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Memorandum

HYDRAULICS BRANCH
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Memorandum

Denver, Colorado

TO : Open and Closed Conduit Systems Committee

DATE: February 23, 1977

FROM : Chief, Hydraulics Branch

SUBJECT: Research into Water Surface Profiles at Grade Changes in Steep Canal Laterals

In compliance with our agreement, Mr. Perry L. Johnson has undertaken and completed the subject research. The work was done in the Hydraulics Branch of the Division of General Research.

The purpose of the study was to develop sufficient data describing the water surface to allow the evaluation of required lining heights for steep chute canal laterals. Of particular concern in this study were water surface profiles at locations in the chute where grade changes occur. It was felt that at these stations, the potential exists for the formation of rough water which could overtop the lining. Overtopping could result in failure of the lateral. At present, the linings are arbitrarily increased at these critical stations. Arbitrary increases may result in over design and, thus, added cost. Likewise, if potential water surfaces are not clearly understood, it is possible situations may develop where arbitrary lining increases are not adequate. Results from these studies could, therefore, yield less expensive and yet better designed linings.

In the past, to transport water over land exceeding maximum design slope for normal canals, the Bureau has provided near level canal sections with drop structures to dissipate the excess energy. This results in canal sections with relatively large cuts and fills. Not only are such sections more expensive to build, but they also result in the placing of concrete lining in deep cuts where ground water and freezing can combine to break up or displace the lining. The deep cuts also often drift in with snow or weeds which result in increased O&M costs. These problems would be reduced if canal grades were allowed to follow or be slightly above the ground surface. Depending on the terrain, such canals may develop flow velocities of up to 15 ft/s. These canals will be referred to as steep chute laterals. The changing slopes of such laterals result in changing flow velocities. It is quite likely that both supercritical and subcritical flow conditions would occur in the same lateral. The opportunity, therefore, exists for acceleration of flows from subcritical to supercritical or from supercritical to faster supercritical and for deceleration of flows from supercritical to subcritical with a hydraulic jump or from supercritical to slower supercritical. Relating to this situation, the following questions are to be resolved in the study:

1. What unstable flow conditions can be expected from such flow accelerations and decelerations?

2. What lining freeboard is required to contain these flows?

3. Would vertical curve transitions result in smoother flow conditions and, if so, what lengths of curve would be required?

4. What magnitude of vertical slope changes can be used while still maintaining satisfactory flow conditions?

As the study progressed, it was found that existing literature contained most of the desired information. However, a portion of the data found was collected in studies of somewhat different flow conditions. Data were obtained from a hydraulic model to augment and verify the data found in the literature.

A two-dimensional hydraulic model was used. The model was basically a 6-inch-wide, 11-foot-long, rectangular cross-section flume (figure 1). The first 6 feet of the floor could be set at any desired slope, the last 5 feet could then be independently set at any other desired slope. The walls of the flume were made of clear plastic which allowed visual observation of the water surface profile. Flow was supplied to the flume from a head box through a 6-inch by 8-inch slide gate. The head box could establish enough head to create flume flow velocities up to 16 ft/s.

The model represented only rectangular channel flow conditions and centerline flow conditions for trapezoidal channels. Flow conditions at the side walls of trapezoidal channels were not evaluated. Thus the effects of irregularities or breaks in the side walls were not evaluated. There is always a very real potential for side irregularities to cause the formation of large waves in high-velocity flows. Therefore, it is desirable to maintain hydraulically clean flow lines along the side walls if at all possible.

Flow acceleration. - Flow accelerations, both from subcritical to supercritical and from supercritical to faster supercritical were found to have stable, smooth, water surface profiles. These profiles conform to computable water surface drawdown curves. Standard lining freeboard should be adequate to contain these flows. Procedures for computation of water surface profiles are typically presented in King and Brater (pages 8-39 to 8-44) (ref. 1, Bibliography), and Chow (pages 217 to 326) (ref. 2).

