AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 3511
(Vol. 128, 1963, Part III)

UNIFICATION OF PARSHALL FLUME DATA

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SYNOPSIS

The Parshall critical depth flume has been used extensively in the western United States and other parts of the world since about 1925 and is accepted as a standard measuring device in open channel irrigation.

The original design, which was somewhat empirical, has been maintained until now; also the results have been presented as a number of empirical formulae covering widths ranging from 1 in. to 50 ft. For these reasons engineers have hesitated to use sizes of flumes other than those specified and have felt that great care must be taken to build the flumes according to the dimensions given.

In this paper, dimensional methods have been used to develop a semi-theoretical equation relating flow and depth for all flumes from 1 in. to 50 ft. Excellent agreement between this equation and all published data is found. This will permit using flumes of non-standard sizes and will broaden the field of application for this type of measuring device.

INTRODUCTION

The Parshall flume as used in 1961 is based on experiments that were continued over several years. The results of the first experiments for flumes of

Note.—Published essentially as printed here in December 1961 in the Journal of the Irrigation and Drainage Division as Proceedings Paper 3006. Positions and titles given are those in effect when the paper or discussion was approved for publication in Transactions.

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throat widths 1 ft to 8 ft were presented[2] in 1926. Subsequently, data were given[3,4] for flumes of sizes 10 ft to 50 ft, 3 in., 6 in. and 9 in. and in[5] 1957 1 in. and 2 in. All the data was presented as empirical formulas, seven in number, of the form

$$Q = K_e\, b\, y_1^n \qquad (1)$$

with the values of K_e and n are given in Table 1. In Eq. 1 Q refers to the discharge, K_e is a discharge coefficient to be defined, b denotes the throat width, and y_1 is the depth at measuring section. The "inch" flumes were constructed of steel, the 1 ft to 8 ft flumes, of wood and the largest 10 ft to 50 ft flumes of concrete.

The flume dimensions were chosen in an arbitrary manner. Referring to Fig. 1, all flumes have the same side angle θ - $\tan^{-1}$ 0.2, and dropdown angle ϕ - $\tan^{-1}$ 0.375. The other important dimension as far as this paper is concerned is x_1, the centerline distance from the throat crest to the upstream

TABLE 1.—VALUES OF K_e AND n IN EQ. 1

Throat Width	K_e	n
1 in.	4.06	1.55
2 in.	4.06	1.55
3 in.	3.97	1.547
6 in.	4.12	1.580
9 in.	4.09	1.530
1 ft to 8 ft	4	$1.522\, b^{0.026}$
10 ft to 50 ft	$3.6875 + \frac{2.5}{b}$	1.6

measuring section. For the "foot" flumes this is related to the throat width by the equations,

$$x_1 = 0.327\, b + 2.615 \qquad (2a)$$

and

$$b_1 = 1.131\, b + 1.046 \qquad (2b)$$

For the "inch" flumes there are no corresponding relationships.

The empirical nature of this work has discouraged theoretical analysis, and such attempts as have been made seem to have been unsuccessful.[6] For the

[2] "The Improved Venturi Flume," by Ralph L. Parshall, Transactions, ASCE, Vol. 89, 1926, p. 841.

[3] "Parshall Flumes of Large Size," by Ralph L. Parshall, Colo. Agric. Experiment Sta., Bulletin No. 386, May, 1932.

[4] "The Parshall Measuring Flume," by Ralph L. Parshall, Colo. Agric. Experimental Sta., Bulletin No. 423, March, 1936.

[5] "Parshall Measuring Flumes of Small Sizes," by A. R. Robinson, Tech. Bulletin No. 61, Agric. Experimental Sta., Fort Collins, Colo., October, 1957.

[6] "Critical Flow Meters (Venturi Flumes)," by A. Balloffet, Proc. Paper No. 743, ASCE, Vol. 81, July, 1955.

same reason it is necessary to keep rigidly to the specified dimensions,[7] which has obvious disadvantages. In this paper the seven empirical formulas are reduced to one comprehensive formula which, as will be explained, gives a degree of flexibility in the choice and design at the flume.

All calculations are based on original data which differ slightly from those published in tables.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

THEORY

The portion of a Parshall flume between the upstream measuring point and the throat is a transition section with straight converging walls and a horizontal floor. However, the hydraulics of the flume is complicated by the dropdown at the throat causing curvature of the stream lines and a resulting reduction in pressure below hydrostatic at and near the throat.

Research carried out on the free overfall in parallel channels[8,9] shows the pressure reduction causes the critical depth for hydrostatic flow to occur upstream of the drop at a section at which the pressure is approximately hydrostatic. The critical depth is given by

$$y_{cn} = \sqrt[3]{\frac{Q^2}{b_{cn}^2\, g}} \qquad \qquad (3)$$

in which b_{cn} is the channel width at which critical depth occurs, Q denotes the discharge, and g is the acceleration due to gravity. In the present case, a similar effect may be expected but with the complication that y_{cn} occurs at a section of unknown width b_{cn}. This has been shown experimentally by V. L. Hauser,[10] (See also Fig. 1). The problem is made more difficult by possible frictional effects near the throat where the velocity is very high.

Neglecting scale effect it may be assumed that the variables involved in the Parshall flume are: Q, b, y_1, x_1, g, θ, and ϕ (See Fig. 1) in which y_1 is the depth at distance x_1 from the throat crest, that is

$$f_1\,(Q, b, y_1, x_1, g, \theta, \phi) = 0 \qquad \qquad (4)$$

7 "Water Measurement Manual," Bur. of Reclamation, May, 1953.

8 "Analyzing Hydraulic Models for Effect of Distortion," by M.P. O'Brien, Engineering News-Record, McGraw-Hill Publishing Co., Inc., New York, Vol. 109, September 15, 1932.

9 "Discharge Characteristics of the Free Overfall," by Hunter Rouse, Civil Engineering, Vol. 6, No. 4, 1936.

10 "The Relationship Between the Length of the Converging Section and the Free Flow Equation for the Venturi Flume," by V. L. Hauser, thesis presented to the Univ. of California, in Davis, Calif., in 1957, in partial fulfilment of the requirements for the degree of Master of Science.

