MEMORANDUM

Denver, Colorado
October 14, 63

To : Chief Engineer

Through : Chief , Hydraulic Laboratory

From : Chian-Min Wu

Subject : Final Report

At the conclusion of my training program , I am requested to submit a final report in five copies .

Enclosed please find a copy of my final report , several technical articles are involved . Among which " Flow Characteristics of Spillway Buckets," is an interesting one . Academically, this will lead to a better understanding of the hydraulic jump in Fluid Mechanics and practically summarize the studies of spillway bucket since construction of Grand Coulee Dam in 1933 .

Prior to my departure from the Hydraulic Laboratory , I have solved another interesting problem , " Hydraulics of Vertical Wells" . The result of the analysis is attached as Appendix 4 in my final report .

I believe you will find this report interesting and informative , and will lead to comprehensive discussion of the extensive research program in the near future .

I am sure and you would agree that the training program will lead to a rewarding experience , and that my fellowship will effectively help me to contribute to the development of my country .

Chian Min Wu

cc. Mr. Howard ,M.A., Head , Training and Visitors Section

Enclosure

Final Report

on training on assignmet with the
United States Bureau of Reclamation ,
Hydraulic Laboratory for the United
Nations Technical Assistance Operations .

by

Chian - Min Wu

September 1963

OPTIONAL FORM NO. 10
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Appendix 4 PAP - PAP 180

UNITED STATES GOVERNMENT

Memorandum

Memorandum
TO : Chief, Hydraulics Branch

Denver, Colorado
DATE: October 11, 1963

FROM : C. M. Wu

SUBJECT: Hydraulics of vertical stilling wells

Hydraulics of vertical stilling wells has been studied in the attached paper.

The investigation has proceeded since 1906 and with aid of model tests, extensive studies have been conducted in this laboratory since 1949 to 1962 as one of the research problems.

I am encouraged by my preliminary theoretical analysis, which I submitted to you on September 27 and under the direction of Mr. P. F. Enger, I completed the draft in last 2 weeks. Hoping this will lead to a short cut in solving the problem.

Chian Min Wu

Enclosure

CONTENTS

Final Report

on training on assignment with the
United States Bureau of Reclamation,
Hydraulic Laboratory for the United
Nations Technical Assistance Operations.

by

Chian Min Wu

September 1963

This report presents technical articles on subjects studied during the 6 month's training program in the United States Bureau of Reclamation Hydraulic Laboratory as fulfillment of a fellowship from the United Nations Technical Assistance Operations.

The assignment covered the period from March 28 to September 16, 1963.

SUMMARY

1. Engineering science is an art rather than an exact science. Particularly in the field of engineering hydraulics, many empirical coefficients and constants are introduced, emphasizing the world-wide fundamental need for hydraulic laboratories.

2. There are no borders for scientific studies and hydraulic studies are no exception. Accomplishments of the United States in modern instruments, such as oscillographs, oscilloscopes, pressure cells, high speed photography, electronic computers, radio active isotopes as tracers, radar and sonar are worthy of mention and of further study.

3. Hydraulic laboratory practice should not be limited to experiment. A combination of mathematical analysis and experimental techniques is the only way to obtain basically sound result.

4. Academic education combined with technical training is the fundamental approach for the Asian countries to catch up with modern developments.

INTRODUCTION

Engineering science is an art rather than an exact science. Especially in the field of engineering hydraulics many empirical coefficients and constants are introduced. Thus, there is a worldwide fundamental need for hydraulic laboratories.

Considerable progress has been made in the past decade in the field of laboratory study, particularly the basic theory of model similarity. Many hydraulic model tests such as those for spillway structures, scour, deposition and waves in harbors, flow in closed conduits, and movable and fixed-bed river models have been conducted. Perhaps the most outstanding contribution to the hydraulic laboratory study is the rapid development of instruments, such as oscillographs, oscilloscopes, pressure cells, high speed photography and electronic computers. These have made it possible to solve and observe the complex matter of hydraulic problems.

Because of the wide scope of the subject matter available, it has been necessary to limit this report to some of the problems in which the writer has experienced the need of more research and which must be introduced to aid hydraulic development in his mother country.

HYDRAULIC STRUCTURE MODEL

Spillway Model

Although spillways have been the subject of more research than perhaps any other hydraulic structure, there are still deficiencies in our knowledge of their functioning. There is more to be learned about the process of energy loss on the spillway face, pressure distribution, cavitation, and the effects of submergence of the spillway jet.

Basic information concerning the characteristics of spillway flow is not sufficient. The solution to the problem has been approached by means of hydraulic models.

Model investigations are sometimes extensive and time consuming and, in addition, a nontheoretically guided-model study will sometimes introduce questionable results. Model investigations can be supplemented with mathematical analysis to obtain more understandable results. Therefore, it is apparent that a combination of analytical and experimental study is desirable.

An example study of the theoretical and experimental approach has been conducted by the writer during his assignment and is shown in Appendix 1 ("Flow Characteristics of Spillway Buckets").

Dimensional Analysis

The proper interpretation of a set of observations is much more difficult than the mere accumulation of observations in both fixed and movable hydraulic models, since the interpretation involves consideration of the relative importance of all factors that cause variations in the observations.

Dimensional analysis is a tool to obtain the proper interpretation and indicate the general trend or law. An example study of the dimensional analysis has been conducted and is shown in Appendix 2 ("Model Study of Vertical Stilling Well Basin").

MOVABLE-BED HYDRAULIC MODELS AND SEDIMENT STUDY

The rivers in Taiwan, Republic of China, and similar Asian countries, are always subjected to serious sediment problems because of the action of "Earthquakes" and "Typhoon Rains" on the weak and easily eroded rock formations. It is found that there are many areas in which the tools available to the engineer for solution of such problems are inadequate. The fields in which research is needed are herein discussed.

Empirical Approach of Model Similarity

Although the utility of movable-bed river models is generally accepted there remains many problems. The most fundamental but unsolved problem is the establishment of model similarity. Many laws have been established among which the bottom velocity, settling velocity[1/], tractive force[2/], regime theory[3/], and sediment transport theories[4/] are familar to hydraulic engineers, yet the prototype record is the only criterion of movable-bed analogy law. Past prototype records must be repeatedly checked by model, otherwise, many quantitative similarities are not free from model effects.

The most direct and reliable method of selecting the various scales of a movable-bed model is the so-called "Historical Verification

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1/ "Model Studies of Sediment Control Structures on Diversion Dams," by H. M. Martin and E. J. Carlson, August 1953

2/ "Hydraulic Laboratory Practice," by J. R. Freeman, 1929

3/ "Scale Relations Among Sand-Bed Rivers Including Models," by T. Blench, Proc. ASCE Vol. 81, Separate No. 667, April 1955

4/ "Similarity of Distorted River Models with Movable Bed," by H. A. Einstein and N. Chien, Proc. ASCE Vol. 80, Separate No. 566, December 1954

Method." It is to adjust the model scales to simulate some known prototype occurrences similar to that of the model. Upon application of the "Historical Verification Method" several tests were conducted and results showed that temperature change influenced both prototype and model.[5] Hydraulic model tests using the "Historical Verification Method" as a criterion, should take into account this critical phenomenon. Model verifications which have been made with high temperatures are not necessarily valid at low water temperatures.

Further tests should be conducted to determine temperature effects on bed conditions and roughness.

As described above the first and foremost in a movable-bed study is the matter of reliable basic data. Unfortunately hydraulic and hydrologic data are limited, so that theoretical investigations of similarity play a great roll in movable-bed hydraulic studies as a supplementary tool to the empirical approach of movable-bed model studies.

Theoretical Approach of Movable-bed Similarity

There are many problems for which the basic equations are known but these are mathematically so complicated that their direct application becomes practically impossible. Many such problems can be solved by the use of models and the laws of similarity. The even more important method of applying similarity is by means of general relationships between dimensionless variables and dimensional homogeneity. Based upon these facts Einstein, Blench, and others have derived methods of determining similarity of distorted movable-bed models. Einstein's model law is more theoretical than those of others.

Einstein solved the flow and sediment similarity equations by choosing one of the following ratios: (1) horizontal ratio, (2) vertical ratio, and (3) density ratio. Further investigations were made by the Hydraulic Laboratory, Taipei, Taiwan, and the result is shown in Appendix 3.

The similarity conditions for distorted river models with movable-bed materials are thus derived from the theoretical and empirical equations to describe the hydraulic and sediment phenomena in rivers. Rivers, in general, are not susceptible to mathematical or theoretical analysis except for certain assumptions. In the preceding approach several assumptions were made. Further advanced study is necessary for more exact and sound approximation.

[5] "Effective of Temperature on Sediment Transportation 2," by E. W. Lane, E. J. Carlson, and O. S. Hanson

SEDIMENT TRANSPORTATION

Adequate basic data on sediment transportation are not available in many countries. Much more field and laboratory studies must be made for correlation and then for extension to expedite solutions.

Suspended Load Sampling

In past years, in Taiwan and several Asian countries, suspended sediment was sampled by using vertical-type samplers. Sampling is currently being done by using U S DH-48, D 49, and P-46[6], horizontal-type samplers.

Because standard samplers were not available in the past, calibration of the sampling apparatus previously used for collecting field data should be carried out in the hydraulic laboratory. Recently the efficiency of vertical-type suspended load samplers was determined in a laboratory and a value of about 80 percent was obtained.[7] Therefore, the efficiency of uncalibrated suspended sediment samplers should be determined in hydraulic laboratories. This would make much of the data collected in the past usable and would add greatly to the reliability of any data developed with nonstandard equipment.

Distribution Study

In the determination of the total suspended load transported, it is usually necessary to combine field data with empirical computations or theoretical extrapolation, except when it is possible to sample through the total depth. Unfortunately, the sampling depth is usually limited and a theoretical correction of the nonsampled sediment load is based on a vertical distribution formula of suspended sediment, which is[8]

$$\frac{C_y}{C_a} = k\left(\frac{d-y}{y} \cdot \frac{a}{d-a}\right)^z$$

$$z = \frac{W}{k}\sqrt{gRS}$$

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[6] "The Design of Improved Types of Suspended Sediment Samplers," Report No. 6 of a Study of Methods used in Measurement and Analysis of Sediment Loads in Streams, by Subcommittee on Sedimentation, Federal Inter-Agency River Basin Committee, June 1946

[7] "On the Efficiency of Vertical Type Suspended Load Sampler," Hyd-Lab Report No. 5, National Taiwan University, 1955

[8] "The Present Status of Research on Sediment Transport," by Ning Chien, Proc. ASCE, Separate No. 565, Vol. 80, December 1954

where C_y and C_a are the concentrations of a grain size with settling velocity w at distances y and a from the bed, respectively; k is the von Kaiman universal constant; d is the depth of the flow; g is the gravitational acceleration; R is the hydraulic radius; and S is the slope.

The controlling parameter z can be corrected from either field or laboratory data. As there are many disadvantages in field operations, laboratory studies of the above parameter, though several tests have been conducted[8/], is still necessary.

Standard Sampler

A physical limitation of modern samplers is the distance between the sampling nozzle intake and the stream bottom. To obtain sediment samples near the bed, several samplers have been developed. A horizontal tube sampler for sampling close to hard stream bottoms and for large sediment has been widely applied. Drag flume studies of plastic tube samplers have contributed greatly to the development of standard samplers suitable for special river conditions such as in Taiwan.

The photoelectric cell is used in several American laboratories. Perhaps the use of a photocell will improve the sampling equipment more rapidly than sampling techniques will improve.

Bedload Movement

On estimating the sediment loads of streams for reservoir projects, empirical formulae have been widely used. Most bedload formulae are based on drag flume experiments and when applied to river problems may not give as accurate results as expected.

Owing to a wide variation of physical aspects of sediments from one geographic area to another, study in the hydraulic laboratory, where flow conditions can be controlled, is regarded as a necessity.

The bedload sampler is a fundamental tool to sediment movement study. Unfortunately, existing bedload samplers were developed in countries where the rivers were much more gentle than Taiwan. The nature of the rivers on Taiwan and other rugged countries will probably require new equipment for field measurement of bedloads.

Development and calibration of such equipment can best be conducted under controlled conditions in the laboratory to simplify the problems.

ELECTRONIC COMPUTER

Programing

Use of computers is a revolution in mathematic science. Laboratory experiments often produce so much data that a computer becomes necessary, in order to apply correction factors, to eliminate invalid readings, to establish general trends, and to reduce the data to meaningful.

Fortran programing is one of the most simple languages in computer works. The method of programing had been studied and several examples are shown in Appendix 1.

APPENDIX - 1

Subject: Flow characteristics of spillway buckets

PURPOSE

To investigate the bucket flow characteristics considering the radial effect of the bucket and to develop theoretical and experimental equations for the main-flow parameters.

CONCLUSIONS

1. A theoretical analysis and study of experimental data show a pronounced centrifugal effect on the action of the radial flow through a roller bucket and suggest that the flow characteristics in spillway buckets can be analyzed by use of the equations of pressure plus momentum with the addition of the centrifugal force.

2. A theoretical equation is developed to relate the bucket radius to the flow characteristics of two different flow conditions described as a sweepout jump-flow condition and a submerged jump-flow condition. Also, a theoretical equation is derived for the energy loss in both. These equations are experimentally verified using data obtained by the USBR, Hydraulic Laboratory, within the

limits of Froude numbers up to 9 and backup factors (ratio of flow depth above the invert, y_3, to the supercritical depth at the invert, y_1) to approximately 25. A theoretical equation is developed for the backup depth factor at the bucket invert for the submerged jump-flow condition. This equation is also experimentally verified within these limits by applying a correction coefficient for air-entrainment.

3. Curvilinear effect on Froude number and critical depth of bucket flow is analyzed and it is shown that a correction factor must be applied to the originally defined Froude number, i.e., $F = \frac{V}{\sqrt{gy}}$ and it is also shown that the hydraulic jump can be formed in the bucket even though the Froude number is less than 1.

4. Considerable information concerning jump, surge length, surge height, and other flow parameters is analyzed, and it is shown that all of the flow characteristics may be expressed as functions of the flow parameters (Froude number, tailwater factor, submerged factor) and the shape parameters (abrupt factor, baffle factor, and radius factor).

5. Using the equations obtained, it is found that the energy loss in a submerged jump within a bucket is more than in the corresponding free jump on a horizontal floor, but constantly less than that

of a corresponding sweepout jump-flow condition within the bucket. However, due to unstable characteristics of the sweepout jump-flow condition, a slight submergence is recommended.

6. Comparison of energy dissipation capacity of various hydraulic jump basins and the lengths of these basins with that of the bucket shows that the bucket is more efficient in dissipating the energy and does so in a shorter length of structure.

INTRODUCTION

If an initial hydraulic jump depth, y_1, is to be formed at the toe of a bucket invert for a supercritical stream discharging over a spillway, the tailwater depth should be equal to the subcritical sequent depth, y_4, given by the momentum equation. If the tailwater depth, y_5, is less than y_4, the jump is swept out of the bucket. This is known as a flip-bucket flow condition (Figure 1-a*). If $y_5 = y_4$, the hydraulic jump is on the verge of sweeping out of the basin. This is defined here as a sweepout jump-flow condition (Figure 1-b). If, however, y_5 is greater than y_4, the jump is submerged or drowned as shown in Figure 1-c. This is defined as a submerged jump-flow condition.

Basic information concerning the characteristics of the bucket flow is published in many hydraulic and fluid mechanics books, yet the

*All figures refer to figures in this appendix, only.

solution of the problem has been approached by means of hydraulic models. Therefore, it is apparent that an analytical study of bucket-flow phenomenon would be helpful to at least provide a theoretical guide for the model study.

In 1933 and 1945 bucket-flow characteristics were studied by the Bureau of Reclamation with the aid of hydraulic models for the Grand Coulee and Angostura Dam buckets.[1/] [2/] In 1953-1954 extensive hydraulic model tests were conducted by the Bureau of Reclamation's Hydraulic Laboratory to establish general design procedures.[3/] Similar studies were conducted by Lehigh University to define the general performance characteristics.[4/]

The radial effect on the flow through the bucket for both sweepout jump-flow and submerged jump-flow conditions has not been conclusively established. No doubt the complexity of the behavior of the rolling body, including its turbulence and surge, has discouraged the earnest efforts of many investigators, and perhaps the perspective on the main issues of the problem has been obstructed by minor

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[1/] Jacob E. Warnock, "Spillway and Outlets for Grand Coulee Dam," Hydraulic Laboratory Report No. 103, USBR, 1935

[2/] "Hydraulic Model Studies on the Spillway for the Angostura Dam," Hydraulic Laboratory Report No. 192, USBR, 1946

[3/] G. L. Beichley and A. J. Peterka, "The Hydraulic Design of Slotted Spillway Buckets," Proc. ASCE, Vol. 85, No. Hy-10, October 1959

[4/] M. B. McPherson and M. H. Karr, "A Study of Bucket-Type Energy Dissipator Characteristics," Proc. ASCE Vol. 83, No. Hy-3, June 1957

inconsistencies of flow behavior. It is suspected that difficulties in the past have been experienced largely due to efforts to merge the phenomenon into complex theory, whereas, guidance by the simple fundamental laws of mechanics and application of empirical formulae based upon analyses of experimental data would have probably yielded substantial progress.

In order to gain a more rational understanding of this phenomenon, a combination of theory and experiments are used in this study.

Notation: The letter symbols adopted for use in this study are defined where they first appear and are arranged alphabetically in Appendix A.*

SWEEPOUT JUMP-FLOW CONDITION

Review of Theory

The equations of pressure plus momentum are expressions of basic theories and must be satisfied in the simultaneous algebraic calculations required for the solution of jump problems. Recently a concept of the hydraulic horizontal jump was used for analysis of the bucket flow where an apron extended horizontally from the bucket invert.[5/] Centrifugal force was not taken into consideration. This type of bucket is called a half bucket in this analysis. Further developments of solid and slotted buckets have shown that

[5/] Serge Leliavsky, "Irrigation and Hydraulic Design," Chapman and Hall, Ltd., London, Volume 1, 1955

the centrifugal effect on the pressures in the bucket (Figure 2) may become so pronounced that it must be included in the analysis.

The sweepout jump-flow condition, Figure 1-b, is not a recommended operating condition since it creates much water surface roughness and is unstable but is analyzed here only as preparation for analysis of the submerged jump-flow condition.

Definition Sketch

Based on recent model studies[1/] [2/] [3/] [4/] [6/] a definition sketch for the bucket flow is presented.

