can be used to analyze the elements required for stability. The experience with automated canals has shown that a rigorous analysis is required if the effects of modifications to the feedback loop are to be properly predicted. Sets of parameters which work on one system are completely inappropriate on other similar systems. Finding the correct elements for the valves and their proper adjustments experimentally has been an expensive and time-consuming process. If the development of the mathematical model is completed and verified with the existing field data, the model could be used to determine the necessary elements during the design process. Subsequently, these elements would require a minimum of field adjustment.

The amplitude of the surges can be kept within acceptable limits by the application of pressure-actuated bypasses around the valves or by the addition of surge tanks or accumulators to the line. The addition of bypasses is advisable in any case, since they provide a relatively inexpensive factor of safety against failures in the feedback loop of the pressure-reducing valves.

Appendix A presents a description of the computer program for the impedance analysis. It can be used for any type of series pipeline system including penstocks.

Appendix B presents the electrical analogs for several typical hydraulic elements.

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Memorandum

Memorandum

TO : Chief, Mechanical Branch
Chief, Water Conveyance Branch

Denver, Colorado
DATE: January 12, 1978

THROUGH: Chief, Hydraulics Branch
Chief, Division of General Research
Chief, Division of Design

FROM : Henry T. Falvey, Technical Specialist

SUBJECT : Pleasant Oak Main Surge Problem

Dr. E. B. Wylie, coauthor of the book "Hydraulic Transients" consulted with the Bureau on December 27-28, 1977, relative to gate stroking for the Central Arizona Project. While he was here, I discussed with him the surging that has been experienced on the Pleasant Oak Main. He was given a profile of the pipeline and four representative records to study the night of the 27th.

After reviewing the data, Wylie concluded that the control concept used on the pipeline is inherently unstable. The valve characteristics are very similar to the characteristics of a leaky valve discussed in his book. That is, when the discharge increases, the head upstream of the valve decreases and vice versa. Figure 8.17(b) from Wylie's book illustrates how the magnitude of the fluctuating increases for this case. On the other hand, a fixed valve is inherently stable, figure 8.17(a).

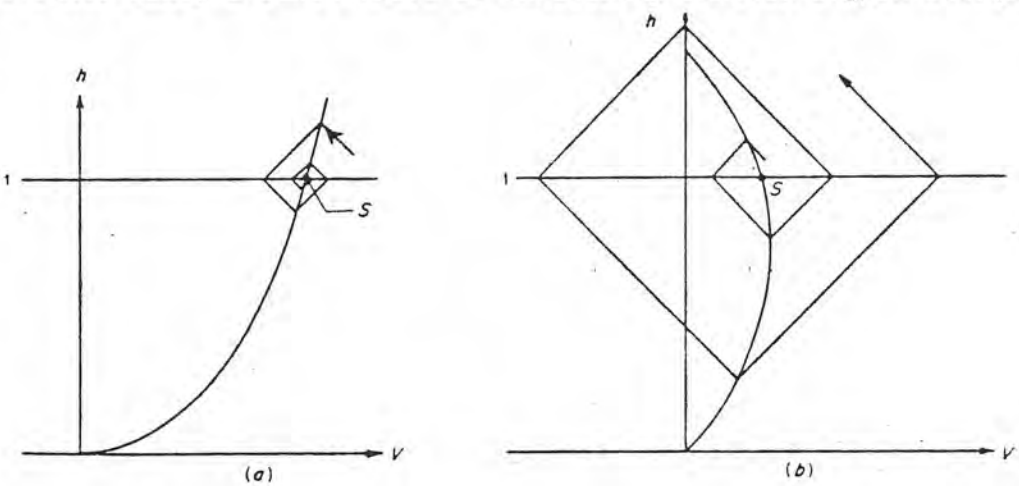


Fig. 8.17 Graphical representation of valve leakage.

According to Wylie, there are two solutions to the problem. One is to make the valve have characteristics which approximate those of a fixed orifice. This is accomplished through introducing lag into the valve motion. The studies of Rudd fit into this category. The other solution is the "brute force" approach in which the instability is controlled through modifications to the pipeline system itself. The pressure relief valves designed by Warren Thomas are representative of this type of solution.

In subsequent telephone conversations, Wylie expressed some doubt about the ability to control the transient through modifications to the valve. His reasoning was that if the valve is modified so that it does not surge, then it also may not control the downstream pressure as intended.

The doubts of Wylie may have already been substantiated. The laboratory developed a device which acted upon the pilot valve to dampen sinusoidal variations in downstream pressure. This device performed as designed in the laboratory. That is, it allowed the main valve to respond quickly to changes in downstream pressure, but it held the valve in a fixed position for sinusoidal variations. In the field, the device was completely ineffectual in supressing the steady surging.

To assist in analyzing the effect of changes to the pipeline, the Hydraulics Branch is developing a computer program of generalized pipeline systems using the impedance method. This method was developed by Wylie and is discussed in his book. However, the case of inline valves, as used on the Pleasant Oak Main, was not covered in Wylie's book. This was discussed and he derived the necessary equations while he was here.

In summary, Wylie's analysis confirmed the suspicions that the control concept employed on the Pleasant Oak Main is inherently unstable. The types of analyses presently being conducted by the Bureau should lead to a method of controlling the magnitude of the surge, but the problem probably cannot be eliminated. Finally, the pressure relief valves installed upstream of the main valves were an excellent idea. However, they would function better if the flow was reintroduced into the pipeline downstream of the valves instead of being wasted. This bypass concept should be considered for the El Dorado Main, No. 2, presently being constructed.

Henry T. Falvey

Copy to: 254 (Ruud)
253 (Thomas)

APPENDIX A

IMPEDANCE PROGRAM

INTRODUCTION

Impedance is defined as the ratio of a force-like quantity (voltage, force, pressure) to a flow-like quantity (current, velocity, heat flux). Since impedance applies only to sinusoidal variations of the two quantities, the ratio is usually represented as a complex number. From a very simplistic viewpoint, impedance can be considered as the apparent resistance to an alternating flow quantity. The corresponding quantity with constant flow is called true resistance.

In a pipeline system, the impedance can be computed for each point in the line. In this case, the impedance is the ratio of the pressure head fluctuations to the discharge variations. (Note: For an electrical analogy it is more convenient to define impedance as the ratio of the pressure head intensity fluctuations to the discharge fluctuations.) Since the pipeline is elastic, the magnitude of the discharge fluctuation is not necessarily constant along the length of the line. To avoid any ambiguity, the value of the impedance relates the head fluctuations at a location on the line to the discharge fluctuations at that location.

The magnitude of the head fluctuation at a given point in the pipeline varies with discharge and with the frequency of the discharge fluctuations. At certain frequencies, the head fluctuations attain maximum values. These frequencies and their corresponding periods are referred to as critical conditions. If the impedance is determined for a range of frequencies, it is possible to identify the critical periods of the pipeline. For a given amplitude of flow rate fluctuations, large values of impedance are equivalent to large values of pressure head variations. Therefore, resonant conditions in the pipeline can be readily identified as maximum values on a plot of the impedance modulus versus frequency or period.

In lines of finite length, the steady oscillatory motion is made up of a forward and a reflected wave. The amplitude of the wave resulting from the forward and reflected waves in the pipeline corresponds with the impedance modulus of the line. The resulting wave can also be considered by its ability to transmit energy. If no energy is transmitted, the wave is a pure standing wave. If energy is transmitted, the wave is called a traveling wave. The real part of the impedance value represents the energy-transmitting or traveling portion of the wave and the imaginary part represents the energy-storing portion of the wave. This energy-storing portion is frequently referred to as the standing wave.

The end of each pipe segment which is closest to the source of disturbances is called the sending end of the segment. The opposite end of each segment is known as the receiving end. In general, the computations of impedance in a pipeline system begin at the end of the system farthest from the disturbance. Then, the impedance is determined for each segment by systematically working back toward the exciter.

If the line is infinitely long, no reflections are returned to the exciter. The impedance of this line is called the characteristic impedance. A finite length line which terminates in the characteristic impedance is called a matched, nonresonant or flat line. The terminator element which equals the characteristic impedance is called a matched load. It should be noted that the characteristic impedance is a real number if the line losses are negligible. However, if line losses are appreciable, the characteristic impedance will have both real and imaginary parts. This indicates that an infinitely long line with losses will both transmit and store energy.

In designing a pipeline system, it is desirable for the impedance of the line to match the characteristic impedance as closely as possible. If this goal cannot be achieved, then the frequencies of any forcing functions should not be near any of the critical frequencies of the pipeline system.

DESCRIPTION OF PROGRAM

A program was developed to compute the impedance characteristics of a pipeline which can contain the following elements connected in series:

A reservoir

A fixed inline orifice

A change in line diameter

A branching pipeline with either an open or a closed end

An accumulator

A standpipe

The order in which the elements are connected must correspond with the order in which the elements appear in the actual pipeline system.

The program consists of several parts. These are as follows:

