OPTIONAL FORM NO. 10
5010-104

UNITED STATES GOVERNMENT

Memorandum

Memorandum

Denver, Colorado

TO : Chief, Hydraulics Branch

DATE: October 3, 1963

FROM : C. M. Wu

SUBJECT: Flow characteristics of spillway buckets

Submitted please find a revised draft of "Flow Characteristics of Spillway Buckets."

A new article, curvilinear effect on Froude number and critical depth of bucket flow is added and it is shown that the hydraulic jump can be formed in the bucket even though the Froude number is less than 1. This may be interesting for hydraulic engineers.

Chian Min Wu

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

Office of Chief Engineer
Division of Research
Hydraulics Branch
Structures and Equipment Section
Denver, Colorado
July 26, 1963

Laboratory Report No. Hyd-
Written by: Chian-Min Wu
G. L. Beichley
Checked by: T. J. Rhone
Reviewed by: W. E. Wagner
Submitted by: H. M. Martin

Subject: Flow characteristics of spillway buckets

PURPOSE

To investigate the bucket flow characteristics considering the radial effect of the bucket and to develop theoretical and experimental equations for the main-flow parameters.

CONCLUSIONS

1. A theoretical analysis and study of experimental data show a pronounced centrifugal effect on the action of the radial flow through a roller bucket and suggest that the flow characteristics in spillway buckets can be analyzed by use of the equations of pressure plus momentum with the addition of the centrifugal force.

2. A theoretical equation is developed to relate the bucket radius to the flow characteristics of two different flow conditions described as a sweepout jump-flow condition and a submerged jump-flow condition. Also, a theoretical equation is derived for the energy loss in both. These equations are experimentally verified using data obtained by the USBR, Hydraulic Laboratory, within the

limits of Froude numbers up to 9 and backup factors (ratio of flow depth above the invert, y_3, to the supercritical depth at the invert, y_1) to approximately 25. A theoretical equation is developed for the backup depth factor at the bucket invert for the submerged jump-flow condition. This equation is also experimentally verified within these limits by applying a correction coefficient for air-entrainment.

3. Curvilinear effect on Froude number and critical depth of bucket flow is analyzed and it is shown that a correction factor must be applied to the originally defined Froude number, i.e., $F = \frac{V}{\sqrt{gy}}$ and it is also shown that the hydraulic jump can be formed in the bucket even though the Froude number is less than 1.

4. Considerable information concerning jump, surge length, surge height, and other flow parameters is analyzed, and it is shown that all of the flow characteristics may be expressed as functions of the flow parameters (Froude number, tailwater factor, submerged factor) and the shape parameters (abrupt factor, baffle factor, and radius factor).

5. Using the equations obtained, it is found that the energy loss in a submerged jump within a bucket is more than in the corresponding free jump on a horizontal floor, but constantly less than that

of a corresponding sweepout jump-flow condition within the bucket. However, due to unstable characteristics of the sweepout jump-flow condition, a slight submergence is recommended.

6. Comparison of energy dissipation capacity of various hydraulic jump basins and the lengths of these basins with that of the bucket shows that the bucket is more efficient in dissipating the energy and does so in a shorter length of structure.

INTRODUCTION

If an initial hydraulic jump depth, y_1, is to be formed at the toe of a bucket invert for a supercritical stream discharging over a spillway, the tailwater depth should be equal to the subcritical sequent depth, y_4, given by the momentum equation. If the tailwater depth, y_5, is less than y_4, the jump is swept out of the bucket. This is known as a flip-bucket flow condition (Figure 1-a*). If $y_5 = y_4$, the hydraulic jump is on the verge of sweeping out of the basin. This is defined here as a sweepout jump-flow condition (Figure 1-b). If, however, y_5 is greater than y_4, the jump is submerged or drowned as shown in Figure 1-c. This is defined as a submerged jump-flow condition.

Basic information concerning the characteristics of the bucket flow is published in many hydraulic and fluid mechanics books, yet the

*All figures refer to figures in this appendix, only.

solution of the problem has been approached by means of hydraulic models. Therefore, it is apparent that an analytical study of bucket-flow phenomenon would be helpful to at least provide a theoretical guide for the model study.

In 1933 and 1945 bucket-flow characteristics were studied by the Bureau of Reclamation with the aid of hydraulic models for the Grand Coulee and Angostura Dam buckets.[1/] [2/] In 1953-1954 extensive hydraulic model tests were conducted by the Bureau of Reclamation's Hydraulic Laboratory to establish general design procedures.[3/] Similar studies were conducted by Lehigh University to define the general performance characteristics.[4/]

The radial effect on the flow through the bucket for both sweepout jump-flow and submerged jump-flow conditions has not been conclusively established. No doubt the complexity of the behavior of the rolling body, including its turbulence and surge, has discouraged the earnest efforts of many investigators, and perhaps the perspective on the main issues of the problem has been obstructed by minor

---

[1/] Jacob E. Warnock, "Spillway and Outlets for Grand Coulee Dam," Hydraulic Laboratory Report No. 103, USBR, 1935

[2/] "Hydraulic Model Studies on the Spillway for the Angostura Dam," Hydraulic Laboratory Report No. 192, USBR, 1946

[3/] G. L. Beichley and A. J. Peterka, "The Hydraulic Design of Slotted Spillway Buckets," Proc. ASCE, Vol. 85, No. Hy-10, October 1959

[4/] M. B. McPherson and M. H. Karr, "A Study of Bucket-Type Energy Dissipator Characteristics," Proc. ASCE Vol. 83, No. Hy-3, June 1957 inconsistencies of flow behavior. It is suspected that difficulties in the past have been experienced largely due to efforts to merge the phenomenon into complex theory, whereas, guidance by the simple fundamental laws of mechanics and application of empirical formulae based upon analyses of experimental data would have probably yielded substantial progress.

In order to gain a more rational understanding of this phenomenon, a combination of theory and experiments are used in this study.

Notation: The letter symbols adopted for use in this study are defined where they first appear and are arranged alphabetically in Appendix A.*

SWEEPOUT JUMP-FLOW CONDITION

Review of Theory

The equations of pressure plus momentum are expressions of basic theories and must be satisfied in the simultaneous algebraic calculations required for the solution of jump problems. Recently a concept of the hydraulic horizontal jump was used for analysis of the bucket flow where an apron extended horizontally from the bucket invert.[5/] Centrifugal force was not taken into consideration. This type of bucket is called a half bucket in this analysis. Further developments of solid and slotted buckets have shown that

[5/] Serge Leliavsky, "Irrigation and Hydraulic Design," Chapman and Hall, Ltd., London, Volume 1, 1955 the centrifugal effect on the pressures in the bucket (Figure 2) may become so pronounced that it must be included in the analysis.

The sweepout jump-flow condition, Figure 1-b, is not a recommended operating condition since it creates much water surface roughness and is unstable but is analyzed here only as preparation for analysis of the submerged jump-flow condition.

Definition Sketch

Based on recent model studies[1/ 2/ 3/ 4/ 6/] a definition sketch for the bucket flow is presented.

Referring to Figure 1-b, the factors involved in the analysis of the sweepout jump-flow conditions are defined as follows:

Sequent depth factor (ψ) is defined as the ratio of the sequent depth y_4 to the supercritical initial depth, y_1.

$$\psi = \frac{y_4}{y_1} \qquad (1)$$

Tailwater factor (S_1) is defined as the ratio

$$S_1 = \frac{(y_4 - y_2)}{y_2} \qquad (2)$$

[6/] Atsuya Okada and Takeshi Ishibashi, "Hydraulic Model Tests on Flood Spillway of Sakuma Dam," Central Research Institute, Electric Power Industry, Japan, June 1, 1956 where y_2 is the subcritical sequent depth of normal free jump on horizontal floor.

Abrupt factor (Z) is defined as the ratio of abrupt rise, z , to initial depth, y_1 ,

$$Z = \frac{z}{y_1} \tag{3}$$

Baffle factor (k) is defined as the ratio of the total width of baffle, b_t, to that of bucket width, b ,

$$k = \frac{b_t}{b} \tag{4}$$

Radius factor (R) is defined as the ratio of the initial depth, y_1, to bucket radius, r_1 ,

$$R = \frac{y_1}{r_1} \tag{5}$$

For slotted bucket similar to the Angostura Spillway bucket, $0 < k < 1$ and $Z > 0$.

For solid bucket similar to the Grande Coulee bucket without baffle, $k = 1$ and $Z > 0$.

For half bucket where a horizontal apron extends horizontally from the bucket invert, either k or $Z = 0$.

Drag Forces on Bucket Baffles

If the shear stress along the horizontal solid boundaries of a solid bucket between Sections 1 and 4, Figure 1-b, can be neglected and the momentum coefficients, β_1 and β_4, are assumed as unity, then by use of force and momentum theory,

$$\left(\frac{1}{2}\right) \gamma y_4^2 - \left(\frac{1}{2}\right) \gamma y_1 \left(y_1 + \frac{y_1 v_1^2}{g r_1}\right) + p$$

$$= \left(\frac{q \gamma}{g}\right)\left(\frac{q}{y_1} - \frac{q}{y_4}\right) \qquad (6)$$

in which γ is the specific weight of water, q is the discharge per unit width of bucket, g is the acceleration due to gravity, $\frac{y_1 v_1^2}{g r_1}$ is the static head correction factor of the bucket due to the radial flow.[7/] [8/]

p is the force acting on the baffle unit of the solid bucket and can be written in the form of the basic drag equation

$$p = Cd \frac{\rho v_1^2}{2} \Delta Z \qquad (7)$$

where Cd is the coefficient of drag, ρ is the mass density of water.

