PAP 292

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Phoresis scavenging of small aerosol particles

Study of the theory of diffusiophoretic and thermophoretic forces near an evaporating water droplet leads to the following expression for the efficiency of a droplet of radius R to collect a particle of radius a

$$E(a,R) = 4 v_p / UT(R)$$

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where UT(R) is the terminal velocity of a droplet of radius R and

$$v_p = (2.25 \times 10^{-5})(TA - TD)\, V/R$$

is the combined phoretic velocity of a particle with a = 0.1 micron, V is a ventilation factor and (TA - TD) is the temperature difference between the ambient and the droplet. At 10°C and 60 percent relative humidity the temperature difference is approximately 6-7°C.

In Figure 1 values of E are plotted versus R for a = 0.1 micron. Also shown (dashed curve) is a plot of E versus height for a droplet that completely evaporates by the time it reaches the ground.

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The percentage of particles removed can be calculated from the following expression

$$m = N(1 - e^{-Wt})$$

where N is the original number of particles per CC and W is the washout coefficient which is determined from the efficiency and droplet number and size distribution.

In a typical experiment we consider a volume of air 10 m square and 100 m above the surface, (10^{10} cm^3). Assuming we inject water at the rate of 10^4 gm/sec (approximately 160 gal/min) at 10°C and 60 percent relative humidity it would take 10^4 seconds to remove 70 percent of the particulates or 26,500 gallons of water. Figure 2 shows the percentage of particles removed versus time. Such inefficiency can be explained by the height versus efficiency curve in Figure 1. Of the entire 100 m only the last 1-2 meters are efficiently cleaned, the continuous spraying is needed to clear the upper 98 percent of the volume where the efficiency is low.

An alternative would be to spray 30 micron radii droplets at 1 m intervals all along the height of the column. Calculations using the same water input rate showed no increase in removal efficiency.

In conclusion the proposed study would be highly inefficient for particulate removal.

Note: One or two possible alternatives have not as yet been investigated; however, my present knowledge of their potential for increasing efficiency does not warrant expenditure of any further work time in developing them. I will, however, investigate these areas on my own time and report any conclusions which would justify submitting a proposal for continued research and application.

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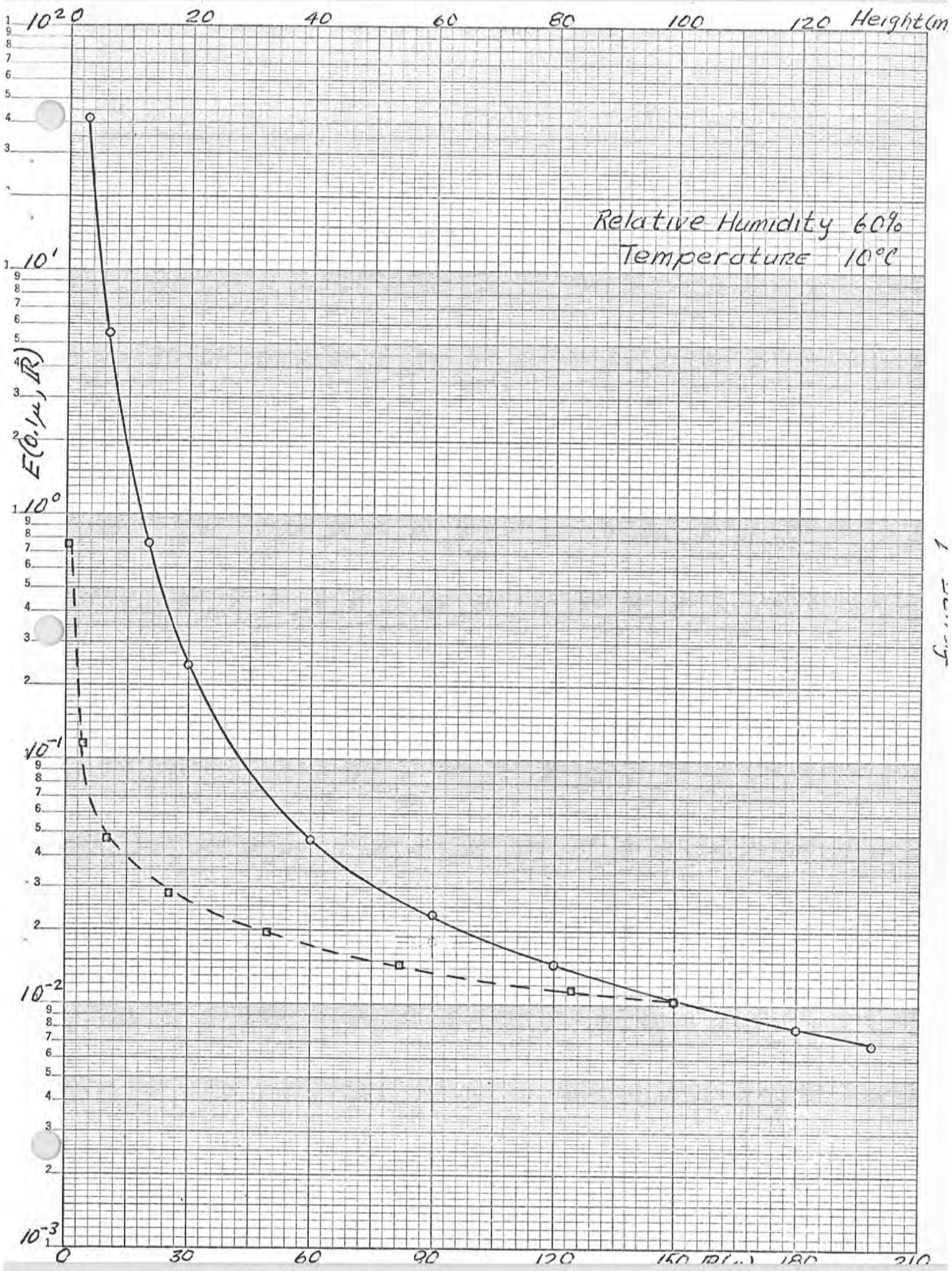
Height (m)
Relative Humidity 60%
Temperature 10°C
$E(0.1\mu, \vec{R})$
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
0
20
40
60
80
100
120
30
90
150
180
210