HFIMP - The main calling program

INPUTD - The necessary data to describe the physical system

RIMP - Definition of the receiving impedance for the various element types

IMPEDA - Calculates the sending impedance for each element

CMULT - Complex multiplication routine

CDIV - Complex division routine

PLØT - Creates a plot of the results

RANGE - Determines maximum and minimum values for the plot

LINPL - Determines the origin and step increment for a linear plot

LØGPL - Determines the origin and step increment for a logarithmic plot

The required inputs are described in detail in subroutine INPUTD. The program produces two outputs. One is a plot file which is created by procedure file HFPLT. This plot file called HF1 can be disposed to any of the peripherals available at the E&R Center. The attached figures 2 through 15 were disposed to the 35-mm camera after being scaled to produce slide-size films. The second output is a listing of all frequencies, periods, line impedance (modulus, real part, imaginary part, phase angle in degrees), and characteristic impedance (real and imaginary parts).

```
      PROGRAM HFIMP(INPUT,OUTPUT)
C
C       SOLUTION OF TERMINAL IMPEDANCE IN SIMPLE SYSTEM
C               INCLUDING FRICTION
C
      COMMON PI,G,AS,BASEOM,NHARM,POLY,BLENGT,BDIA,BA
      COMMON A,D,F,HBAR,ITYPE,QBAR,VOL,XLENGT,IELEMT
      DIMENSION OM(200),PHIQ(200),T(200),ZC(200),ZS(200)
      G= 32.2
      PI= 3.1416
      IELEMT= 0
      CALL INPUTD
      N=IELEMT
      CALL ID(20HFALVEY,MC1532;X3760 ,20)
      CALL COMPRS
      DO 10 J=1,N
      IELEMT= J
      CALL INPUTD
      CALL IMPEDA(OM,PHIQ,T,ZC,ZS)
      CALL PLOT(T,PHIQ,ZC,ZS,IELEMT)
   10 CONTINUE
      CALL DONEPL
      CALL EXIT
      END
```

```
      SUBROUTINE INPUTD
      COMMON PI,G,AS,BASEOM,NHARM,POLY,BLENGT,BDIA,BA
      COMMON A,D,F,HBAR,ITYPE,QBAR,VOL,XLENGT,IELEMT
C       DATA FORMAT
C           ITYPE - RECEIVING IMPEDANCE TYPE
C              ITYPE=1   --  RESERVOIR
C              ITYPE=2   --  FIXED IN-LINE ORIFICE
C              ITYPE=3   --  SERIES LINE
C              ITYPE=4   --  ACCUMULATOR
C              ITYPE=5   --  STANDPIPE
C              ITYPE=6   --  CLOSED END BRANCH PIPE
C              ITYPE=7   --  OPEN END BRANCH PIPE
C           F - FRICTION FACTOR IN PIPE
C           A - WAVE VELOCITY
C           D - PIPE DIAMETER
C           XLENGT - PIPE LENGTH
C           QBAR - MEAN DISCHARGE
C
C      WITH A PIPE BRANCH THE INPUT MUST INCLUDE
C
C           BLENGT - LENGTH OF BRANCH PIPE
C           BDIA - DIAMETER OF BRANCH PIPE
C           BA - WAVE VELOCITY IN BRANCH PIPE
C
C
C     WITH A FIXED ORIFICE THE INPUT MUST ALSO INCLUDE
C
C           HBAR - THE HEAD ACROSS THE ORIFICE
C
C     WITH AN ACCUMULATOR OR A STANDPIPE, THE IMPEDANCE
C     COMPUTATIONS MUST BE REPEATED FOR EACH OF THE SIGNIFICANT
C     HARMONICS THAT EXIST WITHOUT THE ACCUMULATOR.  THE INPUT
C     MUST ALSO INCLUDE THE FOLLOWING;
C           VOL - VOLUME OF ACCUMULATOR
C           AS - AREA OF STANDPIPE
C           BASEOM - FREQUENCY (RADIANS/SEC) CORRESPONDING
C                TO THE BASE PERIOD OF OSCILLATION
C           NHARM - NUMBER OF HARMONIC
C           HBAR - MEAN HEAD AT ACCUMULATOR OR STANDPIPE
C           POLY - POLYTROPIC GAS CONSTANT FOR THE AIR IN THE
C                  ACCUMULATOR
C
C
      IF(IELEMT.NE.0)GO TO 100
C
C     SET IELEMT EQUAL TO THE NUMBER OF ELEMENTS MAKING UP THE SYSTEM
C
      IELEMT= 7
      RETURN
C
C     ADD THE APPROPRIATE STATEMENT NUMBERS TO THE FOLLOWING DO
C     LOOP STRING AS SYSTEM ELEMENTS ARE ADDED TO THE PROGRAM
C
  100 GO TO(1,2,6,3,7,4,5),IELEMT
C
C     FIRST PIPE DOWNSTREAM OF RESERVOIR
    1 ITYPE=1
```

```
      F= 0.015
      A= 3193.
      D= 2.0
      XLENGT= 7991.
      QBAR= 32.3
      RETURN
C
C     SECOND PIPF DOWNSTREAM OF RESERVOIR
    2 ITYPE= 3
      F= 0.015
      A= 2701.1
      D= 1.75
      XLENGT= 1040.
      QBAR= 29.
      RETURN
C
C      STATION PRV-3 WITH NO PIPE DOWNSTREAM
    6 ITYPE= 2
      F= 0.015
      A= 3079.5
      D= 1.75
      XLENGT= 0.
      HBAR= 80.1
      QBAR= 29.
      RETURN
C
C     PIPE BETWEEN PRV-3 AND PRV-4
    3 ITYPE= 3
      F= 0.015
      A= 3079.5
      D=1.75
      XLENGT= 3557.
      QBAR= 29.
      RETURN
C
C     STATION PRV-4 WITH NO PIPE DOWNSTREAM
    7 ITYPE= 2
      F= 0.015
      A= 3039.1
      D= 2.0
      XLENGT= 0.
      HBAR= 85.4
      QBAR= 29.
      RETURN
C
C     FIRST PIPE DOWNSTREAM CF PRV-4
    4 ITYPE= 3
      F= 0.015
      A= 3039.1
      D= 2.0
      XLENGT= 8191.
      QBAR= 27.55
      RETURN
C
C     SECOND PIPE DOWNSTREAM OF PRV-4
    5 ITYPE=3
      F= 0.015
```

```
A= 3467.3
D= 1.75
XLENGT= 1335.
QBAR= 26.1
RETURN
END
```

```
      SUBROUTINE IMPEDA(OM,PHIQ,T,ZC,ZS)
      COMMON PI,G,AS,BASEOM,NHARM,POLY,BLENGT,BDIA,BA
      COMMON A,D,F,HBAR,ITYPE,QBAR,VOL,XLENGT,IELEMT
      REAL K1,K2,K3,K4,K5,K6,K7,K8,L
      DIMENSION OM(200),PHIQ(200),T(200),ZC(200),ZS(200)
C
C       START READ AND PRINT DATA
      OM(1)= 0.02
      DOM= 0.02
C       DETERMINATION OF CHARACTERISTIC IMPEDANCE
      AR=(PI/4.)*D*D
      R=F*QBAR/(G*D*AR*AR)
      L=1./(G*AR)
      C=G*AR/(A*A)
      N= 0
      WRITE 6, IELEMT
    6 FORMAT(1H1,39X,7HELEMENT,I4//
     18X,2HOM,9X,1HT,9X,2HZS,9X,3HZSR,7X,3HZSI,8X,5HPHI Q,
     27X,3HZCR,8X,3HZCI/)
      DO 9 I=1,200
      K1=.5*ATAN(R/(L*OM(I)))
      K2=SQRT(C*OM(I))*((L*OM(I))**2+R**2)**.25
      K3=SIN(K1)
      K4=COS(K1)
      AL=K2*K3
      BE=K2*K4
      ZCR=BE/(C*OM(I))
      ZCI=-AL/(C*OM(I))
      ZC(I)=SQRT(ZCR*ZCR+ZCI*ZCI)
C
C       DETERMINATION OF HYPERBOLIC FUNCTIONS
      Y=EXP(AL*XLENGT)
      SINH=.5*(Y-1./Y)
      COSH=.5*(Y+1./Y)
      SI=SIN(BE*XLENGT)
      CO=COS(BE*XLENGT)
      K5=SINH*CO
      K6=COSH*SI
      K7=COSH*CO
      K8=SINH*SI
      CALL CDIV(THR,THI,K5,K6,K7,K8)
C
C       DETERMINATION OF SENDING IMPEDANCE
      CALL RIMP(ZRR,ZRI,ZS,PHIQ,I,OM(I))
      CALL CMULT(ZCTHR,ZCTHI,ZCR,ZCI,THR,THI)
      CALL CDIV(ZRZCR,ZRZCI,ZRR,ZRI,ZCR,ZCI)
      CALL CMULT(RCTR,RCTI,ZRZCR,ZRZCI,THR,THI)
      RCTR=1.-RCTR
      RNUMR=ZRR-ZCTHR
      RNUMI=ZRI-ZCTHI
      CALL CDIV(ZSR,ZSI,RNUMR,RNUMI,RCTR,RCTI)
      ZS(I)=SQRT(ZSR*ZSR+ZSI*ZSI)
      PHIQ(I)= ATAN2(ZSI,ZSR)*180./PI
      T(I)=2.*PI/OM(I)
C
C       OUTPUT OF RESULTS
      ILOOP=(I-1)/50-N
```

```
      IF(ILOOP.NE.1)GO TO 3
      N=N+1
      WRITE 5
    5 FORMAT(1H1,8X,2HOM,9X,1HT,9X,2HZS,9X,3HZSR,7X,3HZSI,8X,5HPHI Q,
     17X,3HZCR,8X,3HZCI/)
    3 WRITE 4,OM(I),T(I),ZS(I),ZSR,ZSI,PHIQ(I),ZCR,ZCI
    4 FORMAT(1X,8F11.3)
      IF(I.LT.200)OM(I+1)=OM(I)+DOM
    9 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RIMP(ZRR,ZRI,ZS,PHIQ,N,OM)
      COMMON PI,G,AS,BASEOM,NHARM,POLY,BLENGT,BDIA,BA
      COMMON A,D,F,HBAR,ITYPE,QBAR,VOL,XLENGT,IELEMT
      DIMENSION ZS(1),PHIQ(1)
C       ITYPE DEFINES THE RECEIVING IMPEDANCE
C       ITYPE=1  --  RESERVOIR
C       ITYPE=2  --  FIXED IN-LINE ORIFICE
C       ITYPE=3  --  SERIES LINE
C       ITYPE=4  --  ACCUMULATOR
C       ITYPE=5  --  STANDPIPE
C       ITYPE=6  --  CLOSED END BRANCH PIPE
C       ITYPE=7  --  OPEN END BRANCH PIPE
C
      PHIQ(N)= PI*PHIQ(N)/180.
      ZSR= ZS(N)*COS(PHIQ(N))
      ZSI= ZS(N)*SIN(PHIQ(N))
      GO TO (1,2,3,4,5,7,7),ITYPE
C
C         RESERVOIR
    1 ZRR=0.
      ZRI=0.
      RETURN
C
C         FIXED IN-LINE ORIFICE
    2 ZRR=ZSR-2.*HBAR/QBAR
      ZRI=ZSI
      RETURN
C
C         SERIES LINE
    3 ZRR=ZSR
      ZRI=ZSI
      RETURN
C
C         ACCUMULATOR
    4 Z2R= ZSR
      Z2I= ZSI
      HARM= FLOAT(NHARM)
      CTE= HARM*BASEOM*VOL/(POLY*HBAR)
      DENR= 1.-CTE*Z2I
      DENI= CTE*Z2R
      CALL CDIV(ZRR,ZRI,Z2R,Z2I,DENR,DENI)
      RETURN
C
C         STANDPIPE
    5 HARM= FLOAT(NHARM)
      Z2I= -1./(HARM*BASEOM*AS)
      Z2R= 0.
      Z3I= -(G+HARM*BASEOM*BASEOM*HBAR)*A*A/
     1((BASEOM*G*AS)*(HARM*A*A-G*HBAR))
      Z3R= 0.
      Z4R= ZSR
      Z4I= ZSI
      ZDENR= Z3R-Z4R
      ZDENI= Z3I-Z4I
      CALL CMULT(ZPR,ZPI,Z3R,Z3I,Z4R,Z4I)
      CALL CDIV(ZRR,ZRI,ZPR,ZPI,ZDENR,ZDENI)
      RETURN
```

```
C
C          BRANCH PIPE
    7 BAREA= (PI/4.)*BDIA*BDIA
      ZS2R= 0.
      ZS2I= BA*BA/(G*BAREA*OM*BLENGT)
C
C          THIS STEP REQUIRED FOR AN OPEN END PIPE,
C          OTHERWISE THE PIPE IS A CLOSED END
C
      IF(ITYPE.EC.7)ZS2I= -(OM*BLENGT)/(G*BAREA)
      ZS1R= ZSR
      ZS1I= ZSI
      DENR= ZS2R-ZS1R
      DENI= ZS2I-ZS1I
      CALL CMULT(ZPR,ZPI,ZS2R,ZS2I,ZS1R,ZS1I)
      CALL CDIV(ZFP,ZRI,ZPR,ZPI,DENR,DENI)
      RETURN
      END
```

```
      SUBROUTINE CMULT(PR,PI,AR,AI,BR,BI)
C       COMPLEX MULTIPLICATION
C
C         PR= PRODUCT (REAL PART)
C         PI=    #      (IMAGINARY PART)
C         AR= FIRST TERM (REAL PART)
C         AI=      #       (IMAGINARY PART)
C         BR= SECOND TERM (REAL PART)
C         BI=       #       (IMAGINARY PART)
C
      PR=(AR*BR-AI*BI)
      PI=(AI*BR+AR*BI)
      RETURN
      END
```

```
      SUBROUTINE CDIV(QR,QI,AR,AI,BR,BI)
C        COMPLEX DIVISION
C
C         QR= QOUTIENT (REAL PART)
C         QI=     #      (IMAGINARY PART)
C         AR= NUMERATOR (REAL PART)
C         AI=      #      (IMAGINARY PART)
C         BR= DENOMINATOR (REAL PART)
C         BI=       #       (IMAGINARY PART)
C
      RDIV=BR*BR+BI*BI
      QR=(AR*BR+AI*BI)/RDIV
      QI=(AI*BR-AR*BI)/RDIV
      RETURN
      END
```

```
      SUBROUTINE PLOT(T,PHIQ,ZC,ZS,IELEMT)
      DIMENSION T(1),PHIQ(1),ZC(1),ZS(1)
C       PLOTTING ROUTINE
C       INITIALIZATION OF GRAPH
      CALL BGNPL(-1)
      CALL PAGE(10.,8.)
      CALL NOCHEK
C       PLOT OF T VS. PHI Q
C
C       SELF-SCALING OF AXES
      CALL RANGE(T,TMINI,TMAXI)
      GLENGT=9.
      CALL LOGPL(TMINI,TMAXI,TORIG,TINCR,GLENGT)
      CALL RANGE(ZS,ZSMINI,ZSMAXI)
      CALL RANGE(ZC,ZCMINI,ZCMAXI)
      IF (ZSMINI.LT.ZCMINI) GOTO 20
      ZMINI=ZCMINI
      GOTO 22
   20 ZMINI=ZSMINI
   22 IF(ZSMAXI.GT.ZSMINI) GOTO 24
      ZMAXI=ZCMAXI
      GOTO 25
   24 ZMAXI=ZSMAXI
   25 GLENGT=3.5
      CALL LINPL(ZMINI,ZMAXI,ZORIG,ZINCR,GLENGT)
      CALL PHYSOR(.7,5.)
      CALL TITLE(0,0,"T (SEC)$",100,"PHI Q$",100,9.,2.)
      CALL HEIGHT(.21)
      CALL MESSAG("HYDRAULIC IMPEDANCE DATA$",100,2.1,2.4)
      CALL HEIGHT(.14)
      CALL MESSAG("GRAPH NO.$",100,7.,2.5)
      CALL INTNO(IELEMT,8.2,2.5)
      CALL XLOG(TORIG,TINCR,-180.,180.)
      CALL GRID(1,2)
      CALL CURVE(T,PHIQ,200,0)
      CALL ENDGR(0)
C       PLOT OF T VS. ZC AND ZS
      CALL PHYSOR(.7,.7)
      CALL TITLE(0,0,"T (SEC)$",100,"ZC(SOLID)  ZS(DASHED)     $",100,
     19.,3.5)
      CALL XLOG(TORIG,TINCR,ZORIG,ZINCR)
      CALL GRID(1,2)
      CALL CURVE(T,ZC,200,0)
      CALL DASH
      CALL CURVE(T,ZS,200,0)
      CALL ENDGR(0)
      CALL ENDPL(0)
      RETURN
      END
```

```
      SUBROUTINE RANGE(Q,QMINI,QMAXI)
      DIMENSION Q(200)
C
C       DETERMINATION OF MAXIMUM AND MINIMUM VALUES OF X AND Y
C
      DO 14 IQ=1,200
      IF(IQ.GT.1) GOTO 15
      QMINI=Q(IQ)
      QMAXI=Q(IQ)
      GOTO 14
   15 IF (Q(IQ).GE.QMINI) GOTO 16
      QMINI=Q(IQ)
   16 IF (Q(IQ).LE.QMAXI) GOTO 14
      QMAXI=Q(IQ)
   14 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LINPL(QMINI,QMAXI,QORIG,QINCR,GLENGT)
C       LINEAR AXIS VALUES
      QORIG=QMINI
      QINCR=(QMAXI-QMINI)/GLENGT
C       ROUNDING OF LINEAR AXIS POINTS
      TENPWR=10**(AINT(ALOG10(QINCR)))
      IF(QINCR.LT.1.) TENPWR=TENPWR/10.
      FACTOR=.5*AINT(2.*QINCR/TENPWR)
      QINCR=FACTOR*TENPWR
      QORIG=QINCR*AINT(QMINI/QINCR)
      IQ=1
      DO 85 IQ=1,20
      IF(QORIG.GT.QMINI) QORIG=QORIG-QINCR
      IF(QORIG+GLENGT*QINCR.GE.QMAXI) GOTO 86
      FACTOR=FACTOR+.5
      QINCR=FACTOR*TENPWR
      QORIG=QINCR*AINT(QMINI/QINCR)
   85 CONTINUE
      WRITE 87
   87 FORMAT(1X,37HBAD DATA OR SUBROUTINE ERROR IN LINPL)
   86 RETURN
      END
```

```
      SUBROUTINE LOGPL(QMINI,QMAXI,QORIG,QINCR,GLENGT)
C       LOG AXIS VALUES
      IF (QMINI.LE.0.) GOTO 50
      IF (QMINI/QMAXI.LT..1E-10) GOTO 50
      QORIG=10**(AINT(ALOG10(QMINI)))
      QINCR=GLENGT/(AINT(ALOG10(QMAXI))-AINT(ALOG10(QMINI))+1)
      RETURN
C       ERROR MESSAGES
   50 WRITE 52
   52 FORMAT(1X,47HZERO OR NEGATIVE VALUES ENCOUNTERED IN LOG PLOT)
      CALL EXIT
      END
```

ELEMENT 1

OM	T	ZS	ZSR	ZSI	PHI Q	ZCR	ZCI
.020	314.160	6.298	-6.101	-1.562	-165.644	49.823	-38.549
.040	157.080	6.885	-6.132	-3.131	-152.953	39.749	-24.159
.060	104.720	7.776	-6.183	-4.715	-142.672	36.184	-17.693
.080	78.540	8.895	-6.257	-6.323	-134.698	34.497	-13.919
.100	62.832	10.187	-6.353	-7.963	-128.586	33.574	-11.441
.120	52.360	11.616	-6.475	-9.644	-123.879	33.019	-9.694
.140	44.880	13.164	-6.625	-11.376	-120.214	32.663	-8.400
.160	39.270	14.824	-6.805	-13.170	-117.325	32.421	-7.405
.180	34.907	16.596	-7.019	-15.038	-115.021	32.250	-6.617
.200	31.416	18.487	-7.274	-16.996	-113.168	32.125	-5.978
.220	28.560	20.510	-7.574	-19.060	-111.671	32.031	-5.451
.240	26.180	22.680	-7.928	-21.249	-110.461	31.959	-5.008
.260	24.166	25.019	-8.347	-23.585	-109.489	31.902	-4.631
.280	22.440	27.556	-8.843	-26.098	-108.719	31.856	-4.306
.300	20.944	30.325	-9.435	-28.820	-108.126	31.819	-4.024
.320	19.635	33.372	-10.144	-31.792	-107.696	31.789	-3.776
.340	18.480	36.753	-11.001	-35.067	-107.418	31.764	-3.557
.360	17.453	40.542	-12.050	-38.709	-107.291	31.742	-3.361
.380	16.535	44.835	-13.347	-42.802	-107.319	31.724	-3.186
.400	15.708	49.760	-14.978	-47.453	-107.517	31.709	-3.028
.420	14.960	55.492	-17.063	-52.804	-107.908	31.696	-2.885
.440	14.280	62.273	-19.789	-59.046	-108.528	31.684	-2.755
.460	13.659	70.450	-23.443	-66.435	-109.437	31.674	-2.636
.480	13.090	80.531	-28.499	-75.320	-110.725	31.665	-2.527
.500	12.566	93.293	-35.761	-86.168	-112.539	31.657	-2.427
.520	12.083	109.962	-46.682	-99.561	-115.121	31.650	-2.334
.540	11.636	132.530	-64.038	-116.031	-118.894	31.644	-2.248
.560	11.220	164.264	-93.382	-135.139	-124.644	31.638	-2.168
.580	10.833	209.945	-145.553	-151.298	-133.891	31.633	-2.094
.600	10.472	271.783	-234.009	-138.223	-149.430	31.629	-2.024
.620	10.134	325.448	-323.711	-33.575	-174.078	31.625	-1.959
.640	9.818	311.181	-287.183	119.830	157.351	31.621	-1.898
.660	9.520	248.292	-178.502	172.586	135.965	31.618	-1.841
.680	9.240	191.574	-104.411	160.620	123.026	31.615	-1.787
.700	8.976	151.384	-64.491	136.960	115.214	31.612	-1.736

With five elements, the program takes 10 seconds to execute. The central memory required to load the program is 110K.

The program computes 200 values of impedance. The starting value of frequency (radians/second) and the step increment are located in subroutine IMPEDA. These two have both been set equal to 0.02. For some pipelines, this choice of values should be changed to cover a better range of critical periods. With the present values, the range of critical periods that can be examined ranges from 1.6 to 314 seconds.

The most serious restriction in the program is the lumped-parameter approximation for the branch pipe. Essentially, this assumption is valid if:

a. Frictional losses are small and a low oscillating frequency exists, or

b. A short pipeline is being considered.

Since the branch pipes will usually be laterals with relatively small discharges, the approximation should not be critical for most municipal pipeline systems.

If standpipes or accumulators are investigated, it is necessary to repeat the computations for each of the frequencies that exist in the forcing function. The frequency corresponding to the basic period of oscillation of the forcing function is given by BASEØM in subroutine INPUTD. Harmonics can be easily investigated by changing NHARM by integer values. NHARM = 1 is equivalent to the fundamental frequency. If the periodic forcing function is not sinusoidal, it can be described by a Fourier series. Each harmonic of the series is then treated as an individual forcing function at its own particular frequency.

The computer coding was developed from equations presented in:

"Hydraulic Transients"
Streeter, V. L.; Wylie, E. B.
McGraw-Hill Book Company
New York, 1967

VERIFICATION

As noted in the introduction, the impedance method analyzes the magnitude of sinusoidal fluctuations in head and discharge. Normally, these are impressed upon a steady-state flow. In deriving the equations of motion for the impedance method, it is necessary to linearize the friction terms for the unsteady motion. However, the friction for the steady component of the flow is not linearized. The linearization process results in the term containing q^2 being dropped, where q is the fluctuating discharge. Based upon this approximation, it can be shown that the impedance method is valid only when $q < Q/2$ where Q is the steady-state flow.

The impedance program was verified by simulating a simple system using the method of characteristics. These computations yield the time history of the discharge and head variations. From these, it is possible to compute the impedance values which are compared with the values obtained from the impedance program. The simple system consisted of a reservoir, 1125 m of 1270-mm pipe, and an oscillating valve. the basic water-hammer program given by Streeter and Wylie was used to obtain a solution with the method of characteristics. Two valve fluctuation magnitudes were investigated for a rotational frequency of oscillation of 1.7 rd/s. The magnitudes were 1 percent and 10 percent of full open.

The valve motions produced the following discharge variations:

Valve motion percent	2q/Q at reservoir percent	2q/Q at valve percent
1	1.25	0.06
10	12.41	1.79

It can be seen that the 1 percent valve motion satisfies the discharge criteria much better than the 10 percent motion.

For the 1 percent motion, the correlation between the impedance method and the characteristic method was very good. The value of the impedance modulus agreed within 2 percent and the phase angle was within 5 percent. For the 10 percent motion, the agreement was not good. The impedance modulus as determined by the method of characteristics was 32 percent of that predicted by the impedance method.

From these results, it can be concluded that the impedance method will yield good results when the double amplitude of the discharge variation at all points in the line is less than or equal to 1 percent of the mean discharge. Otherwise, the impedance method will tend to overpredict the head variations.

PLEASANT OAK MAIN SIMULATION

The existing configuration at Pleasant Oak Main was simulated by assuming that a fluctuating discharge exists at pressure-reducing station 5, figure 1. All other stations were assumed to behave as fixed orifices. In this fashion, it is possible to separate the dynamic response of the values at stations 3 and 4 from the response of the pipeline itself. The only deviation from the existing configuration was that no laterals were simulated. The geometry of these laterals was not available during the development of the program.

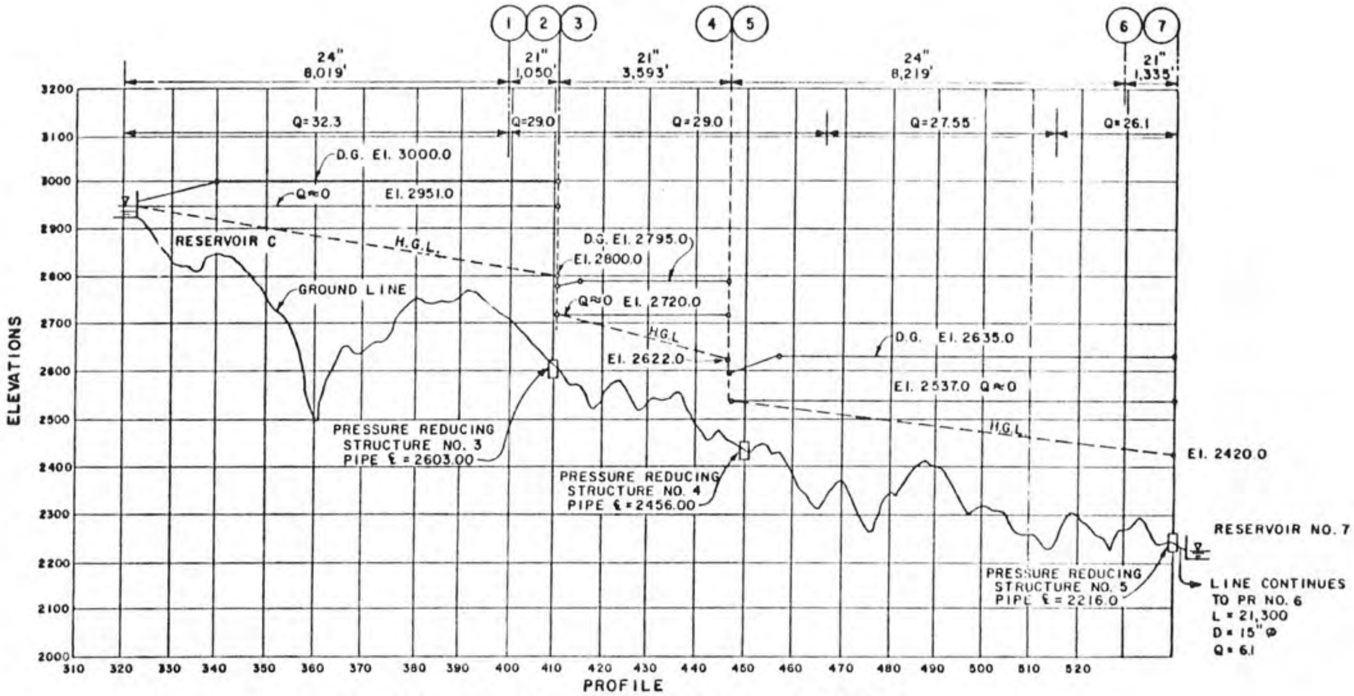
Since the response characteristics of the main are strongly dependent upon the wave travel time, the field test results were analyzed to obtain accurate wave speed values. From field tests 11 and 25, the values for the mean wave speed and the standard deviation are as follows:

Diameter mm	Wave speed m/s
610	960 $\pm$ 20
535	890 $\pm$ 20

The results of the computer program are summarized in table I and in figures 2-15. It is obvious that the pipeline upstream of station 3 is susceptible to resonance (high impedance values) when the discharge is small. At design discharge, the pipeline between stations 4 and 5 is the segment most susceptible to resonance. Furthermore, as the discharge increases, the number of critical periods increases.

