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ANALYSIS OF RANDOM PRESSURE FLUCTUATIONS IN STILLING BASINS

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SYNOPSIS

Knowledge of the amplitude-frequency spectrum of pressure fluctuations caused by turbulence in hydraulic structures allows the determination of dynamic loading in structural design. Recent experience with prototype structures has indicated fatigue failure caused by vibration. This paper describes pilot studies for electronic analysis of random pressure fluctuations in models of two distinct types of energy dissipators. The studies were restricted to determination of dynamic loads acting on the structures, without regard to the dynamic response of the structures. The results are discussed with respect to their application to design problems and possible sources of error are described.

RÉSUMÉ

Une connaissance du spectre de fréquence de l'amplitude des fluctuations de pression dans les structures hydrauliques permet de déterminer le chargement dynamique pour les calculs structurals. L'expérience récente avec des structures prototypes a présenté des destructions dues à la fatigue causée par les vibrations. Le présent travail décrit des études initiales pour l'analyse électronique des fluctuations aléatoires de pression dans des modèles de deux types distincts de dissipation d'énergie. Les études ont été limitées à la détermination des efforts dynamiques agissant sur les structures, sans considération de la réponse dynamique des structures. Les résultats sont discutés relativement à leur application dans les problèmes présentés par les projets, et on décrit les sources possibles d'erreur.

INTRODUCTION

This paper describes the use of electronic equipment in analyses of pressure fluctuations in hydraulic structures. The analyses determine the amplitude-frequency spectrum of the fluctuations for use in structural design. Previously, design information has been provided by oscillograph records obtained from hydraulic models. The records exhibit a mixture of amplitudes and frequencies which cannot be separated by simple inspection. Also, the relatively short periods of record are insufficient for a useful analysis.

Stochastic analysis of the random pressure fluctuations can be used to develop a more usable presentation of the forces occurring in these energy dissipators. Two methods are generally available for this analysis. First, the data can be digitized and processed on an electronic digital computer. Or, voltages from the transducer can be examined with an analog device. The second approach was used in these studies.

Immediately after equipment for spectral analysis became available, the Hydraulics Branch, Division of Research, began pilot studies to determine methods and applications for data analysis of pressure fluctuations in stilling basins. This paper includes results of the early tests.

Pressure fluctuations were examined in models of two distinct types of energy dissipators. Morrow Point Dam features a free-fall spillway with an impact-type, energy absorbing, pool at the base of the dam. Crystal Dam outlet works includes a hydraulic jump stilling basin with a center wall separating the jets issuing from two high-pressure slide gates. The models and locations of piezometers used in these tests are shown in Figure 1.

INSTRUMENTATION

Pressures were converted to voltages with flexible diaphragm pressure transducers. The transducer signal was amplified by a direct writing oscillograph, with output to a commercial spectrum analyzer and accompanying spectral density analyzer. The instrumentation is shown in Figure 2. The spectrum analyzer is capable of determining the amplitude-frequency relationship over a frequency range of approximately 1/2 to 2,000 cps. The spectral density analyzer conditions the signal from the spectrum analyzer to determine the average or peak value of the voltage or power at a given frequency, or the integral of the voltage or power over a certain frequency interval.

Two modes of instrument operation were used in the analysis. The signal amplitude versus frequency was plotted to determine the coefficients of a Fourier series, and the integral $\int_{t_1}^{t_2} (\text{amplitude})^2 \, dt$ was obtained to determine the RMS (root-mean-square) value of the amplitude in various frequency bands.

The frequency range 0-50 cps was examined in both models, using scan times of 2 hours, which showed acceptable repeatability. Some preliminary runs showed that much longer scan times, beyond the 16-hour range of the instrument, would be required for greater repeatability of separate runs. The analyses substantiated the estimate that all frequencies present were within the 0-50 cps band.

INVESTIGATION

Data from two hydraulic models were recorded and analyzed. Pressures were measured on the stilling basin floor of the 1:24 scale model of the spillway for Morrow Point Dam and on the center wall of a 1:12 model of the outlet works stilling basin for Crystal Dam. Energy dissipation in the Crystal outlet works occurs in a normal hydraulic jump. The energy of the Morrow Point spillway jets is dissipated in an impact pool at the base of the dam. Data from these two different types of structures revealed differing patterns of turbulent pressure variations.