Referring to Figure 1-b, the factors involved in the analysis of the sweepout jump-flow conditions are defined as follows:

Sequent depth factor (ψ) is defined as the ratio of the sequent depth y_4 to the supercritical initial depth, y_1.

$$\psi = \frac{y_4}{y_1} \tag{1}$$

Tailwater factor (S_1) is defined as the ratio

$$S_1 = \frac{(y_4 - y_2)}{y_2} \tag{2}$$

[1/] Atsuya Okada and Takeshi Ishibashi, "Hydraulic Model Tests on Flood Spillway of Sakuma Dam," Central Research Institute, Electric Power Industry, Japan, June 1, 1956

where y_2 is the subcritical sequent depth of normal free jump on horizontal floor.

Abrupt factor (Z) is defined as the ratio of abrupt rise, z, to initial depth, y_1,

$$Z = \frac{z}{y_1} \tag{3}$$

Baffle factor (k) is defined as the ratio of the total width of baffle, b_t, to that of bucket width, b,

$$k = \frac{b_t}{b} \tag{4}$$

Radius factor (R) is defined as the ratio of the initial depth, y_1, to bucket radius, r_1,

$$R = \frac{y_1}{r_1} \tag{5}$$

For slotted bucket similar to the Angostura Spillway bucket, $0 < k < 1$ and $Z > 0$.

For solid bucket similar to the Grande Coulee bucket without baffle, $k = 1$ and $Z > 0$.

For half bucket where a horizontal apron extends horizontally from the bucket invert, either k or $Z = 0$.

Drag Forces on Bucket Baffles

If the shear stress along the horizontal solid boundaries of a solid bucket between Sections 1 and 4, Figure 1-b, can be neglected and the momentum coefficients, β_1 and β_4, are assumed as unity, then by use of force and momentum theory,

$$\left(\frac{1}{2}\right)\gamma y_4^2 - \left(\frac{1}{2}\right)\gamma y_1\left(y_1 + \frac{y_1 v_1^2}{g r_1}\right) + p$$

$$= \left(\frac{q\gamma}{g}\right)\left(\frac{q}{y_1} - \frac{q}{y_4}\right) \qquad (6)$$

in which γ is the specific weight of water, q is the discharge per unit width of bucket, g is the acceleration due to gravity, $\frac{y_1 v_1^2}{g r_1}$ is the static head correction factor of the bucket due to the radial flow.[7/ 8/]

p is the force acting on the baffle unit of the solid bucket and can be written in the form of the basic drag equation

$$p = Cd \frac{\rho v_1^2}{2} \Delta Z \qquad (7)$$

where Cd is the coefficient of drag, ρ is the mass density of water.

7/ Ven Te Chow, "Open-Channel Hydraulics," 1959

8/ D. B. Gumensky, "Design of Side Walls in Chute and Spillway," Trans. ASCE Vol. 119, 1954

Introducing Equation 7 into Equation 6

$$\frac{\gamma y_4^2}{2} + Cd\,\frac{\rho v_1^2}{2}\,\Delta Z - \frac{1}{2}\gamma y_1\left(y_1 + \frac{y_1\, v_1^2}{g r_1}\right)$$

$$= \frac{q\gamma}{g}\left(\frac{q}{y_1} - \frac{q}{y_4}\right) \tag{8}$$

Equation 8 could be reduced to form

$$Cd = \frac{1}{F_1^2 Z}\left\{ 2F_1^2\left(1 - \frac{1}{\psi}\right) - \psi^2 + (1 + F_1^2 R)\right\} \tag{9}$$

where F_1 is the Froude number for the corresponding initial flow condition for the horizontal hydraulic jump, i.e., $F_1 = \frac{v_1}{\sqrt{g y_1}}$.

Equation 9 is the drag coefficient relationship for solid bucket. In the case of slotted bucket (Figure 1-c), the drag equation is written as

$$p = Cd \cdot C \cdot k \cdot \frac{\rho\, v_1^2}{2}\,\Delta Z \tag{10}$$

where Cd is the drag coefficient for the corresponding solid bucket with abrupt factor Z and C is the spacing parameter for drag coefficient of slotted bucket. Then, the momentum equation for slotted bucket could be written as

$$\frac{1}{2}\gamma y_4^2 + C \cdot k \cdot Cd \frac{\rho v_1^2}{2} - \gamma y_1 \left(y_1 + \frac{y_1 v_1^2}{g r_1} \right)$$

$$= \frac{q\gamma}{g} \left(\frac{q}{y_1} - \frac{q}{y_4} \right) \qquad (11)$$

and simplifying

$$C = \frac{1}{k\, Cd\, F^2 Z} \left\{ 2 F_1^2 \left(1 - \frac{1}{\psi} \right) - \psi^2 + 1 + F^2 R \right\} \qquad (12)$$

Equation 11 is the general expression of pressure plus momentum equation for spillway bucket, and could be used in evaluating flow parameters, such as sequent depth factor, radius factor, etc.

Experimental Studies of Drag Coefficient

Experimental data, taken by the USBR Hydraulic Laboratory from Grand Coulee,[1] Angostura,[2] and "general performance studies[3] were used in evaluating the drag coefficient of the bucket baffles.

Plot of test data shows that the drag coefficient of the bucket baffle is mainly controlled by dimensionless parameters; Z and k. The best fitted curves for different types of bucket are shown in Figure 3-a. From Figure 3-a, it is seen that Cd decreases as Z increases, and for values of Z larger than 2.5, the effect of baffle is not significant. Therefore, for design purpose, a Z value of less than 2 is recommended.

To evaluate spacing parameter, C, test data from Angostura models[2/] is plotted against baffle factor, k, in Figure 3-b. It is seen that C value increases from $C = 1$ at $k = 100$ percent to maximum of 1.60 at $k = 67$ percent and then decreases to 0 at $k = 0$. Thus, from the viewpoint of drag force, it is found that the slotted bucket of $k = 67$ percent will give the best effect.

Evaluation of Sequent Depth Factor

Substituting Equation 1 into Equation 11, using the equation of continuity, a cubic equation for sequent depth factor in a sweepout jump flow condition can be evaluated,

$$\psi^3 + \left\{ kC\,Cd\,F_1^2\,Z - (1 + F^2 R) - 2F_1^2 \right\} \psi + 2F_1^2 = 0 \qquad (13)$$

A mathematic solution of the sequent depth factor, $\frac{y_4}{y_1}$, is possible but complicated, and usually introduces some imaginary root. This makes the analysis of flow characteristics much more complicated, yet the solution of the equation shows

$$\psi = f(F_1, R, k, \text{and } Z) \qquad (14)$$

or simply the sequent depth factor in the sweepout jump can be shown as a function of flow parameter, F_1, and shape factors R, k, Z. This relationship can be used in solving for tailwater depth in the sweepout for known values of R, K, and Z conditions.

Again a factor of

$$G = \frac{F_1}{\sqrt{1 + \frac{k\,C\,Cd\,F_1^2\,y_1\,\Delta Z - \frac{y_1^2\,v_1^2}{gr_1}}{y_4^2 - y_1^2}}} \tag{15}$$

can be adopted into Equation 11 and simplifying

$$\psi = \frac{y_4}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8G^2} - 1\right) \tag{16}$$

This is the general equation for hydraulic jump and from the above analysis it is apparent that

$$G = f(F_1,\ R,\ k \text{ and } Z) \tag{17}$$

However, a Fortran program is given in Appendix B to simply the computation of Equation 13.

Evaluation of Radius Factor

For known tailwater condition, Equation 11 can be used in evaluating the radius factor, however, a tailwater factor, S_1 is adopted as a tool of comparison, then

$$R = \frac{y_1}{\gamma_1} = \frac{1}{F_1^2} \left\{ k C\, Cd\, F_1^2 Z + \psi^2 - 2F_1^2 \left(1 - \frac{1}{\psi} \right) - 1 \right\}$$

$$= \frac{1}{F_1^2} \left\{ k C\, Cd\, F_1^2 Z + (1 + S_1)^2 \varphi^2 - 2 F_1^2 \left(1 - \frac{1}{(1 + S_1)\varphi} \right) - 1 \right\} \qquad (18)$$

where

$$\varphi = \frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8 F_1^2} - 1 \right) \qquad (19)$$

and

$$R = f(F_1, S_1, Z, K) \qquad (20)$$

Equation 18 is the general radius factor equation for the bucket flow in the slotted-type bucket.

For the solid bucket in the sweepout jump-flow condition, Equation 18 can be reduced to

$$R = \frac{1}{F_1^2} \left\{ k Cd\, F_1^2 Z + \psi^2 - 2 F_1^2 \left(1 - \frac{1}{\psi} \right) - 1 \right\} \qquad (21)$$

$$= \frac{1}{F_1^2} \left\{ k Cd\, F_1^2 Z + (1 + S_1)^2 \varphi^2 - 2 F_1^2 \left(1 - \frac{1}{(1 + S_1)\varphi} \right) - 1 \right\} \qquad (22)$$

and can be expressed as function of F_1, S_1, and Z,

$$R = f(F_1, S_1, Z) \qquad (23)$$

If either Z or $K = 0$, as for the half bucket Equation 18 again can be reduced to

$$R = \frac{1}{F_1^2}\left[(1+S_1)^2\,\varphi^2 - 1 - 2F_1^2\left(1-\frac{1}{[(1+S_1)\,\varphi]}\right)\right] \tag{24}$$

which means that the radius factor can be expressed as a function of only F_1 and S_1;

$$R = f(F_1, S_1) \tag{25}$$

Energy Loss

The initial energy at the toe of the jump or invert of the bucket can be written as

$$E_1 = y_1 + y_1\, v_1^2 / gr_1 + v_1^2 / 2g$$

$$= y_1 + y_1^2\, F_1^2 + F_1^2\, y_1 / 2 \tag{26}$$

Introducing R,

$$\frac{E_1}{y_1} = 1 + F_1^2\, R + F_1^2 / 2 \tag{27}$$

The energy at the end of jump is

$$E_4 = y_4 + \frac{v_4^2}{2g} = (1 + S_1)y_2 + \frac{q^2}{2g\,(1+S)^2\,y_2^2} \tag{28}$$

in which v_4 is the mean velocity at the tailwater.

The energy loss E_L is given by

$$E_L = E_1 - E_4 \tag{29}$$

Substituting Equations 26 and 28 into Equation 29 and simplifying,

$$\frac{E_L}{E_1} = 1 - \frac{(1+S_1)\varphi - \frac{1}{2}F_1^2/(1+S_1)^2\varphi^2}{1 + F_1^2 R + F_1^2/2} \tag{30}$$

that is

$$\frac{E_L}{E_1} = f(F_1, S_1, Z, k) \tag{31}$$

or either

$$\frac{E_L}{E_1} = f(F_1, R, Z, k) \tag{32}$$

and

$$\frac{E_L}{E_1} = f(S_1, R, Z, k) \tag{33}$$

Thus for the relative loss of energy in various types of buckets, Equation 30 has been shown to be a function of Froude number (F_1) tailwater factor (S_1) and the bucket shape factors (R, Z, and k). It is apparent that the energy dissipating capacity of the bucket is much more than that of the corresponding horizontal jump. In order to figure out the efficiency of energy dissipation, an index n is used,

$$n = \frac{\left(\frac{E_L}{E_1}\right)_{b.j}}{\left(\frac{E_L}{E_1}\right)_{f.j}} \tag{34}$$

where b.j represents the bucket flow in sweepout condition and f.j represents the corresponding horizontal jump condition. All the above-mentioned flow parameters can be obtained from the Fortran program in Appendix C.

Curvilinear Effect on Froude Number and Critical Depth

The general specific energy equation at the toe of the jump or invert of the bucket can be written as

$$E = y + y\frac{v^2}{gr} + \frac{v^2}{2g} = y + \frac{q^2}{gry} + \frac{q^2}{2gy^2} \tag{26}$$

The critical state of flow has been defined as the state of flow at which the specific energy is minimum for a given discharge.[3/]

[3/] Paul Boss, "Berechnung der Wasserspiegellage beim Wechsel des Fliesszustandes," (Computation of Water Surface with Change of the Flow Type"), Springer-Verlag, Berlin, 1919, pp 20 and 52.

theoretical criterion for critical flow may be developed by differentiating Equation 26 with respect to y and noting q as a constant thus

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}\left(1 + \frac{y}{r}\right) = 1 - \frac{v^2}{gy}\left(1 + \frac{y}{r}\right) \tag{35}$$

At the critical state of flow the specific energy is a minimum, or the depth where minimum energy occurs $\frac{dE}{dy} = 0$. Let y_{cb} represents the critical depth for the bucket flow, then y_{cb} must satisfy the following condition, that is

$$1 - \frac{q^2}{g\, y_{cb}^3}\left(1 + \frac{y_{cb}}{r}\right) = 0 \tag{36}$$

or

$$y_{cb}^3 - \frac{q^2}{gr}\, y_{cb} - \frac{q^2}{g} = 0 \tag{36'}$$

The solution of y_{cb} from Equation 36 can be obtained, but is not a matter of interest. However, the critical depth of the linear channel, $y_c = \sqrt[3]{\frac{q^2}{g}}$, is introduced in Equation 36 to get a better comparison, that is

$$\frac{y_c^3}{y_{cb}^3} = \left(\frac{1}{1 + \frac{y_{cb}}{r}}\right) \tag{37}$$

Furthermore, the combined effect of gravity and curvature of bucket upon the state of flow, or Froude number for the bucket can be defined as

$$F_b = \sqrt{\frac{v^2}{gy}\left(1 + \frac{y}{r}\right)} \tag{38}$$

or

$$F_b = F\left(\sqrt{1 + \frac{y}{r}}\right) \tag{39}$$

when $F_b = 1$, the flow state is corresponding as defined by Equation 36, and the relationship between the Froude number for linear channel, $F = \frac{v}{\sqrt{gy}}$, and linear channel is

$$F_b = F\sqrt{1 + \frac{y_{cb}}{r}} = 1$$

where $\frac{y_{cb}}{r}$ varies $0 \leq \frac{y_{cb}}{r} \leq 1$, thus for the critical state of the bucket flow, the corresponding Froude number of the nonbucket, $F = \frac{v}{\sqrt{gy}}$, will vary from 1 to $\frac{1}{\sqrt{2}}$. This means that hydraulic jump can be formed even though the corresponding Froude number, $F = \frac{v}{\sqrt{gy}}$, is less than 1.

SUBMERGED JUMP FLOW CONDITION

Definition Sketch:

Referring to Figure 1-c, the all factors involved in the analysis of the submerged jump-flow condition are defined

as follows:

Backedup depth factor (H_b) is defined as the ratio of the depth of flow in the bucket above the invert, y_3, to the supercritical depth, y_1,

$$H_b = \frac{y_3}{y_1} \tag{40}$$

Submerged factor (S_2) is defined as the ratio

$$S_2 = \frac{(y_5 - y_4)}{y_4} \tag{41}$$

Tailwater factor (S_3) is defined as the ratio

$$S_3 = \frac{(y_2 - y_5)}{y_2} \tag{42}$$

Surge factor (H_s) is defined as the ratio

$$H_s = \frac{y_6}{y_5} \tag{43}$$

where y_6 is the average surge peak depth.

Evaluation of Backed-up Depth Factor

The submerged hydraulic jump in a bucket is a system of two distinct streams; one, the principal stream carrying the lower portion

of the jump body, and the other a rotating mixture of water and air supported both in motion and position by forces imparted to it by the principal stream. By the presentation of J. H. Douma,[10] the depth of the principal stream in a bucket invert, y_1, can be shown as a function of a dimensionless parameter, $\frac{q}{r_1 \sqrt{2gh}}$. (Appendix D) Thus the radius factor, R, can be easily determined for the submerged jump-flow condition.

Then refer to Figure 1-c, cut free body through bucket invert, the forces plus momentum equation can be written as:

$$\frac{1}{2}\gamma y_5^2 + kC\,Cd\,\Delta Z \frac{\rho v_1^2}{2} - \frac{1}{2}\gamma (y_3 - y_1)^2 - \frac{1}{2}\left(2y_3 + \frac{y_1 v_1^2}{gr_1} - y_1\right) y_1$$

$$= \frac{q\gamma}{g}\left(\frac{q}{y_1} - \frac{q}{y_5}\right) \tag{44}$$

Substituting the equation of continuity and simplifying,

$$H_b = \frac{y_3}{y_1} = \sqrt{\psi^2 + kC\,Cd\,Z\,F_1^2\,R - 2F_1^2\left(1 - \frac{1}{\psi'}\right)} \tag{45}$$

where $\psi' = \frac{y_5}{y_1} = (1+S_2)\frac{y_4}{y_1} = (1+S_2)(1+S_1)\frac{y_2}{y_1}$

$$= (1+S_3)\frac{y_2}{y_1} = \frac{1}{2}(1+S_1)\left(\sqrt{1+8F_1^2} - 1\right)(1+S_2) \tag{46}$$

---

[10] Discussed by J. H. Douma, "Design of Side Walls in Chutes and Spillways," Trans. ASCE, Vol. 119, 1954

H_b can be shown to be function of F_1, S_2 and shape factors, R, k, Z, that is

$$H_b = f(F_1, S_2, R, k, Z) \tag{47}$$

for a solid bucket

$$H_b = f(F_1, S_2, R, Z) \tag{48}$$

and for a half bucket,

$$H_b = f(F_1, S_2, R) \tag{49}$$

To simplify the computation, all flow parameters can be compared with the horizontal jump, and the third tailwater factor S_3 is used in evaluation of the backedup depth factor, then Equations 47 to 49 can be expressed as

$$H_b = f(F_1, S_3, R, k, Z) \tag{50}$$

Further more, the depth of the principal stream in a bucket invert,[11] y_1, can be approximately determined, and backedup depth selected. Insuch a case, determination of radius factor, R, becomes necessary. Then, by use of Equation 44, the radius factor, R, can be expressed as a function of F_1, S_3, H_b and shape factors k, Z. That is

$$R = \frac{1}{F_1^2}\left[\psi'^2 + kC\,Cd\,Z\,F_1^2 - H_b^2 - 2F_1^2\left(1 - \frac{1}{\psi'}\right)\right] \tag{51}$$

[11] P. M. Stepanov, "O podtoplennom pryzhke vody," Gidrotekhnika i Melioratsiya, Vol. X, No. 1, 1958, pp 43-53

or

$$R = f(F_1, S_3, H_b, k, Z) \tag{52}$$

When $H_b = 1 (y_3 = y_1)$, Equation 51 can be reduced to Equation 18. This means that the sweepout condition is a special case of the submerged condition. Equation 44 can be used in evaluating drag coefficient for spillway buckets for submerged jump-flow condition. The resulting equation is

$$C\ Cd = \frac{1}{k \cdot F_1^2 Z} \left\{ F_1^2 R - \psi'^2 + H_b^2 + 2F_1^2 \left(1 - \frac{1}{\psi'} \right) \right\} \tag{53}$$

Numerical values can be obtained from experimental data. However, due to air entraining facility of the bucket roller and unstable characteristics of water surface, it is reasonable to assume that the drag coefficient for submerged jump-flow condition will remain the same as for corresponding sweepout jump-flow condition.