Table I. - Pleasant Oak Main impedance characteristics

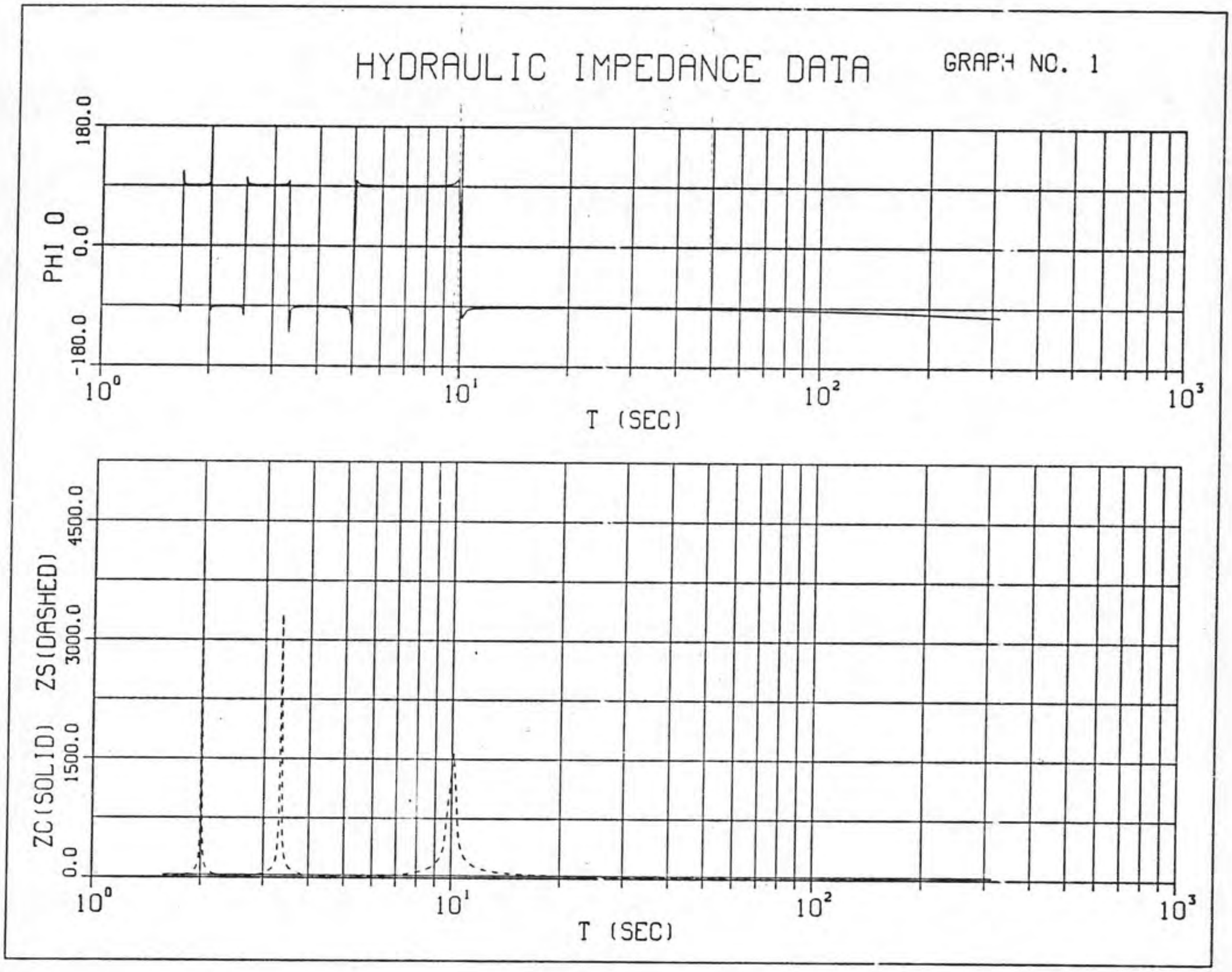
Station	Q = 0.057 m^3/s		Design Q in all sections	
	T (s)	Z ($m/m^3/s$)	T (s)	Z ($m/m^3/s$)
Upstream of 3	11.2	11 800	11.6	3 000
	3.8	14 000	3.8	3 200
	2.3	23 500	2.3	3 400
	1.6	15 100	1.6	3 400
Upstream of 4			15.0	3 500
	2.3	10 600	2.3	6 000
Upstream of 5	4.8	3 000	4.8	11 100
	2.6	10 500	3.9	6 700
			3.21	9 100



PLEASANT OAK MAIN-PLACERVILLE, CALIFORNIA

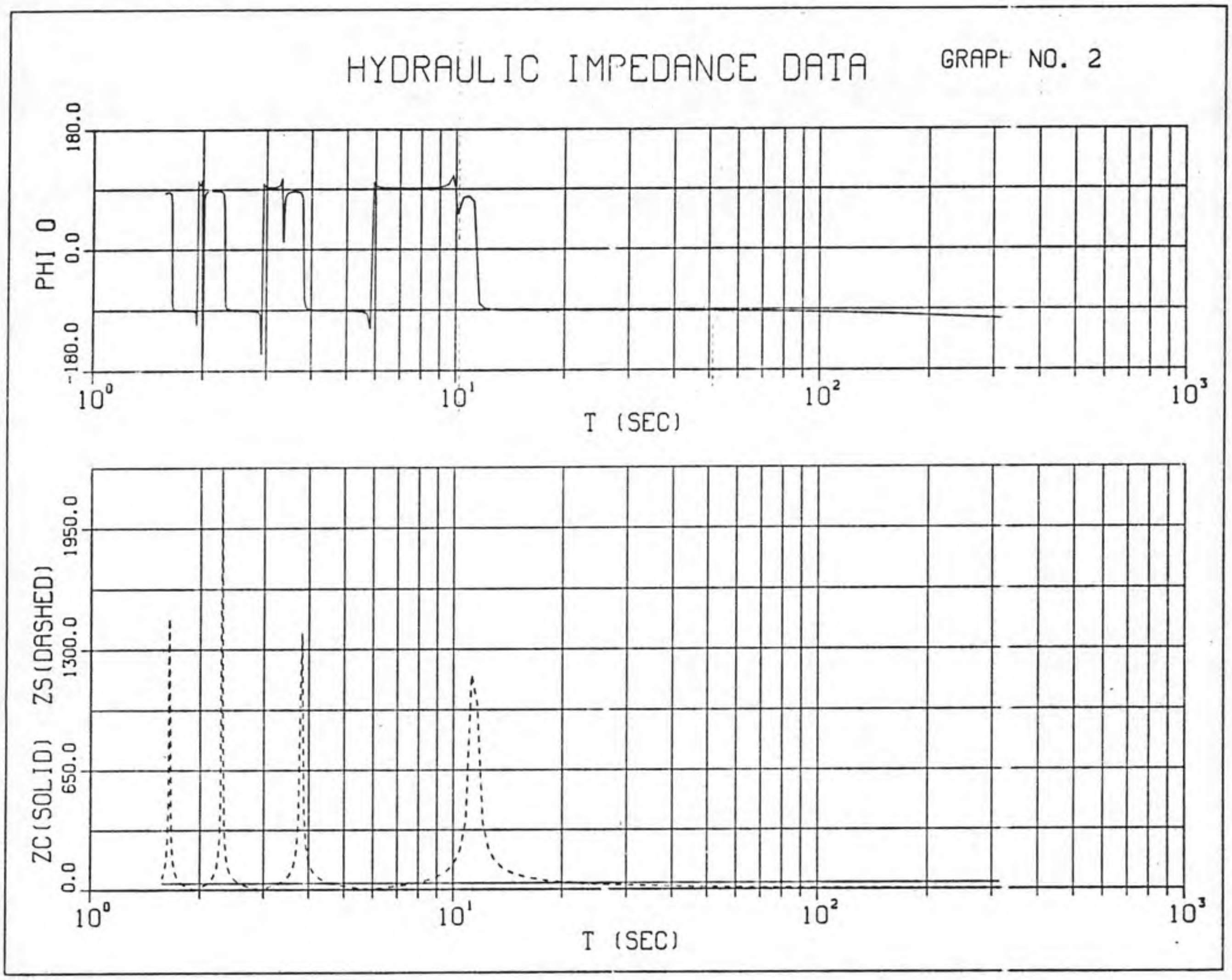
Figure 1

Figure 2



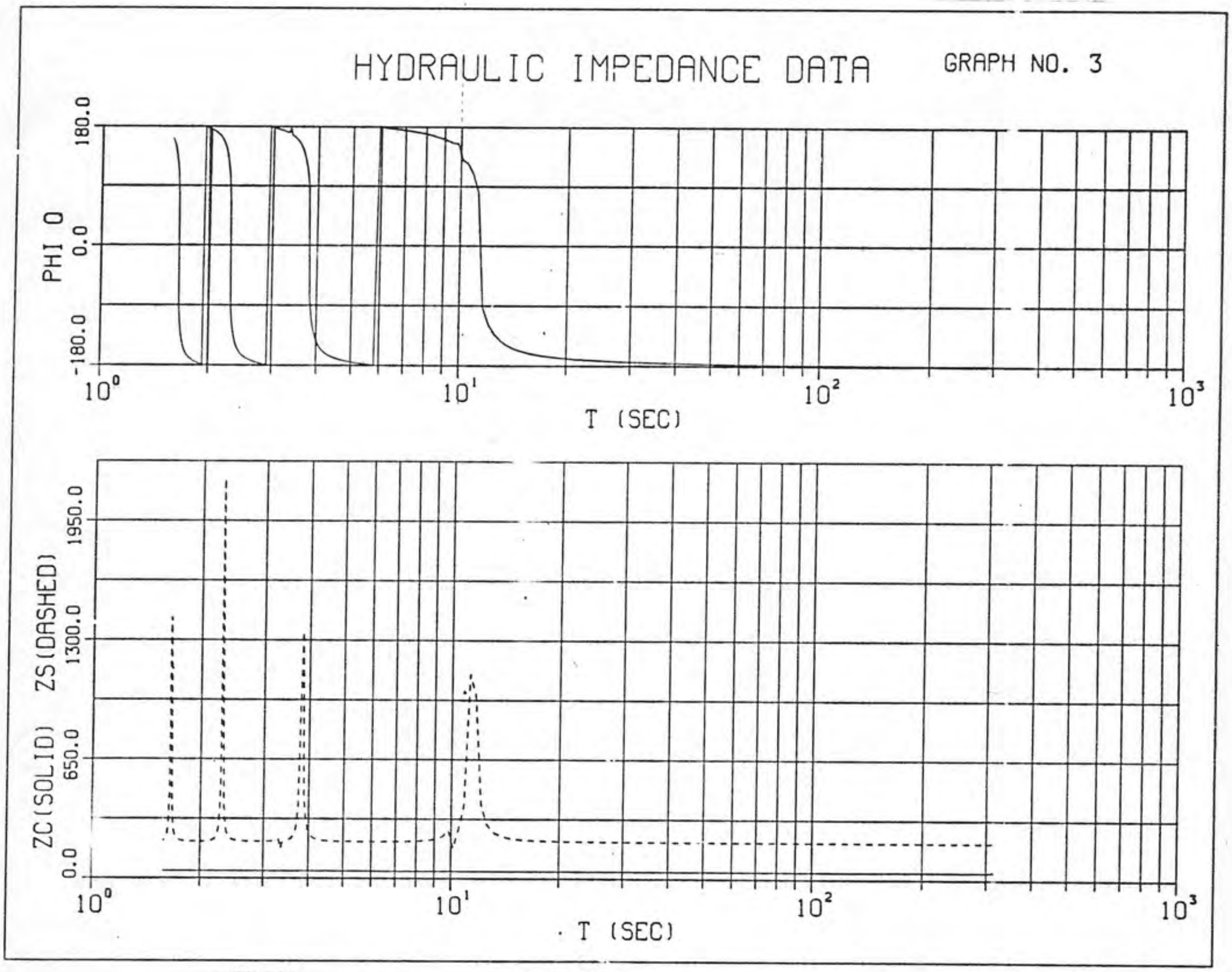
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Change in Pipe Diameter Upstream of
Pressure Reducing Station 3

Figure 3



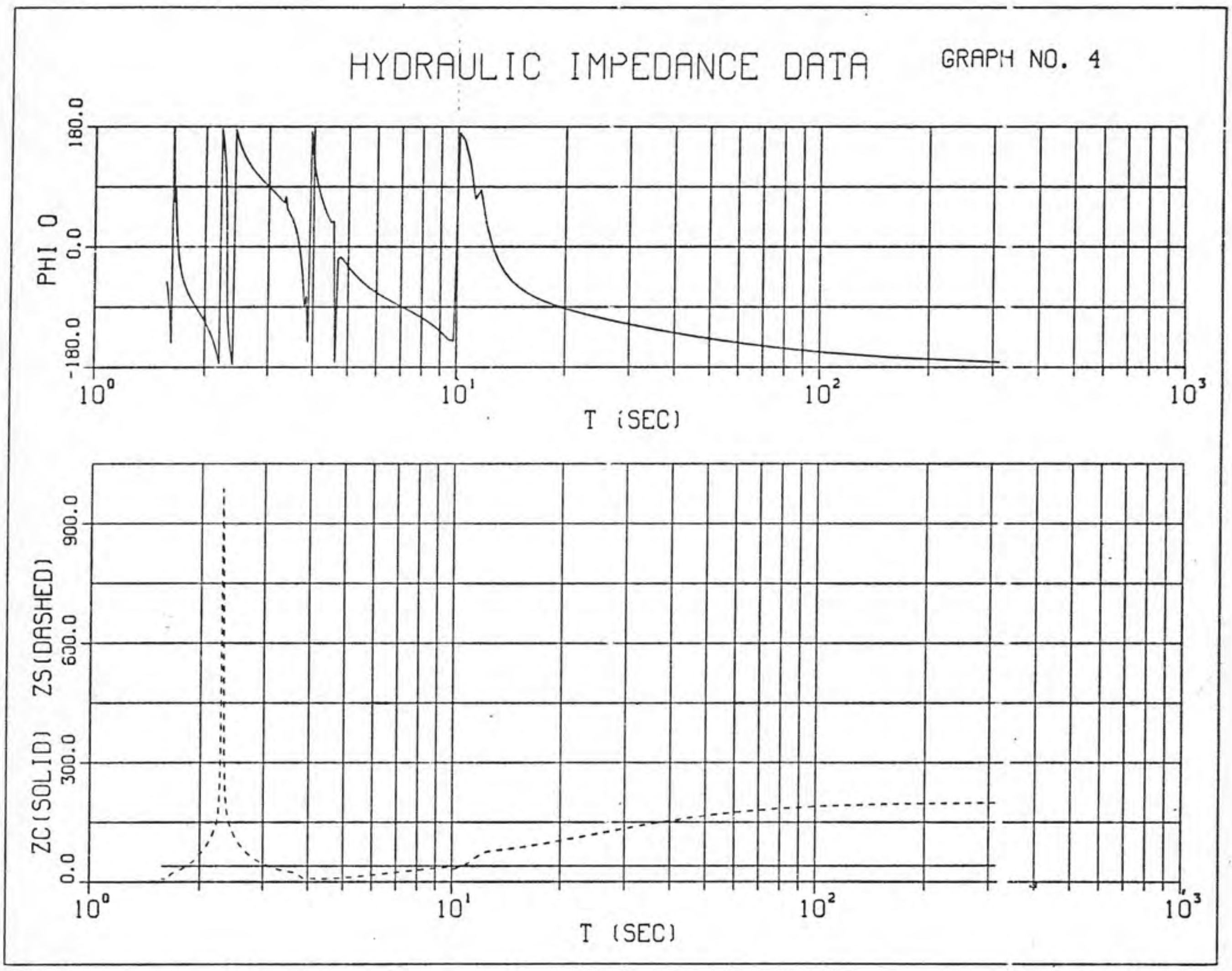
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Upstream of Pressure Reducing Station 3

