THE MECHANISM OF AQUIFER SWEETENING

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ABSTRACT

Where drains are installed in an aquifer containing saline water, the density difference between the saline water and a better quality freshening water influence the rate of removal of the saline water. An interface separates the 2 waters. Computed and observed positions of these interfaces are shown for parallel drain installations in uniform and 2-part aquifers. In all cases the head driving the water toward the drain reaches its maximum midway between drains. In the 2-part aquifer with a lower layer decidedly more permeable than the upper, the flow breaks through into the lower layer and continues in its to a point near the drain and then flows up to the drain. The interface is then in the lower layer. In sweetening an aquifer by a well, the interface goes down with the cone of depression as pumping is started. As saline water is pumped out and replaced by water from above, the interface lowers until the aquifer is sweetened. The interface positions as developed by dyes used in the laboratory tests substantiated the computations.

DESCRIPTORS-- *aquifers/ ground water/ *saline water/ *drainage systems/ drainage/ *water quality/ water table/ irrigation/ irrigation O&M/ soils/ demineralization/ hydrostatic pressures/ *density/ drainage wells/ infiltration/ mathematical analysis/ laboratory tests/ dyes/ waterlogged land/ movements
IDENTIFIERS-- soil salinity/ *interfaces/ depression cone/ drain spacing/ aquifer sweetening

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Waters of natural streams always contain a certain amount of salt. When these waters are diverted and used for irrigation, the water percolating below the root zone has a higher salinity than the water applied, due to transpiration and evaporation. If a permanent irrigation agriculture is to be maintained, water in excess of that needed by the crops must generally be applied to provide drainage to carry away these salt concentrations.

To prevent waterlogging of the land, drainage systems may need to be installed to collect and dispose of these drainage waters. During the passage of the drainage waters through the ground there may be an opportunity provided to pick up additional salt. There is generally no practicable way to dispose of the drainage waters except to return them to the stream from which they originally came. Recirculation of ground water by pumping for irrigation may induce excessive salinity unless provision is made for maintaining a valley salt balance.

Where streams traverse arid areas with little or no tributary inflow and diversions are made for irrigation along their length it is not surprising to find the salinity of their waters increasing downstream. As the water demands approach complete utilization, the salinities in the lower reaches of the stream may be so high that only salt-tolerant crops can be grown.

It is often the case that irrigated areas have some amount of natural drainage, but when expansion of irrigation begins to exceed the natural drainage capability, rising water tables will require the installation of drainage systems, or soil salinity will result. At the time the drainage system is constructed there will be a more saline drainage effluent initially than would occur after equilibrium is reached because of the salts accumulated in the soil.

Whenever the quality of river flows become poor, the problem of salt balance may become critical because of the inability to achieve salt balance with the available water supply. To design and construct an efficient drainage system and to predict within reasonable limits the quality of the effluent, the engineer needs to know the mechanism of ground-water movement within the system. Some analytical developments relating to the mechanism of salt removal are described in the following paragraphs.

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Parallel Drains--Uniform Aquifer

A common type of drainage system employs parallel drains installed at such depths and spacings as will keep the water table clear of the root zone. Either open drain ditches or buried tile lines may be used. As a basis for the analytical treatment it will be assumed that the conditions are as shown in Figure 1. It will be assumed, also, that the salinity of the water stored in the aquifer is greater than that of the replacement water and has a correspondingly greater density. Because turbulent mixing is absent in ground water flow it will be assumed that an interface exists separating the stored saline waters and the less saline replacement waters. The investigation will first be directed toward a search for the ultimate steady state where the saline waters below the interface are stagnant and the freshening waters flow over and along the interface toward the drain. The excess of hydrostatic pressure on the saline water side is then counterbalanced by the pressures sustaining the flow on the fresh water side. (3)* The existence of such a condition has long been known in coastal areas where seawardly flowing ground waters, supplied by rainfall, meet waters of ocean salinity. (1)(2) An interface developed in a glass-walled tank is shown in Figure 1.

Where the flow to unit length of drain is sustained by an infiltration rate i over a drain spacing of length L, continuiy requires that

$$q = i\left(\frac{L}{2} - x\right) \qquad \ldots (1)$$

where x represents the distance from the drain and q represents the flow to the drain from one side.

The density differential between the saline and replacement waters will be expressed by a quantity m defined by the relation

$$m = \frac{1}{(\gamma_s - \gamma_r)} \qquad \ldots (2)$$

where γ_s represents the density of the saline water

γ_r represents the density of the replacement

In both cases the density is expressed as a ratio to the density of distilled water.

*Numbers designate references at end of paper.

If K represents the permeability of the soil and h the head driving the water toward the drain, then the flow to the drain may be expressed in the form:

$$q = K\left(1 + m\right) h \frac{dh}{dx} . \qquad \ldots \quad (3)$$

This expression conforms to the requirement that the superior static pressure of the stagnant saline water must be counterbalanced by the pressures sustaining the flow of the replacement waters to the drain.