7/ Ven Te Chow, "Open-Channel Hydraulics," 1959

8/ D. B. Gumensky, "Design of Side Walls in Chute and Spillway," Trans. ASCE Vol. 119, 1954

Introducing Equation 7 into Equation 6

$$\frac{\gamma y_4^2}{2} + Cd\,\frac{\rho v_1^2}{2}\,\Delta Z - \frac{1}{2}\gamma y_1\left(y_1 + \frac{y_1\,v_1^2}{gr_1}\right)$$

$$= \frac{q\gamma}{g}\left(\frac{q}{y_1} - \frac{q}{y_4}\right) \tag{8}$$

Equation 8 could be reduced to form

$$Cd = \frac{\frac{1}{2}}{F_1^2 Z}\left\{2F_1^2\left(1 - \frac{1}{\psi}\right) - \psi^2 + (1 + F_1^2 R)\right\} \tag{9}$$

where F_1 is the Froude number for the corresponding initial flow condition for the horizontal hydraulic jump, i.e., $F_1 = \frac{V_1}{\sqrt{gy_1}}$.

Equation 9 is the drag coefficient relationship for solid bucket. In the case of slotted bucket (Figure 1-c), the drag equation is written as

$$p = Cd \cdot C \cdot k \cdot \frac{\rho\, v_1^2}{2}\,\Delta Z \tag{10}$$

where Cd is the drag coefficient for the corresponding solid bucket with abrupt factor Z and C is the spacing parameter for drag coefficient of slotted bucket. Then, the momentum equation for slotted bucket could be written as

$$\frac{1}{2}\gamma y_4^2 + C \cdot k \cdot Cd \frac{\rho v_1^2}{2} - \gamma y_1 \left(y_1 + \frac{y_1 v_1^2}{g r_1} \right)$$

$$= \frac{q\gamma}{g} \left(\frac{q}{y_1} - \frac{q}{y_4} \right) \qquad (11)$$

and simplifying

$$C = \frac{1}{k\ Cd\ F^2\ Z} \left\{ 2 F_1^2 \left(1 - \frac{1}{\psi} \right) - \psi^2 + 1 + F^2 R \right\} \qquad (12)$$

Equation 11 is the general expression of pressure plus momentum equation for spillway bucket, and could be used in evaluating flow parameters, such as sequent depth factor, radius factor, etc.

Experimental Studies of Drag Coefficient

Experimental data, taken by the USBR Hydraulic Laboratory from Grand Coulee,[1/] Angostura,[2/] and "general performance studies[3/] were used in evaluating the drag coefficient of the bucket baffles.

Plot of test data shows that the drag coefficient of the bucket baffle is mainly controlled by dimensionless parameters; Z and k. The best fitted curves for different types of bucket are shown in Figure 3-a. From Figure 3-a, it is seen that Cd decreases as Z increases, and for values of Z larger than 2.5, the effect of baffle is not significant. Therefore, for design purpose, a Z value of less than 2 is recommended.

To evaluate spacing parameter, C, test data from Angostura models[2] is plotted against baffle factor, k, in Figure 3-b. It is seen that C value increases from C = 1 at k = 100 percent to maximum of 1.60 at k = 67 percent and then decreases to 0 at k = 0. Thus, from the viewpoint of drag force, it is found that the slotted bucket of k = 67 percent will give the best effect.

Evaluation of Sequent Depth Factor

Substituting Equation 1 into Equation 11, using the equation of continuity, a cubic equation for sequent depth factor in a sweepout jump flow condition can be evaluated,

$$\psi^3 + \left\{ kC\ Cd\ F_1^2\ Z - (1 + F^2 R) - 2 F_1^2 \right\} \psi + 2 F_1^2 = 0 \qquad (13)$$

A mathematic solution of the sequent depth factor, $\frac{y_4}{y_1}$, is possible but complicated, and usually introduces some imaginary root. This makes the analysis of flow characteristics much more complicated, yet the solution of the equation shows

$$\psi = f(F_1, R, k, \text{and } Z) \qquad (14)$$

or simply the sequent depth factor in the sweepout jump can be shown as a function of flow parameter, F_1, and shape factors R, k, Z. This relationship can be used in solving for tailwater depth in the sweepout for known values of R, K, and Z conditions.

Again a factor of

$$G = \frac{F_1}{\sqrt{1 + \dfrac{k\,C\,Cd\,F_1^2\,y_1\,\Delta Z - \dfrac{y_1^2\,v_1^2}{gr_1}}{y_4^2 - y_1^2}}} \tag{15}$$

can be adopted into Equation 11 and simplifying

$$\psi = \frac{y_4}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8G^2} - 1\right) \tag{16}$$

This is the general equation for hydraulic jump and from the above analysis it is apparent that

$$G = f\,(F_1,\ R,\ k \text{ and } Z) \tag{17}$$

However, a Fortran program is given in Appendix B to simply the computation of Equation 13.

Evaluation of Radius Factor

For known tailwater condition, Equation 11 can be used in evaluating the radius factor, however, a tailwater factor, S_1 is adopted as a tool of comparison, then

$$\therefore R = \frac{y_1}{\gamma_1} = \frac{1}{F_1^2} \left\{ k\,C\,Cd\,F_1^2\,Z + \psi^2 - 2F_1^2 \left(1 - \frac{1}{\psi} \right) - 1 \right\}$$

$$= \frac{1}{F_1^2} \left\{ k\,C\,Cd\,F_1^2\,Z + (1+S_1)^2\,\varphi^2 - 2\,F_1^2 \left(1 - \frac{1}{(1+S_1)\,\varphi} \right) - 1 \right\} \quad (18)$$

where

$$\varphi = \frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8\,F_1^2} - 1 \right) \quad (19)$$

and

$$R = f\,(F_1,\ S_1,\ Z,\ K) \quad (20)$$

Equation 18 is the general radius factor equation for the bucket flow in the slotted-type bucket.

For the solid bucket in the sweepout jump-flow condition, Equation 18 can be reduced to

$$R = \frac{1}{F_1^2} \left\{ k\,Cd\,F_1^2\,Z + \psi^2 - 2\,F_1^2 \left(1 - \frac{1}{\psi} \right) - 1 \right\} \quad (21)$$

$$= \frac{1}{F_1^2} \left\{ k\,Cd\,F_1^2\,Z + (1+S_1)^2\,\varphi^2 - 2\,F_1^2 \left(1 - \frac{1}{(1+S_1)\,\varphi} \right) - 1 \right\} \quad (22)$$

and can be expressed as function of F_1, S_1, and Z,

$$R = f\,(F_1,\ S_1,\ Z) \quad (23)$$

If either Z or $K = 0$, as for the half bucket Equation 18 again can be reduced to

$$R = \frac{1}{F_1^2}\left[(1+S_1)^2 \varphi^2 - 1 - 2F_1^2\left(1 - \frac{1}{[(1+S_1)\varphi]}\right)\right] \tag{24}$$

which means that the radius factor can be expressed as a function of only F_1 and S_1 ;

$$R = f(F_1, S_1) \tag{25}$$

Energy Loss

The initial energy at the toe of the jump or invert of the bucket can be written as

$$E_1 = y_1 + y_1 v_1^2 / g r_1 + v_1^2 / 2g$$

$$= y_1 + y_1^2 F_1^2 + F_1^2 y_1 / 2 \tag{26}$$

Introducing R,

$$\frac{E_1}{y_1} = 1 + F_1^2 R + F_1^2 / 2 \tag{27}$$

The energy at the end of jump is

$$E_4 = y_4 + \frac{v_4^2}{2g} = (1 + S_1)y_2 + \frac{q^2}{2g\,(1+S)^2\,y_2^2} \qquad (28)$$

in which v_4 is the mean velocity at the tailwater.

The energy loss E_L is given by

$$E_L = E_1 - E_4 \qquad (29)$$

Substituting Equations 26 and 28 into Equation 29 and simplifying,

$$\frac{E_L}{E_1} = 1 - \frac{(1+S_1)\varphi - \frac{1}{2}F_1^2/(1+S_1)^2\varphi^2}{1 + F_1^2 R + F_1^2/2} \qquad (30)$$

that is

$$\frac{E_L}{E_1} = f(F_1, S_1, Z, k) \qquad (31)$$

or either

$$\frac{E_L}{E_1} = f(F_1, R, Z, k) \qquad (32)$$

and

$$\frac{E_L}{E_1} = f(S_1, R, Z, k) \qquad (33)$$

Thus for the relative loss of energy in various types of buckets, Equation 30 has been shown to be a function of Froude number (F_1) tailwater factor (S_1) and the bucket shape factors (R, Z, and k). It is apparent that the energy dissipating capacity of the bucket is much more than that of the corresponding horizontal jump. In order to figure out the efficiency of energy dissipation, an index n is used,

$$n = \frac{\left(\frac{E_L}{E_1}\right)_{b.j}}{\left(\frac{E_L}{E_1}\right)_{f.j}} \qquad (34)$$

where b.j represents the bucket flow in sweepout condition and f.j represents the corresponding horizontal jump condition. All the above-mentioned flow parameters can be obtained from the Fortran program in Appendix C.