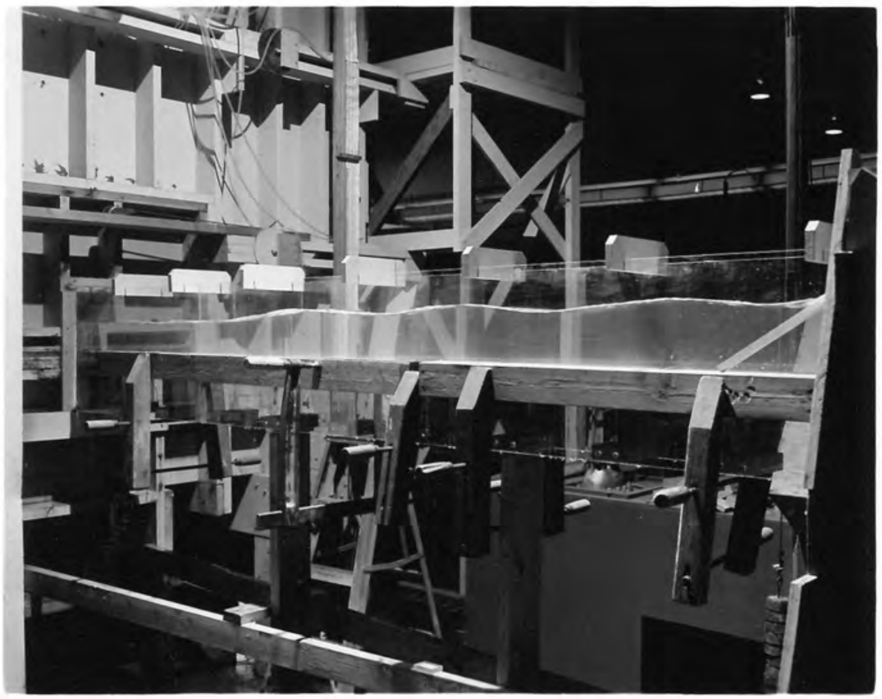

Figure 1   Two dimensional Hydraulic Model

Although ideally increases in grade should follow curves no steeper than those defined by the free trajectory curve of the flow, for the flow velocities considered in this study small angle abrupt grade changes are acceptable. Grade changes of a few degrees at expansion joints should cause no problems. If the flow surface would drop below the free trajectory curve, not only would negative pressure result on the flow surface, but the water would also tend to lift away from the surface. A somewhat unstable condition would thus result which would tend to increase the possibility of sidewall overtopping.

Flow deceleration. - Flow deceleration from supercritical to slower supercritical was likewise found to have stable, smooth, water surface profiles for the flow conditions considered. These profiles can again be calculated using a gradually varied flow analysis. Although grade changes may be quite abrupt, as long as the flow remains supercritical, flow deceleration is gradual. This flow deceleration results when frictional energy losses are greater than the component of gravity that drives the flow. This condition occurs when stabilized flow, moving down a steep slope, passes into a reach of lateral which is at a flatter but still supercritical slope. On the steeper reach the flow has a higher velocity and a higher friction loss. Initially, when the flow passes onto the flatter reach, where the component of gravity that drives the flow is reduced, the higher flow velocity and friction loss still exist. The friction losses, therefore, dominate over the driving component of gravity and the flow starts to slow. As the flow slows the friction losses reduce. This continues until friction losses equal the driving gravitational component and the flow stabilizes. Instantaneous grade changes are acceptable, but gradual changes in grade can be expected to result in more stable flow transitions.

On the other hand, if the flow should enter a reach of lateral which is not steep enough to maintain supercritical flow or which has a high water surface due to a downstream condition, a hydraulic jump with its rough water surface will occur. Depending on the Froude number of the flow entering the hydraulic jump, the jump may take several forms. These forms are shown in Chow (page 395) and "Engineering Monograph 25," (pages 15-16) (ref. 3). Considerable literature exists for predicting the configuration of hydraulic jumps formed for flows with initial Froude numbers greater than 1.7. Such information can be found in most fluid mechanics, hydraulics, and open channel hydraulics texts, as well as in most hydraulic handbooks. On the other hand, very little literature exists for predicting the configuration of the jump formed when the inflow has a Froude number between 1 and 1.7. This is a common range of Froude numbers encountered in steep chute canal laterals.