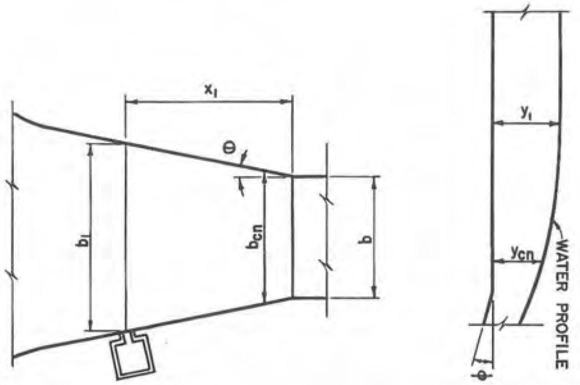

FIG. 1

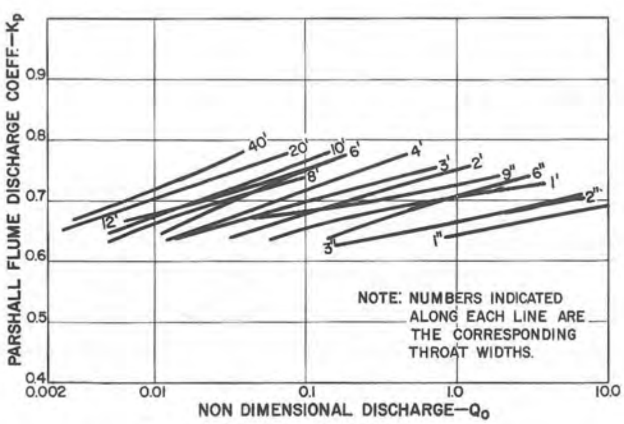

FIG. 2

By dimensional analysis Eq. 4 may be reduced to

$$\frac{Q}{g^{1/2}\ b\ y_1{}^{3/2}} = f_2 \left(\frac{Q}{g^{1/2}\ b^{5/2}}, \frac{x_1}{b}, \theta, \phi \right) \quad \ldots\ldots\ldots \quad (5)$$

For Parshall flumes θ and ϕ are constant, therefore,

$$\frac{Q}{g^{1/2}\ b\ y_1{}^{3/2}} = f_3\ (Q_o, x_o) \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which

$$Q_o = \frac{Q}{g^{1/2}\, b^{5/2}} \quad \text{.................} \quad (7)$$

and

$$x_o = \frac{x_1}{b} \quad \text{...................} \quad (8)$$

in which f_2 and f_3 are unknown functions, so that

$$\frac{Q}{g^{1/2}\, b\, y_1{}^{3/2}} = K_p \quad \text{...............} \quad (9)$$

in which K_p is a discharge coefficient.

Using the published data, K_p has been calculated and plotted against Q_o for various flume widths or for what amounts to the same thing, for various values of x_o, because x_o is different for each flume (see Fig. 2).

It is seen that not only does K_p differ for each flume, but there is a variation with Q_o for the same flume. The reason for the variation will be explained subsequently.

Using dimensional analysis once again, Eq. 4 may be expressed as

$$\frac{Q}{g^{1/2}\, b^{5/2}} = f_4\left(\frac{y_1}{b}, \frac{x_1}{b}\right) \quad \text{............} \quad (10)$$

or

$$Q_o = f_4\left(x_o\, y_o\right) \quad \text{................} \quad (11)$$

in which

$$y_o = \frac{y_1}{b} \quad \text{................} \quad (12)$$

If two geometrically similar flumes A and B were being compared, then x_{oA} and x_{oB} would be equal and Eq. 11 implies that if the depths at the measuring section are such that $y_{oA} = y_{oB}$ then $Q_{oA} = Q_{oB}$.

In general each different size Parshall flume has a different value of x_o. Fortunately, Hauser has measured the flow profiles in a 1 ft flume for four different flows. He used piezometers for measuring the depths, so it should be possible to compare these results with Parshall's which were obtained by the same method.

The flow profiles are shown plotted (non-dimensionally) in Fig. 3, each curve corresponding to the non-dimensional flow parameter Q_o. In addition, values of x_o and y_o have been calculated for each Q_o from different size flumes; a plot of these values is also shown.

Example.—Consider $Q_o = 0.0666$. For the 20 ft flume this corresponds to an actual flow of

$$(0.0666)\left(g^{1/2}\right)\left(20^{5/2}\right) = 677 \text{ cfs}$$

From published data[3] the depth y_1 corresponding to this flow is 3.9 ft, therefore,

$$y_o = \frac{y_1}{b} = \frac{3.9}{20} = 0.195$$

Also for this size flume x = 9.15 ft, therefore,

$$x_o = \frac{x_1}{b} = \frac{9.15}{20} = 0.4575$$

It may be seen from the plot that any particular value of Q_o does not cover the full range of flume sizes.

The excellent agreement between the derived points and the experimental plotting suggests a further step. If, at any measuring section, parallel uniform flow is assumed, then:

$$E_1 = y_1 + \frac{Q_1{}^2}{2\, g\, y_1{}^2\, b_1{}^2} \qquad \text{(13a)}$$

or

$$E_1 = y_1 + \frac{Q_1^2}{2\, g\, y_1{}^2\, (b + 0.4\, x_1)^2} \qquad \text{(13b)}$$

in which E_1 is the specific energy head. Dividing by b yields

$$\frac{E_1}{b} = \frac{y_1}{b} + \frac{\dfrac{Q^2}{g\, b^5}}{2\left(\dfrac{y_1}{b}\right)^2 \left(1 + 0.4\, \dfrac{x_1}{b}\right)^2} \qquad \text{(14a)}$$

or

$$E_o = y_o + \frac{Q_o^2}{2\, y_o{}^2\, (1 + 0.4\, x_o)^2} \qquad \text{(14b)}$$