Curvilinear Effect on Froude Number and Critical Depth

The general specific energy equation at the toe of the jump or invert of the bucket can be written as

$$E = y + y\frac{V^2}{gr} + \frac{V^2}{2g} = y + \frac{q^2}{gry} + \frac{q^2}{2gy^2} \qquad (26)$$

The critical state of flow has been defined as the state of flow at which the specific energy is minimum for a given discharge.[9] A

[9] Paul Böss, "Berechnung der Wasserspiegellage beim Wechsel des Fliesszustandes," (Computation of Water Surface with Change of the Flow Type"), Springer-Verlag, Berlin, 1919, pp 20 and 52.

theoretical criterion for critical flow may be developed by differentiating Equation 26 with respect to y and noting q as a constant thus

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}\left(1 + \frac{y}{\gamma}\right) = 1 - \frac{v^2}{gy}\left(1 + \frac{y}{\gamma}\right) \tag{35}$$

At the critical state of flow the specific energy is a minimum, or the depth where minimum energy occurs $\frac{dE}{dy} = 0$. Let y_{cb} represents the critical depth for the bucket flow, then y_{cb} must satisfy the following condition, that is

$$1 - \frac{q^2}{g\,y_{cb}^3}\left(1 + \frac{y_{cb}}{\gamma}\right) = 0 \tag{36}$$

or

$$y_{cb}^3 - \frac{q^2}{g\gamma}\,y_{cb} - \frac{q^2}{g} = 0 \tag{36'}$$

The solution of y_{cb} from Equation 36 can be obtained, but is not a matter of interest. However, the critical depth of the linear channel, $y_c = \sqrt[3]{\frac{q^2}{g}}$, is introduced in Equation 36 to get a better comparison, that is

$$\frac{y_c^3}{y_{cb}^3} = \left(\frac{1}{1 + \frac{y_{cb}}{\gamma}}\right) \tag{37}$$

Furthermore, the combined effect of gravity and curvature of bucket upon the state of flow, or Froude number for the bucket can be defined as

$$F_b = \sqrt{\frac{v^2}{gy}\left(1 + \frac{y}{r}\right)} \qquad (38)$$

or

$$F_b = F\left(\sqrt{1 + \frac{y}{r}}\right) \qquad (39)$$

when $F_b = 1$, the flow state is corresponding as defined by Equation 36, and the relationship between the Froude number for linear channel, $F = \frac{v}{\sqrt{gy}}$, and linear channel is

$$F_b = F\sqrt{1 + \frac{y_{cb}}{r}} = 1$$

where $\frac{y_{cb}}{r}$ varies $0 \leq \frac{y_{cb}}{r} \leq 1$, thus for the critical state of the bucket flow, the corresponding Froude number of the nonbucket, $F = \frac{v}{\sqrt{gy}}$, will vary from 1 to $\frac{1}{\sqrt{2}}$. This means that hydraulic jump can be formed even though the corresponding Froude number, $F = \frac{v}{\sqrt{gy}}$, is less than 1.

SUBMERGED JUMP FLOW CONDITION

Definition Sketch:

Referring to Figure 1-c, the added factors involved in the analysis of the submerged jump-flow condition are defined as follows:

Backedup depth factor (H_b) is defined as the ratio of the depth of flow in the bucket above the invert, y_3, to the supercritical depth, y_1,

$$H_b = \frac{y_3}{y_1} \tag{40}$$

Submerged factor (S_2) is defined as the ratio

$$S_2 = \frac{(y_5 - y_4)}{y_4} \tag{41}$$

Tailwater factor (S_3) is defined as the ratio

$$S_3 = \frac{(y_2 - y_5)}{y_2} \tag{42}$$

Surge factor (H_s) is defined as the ratio

$$H_s = \frac{y_6}{y_5} \tag{43}$$

where y_6 is the average surge peak depth.

Evaluation of Backed-up Depth Factor

The submerged hydraulic jump in a bucket is a system of two distinct streams; one, the principal stream occupying the lower portion of the jump body, and the other a rotating mixture of water and air supported both in motion and position by forces imparted to it by the principal stream. By the presentation of J. H. Douma,[10] the depth of the principal stream in a bucket invert, y_1, can be shown as a function of a dimensionless parameter, $\frac{q}{r_1 \sqrt{2gH}}$. (Appendix D) Thus the radius factor, R, can be easily determined for the submerged jump-flow condition.

Then refer to Figure 1-c, cut free body through bucket invert, the forces plus momentum equation can be written as:

$$\frac{1}{2} \gamma y_5^2 + kC\, Cd\, \Delta Z \frac{\rho v_1^2}{2} - \frac{1}{2} \gamma (y_3 - y_1)^2 - \frac{1}{2}\left(2y_3 + \frac{y_1 v_1^2}{gr_1} - y_1\right) y_1$$

$$= \frac{q\gamma}{g}\left(\frac{q}{y_1} - \frac{q}{y_5}\right) \qquad (44)$$

Substituting the equation of continuity and simplifying,

$$H_b = \frac{y_3}{y_1} = \sqrt{\psi'^2 + k\,C\,Cd\,Z\,F_1^2 R - 2F_1^2\left(1 - \frac{1}{\psi'}\right)} \qquad (45)$$

where $\psi' = \frac{y_5}{y_1} = (1 + S_2) \frac{y_4}{y_1} = (1 + S_2)(1 + S_1) \frac{y_2}{y_1}$

$$= (1 + S_3) \frac{y_2}{y_1} = \frac{1}{2} (1 + S_1) (\sqrt{1 + 8 F_1^2} - 1) (1 + S_2) \qquad (46)$$

---

[10] Discussed by J. H. Douma, "Design of Side Walls in Chutes and Spillways," Trans. ASCE, Vol. 119, 1954

H_b can be shown to be function of F_1, S_2 and shape factors, R, k, Z, that is

$$H_b = f(F_1, S_2, R, k, Z) \tag{47}$$

for a solid bucket

$$H_b = f(F_1, S_2, R, Z) \tag{48}$$

and for a half bucket,

$$H_b = f(F_1, S_2, R) \tag{49}$$

To simplify the computation, all flow parameters can be compared with the horizontal jump, and the third tailwater factor S_3 is used in evaluation of the backedup depth factor, then Equations 47 to 49 can be expressed as

$$H_b = f(F_1, S_3, R, k, Z) \tag{50}$$

Further more, the depth of the principal stream in a bucket invert,[11/] y_1, can be approximately determined, and backedup depth selected. In such a case, determination of radius factor, R, becomes necessary. Then, by use of Equation 44, the radius factor, R, can be expressed as a function of F_1, S_3, H_b and shape factors k, Z. That is

$$R = \frac{1}{F_1^2}\left(\left[\psi'^2 + kC\,Cd\,Z\,F_1^2 - H_b^2 - 2F_1^2\left(1 - \frac{1}{\psi'}\right)\right]\right) \tag{51}$$

11/ P. M. Stepanov, "O podtoplennom pryzhke vody," Gidrotekhnika i Melioratsiya, Vol. X, No. 1, 1958, pp 43-53 or

$$R = f(F_1, S_3, H_b, k, Z) \tag{52}$$

When $H_b = 1 (y_3 = y_1)$, Equation 51 can be reduced to Equation 18. This means that the sweepout condition is a special case of the submerged condition. Equation 44 can be used in evaluating drag coefficient for spillway buckets for submerged jump-flow condition. The resulting equation is

$$C\,Cd = \frac{1}{k \cdot F_1^2 Z} \left\{ F_1^2 R - \psi'^2 + H_b^2 + 2F_1^2 \left(1 - \frac{1}{\psi'} \right) \right\} . \tag{53}$$

Numerical values can be obtained from experimental data. However, due to air entraining facility of the bucket roller and unstable characteristics of water surface, it is reasonable to assume that the drag coefficient for submerged jump-flow condition will remain the same as for corresponding sweepout jump-flow condition.

In order to verify the assumption, experimental backup depth factors taken by the USBR hydraulic laboratory[3] are plotted against computed values using Equation 45 and drag coefficient values obtain from sweepout jump-flow condition. Because of air entraining facility of the bucket roller, the experimental backup factor seems to be higher than the computed value, Figure 4.

The best fit relationship, Figure 4, may be expressed as

$$H_{b(exp)} = H_{b(the)} + 1.5 \tag{54}$$

This correlation is seen to be good. A correction factor of 1.5 takes care of the simplifications made in theoretical analysis. Hence, the theoretical Equations 45 and 51 could be used to compute the relationship between R, H_b, F_1, and S_3, and a Fortrain program is given in Appendix E to simplify the computation using the program, the required bucket radius for any particular flow can be obtained and the backedup depth of any special flow condition can be easily predicted. This information is not important in hydraulic design of buckets but also important phase in structure design of the walls in chutes and spillways.[8/]

Evaluation of Surge Factor

Approximate surge-height characteristics obtained by the USBR hydraulic laboratory[3/] are shown in Figure 5. The best fit curve shows that the equation of

$$H_s = 1.45\, H_b^{1/12} \qquad (55)$$

can be fitted. This information is helpful in setting top elevations for training walls.

Length of the Submerged Jump in Bucket

As tested by the USBR hydraulic laboratory,[3/] the hydraulic behavior of the submerged bucket dissipator is manifested primarily by the formation of two rollers; one is on the surface moving counterclosewise and is contained within the region above the curved bucket, and the other is a ground roller making a surge moving in a clockwise direction and is situated downstream from the bucket. To find the length of the submerged jump, the bucket invert was taken as the origin, and the end of the jump was taken to be the end of the said roller and surge, where the depth became equal to the tailwater depth, y_5.

If $L_{b.s.j.}$ is the length of the submerged jump, the variation of $L_{b.s.j.}$ with backedup depth factor, H_b, taken by the USBR hydraulic laboratory[3/] is shown in Figure 6a. It was found that the theoretical backedup factors could be satisfied by the linear equation

$$\frac{L_{b.s.j.}}{y_5} = 2.47 - 0.035\, H_{b(the.)} \tag{56}$$

with a correlation coefficient of 0.87.

For experimental backedup factors, the same relationship as given by Equation 54 can be adopted.