A literature search was undertaken to find information describing hydraulic jumps formed for initial flows with Froude numbers ranging from 1 to 1.7. Several references were found; the best of which describe surging flows (or moving hydraulic jumps) and not the stationary hydraulic jumps which are of concern in this memorandum. Although surging is a somewhat different phenomena than stationary hydraulic jumps, it appears that the data found in these studies may still be used to predict resulting wave heights. To verify the applicability of these data, Johnson did limited work with the hydraulic model. He found that the findings in the literature were representative and that computed water heights should be equal to or greater than what would actually occur.

The results of these literature studies are summarized in the graphs of figures 2, 3, and 4. Combined, these studies considered wide rectangular channels and trapezoidal channels with both varying bottom widths and varying side slopes. Thus, the graphs allow evaluation of flow in trapezoidal channels with bottom widths ranging from zero to infinity and with side slopes ranging from vertical to 3 to 1. It was found that in trapezoidal channels, the resulting waves are higher at the bank than they are in mid-channel. The graphs allow evaluation of wave heights at both positions.

Data from three studies are summarized in figure 2. The Lemoine (ref 4.) data and the Preissmann and Cunge (ref. 4) data were theoretically obtained while the Witting (ref. 5) data were experimentally obtained. Preissmann and Cunge considered trapezoidal channels while Lemoine and Witting both dealt with wide rectangular channels. Agreement between the studies appears to be good. Figure 2 predicts the height of waves that result from the hydraulic jumps. Parameters used in figure 2 are:

- a - The height of the initial and greatest wave of the hydraulic jump above the mean downstream flow depth (the subscript m indicates mid-channel, the subscript b indicates bank)
- D_1 - The mean depth of flow upstream of the hydraulic jump
- D_2 - The mean depth of flow downstream of the hydraulic jump
- B - The width of the channel bottom

These parameters are also defined in the drawings included with the graph.

In observing figure 2, it should be noted that the Lemoine analysis and the Preissmann-Cunge analysis both end at a D_2/D_1 value of 1.28. Findings indicate that at this value (which corresponds to a Froude number of 1.23 prior to the jump) the waves formed by the jump will start to break. For higher D_2/D_1 values, and thus for higher initial Froude numbers, breaking becomes more dominant and results in a reduction of the height of waves formed by the jump. This can be seen in the data of Witting which show an increase in wave height for D_2/D_1 values of from 1 to 1.28 and a decrease in wave height for values above 1.28.

The curves, as labeled, yield mid-channel and bank wave heights for trapezoidal channels as well as wave heights for wide rectangular chan-