Examination of the oscillograph records in Figure 3 yields estimates of the maximum, minimum, and average pressure heads. The Crystal record shows a primary frequency of about 15 cps with some superimposed higher and lower frequencies which cannot be determined. The Morrow Point record indicates a frequency of 4 or 5 cps, a superimposed frequency of approximately 25 cps, and some indeterminate frequency less than 1 cps. Little additional information can be extracted from the oscillograph records.

Amplitudes in the amplitude-frequency analysis obtained with the spectrum analyzer correspond to coefficients in the Fourier series:

$$F(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_1 t + b_n \sin n\omega_1 t) \qquad (1)$$

where ω_1 = fundamental angular frequency in radians/sec
$= 2\pi f_1$ (f_1 = fundamental frequency in cps)

In words, an infinite number of periodic signals are superimposed on each other to give an oscillograph trace like those of Figure 3. $a_o/2$ is the steady, static value; for example, $a_o/2$ might be pressure on the bottom of a water pool with the $\sum$ term representing changes in pressure due to wave action on the surface.

The complex signal represented by Equation (1) contains both amplitude and phase characteristics. However, the analyzer equipment yields only the amplitude characteristics; the phase characteristic was not of interest in this study.

Amplitudes in the Fourier analysis represent instantaneous values which vary from zero to some maximum peak. Therefore, work done on the structure should not be calculated from the peaks alone. Some lesser effective value of the pressure or force, namely the RMS value, is used in calculating work.

The effective pressure head in a time interval Δt is:

$$H_{eff.} = \frac{1}{\Delta t}\sqrt{\int_t^{t+\Delta t} \left[\frac{a_o}{2} + \sum_{n=x}^{x'} (a_n \cos n\omega_1 t + b_n \sin n\omega_1 t)^2\right] dt} \qquad (2)$$

The interval x to x' is the frequency interval scanned during the time interval Δt. A simplified form is

$$H_{eff.} = \frac{1}{\Delta t}\sqrt{\int_{t}^{t+\Delta t} [H_t]^2 \, dt} \qquad (3)$$

where H_t represents instantaneous values of pressure head in the frequency interval.

The integral in Equation (3) was evaluated with the spectral density analyzer. Thus, the effective pressure head in any frequency interval is determined. Evaluation of (3) over an infinite time interval and over the entire frequency spectrum gives the effective pressure head corresponding to simultaneous application of all the terms of Equation (1). The static application of a single mean force determined by Equation (3) over the entire frequency spectrum causes a structural deformation which is indicative of the maximum deformation caused by simultaneous application of the periodic dynamic forces in Equation (1). The mean effective pressure head was not evaluated in these tests, but will be determined in future work.

RESULTS

Figure 4 shows the Fourier and spectral density analyses for the Crystal outlet works center wall. Both traces show that the largest pressure heads occur at a frequency of about 18 cps (5.2-cps prototype according to the Froude Law). Other isolated peaks are observed at approximately 3 cps (0.9-cps prototype). These analyses immediately tell the designer that natural frequencies in the vicinity of 5 cps or 1 cps should be avoided.

The maximum effective pressure head at a prototype frequency of 5.2 cps is 4.3 feet (1.3 meters), while the maximum value from the Fourier analysis is 4.0 feet (1.2 meters). The value of a_o is zero. The effective pressure head should be somewhat less than the pressure head indicated by the Fourier coefficient. The discrepancy could be due to the different data sampling intervals for each of the two analyses. If both analyses were made in the same time interval, the results would appear in the correct relationship. The relatively small amplitudes are at first surprising when compared with the larger amplitudes of the oscillograph trace. However, the periodic fluctuations are applied simultaneously and are therefore additive, as indicated by Equation (1). At a point in time, several peaks or portions of signals coincide and either add to or subtract from each other. Also, the large amplitude fluctuations might occur at a very low frequency, below the 1/2 cps lower limit of the equipment. Even though these low frequency forces would not affect the structures with respect to vibration, they could cause short duration "static" loadings. The oscillograph record may, in certain instances, retain importance in defining very low frequency fluctuations.