For each value of Q_o, E_o was calculated for the various values of x_o and y_o. It is seen from Table 2 that E_o is almost constant for each Q_o which means that energy losses in the flumes between $0.392 < x_o < 9.35$ are negligible and that any appreciable loss that may occur must be close to the throat. The non-dimensional energy head was further calculated for flows and flumes of all sizes. The resulting plot of E_o versus Q_o on log-log paper (Fig. 4) produced a straight line whose equation was found to be

$$E_o = 1.351\, Q_o{}^{0.645} \qquad \text{(15)}$$

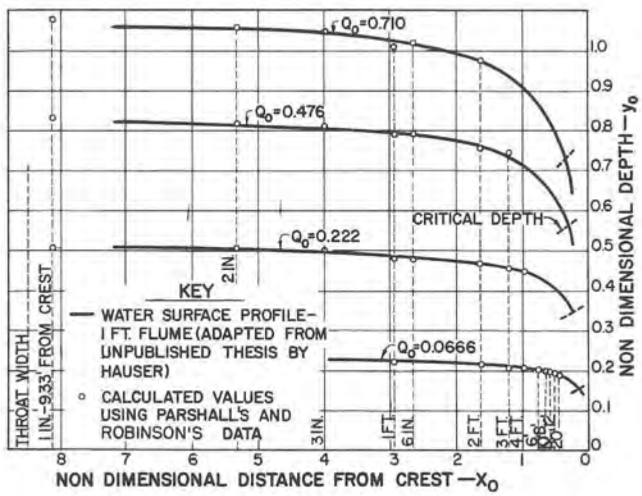
Q_0=0.710
Q_0=0.476
Q_0=0.222
Q_0=0.0666
CRITICAL DEPTH
KEY
WATER SURFACE PROFILE-
I FT. FLUME (ADAPTED FROM
UNPUBLISHED THESIS BY
HAUSER)
CALCULATED VALUES
USING PARSHALL'S AND
ROBINSON'S DATA
THROAT WIDTH
1 IN.-9.33' FROM CREST
2 IN.
3 IN.
1 FT.
6 IN.
2 FT.
3 FT.
4 FT.
NON DIMENSIONAL DISTANCE FROM CREST—X_0
NON DIMENSIONAL DEPTH—y_0

FIG. 3

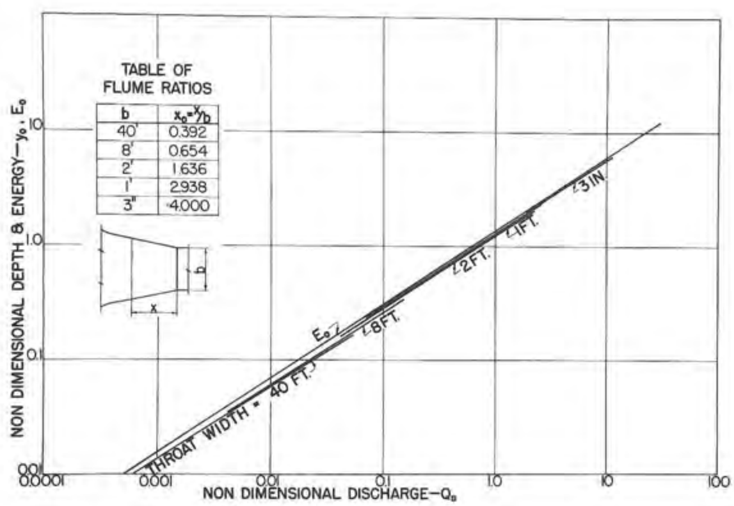
TABLE OF
FLUME RATIOS
b | x_0=x/b
40' | 0.392
8' | 0.654
2' | 1.636
1' | 2.938
3" | 4.000
E_0
THROAT WIDTH = 40 FT.
8 FT.
2 FT.
1 FT.
3 IN.
NON DIMENSIONAL DEPTH & ENERGY—y_0, E_0
NON DIMENSIONAL DISCHARGE—Q_0

FIG. 4

or

$$y_o + \frac{Q_o^2}{2\, y_o^2\, (1 + 0.4\, x_o)^2} = 1.351\, Q_o^{0.645} \qquad \text{(16)}$$

(The points from which Eq. 15 is derived are not shown in Fig. 4; almost without exception the points fall within the thickness of the line.)

Eq. 16 is the general Parshall flume equation relating Q, y, b, and x for flumes of any size. The solution to this equation is laborious, but it has been solved using a computer at the Cornell University Computing Center, Ithaca, N, Y., for various values of x_o. The solution for some values of x_o corresponding to flumes of standard widths are shown plotted in Fig. 4.

TABLE 2.—VALUES OF NON-DIMENSIONAL FLOW, DEPTH AND ENERGY FOR DIFFERENT SIZES OF FLUME

Q_o		0.0666		0.222		0.476		0.710	
b	x_o	y_o	E_o	y_o	E_o	y_o	E_o	y_o	E_o
1 in.	9.31	-	-	0.510	0.514	0.829	0.836	1.08	1.089
2 in.	5.33	-	-	0.500	0.510	0.816	0.833	1.058	1.080
3 in.	4.00	-	-	0.500	0.515	0.811	0.836	1.050	1.084
6 in.	2.67	-	-	0.487	0.511	0.790	0.832	1.020	1.078
9 in.	2.52	-	-	0.485	0.511	0.764	0.812	0.995	1.057
1 ft	2.94	0.220	0.230	0.470	0.494	0.780	0.829	1.01	1.072
2 ft	1.64	0.215	0.233	0.465	0.507	0.755	0.827	0.975	1.070
3 ft	1.20	0.211	0.234	0.444	0.501	0.744	0.837	0.960	1.080
4 ft	0.983	0.208	0.234	0.440	0.505	-	-	-	-
6 ft	0.763	0.206	0.237	-	-	-	-	-	-
8 ft	0.654	0.205	0.238	-	-	-	-	-	-
10 ft	0.589	0.201	0.237	-	-	-	-	-	-
12 ft	0.546	0.199	0.237	-	-	-	-	-	-
20 ft	0.458	0.195	0.235	-	-	-	-	-	-
40 ft	0.392	-	-	-	-	-	-	-	-
		Mean = 0.235			0.508		0.833		1.077

Comparisons between the solution of Eq. 16 and the original data show agreement with an error of less than 1% for the majority of points and less than 2% for the rest with some very few exceptions (see Table 3).

