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SPECTRAL ANALYSIS
A PROTOTYPE STUDY

by

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ABSTRACT

Many critical areas should be considered when planning and performing spectral analyses of fluctuating signals. The analysis of data representing fluctuating water pressures in a prototype stilling basin is used to illustrate these considerations. The prime consideration in planning the analysis is determining how the results will be used. Accuracy of the analyses should be consistent with design requirements. The analog spectral analyzer used in this study cost approximately $9000 in 1967. The equipment is limited to the analysis of the frequency-amplitude spectrum at a single point on the structure during a given time interval. Digital equipment would be more versatile but would cost several times as much as the analog equipment. The capabilities and limitations of the commercially available instrument that was used in this study are discussed. Calibration procedures are described. The prototype stilling basin was designed to dissipate the energy of high-velocity flow from 2 hollow-jet valves. The basin was covered and the divider wall was thus supported at both top and bottom providing the opportunity to measure the pressures without significant modification by flexure of the wall.

DESCRIPTORS--/ *frequency analyzers/ frequency analysis/ accelerometers/ transducers/ hollow jet valves/ hydraulic structures/ pressure measuring instruments/ *outlet works/ *stilling basins/ hydraulics/ calibrations/ *instrumentation/ piezometers/ *spectrum analysis/ test procedures/ vibrations
IDENTIFIERS--/ spectral analysis/ vibration tests/ accuracy

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INTRODUCTION

The purpose of this paper is to describe critical areas which should be considered in planning and performing spectral analyses of fluctuating signals. The analysis of data representing fluctuating water pressures in a prototype stilling basin is used to illustrate these considerations.

The importance of spectral analysis becomes apparent when one reviews the structural failures of divider walls which have been experienced in several prototype stilling basins. These failures have apparently been due to fatigue caused by vibration of the divider walls which were excited to resonance by fluctuating water pressures within the basin. An analysis of the fluctuating pressures observed in hydraulic models of stilling basins will reveal predominant frequencies and amplitudes of the fluctuations which must be considered to adequately design divider walls and prevent future failures. Unfortunately, most of the stilling basin model investigations were performed prior to the development of commercially available equipment to perform spectral analyses.

In recent years, spectral analysis techniques have been applied more and more to hydrodynamic problems. For instance, in 1967 King1/ summarized pilot tests in applying spectral analysis to data obtained from hydraulic models. The analyses revealed certain basic characteristics of the narrow-band random pressure fluctuations. For example, most of the energy is contained in the band of frequencies between zero and 10 Hz (prototype) and the largest amplitudes are present at frequencies less than approximately one-half Hz. (These values assume Froude Law scaling.)

As spectral analysis becomes recognized as an efficient and useful analytical tool, the need for adequate planning of data acquisition methods will increase. In addition, a more detailed understanding of the errors inherent in the various spectral analysis techniques will be required. Hopefully, this paper will be of assistance in planning spectral analyses and serve to stimulate further interest in the subject.

CONSIDERATIONS IN PLANNING THE ANALYSIS

The prime consideration in planning the analysis is determining how the results will be used. The need for information to aid designers in

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1/King, D. L., Analysis of Random Pressure Fluctuation in Stilling Basins, Proceedings, 12th IAHR Congress, Volume 2, pages 210-217, 1967.

analyzing the response of hydraulic structures to dynamic loading is obvious. The method used in converting the raw model data to an applicable form for analyzing prototype performance will be governed by the particular requirements of the designer. For example, will the designer be satisfied with knowledge of the energy spectrum of the hydrodynamic forces at a point and the natural frequency of the structure? Or, is a more detailed correlation of forces acting at various points on the structure necessary?

The nature of turbulence would not lead to an expectation that the frequency band containing most of the energy of the signal would change significantly from point to point. However, the total energy will vary from point to point and, as a result, the amplitudes in each spectrum will be different. Analysis at a single point should produce the general shape of the spectrum and thus warn the designer of possible resonance in the structure. Perhaps the most important limitation in applying spectral analysis at one point on the structure is the inability to determine the net transient forces acting on the structure. This can be overcome in part by using transducers which measure the instantaneous differential pressure between two piezometers. However, the overall effects of transient phenomena (such as surges moving along the structure) still cannot be determined by this technique.

The relationship between the accuracy which is needed for design and the accuracy which can be obtained from the analysis must also be considered. Previous design practice has given little attention to the natural frequency of the structure because information concerning the hydrodynamic frequency spectrum was not available. However, structural analysis techniques have progressed to such a point that the natural frequency of a structure can be calculated quite accurately. Until the present time, effects of hydrodynamic loading have generally been considered by including additional static load on the wall. For example, stilling basin divider walls have sometimes been designed under the assumption of full hydrostatic load on one side of the wall with zero load on the opposite side. Although this procedure seems overconservative, cracks have been observed to develop at the base of prototype stilling basin center walls. This indicates a need to know both the dynamics of the flow and the response of the wall. Perhaps the most important consideration would be a knowledge of the excitation frequencies. This study indicated that by using analog equipment, the amplitude of the true spectral density function could be determined, at best, to within plus or minus 30 percent (See Table 1, page 6). On the other hand, the frequency of spectral peaks could be determined within plus or minus 2 percent. Thus, for a dynamic analysis of flow in stilling basins, the analog analyzer is accurate in those areas where accuracy is required.

Finally, the cost of performing the analysis must be considered. In general, the cost of the processing equipment can be expected to increase as the degree of sophistication of the analysis increases. The cost of the analog spectral analyzer described in this paper was approximately $9,000 in 1967 when the equipment was purchased. The equipment is limited

to the analysis of the frequency-amplitude spectrum at a single point on the structure during a given time interval. The cost of an analog cross correlator, which would allow a more versatile analysis, is in the $1,000 price range.