Figure 4



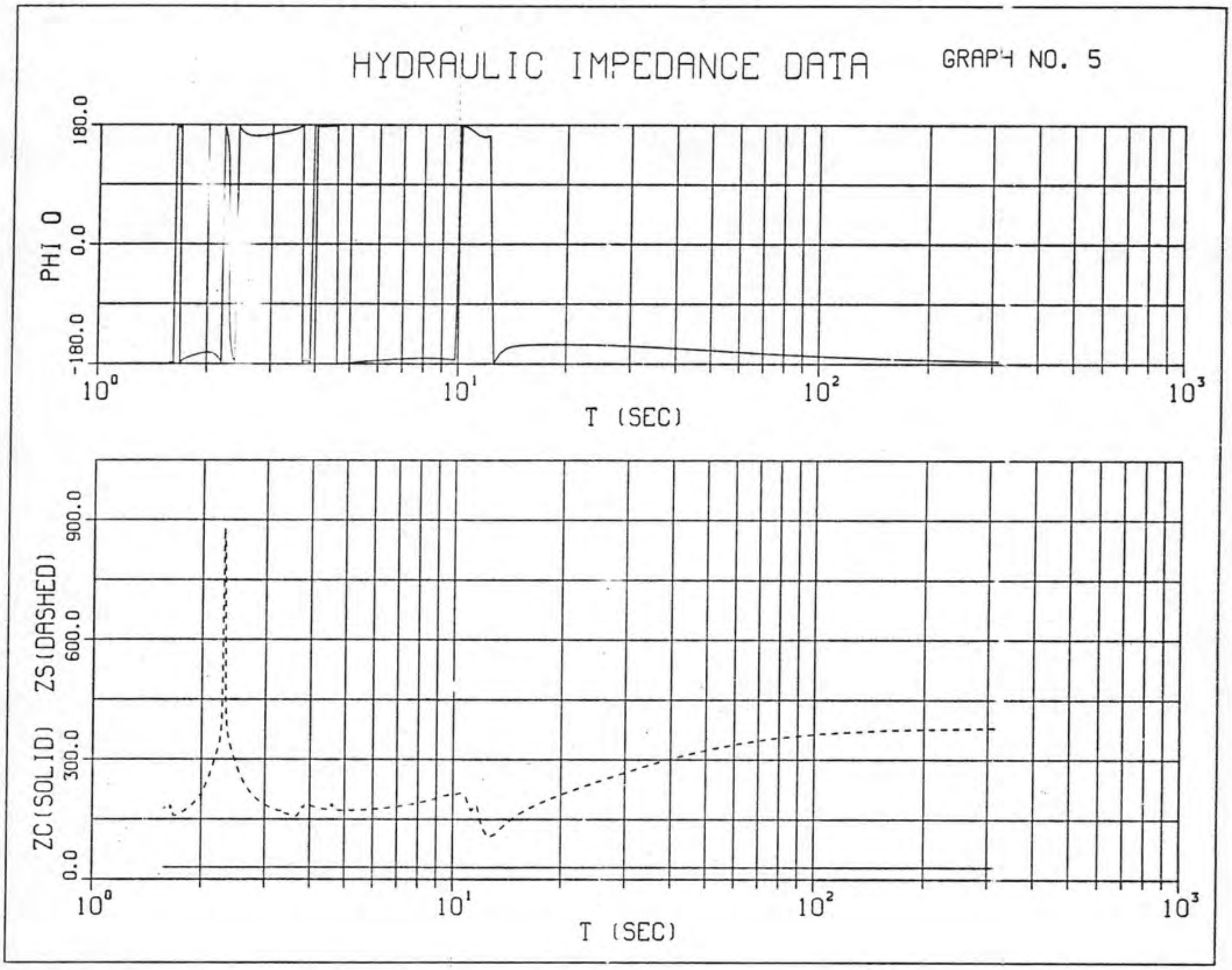
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Downstream of Pressure Reducing Station 3

Figure 5



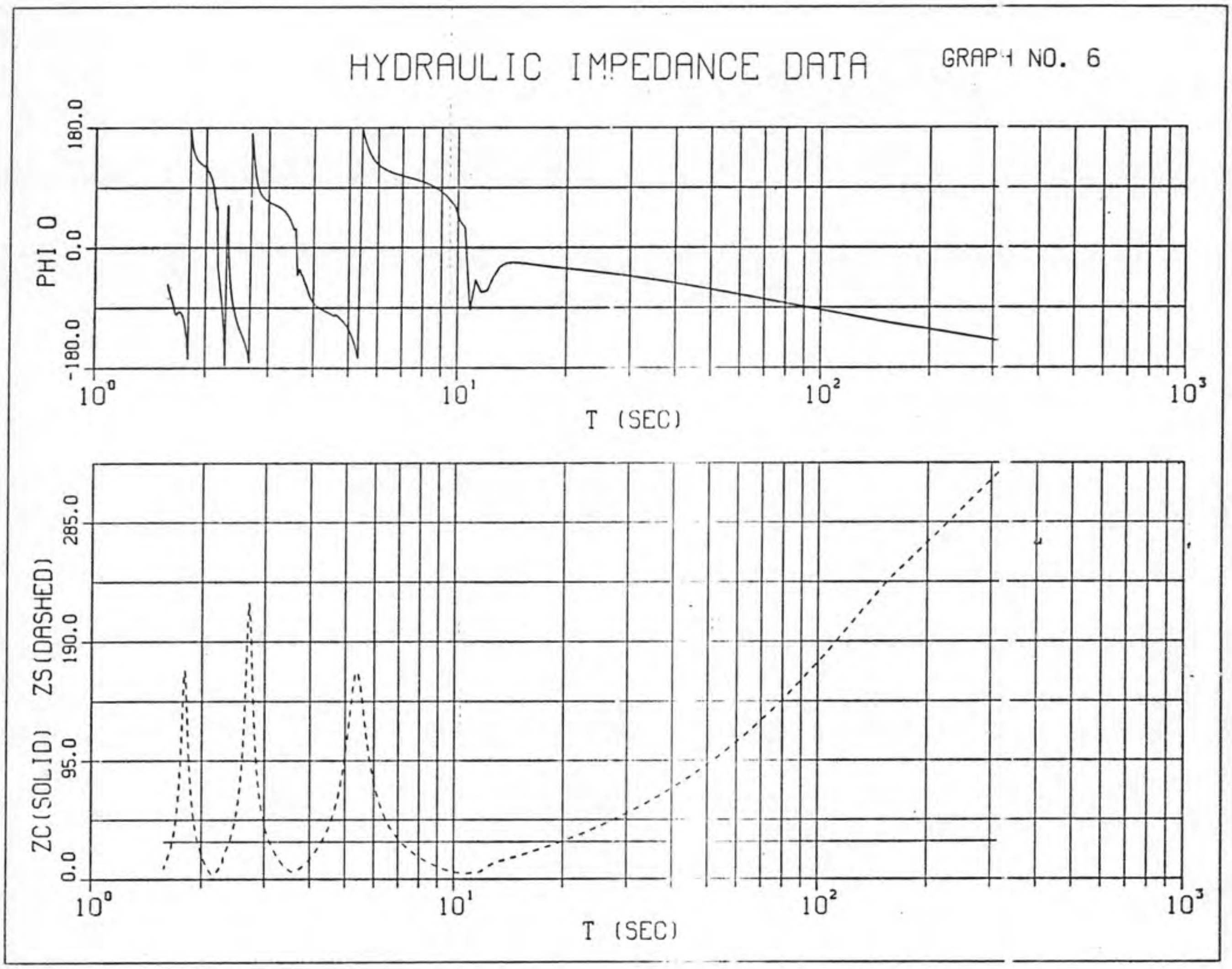
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Upstream of Pressure Reducing Station 4

Figure 6



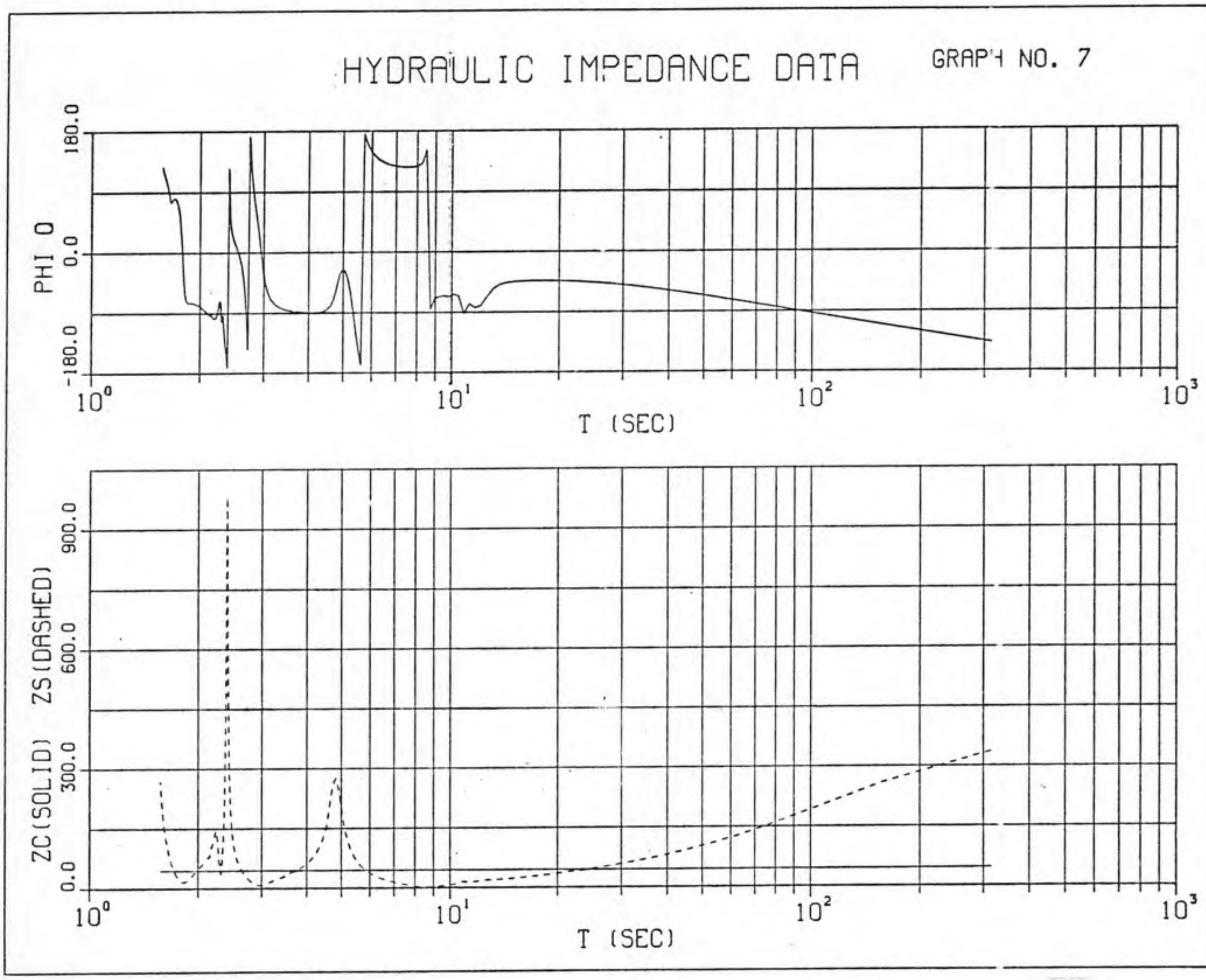
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Downstream of Pressure Reducing Station 4

Figure 7



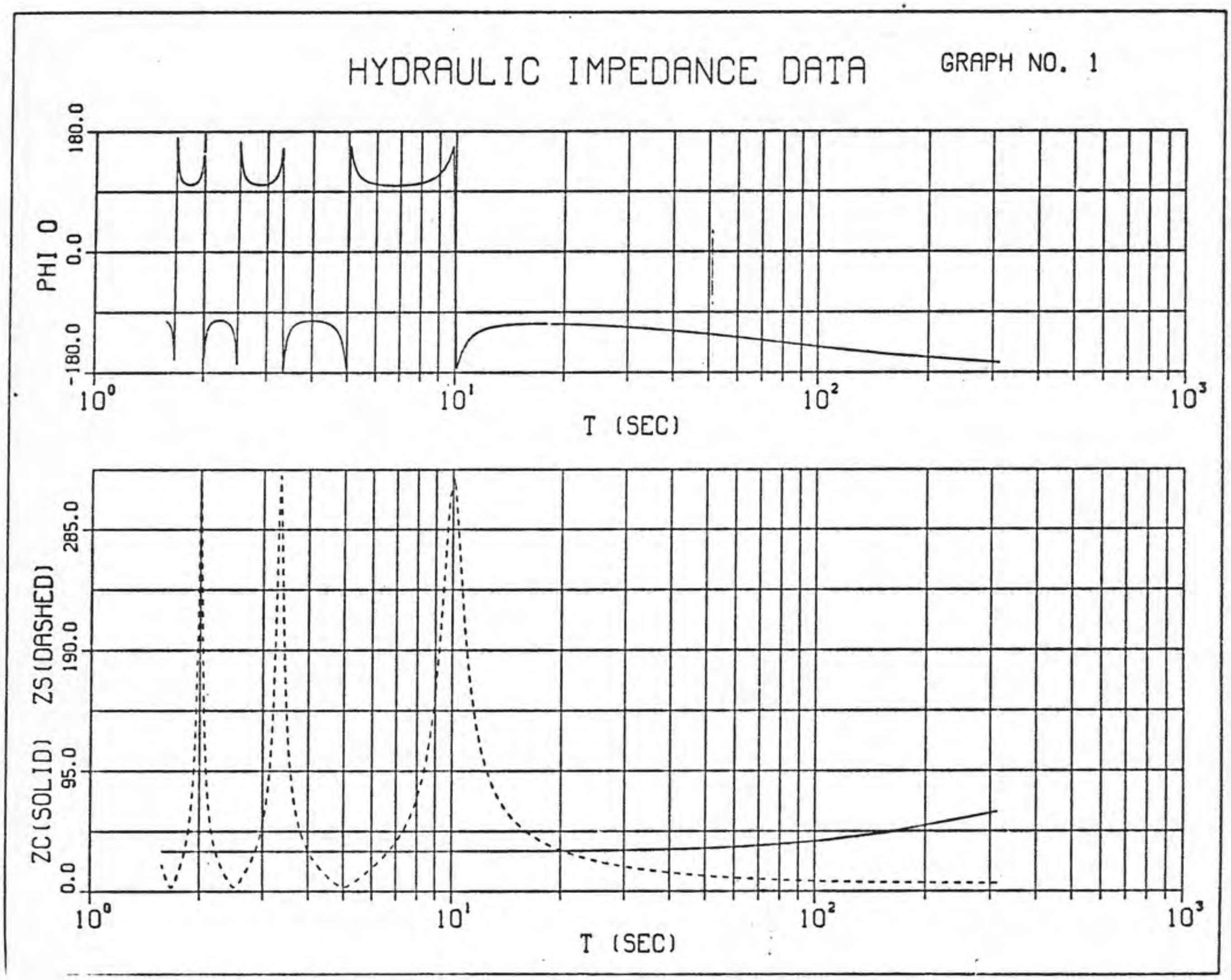
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Change in Pipe Diameter Upstream of
Pressure Reducing Station 5

Figure 8



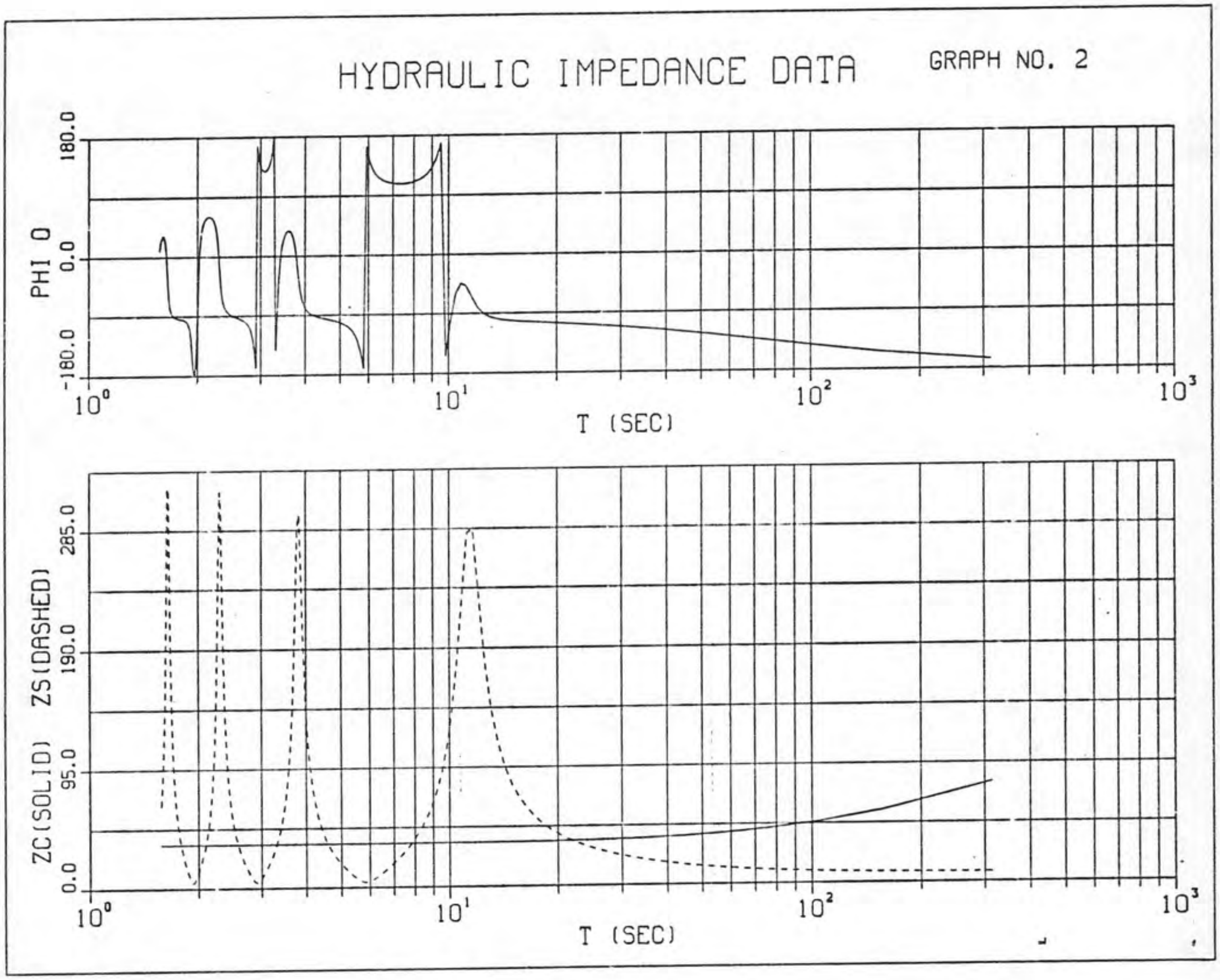
Impedance Diagram
Pleasant Oak Main
Discharge = 0.057 m^3/s
Upstream of Pressure Reducing Station 5

