Optimized Determination of Control Parameters for Canal Automation

by

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Abstract

This report presents the final results of a study conducted in the Hydraulics Branch, Divison of General Research of the U.S. Bureau of Reclamation. It provides a procedure for optimizing selection of coefficients (parameters) to be used with a proportional plus integral (reset) controller for canal automation. The study includes a method for use with a proportional, proportional plus integral and proportional plus integral plus derivative controller. The procedure is illustrated using data from the South Gila canal.

INDEX

I. Introduction

This study is concerned with the control analysis of a system which is designed to regulate the downstream control of canal check gates.

A block diagram of the control system is shown in Fig. 1. In order to reduce the system to a level which will permit an analytical study, the effects of the gate motor dynamics and the deadband of the motor contactor have not been considered in this analysis. In some cases such effects may be lumped into the overall system gain constants.

The type of control which is considered in this study was selected in order to reduce certain undesirable effects such as overshoot of the discharge response to a step input and the steady-state error in the canal level response following such a disturbance. Proportional control alone, cannot reduce such undesirable effects. It is necessary to employ proportional plus integral control to reduce the error in the steady-state canal level response, and proportional plus derivative control to reduce the undesirable transient overshoot effects. To eliminate both transient and steady-state errors, a proportional plus integral plus derivative controller or its equivalent is required. An equivalent controller or compensator is considered in the concluding section entitled "Future Recommendations".

In conclusion, it is usual practice to have the error signal which is acted upon by a proportional plus integral control be the difference between the set point and the output variable. In Fig. 1 the input to the controller is the difference between the set point and the output of the filter. This arrangement could lead to a build-up in the discharge response overshoot.

II. Feedback Control

A reduced block diagram is shown in Fig. 2. The cascade controller is labelled $G_c(s)$ and the lumped model of the canal reach is labelled $G_p(s)$.