Prices for equipment which would enable digital spectral analysis of pressure fluctuations might range from $75,000 to $100,000. Digital equipment is very versatile; cross-correlations, integration of pressure fields and many statistical examinations can be performed with ease. Results are in immediately usable form and calibration is generally not a problem. Accuracy is limited primarily by the quality of the recorded data, the sophistication of the program, and computer memory size.

From the above price considerations it is obvious that several pieces of special purpose analog equipment can be obtained at a much lower cost than that required for digital analysis. However, one must realize that this apparent saving is made with a sacrifice in versatility and probably in accuracy. The cost of performing the analysis is probably comparable with either method. The analog equipment requires a long time for the analysis. The digital analysis requires development of sophisticated programs. Obviously, there is an economic breakpoint between the two methods depending upon the number of analyses which need to be performed and the sophistication of analysis required.

Basic data acquisition equipment which is necessary for either method of spectral analysis should also be considered. Such equipment includes pressure transducers, oscillograph recorders and operational amplifiers, and magnetic tape recorders. Cost of this equipment varies widely. Transducers cost several hundred dollars each; oscillograph recorders range in price from $2,000 to $6,000, and tape recorders cost from $2,000 to $10,000, depending on the number of channels, recording and playback speeds, etc.

It was noted earlier that nearly all of the signal energy in stilling basin studies is contained in frequencies less than about 10 Hz. Therefore, a very large frequency response in transducers and recorders is not necessary if they are to be used only for this purpose.

DESCRIPTION OF THE ANALYZER

The spectral analyzer used for the studies is a commercially available analog instrument, Figure 1. It utilizes a single narrow frequency bandpass filter which is swept through the range of frequencies using the heterodyne principle. A companion unit to the spectrum analyzer was used to condition the output signal from the spectral analyzer. The companion unit, also shown in Figure 1, is known as a power spectral density analyzer and consists of squaring and integrating networks. In the description which follows, the two units are considered to be components of a single piece of equipment, called the analyzer, used to obtain the power spectral density. The range of frequencies which can be analyzed is from 0.5 to 2,500 Hz, covered in several frequency bands. Thus, if it is found

that the frequencies of interest are in only one band, the analyzer can be set to obtain the greatest accuracy for that particular frequency range.

Since this type of analyzer has a lower-frequency limitation, the analysis only applies to the fluctuating component of the signal about some mean. The absolute value of the mean is usually determined by an integrating voltmeter or visually from a strip chart. The lower-frequency limitation can create difficulties when analyzing a signal which is composed of many low frequency components. However, special techniques can be employed to synthetically extend the frequency range to lower values. Recognition of the lower-frequency limitation is necessary in realizing that the spectrum obtained does not correspond to the true spectrum down to zero frequency.

An important factor in the analysis is knowledge of the noise filter band width. Usually the filter band widths are given by the manufacturer of the analyzer. However, a relatively simple procedure allows measurement of the filter band width of the analyzer2/. The analyzer was found to have a noise filter band width of about 0.2 Hz for the scan time and frequency range used in the study.

Since the amplitudes of the individual frequency components are not always constant, some type of amplitude averaging is usually employed in the analysis. The analyzer has two types of averaging. One is an integrated average in which the amplitudes are summed over some time interval. The time intervals can be either preset or triggered by a signal from the tape loop. The other type of averaging available consisted of an RC (resistance and capacitance) circuit which has time constants of 1, 5, 10, 50, and 100 seconds. For all of the analyses used in this study, RC averaging was employed.

To examine the frequency range and still have sufficient resolution, an analysis time of more than 2 hours was necessary. Since only relatively short records were available on tape, a tape loop was utilized. About 20 feet of tape representing about 45 seconds of record were used to form the loop. Although this sample length is relatively short, the assumption was made that the record obtained was a representative sample of the fluctuations.

ACCURACY OF THE ANALYSIS

Signals containing random fluctuations in amplitude and frequency are analyzed by both analog and digital methods. Each method has its own inherent accuracy which depends upon the mathematics of the method and the sample length available for analysis. To obtain a realistic concept of the

2/The Measurement and Interpretation of Ordinary Power Spectra for Vibration Problems, A. G. Piersol, NASA Contractor Report CR-90, 1964 (For Sale by Office of Technical Services, Department of Commerce, Washington, D.C. 20230).

results given by the analysis, the relative accuracy of the results should be examined. This error analysis should include the lowest frequency which can be analyzed, the accuracy of the spectral amplitudes, and the ability to separate spectral peaks from the record. Several references 2/, 3/, 4/ are available which provide guidelines to be used in estimating the accuracy of the results. The application of these guidelines to the analog equipment used in this study is described in the following paragraphs.

The lowest frequency range of the analyzer was 0.5 to 2 Hz. The original recording on the tape was made at 7-1/2 ips. By changing the tape playback speed from 7-1/2 to 30 ips, the frequency range could be synthetically changed to cover 0.125 to 0.5 Hz. This technique is very useful in expanding the frequency range of the analyzer. However, as will be shown shortly, some sacrifice in accuracy of the spectral analysis must be expected when this technique is used.

Since the signal to be analyzed consists of random amplitudes, a statistical uncertainty exists in repeated measurements. The magnitude of the uncertainty is usually expressed by:

$$\sigma = (G_t - G_m), \qquad (1)$$

where σ is a value of the difference which is not expected to be exceeded in two-thirds of the measurements. "G" represents the spectral amplitude at any particular frequency, and the subscripts refer to true and measured values. In dimensionless terms, the standard error is given by:

$$\varepsilon = \sigma/G_t \qquad (2)$$

From Reference 1, the standard error with RC averaging is given by:

$$\varepsilon = 1.04/(BT)^{1/2} \qquad (3)$$

"B" in this expression is the noise band width and "T" is the available record length. The noise band width is one means of describing the filter characteristics of the analyzer. The experimental determination of the noise band width is described in the "calibration" section of this paper. The "RC" time constant in seconds should also be approximately equal to the record length. Greater accuracies cannot be obtained by using time constants longer than the record length. Therefore, to improve accuracies either the sample length must be increased or the noise band width made larger. Note at this point that when lower frequencies are analyzed through an increase in the tape playback speed, greater errors in spectral amplitude will result due to the shorter available record length "T".