Fig. 1

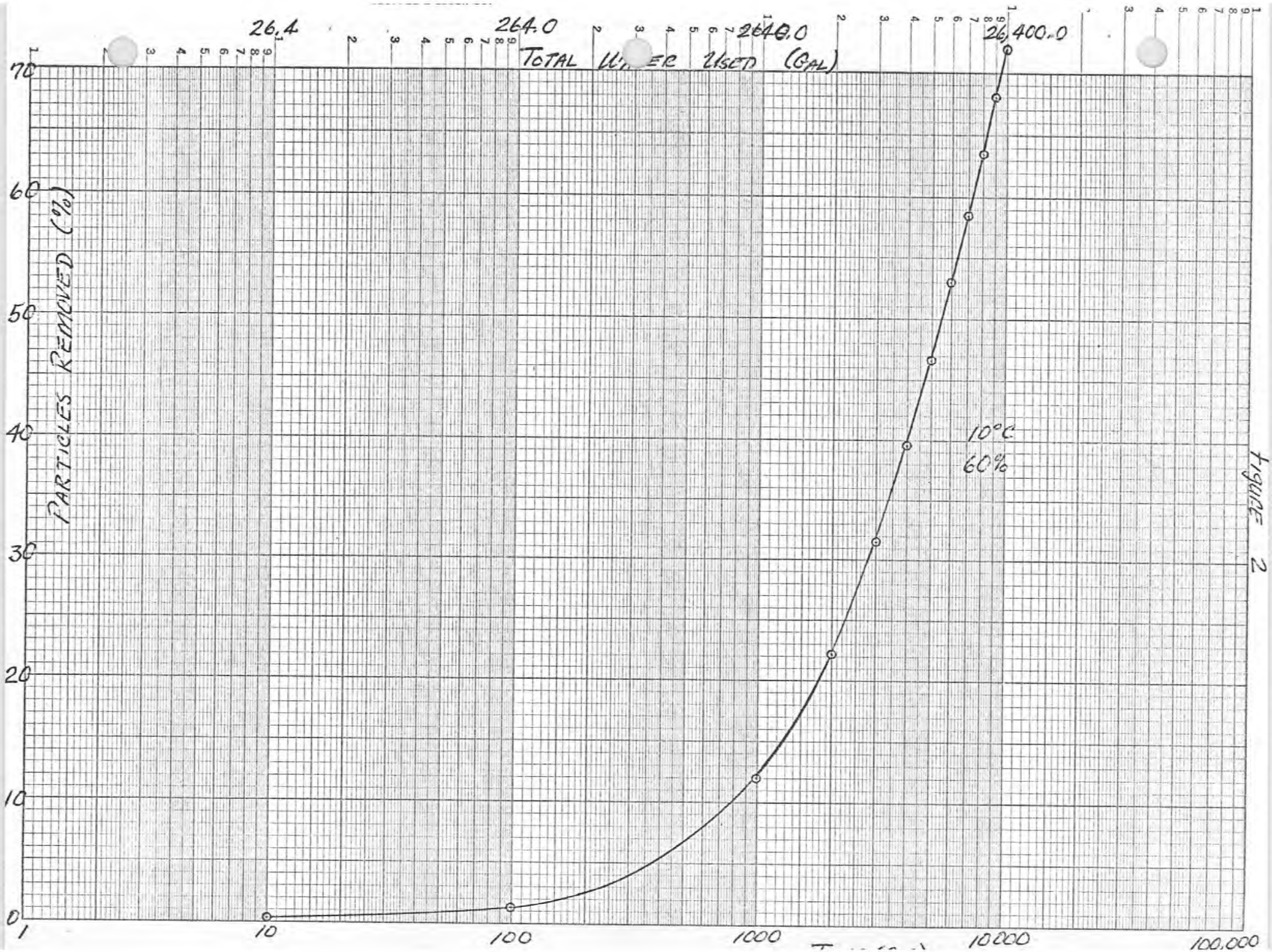
26.4
264.0
2640.0
26,400.0
TOTAL WATER USED (GAL)
PARTICLES REMOVED (%)
70
60
50
40
30
20
10
0
1
10
100
1000
10000
100,000
10°C
60%