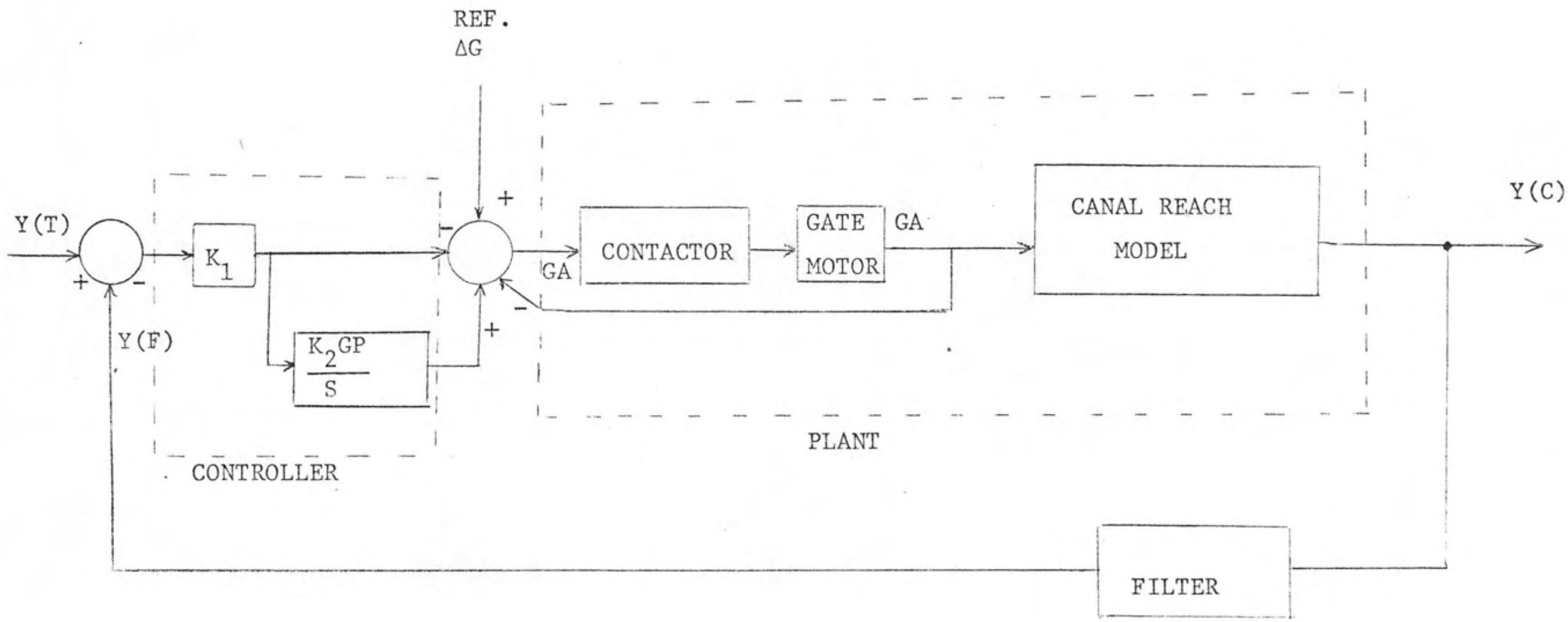


FIG. 1. Block Diagram of Automatic Downstream Control

The notation indicates that the transfer functions are expressed as a function of the Laplace variable s.

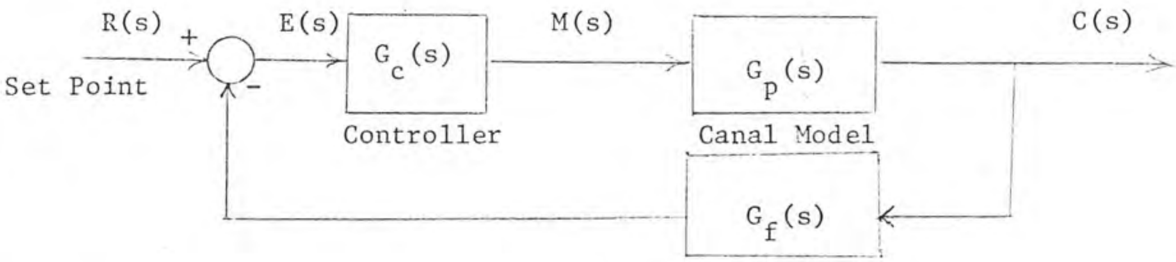


FIG. 2

The overall closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)G_f(s)} \tag{1}$$

For the purpose of analysis the information which concerns transient and steady-state response is contained in the characteristic equation which is defined as

$$1 + G_c(s)G_p(s)G_f(s) = 0 \tag{2}$$

that is, the transfer function of the controller, the canal model and the filter are contained in this equation.

Before considering the characteristic equation in detail a discussion of the control laws which are used in this study will be considered.

From the block diagram, the controller transfer function is

$$\frac{M(s)}{E(s)} = G_c(s) \tag{3}$$

That is, the control law relates the input variable of the canal model to the error signal of the system. The three basic forms of control laws considered here are:

Proportional action

$$G_c(s) = K_1 \tag{4}$$

Proportional plus Integral Action (Reset)

$$G_c(s) = K_1(1 + \frac{1}{T_1 s}) \tag{5}$$

Proportional plus Integral plus Derivative Action

$$G_c(s) = K_1(1 + \frac{1}{T_1 s} + T_2(s)). \tag{6}$$

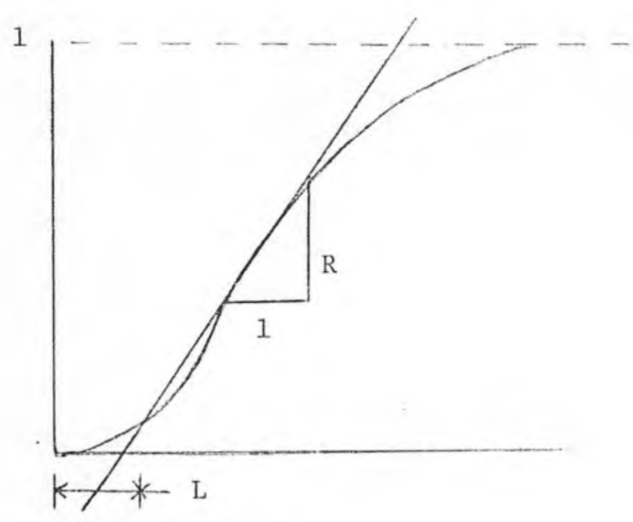
The parameter K_1 is a gain and T_1 and T_2 are integral and derivative action time constants. These are the adjustable parameters which are to be optimized in this study.