3/Blackman, R. B., Tukey, J. W., The Measurement of Power Spectra, Dover Publications, S 507.
4/Lee, Y. W., Statistical Theory of Communication, Wiley and Sons, 1966.

Morton, W., An Engineering Guide to Spectral Analysis, Accuracies Incorporated, June 1969

To insure that the analyzer views the entire signal for each frequency, that it responds to the frequency peaks, and that the "RC" averaging has time to reach a value representing true averaging, rather slow scan rates must be used. In these studies, it was found that the necessity for providing proper averaging placed the greatest restriction on the scan rate. For instance, with a noise filter band width of 0.2 Hz and "RC" averaging time constant (k) of 50 seconds, a scan time of over 2 hours was required. This scan time (ST) can be computed from:

$$ST(\text{hours}) = (4k \times \text{Range})/(3600\ B) \qquad (4)$$

The following table gives a summary of values pertaining to the spectral analyses performed in this study:

Table 1

Figure	Frequency Range Analyzed Hz	Filter Band Width Hz	Standard Error	Rms of signal in band width ft of water
6	0.5 - 10	0.20	0.33	2.0
7	0.25 - 2.5	0.04*	0.78	2.5
8	0.5 - 10	0.22	0.33	4.4

*Equivalent band width with the 30-ips playback speed.

Resolution is the ability of the analyzer to separate spectral peaks from the record. One criterion for proper resolution, from Reference 2, is that the noise band width of the analyzer be "less than one-fourth the band width of the narrowest peak in the spectrum being analyzed." The band width of the narrowest peak is defined at the half power points of the spectral peak. Thus, the analyzer should not be expected to separate or "resolve" any peaks which are narrower than about .8 Hz in Figures 6 and 8, and narrower than .16 Hz in Figure 7. In essence, this means that the many small peaks shown in the analyses should be ignored. Valid statements can only be made for the trends shown in Figures 6, 7, and 8. For instance, in Figure 8, spectral peaks apparently occur near frequencies of about .9, 1.6, and 2.1 Hz, etc. One cannot say with certainty that spectral peaks occur at each of the jagged points around the aforementioned frequencies.

From the resolution considerations, it would appear that the filter band width should be set as small as possible to obtain the best resolution. However, Equation 3 indicates that the filter band width should be made as large as possible to minimize the amplitude errors. A compromise between these two conflicting requirements is usually made in favor of increased resolution.

CALIBRATION

The instructions supplied with the analyzer indicate that the equipment is intended primarily to determine the relative amplitude of the spectral power peaks. However, the absolute amplitude of the spectral function can be obtained through proper calibration. With random signals, a white-noise generator is normally used for an accurate calibration. However, considering the large relative error that is present in the amplitude of the spectral function, a method which would provide an approximate but rapidly obtained calibration seemed justified. The method to be described achieves both the amplitude and the frequency calibration simultaneously using only an oscillator and an rms meter. It should be emphasized that the amplitude calibration with this method is approximate. However, the frequency calibration is as accurate as can be obtained.

The following steps illustrate the calibration procedure:

Step 1. Determine the relationship between the voltage on the tape and the equivalent pressure in feet of water. For example, a 100-mv voltage signal on the tape represents 2.5 feet of water pressure head.

Step 2. With the analyzer, analyze a sinusoidal signal from the oscillator which falls within the frequency band to be analyzed. For example, a 25-mv rms signal at 10 Hz was analyzed for Test T-2, Figure 6. This is equivalent to a mean squared pressure head of 0.39 ft^2 of water (25 mv x 25 mv x 6.25 ft^2 $H_2O/(1. \times 10^4\ mv^2)$).

Step 3. Integrate the area under the analyzed signal and divide by the amplitude of the signal to determine the noise band width. A planimeter can be used for this step. The results of this step gave a noise band width of 0.204 Hz for Test T-2.

Step 4. The calibration for the amplitude of the power spectral function is given approximately by the relationship $G = y^2/B$. In the example of Test T-2, the relationship is $G = 0.391/0.204 = 1.92$ ft^2/Hz.

This procedure can be repeated at other frequencies within the band to be analyzed to determine other frequency markers. If the analysis sweep time was chosen long enough, the calibration of G should not change significantly.

The calibration can be checked by the following method:

First, integrate the area under the power spectrum of the signal from the pressure transducer. The square root of the value obtained is equal to the rms of the fluctuating component.

Second, pass the signal to be analyzed through a true rms meter. If the meter has a low enough frequency response, the two values of the rms should agree.

If the tape speed has been increased as was done for Test T-4, the rms values will not agree. The signal amplitude as determined by the rms meter will appear to be too large. However, the equivalent rms value at the original speed can be obtained from the relationship

$$rms_o = (S_o/S_1)^{1/2}\ rms_1 \qquad (5)$$

where S is the tape speed. This equivalent rms value is the one which should be used in the comparison. The comparison just described was not successful in these tests since the rms meter used had a lower-frequency response of about 2 Hz. Most of the energy content in the signal of Test T-2 was in frequencies below 2 Hz, Figure 6.

PROTOTYPE APPLICATION

Spectral analyses were performed on pressure data from a prototype stilling basin designed to dissipate the energy of high-velocity flow from two hollow-jet valves. Transducers were installed in both sides of the divider wall of the basin, Figure 2, to determine the magnitude of pressure fluctuations acting on the structure.

Previous experience has shown structural damage in stilling basins with unsupported divider walls. In the case described in this paper, however, the basin was covered and the divider wall was thus supported at both top and bottom. This arrangement provided the unique opportunity to measure the pressures without significant modification by the flexure of the wall. Measurement of vibration with accelerometers located on the wall, Figure 2, supported the assumption that the wall was rigid.

