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THE POSITION OF THE LOWER SECTION OF THE JET ON DROPS AND SKI-JUMP SPILLWAYS

(O polozhenii nizhnego secheniia strui na perepadakh i konsol'nykh sbrosakh)

By

S. E. El'iasberg

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By

Danny L. King
Hydraulics Branch
Division of Research

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ABSTRACT

Formulas are presented for calculation of the path of a submerged jet for drops and ski-jump spillways. The jet path is used in designing the apron length in stilling basins and in determining the area of maximum scour in erodible beds. The formulas are based on the premise that the jet continues in a straight line upon entering the tailwater pool, tangent to the trajectory at the point of entry. Comparison is made with the usual assumption that the free trajectory continues through the pool. A sample problem is included. (Translator)

DESCRIPTORS-- design criteria/ aprons/ *drops/ structures/ *spillways/ stilling basins/ free fall/ flip buckets/ *jets/ open channels/ scour/ foreign design practices/ submergence/ mathematical analysis

IDENTIFIERS-- USSR/ ski-jump spillways/ tailwater

THE POSITION OF THE LOWER SECTION OF THE JET
ON DROPS AND SKI-JUMP SPILLWAYS

By Assistant Professor S. E. El'iasberg

In a number of cases the distance to the lower section of a falling jet must be determined--in particular, for calculating the length of a stilling basin below the wall of a drop (Fig. 1) and for designing the cantilever spillway of a low head dam and cantilevered drops (Fig. 2).

Usually this distance is determined according to the formula of the free fall jet which does not consider the change in path of the jet after its submergence in the tailwater.

As was noted by M. A. Mikhalev,a/ upon entering the water the jet continues its movement straight, tangent to the trajectory of the free fall at the point of intersection of the trajectory with the free water surface. He proposed to find the direction of the axis of the submerged jet by a graphic means of constructing the indicated tangent

The formulas are given below for calculating the position of the jet at section C-C (Fig. 1 and 2). The limits of application of these formulas are also established.

a/ Author's Note: M. A. Mikhalev, "Determination of the Depth of Scour in Erodible Foundations by a Falling Jet," Gidrotekhnicheskoe Stroitel'stro, No. 9, 1960

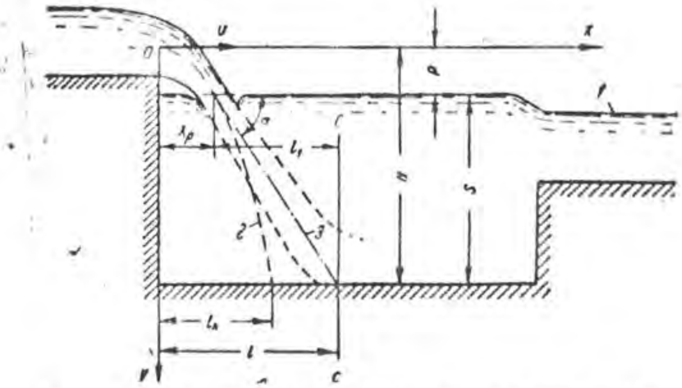

Figure 1. Section of a stilling basin

1 - Tailwater level; 2 - axis of jet for free fall; 3 - axis of submerged jet

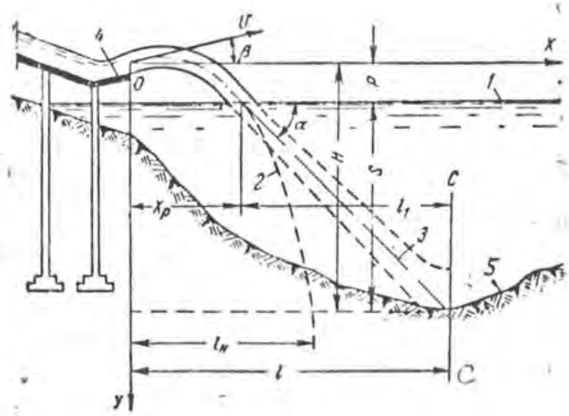

Figure 2. Section of a cantilevered drop

1 - Tailwater level; 2 - axis of jet for free fall; 3 - axis of submerged jet; 4 - downstream end of the drop; 5 - bottom of the erosion crater

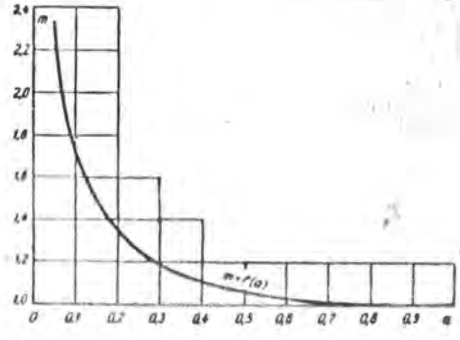

Figure 3. Graph of m=f(a)

For the general case, with arrangement of the downstream part of the structure at angle β to the horizontal (Fig. 2), for the coordinate system shown in Figures 1 and 2, the equation for a free fall jet may be written in the following form:

$$\left.\begin{aligned} x &= \frac{v^2 \sin\beta \cos\beta}{g} + v\cos\beta \sqrt{\frac{v^2 \sin^2\beta}{g^2} + \frac{2y}{g}}, \\ y &= \frac{gx^2}{2v^2\cos^2\beta} - x \operatorname{tg}\beta. \end{aligned}\right\} \quad (1)$$

Equations (1) apply only down to the tailwater level, at this level we have:

$$x_p = \frac{v^2 \sin\beta \cos\beta}{g} + v\cos\beta \sqrt{\frac{v^2 \sin^2\beta}{g^2} + \frac{2p}{g}}. \quad (2)$$