FIGURE 2

THERON- DIFFUSIOPHORETIC SCAVENGING OF SMALL AEROSOL PARTICLES.

Diffusiophoresis – A particle in an isothermal gas mixture will move due to concentration gradients of the constituents.

A special case of this is the so called Stefan flow which occurs near an evaporating or condensing surface. Since we are interested in the scavenging of small aerosol particles by falling evaporating water droplets this area will be investigated in more detail.

Stefan flow – Stefan postulated that near an evaporating (or condensing surface) there exists a hydrodynamic flow of the carrier gas away (or toward) the surface. Consider a small evaporating water droplet at rest with respect to its environment. In order to maintain a constant total pressure about the droplet there exists a pressure gradient of the carrier gas (air) equal and opposite to the water vapor pressure gradient. This can be seen from the fact that

$$p_a + p_v = \mathbb{P} \tag{1}$$

so

$$\frac{dp_a}{dr} + \frac{dp_v}{dr} = \frac{d\mathbb{P}}{dr} = 0 \tag{2}$$

and hence

$$\frac{dp_a}{dr} = -\frac{dp_v}{dr} . \tag{3}$$

To visualize this situation consider the diagram in fig. 1. The hydrodynamic Stefan flow is explained by the fact that air molecules diffusing toward the surface have no place to go once they reach the surface. Stefan postulated that a hydrodynamic flow must exist to compensate for this.

Thermo-diffusiophoretic scavenging of small aerosol particles.

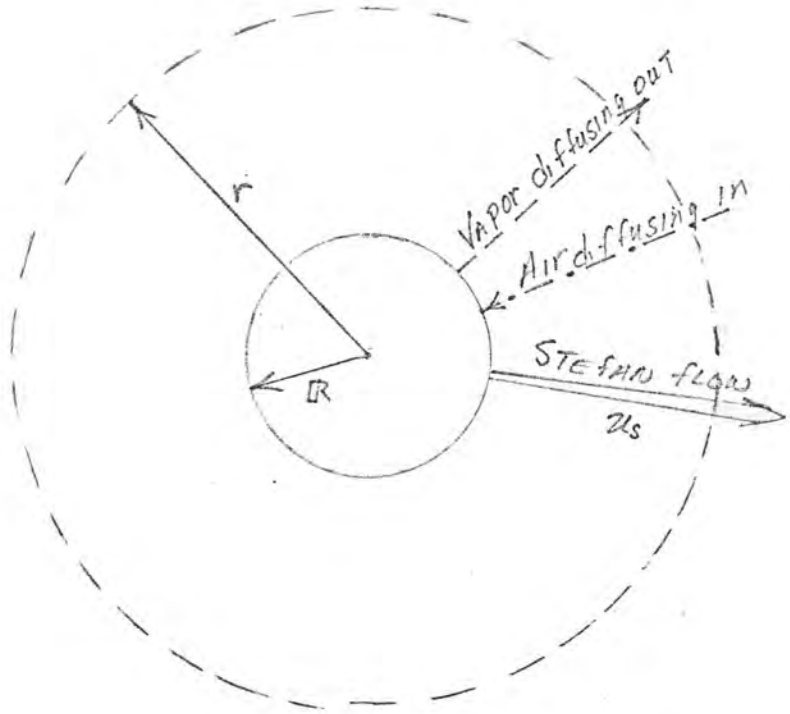

fig. 1

To obtain an expression for the velocity of the Stefan flow let us look at Ficks Law of Diffusion for the vapor and air. That is

$$-\frac{dm_v}{dt} = 4\pi r^2 \bar{\rho} \, D \, \frac{dC_v}{dr} \qquad (4)$$

and

$$\frac{dm_a}{dt} = 4\pi r^2 \bar{\rho} \, D \, \frac{dC_a}{dr} \qquad (5)$$

where m_v and m_a are respectively the total mass of vapor diffusing out from the surface