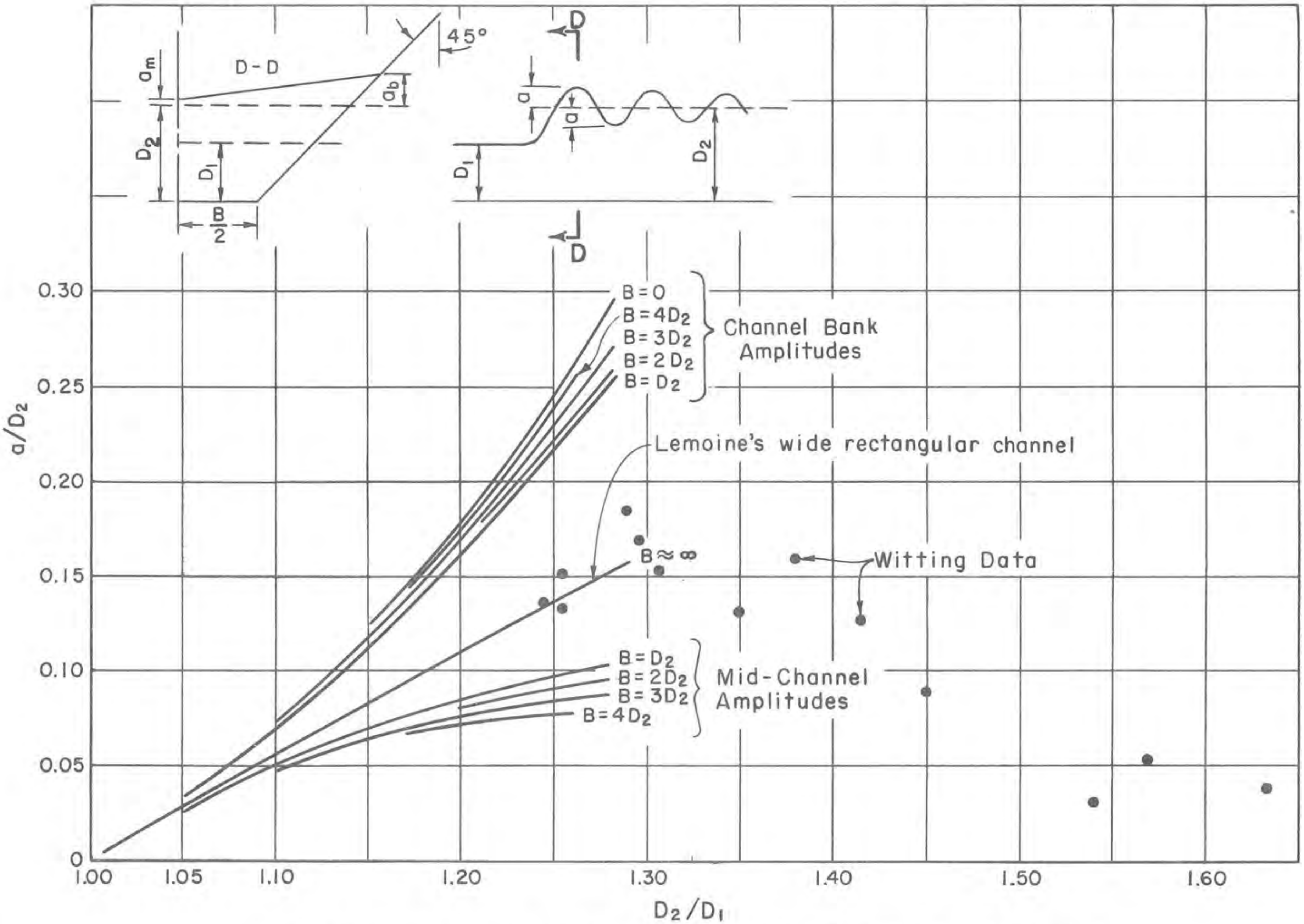

FIGURE 2 WAVE AMPLITUDE PREDICTION DIAGRAM

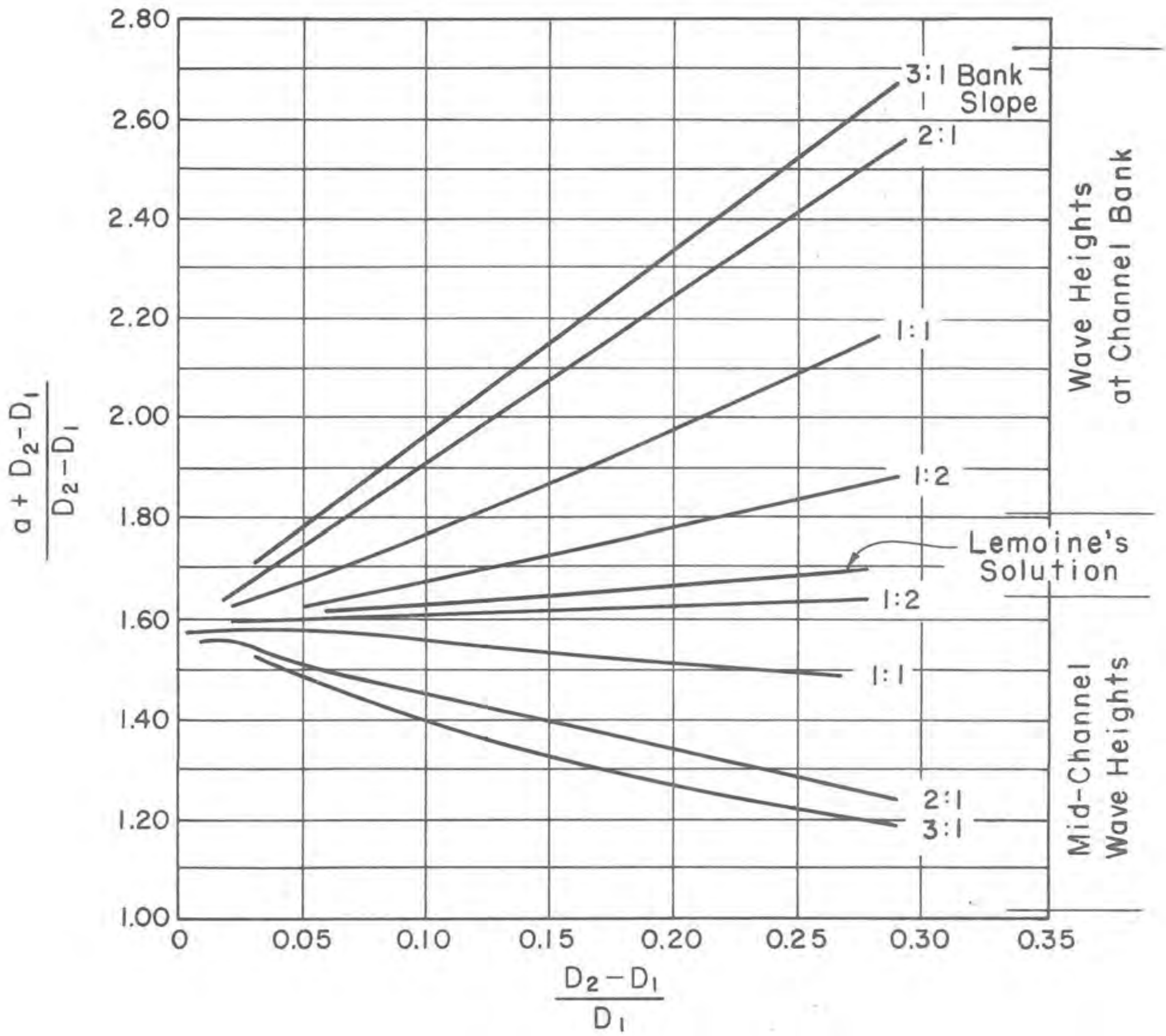

FIGURE 3 CHANNEL SIDE SLOPE INFLUENCE DIAGRAM

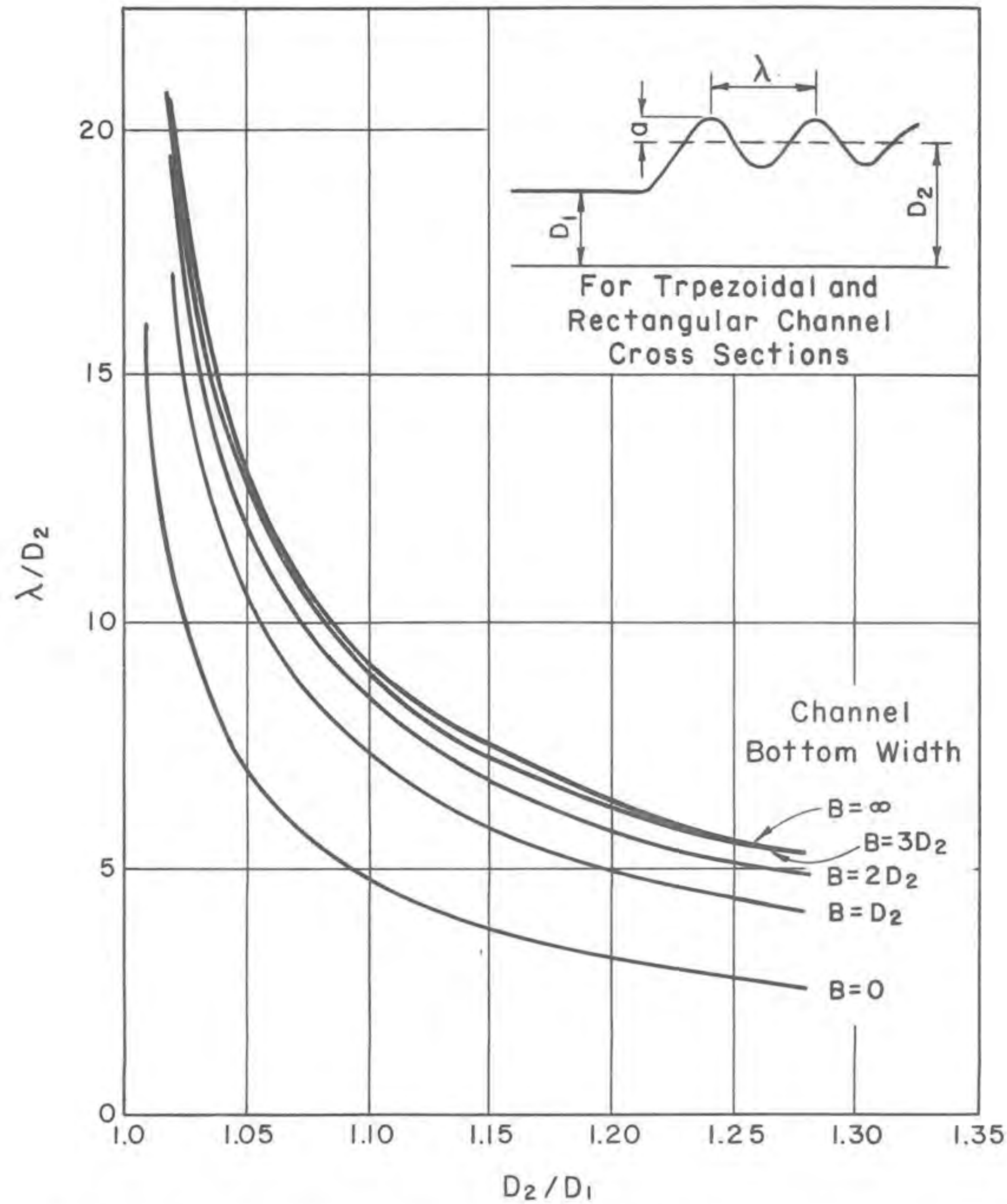

FIGURE 4 WAVE LENGTH PREDICTION DIAGRAM

nels. Only trapezoidal channels with 1 to 1 side slopes are considered in figure 2. As indicated, trapezoidal channel bottom widths ranging from zero to infinity can be analyzed.

Figure 3 allows evaluation of the effect of trapezoidal bank side slope on the height of waves formed. Side slopes considered range from vertical walls (Lemoine's rectangular channel) to a 3 to 1 (horizontal to vertical) slope. Again, both mid-channel and bank wave heights can be evaluated. Figure 3 applies only to trapezoidal sections whose bottom width is equal to the average water surface depth downstream of the jump (D_2). Thus, figure 3 cannot be