STABILITY OF AUTOMATIC CANAL SYSTEMS UTILIZING THE FREQUENCY RESPONSE METHOD

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Introduction

Fluid motion stability is a significant factor in the design of an adequate automatic feedback control system for typical canals. Rapid response times are limited to the practical canal water level recovery characteristics and to stability considerations. Methods to study and analyze fluid motion stability in an entire canal system for all possible flow conditions has in the past not been readily available. A great deal of judgment was left to the designer of the automatic feedback control system to select the final control parameters. The utilization of the frequency-response method recently developed specifically to obtain the canal system response characteristics, facilitates the analysis of flow stability and the selection of control parameters.

The practical application of the frequency-response method is made to the Coalinga Canal located near Coalinga, California. The Coalinga Canal has installed the closed-loop automatic downstream control system known as the Electronic Filter Level Offset (EL-FLO) plus RESET method (proportional plus proportional reset mode of control). The control parameters and stability of control were developed by the transient-response method prior to the availability of the frequency-response method.

Subsequent paragraphs will (1) describe briefly the EL-FLO plus RESET method of automatic feedback control, (2) discuss the selection of control parameters and stability of control considerations of the Coalinga Canal automatic control system utilizing the transient-response method, (3) discuss the frequency-response method, (4) apply the frequency-response method to the automated Coalinga Canal system, and finally (5) summarize and offer conclusions.

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EL-FLO Plus RESET Control System

The EL-FLO plus RESET method of control is a basic proportional plus proportional reset mode of control (1).* It is applied to the Coalinga Canal to achieve automatic downstream control of the canal check gates. "Downstream control" means that the control is from downstream toward the head of the canal. Thus, the control concept, although contrived electrically/mechanically through sensors and motor controls, is parallel to the natural control (in subcritical flow) observed in backwater curves, i.e., the control is from downstream to upstream (2). A longitudinally compressed canal reach as the controlled SYSTEM and the elements of the EL-FLO plus RESET control system FEEDBACK PATH are illustrated in figure 1. The control algorithm for the EL-FLO plus RESET controller to obtain the total desired upstream gate opening, GD, is:

$$GD = GP + GR \tag{1}$$

or

$$GD = K1*(YT-YF)+K2\int_{0}^{t} K1*(YT-YF)*dt \tag{2}$$

The feedback path is completed by the comparator and actuator elements. The total desired gate opening, GD, is compared to the actual gate opening, GA. Whenever the difference, $\pm\Delta G$ exceeds a referenced value typically 0.03 metres, the actuator will (depending on the sign of the difference) start the gate hoist motor to raise or lower the canal gate.

Transient-Response Method

Transient-response refers to the behavior of the SYSTEM, figure 1, as a result of the OUTPUT changing from one steady-state flow or level to another steady-state. The steady-state response refers to the behavior of the OUTPUT as time becomes infinite. The transient-response method analyzes the behavior of the SYSTEM or response characteristics during the unsteady flow conditions after a change of the downstream flow demand has occurred. The practice is to increase or decrease the flow demand at various magnitudes using step, ramp, or impulse-type changes. A simple mathematical expression for complicated systems, such as canals, is almost impossible to find. The unsteady flow in an open channel can be expressed by the following differential equation:

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* Parenthetical numbers refer to literature cited.