In order to verify the assumption, experimental backup depth factors taken by the USBR hydraulic laboratory[3/] are plotted against computed values using Equation 45 and drag coefficient values obtain from sweepout jump-flow condition. Because of air entraining facility of the bucket roller, the experimental backup factor seems to be higher than the computed value, Figure 4.

The best fit relationship, Figure 4, may be expressed as

$$H_{o(exp)} = H_{o(the)} + 1.5 \tag{54}$$

This correlation is seen to be good. A correction factor of 1.5 takes care of the simplifications made in theoretical analysis. Hence, the theoretical Equations 45 and 51 could be used to compute the relationship between R, H_b, F_1, and S_3, and a Fortrain program is given in Appendix E to simplify the computation using the program, the required bucket radius for any particular flow can be obtained and the backedup depth of any special flow condition can be easily predicted. This information is not important in hydraulic design of buckets but also important phase in structure design of the walls in chutes and spillways.[8]

Evaluation of Surge Factor

Approximate surge-height characteristics obtained by the USBR hydraulic laboratory[3] are shown in Figure 5. The best fit curve shows that the equation of

$$H_s = 1.45\ H_b^{1/12} \tag{55}$$

can be fitted. This information is helpful in setting top elevations for training walls.

Length of the Submerged Jump in Bucket

As tested by the USBR hydraulic laboratory,[3] the hydraulic behavior of the submerged bucket dissipator is manifested primarily by the formation of two rollers; one is on the surface moving counterclockwise and is contained within the region above the curved bucket,

and the other is a ground roller making a surge moving in a clockwise direction and is situated downstream from the bucket. To find the length of the submerged jump, the bucket invert was taken as the origin, and the end of the jump was taken to be the end of the said roller and surge, where the depth became equal to the tailwater depth, y_5.

If $L_{b.s.j.}$ is the length of the submerged jump, the variation of $L_{b.s.j.}$ with backedup depth factor, H_b, taken by the USBR hydraulic laboratory[3/] is shown in Figure 6a. It was found that the theoretical backedup factors could be satisfied by the linear equation

$$\frac{L_{b.s.j.}}{y_5} = 2.47 - 0.035\, H_{b(\text{the.})} \tag{56}$$

with a correlation coefficient of 0.87.

For experimental backedup factors, the same relationship as given by Equation 54 can be adopted.

If Equation 45 is substituted into Equation 56, jump length factor, $\frac{L_{b.s.j.}}{y_5}$ can be shown as a function of F_1, S_2, or S_3, and shape factors R, Z, k. That is

$$L_{b.s.j.}/y_5 = 2.47 - 0.035\sqrt{\psi'^2 + kC\,Cd\,Z\,F_1^2 - F_1^2\,R - 2F_1^2\left(1 - \frac{1}{\psi'}\right)} \tag{57}$$

or

$$\frac{L_{b.s.j.}}{y_5} = f(F_1, S_s, R, Z, k) \tag{58}$$

For $H_b = 1$, that is, when flow begins to sweepout, $\frac{L_{b.s.j.}}{y_5}$ will be as small as 2.44 and after $H_b > 1$, $\frac{L_{b.s.j.}}{y_5}$ will be decreased, gradually. A comparison of the jump lengths on the different types of energy dissipators[12/] shows that the bucket energy dissipators are the shortest.

Length of the Surge in Submerged Jump

The length from bucket invert to the average surge peak is taken as the length of the surge, L_s, and the relationship of $\frac{L_s}{y_5}$ to H_b is shown in Figure 6-b. It was found that all the best fit curve through the points could be satisfied by the equation

$$\frac{L_s}{y_5} = 1.47 - 0.019\, H_b \tag{59}$$

with a correlation coefficient of 0.84.

Comparison of equations 56 and 59 shows that the length between average surge peak and end of jump has a variation of

$$\frac{(L_{b.s.j.} - L_s)}{y_5} = 0.016\, H_b + 1 \tag{60}$$

12/ "Hydraulic Design of Stilling Basins and Other Energy Dissipators," Engineering Monographs No. 25, USBR, 1963

Length of the Roller

Due to the bucket action, the length of the roller was not so pronunced as in the horizontal hydraulic jump. The relationship between the ratio of the roller length, L_r, to tailwater depth, y_5, and backedup depth factor, H_b, is shown in Figure 6-c. The correlation is not good and a variation of 0. to 0.4 is obtained.

Energy Loss

Referring to Figure 1-c, the energy at the invert section of the bucket can be written as

$$E_1 = y_3 + \frac{v_1^2}{2g} + \frac{y_1 \; v_1^2}{g \, r_1} \tag{61}$$

or

$$\frac{E_1}{y_1} = H_b + \frac{F_1^2}{2} + F_1^2 \, R \tag{62}$$

The energy at end of jump is

$$E_5 = y_5 + \frac{v_5^2}{2g} = y_5 + \frac{F_1^2 \, y_1^3}{2 y_5^2} \tag{63}$$

or

$$\frac{E_5}{y_1} = \frac{y_5}{y_1} + \frac{F_1^2 \, y_1^2}{2 \, y_5^2} = \psi' + \frac{F_1^2}{2\psi'^2} \tag{64}$$

Then the energy loss E_L is given by

$$E_L = E_1 - E_S \tag{65}$$

Substituting Equations 61 and 63 into Equation 65, simplifying

$$\frac{E_L}{E_1} = 1 - \frac{\left(\psi' + \frac{F_1^2}{(2\psi'^2)}\right)}{\left(H_b + \frac{F_1^2}{2} + F_1^2 R\right)} \tag{66}$$

Again it is shown that

$$\frac{E_L}{E_1} = f(F_1, S_s, R, H_b, Z, k) \tag{67}$$

and when $H_b = 1$, Equation 66 can be reduced to Equation 30, as stated above, similar conclusion can be made for the relationship between sweepout and submerged jump condition in bucket flow.

Verification of Equation 66 is obtained by using experimental losses computed from data taken by the USBR hydraulic laboratory[3/] compared with theoretical lossed computed using Equations 45 and 66 with Cd value obtained from Figure 3. The results of the comparison are shown in Figure 7. The correlation is seen to be good.

The backedup depth factor is a element in the energy loss Equation 66, yet, the experiment results show that the variation due to error in

measurement of backedup depth has little effect on the relative energy loss and the relative loss curve, Figure 7 shows a better correlation than that of the backedup depth factor curve, Figure 4.

Through use of Equation 66, the energy dissipating capacity of the submerged bucket is seen to be more or less than that of the corresponding free jump on horizontal floor, depending upon the particular value of F_1, S_3 and shape factor R, Z, k. Again the index of relative dissipation, Equation 34 is used for evaluation of dissipation efficiency. For any given F_1, k, Z, and S, n could be more or less than unity, depending on R and S_3.

By differentiating Equation 34 with respect to either S_3, F_1, or R, and treating the resulting equation equal to 0, then a maximum value of n, for any particular flow condition can be theoretically obtained. However, the resulting equation is a complicated function of F_1, S, R and shape factors k, Z. A sample computor computation shows that the equation is an increasing function and becomes infinity as H_b increases.

In the preceeding consideration of the energy loss, it should be always be remembered that the computed energy loss in the submerged jump occurs in a length much less than that of the corresponding bucket jump in sweepout condition or free jump on a horizontal floor. Comparison of the energy dissipation capacity is constantly lower

than that of the sweepout condition. From these considerations, it is concluded that the submerged jump could not be preferred to the sweepout condition for energy dissipation purposes unless the backedup factor is less than approximately 20 percent of the tailwater depth. This agrees with the minimum tailwater recommended by USBR and the test results given by McPherson and others.[12] Hence, the equations mentioned above can be adopted in buckets design, graphical or tabular representation of equations is possible but due to too many variables included in the analysis, and in view of the extensive and rapidly growing use of digital computers, such tables or graphs would not be of widespread interest. Instead, a Fortran program was written for use on an IBM 7090 and is shown in Appendix E.

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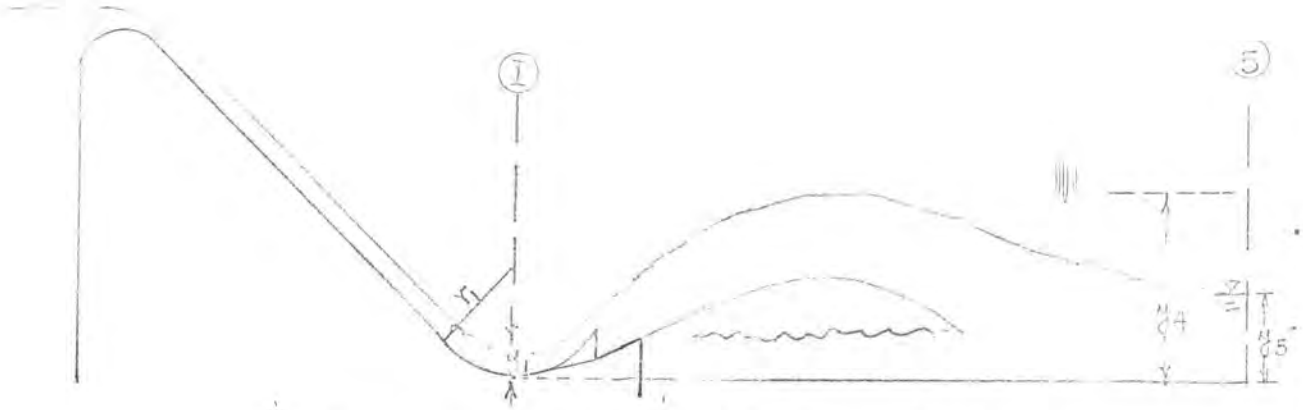
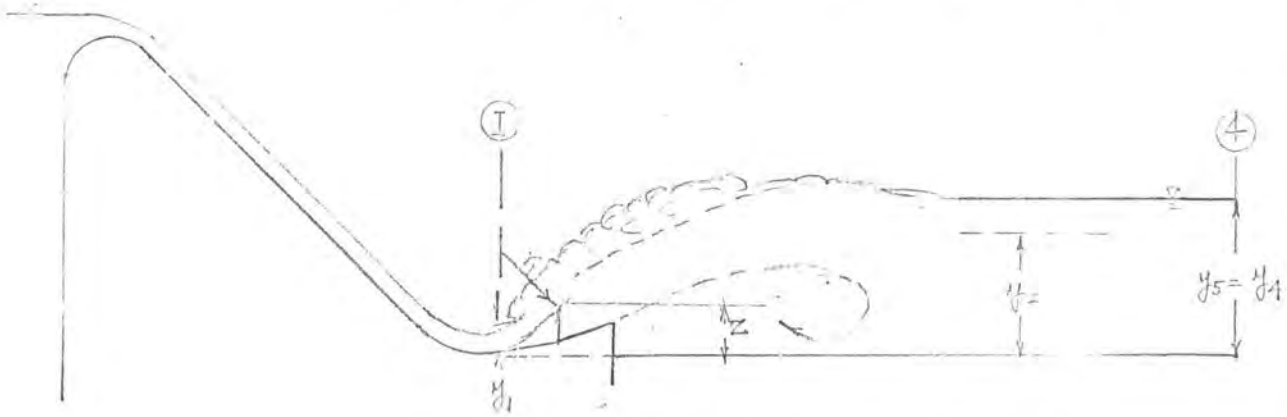
[12] "Symposium on Energy Dissipators," Central Board of Irrigation and Power, Publication No. 70, New Delhi, India, August 19[illegible]

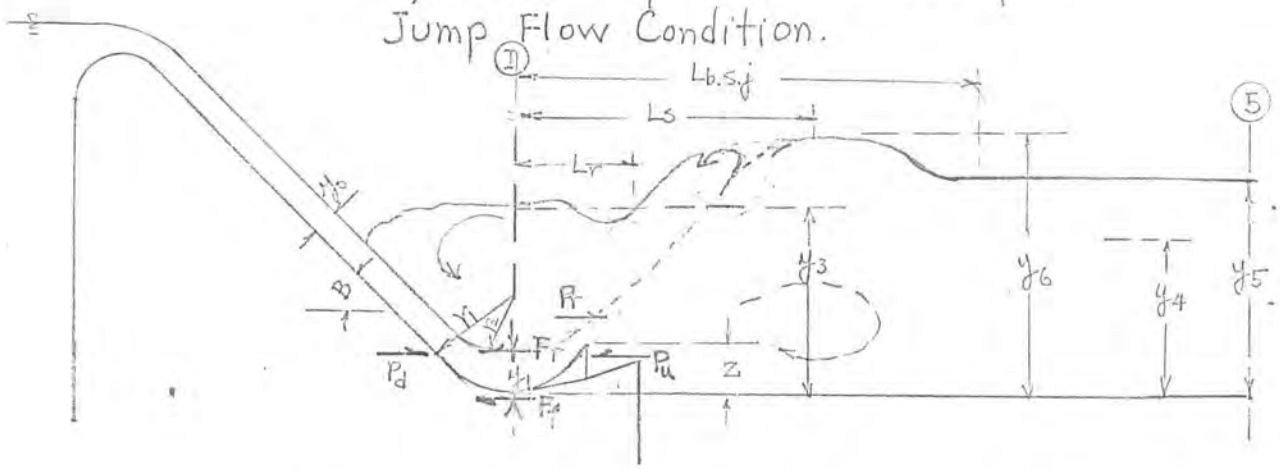
FIGURE 1.
REPORT HYD.



A. Flip Bucket or Jet Flow Condition.

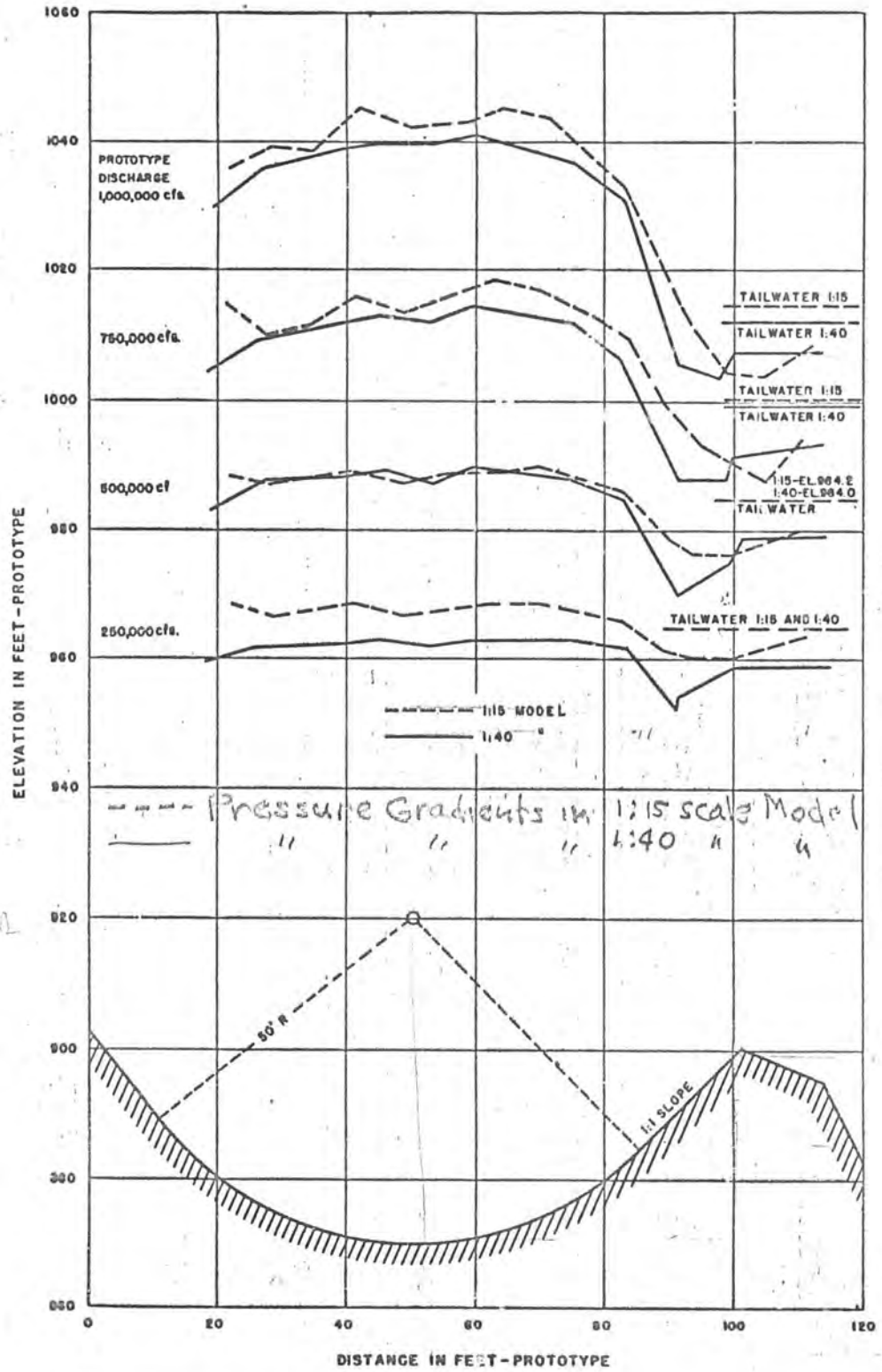


B. Hydraulic Jump Bucket or Sweepout Jump Flow Condition.



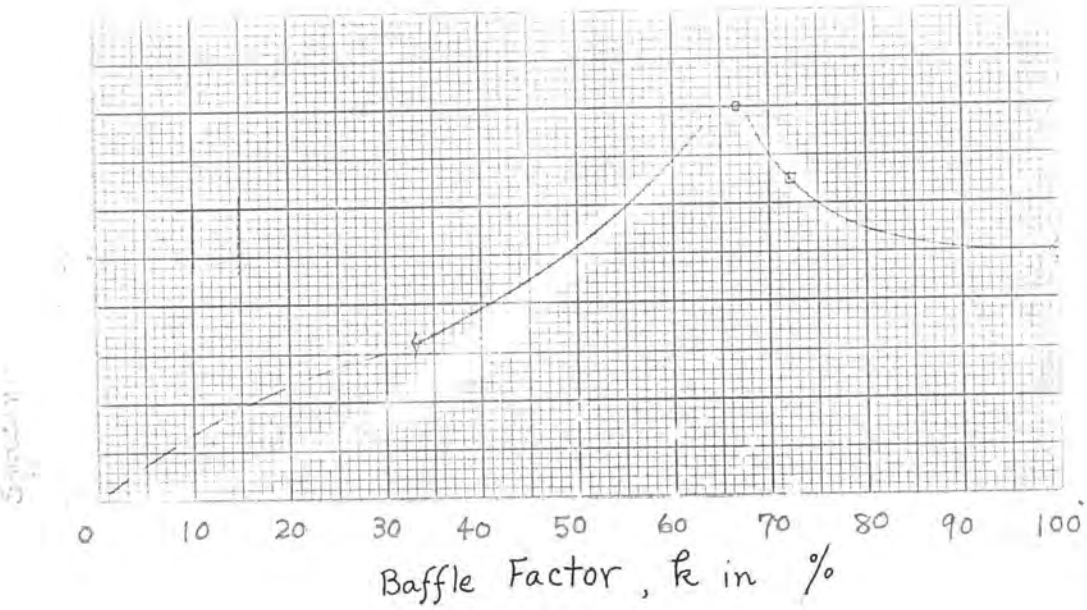
C. Roller Bucket or Submerged Jump Flow Condition.