Elimination of q between Equations (1) and (3) yields the differential equation:

$$h \frac{dh}{dx} = \frac{i\left(\frac{L}{2} - x\right)}{K(1 + m)} . \qquad \ldots \quad (4)$$

An integration, subject to the requirement that $h = h_o$ when $x = 0$, yields the solution:

$$h = \sqrt[2]{\frac{iL^2}{4(1 + m)K} - \frac{i\left(\frac{L}{2} - x\right)^2}{(1 + m)K} + h_o^2} . \qquad \ldots \quad (5)$$

In most cases $h_o = 0$.

A comparison of interface positions as computed from this relationship and as observed in a laboratory test is shown in Figure 1. The quantity h reaches its maximum value midway between the drains where $x = \frac{L}{2}$. This value is, when $h_o = 0$,

$$h_m = \sqrt[2]{\frac{iL^2}{4(1 + m)K}} . \qquad \ldots \quad (6)$$

Parallel Drains--Two-Part Aquifer

It is common to find river valley aquifers which show an increasing permeability with depth. In some cases the transition between the two permeabilities is sharp. The case of the sharp transition will be considered here. When flow conditions are such that the interface remains in the upper, less permeable, layer, then the relations described above will apply. However, if the lower layer is decidely more permeable than the upper layer and flow conditions are such that the interface makes contact with the upper surface of the lower layer, then laboratory tests indicate that the flow breaks through into the lower layer and continues in it. Thereafter the infiltration takes an essentially vertical course down through the upper layer, and the position of the interface can then be estimated by use of Formula (5) by placing the drain location at the top of the lower layer.

A comparison of computed and observed configurations for this case is shown in Figure 2.

Laboratory tests show that where the flow breaks through into the lower bed it follows it to a point near the drain and then flows up to the drain through a narrow zone below the drain. When the saline waters are made visible by use of a dye and the experiment is run until terminal conditions are approached, a stagnation line shows up. Such a line is shown in Figure 2. It separates the downward flowing waters in the upper bed, supplied by infiltration, and the upward flowing waters from the lower bed moving toward the drain. As there is no flow at the stagnation line, color can remain there after it has been washed out of the adjacent areas. The location of this line indicates that substantially all of the infiltration passes through the upper bed into the lower bed before flowing to the drain.

Salt Removed by Drains

In field installations where drains are installed at depths and spacings chosen to keep the water table out of the root zone a check by Formula (5) will generally indicate that the ultimate position of the interface will be below the bottom of the aquifer at a point midway between the drains. In such cases only a wedge of saline water remains below the drain. At some shorter spacing the two wedges join and the interface is continuous between drains. If the drain spacing is still further shortened until the drain spacing begins to be comparable with the aquifer depth, then considerable amounts of salt could be left in semipermanent storage. Laboratory and analytical investigation indicate this possibility, but there is some question that it could be realized in the field because of the presence there of disturbing influences not accounted for in the laboratory and analytical studies. In river valley aquifers, for example, there is a down-valley movement of the ground water due to the valley gradient. In addition, the irregular pattern of irrigation on adjacent parcels of land creates ground-water mounds that cause a continual shifting of the ground waters. When an interface between saline and replacement waters is moved in this way, there is a diffusion of salinity across the interface which is favorable to the removal of salt. (4)(5) Molecular diffusion across the interface also favors salt removal.

There are, therefore, slow mechanisms of salt removal other than the relatively rapid one treated above. It seems certain that all of the saline water will ultimately be removed, but it is also reasonable to suppose that aquifer sweetening is complete for all practical purposes when the first phase, as treated analytically herein, has come to an end.

Quality of the Effluent from Drains

As a basis for making an estimate of the quality of water issuing from the drains it is possible to use the reasonable assumption that the rate of flow of saline water is proportional to the amount of removable saline water remaining in the aquifer. It will first be necessary to determine the amount of saline water to be removed. Formula (5) with $h_o = 0$ can be written in the form

$$h = \sqrt{\frac{iL^2}{4(1 + m)K}} \sqrt{1 - \left(1 - \frac{2x}{L}\right)^2} \quad . \qquad \ldots \quad (7)$$

The saline water volume to be removed is

$$W = \int_o^L mh \, dx \qquad \ldots \quad (8)$$

then

$$W = m \sqrt{\frac{iL^2}{4(1 + m)K}} \int_o^L \sqrt{1 - \left(1 - \frac{2x}{L}\right)^2} \, dx \; .$$

By introducing the change of variable

$$u = \left(1 - \frac{2x}{L}\right) \qquad du = - \frac{2dx}{L} .$$

The above expression takes the form:

$$W = \frac{mL^2}{4} \sqrt{\frac{1}{(1+m)K}} \int_{-1}^{+1} \sqrt{1 - u^2}\, du .$$