III. Parameter Adjustment

In process control, the two most widely used methods to optimize control parameters are given in the following paragraphs. The analysis upon which these methods are based and the optimization approach which is used are given in an appendix.

[1] ? see page 9

Method I of the technique due to Ziegler and Nichols [2] depends on the process response for the open loop system using a proportional controller. Two parameters, the maximum slope R and the lag time L, for a unit step input are measured on a computer model graphical print out.



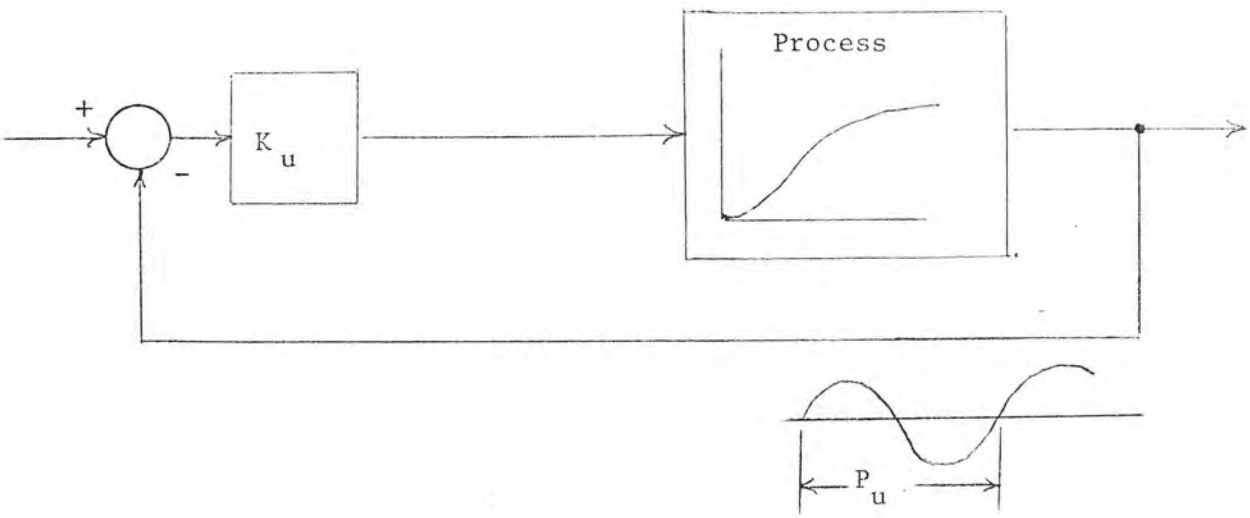
Unit-step input
response-open loop.

In terms of these parameters for proportional control

$$K_1 = \frac{1}{RL} \text{ is}$$

usually optimum.

Method II is based on the response of a closed-loop system, where the process is under proportional control, and its gain K_1 is at the stability limit



K_u; that is, the loop has sustained oscillations of a period P_u. The two parameters K_u and P_u may be determined by actual tests or estimated by simulation. With these data the optimum adjustment are:

Proportional Control (P)

$$K_1 = \frac{1}{2} K_u$$

Proportional plus Integral Control (PI)

Method I	Method II
$K_1 = \frac{0.9}{RL}$	$K_1 = 0.45\ K_u$
$T_1 = 3.3\ L$	$T_1 = 0.83\ P_u$
$K_2 = K_1/T_1$	

Proportional plus Integral plus Derivative Control (PID)

Method I	Method II
$K_1 = \frac{1.2}{RL}$	$K_1 = 0.36\ K_u$
$T_1 = 2L$	$T_1 = 0.83\ P_u$
$K_2 = K_1/T_1$	$K_2 = K_1/T_1$
$T_2 = 0.5L$	$T_2 = 2\ P_u$

The filter used in this system has essentially the same characteristics as a PD controller. The additional gain and derivative action is probably the reason for the overshoot of the first and second peaks of the discharge response. To compensate for this effect, modifications were made in the above procedure.

IV. Ultimate Parameters

The term ultimate is used here to mean values which are critical or on the verge of instability. The gain and frequency to be calculated are values which occur on the $i\omega$ axis in the complex frequency plane, and are therefore values which relate to the natural frequency and corresponding gain of the closed-loop system.