FIGURE 2
REPORT HYD.



PRESSURES IN THE
GRAND COULEE SPILLWAY
BUCKET

FIGURE 3
REPORT HYD



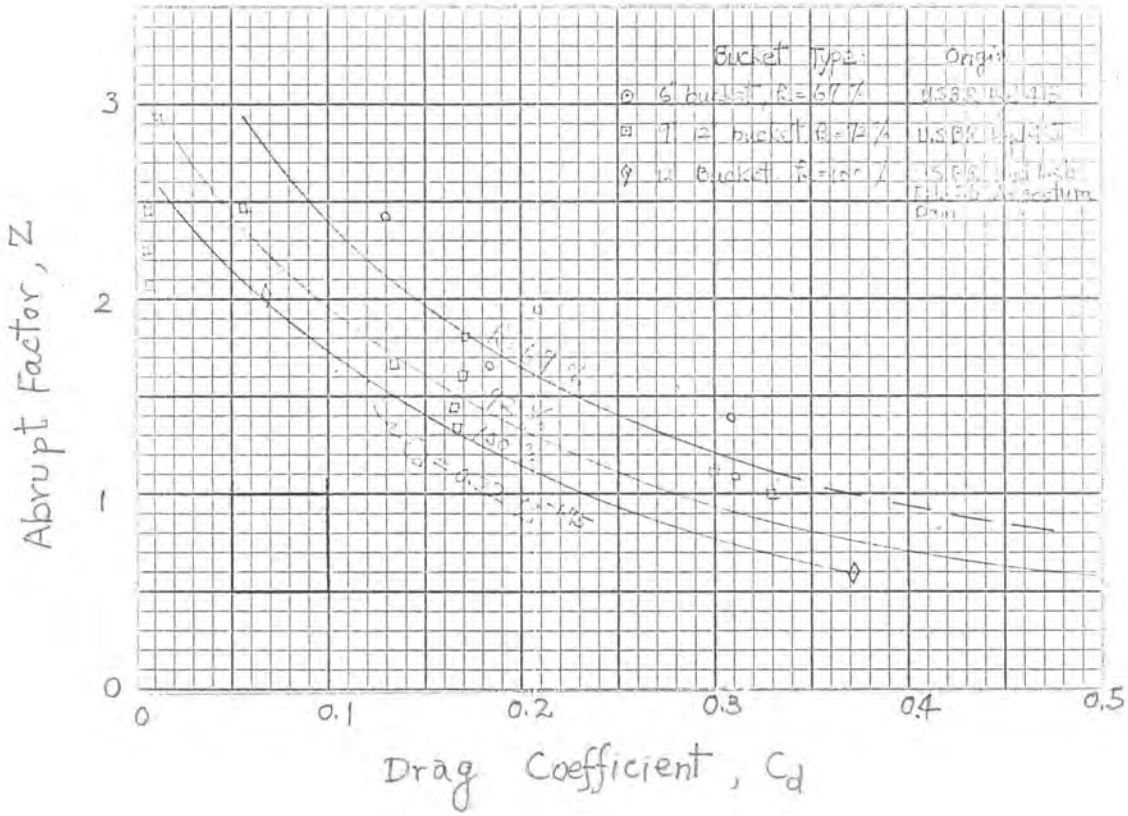


FIGURE 4
REPORT HYD.

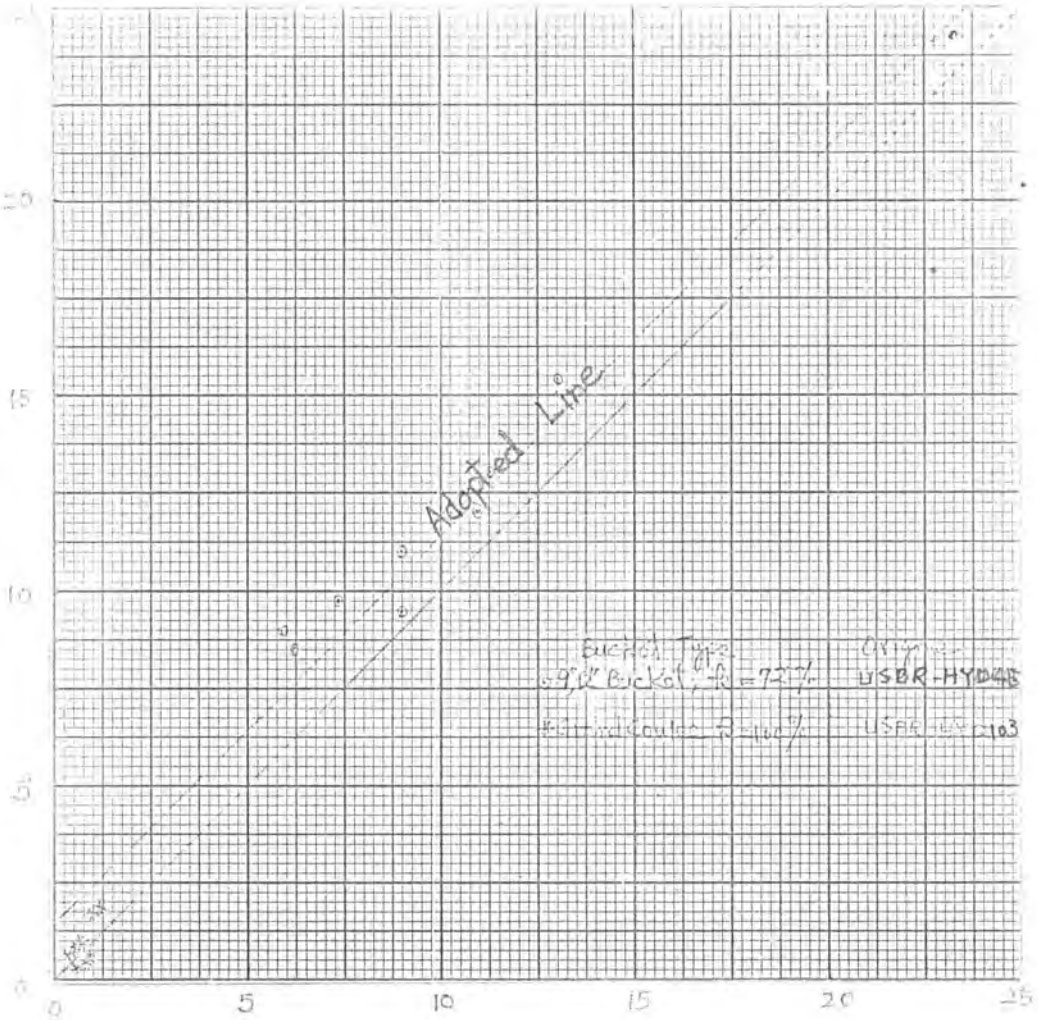
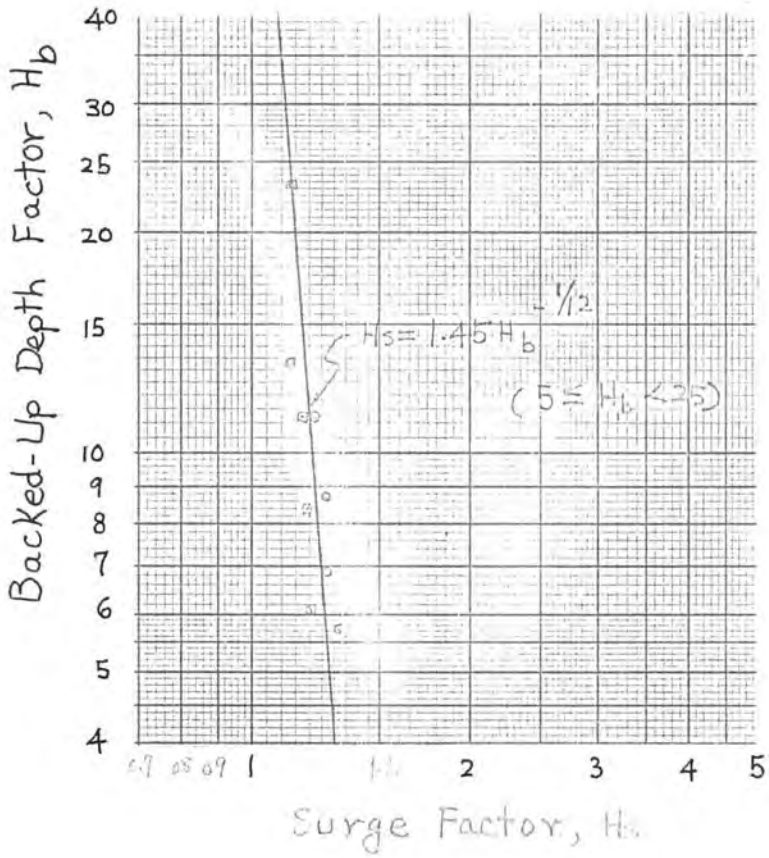
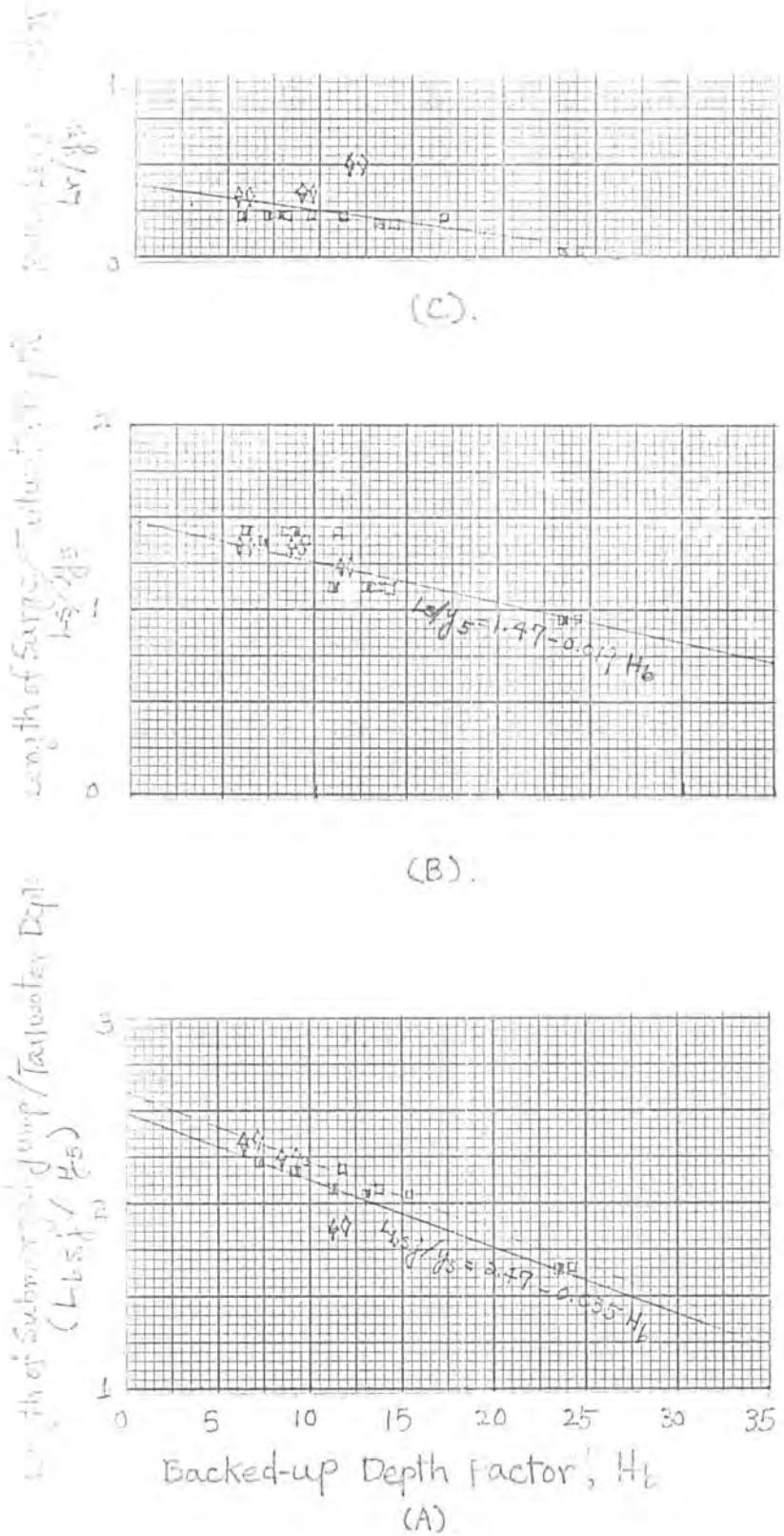
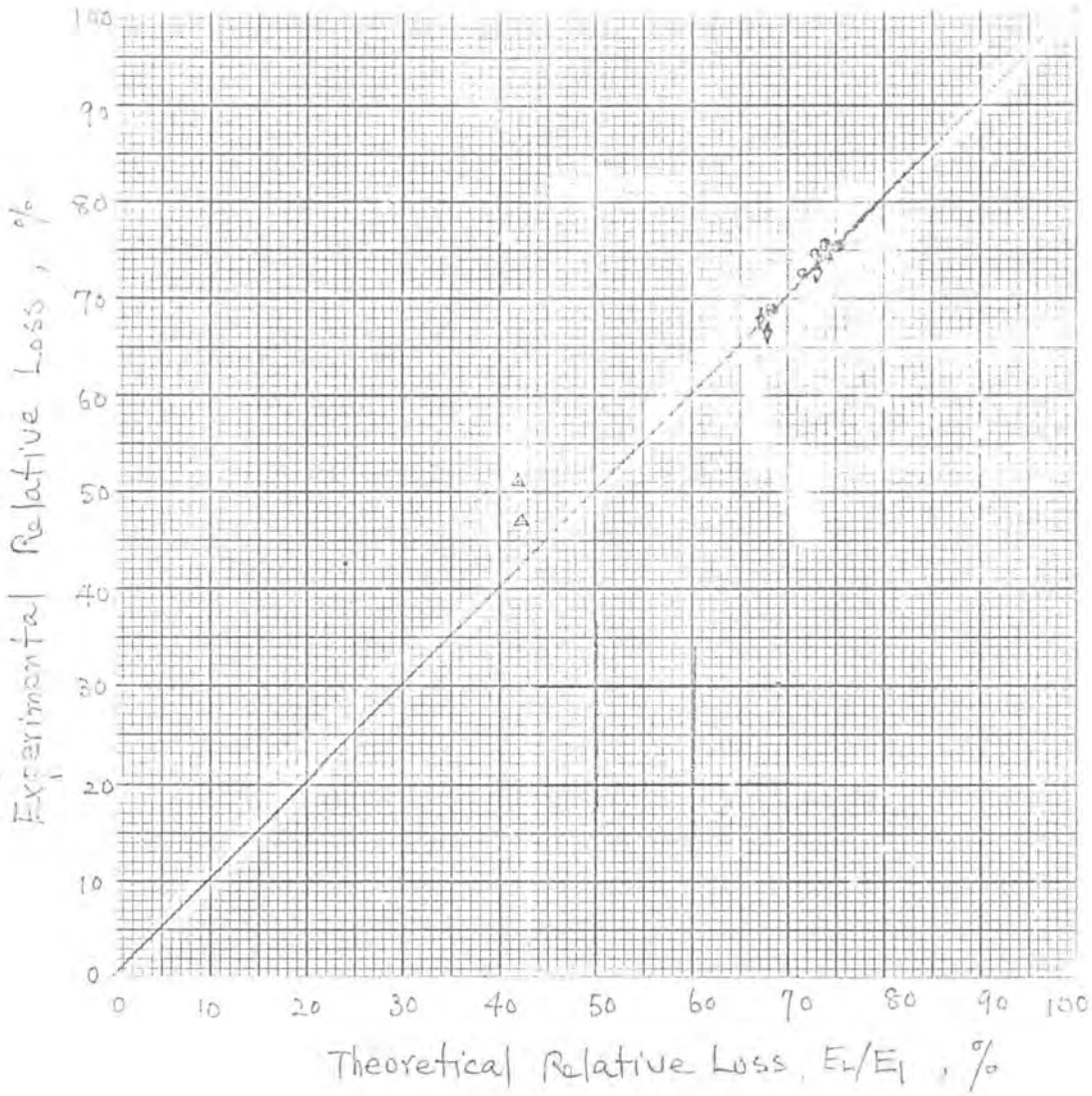


FIGURE 5
REPORT HYD





LENGTH PARAMETERS FOR SUBMERGED JUMP FLOW CONDITION.



○ 9" Slotted bucket

◊ 12" Slotted bucket

△ Grand Coulee Solid bucket.

EXPERIMENTAL VERIFICATION OF ENERGY LOSS EQUATIONS FOR SUBMERGED JUMP-FLOW CONDITION.