Thermo-diffusiophoretic scavenging of small aerosol particles

and the total mass of air diffusing in, $\bar{\rho}$ is the mean density of the environment, D is the mutual diffusion coefficient of the two components and C_v and C_a are the respective concentrations defined by

$$C_v = \frac{\rho_v}{\bar{\rho}} = \frac{\frac{M_v \rho_v}{\mathcal{R} T}}{\frac{\bar{M} \mathbb{P}}{\mathcal{R} T}} = \frac{M_v \rho_v}{\bar{M} \mathbb{P}} \tag{6}$$

and

$$C_a = \frac{\rho_a}{\bar{\rho}} = \frac{\frac{M_a \rho_a}{\mathcal{R} T}}{\frac{\bar{M} \mathbb{P}}{\mathcal{R} T}} = \frac{M_a \rho_a}{\bar{M} \mathbb{P}} \tag{7}$$

where M's are the molecular weights and $\mathcal{R}$ is the Universal gas constant. For the Stefan flow we can write

$$\frac{dm_a}{dt} = 4\pi r^2 u_s \rho_a . \tag{8}$$

Combining equations (5) and (8) yields

$$4\pi r^2 \bar{\rho} D \frac{dC_a}{dr} = 4\pi r^2 u_s \rho_a \tag{9}$$

or

$$u_s = \frac{D}{\left(\frac{\rho_a}{\bar{\rho}}\right)} \frac{dC_a}{dr} = \frac{D}{C_a} \frac{dC_a}{dr} \tag{10}$$

from equations (3), (6) and (7) we may write

$$\frac{dC_a}{dr} = \frac{M_a}{\bar{M} \mathbb{P}} \frac{d\rho_a}{dr} \tag{11}$$

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and

$$\frac{dC_v}{dr} = \frac{M_v}{\bar{M}\,\mathbb{P}}\,\frac{dp_v}{dr} = -\frac{M_v}{\bar{M}\,\mathbb{P}}\,\frac{dp_a}{dr} \qquad (12)$$

so that

$$\frac{dC_a}{dr} = -\frac{M_a}{\bar{M}\,\mathbb{P}}\cdot\frac{\bar{M}\,\mathbb{P}}{M_v}\,\frac{dC_v}{dr} = -\frac{M_a}{M_v}\,\frac{dC_v}{dr} \qquad (13)$$

and

$$u_s = -\frac{M_a}{M_v}\,\frac{D}{C_a}\,\frac{dC_v}{dr} = -\frac{D}{p_a}\,\frac{dp_v}{dr} \qquad (14)$$

Goldsmith, et. al., have found in experiments at standard temperature and pressure where $D = 0.24\ cm^2/sec$ and $p_a = 990\ mb$. that

$$u_s = -1.89 \times 10^{-4}\,\frac{dp_a}{dx}\ cm/sec$$

which does not agree with the value of $-2.4 \times 10^{-4}\ dp_a/dx$ given by equation (14) for equivalent values.

This difference can be resolved if we consider the forces on the particles due to the mutual diffusion of the two gases. Waldmann and Schmidt give as the velocity of particles suspended in a mutually diffusing gas system the value

$$V_d = -\frac{\sqrt{m_1} - \sqrt{m_2}}{\gamma_1\sqrt{m_1} + \gamma_2\sqrt{m_2}}\,D\,\nabla\gamma_1 \qquad (15)$$

where D is as before and m_1 and m_2 are the

Thermo-diffusiophoretic scavenging of small aerosol particles.

masses of the water vapor and air molecules respectively and γ_1 and γ_2 are the molar fractions of the vapor and carrier gas given by

$$\gamma_1 = \gamma_v = \frac{p_v}{\Pi} \tag{16}$$

and

$$\gamma_2 = \gamma_a = \frac{p_a}{\Pi} \tag{17}$$

Assuming the diffusion to be only in the radial direction then

$$\nabla \gamma_v = \frac{d\gamma_v}{dr} = \frac{1}{\Pi}\frac{dp_v}{dr} = \frac{\gamma_a}{p_a}\frac{dp_v}{dr} \tag{18}$$

so that

$$V_d = -\left(\frac{\sqrt{m_v} - \sqrt{m_a}}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}}\right)\frac{\gamma_a}{p_a} D \frac{dp_v}{dr} . \tag{19}$$