Since

$$\int_{-1}^{+1} \sqrt{1 - u^2}\; du = \frac{1}{2} \left[u \sqrt{1 - u^2} + \sin^{-1} u \right]_{-1}^{+1} = \frac{\pi}{2}$$

then

$$W = \frac{m \pi L^2}{8} \sqrt[2]{\frac{1}{(1+m)K}} . \qquad \ldots (9)$$

The condition that salt removed from the aquifer must appear in the drainage water is:

$$\frac{ds}{dt} + \frac{2qs}{VW} = 0 . \qquad \ldots (10)$$

This is a differential equation whose solution is

$$s = s_o e^{\frac{-2qt}{VW}} \qquad \ldots (11)$$

where s_o represents the salinity s at the time $t = 0$. The factor 2 appears in the expression because of the assumption that the drainage flow passes to two drains. This development applies, without modification, to a single part aquifer in which the ultimate interface is continuous between the drains. In many field situations there may remain only a wedge of saline water under the drains and W will then approach the whole volume of water in the aquifer.

A comparison of observed and computed effluent qualities is shown in Figure 3. Data were taken from Test No. 7 in which a high salt concentration was used to get a continuous interface between drains. A second comparison is shown in Figure 5 for a two-part aquifer.

Drainage by Wells

As a basis for an analytical treatment of this case, it will be supposed that the well penetrates the full thickness of the aquifer and that the casing is perforated for a length λ. The well is supplied by infiltration as shown in Figure 6. It will be supposed, again, that the aquifer is originally filled with saline water and that the replacement water is of a lesser salinity and that it forms an interface with the more saline waters below. After the pump is started a cone of depression begins to develop which carries the interface down with it. Saline water only issues from the well until the interface reaches the perforations after which saline and freshening waters enter the well in proportions dictated by the position of the interface with respect to the perforations. As saline water is pumped out and replacement water is supplied from above, the interface lowers. Eventually, the aquifer is sweetened to the bottom.

It will first be necessary to determine the drawdown y produced by pumping a well supplied by infiltration. If the well flow is Q and its supply comes from an infiltration at the rate i received on a circular area of radius R surrounding the well, then if an ultimate steady state is to exist

$$Q = \pi R^2 i \quad R = \sqrt{\frac{Q}{\pi i}} \, . \qquad \ldots \quad (12)$$

In terms of a permeability K and a saturated depth D, the flow q across a cylindrical surface of radius r distant from the well will be

$$q = -2\pi r \quad KD \frac{dy}{dr} \, . \qquad \ldots \quad (13)$$

The flow across this surface supplied by the infiltration i received on the area between the radii r and R will be

$$q = i\pi(R^2 - r^2) . \qquad \ldots \quad (14)$$

Elimination of q from these two expressions yields the differential equation:

$$\frac{dy}{dr} = -\frac{i(R^2 - r^2)}{2KDr} . \qquad \ldots \quad (15)$$

A solution subject to the requirement that $y = 0$ when $r = R$ is

$$y = \frac{Q}{2\pi KD}\left[\log_e \frac{R}{r} + \frac{r^2}{2R^2} - \frac{1}{2}\right] . \qquad \ldots \quad (16)$$

This expression represents an ultimate steady state where the flow of the well Q is supplied by infiltration. Initiation of pumping is followed by a transient condition which terminates in this steady state. A separate investigation indicates that the steady state will be essentially attained when the time t after pumping begins satisfies the relation:

$$\frac{\alpha t}{R^2} = 0.5 \quad \text{or} \quad t = \frac{R^2}{2\alpha} , \qquad \ldots \quad (17)$$

where

$$\alpha = \frac{KD}{V} \qquad \ldots \quad (18)$$

is the aquifer constant.

Having established the position of the interface, it is now possible to determine the quality of the effluent issuing from the well as a function of time. The rate of descent of the interface is i/V_o where i represents the infiltration rate and V_o the ratio of the gross voids to the total volume of the aquifer.

Before the interface reaches the perforations, saline water only issues from the well. After the interface reaches the perforations, saline water enters over the length h and replacement water enters over the length $(\lambda - h)$ where h represents the depth of saline water against the well perforations. After the interface reaches the perforations, the rate of descent of the interface is given by the relation

$$\frac{dh}{dt} = \frac{-i}{V_o} \frac{h}{\lambda} \ . \qquad \ldots \quad (19)$$

Let

$$\beta = \frac{i}{V_o \lambda} \ . \qquad \ldots \quad (20)$$

Then a solution meeting the requirement that $h = \lambda$ when $t = 0$ is

$$h = \lambda e^{-\beta t} \ . \qquad \ldots \quad (21)$$

The quality of the issuing water is

$$s = \frac{s_1(\lambda - h) + s_o h}{\lambda} \ . \qquad \ldots \quad (22)$$

A comparison of observed and computed effluent qualities is shown in Figure 5.