Figure 9



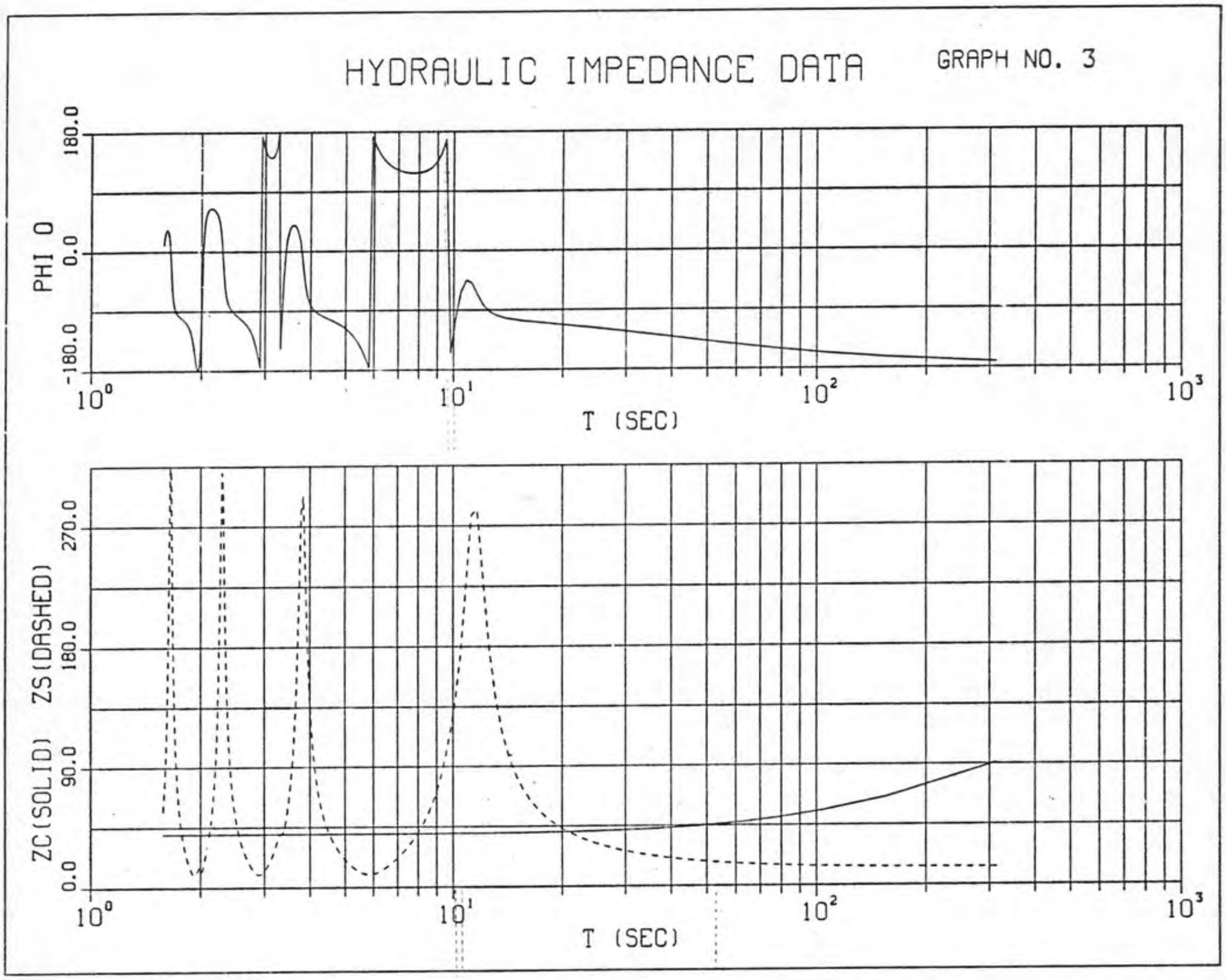
Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Change in Pipe Diameter Upstream of
Pressure Reducing Station 3

Figure 10



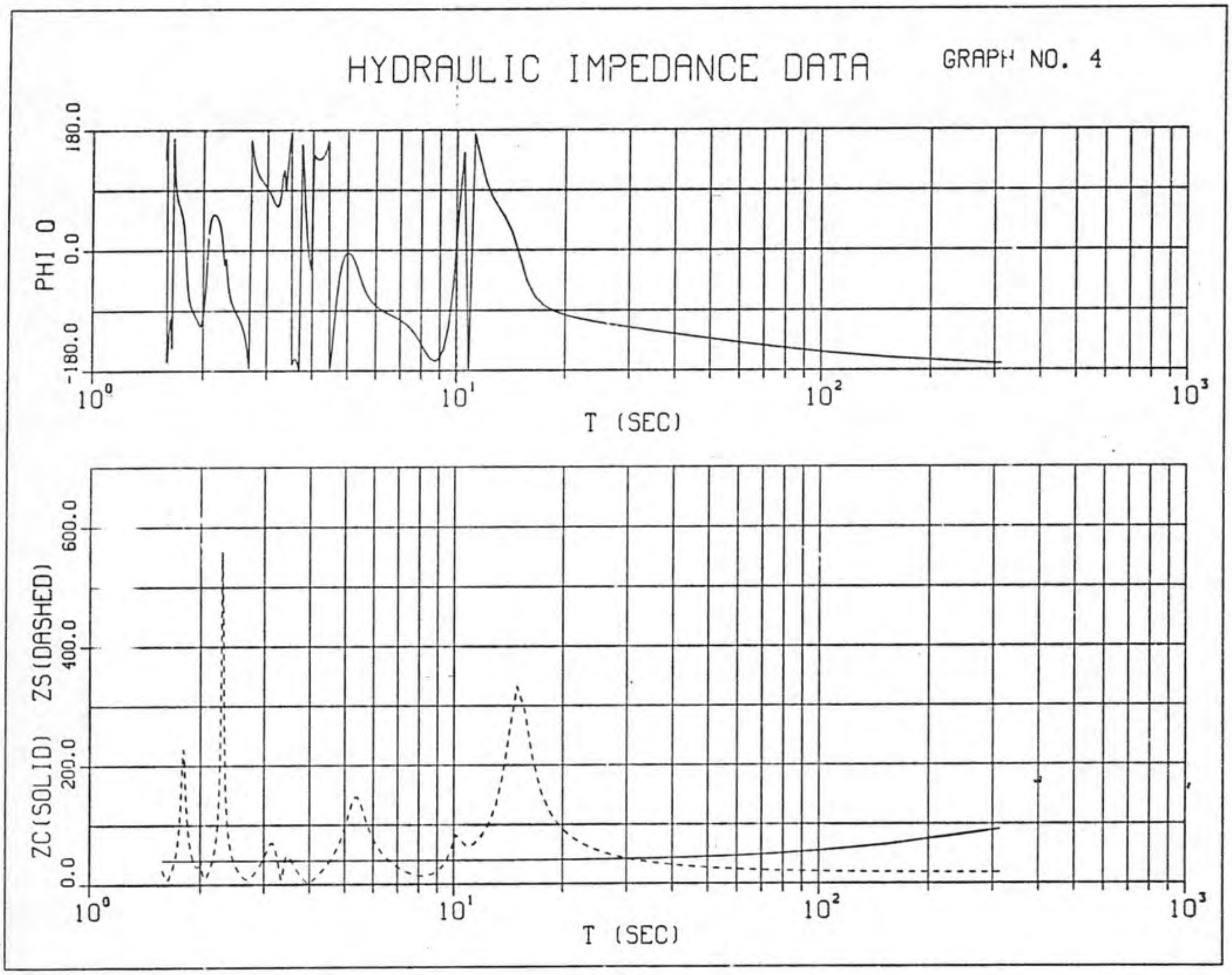
Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Upstream of Pressure Reducing Station 3

Figure 11



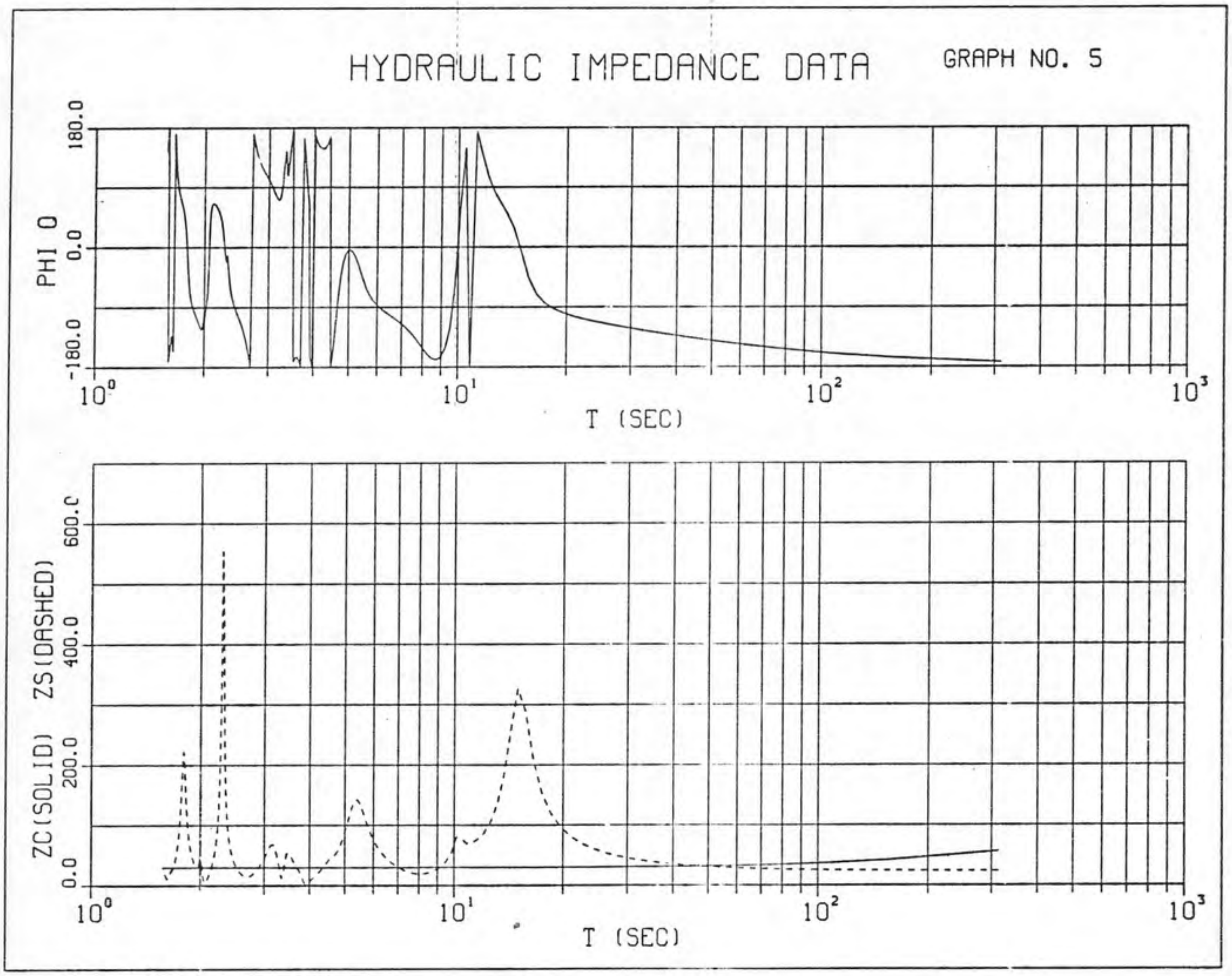
Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Downstream of Pressure Reducing Station 3

Figure 12



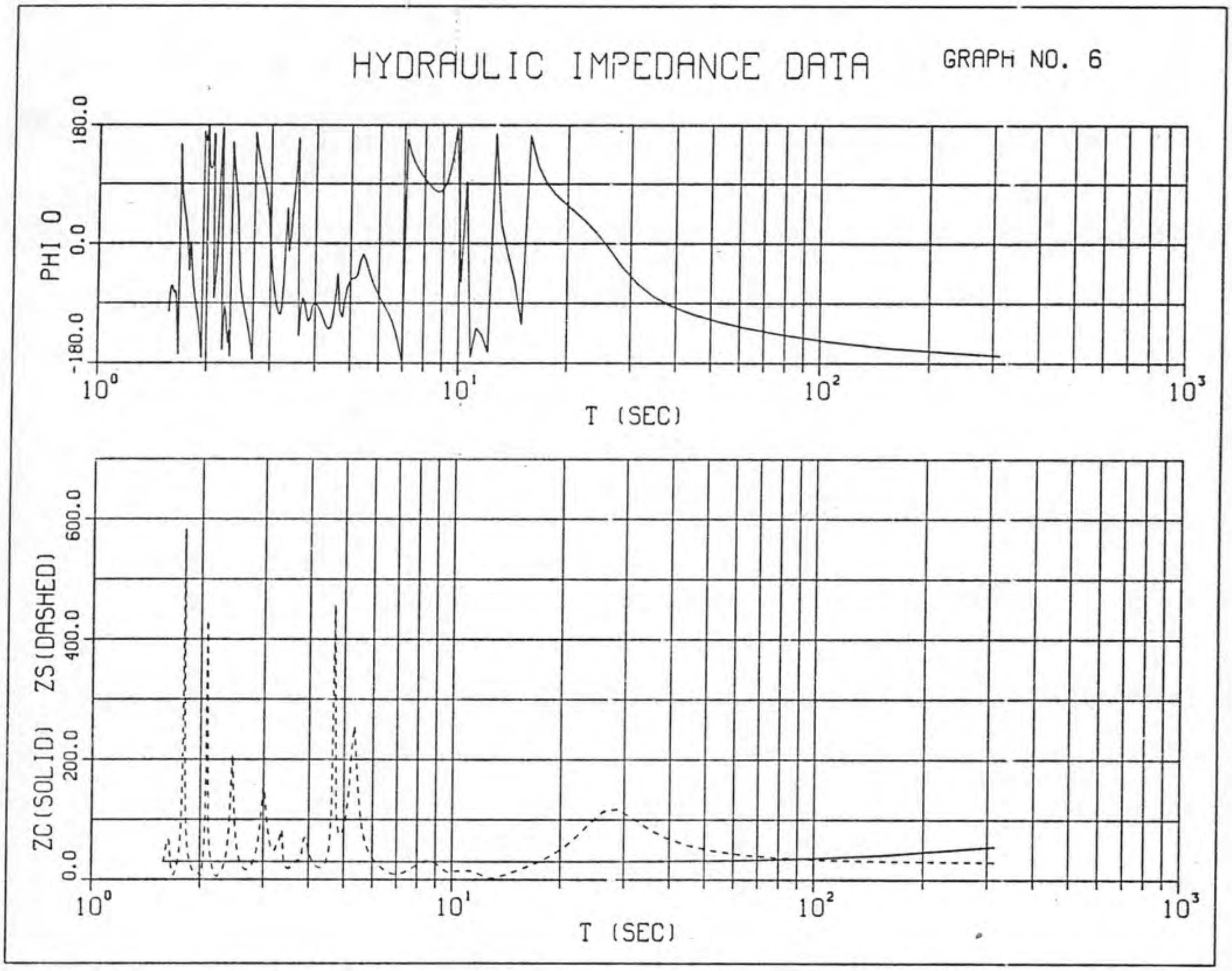
Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Upstream of Pressure Reducing Station 4

Figure 13



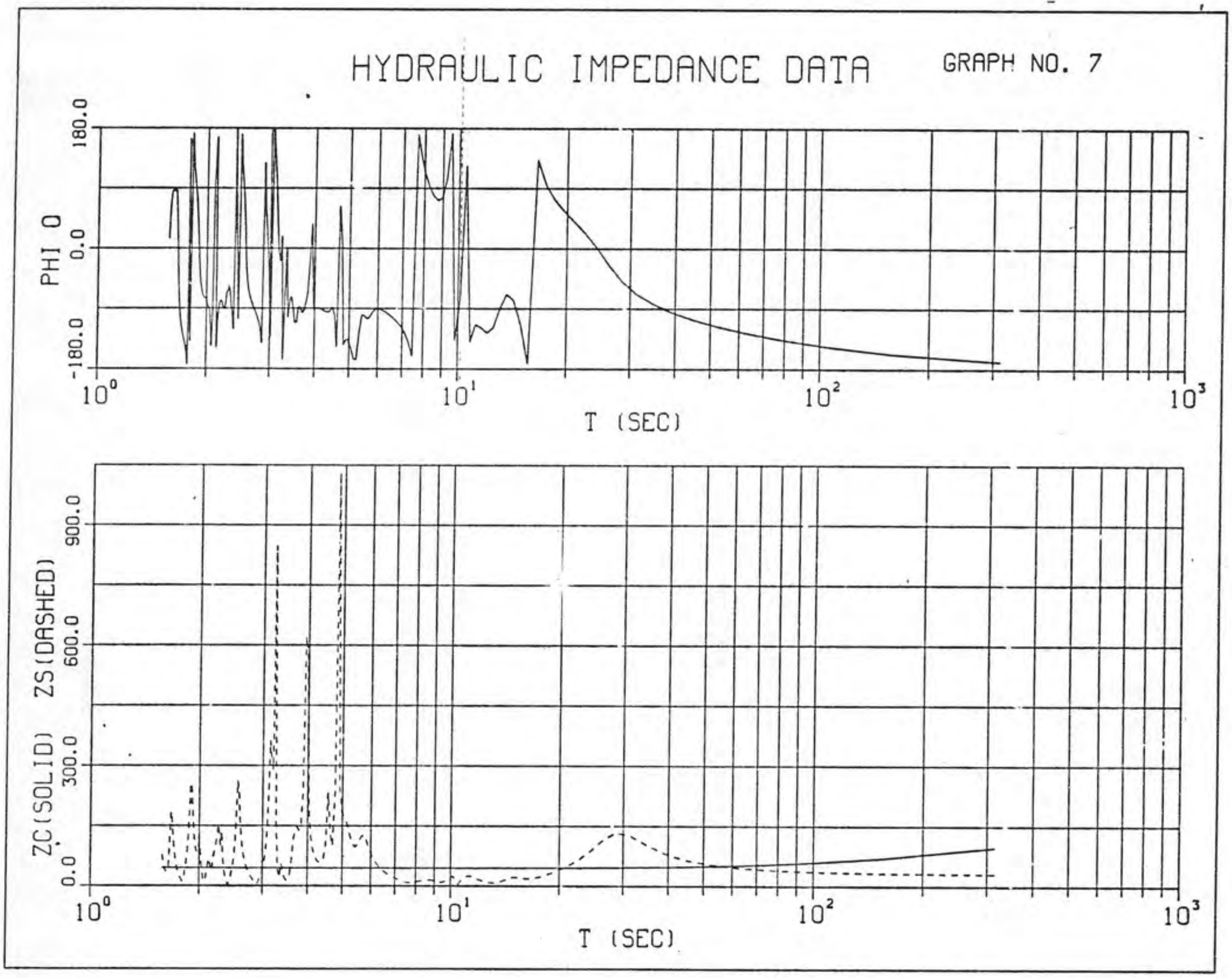
Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Downstream of Pressure Reducing Station 4

Figure 14



Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Change in Pipe Diameter Upstream of
Pressure Reducing Station 5

Figure 15



Impedance Diagram
Pleasant Oak Main
Design Discharge in Each Pipe Segment
Upstream of Pressure Reducing Station 5

APPENDIX B

ELECTRICAL EQUIVALENTS

INTRODUCTION

Very frequently, it is possible to transfer developments in one engineering field to another. One example of this is the methods that have been developed in the field of electrical engineering to investigate the response characteristics of circuits. If proper electrical equivalents can be found for hydraulic and mechanical devices, then the response characteristic of these hydraulic or mechanical systems can be determined by analyzing the equivalent electrical circuit. The electrical equivalent is frequently called the electrical analog.