Adding the diffusiophoretic term to U_s yields

$$V = V_d + U_s = -\left(1 + \left(\frac{\sqrt{m_v} - \sqrt{m_a}}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}}\right)\gamma_a\right)\frac{D}{p_a}\frac{dp_v}{dr} \tag{19a}$$

or

$$= -\left(\frac{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a} + \gamma_a\sqrt{m_v} - \gamma_a\sqrt{m_a}}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}}\right)\frac{D}{p_a}\frac{dp_v}{dr}$$

$$= -\left(\frac{\sqrt{m_v}(\gamma_v + \gamma_a)}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}}\right)\frac{D}{p_a}\frac{dp_v}{dr} .$$

Thermo-diffusiophoretic scavenging of small aerosol particles

Since $\gamma_v + \gamma_a = \frac{p_v}{P} + \frac{p_a}{P} = 1$ by equation (1)

then

$$V = -\frac{\sqrt{m_v}}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}}\,\frac{D}{p_a}\,\frac{dp_v}{dr}. \qquad (20)$$

Now going back to the experimental value given by Goldsmith ($-1.89 \times 10^{-4}\, dp_v/dr$), if $m_v = 3.0 \times 10^{-23}$ gm and $m_a = 4.8 \times 10^{-23}$ gm, and since $\gamma_a >> \gamma_v$ then

$$V = -\frac{\sqrt{m_v}}{\sqrt{m_a}}\left(2.4 \times 10^{-4}\,\frac{dp_v}{dr}\right)$$

$$= -0.79\left(2.4 \times 10^{-4}\,\frac{dp_v}{dr}\right) = -1.9 \times 10^{-4}\,\frac{dp_v}{dr}$$

which is excellent agreement between theory and experiment.

Up till now we have concentrated only on diffusion forces. In the case of an evaporating droplet falling through the atmosphere the temperature of the droplet is lower than that of the surrounding air hence a thermal gradient exists. If a particle is placed in this thermal gradient it will move toward the lower temperature. This phenomenon is known as thermophoresis and will be considered now.

Thermophoresis – A simplified explanation of thermophoresis can be given by considering the drawing in fig 2.

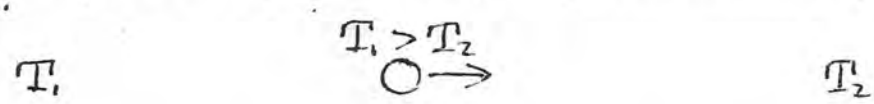

fig. 2.

Thermo-diffusiophoretic scavenging of small aerosol particles.

Gas molecules to the left of the particle have a higher temperature than those to the right. Hence their velocity is greater and on the average will transfer more momentum to the particle resulting in the particle moving toward the lower temperature region.

Waldmann and Schmidt give as the force on a small aerosol particle in a temperature gradient the value (originally due to Brock)

$$\vec{F}_{th} = -6\pi \eta a \, \mathbb{A} \frac{k}{\mathbb{P}} \nabla T \tag{21}$$

where

$$\mathbb{A} = \frac{2\sigma\left(k + C_t \frac{l}{a} k_p\right)}{\left(1 + 3C_m \frac{l}{a}\right)\left(2k + k_p + 2C_t \frac{l}{a} k_p\right)} \tag{22}$$

where

$$K_m = \frac{l}{a} \quad \text{(Knudsen number)}$$

and a is the radius of the particle, k and k_p are the thermal conductivities of the gas and particles respectively, l is the mean free path of the gas, η is the dynamic viscosity of the gas and σ, C_m and C_t are constants with the values

$$\left.\begin{aligned} \sigma &= 1/5 \\ C_m &= 1.0 \\ C_t &= 2.5 \,. \end{aligned}\right\} \tag{23}$$

We see from equations (20) and (21) that the diffusion forces tend to keep particles away

Thermo-diffusiophoretic scavenging of small aerosol particles.

from the surface of the drop where as thermal forces tend to draw the particle unto the surface. We must now investigate under what conditions and for what sized particles the thermal forces will dominate.