If Equation 45 is substituted into Equation 56, jump length factor, $\frac{L_{b.s.j.}}{y_5}$ can be shown as a function of F_1, S_2, or S_3, and shape factors R, Z, k. That is

$$L_{b.s.j.}/y_5 = 2.47 - 0.035\sqrt{\psi'^2 + kC\,Cd\,Z\,F_1^2 - F_1^2 R - 2F_1^2\left(1 - \frac{1}{\psi'}\right)} \tag{57}$$

or

$$\frac{L_{b.s.j.}}{y_5} = f(F_1, S_3, R, Z, k) \tag{58}$$

For $H_b = 1$, that is, when flow begins to sweepout, $\frac{L_{b.s.j.}}{y_5}$ will be as small as 2.44 and after $H_b > 1$, $\frac{L_{b.s.j.}}{y_5}$ will be decreased, gradually. A comparison of the jump lengths on the different types of energy dissipators[12] shows that the bucket energy dissipators are the shortest.

<u>Length of the Surge in Submerged Jump</u>

The length from bucket invert to the average surge peak is taken as the length of the surge, L_s, and the relationship of $\frac{L_s}{y_5}$ to H_b is shown in Figure 6-b. It was found that all the best fit curve through the points could be satisfied by the equation

$$\frac{L_s}{y_5} = 1.47 - 0.019\, H_b \tag{59}$$

with a correlation coefficient of 0.84.

Comparison of equations 56 and 59 shows that the length between average surge peak and end of jump has a variation of

$$\frac{(L_{b.s.j.} - L_s)}{y_5} = 0.016\, H_b + 1 \tag{60}$$

[12] "Hydraulic Design of Stilling Basins and Other Energy Dissipators," Engineering Monographs No. 25, USBR, 1963

Length of the Roller

Due to the bucket action, the length of the roller was not so pronunced as in the horizontal hydraulic jump. The relationship between the ratio of the roller length, L_r, to tailwater depth, y_5, and backedup depth factor, H_b, is shown in Figure 6-c. The correlation is not good and a variation of 0. to 0.4 is obtained.

Energy Loss

Referring to Figure 1-c, the energy at the invert section of the bucket can be written as

$$E_1 = y_3 + \frac{v_1^2}{2g} + \frac{y_1 \; v_1^2}{g \, r_1} \tag{61}$$

or

$$\frac{E_1}{y_1} = H_b + \frac{F_1^2}{2} + F_1^2 \, R \tag{62}$$

The energy at end of jump is

$$E_5 = y_5 + \frac{v_5^2}{2g} = y_5 + \frac{F_1^2 \, y_1^3}{2y_5^2} \tag{63}$$

or

$$\frac{E_5}{y_1} = \frac{y_5}{y_1} + \frac{F_1^2 \, y_1^2}{2\, y_5^2} = \psi' + \frac{F_1^2}{2\psi'^2} \tag{64}$$

Then the energy loss E_L is given by

$$E_L = E_1 - E_5 \tag{65}$$

Substituting Equations 61 and 63 into Equation 65, simplifying

$$\frac{E_L}{E_1} = 1 - \frac{\left(\psi' + \frac{F_1^2}{(2\psi'^2)}\right)}{\left(H_b + \frac{F_1^2}{2} + F_1^2 R\right)} \tag{66}$$

Again it is shown that

$$\frac{E_L}{E_1} = f\left(F_1, S_3, R, H_b, Z, k\right) \tag{67}$$

and when $H_b = 1$, Equation 66 can be reduced to Equation 30, as stated above, similar conclusion can be made for the relationship between sweepout and submerged jump condition in bucket flow.

Verification of Equation 66 is obtained by using experimental losses computed from data taken by the USBR hydraulic laboratory[3] compared with theoretical lossed computed using Equations 45 and 66 with Cd value obtained from Figure 3. The results of the comparison are shown in Figure 7. The correlation is seen to be good.

The backedup depth factor is a element in the energy loss Equation 66, yet, the experiment results show that the variation due to error in measurement of backedup depth has little effect on the relative energy loss and the relative loss curve, Figure 7 shows a better correlation than that of the backedup depth factor curve, Figure 4.

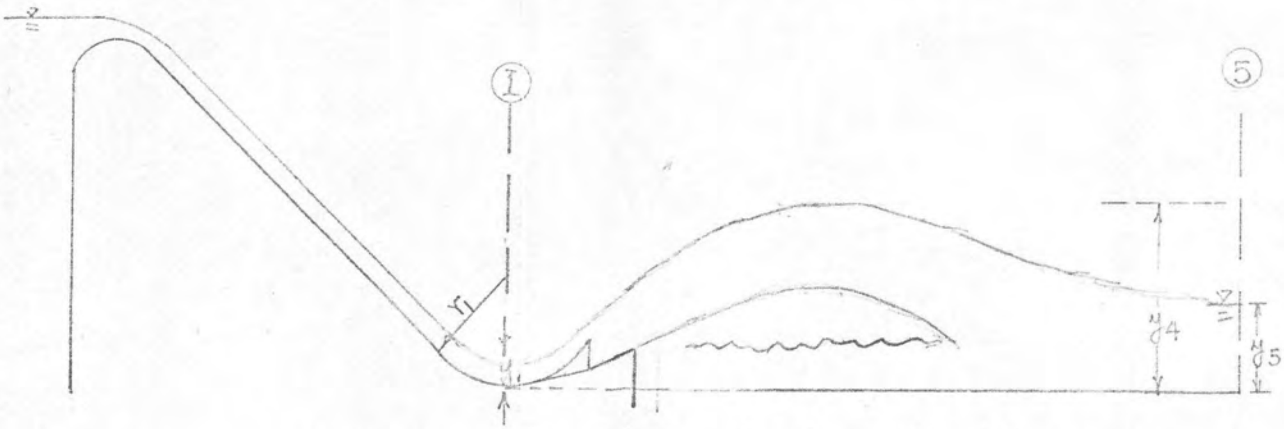
Through use of Equation 66, the energy dissipating capacity of the submerged bucket is seen to be more or less than that of the corresponding free jump on horizontal floor, depending upon the particular value of F_1, S_3 and shape factor R, Z, k. Again the index of relative dissipation, Equation 34 is used for evaluation of dissipation efficiency. For any given F_1, k, Z, and S, n could be more or less than unity, depending on R and S_3.

By differentiating Equation 34 with respect to either S_3, F_1, or R, and treating the resulting equation equal to 0, then a maximum value of n, for any particular flow condition can be theoretically obtained. However, the resulting equation is a complicated function of F_1, S, R and shape factors k, Z. A sample computor computation shows that the equation is an increasing function and becomes infinity as H_b increases.

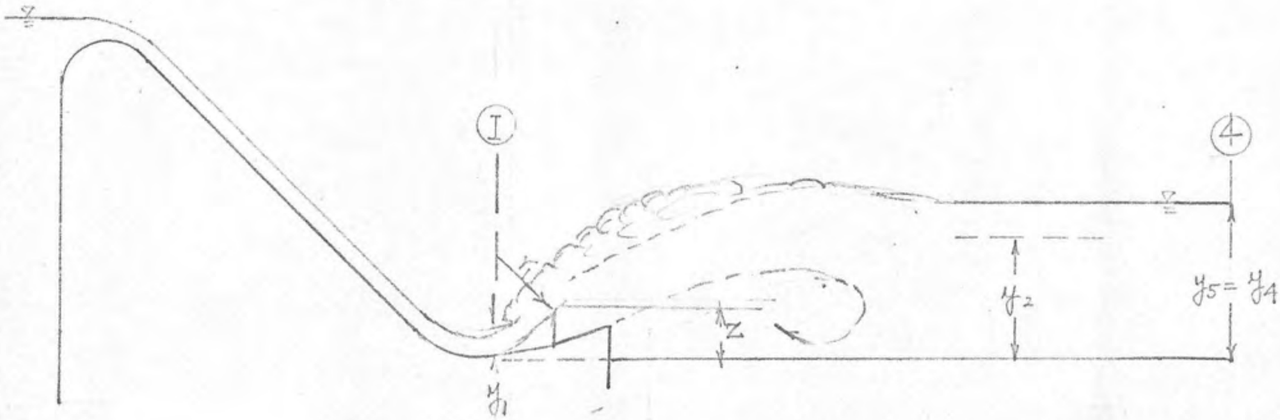
In the preceeding consideration of the energy loss, it should be always be remembered that the computed energy loss in the submerged jump occurs in a length much less than that of the corresponding bucket jump in sweepout condition or free jump on a horizontal floor. Comparison of the energy dissipation capacity is constantly lower than that of the sweepout condition. From these considerations, it is concluded that the submerged jump could not be preferred to the sweepout condition for energy dissipation purposes unless the backedup factor is less than approximately 20 percent of the tailwater depth. This agrees with the minimum tailwater recommended by USBR and the test results given by McPherson and others.[13] Hence, the equations mentioned above can be adopted in buckets design, graphical or tabular representation of equations is possible but due to too many variables included in the analysis, and in view of the extensive and rapidly growing use of digital computers, such tables or graphs would not be of widespread interest. Instead, a Fortran program was written for use on an IBM 7090 and is shown in Appendix E.