Unfortunately, the higher amplitudes measured at a model frequency of about 18 cps are not valid. Excitation of a similar pressure measuring system (piezometer tubing, transducer, and connecting tubing) with an oscillator indicated a probable resonant condition in the vicinity of the 18-cps peaks. Therefore, these peaks should be disregarded in the analysis. These findings emphasize the necessity for determining the dynamic response characteristics of any pressure measuring system. Careful selection of tube lengths and diameters and pressure transducers can avoid natural frequencies in critical ranges of measurement.

The Fourier and spectral density analyses for the Morrow Point stilling basin floor are shown in Figure 5. The patterns are markedly different from those of Figure 4. No well-defined isolated regions of relatively large amplitudes are present; the amplitudes generally increase with decreasing frequency. The maximum Fourier coefficient of approximately 3 feet of water (0.9 meter) is shown very close to zero frequency.

For the Morrow Point stilling basin, $a_o/2$ in Equation (1) has a value of 57.6 feet of water (17.6 meters). For example, at a prototype frequency of 0.4 cps (about 2-cps model), the effective pressure head is about 57.7 feet ($\sqrt{57.6^2 + 2.3^2}$). The contribution by dynamic forces is essentially negligible. However, larger fluctuations with frequencies below the range of the analyzer are probably present.

In these studies, both the pressure heads and frequencies were scaled according to the Froude similarity criterion. However, the applicability of this criterion to scaling of the frequency might be questioned. Low frequency pressure fluctuations are due primarily to surface waves, surges, and large scale eddies in which gravity forces dominate. Higher frequency fluctuations are due to small scale turbulence in which viscous forces dominate. Use of the Froude criterion over the frequency range encountered in these studies is probably reasonable. Prototype data should establish correct scaling procedures. Fortunately, the Crystal outlet works stilling basin will contain piezometers in locations corresponding to those in the model.

The analysis described in this paper allows solutions to two problems encountered in the design of hydraulic structures: (1) determination of stresses caused by the application of a periodic force at a given frequency, such as the natural frequency of the structure, and (2) determination of stresses resulting from the complete spectrum of forces and frequencies under nonresonant conditions. A complete analysis of any structure requires that the methods described in this paper be applied at several representative points on the structure.

The surfaces on which the pressures were measured were considered to be sufficiently rigid to minimize the effect of structural response on the spectral analysis. In a structure such as a cantilevered wall or a gate leaf, the movement caused by a pressure field can affect that pressure field. This effect must be considered in design.

CONCLUSIONS

The test results illustrate the utility of electronic equipment in the determination of the frequency spectrum of random pressure fluctuations in stilling basins. Data from models of two distinct types of stilling basins showed a marked difference between the frequency spectra. Details of application of these data are not included.

The studies described in this paper were restricted to determination of dynamic loads acting on a structure, without regard to the dynamic response of the structure or its effect on the measured loads. Future work will give more attention to these problems. Also, the capabilities of the instrumentation will be more fully explored. Addition of a magnetic tape recording system will allow inspection of frequencies less than the present lower limit of 0.5 cps.