Combined Thermo-diffusiophoretic forces- The most convient way to examine the combined influence of thermo- and diffusiophoresis is to determine the resultant velocity on a particle. This may be done by first looking at the friction force of a particle of radius a in a gas which is moving with a velocity $\mathbb{V}$. Waldmann and Schmidt give

$$\vec{F}_{fr} = -6\pi\eta a(\vec{v}_p - \mathbb{V})/\mathbb{D} \tag{24}$$

where

$$\mathbb{D} = 1 + A\frac{l}{a} + B\frac{l}{a}e^{-C\frac{a}{l}} \tag{25}$$

and A, B and C are constants with the values

$$\left.\begin{aligned} A &= 1.45 \\ B &= 0.40 \\ C &= 1.0 \end{aligned}\right\} \tag{26}$$

for steady-state evaporation we can write

$$\vec{F}_{fr} + \vec{F}_{th} = 0 \tag{27}$$

so

$$-6\pi\eta a\mathbb{A}\frac{k}{\mathbb{P}}\nabla T - 6\pi\eta a(\vec{v}_p - \mathbb{V})/\mathbb{D} = 0. \tag{28}$$

Thermo-diffusiophoretic scavenging of small aerosol particles.

Assuming all gradients to have only radial components and using equations (20) written in terms of molar fractions we have

$$-6\pi\eta a \mathbb{A}\frac{k}{\mathbb{P}}\frac{dT}{dr} - \frac{6\pi\eta a v_p}{\mathbb{D}} - 6\pi\eta a\frac{\mathbb{B}}{\mathbb{D}}\frac{D}{\gamma_a}\frac{d\gamma_v}{dr} = 0$$

and solving for v_p we have

$$v_p = -\mathbb{A}\,\mathbb{D}\frac{k}{\mathbb{P}}\frac{dT}{dr} - \mathbb{B}\frac{D}{\gamma_a}\frac{d\gamma_v}{dr} \tag{29}$$

where

$$\mathbb{B} = \frac{\sqrt{m_v}}{\gamma_v\sqrt{m_v} + \gamma_a\sqrt{m_a}} \tag{30}$$

Equation (29) gives the velocity of a particle subjected to thermo- and diffusiophoretic forces near an evaporating droplet. Equation (29) can be rewritten using the following relationships

$$\gamma_v = \frac{\bar{m}}{m_v}\frac{\rho_v}{\bar{\rho}} \qquad \gamma_a = \frac{\bar{m}}{m_a}\frac{\rho_a}{\bar{\rho}} \tag{31}$$

so that

$$\frac{1}{\gamma_a}\frac{d\gamma_v}{dr} = \frac{1}{\rho_a}\frac{m_a}{m_v}\frac{d\rho_v}{dr}. \tag{32}$$

Now

$$v_p = -\mathbb{A}\,\mathbb{D}\frac{k}{\mathbb{P}}\frac{dT}{dr} - \mathbb{B}\frac{D}{\rho_a}\frac{m_a}{m_v}\frac{d\rho_v}{dr}, \tag{33}$$

and since

$$q\,(\text{heat flux density}) = -k\frac{dT}{dr} \tag{34}$$

Thermo-diffusiophoretic scavenging of small aerosol particles

the heat flow for an evaporating droplet is

$$L \frac{dm}{dt} = k \frac{dT}{dr} = -q \qquad (35)$$

where L is the latent heat of vaporization. Equation (4) in terms of mass flux density can be written as

$$\frac{dm}{dt} = -D \frac{d\rho_v}{dr} = -\frac{q}{L} \qquad (36)$$

so that

$$v_p = \mathbb{A} \mathbb{D} \frac{q}{\mathbb{P}} - \mathbb{B} \frac{1}{\rho_a} \frac{m_a}{m_v} \frac{q}{L}$$

or

$$v_p = q \left(\mathbb{A} \mathbb{D} \frac{1}{\mathbb{P}} - \mathbb{B} \frac{1}{\rho_a} \frac{m_a}{m_v} \frac{1}{L} \right) . \qquad (37)$$

The total heat flux for a droplet of radius $\mathbb{R}$ is

$$Q = 4\pi \mathbb{R}^2 q \quad . \qquad (38)$$

Previously we have considered the droplet as stationary, however, we are interested in a falling droplet. In this case we have for the heat flux

$$Q = 2\pi \mathbb{R} k (T_{\mathbb{R}} - T_\infty) Nu \qquad (39)$$

where

$$Nu \text{ (Nusselt number)} = 2(1 + \beta Re^{1/2} Pr^{1/3}) \qquad (40)$$

and

Thermo-diffusiophoretic scavenging of small aerosol particles.

$$\left.\begin{aligned} &\beta \simeq 0.4 \\ &Re\,(\text{Reynolds number}) = \frac{2 u_T \mathbb{R}}{\nu} \\ &Pr\,(\text{Prandtl number}) = \frac{\nu \rho_a c_{pa}}{k} \end{aligned}\right\}. \tag{41}$$