At the stability limit, $s = i\omega_u$ and $\delta = 0$. $K_I = K_u$ is the corresponding value of INCO at this limit. This is the value of INCO at which the discharge is in an oscillatory state. In this case (1) (see Appendix, VIII- 1) becomes

$$1 + \frac{K_u e^{-i\omega_u t_D}}{i\omega t_f + 1} = 0 \tag{1}$$

or

$$1 + i\omega t_f + K_u \cos \omega_u t_D - iK_u \sin \omega_u t_D = 0 \tag{2}$$

Separating this equation into real and imaginary parts,

$$\begin{aligned} 1 + K_u \cos \omega_u t_D &= 0 \\ \omega_u - K_u \sin \omega_u t_D &= 0 \end{aligned} \tag{3}$$

and solving these equations for K_u we have

$$K_u = \frac{-1}{\cos \omega_u t_D} \tag{4}$$

The natural frequency of the system which is ω_u in (15), is determined from the relationship

$$\omega_u = \frac{2\pi}{t_u + t_D} \ .$$

Using the value of $(t_u + t_D)$, for Q = 10 cfs, (from STAGAT) which is

$$(t_u + t_D) = 2108 \text{ sec.}$$

the calculated value of ω_u is

$$\omega_u = \frac{2\pi}{2108} = 0.00298 \text{ rad/sec.}$$

Using this value of ω_u the corresponding value of K_u may be calculated from (15) (using $t_D = 1027$),

$$K_u = \frac{-1}{\cos[(0.00298)(1027)]} = 1.000 \ .$$

This value of K_u is the value of INCO when $\delta = 0$. The corresponding value of GAIN is calculated for a discharge value of Q = 10 cfs. (from STAGAT).

INCO = GAIN x FNGATE (0.78386)

FNGATE = DELTA COED/DELTA GATE (0.094273) + 0.61473

= (-2.057)(0.094273) + 0.61473

= 0.4208

INCO = GAIN (0.4208) (0.78386)

$$\text{GAIN} = \frac{\text{INCO}}{0.3298} = 3.032 \text{ INCO} = 3.032$$

This value of ultimate gain, which is obtained analytically, is close to the value (Table 1, Section V) which was determined by trial.

V. Computer Results

Computer runs were made using program CORNING with South Gila canal data. Both methods described in Section III were used to select the range of values for K_1 and K_2 (with PI control).

M_p = First peak overshoot

H = Ratio of second to first peak

T_s = Water level recovery time (90%)

1. Oscillatory Cases

	K_u	K_2	P_u	M_p%	H%	$T_f$sec
1	3.58	.000	5959	38	100	1120
2	3.58	.006	16619	43.7	100	1120
3	3.58	.015	5959	108	100	1120

The cases listed in this table are oscillatory cases of the discharge response. The period and the gain for Method II and new values of K_1, K_2 and $T_2 = t_f$ were obtained. (See Table 2 and 3.)

2. Filter Time Constant T_f = 1120

	K_1	K_2	M_p%	H%	$t_f$sec	$T_s$hrs
1	0.907	0.0117	50.0	24.2	1120	2.07
2	0.970	0.0107	55.8	20.8	1120	2.11
3	0.970	0.0097	55.8	25.0	1120	2.15
4	0.970	0.0088	44.0	28.5	1120	2.19
5	1.13	0.0137	65.9	16.7	1120	1.64
6	1.13	0.0125	55.8	25.0	1120	1.68
7	1.13	0.0114	57.0	20.8	1120	1.72
8	1.13	0.0104	55.8	25.0	1120	1.77
9	1.29	0.0160	68.2	36.0	1120	1.51
10	1.29	0.0140	65.9	20.7	1120	1.64
11	1.29	0.01 0	64.4	17.8	1120	1.77
12	1.29	0.0118	61.1	26.9	1120	1.90

The 12 cases are typical cases which did not meet specifications. They were obtained by an iterative application of Method II. The filter has essentially the same effect as a derivative PD controller. It is probably this effect which caused the high values of the ratio H (the ratio of the second to first peak). These high values also could be caused by nonlinear effects known as integral saturation and reset windup which are discussed in Section VI.