The stilling basin is 45.5 feet deep, 115.7 feet long, and contains two 17.8-foot-wide bays. The divider wall is 4 feet, 4 inches thick, and the hollow-jet valves are 84 inches in diameter. The stilling basin is designed for a total maximum discharge of 5,000 cfs (2,500 cfs through each valve), Figure 3.

The spectral analyses represent data from Transducers R-1 and R-4 for a discharge of 5,000 cfs. Tailwater depth, measured from the basin floor, was 37.9 feet. The hollow-jet valves were 49 percent open, with a total head of 425 feet immediately upstream from the valves.

Signals from the transducers were recorded simultaneously on a direct-writing oscillograph, Figure 4, and a 14-channel magnetic tape recorder. Upon return to the laboratory, portions of the tape recording were selected for formation of loops and subsequent spectral analysis.

The transducers were of the unbalanced strain gage type, with a range of 0-500 psia. The transducers were mounted with their 5/8-inch stainless steel diaphragms flush with the concrete surface, Figure 5. The natural frequency of the transducers is approximately 8,000 Hz, and they are compensated for temperature over a range of minus 65° F to plus 250° F.

The bridge excitation voltage for the transducers was supplied by carrier preamplifiers as part of a direct-writing recording system. The frequency response of the oscillograph recorder is 0-150 Hz. The recorder has a variable chart speed and heated stylus. Transient response is 5 milliseconds from 10 to 90 percent of final deflection.

The 14-channel (plus voice) magnetic tape recorder (1-inch tape) was equipped with FM record/reproduce inserts. Record and playback speeds of 1-7/8, 3-3/4, 7-1/2, 15, 30, and 60 inches/second are available, giving a possible playback capability of up to 32 times faster or slower than the recording speed. The recorder conforms to standard IRIG (Inter-Range Instrumentation Group) specifications. All recordings were made at 7-1/2 ips.

The results of the analysis are presented in Figures 6 through 8. With Piezometer R-1, the spectrum indicates a peak around 0.4 Hz, Figure 6. However, since this peak is outside the range of the analyzer, a run was made for the frequency range 0.125 to 2.5 Hz, Figure 7. This clearly identified a peak of 0.37 Hz. The significant part of the analysis is the presence of the maximum energy between zero and 2 Hz. In addition, the increase in energy as the frequency decreases should be noted on all the analyses.

With Piezometer R-4, Figure 8, the energy spectrum encompassed a larger band width with peaks around 0.9, 1.6, and 2.1 Hz.

Although the analysis presented here represents data taken at only two points, it does indicate that training walls in stilling basins should be designed to have natural frequencies above about 5.0 Hz. This conclusion is in general agreement with data obtained from model studies1/.

The percentage contribution of the fluctuating pressure to the total effective pressure is given by:

$$\text{Fluctuating component } (\%) = \left(\int_0^\infty G df\right)^{1/2} / \left(\overline{\text{mean}}^2 + \int_0^\infty G df\right)^{1/2} \qquad (6)$$

By this definition, the fluctuating component represented about 10 percent of the total effective pressure.

CONCLUSIONS

The analog spectrum analyzer is a relatively inexpensive piece of equipment which can be used to determine the frequency ranges and approximate spectral densities of fluctuating signals. However, the results of the analysis must be critically examined for their relative accuracy. A knowledge of the equipment's characteristics and limitations is also essential for proper planning of the test program as well as for conducting the analysis. Failure to consider these points can lead to misinterpretation of the results.