[13] "Symposium on Energy Dissipators," Central Board of Irrigation and Power, Publication No. 70, New Delhi, India, August 1961

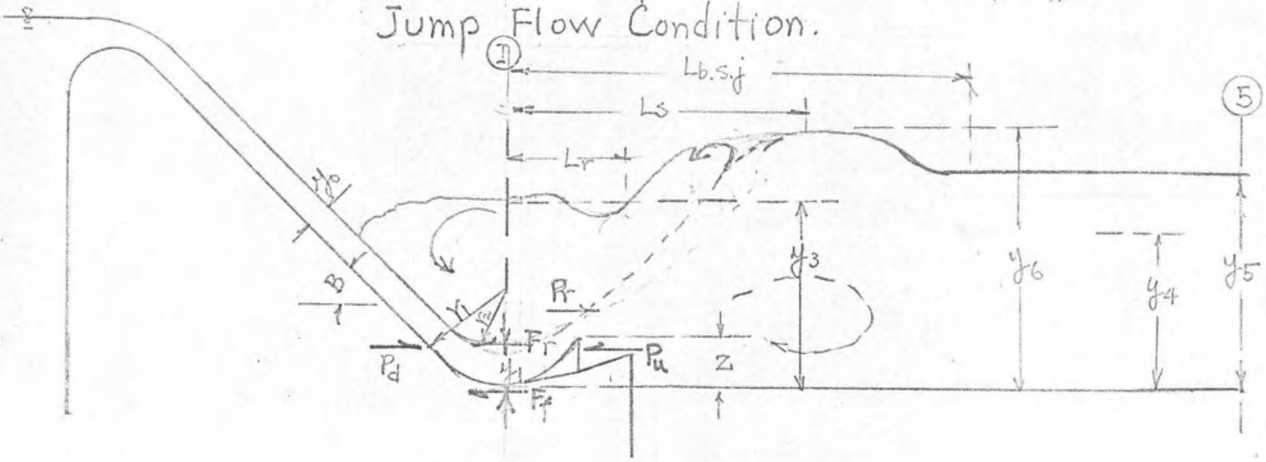
FIGURE 1.
REPORT HYD.



A. Flip Bucket or Jet Flow Condition.

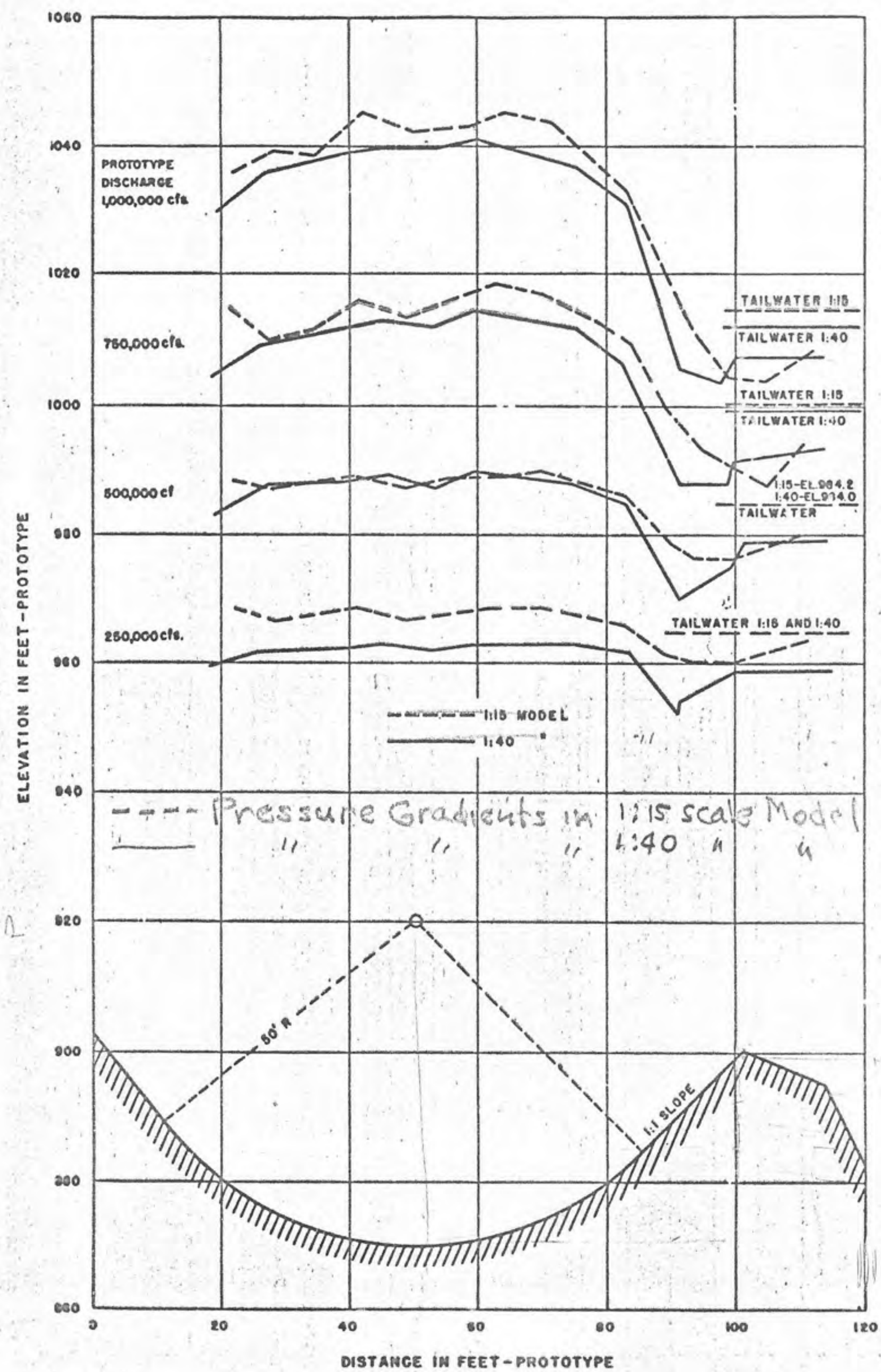


B. Hydraulic Jump Bucket or Sweepout Jump Flow Condition.

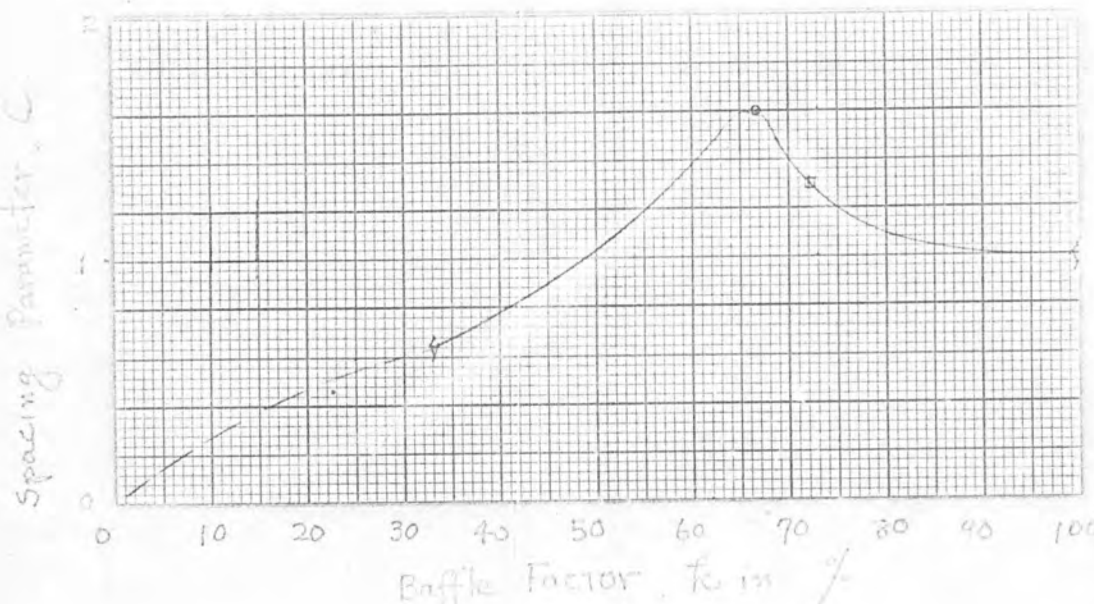


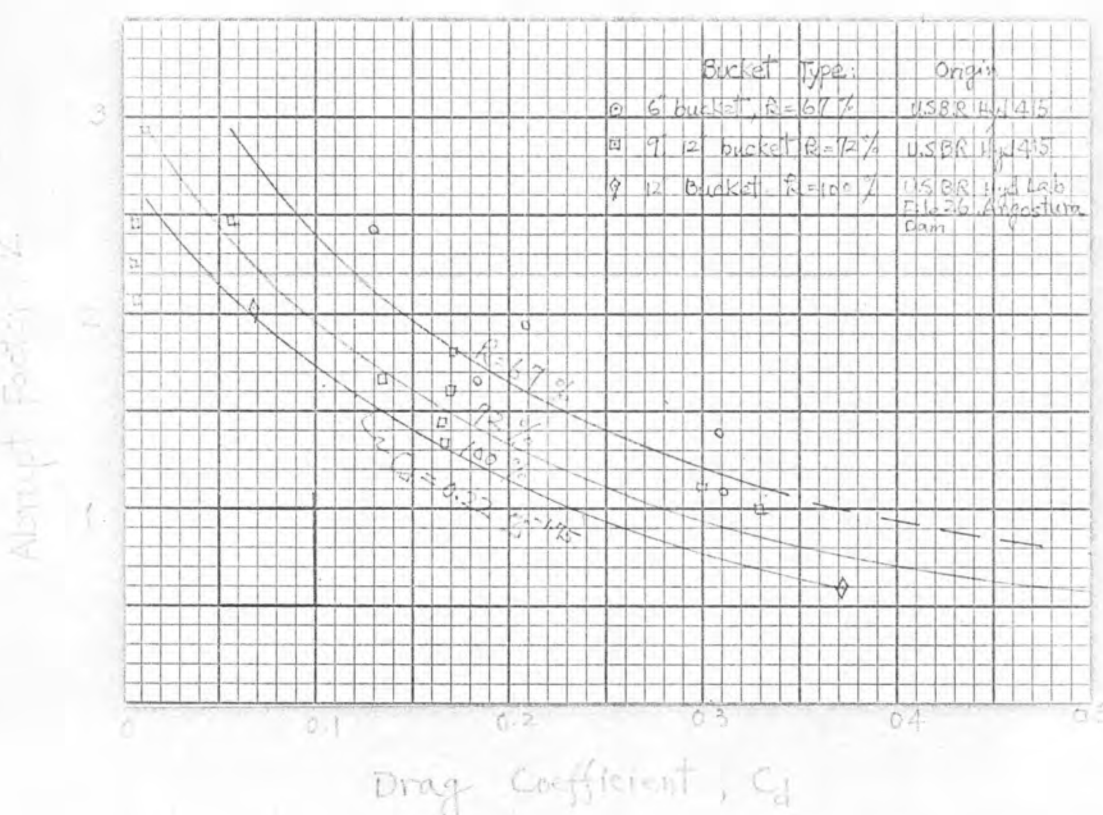
C. Roller Bucket or Submerged Jump Flow Condition.