APPENDIX 1--NOMENCLATURE

b	bucket width
b_t	width of baffle in bucket
C	spacing parameter
Cd	drag coefficient
E	specific energy
E_1, E_2, E_3	specific energy at various sections
E_L	specific energy loss
F_1	Froude number for linear channel
F_b	Froude number for bucket
f	function
g	acceleration due to gravity
G	jump constant
H	difference between reservoir elevation and tailwater elevation
H_b	backedup depth factor
H_s	surge factor
$L_{b.s.j.}$	length of submerged jump in bucket flow
L_r	length of roller in bucket
L_s	length of surge in bucket
n	relative efficiency of energy dissipation
P_r	component of roller weight acting downstream on the principal stream of bucket
P_d	floor reaction
P_u	floor reaction

q	discharge intensity
R	radius factor
r	radius
r_1, r_2	radius at various sections
S	submerged factor or tailwater factor
$S_{1,2,3}$	submerged factor or tailwater factor for various flows
v	mean velocity
$v_{1,2,3}$	mean velocity at different sections
x	longitudinal distance
y	depth of flow
y_c	critical depth of linear channel
y_{cb}	critical depth of bucket
$y_{1,2,3}$	depth of flow at different sections
r	specific weight of water
φ	sequent depth ratio for free jump
ψ	sequent depth factor for bucket jump
$\beta_{1,2,4}$	momentum coefficient
ρ	density of water

1604 FORTRAN CODING FORM

PROGRAM APPENDIX -B SEQUENT DEPTH FØR BUCKETS

ROUTINE

NAME C. M. Wu

PAGE 1/1

DATE

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73
C			PRØGRAM FØR SEQUENT DEPTH	
	1		FØRMAT (6 F 8.4)	
	2		READ INPUT TAPE 5 , 1, X,Y, CK, H, C,CD	
C			X = RADIUS FACTOR	
C			Y = FRØUDE NUMBER	
C			CK=BAFFLE FACTØR	
C			H =ABRUPT FACTØR	
C			C =SPECING PARAMETER FRØM FIG 3	
C			CD=DRAG CØEFFICIENT FRØM FIG 3	
C			EQUATIØN = SD**3 + (CK*C*CD*Y*Y*H - (1.0 + Y*Y*X) * 2.0 * Y*Y) *SD + 2.0 *Y*Y	
	3		A1 = C*CD*Y*Y*H*CK - (1.0 + Y*Y*X) * 2.0 * Y*Y	
			A2 = 2.0 * Y*Y	
			A3 = ((- 0.5) * A2 + SQRTF ((A2*A2/4.0) + (A1*A1*A1/27.0)))** 0.333	
			A4 = ((- 0.5)*A2 - SQRTF ((A2*A2/4.0) + (A1*A1*A1/27.0)))**0.333	
			SD = A3 + A4	
C			SD = SEQUENT DEPTH FACTØR	
			WRITE OUTPUT TAPE 6,4	
	4	0	FØRMAT (80H1 RADIUS FACTØR FRØUDE NØ BAFFLE FACTØR ABRUPT	
		1	FACTØR SEQURNT FACTØR)	
			WRITE OUTPUT TAPE 6,5 , X,Y,CK, H, SD	
	5		FØRMAT (8X,F8.4, 4X, F8.4,4X,F8.4,8X,F10.4,10X,E12.4)	
	6		GØ TØ 2	
			END	

1604 FORTRAN CODING FORM

PROGRAM APPENDIX C RADIUS FACTØR FØR SWEEPØUT BUCKET

ROUTINE

NAME C. M. Wu

PAGE 1/2

DATE

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73–80
C			FLØW PARAMETERS FØR SWEEPØUT BUCKET	
	1		FØRMAT (6 F8.4)	
	2		READ INPUT TAPE 5, 1, Y, SD, CK, H, C, CD	
C			Y = FRØUDE NØ	
C			SD = TAILWATER / INITIAL DEPTH	
C			CK = BAFFLE FACTØR	
C			H = ABRUPT FACTØR	
C			C = SPACING FACTØR FRØM FIG 3	
C			CD = DRAG CØEFFICIENT FRØM FIG 3	
C			X = RADIUS FACTØR	
C			ER = RELATIVE ENERGY LØSS	
C			A = EFFICIENCY	
		0	X=(1.0/Y*Y)*(SD*SD-2.0*Y*Y*(1.0-1.0/SD)-1.0+C*CD*Y*	
		1	Y*H*CK)	
			EL=1.0 + Y*Y*X + 0.5*Y*Y - SD-(0.5*Y*Y)/(SD*SD)	
			E1=1.0 + Y*Y*X + 0.5*Y*Y	
			ER= EL/E1	
		0	E2 = 1.0 - ((8.0*Y*Y+1.0)** 1.5 - 4.0*Y*Y + 1.0) /(8.0*Y*Y	
		1	*(2.0+Y*Y))	
			A= ER/E2	
			WRITE ØUTPUT TAPE 6, 3	
	3	0	FØRMAT (98H1 RADIUS-FAC FRØUDE NØ SEQUENT-FAC	
		1	BAFFLE FAC ABRUPT FAC R-ENERGY LØSS EFFICIENCY)	

1604 FORTRAN CODING FORM

PROGRAM	APPENDIX C. RADIUS FACTØR FØR
ROUTINE	SWEEPØUT BUCKET
NAME	C. M. Wu
PAGE	2/2
DATE	

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73–80
			WRITE ØUTPUT TAPE 6,4, X,Y,SD,CK,H,ER,A	
	4	0	FØRMAT (6X, F8.4, 4X, F8.4, 6X, F8.4, 6X, F8.4, 5X, F8.4,	
		1	5X, E14.5, 2X, E10.5)	
			GØ TØ 2	
			END.	

APPENDIX D

Determination of Depth on the Principal Stream in a Bucket Invert

According to previous model studies of flow in the spillway buckets of Grand Coulee, Pine Flat, and Popolopen Dam, J. H. Douma assumed that the velocity distribution on the bucket would be that of an irrotational vortex; $V = \frac{A}{r}$, in which A is a constant and r is the radius of any streamline. For this flow pattern, all the streamlines are circular and concentric with the buckets. This assumption has also been adopted for determining the depth of the principal stream in a bucket invert.

From Douma's streamline analysis, the following relationship was obtained[1]

$$r_2 \log\left(\frac{r_1}{r_2}\right) = \frac{q}{\sqrt{2gH}} \tag{1}$$

where r_1 is the radius of the bucket;

r_2 is the radius of the streamline in the principal stream of bucket;

H is the difference between reservoir elevation and tailwater elevation.

Dividing both sides of Equation 1 by r_1, a dimensionless equation can be obtained, that is

$$\left(\frac{r_2}{r_1}\right) \log\left(\frac{r_1}{r_2}\right) = \frac{q}{r_1\sqrt{2gH}} \tag{2}$$

[1] Discussed by J. H. Douma, "Design of Side Walls in Chutes and Spillways," Trans. ASCE, Vol. 119, 1954

or $\frac{r_2}{r_1}$ can be expressed as function of a dimensionless parameter $\frac{q}{r_1\sqrt{2gH}}$.

$$\left(\frac{r_2}{r_1}\right) = f\left(\frac{q}{r_1\sqrt{2gH}}\right) \tag{3}$$

where r_1, q, H are all known values.

The depth of the principal stream in a bucket invert, y_1, can be expressed in terms of r_1 and r_2, that is

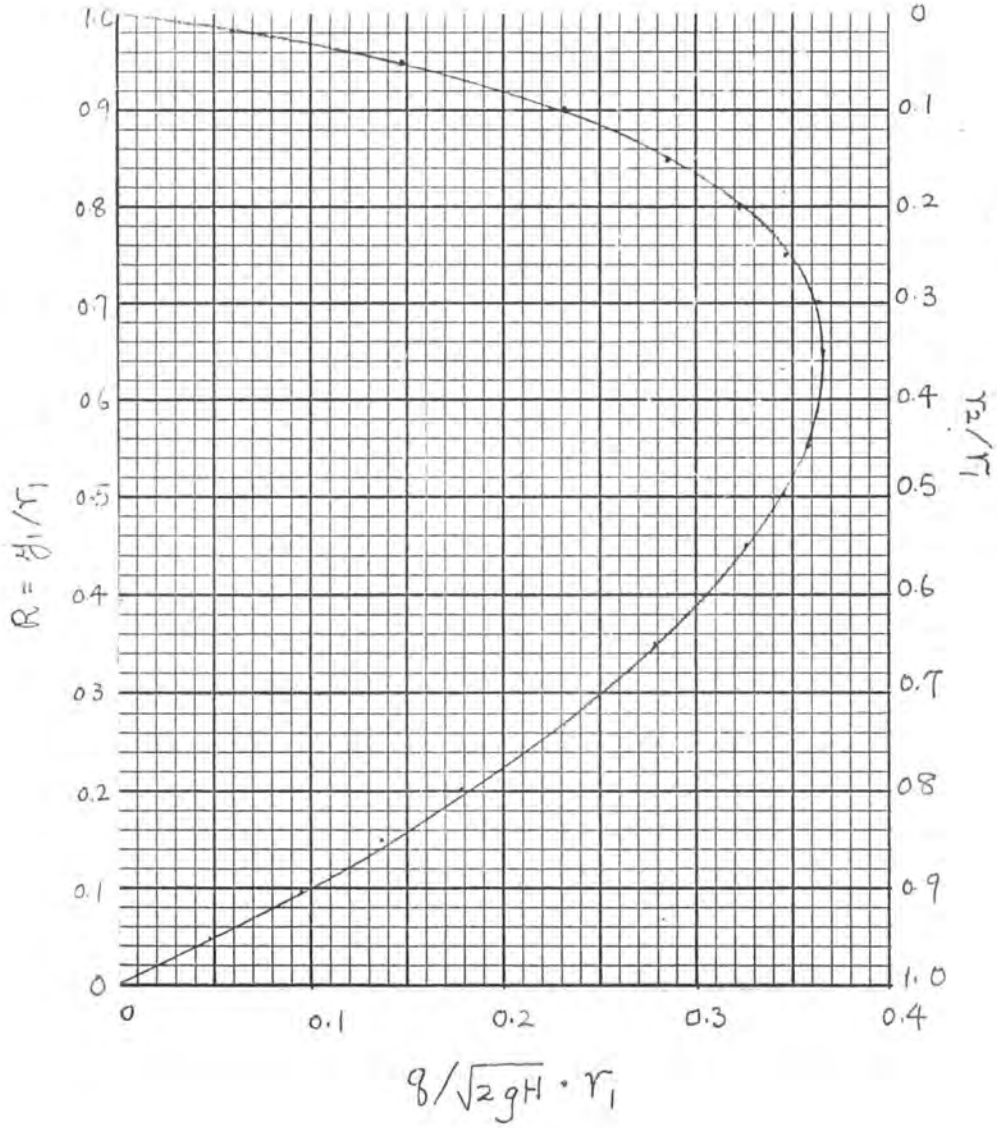
$$y_1 = r_2 - r_1 \tag{4}$$

Again using a dimensionless parameter of $\frac{y_1}{r_1}$ and in terms of $f\left(\frac{q}{r_1\sqrt{2gH}}\right)$, a simple solution of y_1 can be obtained by using the relationship of

$$\left(\frac{y_1}{r_1}\right) = f\left(\frac{q}{r_1\sqrt{2gH}}\right) \tag{5}$$

The result of Equation 5 is shown in Figure 1.

FIGURE 1.
APPENDIX D.



RADIUS FACTOR FOR SUBMERGED BUCKETS.

1604 FORTRAN CODING FORM

PROGRAM APPENDIX E FLOW PARAMETER FOR SUBMERG-
ROUTINE ED BUCKET

NAME C. M. Wu
PAGE 1/2
DATE 16 Aug 163

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73–80
C			FLOW PARAMETERS FOR SUBMERGED BUCKET	
	1		FORMAT (7 F8.4)	
	2		READ INPUT TAPE 5, 1, X, Y, CK, H, SD, C, CD	
C			X = RADIUS FACTOR FROM APPENDIX D	
C			Y = FROUDE NUMBER	
C			CK = BAFFLE FACTOR	
C			H = ABRUPT FACTOR	
C			SD = SEQUENT DEPTH FACTOR	
C			C = SPACING FACTOR FROM FIG 3	
C			CD = DRAG COEFFICIENT FROM FIG3	
C			B = BACKED-UP DEPTH FACTOR	
C			ER = RELATIVE ENERGY LOSS	
C			A = EFFICIENCY	
	3	0	B = SQRTF(SD*SD + CK*C*CD*H*Y*Y - Y*Y*X - 2.0	
		1	*Y*Y*(1.0 - (1.0/SD)))	
			E1 = B + 0.5*Y*Y + Y*Y*X	
			E4 = SD + (0.5*Y*Y)/(SD*SD)	
			DE = E1 - E4	
			ER = DE/E1	
		0	E2 = 1.0 - ((8.0*Y*Y + 1.0)**1.5 - 4.0*Y*Y + 1.0)/(8.0*Y*Y*	
		1	(2.0 + Y*Y))	
			A = ER/E2	
			WRITE OUTPUT TAPE 6, 4	

1604 FORTRAN CODING FORM

PROGRAM Appendix E

ROUTINE

NAME C. M. Wu

PAGE 2/2

DATE 16 Aug '63

TYPE 1	STATEMENT NUMBER 2–5	CONT. 6	FORTRAN STATEMENT 7–72	SERIAL NUMBER 73–80
	4	0	FORMAT (108 H1 RAD-FACTOR FROUDE NO BAFFLE-FAC	
		1	ABRUPT-FAC SEQUENT-FAC BACKED-UP-F RELATIVE LOSS	
		2	EFFICIENCY)	
			WRITE OUTPUT TAPE 6, 5, X, Y, CX, H, SD, B, ER, A	
	5	0	FORMAT (4X, F8.4, 4X, F8.4, 4X, F8.4, 5X, F8.4, 5X, F8.4,	
		1	6X, E11.4, 3X, E12.4, 4X, E10.4)	
			GO TO 2	
			END	
C		0	NOTE: TAPES 5 AND 6 ARE SYSTEM TAPES IN FORTRAN.	
C		1	TAPE 5 USED FOR INPUT, TAPE 6 FOR WRITEOUT	

OPTIONAL FORM NO. 10
5010-104

UNITED STATES GOVERNMENT

Memorandum

Memorandum

TO : Chief, Hydraulics Branch

Denver, Colorado

DATE: September 27, 1963

FROM : C. M. Wu

SUBJECT: Model study of vertical stilling wells

Flow characteristics of vertical stilling wells have been restudied and the original draft, which will be Appendix 2 in my final report to the U.N. and was submitted to you on August 12, 1963, is rewritten.

A theoretical analysis and experimental data show the problem can be classified into two categories: (1) well geometry and (2) fillet effect on wave action. Some preliminary conclusions and recommendations are mentioned in the draft.

Chian M. Wu

Appendix 2

MODEL STUDY OF VERTICAL STILLING WELL

Introduction

A vertical stilling well was considered to be the most economical structure to dissipate kinetic energy of the high velocity flow, yet, there was little information regarding the dimensions required for satisfactory operation. Recently, an equation for determining the basin dimensions was developed by using model tests data of Wanship[1/] and Soap Lake Siphon stilling well.[2/] However, the resulting equation was unbalanced in dimensions. Therefore, the original data were reanalyzed in this study.

Parameter Analysis

A definition sketch is shown in Figure 1, Appendix 2. The variables involved in the problem can be classified into two categories: well geometry and shape factors.

As well geometry is concerned, previous investigations[1/, 2/] had shown that well performance was sensitive to a number of variables. Further studies show that the effectiveness of fillets and their exact location, size, etc., is a matter of shape factor and is investigated separately in this analysis.

For the overall well geometry and flow phenomenon in question, with effects of boundary friction and surface tension considered negligible,

$$d = f(A_p, A_w, Q, H, V_p, V_w, Z, \rho, \gamma, \text{and shape}) \qquad (1)$$

where

d = depth in well;

A_p = area of pipe;

A_w = area of well;

Q = discharge;

H = total head at pipe outlet = $V_p^2/2g + p$;

1/ "Hydraulic Model Studies of the Stilling Well for the Blow-off Structure, Soap Lake Siphon, Columbia Basin Project," by D. Colgate, USBR Hydraulic Laboratory Report No. Hyd-277

2/ "Hydraulic Model Studies of the Wanship Dam Vertical Stilling Wells," USBR Hydraulic Laboratory Report No. Hyd-481, 1962

V_p = velocity in the pipe;

V_w = velocity in the well;

Z = height of pipe outlet;

ρ = density of water;

p = static head of pipe;

γ = unit weight of water

in which several elements are not important and are neglected in the analysis.

One of the several combinations of dimensionless parameters which satisfy the above equation in determination of well dimensions is

$$\frac{d}{H} = f\left(\frac{A_p}{A_w}, \frac{V_p^2}{2gH}, \frac{Z}{H}, \text{ shape factor} \right). \qquad (2)$$

The parameter $V^2/2gH$ can be equated to the function of the ratio of static head to velocity head, but in design computation it is moved directly usable as it stands.

All the parameters mentioned above can be checked by applying force plus momentum equation in free body diagram as shown on Figure 2, Appendix 2.

Under certain conditions, if the slope of the water surface is near horizontal and the effects of vertical well boundary friction and surface tension can be neglected, then:

$$\gamma_p A_p + k_1 \gamma(d-z)(A_w - A_p) + \gamma z A_w - k_2 \gamma d A_w$$

$$+ \gamma \frac{Q}{g} (v_p + v_w) = 0 \qquad (3)$$

where k_1 and k_2 are correction factors of static head due to the shape factor for the pipe outlet and well floor, respectively. This can be simplified into

$$p + \left\{ k_1\left(\frac{A_w}{A_p} - 1\right) - k_2 \frac{A_w}{A_p} \right\} d + \left\{ \frac{A_w}{A_p} - k_1\left(\frac{A_w}{A_p} - 1\right)\right\} z$$

$$+ \frac{V_p^2}{g}\left(1 + \frac{A_p}{A_w - A_p}\right) = 0 \qquad (4)$$

or

$$f_1\left(\frac{A_w}{A_p}, k\right) d = \frac{V_p^2}{g}\left(1 + \frac{A_p}{A_w - A_p}\right) + p + f_2\left(\frac{A_w}{A_p}, k\right) z \qquad (5)$$

Dividing both sides by total head, H, to make the equation dimensionless, then

$$f_1\left(\frac{A_w}{A_p}, k\right)\frac{d}{H} = \frac{V_p^2}{2gH}\left(1 + 2\frac{A_p}{A_w - A_p}\right) + 1 + f_2\left(\frac{A_w}{A_p}, k\right)\frac{Z}{H}$$

or

$$\frac{d}{H} = f\left(\frac{A_p}{A_w}, \frac{V_p^2}{2gH}, \frac{Z}{H}, k\right) \qquad (6)$$

where k represents the shape factor.

Investigation

Previously constructed models and test data are used in this study. The model, a box 4 by 4 by 6 foot deep with 4-1/2-inch pipe was tested with discharges between 1.5 and 7.13 cfs, velocities of 9.0 ~ 64.5 ft/sec, and total heads of 7.5 ~ 71.0 ft.

Using the parameters previously analyzed, the test results are presented in Figure 3, Appendix 2, which is a plot of $V^2/2gH$ versus d/H and Z/H.