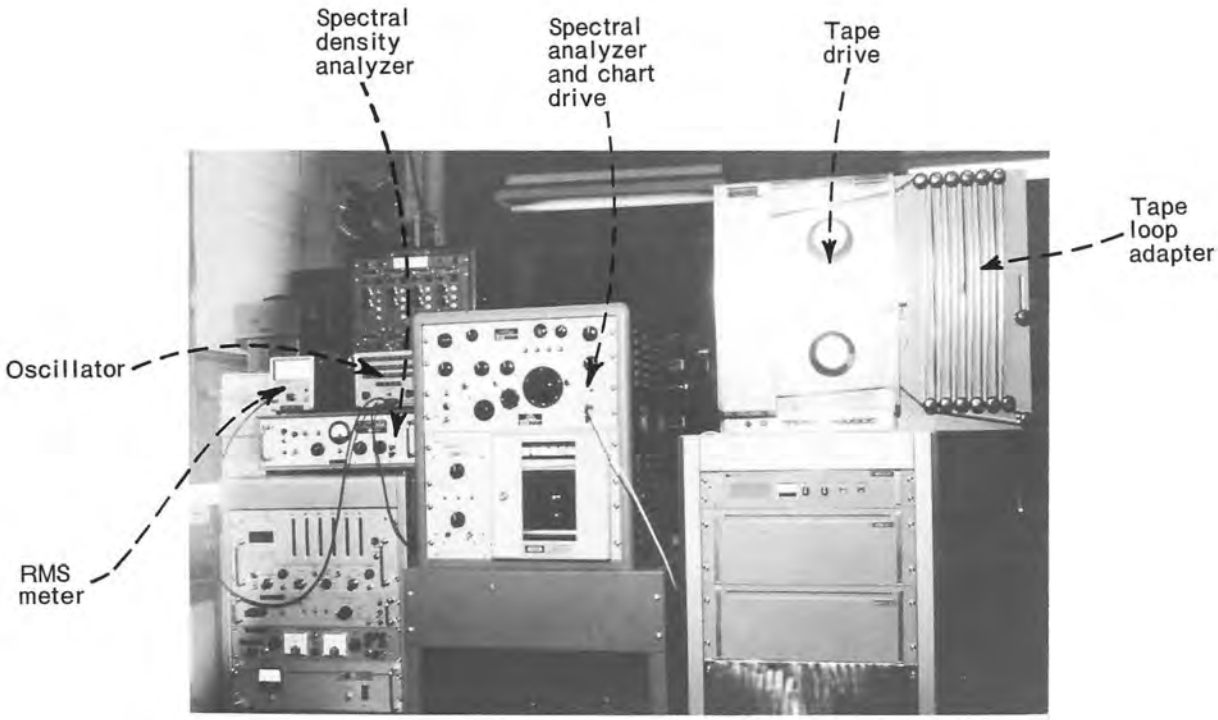

Figure 1. Magnetic tape and spectral analysis equipment. Photo PX-D-60408 NA

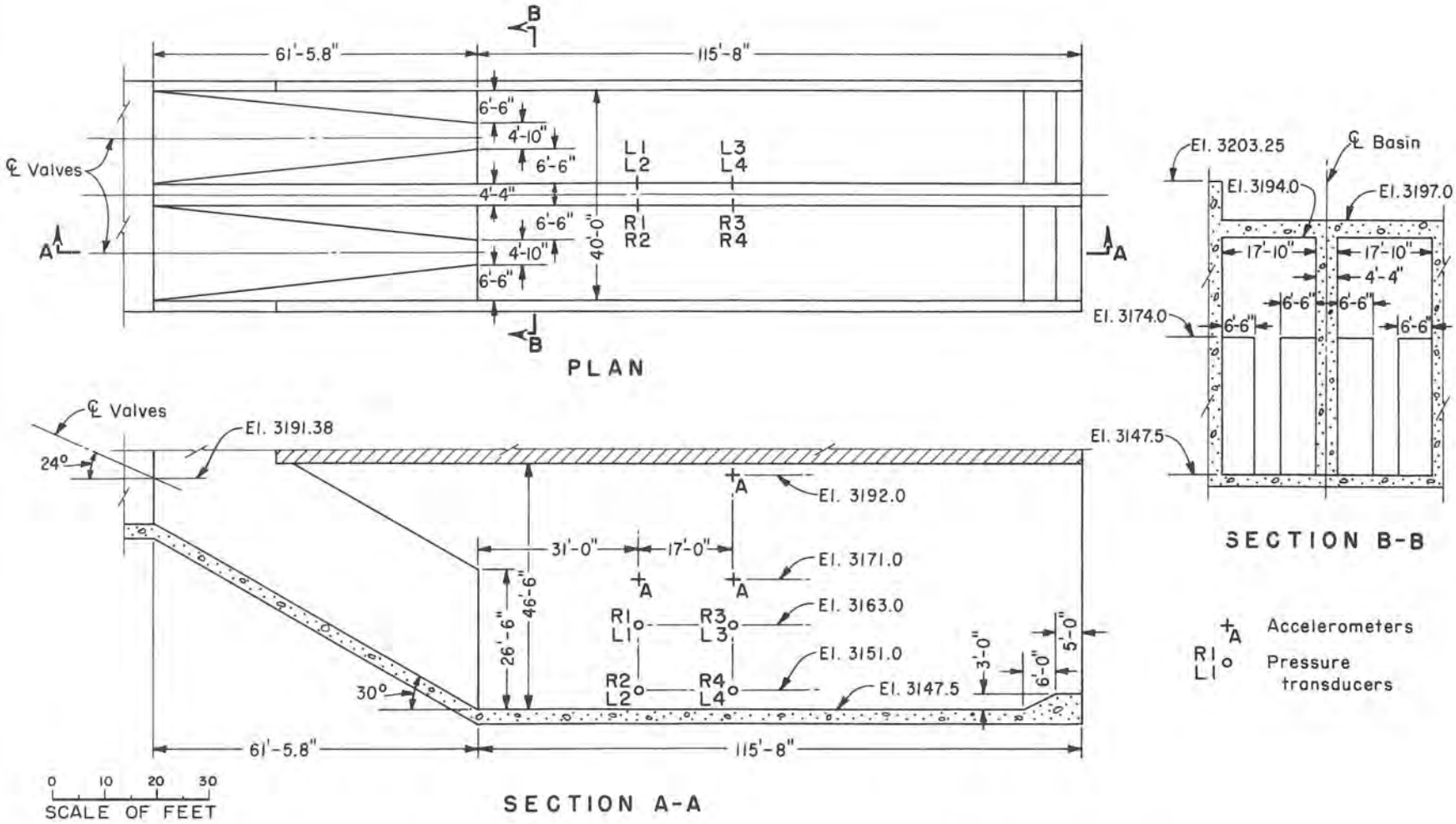

FIGURE 2 - LOCATION OF PRESSURE TRANSDUCERS AND ACCELEROMETERS IN OUTLET WORKS STILLING BASIN

Figure 3. Outlet works discharging 5,000 cfs (2,500 cfs through each hollow-jet valve). Photo PX-D-48578