ACKNOWLEDGMENTS

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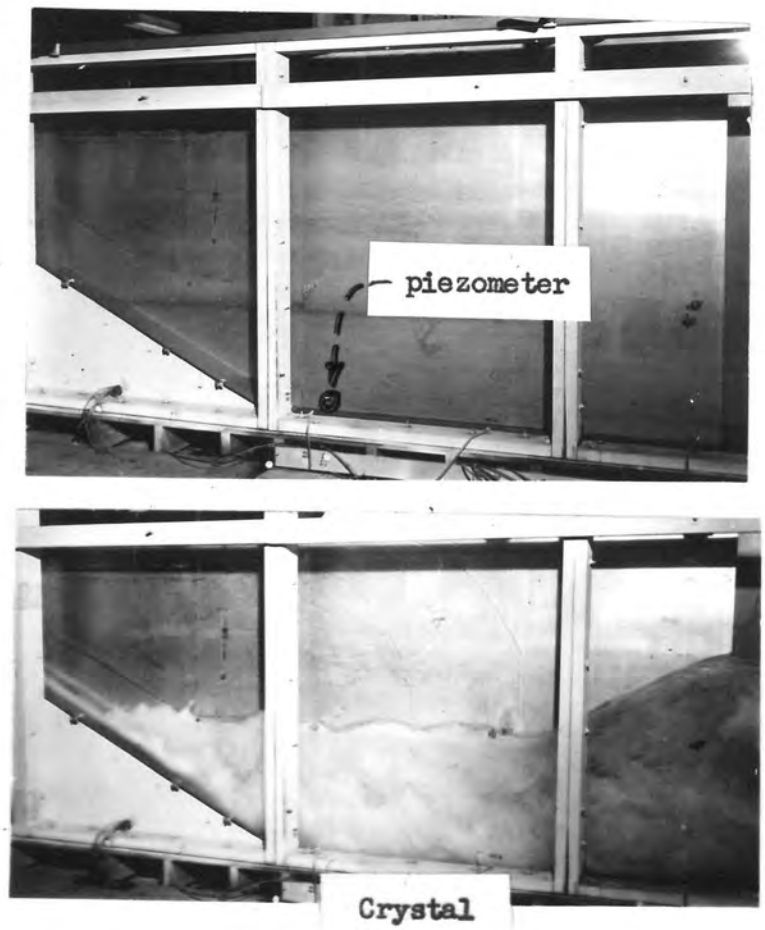


Figure 1. Model operation and piezometer locations

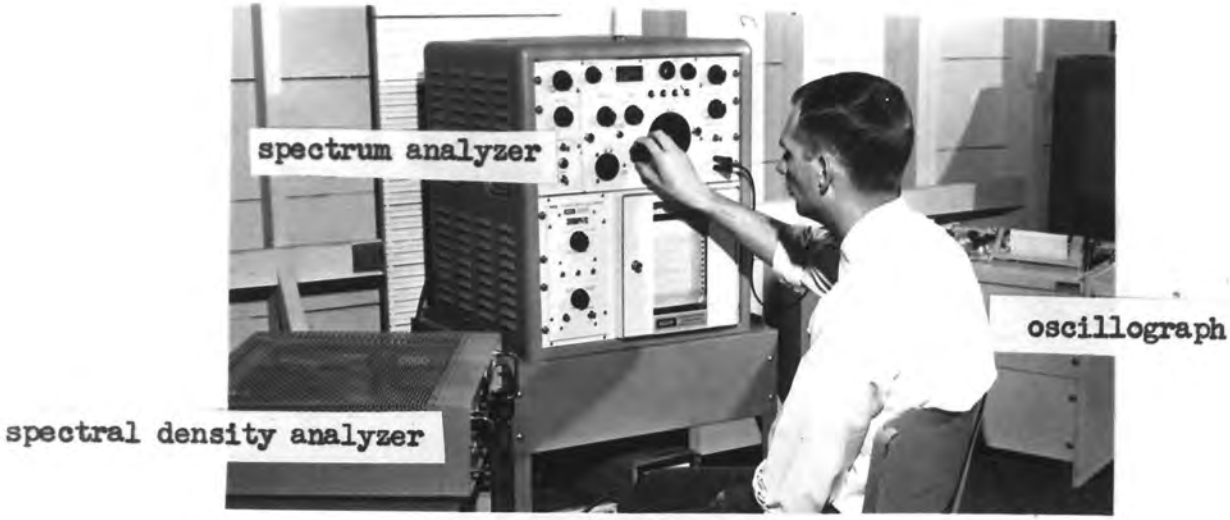
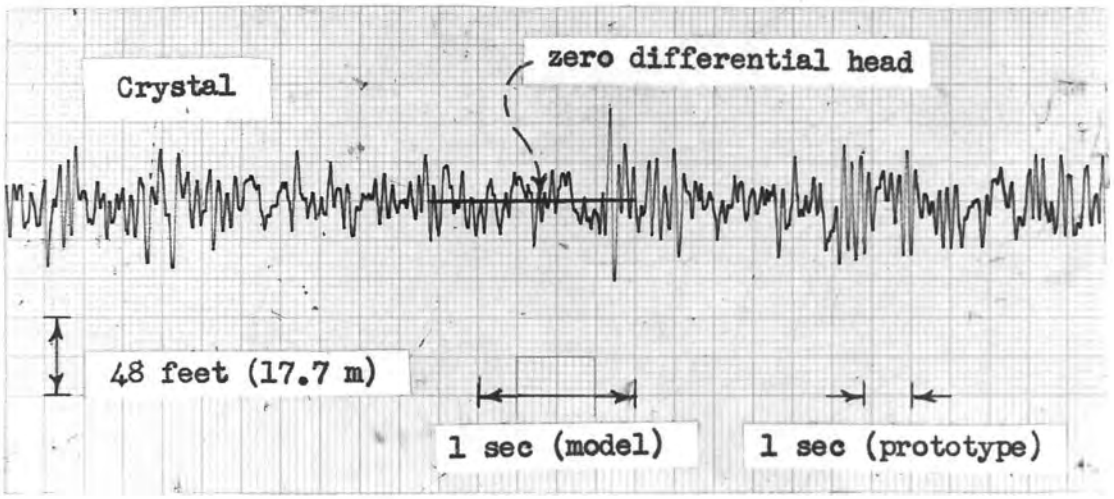