Comments

The ultimate position of the interface between the saline and replacement waters, where drains are used, is determined by a balance between very small forces. This position, while determinate, is a very unstable one. The laboratory tests show evidence of this in several ways. The effluent qualities from different drains show considerable variations in quality, although care was taken to get a uniform distribution of infiltration over the surface of the model. Slight variations in the arrangement of the drains also seem to produce quality variations out of all proportion to their size. It is believed that similar field variations must be expected because here uncontrolled variations are sure to be present.

Although the formulas presented herein are approximate, it is believed that they represent the major factors closely enough to be useful for engineering purposes.

Notation

The notation used is shown below. The appropriate physical dimensions are also indicated.

a	an effective drain radius	(ft)
b	in a two-part aquifer, the height of the drain above the bottom of the upper bed	(ft)
D	depth of a permeable layer	(ft)
e	base of the natural logarithms	(dimensionless)
h	drainable depth	(ft)
i	infiltration rate	(ft/sec)
K	permeability	(ft/sec)
L	distance between drains	(ft)
m	$= \left(\frac{1}{\gamma_s - \gamma_r}\right)$	
Q	flow of a well	(ft^3/sec)
q	flow to a drain, per unit length of drain	(ft^2/sec)
r	radius	(ft)
R	an outer radius	(ft)
s	salinity, parts per million	(ppm)
s_o	an initial salinity	(ppm)
s_1	salinity of replacement water	(ppm)
t	time	(seconds)
u	a variable of integration	(dimensionless)
V	ratio of drainable void volume to the gross volume	(dimensionless)
V_o	ratio of the total void volume to be gross volume	(dimensionless)

α	$= KD/V$	(ft/sec)
β	$= i/V_{\sigma}\lambda$	(1/sec)
γ_r	density of replacement water expressed as a ratio to the density of distilled water	(dimensionless)
γ_s	density of saline water expressed as a ratio to the density of distilled water	(dimensionless)

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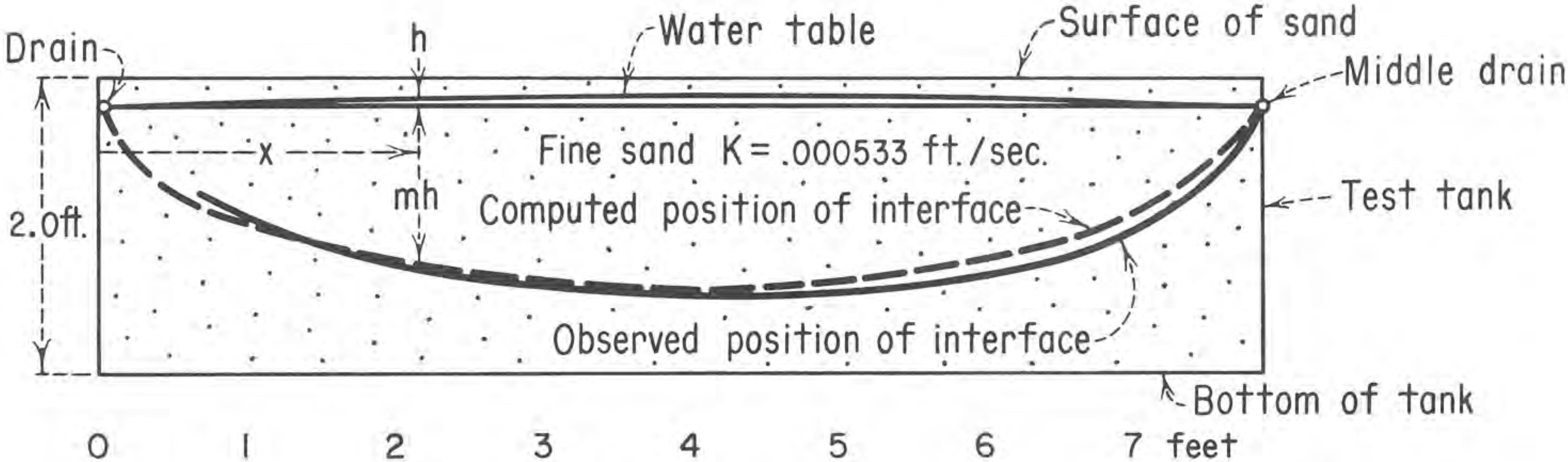
Test No. 7	s_0 = 78000 ppm	Uniform sand	Parallel drains
m = 18.18	L = 7.86 feet	i = 000,002,61 ft./sec.	3 Drains
γ_s = 1.055	γ_r = 1.000 feet	Tank 2 ft. by 15.72 ft. in plan, 2 ft. deep	



Computation by E. J. Carlson

FIG. 1 INTERFACE POSITION IN UNIFORM SAND

Test No. 3 — s_0 = 6300 ppm — Two part aquifer — Parallel drains

m = 222 — L = 7.86 ft. — i = 0000123 ft./sec. — 3 Drains

γ_s = 1.0045 — γ_r = 1.000 — Tank 2 ft. by 15.72 ft. in plan, 2 ft. deep.