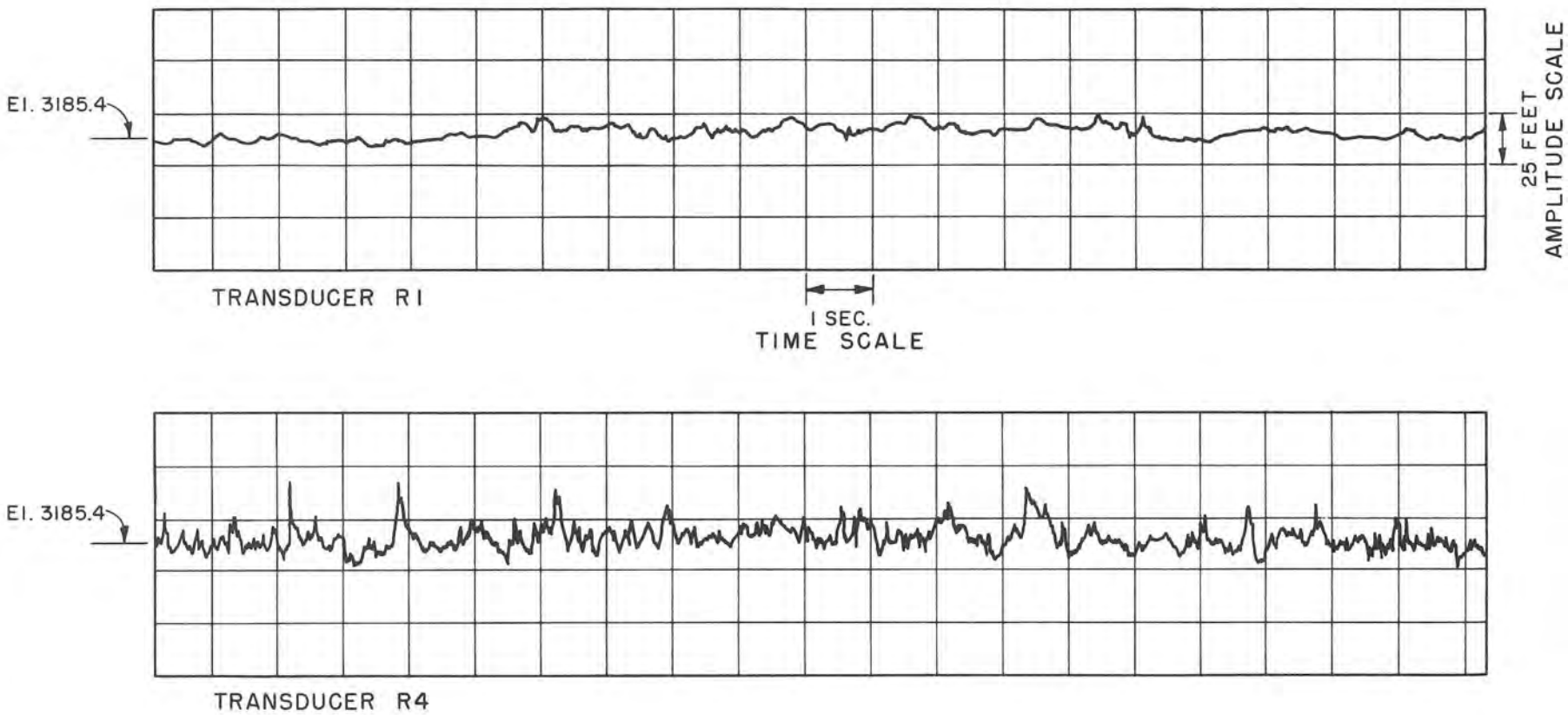

FIGURE 4 - OSCILLOGRAPH TRACES OF ANALYZED SIGNALS

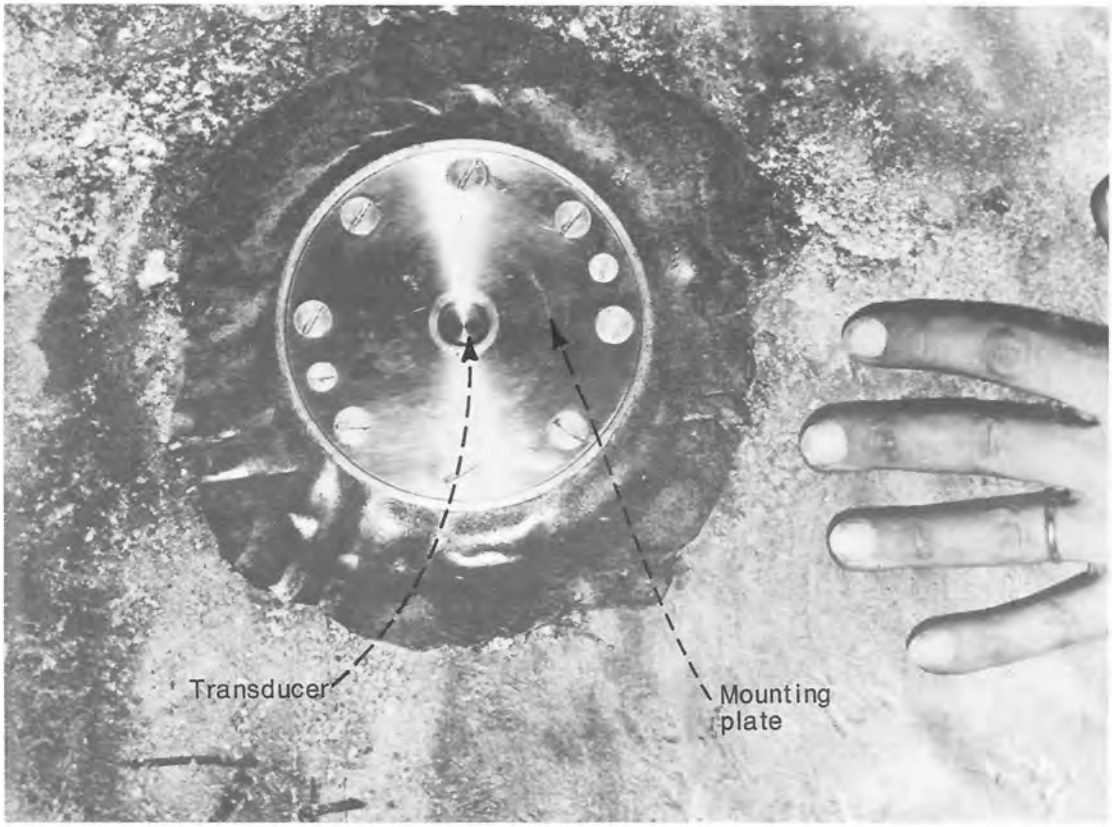

Figure 5. Installation of pressure transducer in divider wall. Photo P-459-640-3089