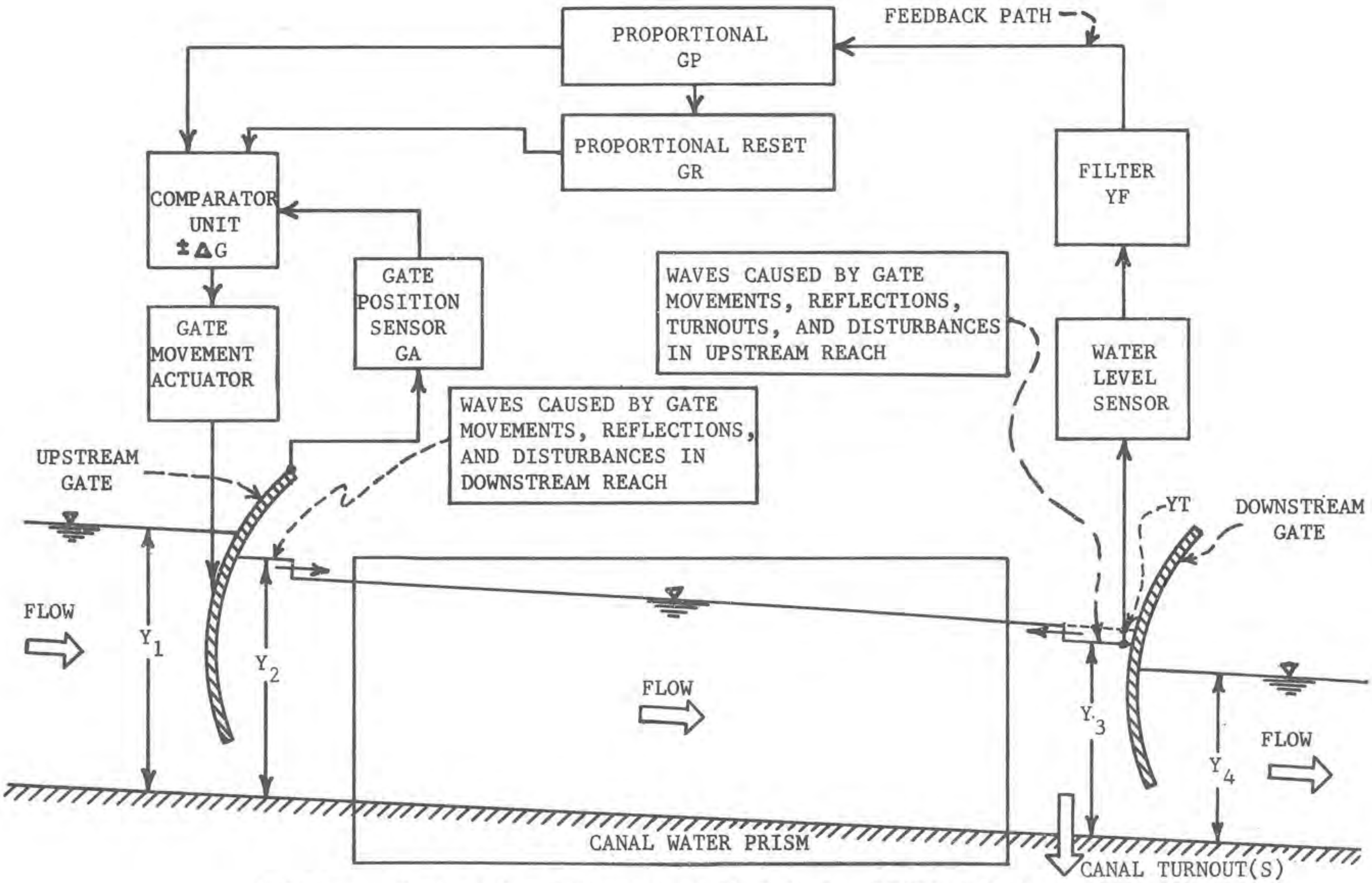

FIGURE 1 - Typical Canal Reach with EL-FLO plus RESET Method of Automatic Downstream Control

$$\frac{dV}{dt} \pm \frac{g}{c} * \frac{dy}{dt} = g(S_o - S_f) \qquad (3)$$

where V is the average velocity of flow, t is time, g is acceleration of gravity, y is the depth of flow, S_o is the bottom slope of the canal invert, and S_f is the friction slope using Manning's equation, and C is wave celerity.

The method of characteristics using a digital computer provides one means of solving the differential equations for unsteady flow in an open channel (3). The canal gates are boundaries between canal reaches. The automatic control system algorithms can be easily added to the numerical model to establish the new gate openings at the boundaries using the canal water level, Y3, solved from the differential equation (3) for the sensor input, figure 1.

Properly selected control parameters of the feedback control system will produce a damped oscillation, figure 2, when a unit step change of the downstream demand occurs. If the selected control parameters have too much "gain" or not enough time lag compensation for example, sustained oscillations will result and may even amplify creating an unstable canal system. The oscillation, figure 2, or the behavior of the canal reach from the time the downstream flow disturbance has occurred until the canal system has reached the new steady-state flow is the "transient-response" curve. The shape or characteristics of the transient-response curve is of primary concern to the designer of the feedback control system. The transient-response characteristics can be essentially determined by six points shown in figure 2.