Figure 2. Instrumentation



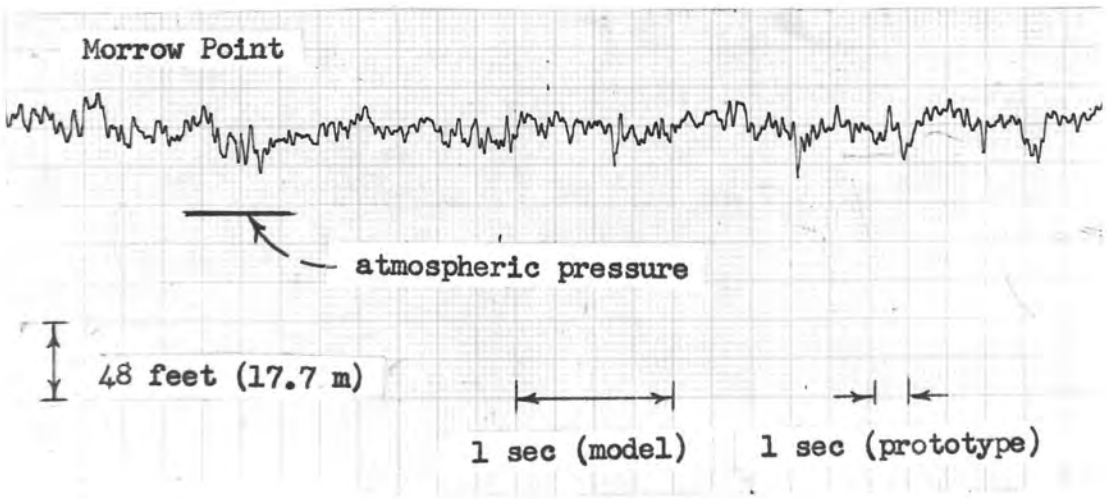


Figure 3. Oscillograph records

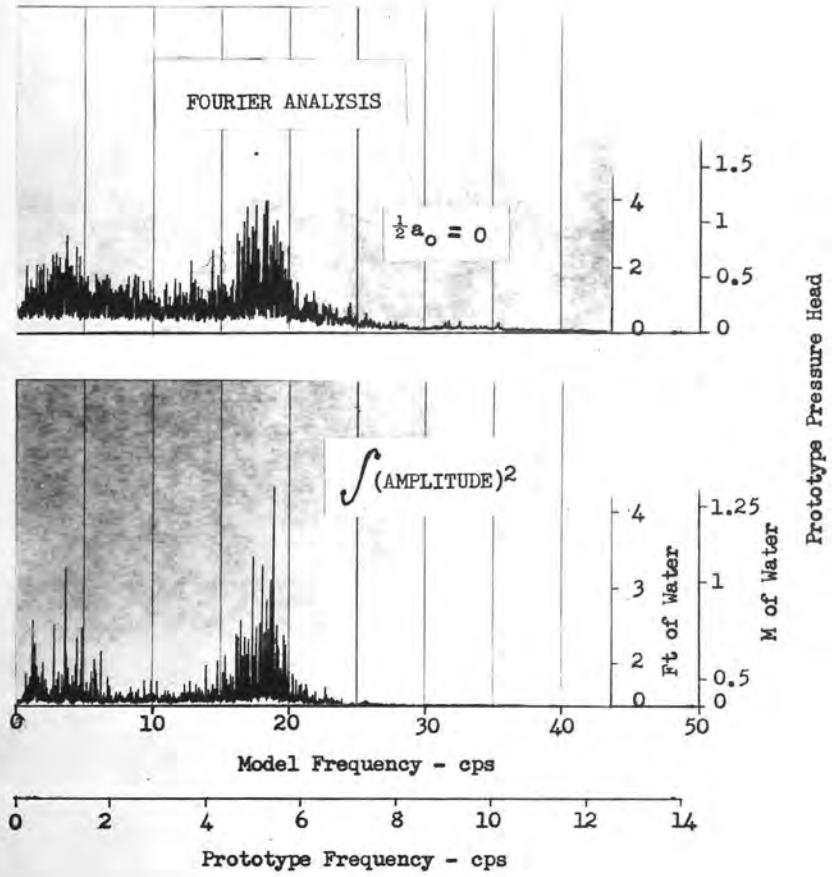