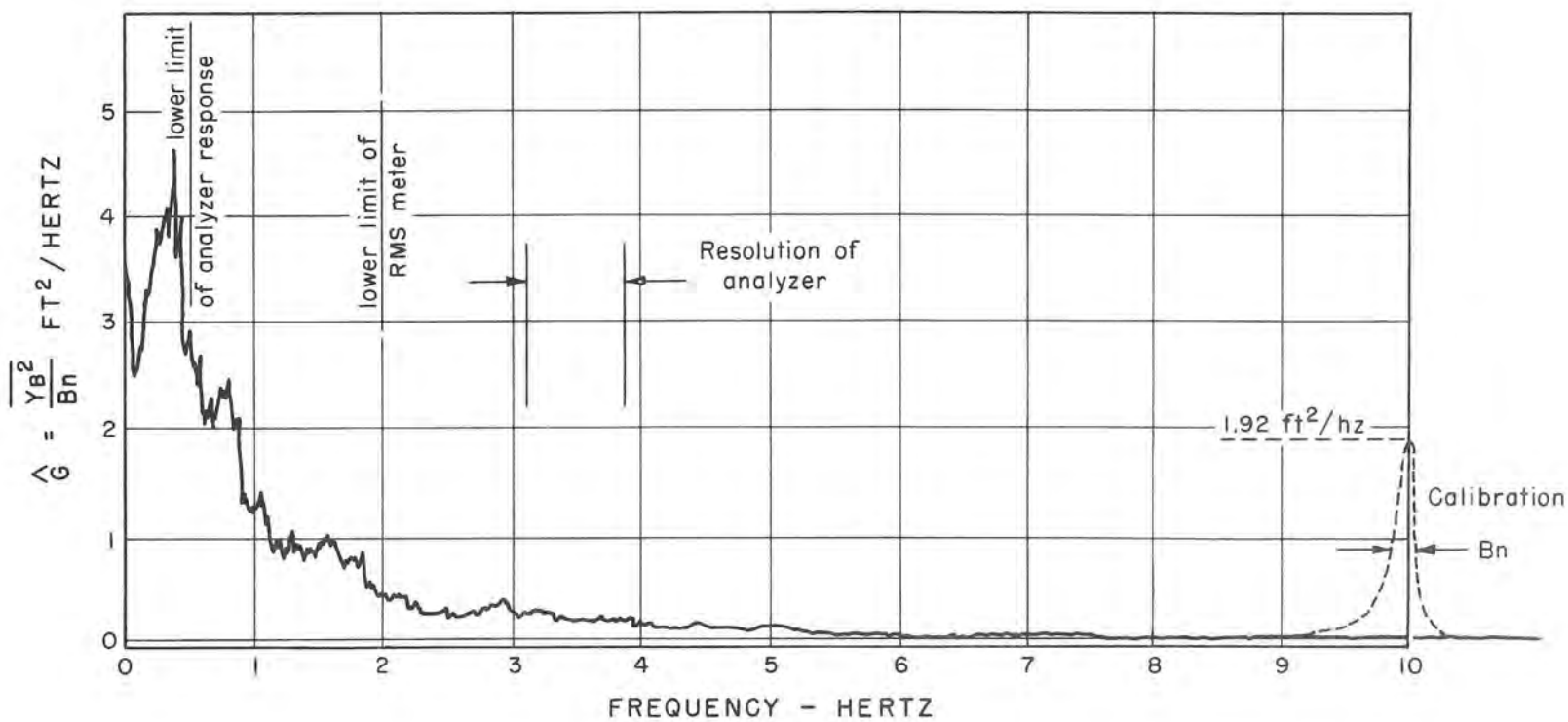

FIGURE 6 - SPECTRAL DENSITY ANALYSIS FOR PIEZOMETER R1.
TAPE SPEED = 7½ ips, SWEEP WIDTH = 10 hz,
AVERAGE AMPLITUDE SQUARED, RC TIME CONSTANT = 50 sec.,
ANALYSIS TIME = 2 hr. Bn = .204 hz RMS = 2.0 ft.