Although electrical analogs have been derived for certain hydraulic and mechanical systems 1/ 2/, it is worthwhile to present in one location all of the derivations that apply to a pressure-reducing valve. The procedure to derive the electrical equivalents is as follows:

1. The unsteady equation of motion is written for the hydraulic or mechanical device. In this equation, the appropriate terms have been given a small perturbation of the form $dx = \delta \sin (\omega + \phi)$.

2. The steady-state equation is subtracted from the unsteady equation.

1/ Jewusiak, H., and Bigley, W. J., Mechanical Network Analysis, Machine Design, 1964.
2/ Braunagel, M. V., Electrical Analog for Open Canals, Paper prepared for Division of Design, Bureau of Reclamation, August 1968.

3. The resulting equation is rearranged into a form which corresponds with a functional description of an electrical element.

4. The coefficients of the resulting equation are equated with their corresponding electrical quantities.

After the electrical equivalents have been determined for each hydraulic and mechanical element, they are combined into an electrical circuit which describes the entire process. This circuit can then be simulated electrically or analyzed mathematically to determine the response characteristics of the system.

Some freedom is possible in developing a system of consistent analog quantities. For instance, for open channel flow Braunagel 2/ found it convenient to equate electrical voltage with the force acting on the cross sectional area of the canal. A different tack was taken in the equations which follow. For the derivations presented in this report, it is assumed that 1 coulomb of electricity is equivalent to a cubic meter of liquid. With this assumption, the following equivalents can be defined:

Electrical (SI)		Hydraulic/Mechanical (SI)	
Potential	(N.m/C,V)	Pressure intensity	(Pa)
Current	(C/s,A)	Discharge	(m^3/s)
Resistance	(N.m s/C^2,Ω)	Impedance*	(Pa.s/m^3)
Capacitance	(C^2/N.m,F)	Volume change per unit stress	(m^5/N)
Inductance	(N.m s^2/C^2,H)	Density per unit length	(kg/m^4)
Power	(N.m/s,W)	Power	(N.m/s)

* Note: This impedance is defined as pressure intensity divided by flow rate.

With these definitions of the electrical and hydraulic or mechanical analogs, it is possible to express the functional relationships of these equivalents as follows:

	Electrical	Hydraulic
Resistance	$\dfrac{E}{I}$	$\dfrac{p}{Q}$
Capacitance	$\dfrac{\int_0^t I\,dt}{E}$	$\dfrac{\int_0^t Q\,dt}{p}$
Inductance	$\dfrac{E}{dI/dt}$	$\dfrac{p}{dQ/dt}$

The sections which follow derive the electrical equivalents for the hydraulic elements which can occur on a pressure-reducing valve. These include a capillary tube, an air chamber, a needle valve, the pilot valve, an actuator, the main valve, and filters.

a. <u>Capillary Tube</u>. - The unsteady flow through a capillary tube is given by:

$$\frac{(V_1 + dV_1)^2}{2g} + \frac{(P_1 + dP_1)}{\gamma} = \frac{(V_2 + dV_2)^2}{2g} + \frac{(P_2 + dP)}{\gamma} + h_f + \frac{L}{g}\frac{dV}{dt}$$

where V = Velocity

P = Pressure

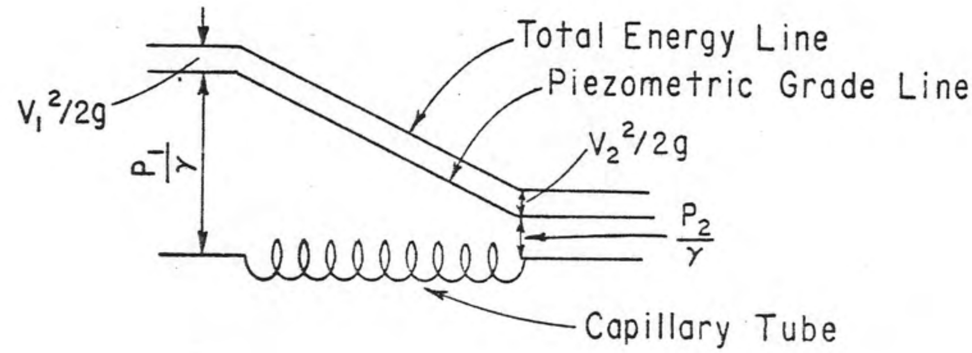
h_f = Friction loss

L = Length of tube

g = Acceleration of gravity

t = Time

The subscripts 1 and 2 refer to the conditions of each end of the tube, see definition sketch.



Definition Sketch

With laminar flow, the resistance is given by:

$$h_f = \frac{128 \nu L}{\pi g D^4} \left(Q + q \right)$$

For turbulent flow, the linearized resistance is:

$$h_f = \frac{f}{g D A} \, q$$

where f = Darcy Weisbach friction factor

ν = Kinematic viscosity of water

D = Inside diameter of capillary tube

Q = Steady-state flow rate

q = Sinusoidal flow rate

A = Cross sectional area of capillary tube

The steady-state equation for the flow is given by:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + \frac{128 \nu L}{\pi g D^4} Q$$

If compressible effects are small, then $V_1 = V_2$. Subtracting the steady flow equation from the unsteady equation gives:

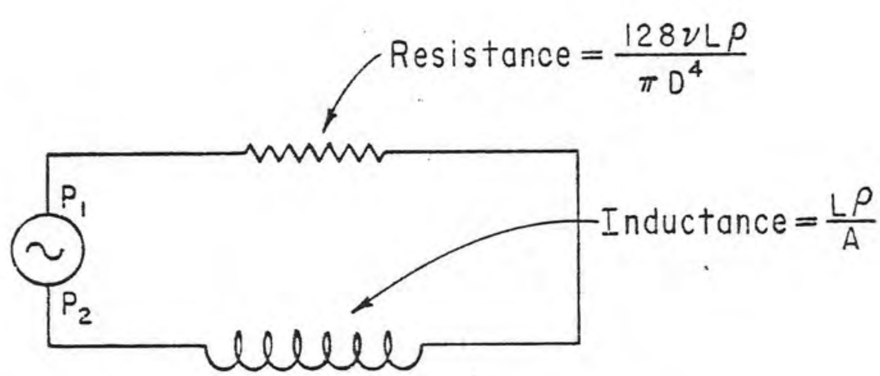
$$\frac{dp_1}{\gamma} - \frac{dp_2}{\gamma} = \frac{128\nu L}{\pi g D^4} \cdot q + \frac{L}{Ag} \cdot \frac{dq}{dt}$$

or

$$dp_1 - dp_2 = \frac{128\nu L\rho}{\pi D^4} \cdot q + \frac{L\rho}{A} \cdot \frac{dq}{dt}$$

where ρ = Density of water

From the equations of motion it is obvious that the electrical equivalent of the capillary tube is given by the following sketch:



Electrical Equivalent of a Capillary Tube

The ratio of resistance to inductance is given by:

$$\frac{\text{Resistance}}{\text{Impedance}} = \frac{32\nu}{D^2}$$

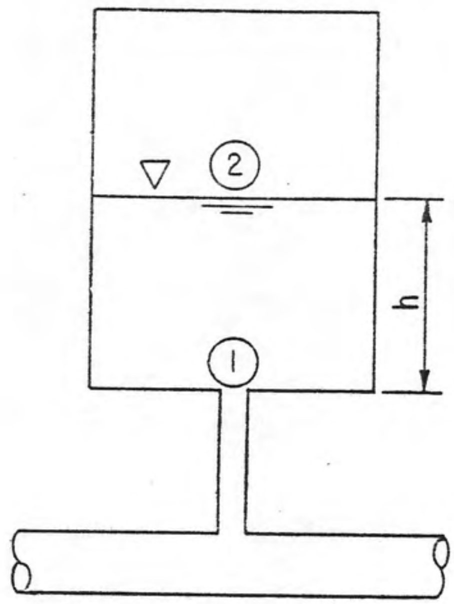
For typical capillary tubes, $\nu = 1.2\ mm^2/s$ and D = 2 mm. Thus, the magnitude of the resistance is about 10 times greater than the inductance.

b. Air Chamber. - The unsteady flow equation for the air chamber is written for the water prism in the chamber only, see definition sketch. If desired, the connecting tubing can be considered to respond as a capillary tube. With the rigid water column assumption and neglecting friction, the unsteady flow equation is:

$$\frac{p_1 + dp_1}{\gamma} = (h + dh) + \frac{p_2 + dp_2}{\gamma} + \frac{h}{g} \cdot \frac{dV}{dt}$$

where h = Initial steady-state depth.

All other variables have been previously defined.



Definition Sketch for Air Chamber

From the perfect gas law:

$$p_2 + dp_2 = \frac{P_2\, c}{c - A\,dh}$$

where c = Steady-state air volume in chamber

A = Cross sectional area of chamber

Solving this equation for dP_2 gives:

$$dp_2 = \frac{P_2\, A\; dh}{c\left(1 - A\; dh/c\right)}$$

Subtracting the steady-state flow equation, gives:

$$dp_1 - dp_2 = \gamma \, dh + \frac{P_2 \, A \, dh}{c\left(1 - A \, dh/c\right)} + \rho h \frac{dV}{dt}$$

With

$$dh = \frac{1}{A} \int_0^t q \, dt$$

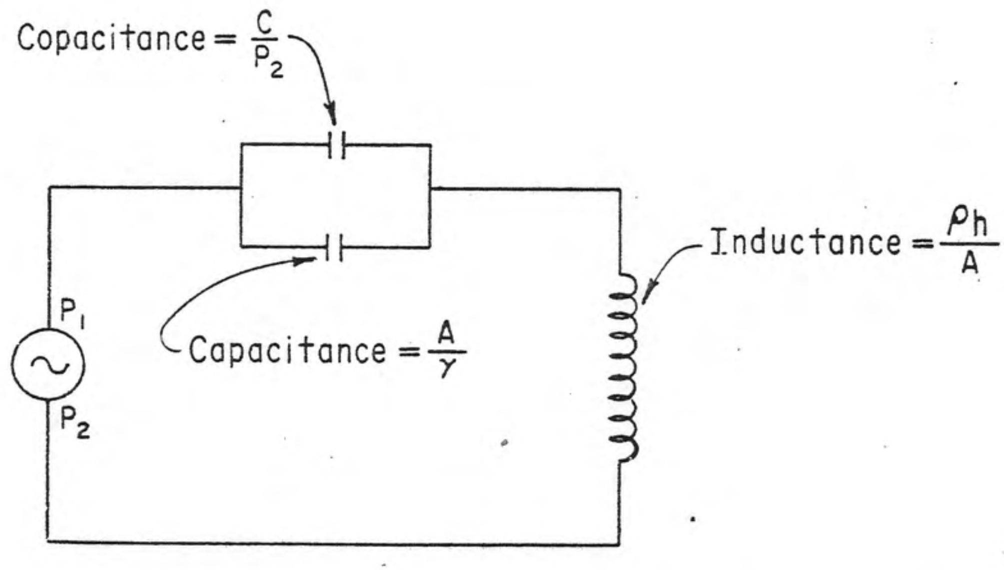
and

$$A \, dh \ll c$$

the equation becomes:

$$dp_1 - dp_2 = \left(\frac{P_2}{c} + \frac{\gamma}{A}\right) \int_0^t q \, dt + \frac{\rho h}{A} \cdot \frac{dq}{dt}$$

Referring to the equations of motion, the electrical equivalent of this relationship is drawn in the following sketch:



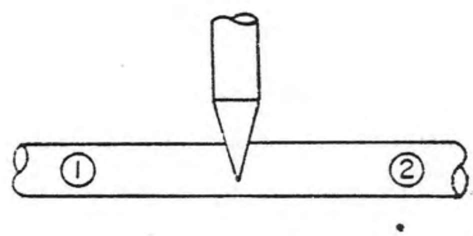
Electrical Equivalent of Air Chamber

c. Needle Valve. - The steady flow equation through a needle valve is given by:

$$Q = C_d S \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

where C_d = Needle valve discharge coefficient

S = Reference area



Definition Sketch

Giving this equation a small perturbation results in:

$$Q + q = (C_d + dC_d)(S + dS)\sqrt{\frac{2(H + dH)}{\rho}}$$

where $H = P_1 - P_2$

$dH = dP_1 - dP_2$

This can be linearized by assuming:

$$H + dH \approx H\left(1 + \frac{dH}{2H}\right)^2$$

With this substitution, one obtains:

$$Q + q = \left(C_d S + C_d\, dS + S dC_d + dC_d\, dS\right)\left(1 + \frac{dH}{2H}\right)\sqrt{\frac{2H}{\rho}}$$

By assuming SdC_d, $dC_d ds$, and $dsdH << 0$, the equation becomes:

$$Q + q = C_d S\sqrt{\frac{2H}{\rho}} + C_d\, dS\sqrt{\frac{2H}{\rho}} + C_d S\,\frac{dH}{2H}\sqrt{\frac{2H}{\rho}}$$

Subtracting the steady-state equation results in:

$$q = C_d S \sqrt{\frac{2H}{\rho}} \left(\frac{dS}{S} + \frac{dH}{2H} \right)$$

Letting $\tau = C_d S$ and $d\tau = C_d ds$ gives:

$$q = Q \left(\frac{d\tau}{\tau} + \frac{dH}{2H} \right)$$

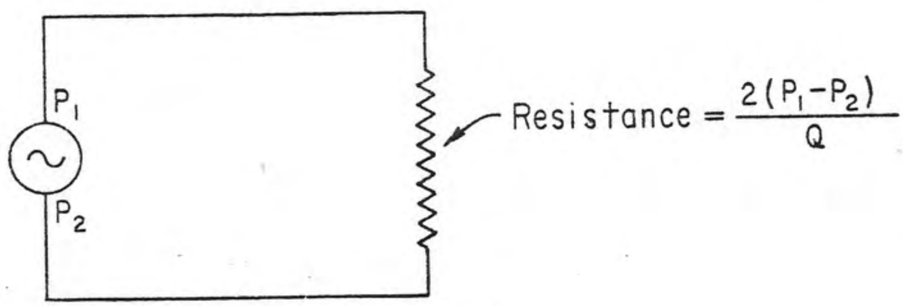
or solving for dH,

$$dp_1 - dp_2 = \frac{2H}{Q} \cdot q - 2H \frac{d\tau}{\tau}$$

If the needle valve is stationary,

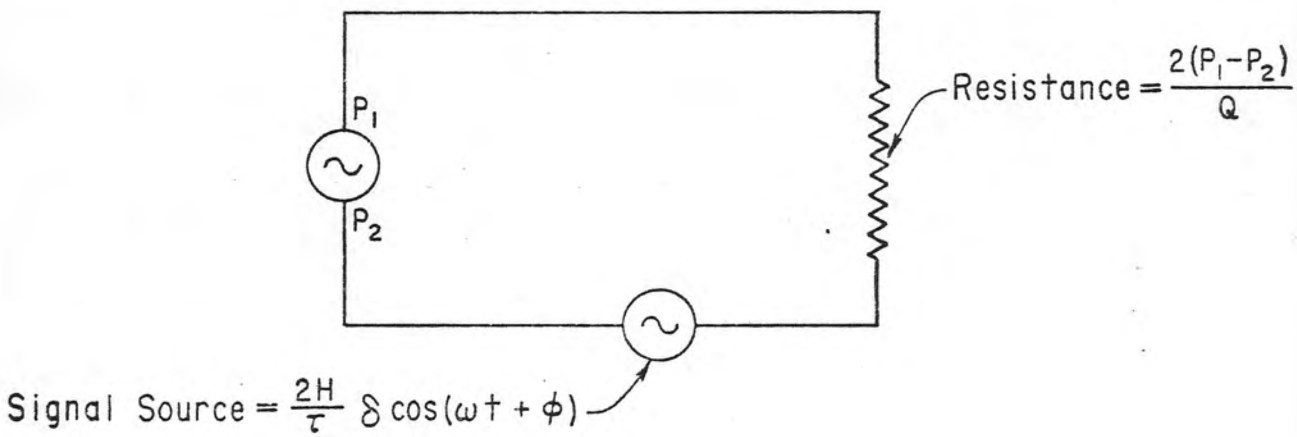
$$dp_1 - dp_2 = \frac{2(p_1 - p_2)}{Q} \cdot q$$

This has the following electrical equivalent:



Electrical Equivalent of a Fixed Needle Valve

The oscillating needle valve acts as a signal source with an arbitrary frequency. The amplitude of the T variation is δ, its rotational frequency is ω, and the phase angle is ϕ. The rotational frequency is not necessarily equal to the forcing frequency of the system.