DETERMINATION OF THEORETICAL FLOW

Considering the flume as an open channel transition, assume (1) flow at the measuring section uniform and parallel, and (2) flow at the throat non-uniform and curved. Having obtained a relationship for the actual flow rate, a theoretical expression will now be derived. Referring to Fig. 1

$$E_1 = y_1 + \frac{V_1^2}{2\, g} \qquad \text{(17a)}$$

TABLE 3.—ACCURACY OF EQ. 16

Flume Size	Error, in percentage	Remarks
1 in.	< 1	
2 in.	< 1	
3 in.	< 1	< - 2% large flows
6 in.	< 1	< + 2% large flows
9 in.	Approx. 5	Original calibration suspected
1 ft	- 2 to 3	Scatter of original data.[a]
2 ft	2	Scatter of original data.[a]
3 ft	< 1	
4 ft	< 1	
6 ft	< 1	< 2% small flows
8 ft	< 1	
10 ft	< 1	
12 ft	< 2	Scatter of original data.
20 ft	< 1	2% small flows
40 ft	< 1	

[a] Parshall reported trouble with ice at time of calibration.

in which E_1 is total energy head at the measuring point x_1, and V_1 is the mean velocity at the measuring section, or

$$\frac{g\,E_1}{V_1^2} = \frac{1}{2\left(1 - \frac{y_1}{E_1}\right)} \quad \text{.............. (17b)}$$

also

$$y_c = \lambda_1 \frac{2}{3} E_1 \quad \text{................. (18)}$$

and

$$V_c = \lambda_2 \sqrt{g\,y_c} \quad \text{................. (19)}$$

in which λ_1 and λ_2 are factors correcting for non-hydrostatic pressure, friction and non-uniform velocity distribution, and V_c is the critical velocity. Substituting for y_c from Eq. 18, Eq. 19 becomes

$$V_c = \lambda_2\,\lambda_1^{\frac{1}{2}} \sqrt{\frac{2}{3}\,g\,E_1} \quad \text{............. (20)}$$

Also

$$Q = b\,V_c\,y_c \quad \text{................ (21)}$$

Substituting from Eqs. 18 and 20 and writing $\lambda_2\,\lambda_1{}^{3/2} = \lambda$

$$Q = \lambda \left(\frac{2}{3}\right)^{\frac{3}{2}} b\,g^{1/2}\,E_1^{3/2} \quad \text{............. (22a)}$$

or

$$Q = \lambda \frac{\left(\frac{2}{3}\right)^{\frac{3}{2}} b\, g^{1/2}\, y_1^{3/2}}{\left(\frac{y_1}{E_1}\right)^{3/2}} \qquad \text{...........(22b)}$$

Comparing with Eq. 9

$$K_p = \frac{\lambda\ (2/3)^{3/2}}{\left(\frac{y_1}{E_1}\right)^{3/2}} \qquad \text{.................. (23)}$$

The value of λ may be found by writing Eq. 22 as

$$Q_o = \lambda \left(\frac{2}{3}\right)^{\frac{3}{2}} E_o^{\frac{3}{2}} \qquad \text{................. (24)}$$

and comparing with Eq. 15

$$\lambda = 1.17\ Q_o^{0.0325} \qquad \text{................. (25)}$$

Eq. 23 then becomes

$$K_p = 1.17\ Q_o^{0.0325} \frac{(2/3)^{3/2}}{\left(\frac{y_1}{E_1}\right)^{3/2}} \qquad \text{........... (26)}$$

It is thus seen that K_p is a function of $\frac{y_1}{E_1}$ and Q_o $\frac{y_1}{E_1}$ can be expressed as a function of β and Q_o as follows:

From continuity

$$V_1\, y_1\, b_1 = V_c\, y_c\, b \qquad \text{............... (27)}$$

therefore

$$y_1 = \frac{V_c}{V_1} y_c \frac{b}{b_1} \qquad \text{................. (28)}$$

or

$$y_1 = \frac{V_c}{V_1}\ y_c\, \beta \qquad \text{................. (29)}$$

Substituting in Eq. 29 from Eqs. 19, 18, and 17b and rearranging gives

$$\left(1 - \frac{y_1}{E_1}\right)\left(\frac{y_1}{E_1}\right)^2 = \frac{4}{27} \lambda^2\, \beta^2 \qquad \text{............. (30)}$$

Solving the cubic equation yields, for the relevant value,

$$\frac{y_1}{E_1} = \frac{2}{3} \cos \left[1/3 \cos^{-1} \left(1\text{-}2\,\lambda^2\,\beta^2\right)\right] + 1/3 \quad \ldots\ldots\ (31)$$

and eliminating $\frac{y_1}{E_1}$ from Eq. 23

$$K_p = \frac{\lambda}{\left\{\cos\left[1/3 \cos^{-1}\left(1 - 2\,\lambda^2\,\beta^2\right)\right] + 1/2\right\}^{3/2}} \quad \ldots\ldots\ (32)$$

Because λ is a function of Q_o, it is seen that K_p depends on both β and Q_o. β changes from flume to flume, depending only on the geometry, while Q_o is a function of discharge and flume width. This accounts for the variation of K_p shown in Fig. 2.

Note that for a parallel throated flume with friction neglected $\lambda = 1$ then, Eq. 30 becomes

$$\left(1 - \frac{y_1}{E_1}\right)\left(\frac{y_1}{E_1}\right)^2 = \frac{4}{27}\,\beta^2 \quad \ldots\ldots\ (33)$$

and Eq. 23 becomes

$$K = \frac{(2/3)^{3/2}}{\left(\frac{y_1}{E_1}\right)^{3/2}} \quad \ldots\ldots\ (34)$$

The relationship between β, $\frac{y_1}{E_1}$ and K as given by Eqs. 33 and 34 are shown plotted in Fig. 5.

Also Eq. 32 becomes

$$K = \frac{1}{\left\{\cos\left[1/3 \cos^{-1}\left(1 - 2\,\beta^2\right)\right] + 1/2\right\}^{3/2}} \quad \ldots\ldots\ (35)$$

It will now be shown that λ is connected with the location of the nominal critical depth y_{cn}.