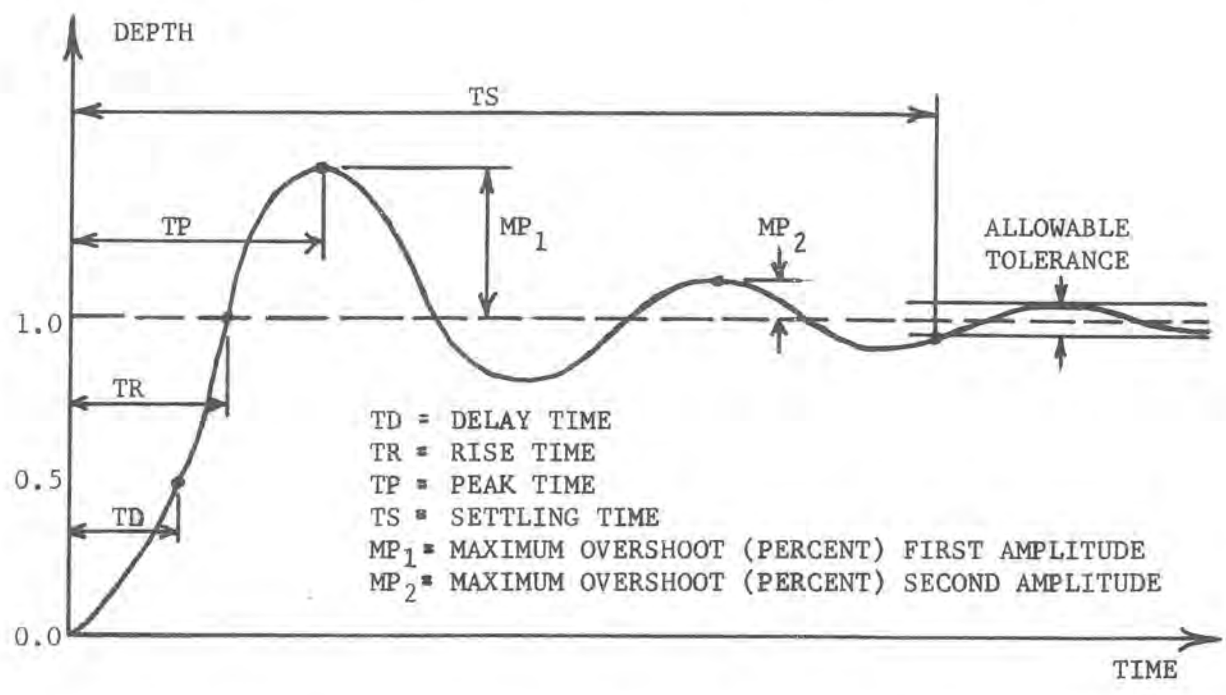

Figure 2 - Transient-Response Curve of a Second Order System to a Unit Step Change

A great number of computer mathematical model test runs are required to obtain the transient-response characteristics of an entire canal system for various control parameters at the many possible flow conditions. Utilizing the transient-response methods to select optimum control parameters becomes a matter of trial and error. The analysis of flow stability is sometimes inconclusive. A great deal of judgment by the designer is required to select final control parameters. Other methods are needed that would readily give an analysis of fluid motion stability so the designer could easily evaluate the success of the selected control parameters.

The Frequency-Response Method

The frequency-response method analyzes the steady-state response when the system is subjected to a sinusoidal disturbance at the upstream end of the canal reach (4). The sinusoidal input can be described by a vector that rotates with a constant angular speed which can be represented in either complex or in a real notation. The frequency-response method essentially utilizes techniques which are available but have not been extensively exploited in the analysis of a canal system to describe the representation of the sinusoidal input. Such techniques available are the Bode Diagram, Polar or Nyquist Diagram, and the Nichols Diagram each having its own advantage. The Bode Diagram is used in this paper and is a commonly used graphical plot associated with the complex representation of the expoential type to determine the relative stability of the system's open-loop frequency response.