Electrical Equivalent of an Oscillating Needle Valve

If C_d is not constant, then the equation of motion becomes:

$$dp_1 - dp_2 = \frac{2(p_1-p_2)}{Q}\cdot q - \frac{2(p_1-p_2)}{T}\cdot dT - \frac{2(p_1-p_2)}{C_d}\cdot dC_d$$

$$= \frac{2(p_1-p_2)}{Q}\cdot q - \frac{2(p_1-p_2)}{T}\,dT\left(1+\frac{T}{C_d}\cdot\frac{dC_d}{dT}\right)$$

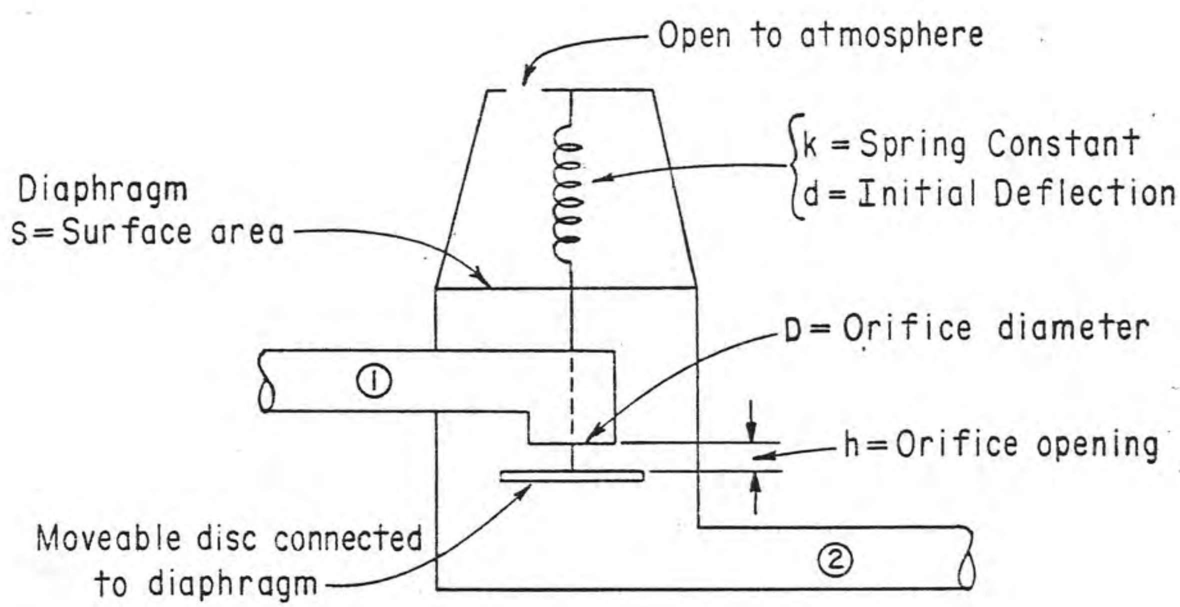
For this case, the electrical equivalent is still a signal source. However, the amplitude of the source is equal to:

$$\frac{2(p_1 - p_2)\delta}{T}(1 + Sm)$$

where m = Slope of C_d versus T curve at the mean valve opening

$$= \left(\frac{dC_d}{dT}\right)_T$$

d. Pilot Valve. - The general configuration of the pilot valve is shown in the definition sketch.



Pilot Valve Definition Sketch

The unsteady flow equation can be derived in the same form as the needle valve if the volume change per unit time in the pilot valve is small. Mathematically, the following condition must be satisfied:

$$S \frac{dh}{dt} < 0$$

In terms of the pilot valve parameters:

$$dp_1 - dp_2 = \frac{2(p_1 - p_2)}{Q} \cdot q - \frac{2(p_1 - p_2)}{\tau} d\tau \left(1 + \frac{\tau}{C_d} \frac{dC_d}{d\tau} \right)$$

where $\tau = C_D \pi Dh$

$d\tau = C_D \pi D dh$

or $$dp_1 - dp_2 = \frac{2(p_1 - p_2)}{Q} \cdot q - \frac{2(p_1 - p_2)}{h} dh \left(1 + \frac{h/D}{C_d} \cdot \frac{dC_d}{d(h/D)} \right)$$

From discharge coefficient curves of a representative pilot valve, the additive term in the parentheses was found to be:

$$\frac{h/D}{C_d} \cdot \frac{dC_d}{d(h/D)} = 0.6 \text{ to } 1.0$$

The motion of the movable disk can be estimated by applying Newton's equation to the mechanical parts of the valve. This is:

Summation of forces   Mass · Acceleration

$$\left(p_1 + dp_1 \right) S - p_{atm} S - k\left(d + dh\right) = M \frac{dV_d}{dt}$$

where P_{atm} = Atmospheric pressure

M = Mass of all moving parts

V_d = Velocity of diaphragm

Subtracting the steady-state equation gives:

$$S\, dp_2 - k\, dh = M \frac{dV_d}{dt}$$

Or

$$\frac{d^2(dh)}{dt^2} + \frac{k}{M} dh = \frac{S}{M} dp_2 \qquad (1)$$

The natural period of the pilot valve is given by:

$$T = 2\pi \sqrt{\frac{M}{k}}$$

If the period of the forcing function given by

$$dp_2 = \delta \sin(\omega t + \phi)$$

is equal to the natural period, the amplitude of the oscillations will grow until the valve alternates between fully open and fully closed. However, this condition is not likely to occur since typical values for the natural period are

$$T = 2\pi \sqrt{\frac{0.105 \text{ kg}}{51000 \text{ N/m}}} = 0.009 \text{ s}$$

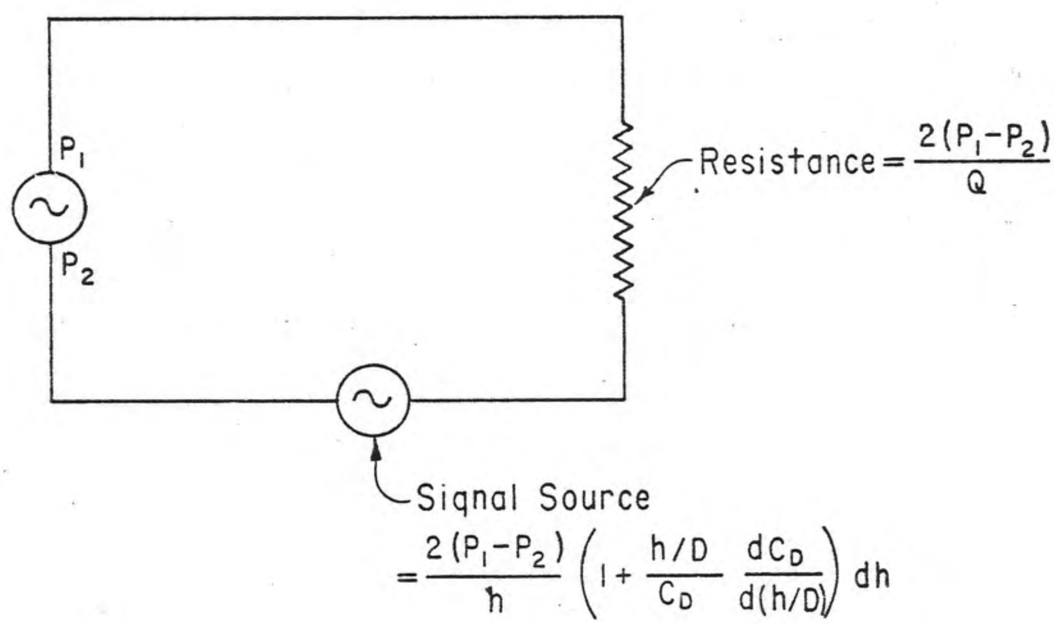
and typical periods for the forcing function are in the order of seconds.

The general solution of equation (1) is:

$$dh = \sqrt{\frac{S}{M}} \int_0^t \delta \sin(\omega t + \phi) \sin\left(\sqrt{\frac{k}{M}}(t-\lambda)\right) d\lambda$$

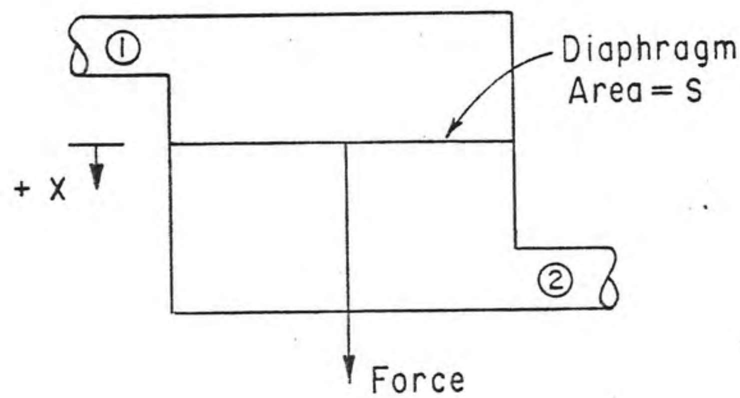
where $\delta \sin(\omega + \phi)$ is the variation of dP_2.

The electrical equivalent of the pilot valve is given in the following sketch:



Electrical Equivalent of Pilot Valve

e. <u>Actuator</u>. - A device used to transform a pressure difference into a force is called an actuator, see definition sketch.



Definition Sketch of an Actuator

From continuity

$$S\,dx = \int_0^t q_1\,dt = \int_0^t q_2\,dt$$

The application of Newton's equation gives:

$$S(p_1 + dp_1) - S(p_2 + dp_2) + F = M\,\frac{dV_d}{dt}$$

where M = Mass of moving parts

F = Resultant force

V_d = Velocity of diaphragm

Subtracting the steady-state conditions results in:

$$S(dp_1 - dp_2) = M \frac{dV_d}{dt}$$

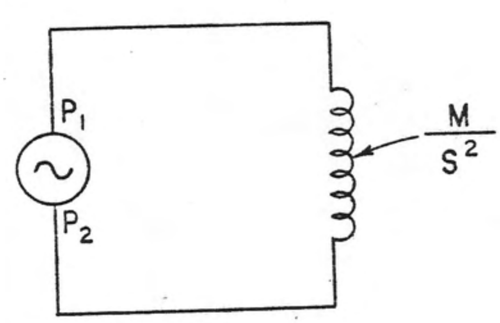
However,

$$S \frac{dV_d}{dt} = \frac{dq_1}{dt} = \frac{dq_2}{dt}$$

Thus,

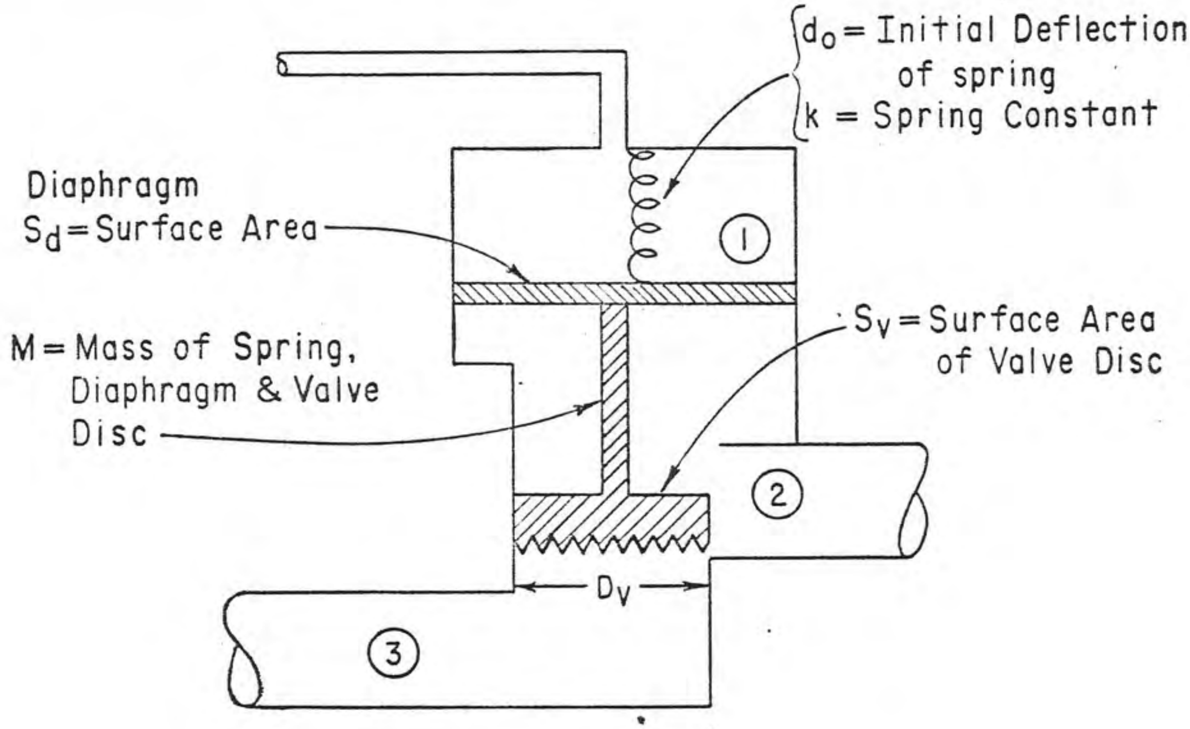
$$dp_1 - dp_2 = \frac{M}{S^2} \frac{dq}{dt}$$

The electrical equivalent of this equation is shown in the following sketch:



Electrical Equivalent of an Actuator

f. Main Valve. - The main valve is positioned by the pressure exerted on a diaphgram, see schematic.



Schematic of Main Valve

From Newton's equation:

$$S_d\left(p_2 + dp_2 - p_1 - dp_1\right) + S_v\left(p_3 + dp_3 - p_2 - dp_2\right) - k\left(d_0 + dx\right) =$$

$$M \frac{dV_d}{dt}$$

At equilibrium,

$$S_d\left(p_2 - p_1\right) + S_V\left(p_3 - p_2\right) - k\, d_o = 0$$

The unsteady equation is obtained by subtracting the equilibrium equation from Newton's equation. The result is:

$$S_d\, dp_2 - S_d\, dp_1 + S_V\, dp_3 - S_V\, dp_2 - k\, dx = M \frac{dV_d}{dt}$$

From continuity

$$dx = \frac{1}{S_d} \int_0^t q_1 \cdot dt$$

By definition,

$$\frac{dq_1}{dt} = \frac{d\left(S_d V_d\right)}{dt} = S_d \frac{dV_d}{dt}$$

With these substitutions, the unsteady equation becomes:

$$\left(1 - \frac{S_v}{S_d}\right) dp_2 - dp_1 + \frac{S_v}{S_d} dp_3 - \frac{k}{S_d^2} \int_0^t q_1\, dt = \frac{M}{S_d^2} \frac{dq_1}{dt}$$

If Sv$\approx$Sd, then

$$dp_3 - dp_1 = \frac{M}{S_d^2} \frac{dq_1}{dt} + \frac{k}{S_d^2} \int_0^t q_1\, dt$$

From the orifice equation which was derived under c. Needle Valve,

$$dp_3 - dp_2 = \frac{2H}{Q} \cdot q - 2H \frac{dT}{T}$$

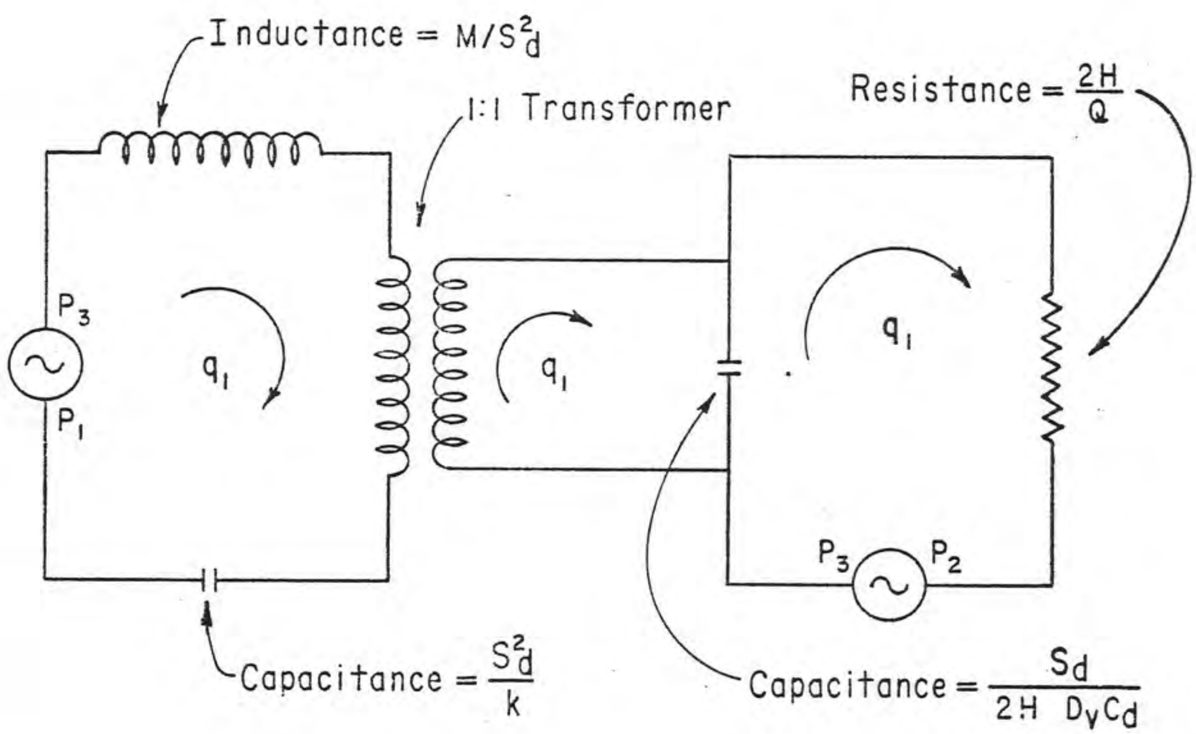
However, $d\tau = \pi D_v C_d Dx$

$$= \frac{\pi D_v C_d}{S_d} \int_0^t q_1\, dt$$

Thus,

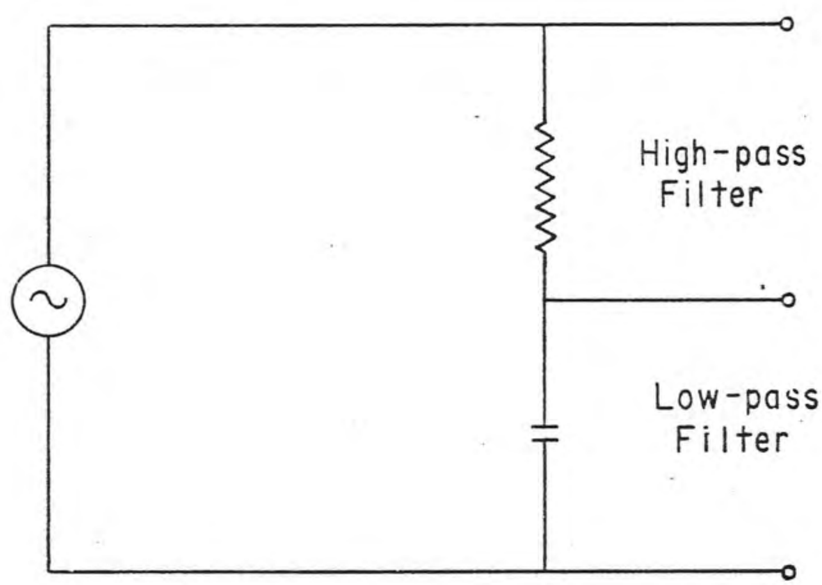
$$dp_3 - dp_2 = \frac{2H}{Q} \cdot q - \frac{2H\pi D_v C_d}{S_d} \int_0^t q_1 \, dt$$