Dividing both sides of Eq. 3 by b yields

$$\frac{y_{cn}}{b} = \sqrt[3]{\frac{{Q_o}^2}{\left(\frac{b_{cn}}{b}\right)^2}} \quad \ldots\ldots\ (36a)$$

or

$$\frac{b_{cn}}{b} = \frac{Q_o}{\left(\frac{y_{cn}}{b}\right)^{3/2}} \quad \ldots\ldots\ (36b)$$

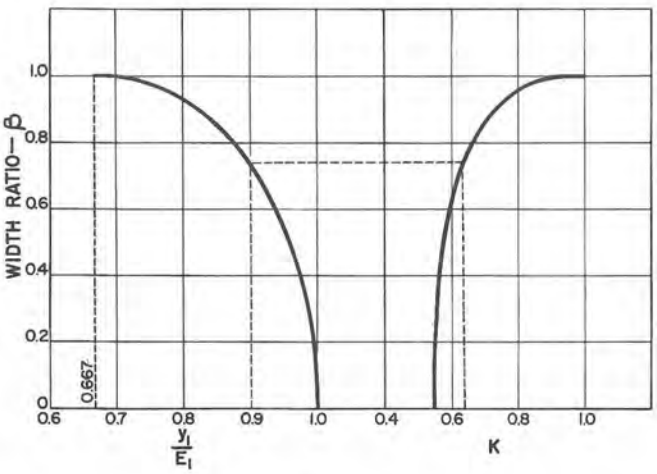

FIG. 5

Referring once again to Fig. 3 the critical depth y_{cn} for each flow profile was estimated by Hauser[10] using Eq. 3. Because b_{cn} was unknown, this was done by a trial process. The locations of y_{cn} are shown in the figure at the intersections of the flow profile and the oblique broken lines.

Using these four values the relationship between $\frac{y_{cn}}{b}$ and Q_o is found to be

$$\frac{y_{cn}}{b} = 0.900\ Q_o^{0.645} \qquad (37)$$

This checks with Eq. 15 for, neglecting friction loss, obviously

$$\frac{y_{cn}}{b} = 2/3\ E_o \qquad (38)$$

Substituting Eq. 37 into Eq. 36 gives

$$\frac{b_{cn}}{b} = 1.17\ Q_o^{0.0325} \qquad (39)$$

that is

$$\lambda = \frac{b_{cn}}{b} \qquad (40)$$

It is thus seen that the variation of the Parshall coefficient is due, at any rate for the range of Eq. 26 (Q_o = 0.0666 - 0.710), to the hydrostatic critical depth occurring upstream of the throat and that friction losses are negligible up to this section.

For smaller values of Q_o Eq. 26 may not apply because friction becomes more appreciable as Q_o diminishes, however further experiments are needed to verify this.

CONCLUSIONS

Eq. 16

$$y_o + \frac{Q_o{}^2}{2\, y_o{}^2\,(1 + 0.4\, x_o)^2} = 1.351\, Q_o{}^{0.645} \quad \ldots\ldots\ldots \quad (16)$$

closely fits all data published on Parshall flumes and may be used for calculating the flow for flumes of any size, not only those published in the literature but also for intermediate sizes. It may also be used for correcting the calibration curves of standard size flumes that do not conform with the specified dimensions of throat width or upstream measuring distance.

A further investigation is needed to examine the effect of varying the side angle or bottom angle but by use of this non-dimensional method, the effect of these variables could be found by experiments on perhaps not more than three sizes of flume.

ACKNOWLEDGMENTS

The writer wishes to thank Peter L. Monkmeyer for his useful suggestions.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

b, b_{cn}, b_1 = Channel widths (see Fig. 1);

E_1 = total energy head at the measuring point x_1;

E_o = non-dimensional energy head = $\frac{E_1}{b}$;

f_1, f_2, and so forth = arbitrary functions;

g = acc^n due to gravity;

K = discharge coefficient for parallel flow at throat;

K_e = as defined in formula $Q = K_e\, b g^{1/2}\, y_1{}^n$;

K_p = discharge coefficient for curved flow at throat;

n = as defined in formula $Q = K_e\, b g^{1/2}\, y_1{}^n$;

Q = discharge;

Q_o = non-dimensional discharge $\frac{Q}{g^{1/2}\, b^{5/2}}$;

V_1 = velocity at measuring section;

V_c = true critical velocity at throat;

x_1 = distance from throat crest to measuring section (see Fig. 1);

x_o = non-dimensional distance = $\frac{x_1}{b}$;

y_1 = depth at measuring section;

y_o = non-dimensional depth = $\frac{y_1}{b}$;

y_c = true critical depth at throat;

y_{cn} = nominal critical depth given by $y_{cn} = \sqrt[3]{\frac{Q^2}{b_{cn}^2\, g}}$;

λ_1, λ_2 = factors correcting for non-hydrostatic pressure, friction and non-uniform velocity distribution;

β = width ratio = $\frac{b}{b_1}$;

ϕ = bottom angle (see Fig. 1); and

θ = side wall angle (see Fig. 1).

DISCUSSION

W. L. FOSS,[11] M. ASCE.—This treatment of flume data serves to point up the complications that arise when one attempts to fit an arbitrarily dimensioned structure to an accepted theory. Although Davis's success in this effort can be appreciated, the resulting complex equation (Eq. 16) seems to be of questionable value to the average user of Parshall flumes.

Almost any person with a knowledge of hydraulics is familiar with the equation:

$$Q = C\, b\, H^{3/2} \quad \text{(41)}$$

The user of a Parshall flume is interested in only two quantities, y_1 and Q_1 and any formula of the form of Eq. 41, with y_1 replacing H, would be acceptable to him.

[11] Proj. Engr., Canada Dept. of Agric., Prairie Farm Rehabilitation Admin., Lethbridge, Alta., Canada.