FIGURE 2
REPORT HYD.

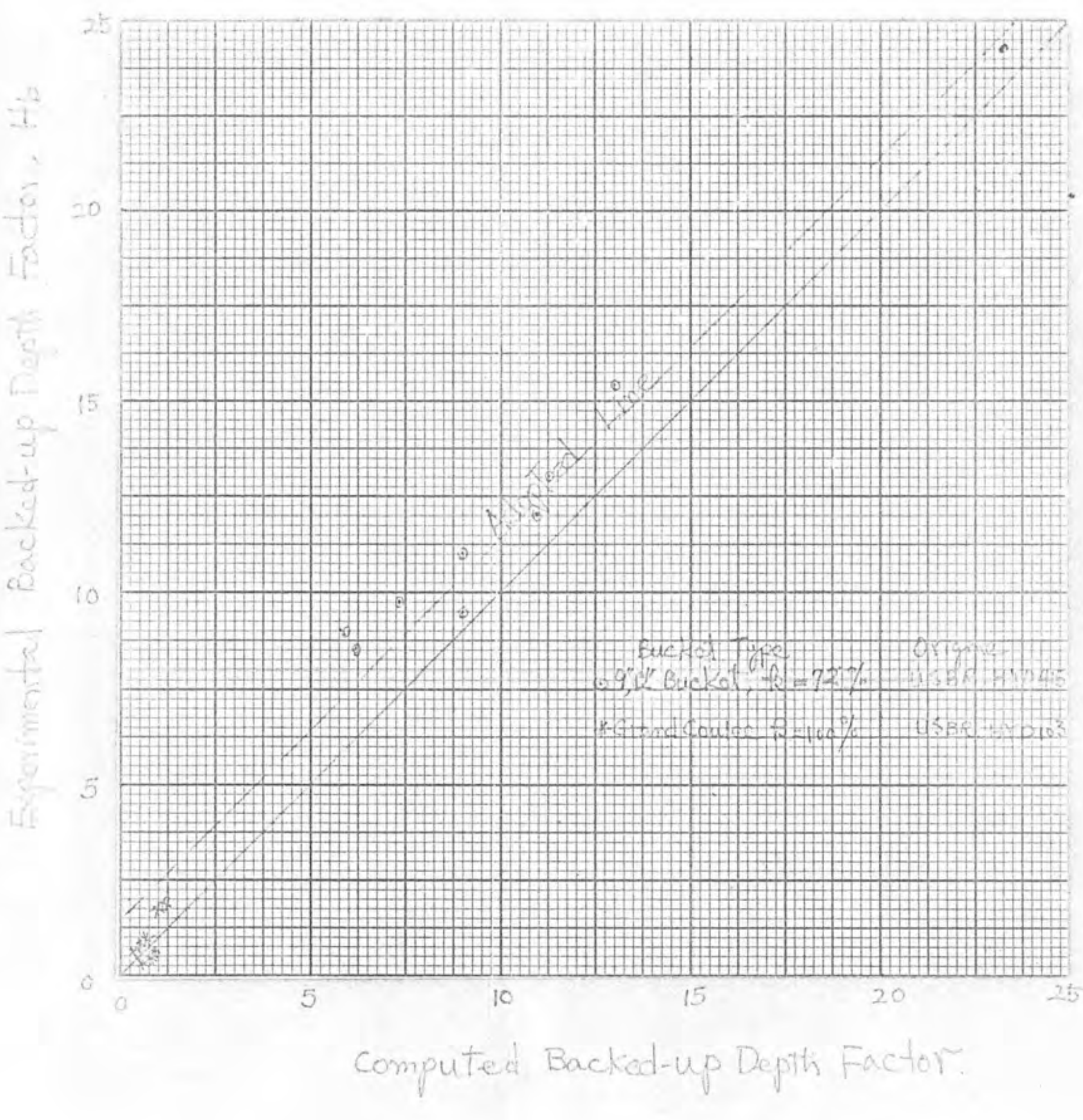


PRESSURES IN THE GRAND COULEE SPILLWAY BUCKET

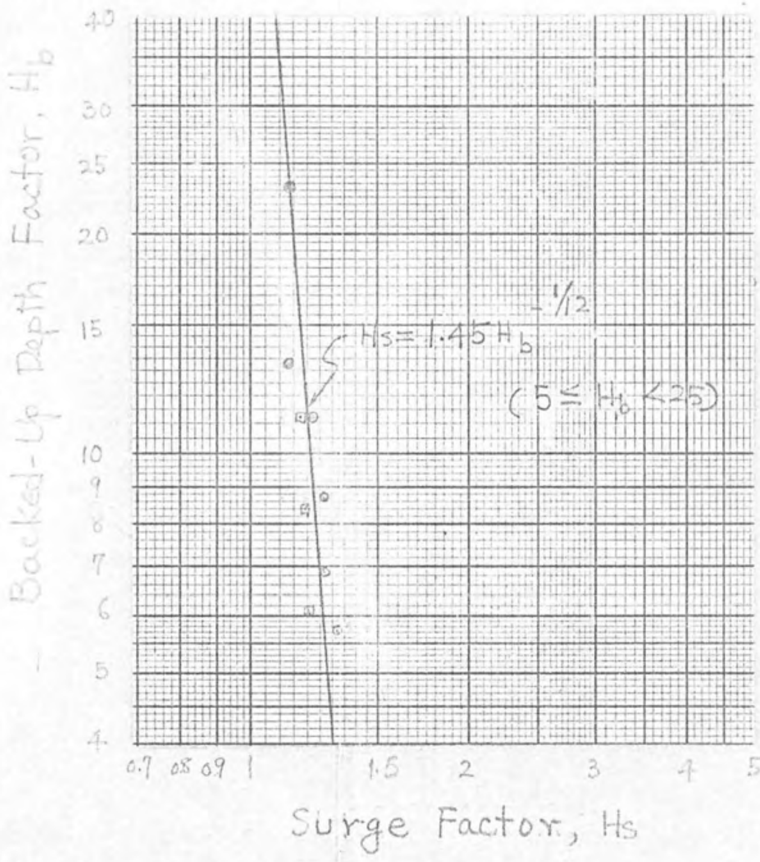




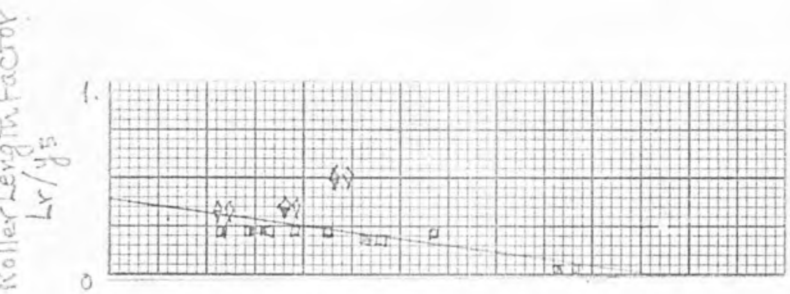
COEFFICIENTS OF DRAG FOR SPILLWAY BUCKETS



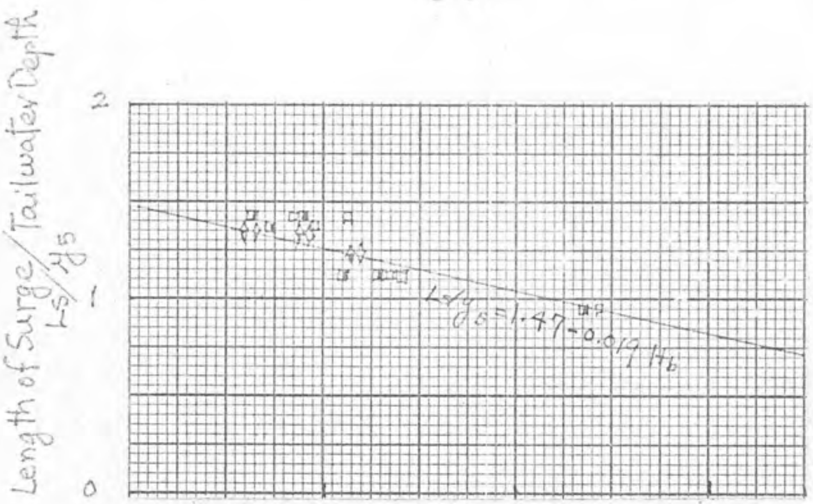
EXPERIMENTAL VERIFICATION OF BACKUP DEPTH FACTOR



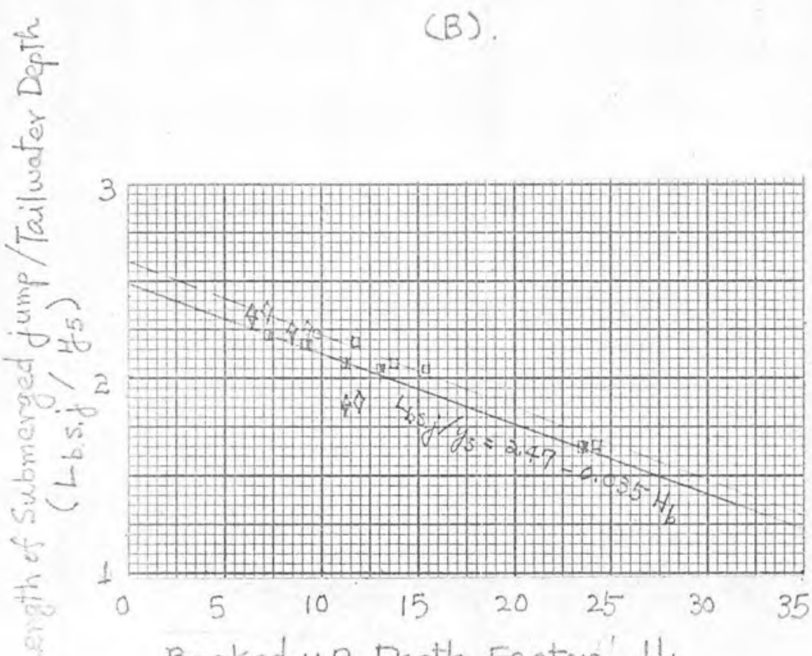
BACKED-UP DEPTH VS. SURGE DEPTH



(C).