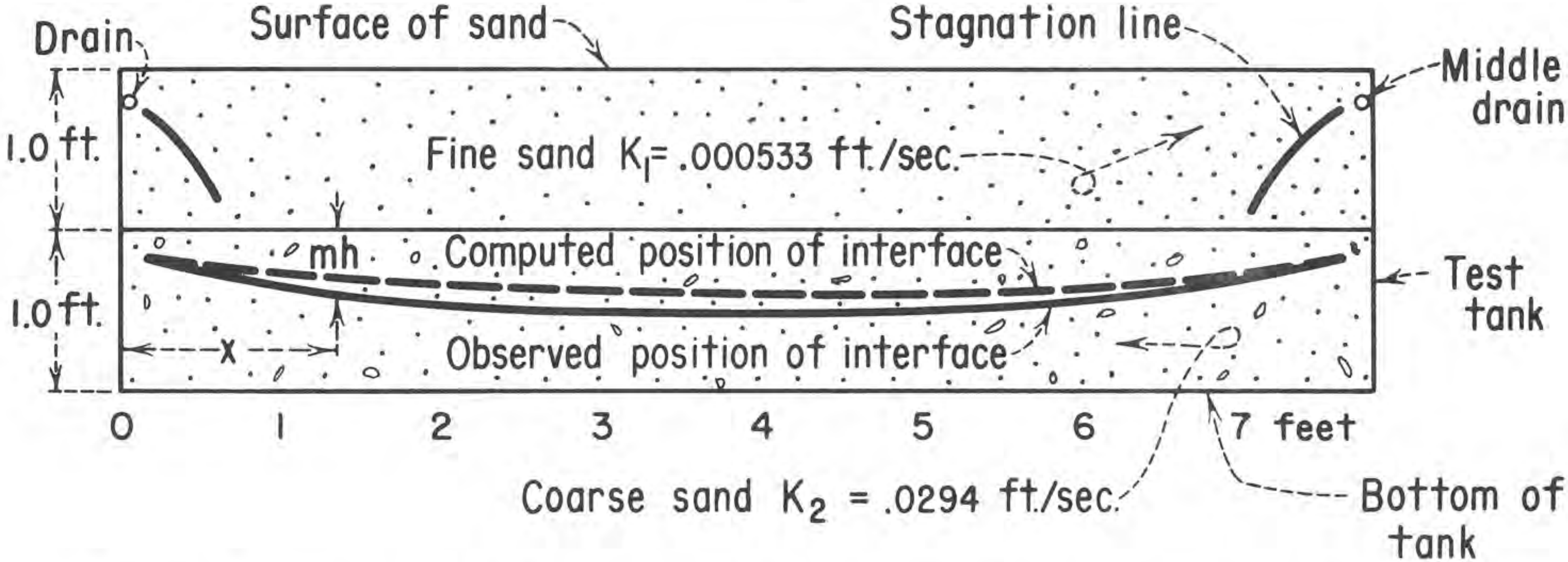


FIG. 2 INTERFACE POSITION IN TWO-PART SAND

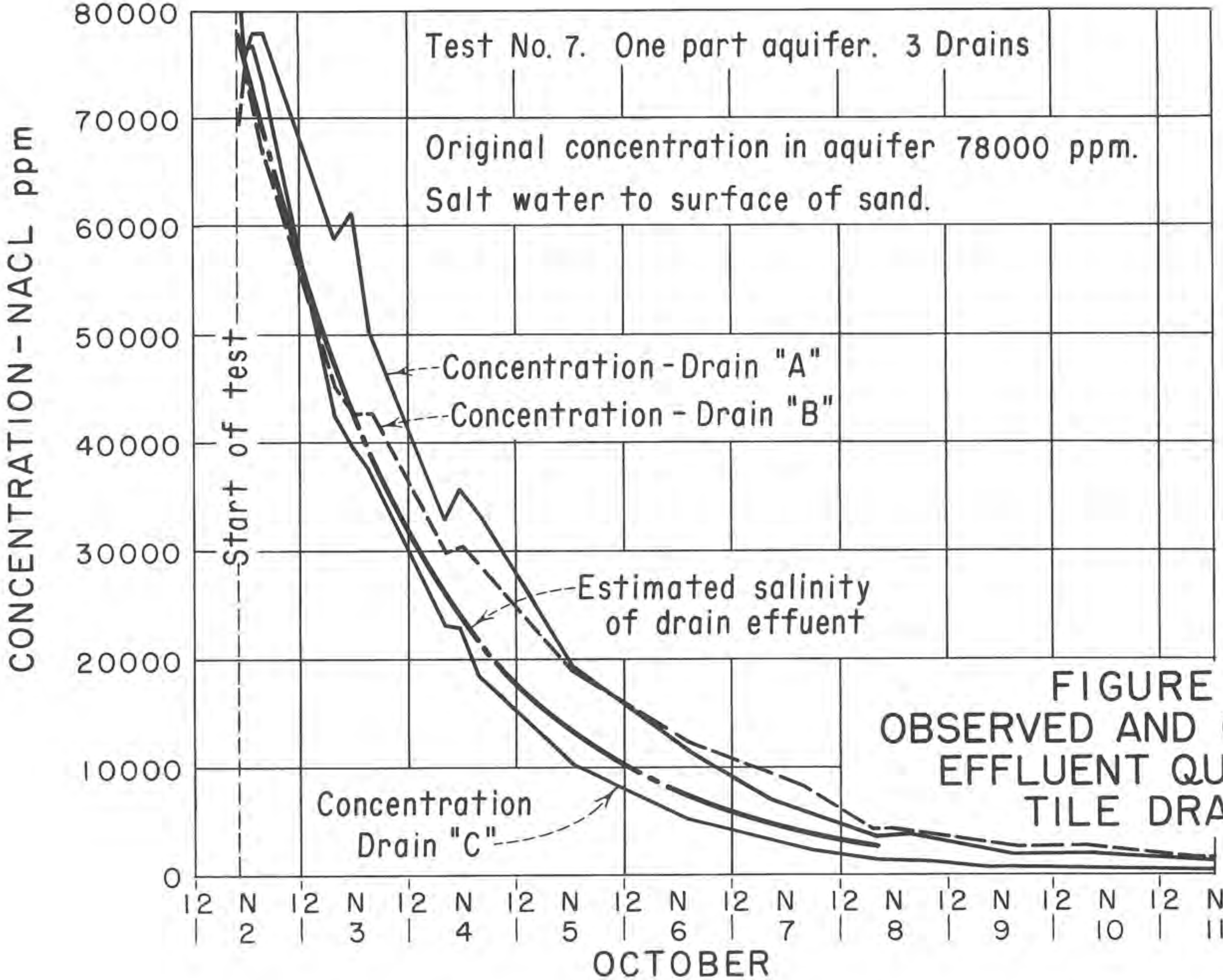


FIGURE 3
OBSERVED AND COMPUTED EFFLUENT QUALITY-TILE DRAINS

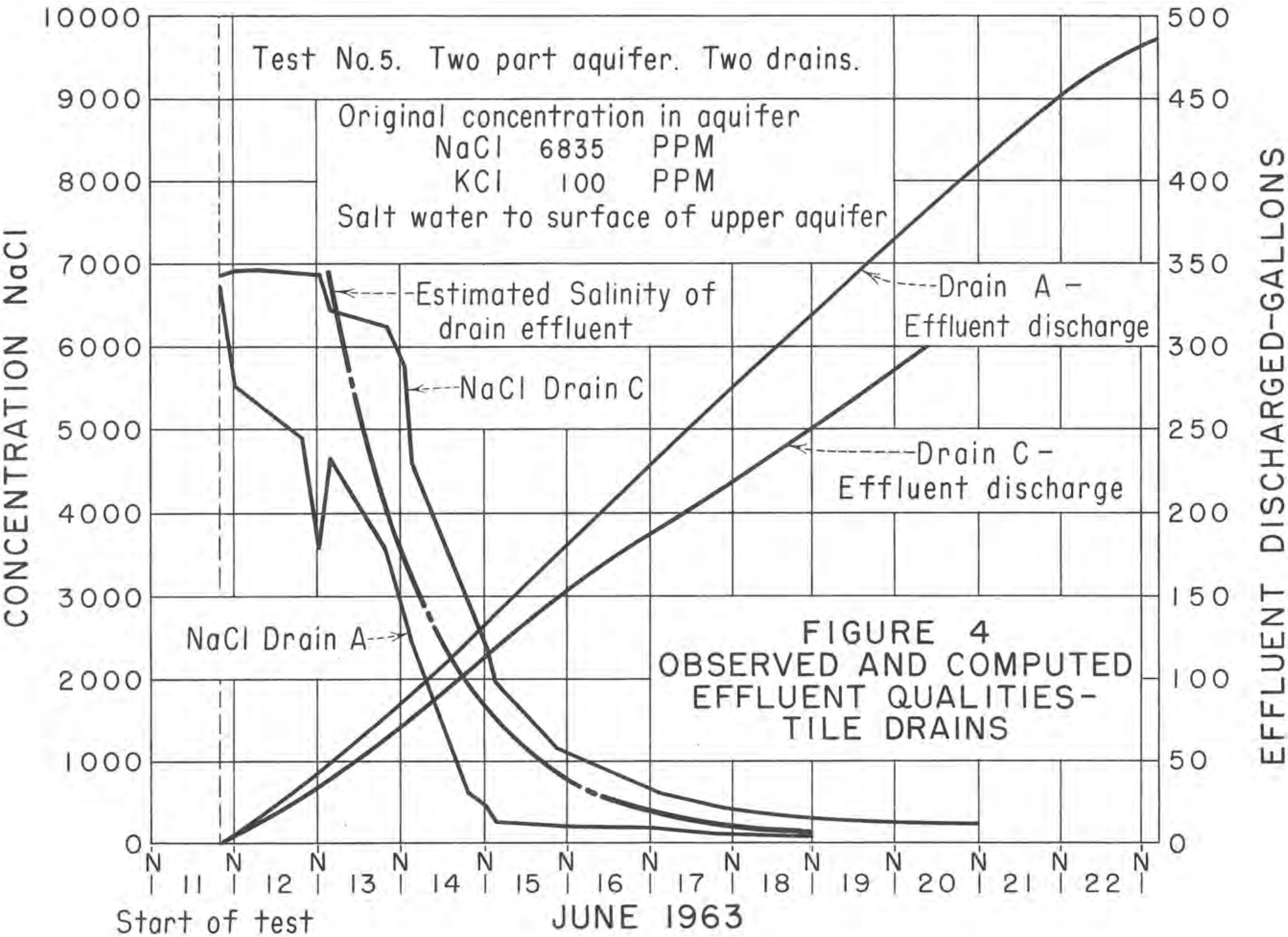


FIGURE 4
OBSERVED AND COMPUTED
EFFLUENT QUALITIES-
TILE DRAINS

Test No. 1 Well drainage series.
Two-part aquifer.
Salt water in lower member 5790 p.p.m.
Well casing perforated in lower member.
Permeabilities: Upper member (2 ft. deep) .000533 ft./sec.
Lower member (2 ft. deep) .0294 ft./sec.
Flow: 3 Gal. in 8 min. on 2 1/2 ft. x 16 ft. area.
Tank 2 1/2 ft. by 16 ft. in plan 4 ft. high with a diagonal diaphragm. Model well drained one-half of the 2 1/2 ft. by 16 ft. surface.