However, published data indicates that C in Eq. 41 is a function of both b and y_1, or

$$C = K\, f(b, y_1) \qquad (42)$$

If Davis's value of Q_o from Eq. 7 is substituted into Eq. 15, there results

$$Q = 3.56\, b^{0.95}\, E_1^{1.55} \qquad (43)$$

However, y_1, not E_1, is actually measured, and although y_1/E_1 varies for different flows and different flumes, Eq. 43 suggests that one might develop a formula in the form of

$$Q = K\, b^n\, y_1^m \qquad (44)$$

that would apply to all flumes, with acceptable accuracy.

Procedure.—Therefore,

$$f(b, y_1) = b^p\, y_1^s \qquad (45)$$

so that Eq. 42 becomes

$$C = K\, b^p\, y_1^s \qquad (46)$$

TABLE 4.

b, in feet	y_1, in feet	Q, from Tables 1 and 2	Q, from Eq. 48	Difference, in percentage
0.5	.243	.223	.225	+0.90
0.5	.395	.477	.486	+1.90
1.0	.220	.377	.371	-1.60
4.0	.832	12.07	12.01	-0.50
20.0	3.90	677.00	680.00	+0.44
40.0	2.00	454.00	453.00	-0.22
40.0	8.00	4179.0	4218.0	+0.92

To find s for a flume with a width of 4 ft, Table 2 may be used to compare the flow characteristics, with Q_o = 0.222 and 0.0666. From Eq. 41, replacing H with y_1, one gets for the flume with Q_o = 0.222,

$$C_1 = \frac{40.27}{4}\,(1.76)^{3/2} = 4.31$$

and, similarly, for the flume with Q_o = 0.0666,

$$C_2 = 3.97.$$

Because the width ratio b_r is 1, and K is a constant, $C_r = y_{1r}^s$, and with C_r = 0.921 and y_{1r} = 0.4727, one gets

$$s = \frac{\log 0.921}{\log 0.4727} = 0.11$$

Now, to find p, take a 1-ft flume from Table 2 with Q_o = 0.222 and y_1 = 0.47, so as to keep y_o in the same order of magnitude as the base flume. Because it has been established that $Q \propto y_1^{0.11}\ y_1^{3/2}$, then, for the 1-ft flume,

$$Q = K\,b^n\,y_1^{1.61} \qquad (47)$$

But because, by Eq. 7, Q = 1.26, $K\,b^n = 4.25$, or, because b = 1, K = 4.25, and n in Eq. 44 can be determined directly, returning to the flume with $Q_o = 0.222$,

$$b^n = \frac{40.27}{4.25}\,(1.76)^{1.61} = 3.81$$

and

$$n = \frac{\log 3.81}{\log 4.0} = 0.963$$

The formula sought is, therefore,

$$Q = 4.25\,b^{0.963}\,y_1^{1.61} \qquad (48)$$

Applying Eq. 48 to various flumes whose capacities are computed from data in Tables 1 and 2, Table 4 is obtained.

ARMANDO BALLOFFET,[12] F. ASCE.—The author has succeeded in presenting an equation to unify the various empirical formulas for the discharge measured by Parshall flumes. This equation, however, may prove somewhat cumbersome for use in practical applications, and the writer feels that its merit lies rather in the fact that it shows the similarity of the flow conditions for different Parshall flumes. As regards the application of the critical flow theory to this measuring device, the writer believes that the assumption of rectilinear and parallel flow made by the author is not warranted in this case. In the writer's opinion, the empirical formulas originally presented by Parshall are simpler than the author's. They might also provide a unified expression and accurate results for intermediate sizes of throat, without recourse to the theory.

In the investigations mentioned in the writer's paper quoted by the author,[6] the writer became convinced that, if the flow at both the measuring section and at the throat could not be considered as rectilinear and parallel, the application of the simple critical flow theory would not produce formulas devoid of functional empirical corrections. The writer studied meters formed by a rectangular prismatic canal with an abrupt transition to a rectangular throat, maintaining the channel bottom horizontal throughout the meter. Two different series were used, one with a throat length equal to the upstream width of the canal and another with the throat formed simply by a plane sheet at right angles with the canal sides. Upstream of the throat, the flow in the models of both series appeared quite in agreement with the assumption of rectilinear and parallel trajectories. However, the flow at the throat was obviously more uniform for the models of the first series than for those of the second. In fact, for the latter, the theoretical critical depth (that is, referred to the width of

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the throat) was obtained downstream of the throat plane, at a place in which the water was flowing as a diverging jet. Whereas for the first series, the critical depth was measured within the throat.

Having obtained the theoretical equations for the first series prediction of the measured discharge within a difference of 4% to 5% was possible. This difference was practically constant for each model, that is, independent of the discharge. Thus, the theory was, in the writer's opinion, quite accurate to describe the flow in the models of the first series.

In the case of the first series, the departure between the theoretical equation and the measured discharges was not accidential and an empirical coefficient, dependent on the upstream depth (or the discharge) had to be applied to the theoretical formula. This was due to the fact that the flow past the throat was both curvilinear and non-parallel, as is the case in the Parshall flume.

For the models of the first series, the critical submergence ratio (that is, the maximum ratio of the downstream and upstream depths compatible with control section at the throat) could be predicted with surprising accuracy by combining the momentum and energy equations for the flow through the meter. The computed value was 0.88 and the observed values varied between 0.83 and 0.89. This proved again that the theory was, in this case, able to describe the flow satisfactorily.

For the models in the second series, the observed critical submergences were similar to those measured by Parshall and ranged between 0.52 and 0.76, completely at variance with the theoretical results.

In the writer's opinion, in the application of the theory to Parshall flumes, the effects of curvilinear flow should be taken into consideration and, to the writer's knowledge, this investigation has not yet (1962) been made.

A. R. ROBINSON,[13] A. M. ASCE, and HENRY LIU.[14]—This paper represents a contribution to understanding the operation of the Parshall measuring flume. The dimensionless approach in the development of the unified equation is unique and the excellent agreement with existing data for all sizes of Parshall flumes is gratifying. However, it should be pointed out that Eq. 16 is essentially empirical, because experimental data have been used to determine the constants.