3. Cases with Variable Time Constant

	K_1	K_2	M_p%	H%	$t_f$sec	$t_s$hrs
1	1.29	0.0156	93.1	70.7	1990	1.58
2	1.29	0.0156	91.9	45.0	1770	1.64
3	1.29	0.0156	87.1	48.6	1608	1.70
4	1.29	0.0156	78.2	47.1	1446	1.75
5	1.29	0.0156	66.6	37.5	1283	1.79
6	1.29	0.0156	66.6	23.8	1120	1.89
7	1.29	0.0156	66.6	23.4	1000	1.96
8	1.29	0.0156	58.8	28.0	840	2.03
9	1.29	0.0156	52.9	26.1	680	2.08
10	1.29	0.0156	43.4	35.7	520	2.22
11	1.29	0.0156	43.4	35.7	360	2.11
12	1.29	0.0156	38.7	12.0	200	2.25
13	1.29	0.0156	32.5	52.4	001	2.00

These cases represent computer runs with constant gains. The filter time constant was varied over a range from 001 to 1990 seconds. Case 12 meets the specifications and these data will be used in a sample calculation. The PID formula is used because the filter is considered as a derivative controller in this analysis.

4. Sample Calculation

The values from case 12 are:

$K_1 = 1.29$ $\quad K_2 = 0.0156$ $\quad t_f = 200$

Using the PID formula (Method II)

$$K_1 = 0.36\ K_u$$

The measured value of K_u is 3.58

$$K_1 = 0.36\ (3.58) = 1.28$$

$$T_1 = 0.83\ P_u$$

$$T_1 = \frac{0.83(5959)}{60} = 83 \text{ sec}$$

$$K_2 = \frac{K_1}{T_1} = \frac{1.28}{83} = 0.015 \text{ min}^{-1}$$

$$T_2 = 2\ P_u$$

$$= 2\ \frac{(5959)}{60} = 198 \text{ sec}$$

These values are close to the measured values in case 12.

VI. Future Recommendations

A brief summary of elements of the control system which indicate problem areas are listed for consideration.

1. The filter transfer function in the feedback loop has essentially the same characteristics as a PI controller in the forward branch of the

control system. A more compact and more easily adjusted controller would result if the filter and the present PI controller were combined into a single unit. An alternative approach, would be to design a replacement lead-lag compensator which would also give the desired steady-state and transient performance.

2. The present gate motor and contactor is designed with position feedback only. Velocity feedback should be added to this unit to prevent observed limit cycle operation of the canal gates.
3. In a PI controller two nonlinear effects may occur. For large changes in set point, the output of the integrator will saturate and stay at its maximum value until the error signal comes within the proportional band. In addition, the phenomenon known as reset windup also occurs. That is, since the error does not change sign the integral action continues to add to the already saturated value it is holding, thus keeping the total controller output at or near its saturation value even though the variable is close to the new set point. This can cause a large overshoot in the system response. A common approach to avoiding reset windup is to design the controller with components which will ensure that the integrator maintains a zero initial state until the error signal is within the linear range. It should be noted that both saturation and reset windup are nonlinear phenomena which do not appear in the transfer function description of the controller.
4. In order to minimize pumping power, the design of a deadbeat controller should be considered. A deadbeat controller adjusts the gate opening so that overshoot and undershoot does not occur. This type of controller minimizes the discharge response with respect to

a performance index which is a function of pumping power.

5. Direct digital control is widely used in modern control systems. Solutions to the problems discussed above, after they are identified by engineering analysis, could readily be implemented by DDC. That is, a microprocessor control unit could be programmed to include the linear control laws, nonlinear effects, and optimization contraints.

6. Modern state variable theory permits the analysis and synthesis of multiple control systems. Multivariable control permits the integrated operation of interconnected systems which are in cascade, parallel or combinations of these. When coupling effects exist between control systems this overall approach is generally the most satisfactory. Discrete-time multivariable analysis and synthesis would include DDC design and is a well developed analytical tool. Finally, with multivariable control, a single digital computer could be used for the overall system.

VII. References

[1] Shand, M. J., "Final Report Automatic Downstream Control Systems for Irrigation Canals," T.T. HEL-8-4 University of California, Hydraulic Engineering Laboratory, 1971.

[2] Ziegler, J. G., and Nichols, N. B., "Optimum Settings for Controllers," ISEJ, June 1964, pp. 731-734.