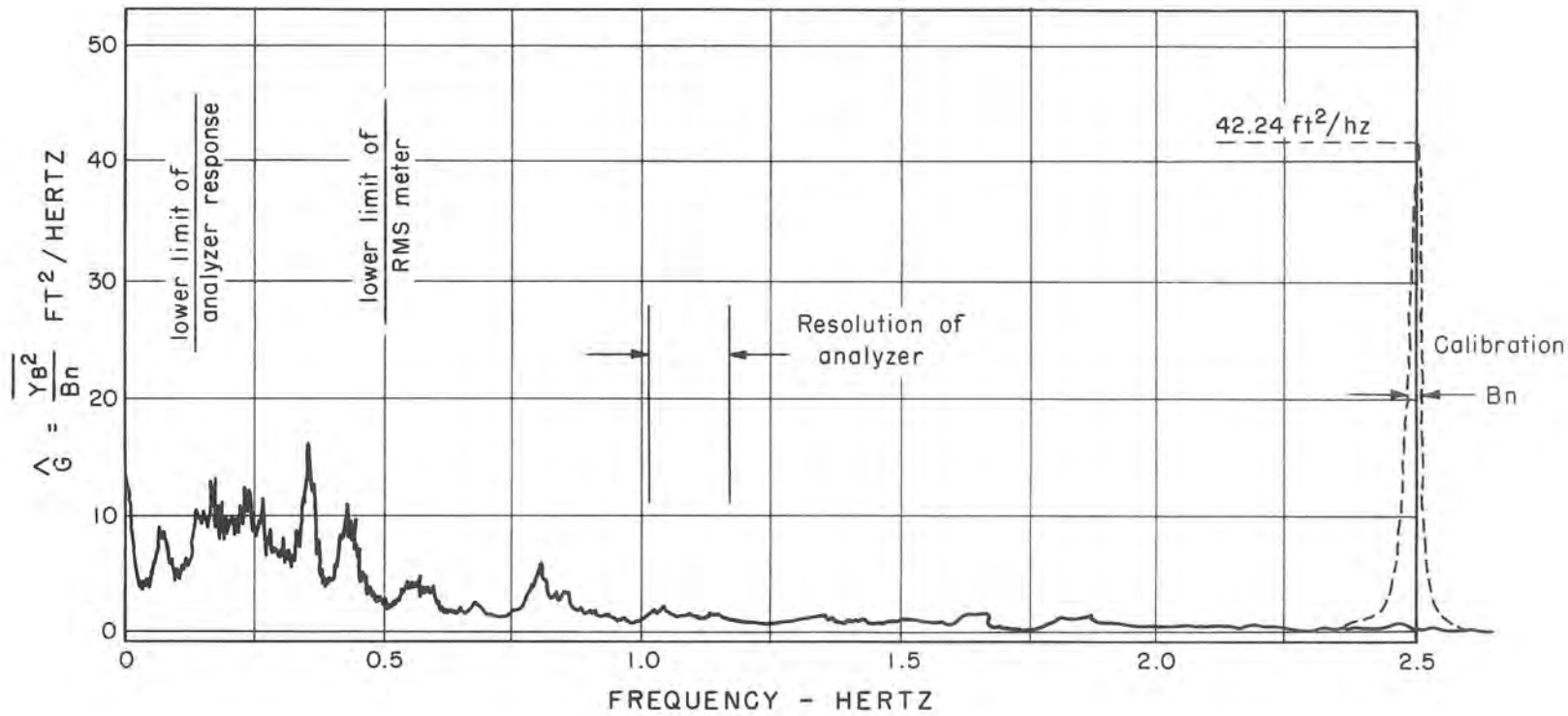

FIGURE 7 - SPECTRAL DENSITY ANALYSIS FOR PIEZOMETER R1.
TAPE SPEED = 30 ips, SWEEP WIDTH = 10 hz (2.5 hz)
AVERAGE AMPLITUDE SQUARED, RC TIME CONSTANT = 10 sec.,
ANALYSIS TIME = 1 hr Bn = .147 hz (.0368 hz) RMS = 2.5 ft

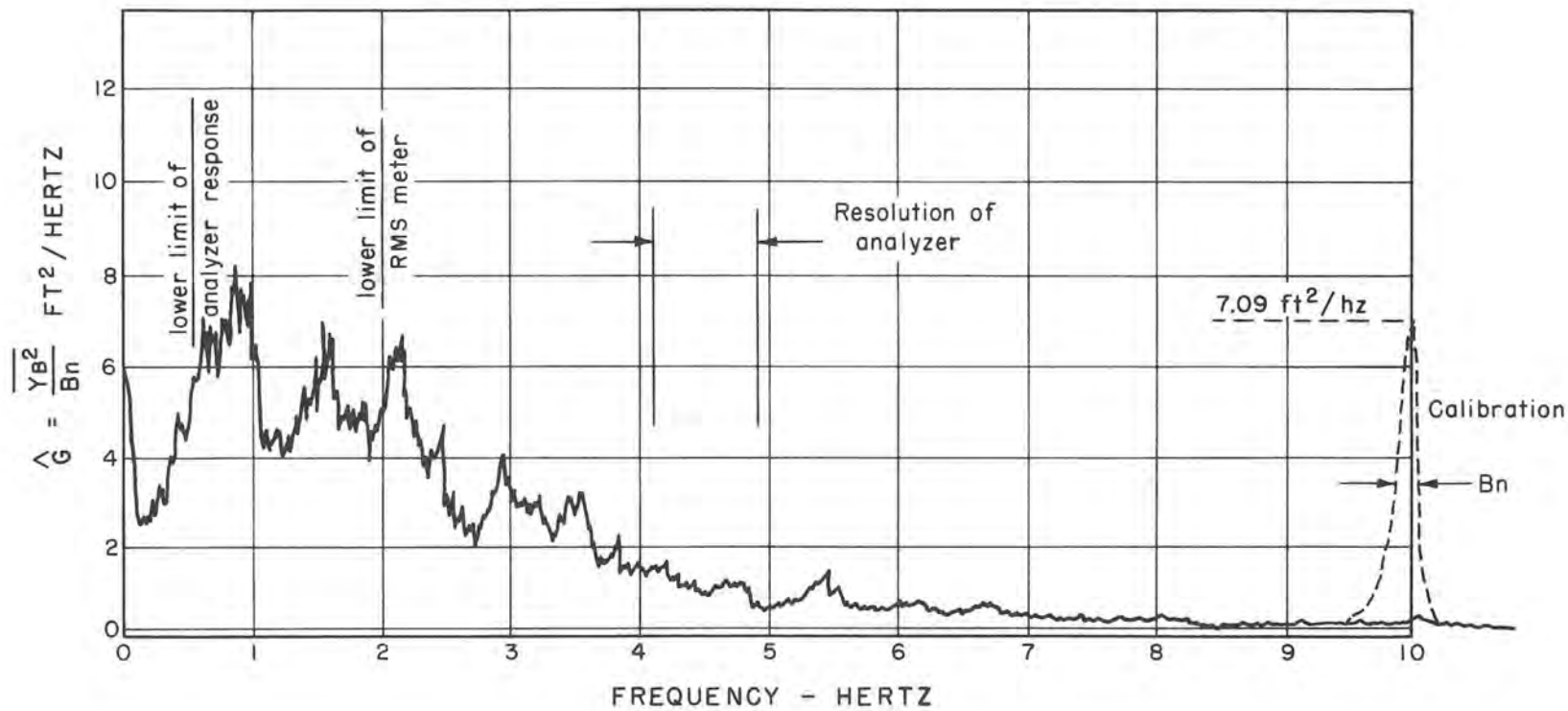

FIGURE 8 - SPECTRAL DENSITY ANALYSIS FOR PIEZOMETER R4.
TAPE SPEED = 7½ ips, SWEEP WIDTH = 10 hz,
AVERAGE AMPLITUDE SQUARED, RC TIME CONSTANT = 50 sec.,
ANALYSIS TIME = 2 hr. Bn = .221 hz RMS = 4.4 ft.