Eq. 9, which Davis obtained by dimensional analyses and reasoning, may also be obtained by using the energy and continuity relationships between point 1 and the beginning of the drop-down angle (point 2). This derivation assumes that $y_2 = y_c$ and

$$y_1 = \frac{3}{2} y_2 \qquad (49)$$

Eq. 49 also presupposes that the velocity head at point 1 is negligible. Critical depth actually occurs in the contraction section upstream from the throat section, as the author has noted. Therefore, the location and magnitude of critical depth is difficult to determine in this case, and Eq. 49 is not entirely correct.

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An examination of Eq. 9 shows that the exponent of y_1 is 1.5. From Table 1, the values of n are always different from 1.5, which indicates that K_p is a function of the depth y_1 for a given size of flume. For the 6-in. flume,

$$Q = 4.12\, b\, y_1^{\,1.58} = K_p\, g^{1/2}\, b\, y_1^{\,1.50} \quad \ldots\ldots\ldots\ldots \quad (50)$$

so that

$$K_p = 0.727\, y_1^{\,0.08} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (51)$$

Fig. 2 is actually the relationship between K_p and y_1 for different sizes of flumes. In other words, an equation similar to Eq. 51 exists for each size of flume. The discharge coefficient K_p is also a function of the geometry of the flume, as shown by the displacement of each curve in Fig. 2.

An examination of Table 1 and Fig. 2 raised a question as to the validity of the original calibration data for some of the flumes. Since the sidewall angle, θ, as well as the drop-down angle, ϕ, are constant for all sizes of Parshall flumes, one would expect a definite trend in the values of K_e and n for the different flumes because of geometric similarity. This should be true for the "foot" flumes and should apply also for the "inch" flumes, despite the slight difference in geometrical relationships. A family of curves—possibly increasing in slope with increasing throat width—would be expected in Fig. 2, but this is not the case.

Because Eq. 16 has been developed from experimental data of flumes with constant contraction and drop-down angles, it should not be applied to those with different angles. Experimental verification is needed to determine the effect of changing the sidewall angle. With Eq. 16, the allowable changes of dimensions from that of the standard ones are throat width, b, and gage location, x_1, only.

Actually the close fit of Eq. 16 to the published data for the Parshall flume is not surprising in view of the relatively small spread in values of K_e and n, as shown in Table 1. An equation of the form

$$Q = 4\, b\, y_1^{\,1.55} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (52)$$

may be used for flumes up to 8 ft with deviations only slightly greater than those using Eq. 16. The importance of Eq. 16 lies in the inclusion of the variable distance x_1 in the discharge formula. With this relationship, the point of measuring upstream head may be changed from the standard one, and a new rating table may be determined. It is hardly foreseeable that intermediate sizes of Parshall flumes, other than those now in published literature, are needed. There is a probable need for flumes in excess of the largest size (50 ft) presently (1962) available.

Eq. 16 applies only to the free-flow condition, with the flow passing through critical depth within the flume. A unified equation for flow under submerged conditions does not presently exist. Instead, empirical relationships using plots are utilized to determine the flow under these conditions. Because submerged flow is difficult to determine accurately, special care should be used in installing the flumes in order that the free-flow condition exists for most of the flows. The elevation of the flume above the bottom of the channel must be set in order that there is a submergence of less than 60% for the "inch" flumes and 70% for the "foot" flumes in order to maintain free flow. The increase in elevation of the crest will raise the water surface upstream and restrict the use of Parshall flumes in channels with very flat slopes.

Tests conducted at Colorado State University, Boulder, Colo., by the Agricultural Research Service have shown that trapezoidal measuring flumes are at times superior in operation to the rectangular-throated Venturi or Parshall flumes. Advantages that were noted include the following:

1. The trapezoidal shape fits the common canal section more closely than does the rectangular flume. For the lined section this simplifies the transition design and construction.
2. Trapezoidal flumes operate under higher degrees of submergence than rectangular flumes without corrections being necessary to the standard rating.
3. A large range of flows may be measured with a relatively small change in depth thus minimizing the amount of freeboard needed on the canal.

Because of these advantages, it is anticipated that greater use will be made of trapezoidal flumes in the future.

R. A. DODGE, JR.[15]—Many hydraulic engineers have felt that the design of Parshall flumes could be lifted from the realm of pure empiricism by the use of standard similitude laws. Davis has demonstrated this possibility.

Eq. 16 appears to be unwieldy and difficult to use because none of the variables can be determined explicitly. This is substantiated by the author's use of a computer in solving the equation. However, a simple equation was derived in the following manner. In Fig. 6, X_o versus Y_o was plotted from Table 2 using Q_o as a parameter. The resulting curves were a set of straight lines of the same slope, having the equation form

$$Y_o = I X_o^n \quad \text{.................} (53)$$

in which $I = Y_o$ when $X_o = 1$. The slope of the curves, n, was found to be 0.0494. I versus Q_o was plotted on log-log paper (Fig. 7) and the equation was determined to be

$$I = 1.19\, Q_o^{0.645} \quad \text{................} (54)$$

Combining Eqs. 53 and 54 results in

$$Y_o = 1.19\, Q_o^{0.645}\, X_o^{0.0494} \quad \text{............} (55)$$

The discharge-depth relationship and the dimensioning in the range of the data in Table 2 for any Parshall flume model or prototype can be determined from