Control system response. It is necessary to derive the complex representation of the exponetial type for each element of the automatic control system feedback path, figure 1, as listed in table 1.

Table 1. - Complex Representation of the Exponential-type for Each Element of the EL-FLO Plus Reset Automatic Feedback Control System.

Element	Amplitude, H	Phase angle, Θ
Filter	$H_f = \frac{1}{(1+(2\pi fT_c)^2)^{1/2}}$ . . (4)	$\Theta_f = -\tan^{-1}(2\pi fT_c)$ . . (5)
Proportional	H_g = Gain, K1' . . . . . . (6)	$\Theta_g = 0$ . . . . (7)
Proportional Reset	$H_r = \frac{K2}{2\pi f}$ . . . . (8)	$\Theta_r = \frac{-\pi}{2}$ . . . . (9)

Where f is frequency-cycles/minute.
T_c is the filter time constant-minutes.
K1' is the ratio of ΔY2 to ΔY3, figure 1.
K2 is the gain of the RESET controller per minute.

The response characteristics of the comparator unit and gate movement are not included in this paper for purposes of clarity and brevity. However, the significance of dead band action should not be ignored. For certain combinations of rate of gate travel and dead band magnitudes, instabilities can develop.

Canal reach response. The frequency response of a canal reach system, G(f), is defined as the ratio of the Fourier Transform of the output from the system, Y(f), to the Fourier Transform of the input to the system, X(f). Mathematically,

$$G(f) = \frac{Y(f)}{X(f)} \tag{10}$$

For analysis with a digital computer, a special form of the Fourier Transform must be used and it is defined as:

$$X\left(\frac{m}{N\Delta T}\right) = \frac{1}{N} \sum_{n=0}^{n=N-1} x(n\Delta T) \exp(-j2\pi mn/N) \tag{11}$$

$m = 0, 1 \ldots, N-1$

To determine the frequency response of the canal reach, any type of input signal can be used such as periodic, random, or a transient. However, it must contain all frequencies of interest. A simple way of determining the frequency-response is to (1) establish a steady-state discharge, (2) input a small impulse into the upstream end of the canal reach (ΔY2, figure 1), and the (3) transform the output. One good input signal is an impulse. The impulse has one fixed value at zero time and another fixed value for all other times. An impulse which produces a unit amplitude at all frequencies can be achieved by making the zero time value equal to $1/\Delta T$ and all other values equal to zero. The procedure is to use the same mathematical model used in the transient-response method setting the ΔT of equation (3) equal to the ΔT in equation (11) taking 1024 samples of the disturbance and then transform the output to get the desired frequency response. Since canal systems are nonlinear, the frequency response will vary with the various established steady-state discharges. The phase angle, Θ, of the canal reach is equal to zero.

Application of the Frequency-Response Method

The automated Coalinga Canal near Coalinga, California, with the EL-FLO plus RESET automatic downstream control system is used for an example of applying the frequency-response method. The physical properties of the Coalinga Canal and Control System parameters are described in appendix I.

The response characteristics of each element, table 1, in the EL-FLO plus RESET control system feedback path (the comparator unit and gate movement response characteristics are not included for clarity purposes) are plotted on the Bode Diagram, figure 3.

Knowing the response characteristics of each element in the feedback path, it is possible to determine the overall response characteristics, H_c, for the EL-FLO plus RESET control system. The combined response of the controller is obtained by multiplying the amplifications (H values) and summing the phases (Θ values) of all the components which make up the controller, figure 3. Since the amplifications (H values) are plotted on a logrithmic scale, the amplifications can be determined graphically by a simple summation of all the components. The graphical summation must be done relative to the H = 1 line. The response characteristics, H_c, of the controller can now be compared to the canal response characteristics, G(f). The open-loop response of the entire canal reach, H_c*G(f), figure 1, can be determined in the same manner that was performed for the individual controller elements to obtain the overall controller response. Figure 3 shows the open-loop frequency response of the system, figure 1, having a characteristic high gain at low frequencies and decreasing gain for the higher frequencies.