directly applied to channels with bottom widths other than D_2. Relative data which may be used to obtain approximate wave height predictions can, however, be obtained by using figures 2 and 3 together. All of the information contained in figure 3 is derived from the theoretical analyses of Lemoine, Preissmann, and Cunge.

Figure 4, which contains only theoretical data of Preissman and Cunge, can be used to predict the wave lengths of the undular waves created by the weak hydraulic jumps. Limited observations indicate that the resulting wave action will dampen out over a distance of 10 wave lengths or less. Thus, figure 4 can be used to approximately predict a maximum length for the wave train.

Prediction of the location at which the hydraulic jump forms was also considered. Methods to predict such locations are again extensively covered in existing hydraulics and fluid mechanics literature. Representative discussions of the subject can be found in Chow (pages 399 to 404) and in King and Brater (pages 8-27 to 8-29). The methods basically consist of evaluating the conjugate depth for the supercritical flow upstream of the jump and then finding the location at which the water surface downstream of the jump is exactly this depth. At this location, the jump will form. Backwater curve calculations are generally required to determine the jump location. Typical calculations predicting jump location as well as resulting wave heights are presented in the following example.

Sample calculations. - If a steep chute lateral with the following properties passes from a supercritical slope of 0.0075 to a subcritical slope of 0.0008, what freeboard is required to contain the hydraulic jump, at what position will the hydraulic jump occur, and what will be the length of the wave train?