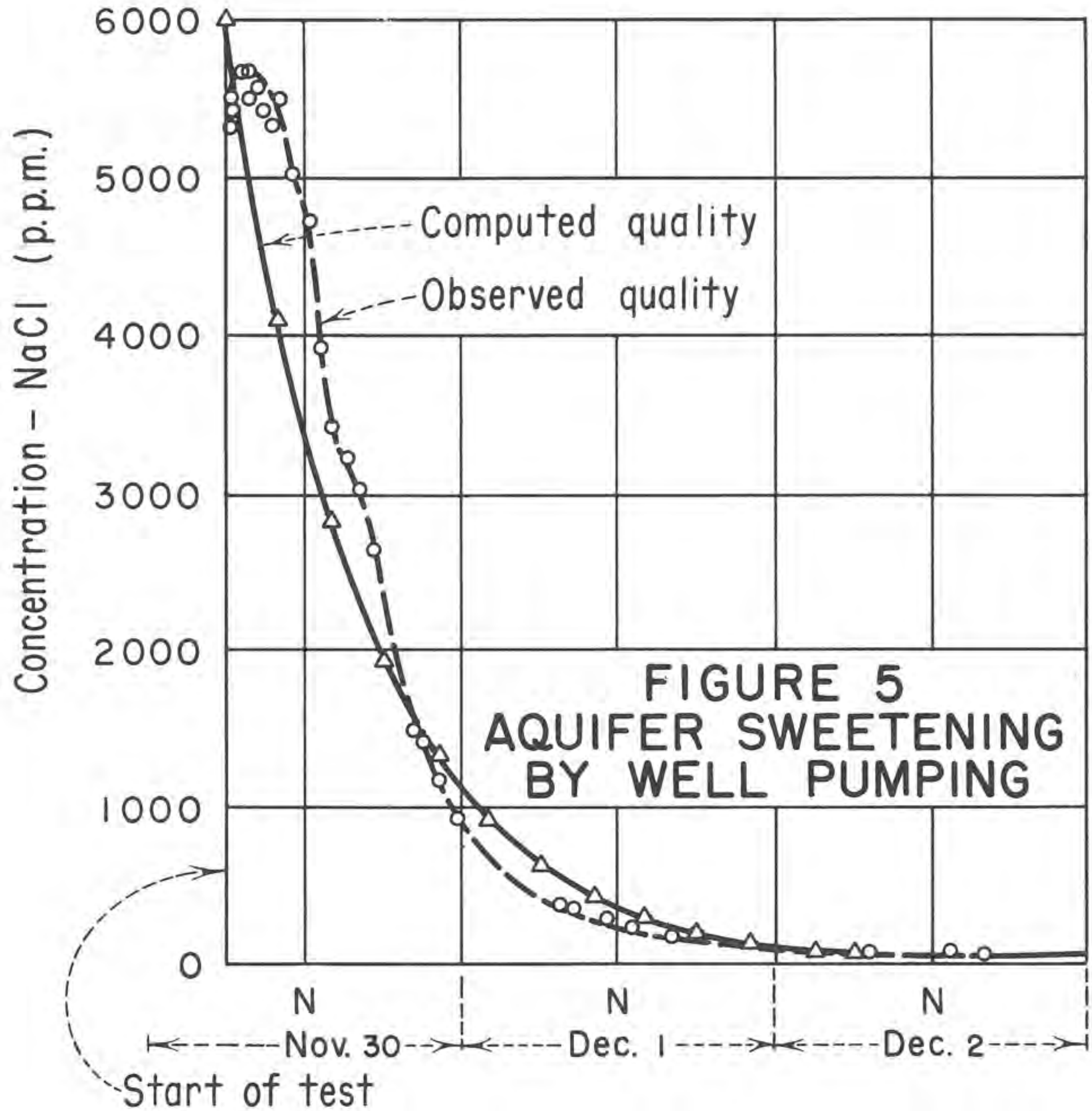
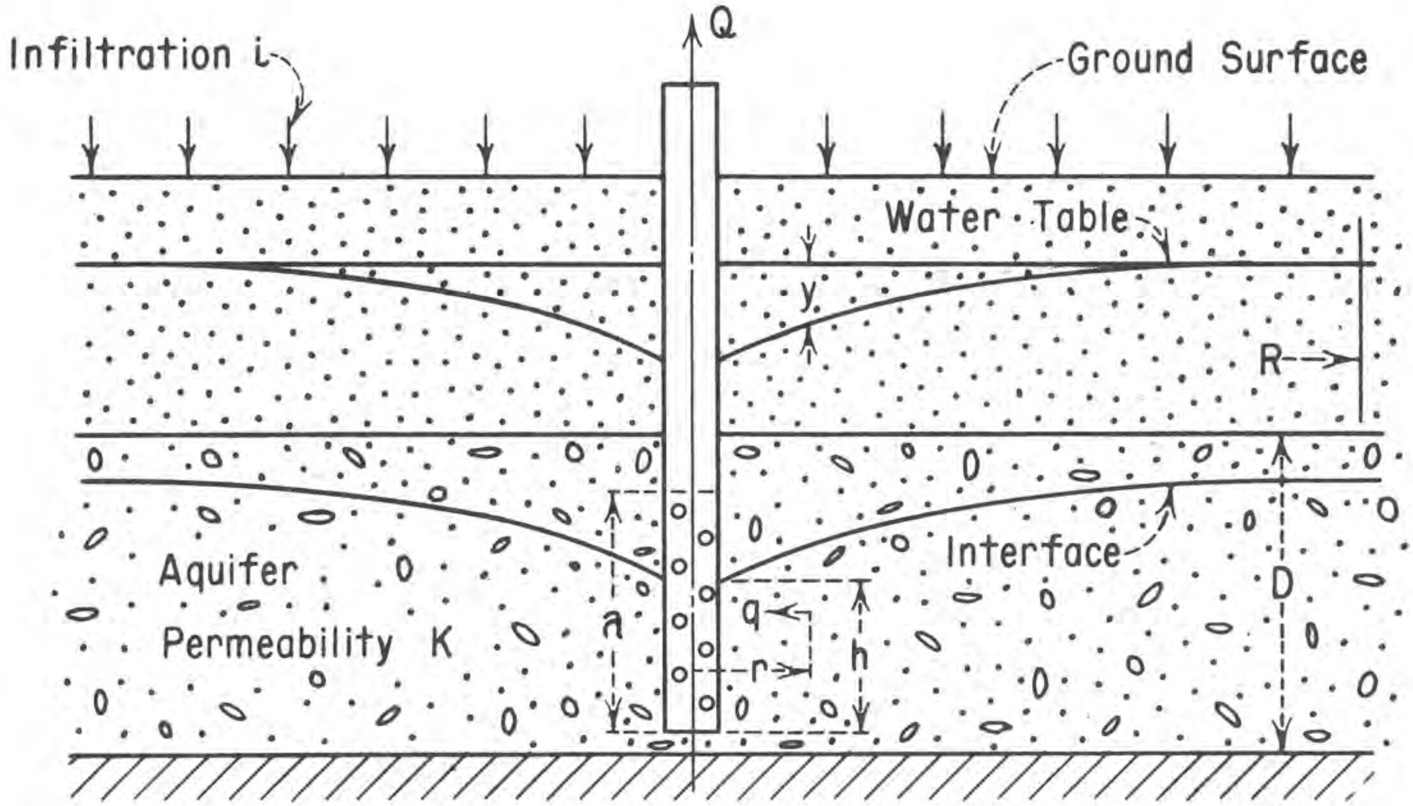


FIGURE 5
AQUIFER SWEETENING BY WELL PUMPING



(Note: In a single part aquifer D extends to the water table.)

FIG. 6 WELL SUPPLIED BY INFILTRATION