The Bode plots illustrate the relative stability of the system. The "gain and phase margins" measure the relative stability and can be determined with a minimum of effort. Referring to the Bode plot, figure 3, the gain margin is the reciprocal of the gain, H_c*G(f), at the phase angle, Θ, of -180° and it is positive being much greater than one. The phase margin is 180° plus the phase angle, Θ, at the gain, H_c*G(f), equal to one and it is a positive 63°. Usually positive gain and phase margins, as determined above, will ensure stability of the closed-loop system (5), figure 1. However, to verify the absolute closed-loop system, the Nyquist or Nichols diagrams should be sketched. The Bode plot has the basic information needed for these latter diagrams.

Figure 3 illustrates that many different response characteristics for each element of the controller can yield the identical overall controller response, H_c. Attention can now be focused on developing a desirable combined response characteristics of the automatic feedback controller and not so much on each individual element as was done using the transient-response method. However, the transient-response analysis is still a necessary step in order to examine the practical recovery characteristics and the speed at which a new steady-state flow can be established.

An analysis, illustrated in figure 3, of one reach of the Coalinga Canal demonstrates the application of the frequency-response method. It appears that critical amplification of disturbances occur at low frequencies and at low steady-state flow conditions

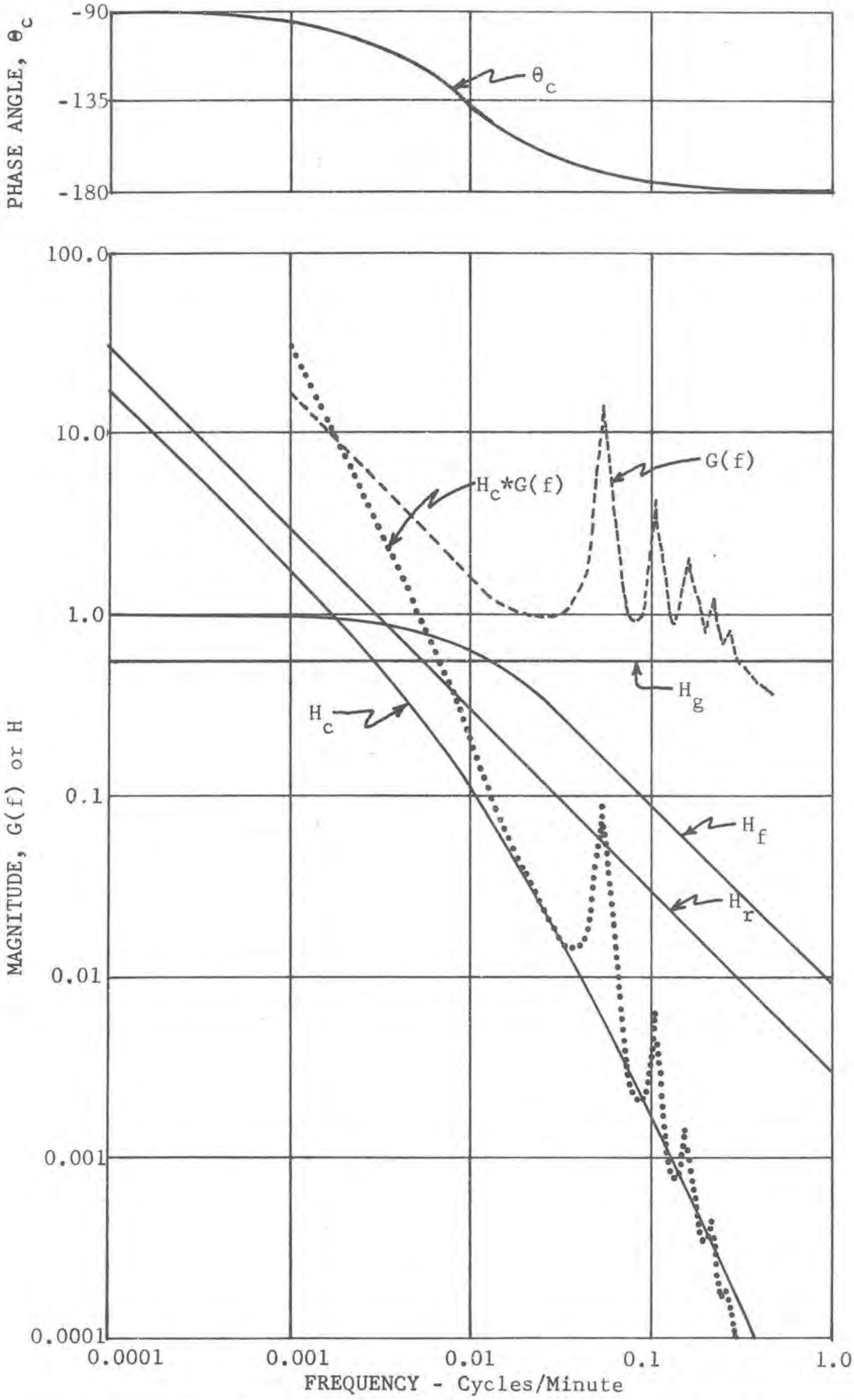

Figure 3 - Frequency-Response Characteristics, EL-FLO plus RESET Control System, Coalinga Canal Reach No. 2

(about 1 percent of the designed channel capacity). The response characteristics were developed for each reach of the Coalinga Canal yielding essentially the same results. Multiple canal reaches were also investigated. The amplitude of the disturbance is considerably attenuated when multiple reaches are in operation. Therefore, only single canal reaches need to be investigated when analyzing control parameters to eliminate instability inherent to control systems of the EL-FLO plus RESET type.

Summary and Conclusions

There are now two methods available to the designer of automatic feedback control for canal systems for determination of control parameters and fluid motion stability. These are the transient-response method and the frequency-response method. The transient-response method was exclusively used in the analysis of the Coalinga Canal automated system yielding satisfactory results but with considerable wasteful computer time and engineering effort and the complete assurance of control stability was lacking.