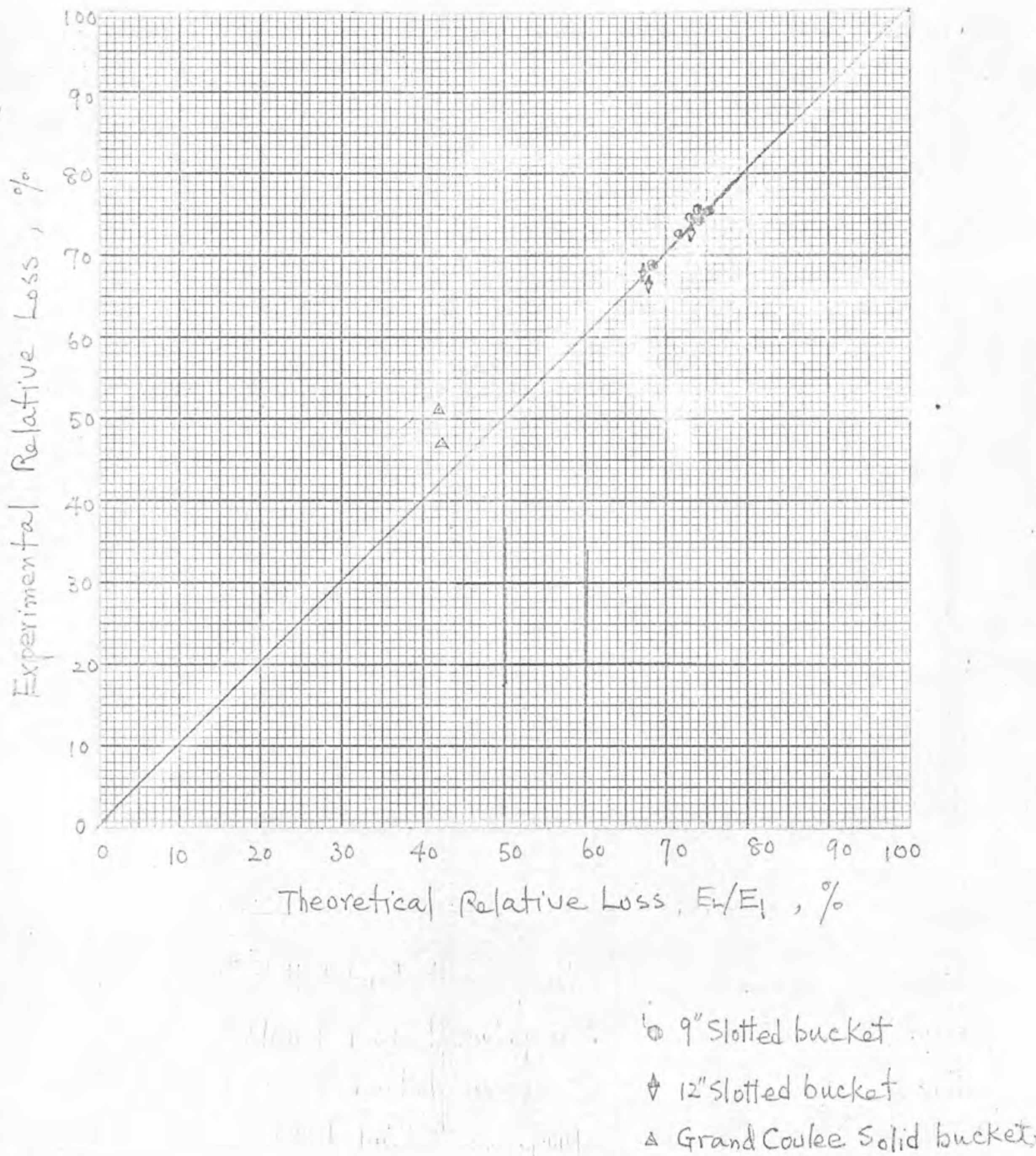


(B).



(A).

LENGTH PARAMETERS FOR SUBMERGED JUMP FLOW CONDITION.



EXPERIMENTAL VERIFICATION OF ENERGY LOSS EQUATIONS FOR SUBMERGED JUMP-FLOW CONDITION.

APPENDIX 1--NOTATIONS

b	bucket width
b_t	width of baffle in bucket
C	spacing parameter
Cd	drag coefficient
E	specific energy
E_1, E_2, E_3	specific energy at various sections
E_L	specific energy less
F_1	Froude number for linear channel
F_b	Froude number for bucket
f	function
g	acceleration due to gravity
G	jump constant
H	difference between reservoir elevation and tailwater elevation
H_b	backedup depth factor
H_s	surge factor
$L_{b.s.j.}$	length of submerged jump in bucket flow
L_r	length of roller in bucket
L_s	length of surge in bucket
n	relative efficiency of energy dissipation
P_r	component of roller weight acting downstream on the principal stream of bucket
P_d	floor reaction
P_u	floor reaction

q	discharge intensity
R	radius factor
r	radius
r_1, r_2	radius at various sections
S	submerged factor or tailwater factor
$S_{1,2,3}$	submerged factor or tailwater factor for various flows
v	mean velocity
$v_{1,2,3}$	mean velocity at different sections
x	longitudinal distance
y	depth of flow
y_c	critical depth of linear channel
y_{cb}	critical depth of bucket
$y_{1,2,3}$	depth of flow at different sections
r	specific weight of water
φ	sequent depth ratio for free jump
ψ	sequent depth factor for bucket jump
$\beta_{1,4}$	momentum coefficient
ρ	density of water

1604 FORTRAN CODING FORM

PROGRAM APPENDIX -B SEQUENT DEPTH FΦR BUCKETS

ROUTINE

NAME C. M. Wu

PAGE 1/1

DATE

```
C      PRΦGRAM FΦR SEQUENT DEPTH
    1  FΦRMAT  (6 F  8.4 )
    2  READ INPUT TAPE  5 , 1, X,Y, CK, H, C,CD
C       X=RADIUS FACTOR
C       Y = FRΦUDE NUMBER
C       CK=BAFFLE FACTΦR
C       H =ABRUPT FACTΦR
C       C =SPECING PARAMETER FRΦM FIG 3
C       CD=DRAG CΦEFFICIENT FRΦM FIG 3
C       EQUATIΦN  =  SD**3 + (CK*C*CD*Y*Y*H  - (1.0 + Y*Y*X ) * 2.0 * Y*Y) *SD  + 2.0 *Y*Y
    3   A1 =  C*CD*Y*Y*H*CK  - ( 1.0 + Y*Y*X ) * 2.0 * Y*Y
        A2 =  2.0 * Y*Y
        A3 =  (( - 0.5) * A2 + SQRTF (( A2*A2/4.0) + (A1*A1*A1/27.0)))** 0.333
        A4 =  (( - 0.5)*A2   - SQRTF ((A2*A2/4.0 ) + (A1*A1*A1/27.0)))**0.333
        SD = A3 + A4
C       SD = SEQUENT DEPTH FACTΦR
        WRITE OUTPUT  TAPE  6,4
    4 0FΦRMAT ( 80H1  RADIUS FACTΦR   FRΦUDE NΦ   BAFFLE FACTΦR   ABRUPT
      1FACTΦR   SEQURNT FACTΦR )
        WRITE OUTPUT TAPE 6,5 , X,Y,CK, H, SD
    5  FΦRMAT (8X,F8.4, 4X, F8.4,4X,F8.4,8X,F10.4,10X,E12.4 )
    6  GΦ TΦ 2
       END
```

1604 FORTRAN CODING FORM

NAME c. m. Wu
PAGE 1/2
DATE

PROGRAM APPENDIX C RADIUS FACTØR FØR SWEEPØUT BUCKET

ROUTINE

```
C     FLØW PARAMETERS FØR SWEEPØUT BUCKET
    1 FØRMAT  ( 6 F8.4 )
    2 READ INPUT TAPE 5, 1, Y, SD, CK, H, C, CD
C     Y = FRØUDE NØ
C     SD = TAILWATER / INITIAL DEPTH
C     CK = BAFFLE FACTØR
C     H = ABRUPT  FACTØR
C     C = SPACING FACTØR FRØM FIG 3
C     CD = DRAG CØEFFICIENT FRØM FIG 3
C     X = RADIUS FACTØR
C     ER = RELATIVE ENERGY LØSS
C     A = EFFICIENCY
     0X=(1.0/Y*Y)*(SD*SD-2.0*Y*Y*(1.0-1.0/SD)-1.0+C*CD*Y*
     1Y*H*CK)
      EL=1.0+Y*Y*X+0.5*Y*Y-SD-(0.5*Y*Y)/(SD*SD)
      E1=1.0+Y*Y*X+0.5*Y*Y
      ER=EL/E1
     0E2=1.0-((8.0*Y*Y+1.0)**1.5-4.0*Y*Y+1.0)/(8.0*Y*Y
     1*(2.0+Y*Y))
      A=ER/E2
      WRITE ØUTPUT TAPE 6, 3
    3 0FØRMAT (98H1   RADIUS-FAC   FRØUDE NØ   SEQUENT-FAC
     1BAFFLE FAC   ABRUPT FAC   R-ENERGY LØSS   EFFICIENCY)
```

1604 FORTRAN CODING FORM

NAME C. M. Wu

PROGRAM APPENDIX C. RADIUS FACTOR FOR

PAGE 2/2

ROUTINE SWEEPOUT BUCKET

DATE

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73–80
			WRITE OUTPUT TAPE 6,4, X, Y, SD, CK, H, ER, A	
	4	0	FORMAT (6X, F8.4, 4X, F8.4, 6X, F8.4, 6X, F8.4, 5X, F8.4,	
		1	5X, E14.5, 2X, E10.5)	
			GO TO 2	
			END.	