The equations

$$\frac{Z}{H} = 0.25\left(1 - \frac{V^2}{2gH}\right)$$

$$\frac{d}{H} = 1.25\left(1 - \frac{V^2}{2gH}\right)$$

$$\frac{A_w}{A_p} = \frac{16}{\left(\frac{9}{24}\right)^2 \frac{\pi}{4}}$$

are found to be good fits for the experimental data. This curve can be used in sizing other installations of

$$\frac{A_w}{A_p} = \frac{16}{\left(\frac{9}{24}\right)^2 \frac{\pi}{4}} .$$

Further Investigation

The present study has investigated the relationship between flow conditions and one pipe-well area ratio, i.e.,

$$\frac{A_w}{A_p} = \frac{16}{\left(\frac{9}{24}\right)^2 \frac{\pi}{4}}$$

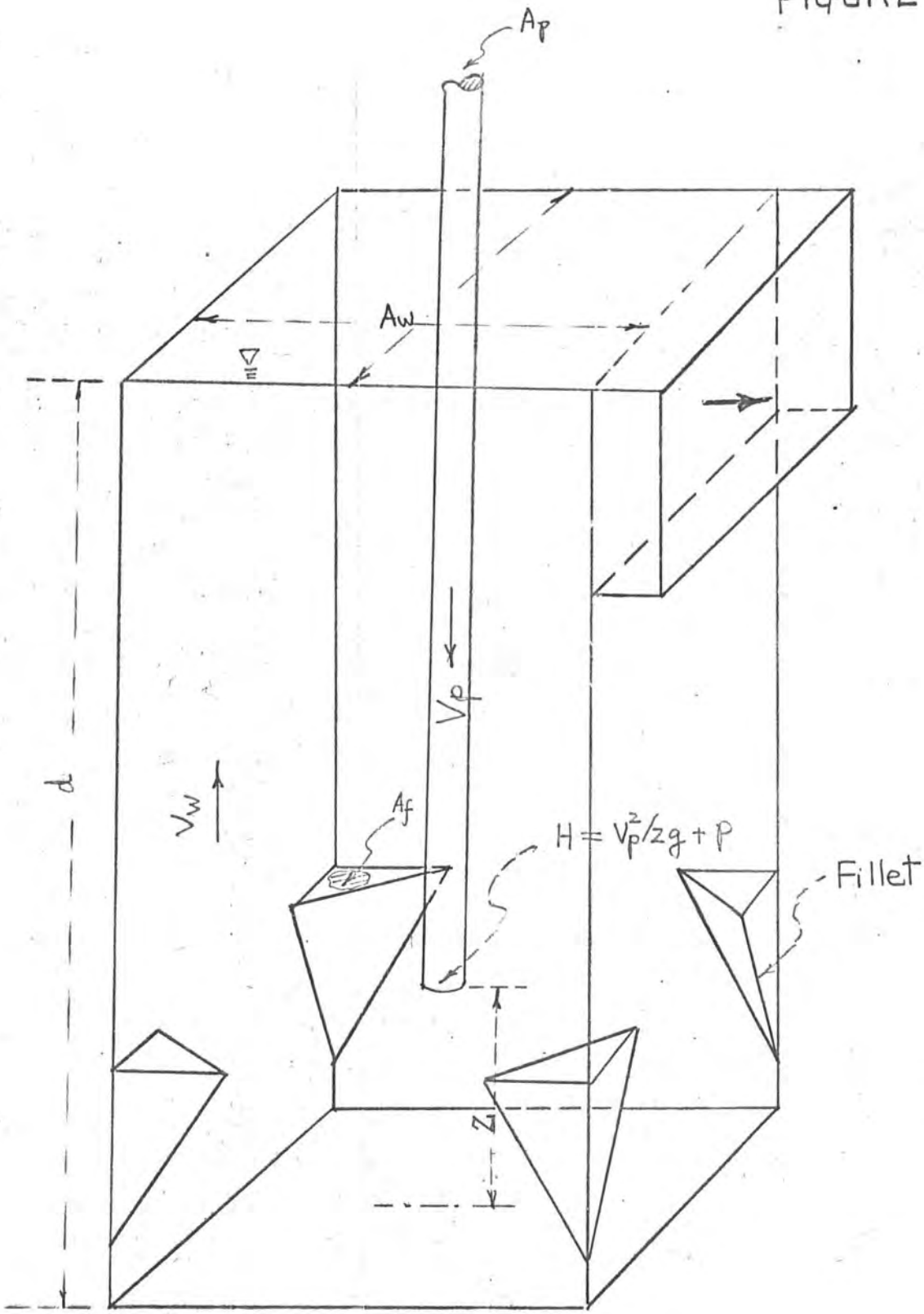
only. Perhaps a family of design charts for different pipe-well area ratios is necessary for economical design of the vertical stilling well.

From what has been investigated already, it is believed that as the well shape, i.e., fillet configuration, is concerned, no significant effects on energy dissipation and flow conditions are mentioned. By

placing fillets in favorable locations, rollers which contain the turbulence in the lower part of the well and reduce the wave action, are formed. Further studies may simplify design criteria by dividing the study into two categories, well geometry and flow condition, fillet parameters and wave height. Perhaps a correlation of the fillet parameters against significant wave height[3/] will result in the best vertical stilling well energy dissipator.

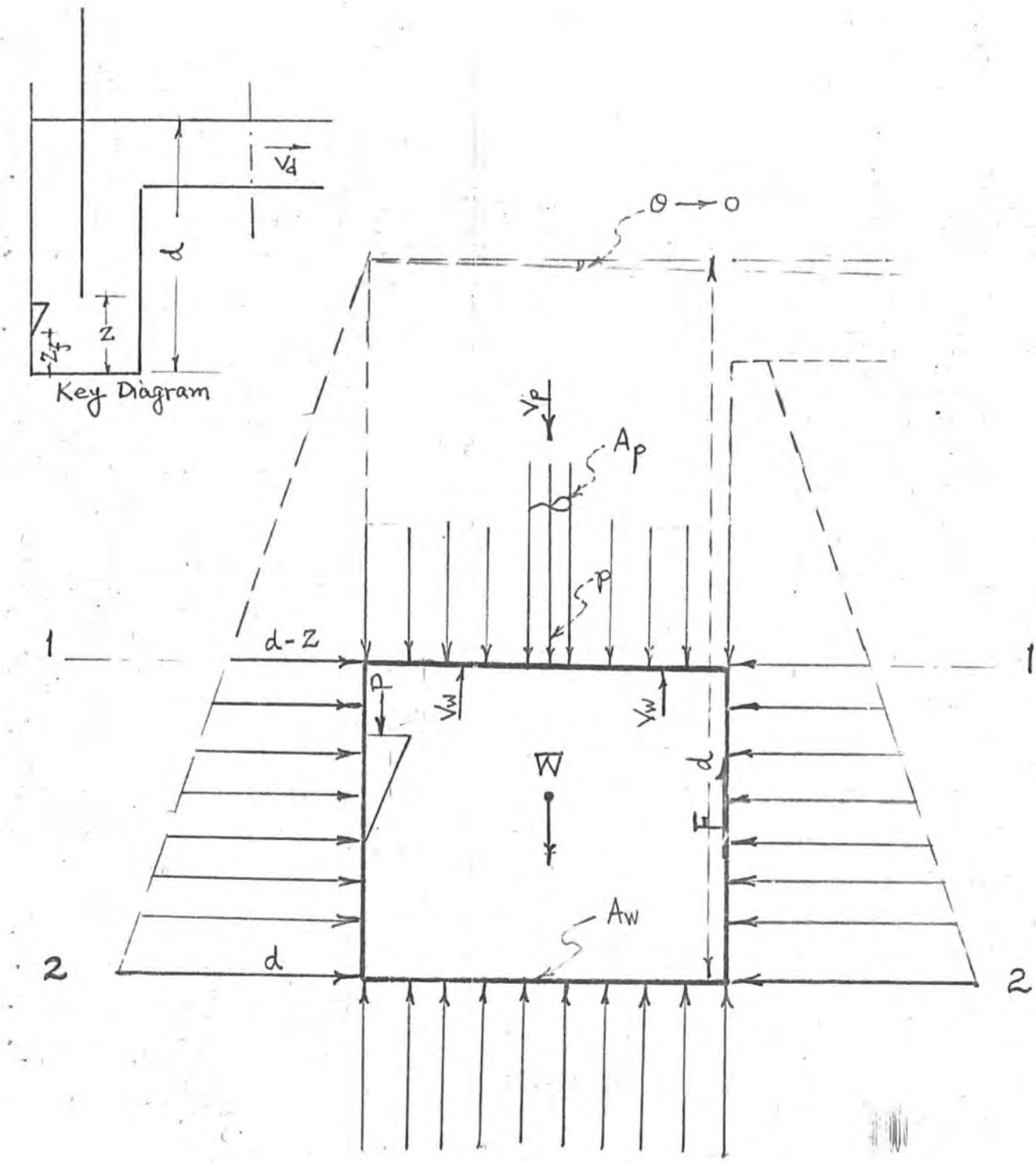
[3/] "Wave Action Below Spillways," by Mohamed Mokhles Abou-Seida, Proc. ASCE Vol. 89, No. Hy 3, May 1963, Part 1, pp 133-152

FIGURE 1



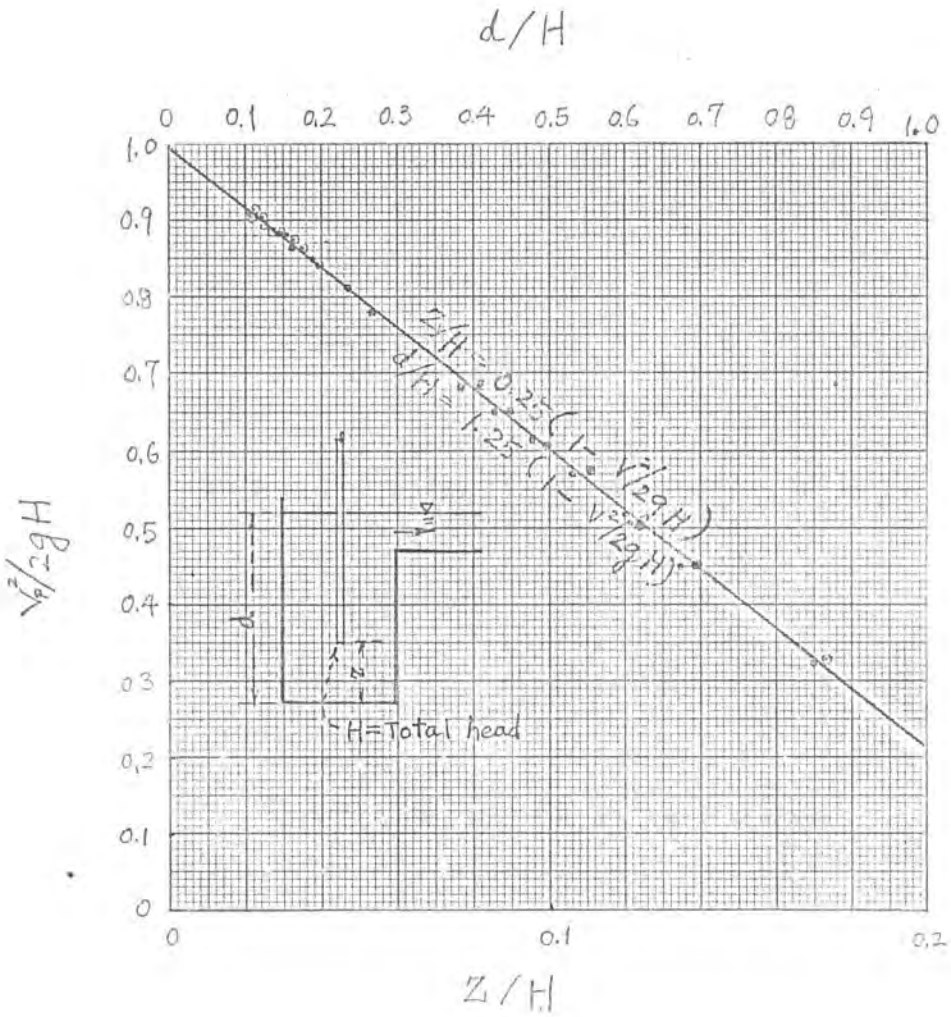
DEFINITION SKETCH OF VERTICAL STILLING WELL BASIN

FIGURE 2



FREE BODY DIAGRAM
VERTICAL STILLING WELL BASIN

FIGURE 3
APPENDIX -2



- $V_p^2/2gH$ vs. Z/H
- $V_p^2/2gH$ vs. d/H

$A_w/A_p = 16/\frac{\pi}{4}\left(\frac{9}{24}\right)^2$

Fillet Angle = 60°

Fillet Size = b/3

Fillet height = 0.05

VERTICAL STILLING WELLS DESIGN CHART

APPENDIX — 3

Appendix 3

THEORETICAL APPROACH OF MOVABLE-BED SIMILARITY

Introduction

Recently, the similarity conditions for distorted river models with movable bed were derived by H. A. Einstein, M. ASCE, and Ning Chien, A.M. ASCE.[1/] In this paper same approach is adopted and further study is made.

Similarity Conditions

Einstein[1/] expressed the relationship of velocity and friction as

$$\frac{V^2}{R_T S_e g} = C^2 \left(\frac{R_T}{K_s} \right)^{2m} \qquad (1)$$

where V is the horizontal flow velocity;

R_T is the hydraulic radius of the total; section with the bottom width as wetted perimenter;

K_s is the grain size of the bed representative for its grain roughness;

g is the gravitational acceleration;

S_e is the slope,

C is the constant in the generalized Manning's equation; and

m the exponent in the generalized Manning's equation,

with the additional of Froude Law

$$\frac{V^2}{2g} + h = \text{constant} \qquad (2)$$

where h is the water depth.

1/ *"Similarity of Distorted River Models with Movable Bed," by Einstein and N. Chien, Proc. ASCE Vol. 80, Separate No. 566, December 1954

1/

His bedload function is

$$\varphi_* = \frac{i_B}{i_b} \frac{q_B}{(\rho_s - \rho_f)g} \left(\frac{\rho_f}{\rho_s - \rho_f} \right)^{1/2} \left(\frac{1}{g\, D^3} \right)^{1/2} \tag{3}$$

and

$$\psi_* = \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{R_b' \, S_e} \xi \, Y \frac{1}{\theta} \left(\frac{\log 10.6}{\log 10.6 \, X/\Delta} \right) \tag{4}$$

where φ_* is the intensity of transport for individual grain size;

ψ_* is the intensity of shear for individual grain size;

i_b is the fraction of bed material in a given grain size range;

i_B is the fraction of bedload in a given grain size range;

D is the grain size;

ρ_f is the density of the fluid;

ρ_s is the density of the solids;

R_b' is the hydraulic radius with respect to the grain;

q_B is the bedload rate in weight under water per unit of time and width;

Y is the pressure correction in transition smooth-rough;

ξ is the hiding factor of grains in a mixture;

X is the characteristics grain size of a mixture; and

Δ is the apparent roughness diameter.

He used the vertical distribution of suspended load equation

$$\frac{C_y}{C_a} = K \left(\frac{h - y}{h} \quad \frac{a}{h - a} \right)^z$$

where C_y and C_a are the concentrations of a grain size with setting velocity w at Points y and a from the bed, respectively, and

K and Z are the constant and exponent for the distribution equation.

The control factor in the distribution equation is

$$Z = \frac{w}{k\sqrt{g R_T S_e}} \tag{5}$$

where w is the settling velocity of a sediment particle;

k is the von Karman universal constant of turbulent exchange,

and the continuity equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{6}$$

where A denotes to the area of a cross section;

t denotes to time;

x denotes to distance; and

Q the discharge,

and continuity equation for sediment discharge

$$\frac{\partial \left(q_T \frac{\rho_f}{\rho_s - \rho_f} \right)}{\partial x} + \gamma_s \frac{\partial Z}{\partial t} = 0 \tag{7}$$

where ∂t denotes to the total-load rate;

γ_s denotes to the unit weight of sediment; and

Z denotes to the depth change of river bed.

Einstein arrived at the solutions of the above similarity equations by choosing one of the following ratios: (1) horizontal ratio, (2) vertical ratio, and (3) density ratio. A simultaneous solution of a combination of two of the above ratios were obtained and the result is shown in Table 1, Appendix 3.

Table 1--Appendix 3

Model Ratios for Open Channel Flows with Sediment Motion

Scale	Ratio	Symbol	Choose hr and Lr							Choose Lr and $(\rho_s-\rho_f)_r$						
			Lr	hr	Cr	B	ΔV	ΔF	ΔS	Lr	$(\rho_s-\rho_f)_r$	hr	B	ΔV	ΔF	ΔS
Horizontal	Lengths	Lr	Chosen							Chosen						
Vertical	Lengths	hr	Chosen							1	$\frac{2m}{2m-1}$	$\frac{2}{2m-1}$	-	$\frac{1}{2m-1}$	$\frac{-2}{2m-1}$	-1
Velocity		Vr	-	1/2	-	-	-	1	-	1/2	$\frac{m}{2m-1}$	$\frac{1}{2m-1}$	-	$\frac{1}{2(2m-1)}$	$\frac{2(m-1)}{2m-1}$	-1/2
Slope		Sr	-1	1	-	-	-	-	1	-	$\frac{m}{2m-1}$	$\frac{2}{2m-1}$	-	$\frac{1}{2m-1}$	$\frac{-2}{2m-1}$	-
Sediment	Size	Dr	$\frac{-1}{2m}$	$\frac{2m+1}{2m}$	$\frac{1}{m}$	-	$\frac{1}{2m}$	$\frac{-1}{m}$	$\frac{1}{2m}$	1	$\frac{2m+1}{2m-1}$	$\frac{4}{2m-1}$	-	$\frac{2}{2m-1}$	$\frac{-4}{2m-1}$	-1
Sediment density under water		$(\rho_s-\rho_f)_r$	$\frac{1-2m}{2m}$	$\frac{2m-1}{2m}$	$\frac{-1}{m}$	-	$\frac{-1}{2m}$	$\frac{1}{m}$	$\frac{2m-1}{2m}$	Chosen						
Bedload ratio per unit width and time by weight under water		q_{Br}	$\frac{-3}{2}$	3	-	-	-	-	3/2	3/2	$\frac{6m}{2m-1}$	$\frac{6}{2m-1}$	-	$\frac{3}{2m-1}$	$\frac{-6}{2m-1}$	-3/2
Total load rate per unit width and time by weight under water		q_{Tr}	$\frac{-3}{2}$	3	-	1	-	-	3/2	3/2	$\frac{6m}{2m-1}$	$\frac{6}{2m-1}$	1	$\frac{3}{2m-1}$	$\frac{-6}{2m-1}$	-3/2
Hydraulic	Time	tr_1	1	$\frac{-1}{2}$	-	-	-	-1	-	1/2	$\frac{-m}{2m+1}$	$\frac{-1}{2m-1}$	-	$\frac{1}{2(2m-1)}$	$\frac{-2(m-1)}{2m-1}$	1/2
Sedimentation	Time	tr_2	$\frac{3m+1}{2m}$	$\frac{-(2m+1)}{2m}$	$\frac{-1}{}$	-1	$\frac{-1}{2m}$	$\frac{1}{m}$	$\frac{-(m+1)}{2m}$	1/2	$\frac{-(2m+1)}{2m-1}$	$\frac{-4}{2m-1}$	-1	$\frac{-2}{2m-1}$	$\frac{4}{2m-1}$	1/2

HYDRAULICS OF VERTICAL STILLING WELL

Purpose

To investigate the flow characteristics and energy dissipation capacities of the vertical stilling well to provide the design performance of the basin as a high velocity energy dissipator.