Now we have

$$-\dot{q} = \frac{k}{\mathbb{R}} (T_\infty - T_{\mathbb{R}}) \left(\frac{Nu}{2}\right) \tag{42}$$

and

$$v_p = -\frac{k}{\mathbb{R}} (T_\infty - T_{\mathbb{R}}) \left(\mathbb{A}\, \mathbb{D} \frac{1}{\mathbb{P}} - \mathbb{B} \frac{1}{\rho_a} \frac{m_a}{m_v L} \frac{1}{} \right) \left(\frac{Nu}{2}\right). \tag{43}$$

Equations (43) is equivalent to equation [9] of Slinn and Hales if we take their g to have a value of 1.0. Slinn and Hales give

$$1.0 > g > 0.85 .$$

This difficulty can be resolved somewhat by investigating equation (19a). The factor in parenthesis is our $\mathbb{B}$

$$\mathbb{B} = \left(1 + \left[\frac{(\sqrt{m_v} - \sqrt{m_a})}{\gamma_v \sqrt{m_v} + \gamma_a \sqrt{m_a}}\right] \gamma_a\right). \tag{44}$$

Schmidt and Wachmann suggest

$$\mathbb{B}' = \left(1 + \left[A' \frac{m_1 - m_2}{m_1 + m_2} + B' \frac{d_1 - d_2}{d_1 + d_2}\right] \gamma_a\right)$$

where $A' = 0.95$ and $B' = -1.05$ and d_1 and d_2 are the molecular diameters of water vapor and air given by

Thermo-diffusiophoretic scavenging of small aerosol particles

$$\left.\begin{aligned} d_1 &= d_v = 4.1\ \text{Å} \\ d_2 &= d_a = 3.7\ \text{Å}\ . \end{aligned}\right\} \quad (45)$$

Using the previous values given for m_a and m_v we have

$$\mathbb{B} = 0.79$$

$$\mathbb{B}' = 0.72\ .$$

We may assume then that

$$g = \frac{\mathbb{B}'}{\mathbb{B}} = 0.91\ . \quad (46)$$

Since Waldmann and Schmidt conclude that $\mathbb{B}'$ shows better agreement with theory we will replace $\mathbb{B}$ with $g\mathbb{B}$ in our equation (43), hence

$$\mathbb{C}_p = -\frac{k}{\mathbb{R}}(T_\infty - T_R)\left(\mathbb{A}\mathbb{D}\frac{1}{\mathbb{P}} - g\mathbb{B}\frac{1}{\rho_a}\frac{m_a}{m_v}\frac{1}{L}\right)\left(\frac{Nu}{2}\right)\ . \quad (43a)$$

The factor which determines whether thermo or diffusiophoretic forces dominate is

$$\mathbb{L} = \left(\mathbb{A}\mathbb{D}\frac{1}{\mathbb{P}} - g\mathbb{B}\frac{1}{\rho_a}\frac{m_a}{m_v}\frac{1}{L}\right). \quad (47)$$

$\mathbb{L}$ depends mainly on a and k_p, therefore we should like determine $\mathbb{L}$ for various values of a or ℓ/a and k_p or k_p/k. Using (22), (23), (25), (26) and the following values

$$\left.\begin{aligned} \mathbb{P} &= 850\ \text{mb} = 8.5\times10^5\ \text{dynes/cm}^2 \\ g &= 0.91 \\ \mathbb{B} &= 0.79 \end{aligned}\right\} \quad (48)$$

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$$\left.\begin{aligned} &\rho_a = 0.0012 \ \text{gm/cm}^3 \\ &m_a/m_v = 1.6 \\ &L = 2.455 \times 10^{10} \ \text{erg/gm}, \end{aligned}\right\} \quad (48)$$

a table of values for $\mathbb{L}$ was made for various values of l/a and k_p/k. This is shown in the following table

$\mathbb{L} \times 10^8$ cm²/dynes

l/a \ k_p/k	0.5	1.0	5.0	10.0
0.1	13.05	10.89		4.62
1.0	9.84	9.46	8.98	8.90
10.0	10.34	10.33	10.32	10.32
100.0	10.57	10.57	10.57	10.57

for

$$v_p = -\frac{k}{\mathbb{R}} (T_\infty - T_R) \mathbb{L} \left(\frac{Nu}{2}\right) \quad (49)$$

Thermo-diffusiophoretic scavenging of small aerosol particles.

In our velocity equation a positive value of $\mathbb{I}$ yields a negative value of v_p which means flow into the droplet for an evaporating case. From the values given, particles much larger than 10 ℓ and with high thermal conductivity with respect to air will not be picked up as easily as smaller particles.

Now that we have an equation for the combined thermo-diffusiophoretic velocity we can investigate the efficiency of this process in removing particles from the atmosphere.