The negative q_1 term in this equation indicates a coupling between the pressure in the valve dome P_1 and the main line pressures. The electrical analog for this coupling is a 1:1 transformer. Through the use of the transformer, the electrical analog of the valve is drawn in the following diagram:



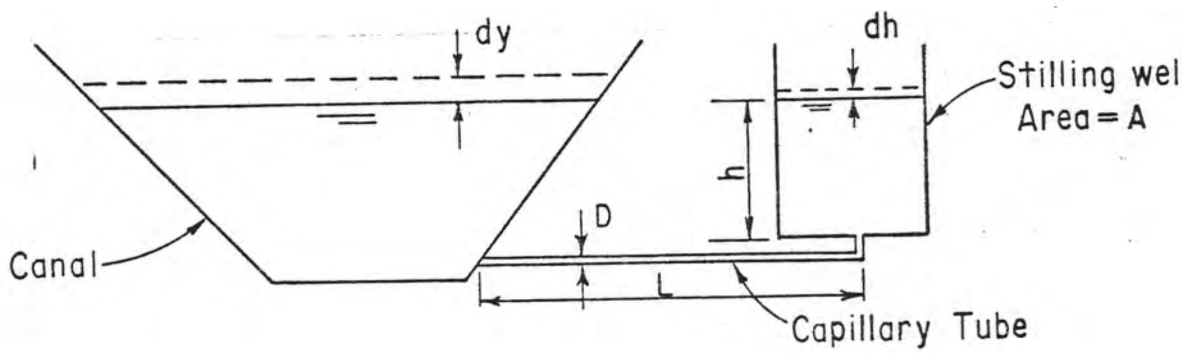
Electrical Analog of Main Valve

g. Filters. - A filter is a device which allows certain frequencies to pass while attenuating the amplitude of other frequencies. A high-pass filter attenuates low frequencies while not effecting the high frequencies. The reverse is true for a low-pass filter. Resistors, inductors, and capacitors are the usual filter elements. A simple circuit which contains both high- and low-pass elements is shown in the following sketch:



Filter Diagram for a Resistor and Capacitor in Series

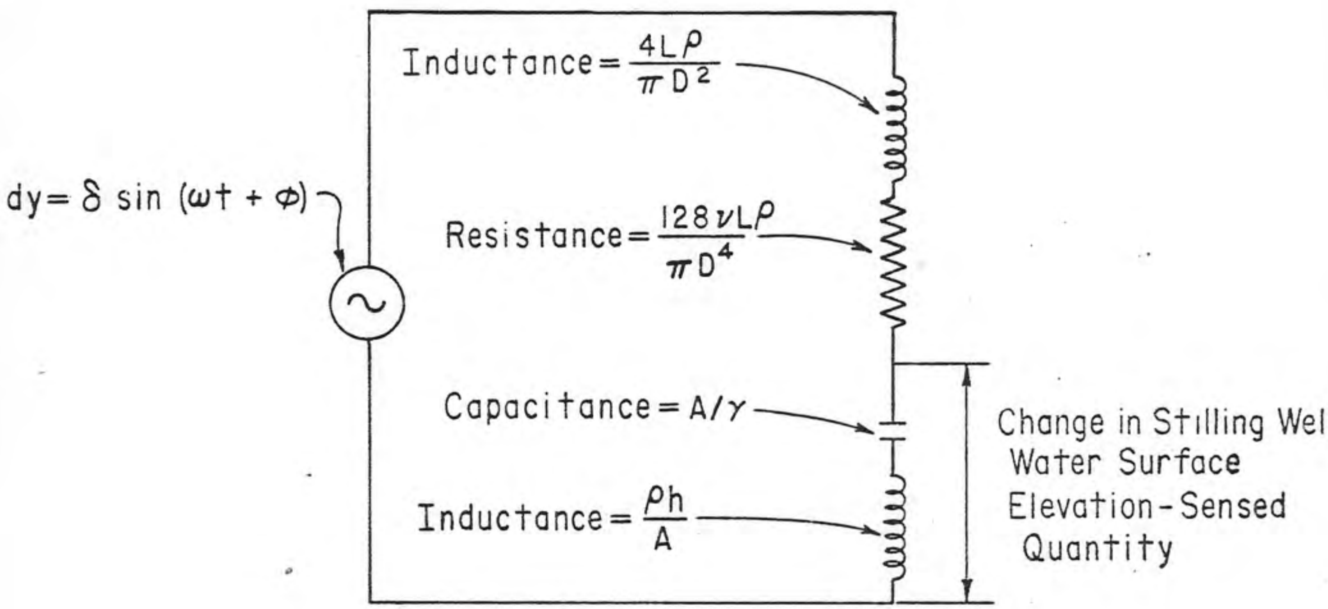
By proper selection of hydraulic elements, it is possible to construct hydraulic filters. Usually, air chambers or surge tanks and capillary tubes are used. A low-pass filter for use on canals has been conceived by Shand, 3/, see definition sketch.



Definition Sketch for Low-pass Filter
(Open-channel)

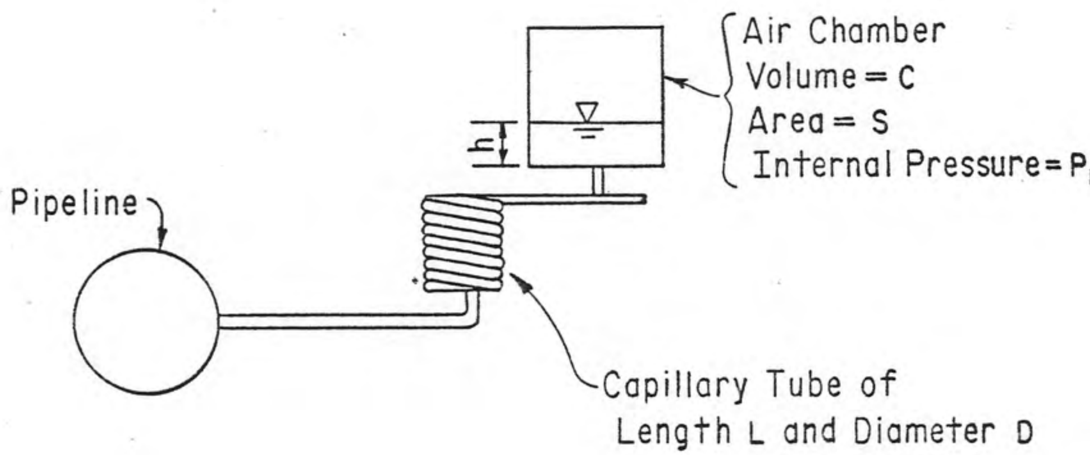
3/ Shand, M. J., Final Report, Automatic Downstream Control Systems for Irrigation Canals, Hydraulic Engineering Laboratory Report HEL-8-4, College of Engineering, University of California, Berkeley, August 1971.

For this case, a sensor detects changes in the water level of the stilling well. The electrical analog is given below. For h small and A large, the value of the stilling well inductance is negligible.

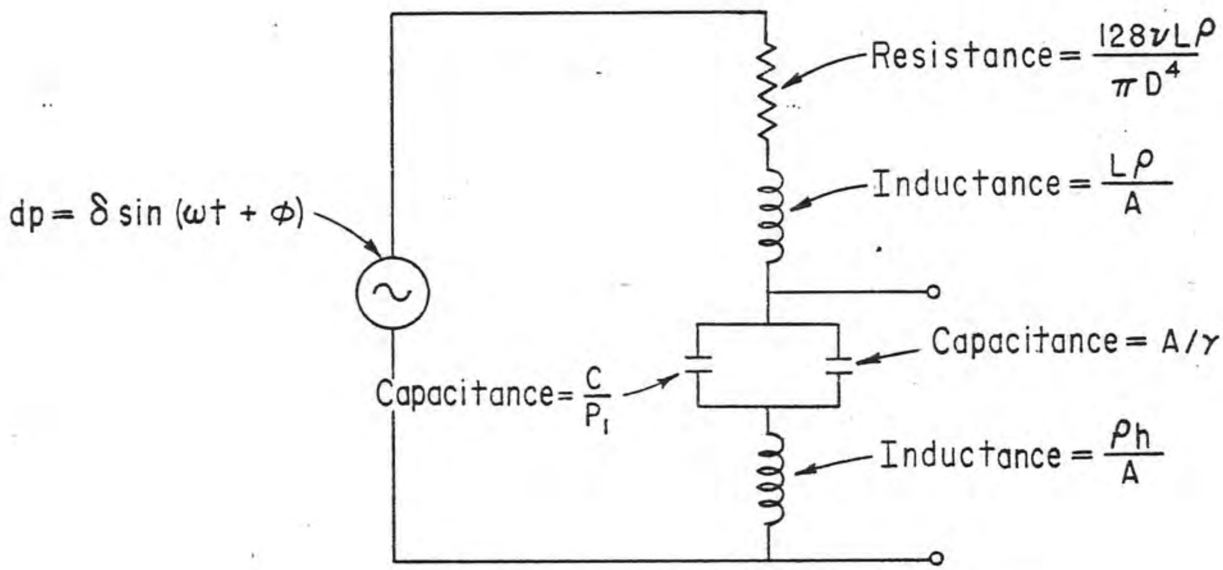


Electrical Equivalent of Open-channel Low-pass Filter

In a closed conduit, the configuration is similar to the open-channel example given above, see definition sketch. Except in this case, the stilling well is replaced with a closed air chamber. The electrical analog is given in the following diagram:



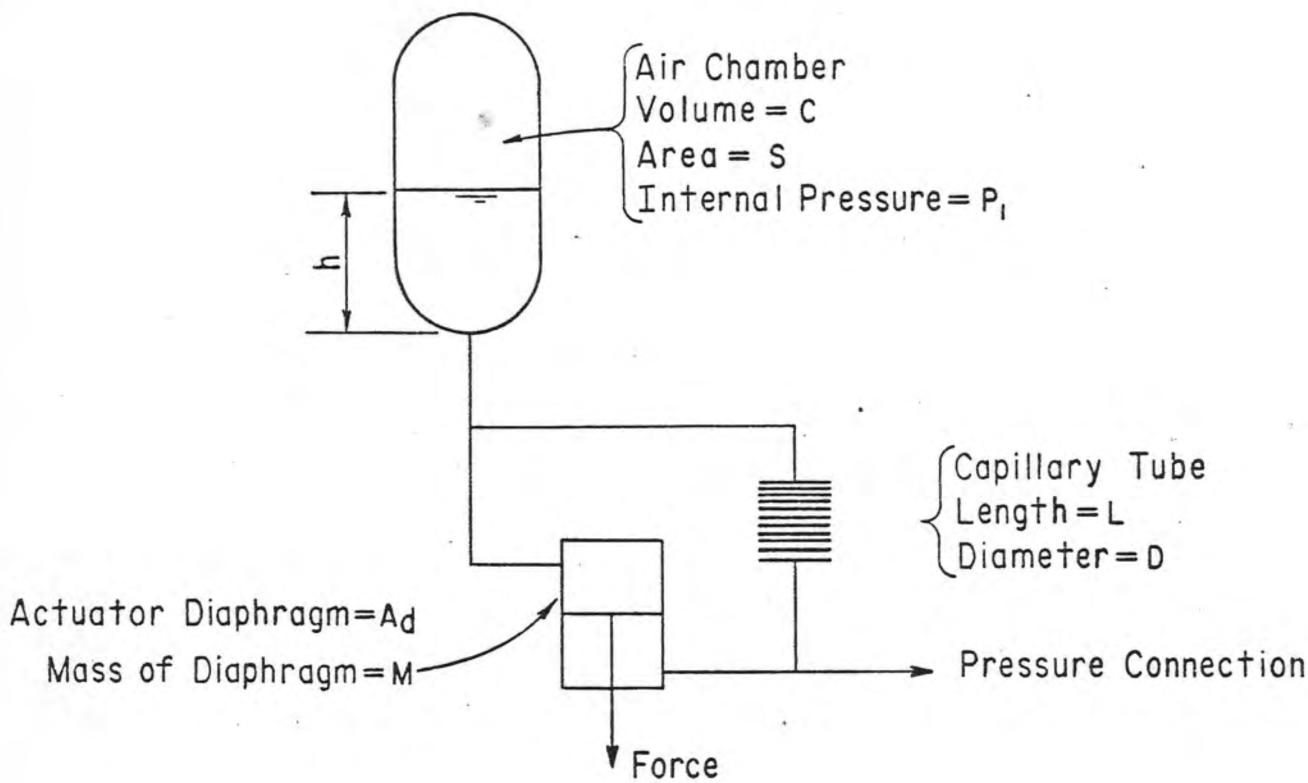
Definition Sketch for Low-pass Filter
(Closed Conduit)



Electrical Equivalent of Low-pass Filter
(Closed Conduit)

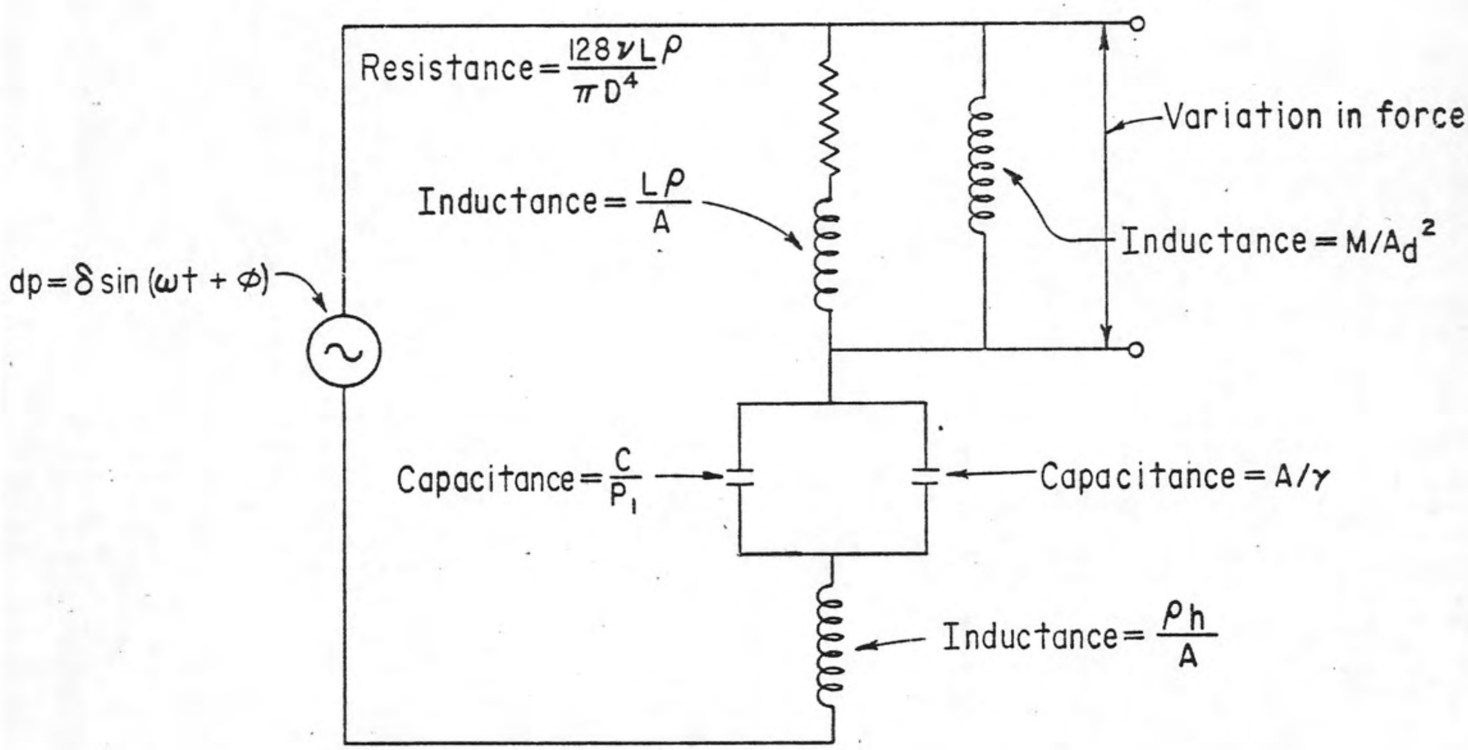
The big difference in this case is the addition of another capacitor in parallel to the air chamber. If the air chamber includes an inflatable bag, the value of the internal pressure P can be varied over large ranges while the volume C remains essentially constant. With an inflatable bag, the capacitor whose value is A/γ is no longer applicable.

A high-pass filter can be constructed by placing an actuator across the capillary tube. In this fashion, pressure differentials are transformed into forces, see definition sketch.



Definition Sketch of High-pass Filter

The electrical analog for the high-pass filter is shown in the following diagram:



Electrical Equivalent of High-pass Filter