Discharge	$= Q = 15\ ft^3/s$
Manning's n	= 0.015
Bottom width	= B = 2 ft
Channel side slope	= 1 to 1

First, to evaluate D_1 (the depth of flow upstream of the hydraulic jump), the steady uniform flow on the 0.0075 slope(s) is studied. From King and Brater (page 7-14), we know that,

$$Q = \frac{K' \, B^{8/3} \, S^{1/2}}{n} \tag{1}$$

where K', is a tabulated constant which is dependent on channel side slope, K' is defined as,

$$K' = \frac{1.486 \, X^{8/3} \, (Z + 1/x)^{5/3}}{[1/x + 2 \, (Z^2 + 1)^{1/2}]^{2/3}} \tag{2}$$

where,

$$X = \frac{D}{B} \text{ and } Z = \frac{e}{D}$$

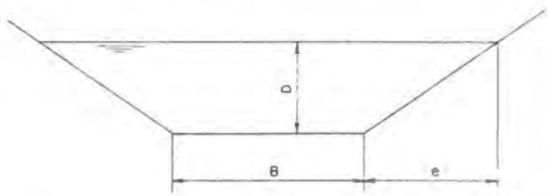

FIGURE 5 TRAPEZOIDAL CHANNEL DIMENSIONS

Using equation (1)

$$15 = \frac{K' \, (2)^{8/3} \, (0.0075)^{1/2}}{(0.015)}$$

$$K' = 0.409$$

solving equation 2 for D_1 (D in this reach of the lateral) yields, $D_1 = 0.884$ feet.

In a similar manner, the steady flow depth in the channel with the 0.0008 slope is determined. Again, using equation 1,

$$15 = \frac{K' \, (2)^{8/3} \, (0.0008)^{1/2}}{(0.015)}$$

$$K' = 1.253$$

solving for X yields, X = 0.804, and a flow depth of 1.608 feet.

If the critical depth is then evaluated for the channel, it is found that D_c = 1.110 feet. Comparing flow depths, it is found that the flow on the 0.0075 slope is supercritical and the flow on the 0.0008 slope is subcritical. Thus, a hydraulic jump occurs. The conjugate depth (D_2) for the 0.0075 slope is then calculated to be 1.138 feet.

Using figure 2, the maximum wave heights produced can then be calculated.

$$D_2/D_1 = 1.138/0.884 = 1.287$$

$$B/D_2 = 2/1.138 = 1.757$$

So, at the bank a/D_2 = 0.260 or a = 0.296 feet.

Thus, wave heights of up to 0.30 feet above the average water surface can be expected to occur at the bank in the hydraulic jump.

To locate the position of the hydraulic jump, the point where the downstream water surface crosses the conjugate depth line is found (figure 6). This requires the calculation of a S_1 backwater curve for the tailwater surface.

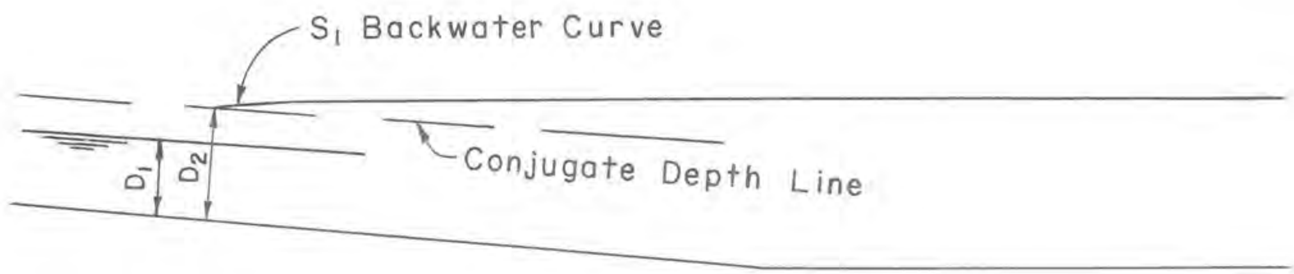

FIGURE 6 BACKWATER CURVE SCHEMATIC

Assuming the flow depth at the station of grade change to be 1.608 feet, the following backwater curve is calculated with the use of King and Brater (ref. 1).

V	D	$\frac{V^2}{2g} + D$	$\overline{D}/B$	$(1/K')^2$	S_{av}	Δ_1
2.585	1.608	1.712				
			0.777	0.728	0.000914	12.9
2.857	1.500	1.627				
			0.725	0.951	0.001194	11.6
3.151	1.400	1.554				
			0.675	1.245	0.001563	10.8
3.497	1.300	1.490				
			0.625	1.67	0.00210	9.8
3.906	1.200	1.437				
			0.585	2.14	0.00269	5.2
4.201	1.138	1.412				
					Total	50.3 ft

So, the jump would start approximately 50 feet upstream of the grade change.