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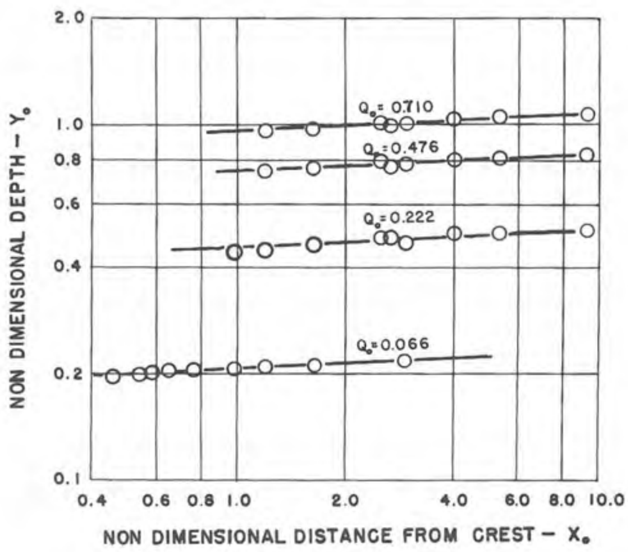
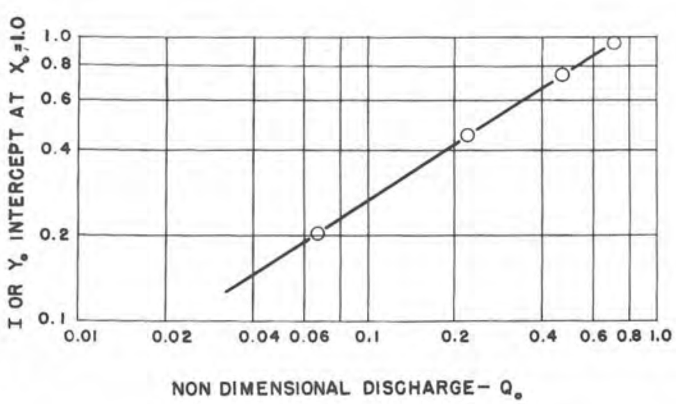


NON DIMENSIONAL DEPTH – Y_o
2.0
1.0
0.8
0.6
0.4
0.2
0.1
Q_o= 0.710
Q_o= 0.476
Q_o= 0.222
Q_o= 0.066
0.4
0.6
0.8
1.0
2.0
4.0
6.0
8.0
10.0
NON DIMENSIONAL DISTANCE FROM CREST – X_o

FIG. 6



I OR Y_o INTERCEPT AT X_o=1.0
1.0
0.8
0.6
0.4
0.2
0.1
0.01
0.02
0.04
0.06
0.1
0.2
0.4
0.6
0.8
1.0
NON DIMENSIONAL DISCHARGE – Q_o

FIG. 7

Eq. 55 and the following dimensionless relationships:

$$Q_o = \frac{Q}{g^{1/2}\, b^{5/2}} \quad (7)$$

$$X_o = \frac{X_1}{b} \quad (8)$$

$$Y_o = \frac{Y_1}{b} \quad (12)$$

All of these relationships can be readily solved by the use of slide rule or by logarithms.

SYDNEY DAVIS,[16] M. ASCE.—Foss, Balloffet, and Dodge comment on the difficulty of using Eq. 16. In the opinion of the writer, this is not serious because, in the case of standard flumes, the average user of Parshall flumes usually resorts to tables. The importance of Eq. 16, as pointed out by Robinson and Liu, is that it allows a rating curve to be determined when the upstream measuring point is changed from the standard one, as well as allowing intermediate throat widths. Thus, Eq. 48 derived by Foss cannot be used for flumes other than those of standard shape and size. Eq. 55 by Dodge is a more elegant solution but can be used only over the range of values given in Table 2. For flumes whose throat widths are 8 ft or larger, the formula overestimates the flow by as much as 5%.

Note that Eq. 2b may be written as

$$\frac{b_1}{b} = 1 + 0.4\,\frac{x_1}{b} \quad (56a)$$

or

$$b_o = 1 + 0.4\, x_o \quad (56b)$$

in which

$$b_o = \frac{b_1}{b} \quad (57)$$

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Then, Eq. 16 becomes

$$y_o + \frac{Q_o^2}{2\,y_o^2\,b_o^2} = 1.351\,Q_o^{0.645} \qquad (58)$$

On the left side of Eq. 58, the side wall angle is now eliminated. The term on the right side does, in general, depend on side wall angle, but experiments carried out at Colorado State University show that, over a certain range of angles, the change in this term is small. It is hoped that the results of these experiments will be published shortly.

Robinson and Liu mention that "K_p is a function of depth y, for a given size of flume." It should be noted that the general expression for K_p was given by Eq. 32. The writer agrees that there is some doubt as to the validity of the original data. It seems that one source of error is the lack of consistent entrance transitions for different size flumes. For example, the "foot" flumes were originally tested with the entrance transitions consisting of straight wing walls at 45°. These walls were later changed to curved transitions, the radii of curvature for the different sizes being chosen, apparently, in a somewhat arbitrary fashion. The flumes were not recalibrated, so far as is known.

Balloffet states his belief "that the assumption of rectilinear and parallel flow made by the author is not warranted in this case." However, as far as the throat section is concerned, the assumption made in the paper, under the heading "Determination of Theoretical Flow," was that the flow was non-uniform and curved. With regard to the measuring section, the assumption of rectilinear and parallel flow seems reasonable. This is seen by studying Table 2. The values for E_o are ultimately based on Eq. 17a. If non-uniform and curved flow were important at the measuring section, E_o would be varying with x_o for each value of Q_o, but the table shows that E_o is essentially constant. Therefore, it would seem that the assumption made by the writer is sufficiently accurate.

Regarding the limit of submergence to which Balloffet has drawn attention, there appears to be at least one error in principle in the derivation of his formula expressing this relationship. For any given rectangular flume, the channel width B and the throat width b are fixed, as is the discharge Q.[6] Moreover, it is assumed that (1) the flow in the throat is critical; (2) the flow immediately upstream in the main channel is subcritical; (3) downstream of the throat, the flow is supercritical but returns to subcritical flow via a hydraulic jump; and (4) at all relevant sections, the stream lines are parallel and the velocity distribution uniform. Under such conditions, there exists only one possible depth and energy head at any given cross section. It follows that the difference between the upstream energy head and the downstream depth is uniquely determined.

In other words, referring to Eq. 19 of Balloffet,[6] Δ is shown to be a function of terms that are themselves functions of B, b, and Q only. Since B, b, and Q are fixed, Δ is also fixed, so that a mathematical minimum for this cannot exist and the further derivation for the limit of submergence involving differentiation is not valid. Furthermore, if one of the limiting values of r, namely, $r = 1$, is substituted into Eq. 24, from Balloffet,[6] the limit of submergence acquires a value of 1.16, approximately, which is impossible because the maximum theoretical value of r is unity.