From the second equation of system (1) we find: $\frac{dy}{dx} = \frac{gx}{v^2\cos^2\beta} - \operatorname{tg}\beta.$

We determine the tangent of the angle at which the jet enters the water by substituting x_p from equation (2). After transformation we get the following formula:

$$\operatorname{tg}\alpha = \sqrt{\operatorname{tg}^2\beta + \frac{2pg}{v^2\cos^2\beta}}. \quad (3)$$

The total length which determines the position of the lower section of the jet, is expressed by the equation

$$l = x_p + l_1 = \frac{v^2 \sin\beta \cos\beta}{g} + v\cos\beta \sqrt{\frac{v^2 \sin^2\beta}{g^2} + \frac{2p}{g}} + \frac{S}{\sqrt{\operatorname{tg}^2\beta + \frac{2pg}{v^2\cos^2\beta}}}, \quad (4)$$

which for $\beta = 0$ takes the following form:

$$l = v\sqrt{\frac{2p}{g}} + \frac{S}{\sqrt{\frac{2pg}{v^2}}}. \quad (5)$$

We will determine how much the results of the calculation from formula (5) differs from the value ℓ, which has been calculated without considering the submergency of the jet, that is by the formula

$$l_H = v\sqrt{\frac{2H}{g}}, \qquad (6)$$

where H = S + p.

Letting $a = \frac{p}{H}$, then p = aH and S = (1-a)H

From equation (5) we obtain:

$$l = v\sqrt{\frac{2aH}{g}} + \frac{(1-a)H}{\sqrt{\frac{2aHg}{v^2}}}.$$

Also letting m = ℓh, then

$$l = ml_H = mv\sqrt{\frac{2H}{g}} \qquad (7)$$

$$m = \frac{v\sqrt{\frac{2aH}{g}} + \frac{(1-a)H}{\sqrt{\frac{2aHg}{v^2}}}}{v\sqrt{\frac{2H}{g}}} = \frac{1+a}{2\sqrt{a}}. \qquad (8)$$

The graph of the function m = f(a) (Fig. 3) shows that m is essentially larger than 1 only when a > 0.5. For values of a < 0.5, the value of ℓ is practically equal to ℓ_H.

Formulas (4), (5), (7) and (8) are derived, starting from a condition of the freely falling jet at the tailwater level, and can be applied only when the structure is not submerged by the tailwater. Moreover, these formulas and graphs (Fig. 3) cannot be applied when the tailwater submerges the structure. Consequently in the computation it is necessary to determine the tailwater conditions. For an approximate verification we can begin with the critical value of $\alpha = 30°$ For $\alpha < 30°$ submergence can occur, whereas within the limits

$15° < \alpha < 30°$ we have an unstable condition for the joining of the levels with a constant alternation of the bottom and surface conditions.b/ 1/ It is necessary to determine the angle of the entrance of the jet into the water from formula (3), which for $\beta = 0$ takes the following form:

$$\mathrm{tg}\,\alpha = \frac{\sqrt{2pg}}{v}, \qquad (9)$$

If angle α according to formulas (3) and (9) is less than 30°, then a surface condition for joining of the basins is possible and formulas (4), (5), (7) and (8) must not be used. Also, the given formulas may be used only down to that depth below which the submerged jet is dispersed; however, this limit does not have a practical value since the bottom of the erosion crater is always higher.

From the results $\alpha = 30°$, one can obtain the value of a_{min} limiting the use of the graph (Fig, 3) for the zone $0.5 > a > 0$.

From formula (9) we get the equation

$$\mathrm{tg}\,30° = \frac{\sqrt{2p_{\text{мин}}g}}{v},$$

from which it follows

$$p_{min} = 0{,}167\frac{v^2}{g}; \qquad (10)$$

$$a_{min} = 0{,}167\frac{v^2}{gH}. \qquad (11)$$

b/ Author's note: M. A. Mikhalev "The problem of kinematic structure of the flow for submerged tailwater condition, according to the type of the spilled jet," Trudy coordinatzonnykh soveshchanii po gidrotekhnike, Vol. VII, Gosenergoizdat, 1963

1/Translator's note: This is a literal translation. The writer refers to a condition in which the jet alternately submerges then sweeps along the surface.

When the value of a decreases further there occurs a change in the conditions from submerged to surface and the graph of the function $m = f(a)$ must not be used.

Example. To determine the distance ℓ from the downstream section of a cantilevered drop with the follow..ng data: v = 10 meter/sec.; p = 2 meters; S = 8 meters, H = p + S = 2 + 8 = 10 meters; $\beta = 0$. Without considering submergence of the jet from formula (6)

$$l = l_n = 10\sqrt{\frac{2\cdot 10}{9,81}} = 14,3 \text{ meters}$$

The angle of entrance of the jet into the water from formula (9)

$$\operatorname{tg}\alpha = \frac{\sqrt{2\cdot 2\cdot 9,81}}{10} = 0,63$$

and $\alpha = 32°$, that is, we have the submerged condition for joining of the tailwaters.

From formula (5)

$$l = 10\sqrt{\frac{2\cdot 2}{9,81}} + \frac{8}{\sqrt{\frac{2\cdot 2\cdot 9,81}{10^2}}} = 19,2 \text{ meters}$$

$$a_{min} = 0,167\,\frac{10^2}{9,81\cdot 10} = 0,17;$$

$$a = \frac{2}{10} = 0,2 > a_{min}$$

From the graph in Fig. 3, m = 1.34. From formula (7) ℓ=1.34 (14.30) = 19.2 meters.