Conclusions

1. Theoretical analysis and study of experimental data show that the momentum theorum can be applied in the vertical stilling well analysis.

2. A theoretical equation is developed to relate the well depth to known flow parameters. Also a theoretical equation is derived for the energy loss for the basin and experimental coefficients are given. For design purposes a simple equation is arrived at in this report.

Introduction

A vertical stilling well (Figure 1) was considered to be the most effective structure to dissipate kinetic energy of the high velocity flow. Many structures, such as Bänninger,[1] Kreuter,[2] Poebing,[3] and Forchheimer[4] dissipators were developed, yet there was little information regarding the dimensions required for satisfactory operation of the basin. In 1949 to 1962, with the aid of hydraulic models, an equation for determining the basin dimensions was

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[1] Bänninger, K., Studien und Versuche über Umsetzung von Geschwindig Keit in Druck, usw, Zeits fur das gesamte Turbinenwesen, Vol. 3, 1906, p 12

[2] Kreuter, F., Hydraulische Bremse, Wasserkraft und Wasserwirtschaft, Vol. 15, 1920, p 65

[3] Poebing, O., Ein neuer Energievernichter, Wasserkraft und Wasserwirtschaft, Vol. 19, 1924, 0.373

[4] Forchheimer, P., Hydraulik, Teubner, Leipzig und Berlin, Ed. 3, 1930, p 319

developed by the Bureau of Reclamation.[5/6/7/8/9/] However, the resulting equation was unbalanced in dimensions and the energy dissipating capacity is not quite understood.

In order to gain a more rational understanding of this phenomenon, a combination of theory and experiments are used in this study.

Hydraulics of Stilling Well

Theoretical Well Depth

According to Newton's second law of motion, the change of momentum per unit of time in the body of water in a stilling well with fillets is equal to the resultant of all the external forces acting on the body. Applying this principle to the free body shown in Figure 2, the following expression for the force plus momentum enclosed between Section 1 and 2, Figure 2, may be written:

$$\gamma p A_p + k_1 \gamma (d - z)(A_w - A_p) + C_d \gamma \frac{V_w^2}{2g} A_f + W$$

$$- \gamma k_2 d A_w + \beta_1 \frac{Q}{g} \gamma v_p + \beta_2 \frac{Q}{g} \gamma v_w - F_f = 0 \qquad (1)$$

where p is the pressure head in the downspout; A and v are area and velocity with subscripts referring to pipe, well and fillet

---

5/ Hydraulic Model Study of the Masonville Siphon Turnout Structure--Horsetooth Feeder Canal, Hydraulic Laboratory Report No. Hyd-237, USBR, February 1949

6/ Hydraulic Model Study of a Proposed Outlet Works Design for Anchor Dam, Hydraulic Laboratory Report No. Hyd-244, USBR, January 1949

7/ Hydraulic Model Studies of the Stilling Well for the Blow-off Structure, Soap Lake Siphon, Columbia Basin Project, Hydraulic Laboratory Report No. Hyd-277, USBR, 1950

8/ Hydraulic Model Studies of the Regulating Gate and Stilling Well--Trenton Dam Canal Outlet Works--Missouri River Basin Project, Hydraulic Laboratory Report No. Hyd-300, USBR, 1951

9/ Hydraulic Model Studies of the Wanship Dam Vertical Stilling Wells, Weber Basin Project, Utah, Hydraulic Laboratory Report No. Hyd-481, USBR, 1962

sections; W is the weight of water enclosed between the pipe outlet and bottom of well floor; γ is the unit weight of water, C_d is the drag coefficient for the fillets; d is the depth of the well; z is the height of the downspout outlet; F_f is the total external force of friction and resistance acting along the well; and k_1 and k_2 are pressure correction factors due to turbulence is Sections 1 and 2, respectively.

In short reach, the external force of friction along the well surface can be ignored. Thus, with $F_f = 0$ and assuming $\beta_1 = \beta_2 = 1$, Equation 1 becomes

$$\left\{k_2\left(\frac{A_w}{A_p}\right) - k_1\left(\frac{A_w}{A_p} - 1\right)\right\} d$$

$$= p + \left\{\frac{A_w}{A_p} - k_1\left(\frac{A_w}{A_p} - 1\right)\right\} Z + C_d \frac{v_w^2}{2g} \frac{A_f}{A_p} + \frac{v_p^2}{g}\left(1 + \frac{A_p}{A_w - A_p}\right)$$

$$= p + \left\{\frac{A_w}{A_p} - k_1\left(\frac{A_w}{A_p} - 1\right)\right\} Z + C_d \frac{v_p^2}{2g} \frac{A_p \cdot A_f}{A_w \cdot A_w}$$

$$+ \frac{v_p^2}{g}\left(1 + \frac{A_p}{A_w - A_p}\right) \qquad (2)$$

or simply can be shown as

$$d = f\left(\frac{A_p}{A_w}, \frac{A_f}{A_w}, p, Z, C_d, k, \frac{v_p^2}{2g}\right) \qquad (3)$$

where k represents shape factor.

In order to make the resulting equation dimensionless, dividing both sides of Equation 2 by total head, H, then becomes

$$\left\{ k_2\left(\frac{A_w}{A_p} \right) - k_1\left(\frac{A_w}{A_p} - 1 \right) \right\} \frac{d}{H}$$

$$= \frac{p}{H} + \left\{ \frac{A_w}{A_p} - k_1\left(\frac{A_w}{A_p} - 1 \right) \right\} \frac{Z}{H} + \frac{C_d}{2gH} \; v_p^2 \; \frac{A_p \cdot A_f}{A_w \cdot A_w}$$

$$+ \frac{v_p^2}{gH}\left(1 + \frac{A_p}{A_w - A_p} \right)$$

$$= 1 + \left\{ \frac{A_w}{A_p} - k_1\left(\frac{A_w}{A_p} - 1 \right) \right\} \frac{Z}{H} + \frac{C_d}{2gH} \; v_p^2 \; \frac{A_p \cdot A_f}{A_w \cdot A_w}$$

$$+ \frac{v_p^2}{2gH}\left(1 + \frac{2A_p}{A_w - A_p} \right) \tag{4}$$

$$\frac{d}{H} = f\left(\frac{A_p}{A_w}, \; \frac{A_f}{A_w}, \; \frac{v_p^2}{2gH}, \; \frac{p}{H}, \; \frac{Z}{H}, \; C_d, \; k \right) \tag{5}$$

Thus the required depth of the vertical stilling well can be related to the total head of the downspout and can be expressed as a function of dimensionless parameters of area ratio (i.e., A_p/A_w, A_f/A_w), velocity head ratio of the downspout (i.e., $v_p^2/2gH$), downspout height ratio (Z/H) and shape factors (C_d and k).

Efficiency of Energy Dissipation

According to the principle of conservation of energy, the total energy at the downspout outlet should be equal to the total energy head at the channel section (Figure 2) plus the loss of energy, h_f, between these two sections, then with respect to the well floor

$$\alpha_p \; \frac{v_p^2}{2g} + p + Z = d + \alpha_d \; \frac{v_d^2}{2g} + h_f \tag{6}$$

where v_d is the velocity of flow in the channel section, α is the energy coefficient.

The loss of energy between well and channel sections can be evaluated by combining Equations 2 and 6, then

$$h_f = \left[k_2\left(\frac{A_w}{A_f}\right) - k_1\left(\frac{A_w}{A_p} - 1\right) - 1\right] d$$

$$+ \left[1 - \frac{A_w}{A_p} + k_1\left(\frac{A_w}{A_p} - 1\right)\right] Z$$

$$- \frac{v_p^2}{2g}\left\{1 + \frac{A_p^2}{A_d^2} + \frac{2\,A_p}{(A_w - A_p)}\right\} - C_d\ \frac{v_p^2}{2g}\ \frac{A_p \cdot A_f}{A_w \cdot A_w} \qquad (7)$$

where A_d is the area of channel section.

The efficiency of the vertical stilling well can be expressed as the ratio of the loss of energy between the well and channel intake sections to the total energy at the downspout outlet. Then

$$n = \frac{h_f}{H} = \frac{1}{p + v_p^2/2g}\ \left\{\left[k_2\left(\frac{A_w}{A_p}\right) - k_1\left(\frac{A_w}{A_p} - 1\right) - 1\right] d\right.$$

$$+ \left[1 - \frac{A_w}{A_p} + k_1\left(\frac{A_w}{A_p} - 1\right)\right] Z$$

$$\left. - \frac{v_p^2}{2g}\left[1 + \frac{A_p^2}{A_d^2} + \frac{2\,A_p}{(A_w - A_p)}\right] - C_d\ \frac{v_p^2}{2g}\ \frac{A_p \cdot A_f}{A_w \cdot A_w}\right\} \qquad (8)$$

where n is the efficiency of the well.

In general, for well without fillets, $A_d \gg A_p$, $A_w \gg A_p$, $k_1 \doteqdot k_2 \doteqdot 1$, then

$$\lim n = \frac{v_p^2}{\left(p + \frac{v_p^2}{2g}\right)}$$

$$\frac{A_p}{A_d} \to 0 \qquad (9)$$

$$\frac{A_p}{A_w} \to 0$$

and when $v_p^2/2g \gg p$, limit $n = 100$ percent. Thus it is shown that a vertical stilling well is one of the most effective structures to dissipate kinetic energy of the high velocity flow.

Experiments

Model

A well 6 feet in depth by 4 feet square was built to provide a testing facility. The downstream side of the well emptied into a trapozoidal channel with movable sloping sides and an adjustable tailgate. The tailgate control provided a total basin depth up to 7-3/4 feet and channel flow depth of up to 1-3/4 feet.

Adjustable corner fillet frames and supports were attached to modify the square well configuration, so as to determine the optimum fillet position and dimensions.

An adjustable downspout of 4-1/2-inch-inside-diameter pipe was installed in the center of the well and the pipe was inserted into a larger 6-inch standard pipe, thus providing adjustment from less than a diameter to approximately 20 diameters above the well floor.

The flow into the system was supplied by 8-inch pipe and controlled through the Venturi meter system with maximum capacity of up to 7-1/2 cubic feet per second.

Thirty piezometer holes were located in a gridlike pattern in the downstream quadrant of the basin floor. With taps connected to a board of differential mercury manometers, the pressure distribution on the basin well can be obtained.

A ring of four piezometers was placed near the end of the downspout to obtain the pressure head in the incoming flow. Three piezometers were located on the channel floor to provide the pressure in the outgoing flow. A capacitance wave probe with half bridge recorder was employed in measuring and recording of surface roughness.

A 1:9.14 scale model (Figure 3) of the larger basin was constructed of clear plastic to provide a means of viewing flow conditions in a geometrically similar well.

Test Procedures

Two separate procedures were used in the previous study. The first procedure consisted of a series of tests without corner fillets for various combination of well depths and heights of downspout. The second procedure consisted of a series of similar tests, but with the corner fillets. Fillets of 45, 50, 55, 60, and 65° (Figure 4) were constructed and tested. To evaluate the optimum location of the fillets, several fillet locations were tested with the other conditions remaining the same as the first procedure so as to get a better comparison.

In both procedures, the flow was adjusted to give a smooth water surface in the well. Pressure readings were taken to obtain the discharge, static head in the downspout, and pressure profile on the well floor.

The tests were repeated in the ranges of discharges from 1 to 7.38 cfs, total heads to 7 to 76 feet of water, and pipe velocities of 9 to 67 feet per second. Well depths of up to 7.90 feet and channel depths of up to 1.90 feet were utilized.

Prior to these two procedures, the small clear plastic model was used to investigate visually the general flow patterns. Various similar fillets of scaled sizes and shapes were tested in the plastic model so as to obtain additional information about their functioning and validity.

Investigation

Previous investigations were based on dimensional analysis. Data were taken according to the result of analysis and were not sufficient to complete a general study. However, the main flow

parameters were evaluated by limited information available. Several curves for main flow parameters were presented. Despite appreciable experimental scatter, the analytical curves are in reasonable agreement with the experimental data over the entire range of the independent variables. In order to save time and effort, the factors, which must be observed or measured experimentally were recommended in this study.

Evaluation of Well Constants

As analyzed in Equations 4 and 6, the parameters involved in well hydraulics were momentum coefficients: β_1, β_2, velocity-distribution coefficients; α_p, α_d, drag coefficient for fillets; C_d, and pressure-distribution coefficients k_1 and k_2. Unfortunately the data for analyzing pressure, velocity and drag coefficients were not available. Pressure readings on the basin floor were available and were used in evaluating the pressure-distribution coefficients in the analysis. It was found that the pressure-distribution coefficient is critical to the well performance.

In order to evaluate the pressure-distribution coefficient on the well floor, k_2, the pressure readings on the floor were integrated by a computer program which calculated a total force on the well floor by $\Sigma p \Delta a$ and required only manometer readings as input data. Then the coefficients k_2 for each different conditions were computed by

$$k_2 = \frac{\Sigma p \Delta a}{\gamma d \, A_w} \tag{10}$$

Plot of test data shows that

$$k_2 = f\left(\frac{A_p}{A_w}, \ \frac{z}{H}, \ \text{and shape factors} \right) \tag{11}$$

The best fitted curves for different conditions are tabulated in Table 1 and a design chart for 60° fillets with size of $1/3 \cdot b$ and height of $z_f/z = 0.26$ is given in Figure 5. Where b is width of the well.

Three trends are noted: the k_2 values will increase as z/H decreases; the k_2 values increase as the fillet angles increase

from 45 to 60°; and k_2 values increase up to optimum fillet height and then decrease gradually.

In order to evaluate pressure-distribution coefficient on the downspout section, piezometer readings around the section are necessary. Due to the lack of readings the coefficient is not analyzed here, but it may be reasonably assumed to be near 1.

Energy and momentum coefficients have been analyzed in many open-channel flows, but limited information was available for vertical flow condition.[10] Velocity-distribution measurements are necessary so that the equations

$$\alpha = \frac{\Sigma v^3 \Delta A}{V^3 A} \tag{12}$$

and

$$\beta = \frac{\Sigma v^2 \Delta A}{V^2 A} \tag{13}$$

can be applied.

Where V is the mean velocity. A is the total area. v and ΔA are the elementary velocity and area, respectively.

Total Force on Well Floor

Pressure distribution is essential for structure design of vertical stilling well. The dimensionless pressure distribution for different conditions was plotted in Figure 6. It was found that the pressure profile could be correlated to total head in the downspout and well depth. The maximum pressure can be related to total head in the downspout and can be shown as a function of z/H. (Figure 7.) It occurs directly beneath the downspout. The profile gradually decreases to 0.8 d for approximately 7 pipe diameters and recovers to 1 d for 4 diameters then becomes larger than 1 d at the edge of the well. For structural design purposes pressure distribution of 1 diameter total head on the center, 1 diameter half the total head around the center of the well, and k_2d the rest of the well section is recommended.

Jet Expansion

The pressure distribution curve is a good index to the investigation of jet expansion. It was found that the jet either side of the

[10] Chow, "Open-Channel Hydraulics," 1959, p 28

centerline varied from approximately 52-1/2° with the downspout 1 downspout diameter from the floor down to approximately 10° with the downspout 7-1/4 diameters from the floor. The curve appeared to become asymptotic to 10° so that a further increase in downspout distance from the well floor will not affect the expansion angle of the jet. This information agrees with angles 12 to 14° on horizontal channels as studied by Hunter Rouse, etc.[11]

Wave Amplitude

Measurements of wave amplitude were made at a point 3 feet downstream from the well and on the channel centerline. It was found that wave amplitude can be correlated to pressure-distribution coefficient, fillet angle, and fillet height. It is shown that wave amplitude decreases as the pressure-distribution coefficient value increases, and the optimum conditions were determined as approximately a 60° fillet with a size of 14-1/2 inches ($\doteq b/3$) and height of 0.4 foot (0.052 d).

Practical Application

Evaluation of Design Equation

Determination of coefficients from the data available was not possible for a completely general study. However, for a particular well, the effect of fillets can be correlated to the pressure-distribution coefficient and the design equation can be simplified from Equation 2 to

$$\left\{\left(k_2 - 1\right)\left(\frac{A_w}{A_p}\right) + 1\right\} d = p + Z + \frac{v_p^2}{g}\left(1 + \frac{A_p}{A_w - A_p}\right) \qquad (2')$$

The pressure-distribution coefficients obtained by the experiments were used in verification of Equation 2'. Despite appreciable experimental scatter, the analytical curve is in reasonable agreement with the measurement over the entire range of the independent variables (Figure 8).

[11] Albertson, Dai, Jensen, and H. Rouse, "Diffusion of Submerged Jets," Trans. ASCE, Vol. 115, 1950, pp 260-273

Furthermore Equation 8 is simplified into

$$n = \frac{1}{p + \frac{v_p^2}{2g}} \left[\left(k_2 - 1 \right) \left(\frac{A_w}{A_p} \right) d - \frac{v_p^2}{g} \left(1 + \frac{2A_p}{A_w - A_p} + \frac{A_p^2}{A_\lambda^2} \right) \right] \qquad (8')$$

and the equation is experimentally verified (Figure 9).