APPENDIX D

Determination of Depth on the Principal Stream in a Bucket Invert

According to previous model studies of flow in the spillway buckets of Grand Coulee, Pine Flat, and Popolopen Dam, J. H. Douma assumed that the velocity distribution on the bucket would be that of an irrotational vortex; $V = \frac{A}{r}$, in which A is a constant and r is the radius of any streamline. For this flow pattern, all the streamlines are circular and concentric with the buckets. This assumption has also been adopted for determining the depth of the principal stream in a bucket invert.

From Douma's streamline analysis, the following relationship was obtained[1/]

$$r_2 \log\left(\frac{r_1}{r_2}\right) = \frac{q}{\sqrt{2gH}} \qquad (1)$$

where r_1 is the radius of the bucket;

r_2 is the radius of the streamline in the principal stream of bucket;

H is the difference between reservoir elevation and tail-water elevation.

Dividing both sides of Equation 1 by r_1, a dimensionless equation can be obtained, that is

$$\left(\frac{r_2}{r_1}\right) \log\left(\frac{r_1}{r_2}\right) = \frac{q}{r_1\sqrt{2gH}} \qquad (2)$$

[1/] Discussed by J. H. Douma, "Design of Side Walls in Chutes and Spillways," Trans. ASCE, Vol. 119, 1954 or $\frac{r_2}{r_1}$ can be expressed as function of a dimensionless parameter $\frac{q}{r_1\sqrt{2gH}}$.

$$\left(\frac{r_2}{r_1}\right) = f\left(\frac{q}{r_1\sqrt{2gH}}\right) \quad (3)$$

where r_1, q, H are all known values.

The depth of the principal stream in a bucket invert, y_1, can be expressed in terms of r_1 and r_2, that is

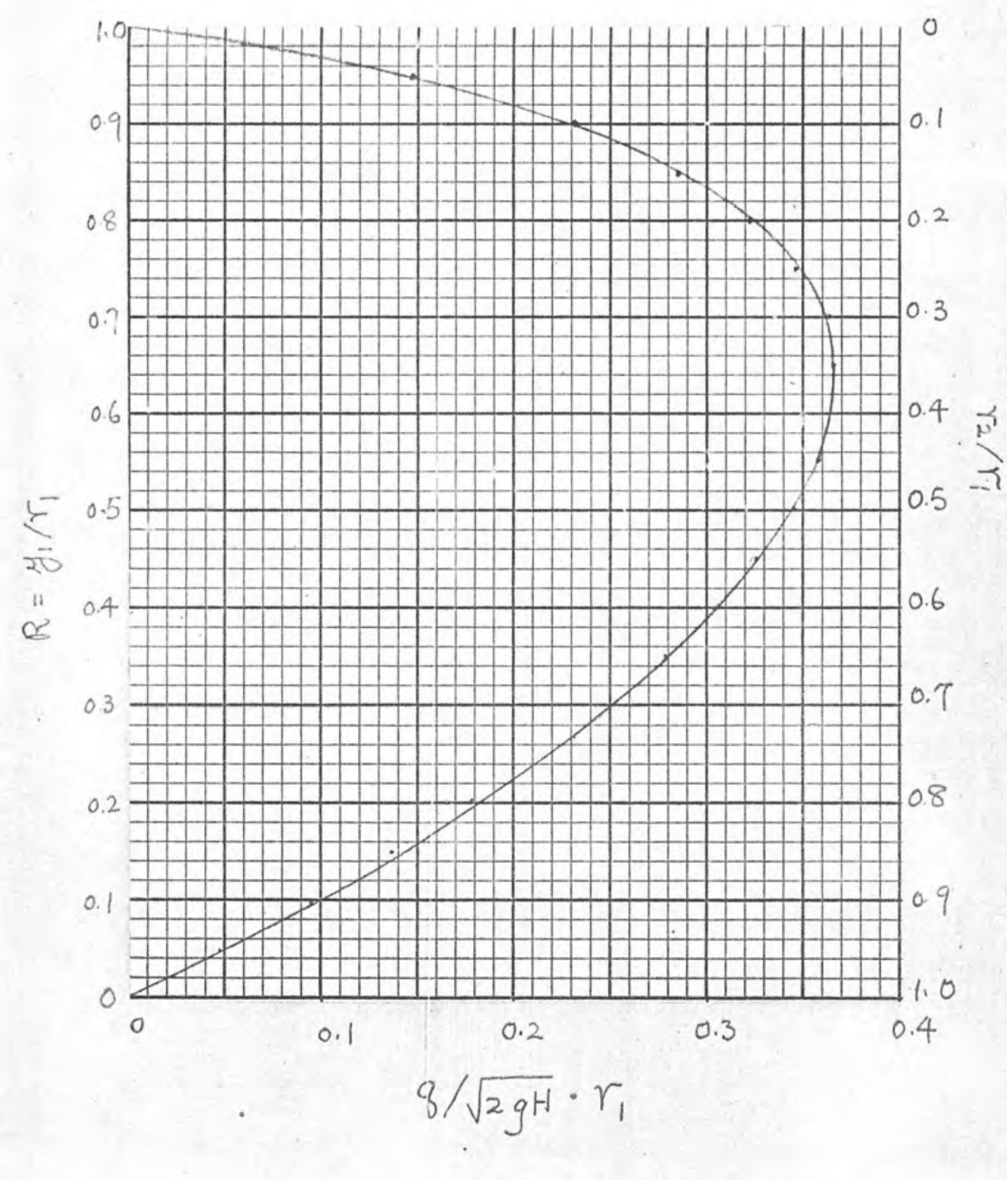
$$y_1 = r_2 - r_1 \quad (4)$$

Again using a dimensionless parameter of $\frac{y_1}{r_1}$ and in terms of $f\left(\frac{q}{r_1\sqrt{2gH}}\right)$, a simple solution of y_1 can be obtained by using the relationship of

$$\left(\frac{y_1}{r_1}\right) = f\left(\frac{q}{r_1\sqrt{2gH}}\right) \quad (5)$$

The result of Equation 5 is shown in Figure 1.

FIGURE 1.
APPENDIX D.



RADIUS FACTOR FOR SUBMERGED BUCKETS.

1604 FORTRAN CODING FORM

PROGRAM APPENDIX E FLØW PARAMETER FØR SUBMERG-
ROUTINE ED BUCKET

NAME C.M. Wu
PAGE 1/2
DATE 16 Aug '63

```
C     FLØW PARAMETERS FØR SUBMERGED BUCKET
    1 FØRMAT (7 F8.4)
    2 READ INPUT TAPE 5, 1, X, Y, CK, H, SD, C, CD
C     X= RADIUS FACTØR FRØM APPENDIX D
C     Y= FRØUDE NUMBER
C     CK = BAFFLE FACTØR
C     H = ABRUPT FACTØR
C     SD = SEQUENT DEPTH FACTØR
C     C = SPACING FACTØR FRØM FIG 3
C     CD= DRAG CØEFFICIENT FRØM FIG3
C     B= BACKED-UP DEPTH FACTØR
C     ER= RELATIVE ENERGY LØSS
C     A = EFFICIENCY
    3 0B = SQRTF(SD*SD + CK*C*CD*H*Y*Y- Y*Y*X-2.0
     1*Y*Y*(1.0-(1.0/SD)))
      E1 = B + 0.5*Y*Y + Y*Y*X
      E4 = SD + (0.5*Y*Y)/(SD*SD)
      DE = E1 - E4
      ER = DE/E1
     0E2= 1.0-((8.0*Y*Y+1.0)**1.5-4.0*Y*Y+1.0)/(8.0*Y*Y*
     1(2.0+Y*Y))
      A= ER/E2
      WRITE ØUTPUT TAPE 6, 4
```

1604 FORTRAN CODING FORM

NAME C. M. Wu

PROGRAM Appendix E

PAGE 2/2

ROUTINE

DATE 16 Aug '63

```
    40 FØRMAT (108 H1  RAD-FACTØR  FRØUDE NØ  BAFFLE-FAC
     1 ABRUPT-FAC  SEQUENT-FAC  BACKED-UP-F  RELATIVE LØSS
     2 EFFICIENCY )
       WRITE ØUTPUT TAPE 6, 5, X, Y, CX, H, SD, B, ER, A
     5 FØRMAT (4X, F8.4, 4X, F8.4, 4X, F8.4, 5X, F8.4, 5X, F8.4,
     1 6X, E11.4, 3X, E12.4, 4X, E10.4)
       GØ TØ 2
       END

C        NØTE: TAPES 5 AND 6 ARE SYSTEM TAPES IN FØRTRAN.
C        TAPE 5 USED FØR INPUT, TAPE 6 FØR WRITEØUT
```