[3] Shand, M. J., "Final Report: The Hydraulic Filter Level Offset Method for the Feedback Control of Canal Checks," T.R. HEL-8-3, University of California, Hydraulic Engineering Laboratory, 1968.

[4] Harder, J. A., Shand, M. J., and Buyalski, C. P., "Automatic Downstream Control of Canal Check Gates by the Hydraulic Filter Level Offset (HyFLO) Method," Preprint, Fifth Technical Conference U.S. Committee on Irrigation, Drainage and Flood Control, Oct. 1970.

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VIII. Appendix

1. Derivation of Formulas

Consider the model developed in Shand [1] for the large discharge case. The characteristic equation is

$$1 + \frac{K_I \, \varepsilon^{-st_D}}{t_f s + 1} = 0 \tag{1}$$

where K_I = INCO, and $s = -\delta + i\omega$ is the Laplace transform variable.

Expanding (1) gives

$$1 + \frac{K_I \, \varepsilon^{\delta t_D} (\cos \omega t_D - i \sin \omega t_D)}{t_f(-\delta + i\omega) + 1} = 0 \tag{2}$$

Separating the variables and setting the real and imaginary parts to zero, gives

$$(1 + \delta t_f) + K_I \, \varepsilon^{\delta t_D} \cos \omega t_D = 0 \tag{3}$$

$$\omega t_f \cos \omega t_D + (1 + \delta t_f) \sin \omega t_D = 0 \tag{4}$$

Rearranging (3) and (4) gives the following expressions:

$$\cos \omega t_D = \frac{-(1 + \delta t_f)}{K_I} \, \varepsilon^{-\delta/\omega(\omega t_D)} \tag{5}$$

$$\sin \omega t_D = \frac{\omega t_D}{\frac{t_D}{t_f} K_I} \, \varepsilon^{-\delta/\omega(\omega t_D)} \tag{6}$$

Combining (5) and (6) results in Shand's equation (65),

$$\frac{\tan \omega t_D}{\omega t_D} = \frac{1}{-\frac{\delta}{\omega}(\omega t_D) - \frac{t_D}{t_f}} \tag{7}$$

2. Optimum Gain Adjustment

In order to derive the optimum expressions for the controller gain from (7), let

$$a = -\frac{\delta}{\omega}(\omega t_D) - \frac{t_D}{t_f}$$

$$b = \omega t_D$$

and from (7)

$$\tan b = \frac{b}{a} \quad . \tag{8}$$

The amplitude ratio of the discharge wave for the second to the first overshoot is defined as

$$H = \frac{Z_2}{Z_1} = \varepsilon^{-aP} \quad , \quad \text{where } P = \frac{2\pi}{b} \quad . \tag{9}$$

An optimum value of H is 15 percent. Therefore

$$H = \frac{3}{20} = \varepsilon^{\frac{-2\pi a}{b}} \quad . \tag{10}$$

Solving for $\frac{a}{b}$ gives

$$\frac{a}{b} = \frac{1}{2\pi} \ln \frac{20}{3} = 0.302.$$

Substituting in (8) gives

$$\frac{b}{a} = \frac{1}{0.302} = \tan b.$$

Then

$$b = \tan^{-1}(3.311) = 1.277$$

$$a = 0.302b = 0.3856 \ .$$

These values of a and b locate the closed loop poles in the complex plane when the optimum ratio H of the first and second peaks is defined to be 15 percent.

The remaining unknown is the ratio t_D/t_f which can be found by solving (5) and (6) simultaneously. The resultant equation is

$$a \sin b \left(\frac{t_D}{t_f}\right) = b \cos b \tag{11}$$

and

$$\frac{t_D}{t_f} = \frac{b \cos b}{a \sin b} = 1 \ . \tag{12}$$

The corresponding value of δ/ω is from

$$a = \frac{\delta}{\omega} b - \frac{t_D}{t_f} \ ,$$

$$\frac{\delta}{\omega} = \frac{a + 1}{b} = \frac{1 + 0.3856}{1.277} = 1.085. \tag{13}$$

In summary, the parameters calculated in this section, that is, the values a, b, δ/ω, and t_D/t_f, are values for closed-loop operation. The assumptions made by Shand [1] in his derivation are also implied.