Figure 4. Spectral analyses for Crystal center wall

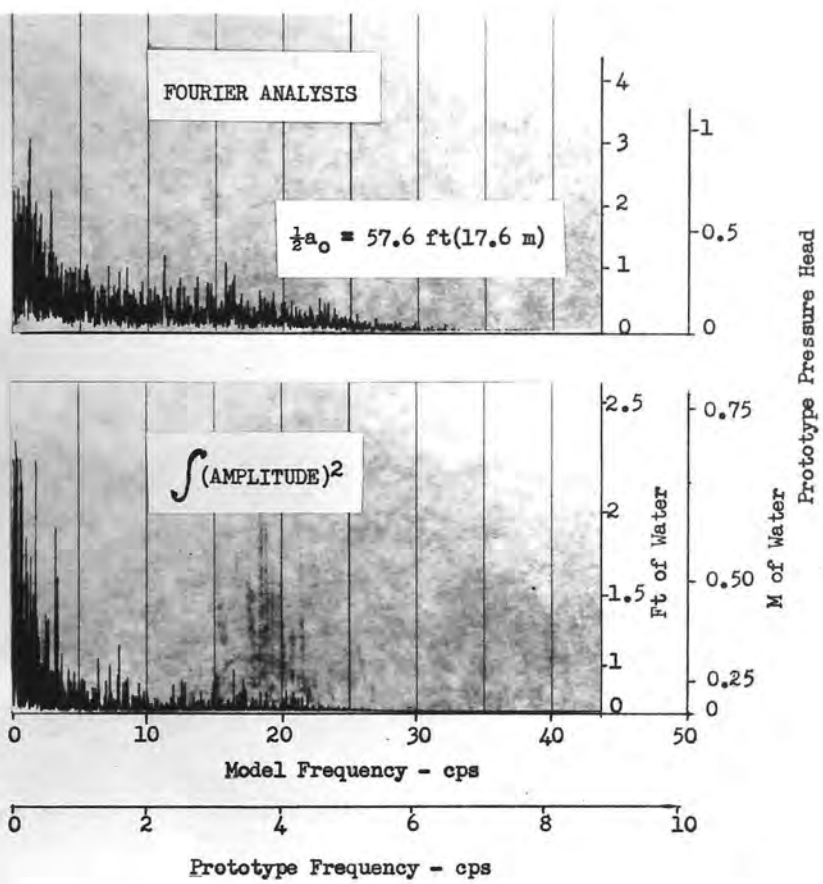


Figure 5. Spectral analyses for Morrow Point stilling basin floor