The addition of the frequency-response method has added a new dimension to the analysis of control parameter selection and system stability. The frequency-response method was described and then applied to one typical Coalinga Canal reach to illustrate its advantages. The main advantage of the frequency-response method is that the designer can readily visualize and evaluate the success of the control parameter selection and control stability for any particular feedback control system or its individual elements. The effects of changing the various control parameters can also be easily studied. Efforts can now be concentrated on developing the best combined controller response characteristics rather than the response for each individual element reducing the number of computer runs needed.

The transient-response and the frequency-response can be used in a complementary fashion. The transient-response analyzes the recovery characteristics from one steady-state flow to another. The frequency-response analyzes the fluid motion stability at steady-state flow conditions and assists in selecting the control parameters. The use of the frequency-response method for multiple reaches of canal has determined the analysis of fluid motion stability need only be investigated for single reaches.

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Appendix I

The Coalinga Canal is 18.34 km long and is concrete lined having a Manning's "n" value of 0.015. There are three check structures equipped with the EL-FLO plus RESET method of automatic downstream control. Each check structure has one gate 5.18 metres wide. Water is supplied from the San Luis Division of the California Aqueduct and is introduced into the Canal by the 33.6 m^3/s Pleasant Valley Pumping Plant. The physical properties of the Coalinga Canal and EL-FLO plus RESET control parameter are listed below:

Physical property	Reach No. 1	Reach No. 2	Reach No. 3	Reach No. 4
Reach length, km	2.17	2.54	5.12	8.51
Accumulated length, km	2.17	4.71	9.83	18.34
Capacity, m^3/s	31.2	28.3	28.3	12.0
Design depth YT, m	3.45	3.30	3.30	2.18
Bottom width, m	3.66	3.66	3.66	3.66
Side slope, m/m	1.5:1	1.5:1	1.5:1	1.5:1
Bottom slope, m/m	0.0001	0.0001	0.0001	0.000067
K1, m/m	1.50*	2.5	3.0	3.8
K2 per minute	0.090*	0.019	0.019	0.000
T_c	17.2*	17.2	17.8	23.6

* The EL-FLO plus RESET control parameters are selected to control the three 3.5 m^3/s pumping units. The three 1.3 m^3/s pumping units are automated by floating control with deadbands of ±0.05 m, ±0.10 m, and ±0.15 m. The three 6.4 m^3/s pumping units are manually controlled.