Figure 4 shows that the wave length for the example hydraulic jump is:

$$\lambda = (1.138)(4.5) = 5.1 \text{ feet}$$

Thus, the maximum probable length of the wave train is (10) x (5.1) or 51 feet. The jump would start at a station 50 feet upstream of the grade change and within 2.6 ($\lambda/2$) feet would result in a standing wave crest whose maximum height at the bank is 0.30 foot. It can be expected that all significant wave action will dampen out by the station of grade change.

The importance of the assumed Manning's n value should be noted. Evaluation of D_1, D_2, λ, and jump position are all dependent on the value of n. The effect of n is significant with three thousandths variation in its value possibly resulting in $\pm$50 feet variation in the predicted location of the hydraulic jump and $\pm$0.2 foot variation in the predicted wave heights resulting from the hydraulic jump. For any specific situation, computations should be carried out using the highest and lowest n values considered possible. Such computations will yield a range which contains all possible water surfaces and all possible jump locations.

A second example follows which shows how to work with channels which have side slopes other than 1:1. Assume again that a reach of lateral with a slope of 0.0075 enters a reach of lateral with a slope of 0.0008. Also assume:

Discharge	=	15 ft^3/s
Manning's n	=	0.015
Channel side slope	=	2:1 (horizontal to vertical)

In the following computations, the maximum wave height is evaluated. Calculations to predict jump location and jump length follow the same procedure outlined in the first example. First D_1 is evaluated.

$$Q = \frac{K' B^{8/3} S^{1/2}}{n} = 15 = \frac{K' 2^{8/3} 0.0075^{1/2}}{0.015}$$

$$K' = 0.4092$$

This yields X = 0.388 and $D_1 = 0.776$. In a similar manner, the stabilized flow depth in the reach of channel with the slope of 0.0008 is calculated to be 1.34 feet. Evaluation of critical depth yields $D_c = 0.842$ feet. Thus, flow in the 0.0075 reach is supercritical, flow in the 0.0008 reach is subcritical, and a hydraulic jump occurs. The conjugate depth for the flow on the 0.0075 slope is then computed and found to be 0.964 feet. Then, using figure 2, the maximum wave height produced at the channel bank is calculated to be 0.255 feet above the average D_2 level. However, figure 2 is directly applicable only to channels with 1 to 1 side slopes. To modify D_2 to represent a 2 to 1 side slope figure 3 is used. For this particular problem,

$$\frac{D_2 - D_1}{D_1} = \frac{0.964 - 0.776}{0.776} = 0.242$$

Thus,

$$\frac{a + D_2 - D_1}{D_2 - D_1} = 2.07 \text{ for a 1:1 side slope}$$

$$= 2.38 \text{ for a 2:1 side slope}$$

The corresponding "a" values are:

$$a = 0.201 \text{ for a 1:1 side slope}$$

$$= 0.259 \text{ for a 2:1 side slope}$$

Thus, the change in side slope from 1:1 to 2:1 results in approximately a 30-percent increase in wave height. Since figure 3 is directly applicable only for channels whose bottom width is equal to D_2, this information may only be used to approximate wave heights that will result in the subject channel. The final computed wave height at the bank is (0.259/0.201)(0.255) or 0.329 feet above the average D_2 level.

REFERENCES

1. King, H. W., and Brater, E. F., "Handbook of Hydraulics," Fifth Edition, 1963, McGraw Hill Book Company

2. Chow, V. T., "Open-Channel Hydraulics," McGraw Hill Book Company

3. Peterka, A. J., "Hydraulic Design of Stilling Basins and Energy Dissipators," Engineering Monograph No. 25, U.S. Bureau of Reclamation, July, 1963

4. Preissmann, A., and Cunge, J. A., "Low-Amplitude Undulating Hydraulic Jump in Trapezoidal Canals," IAHR Journal of Hydraulic Research, Vol. 4, No. 1, 1966

5. Witting, J., "Weak Hydraulic Jumps in Shallow Water," Research Report 260, Everett Research Laboratory, October 1966

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