Washout coefficients:

Collection efficiency – Let n be the number density of particles of radius a in the atmosphere. Then the number of particles collected by a droplet of radius $\mathbb{R}$ in unit time can be written as

$$\frac{dN}{dt} = 4\pi \mathbb{R}^2 v_p n . \tag{50}$$

In general we would have a number of droplets of all sizes collecting particles. Let $\mathbb{N}(\mathbb{R})$ be the volume density of droplets of size $\mathbb{R}$, then

$$\mathbb{N}(\mathbb{R})\frac{dN}{dt} = 4\pi \mathbb{R}^2 v_p n \mathbb{N}(\mathbb{R})$$

which may be integrated over the range of droplet sizes of interest

$$\int \mathbb{N}(\mathbb{R})\frac{dN}{dt}\, d\mathbb{R} = n\int 4\pi \mathbb{R}^2 v_p \mathbb{N}(\mathbb{R})\, d\mathbb{R} = -\frac{dn}{dt} . \tag{51}$$

The term on the left is then the total rate of capture of particles or reversed, the total rate of decrease of particulate material in the atmosphere.

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Then

$$-\frac{1}{n}\frac{dn}{dt} = \int 4\pi \mathbb{R}^2 v_p \, \mathbb{N}(\mathbb{R}) \, d\mathbb{R} \qquad (52)$$

where the integral term on the right is called the washout coefficient, Λ,

$$\Lambda = \int 4\pi \mathbb{R}^2 v_p \, \mathbb{N}(\mathbb{R}) \, d\mathbb{R} \, . \qquad (53)$$

It will be convenient to use the flux density of droplets of size $\mathbb{R}$, defined by

$$\mathbb{F} = \mathbb{N}(\mathbb{R}) \, u_T \quad \left({}^{\#\text{drops}}/_{\text{area} \cdot \text{time} \cdot \text{radius}} \right) \qquad (54)$$

So that equation (53) becomes

$$\Lambda = \int \frac{4\pi \mathbb{R}^2 v_p}{u_T} \, \mathbb{F} \, d\mathbb{R}$$

or upon substitution for v_p from (49)

$$\Lambda = \int \frac{4\pi \mathbb{R} k}{u_T} (T_\infty - T_{\mathbb{R}}) \, \mathbb{L} \left(\frac{Nu}{2} \right) \mathbb{F} \, d\mathbb{R}$$

$$= 4\pi k (T_\infty - T_{\mathbb{R}}) \, \mathbb{L} \, \mathbb{F} \int \frac{\mathbb{R}(1 + A Re^{1/2} Pr^{1/3})}{u_T} \, d\mathbb{R} \qquad (55)$$

In order to evaluate the integral we will need to find an expression for u_T as a function of $\mathbb{R}$ plus the limits of the integration.

For particles or droplets whose radius is less than 40μ we may use Stokes law to relate u_T to $\mathbb{R}$. This may be written as

$$u_T = \frac{2 \rho_c g \mathbb{R}^2}{9 \eta} \qquad (56)$$

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where ρ_L is the density of the droplet, η is the dynamic viscosity and g is the acceleration of gravity.

For larger droplets Dingle and Lee give the following third-degree regression formula for u_T

$$u_T = -17.8951 + 448.9498d + 16.3719d^2 - 45.9516d^3 \qquad (0.1 \leq d \leq 1.4\,mm) \tag{57}$$

where u_T is in cm/sec and d is the diameter in mm. For our calculations $\mathbb{R}$ is in cm, hence (57) can be rewritten as

$$u_T = -17.8951 + 8978.996\mathbb{R} + 6548.76\mathbb{R}^2 - 367612.8\mathbb{R}^3 \tag{58}$$

where d was replaced by $2\mathbb{R}$. The range for (58) is only down to 50μ, so in the calculation (56) was extended past 40μ to 50μ. The use of eqn. (56) for larger droplets does introduce some error however it should affect the calculations appreciably. Now having an expression for u_T in terms of $\mathbb{R}$ we need to determine the limits of integration.

Byers gives the following expression for the evaporation of a droplet with radius r_h at a height h falling to the the ground

$$\int_{r_0}^{r_h} \frac{u_T r}{(1+\beta Re^{1/2} Pr^{1/3})}\,dr = \int_0^h \frac{S-1}{a+b}\,dz \tag{59}$$

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where S is the saturation ratio and a and b are constants determined by the ambient thermodynamic conditions. The left hand side of (59) can be calculated using (56) and (58) and assuming $r_0 = 0$. Then assuming S, a and b do not change over the distance h, a value of h can be determined for the value of r_n chosen.

Once these values had been determined equation (55) could be evaluated. This was the process followed in determining the curve shown in fig 2.

A discussion of the results is given in the attached Memorandum.

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