Table 1

Test No.	Fillet angle	Fillet height ft	Tail-water ft	1/4 dynamic* force on Floor No.	Wave amplitude ft	k_1
1	65°	0.00	1.67	139	0.023	1.072
2		0.50	1.77	195	0.002	1.100
3		1.00	1.68	176	0.011	1.091
4		1.50	1.67	123	0.022	1.064
5	60°	0.00	1.67	128	0.008	1.066
6		0.25	1.67	138	0.003	1.072
7		0.50	1.67	126	0.0025	1.065
8		0.75	1.67	111	0.005	1.057
9		1.00	1.67	93	0.0065	1.048
10	55°	0.00	1.66	118	0.011	1.061
11		0.25	1.66	116	0.0022	1.06
12		0.50	1.66	102	0.0042	1.05
13		0.75	1.67	97	0.0055	1.05
14		1.00	1.67	97	0.0080	1.05
15	50°	0.60	1.66	130	0.0023	1.07
16		0.25	1.66	111	0.0028	1.06
17		0.50	1.66	102	0.0070	1.06
18	45°	0.00	1.66	114		1.06
19		0.25	1.66	99		1.05
20	No fillets		1.67	70		1.03

*1/4 dynamic force = (total force on 1/4 of well floor) minus (dead weight on 1/4 of well floor due to water level in well) = $1/4(\Sigma p \Delta A - dA)$

Q = 5 cfs

d = 4-1/2 inches

Fillet size = 16 inches

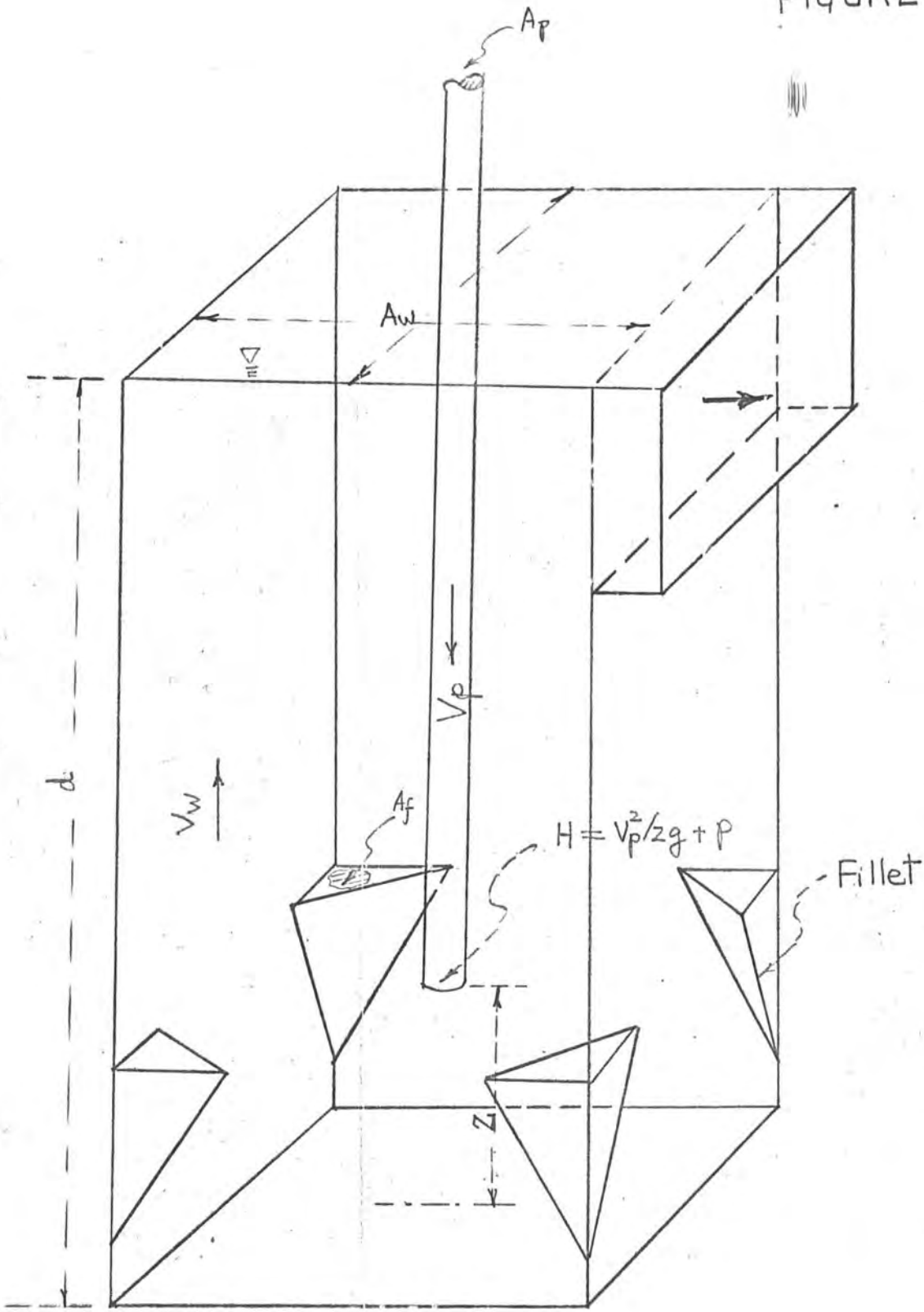
Well area = 16 ft^2

Well depth = 6 feet

Height of downspout = 4-1/2 inches

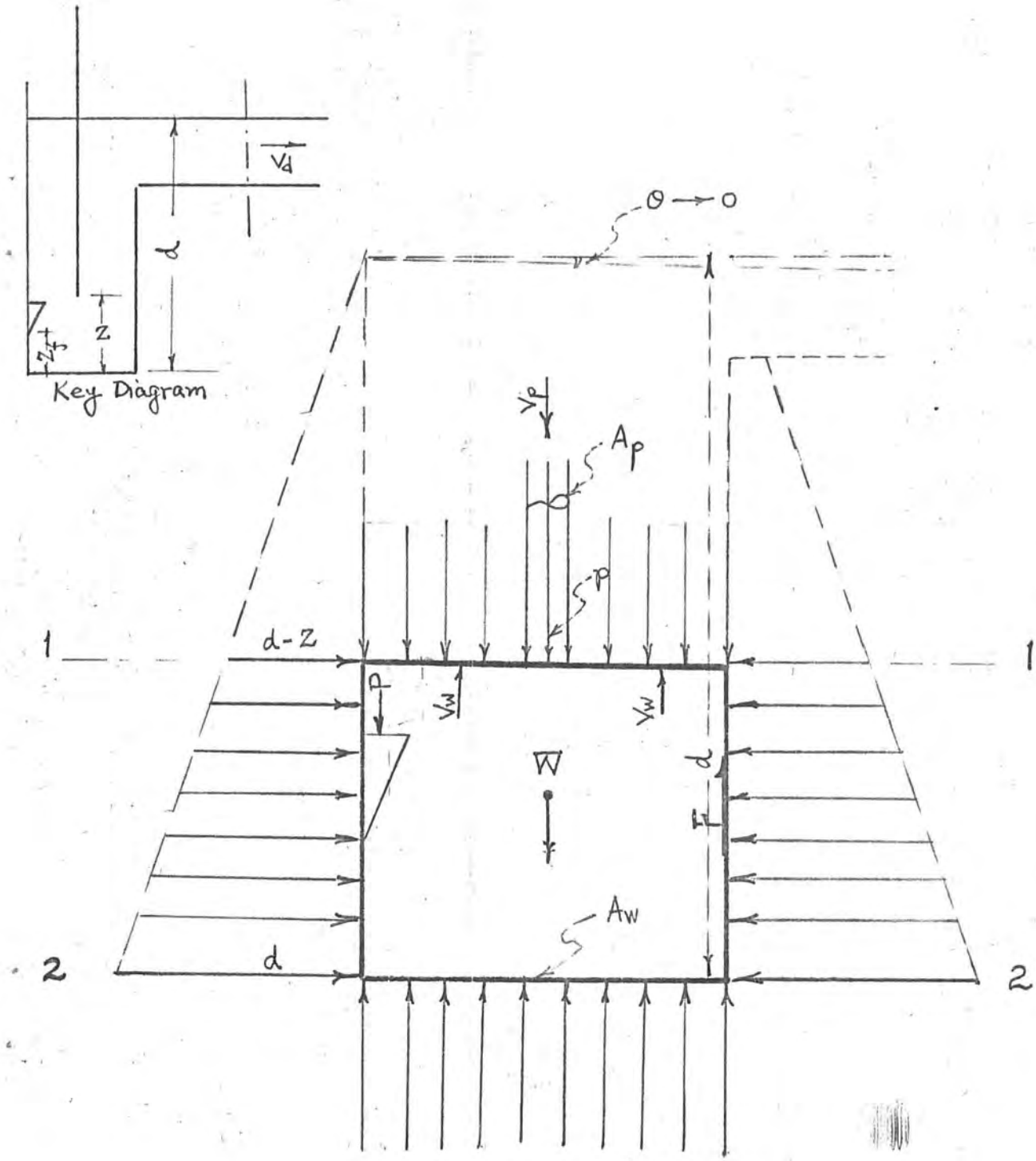
k_1 = pressure-distribution coefficient of well floor

FIGURE 1



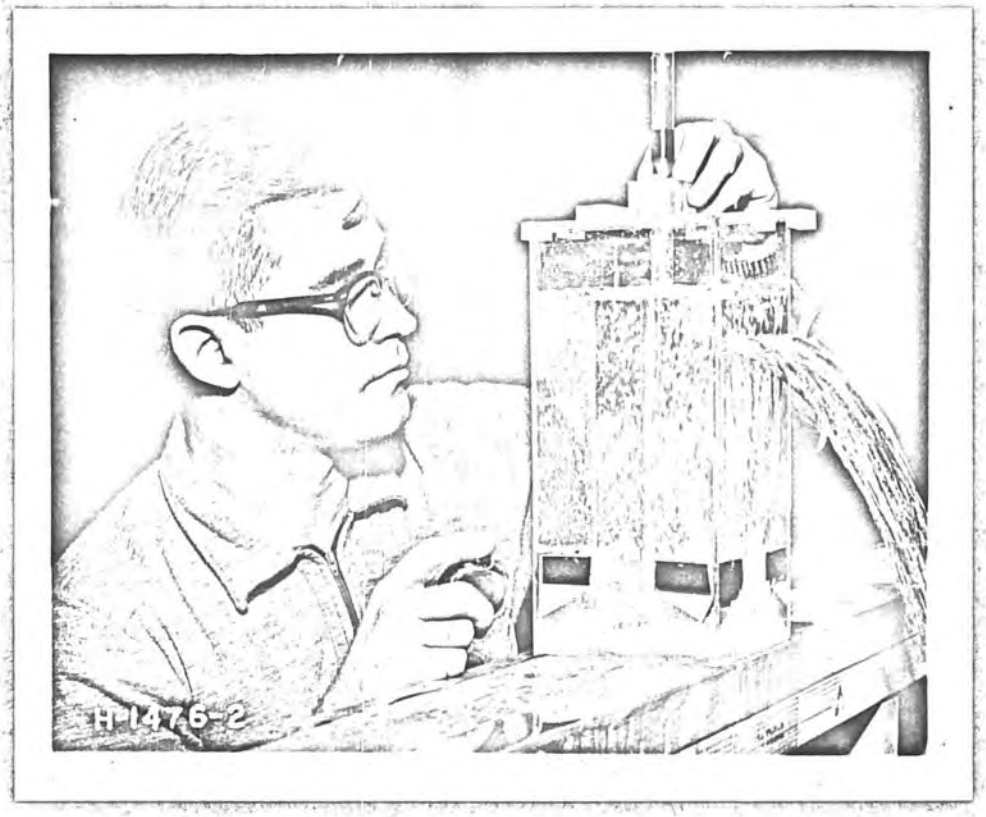
DEFINITION SKETCH OF VERTICAL STILLING WELL BASIN

FIGURE 2



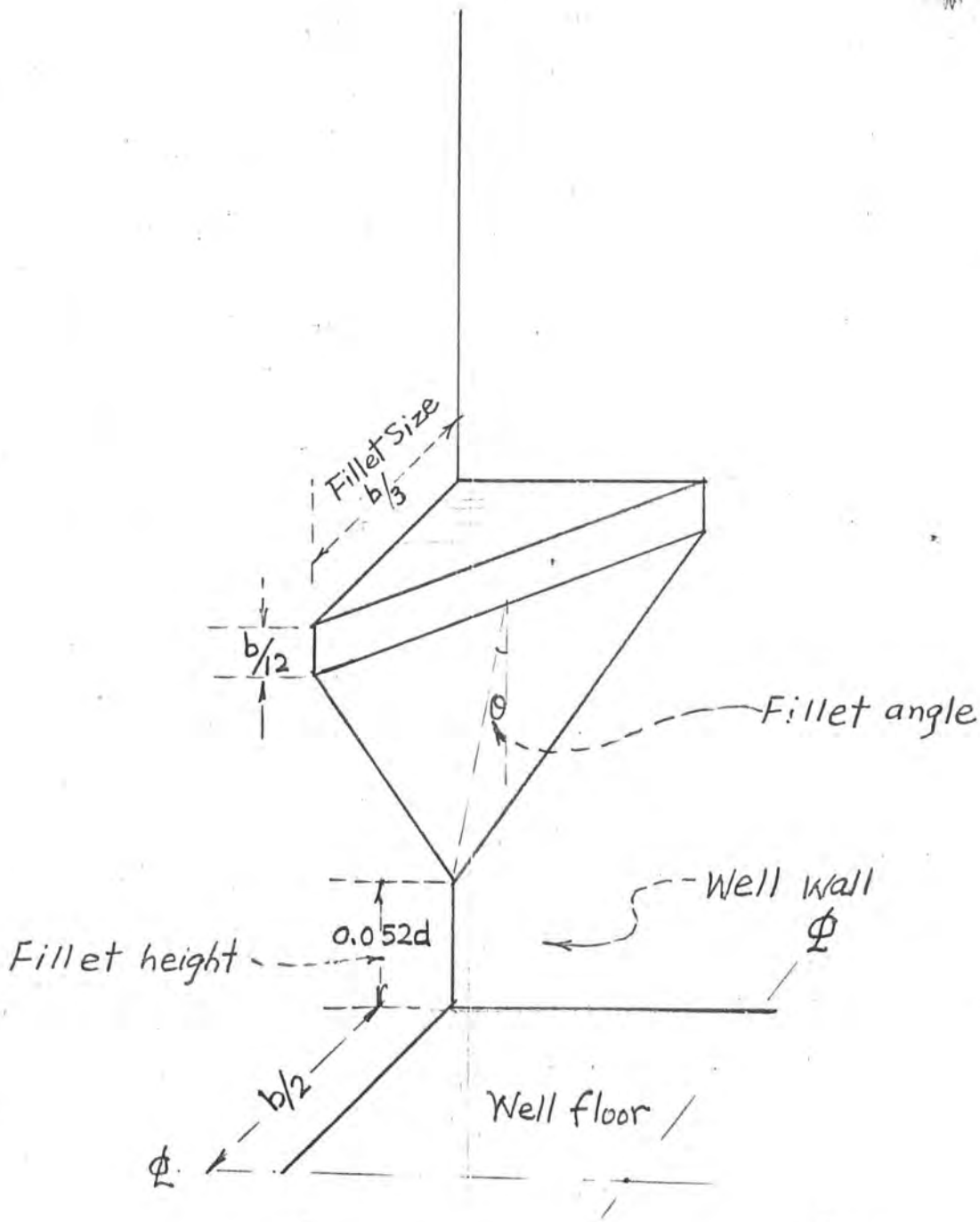
FREE BODY DIAGRAM
VERTICAL STILLING WELL BASIN

FIGURE 3.

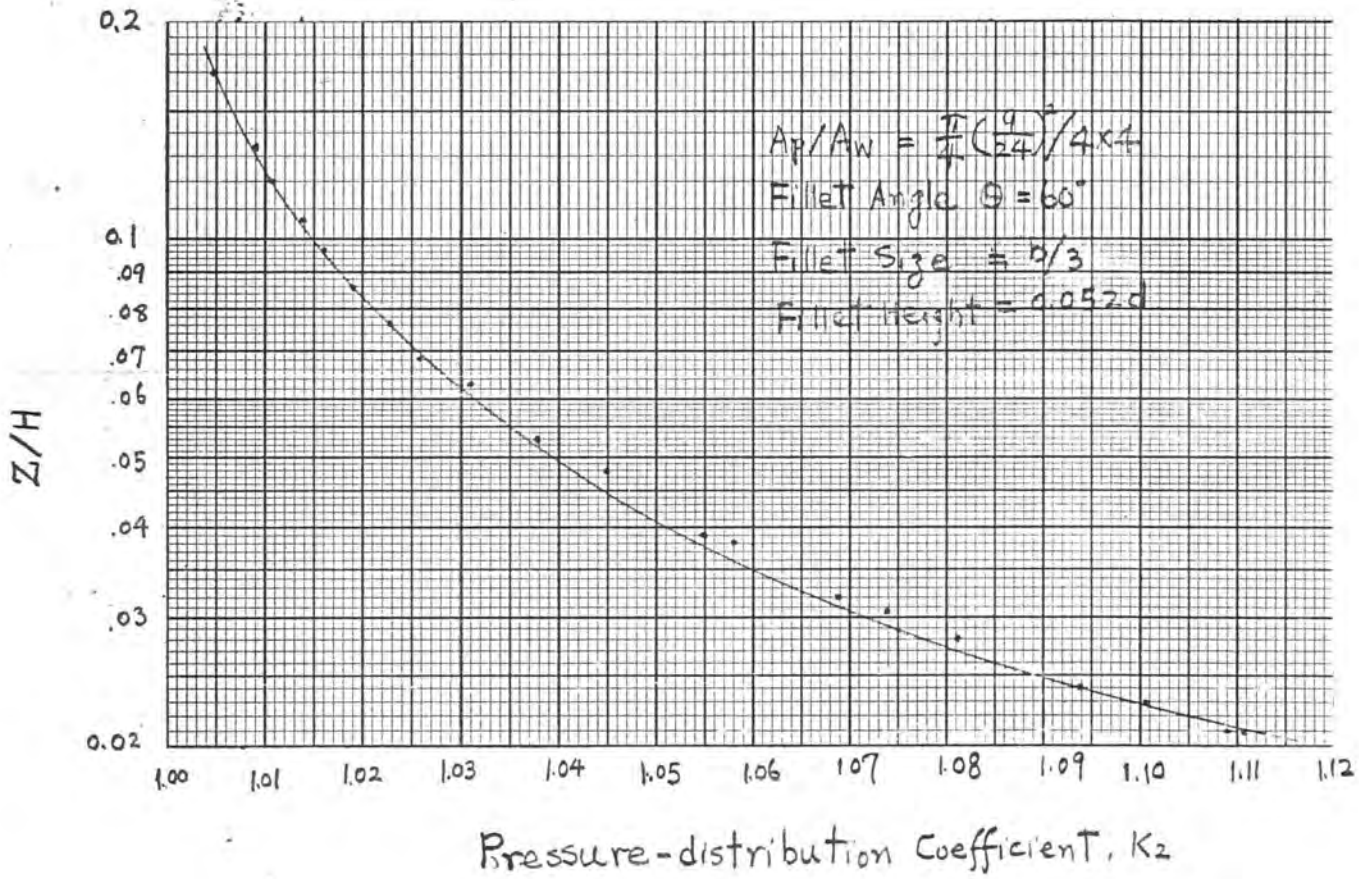


VERTICAL STILLING WELL
PLASTIC MODEL.

FIGURE. 4

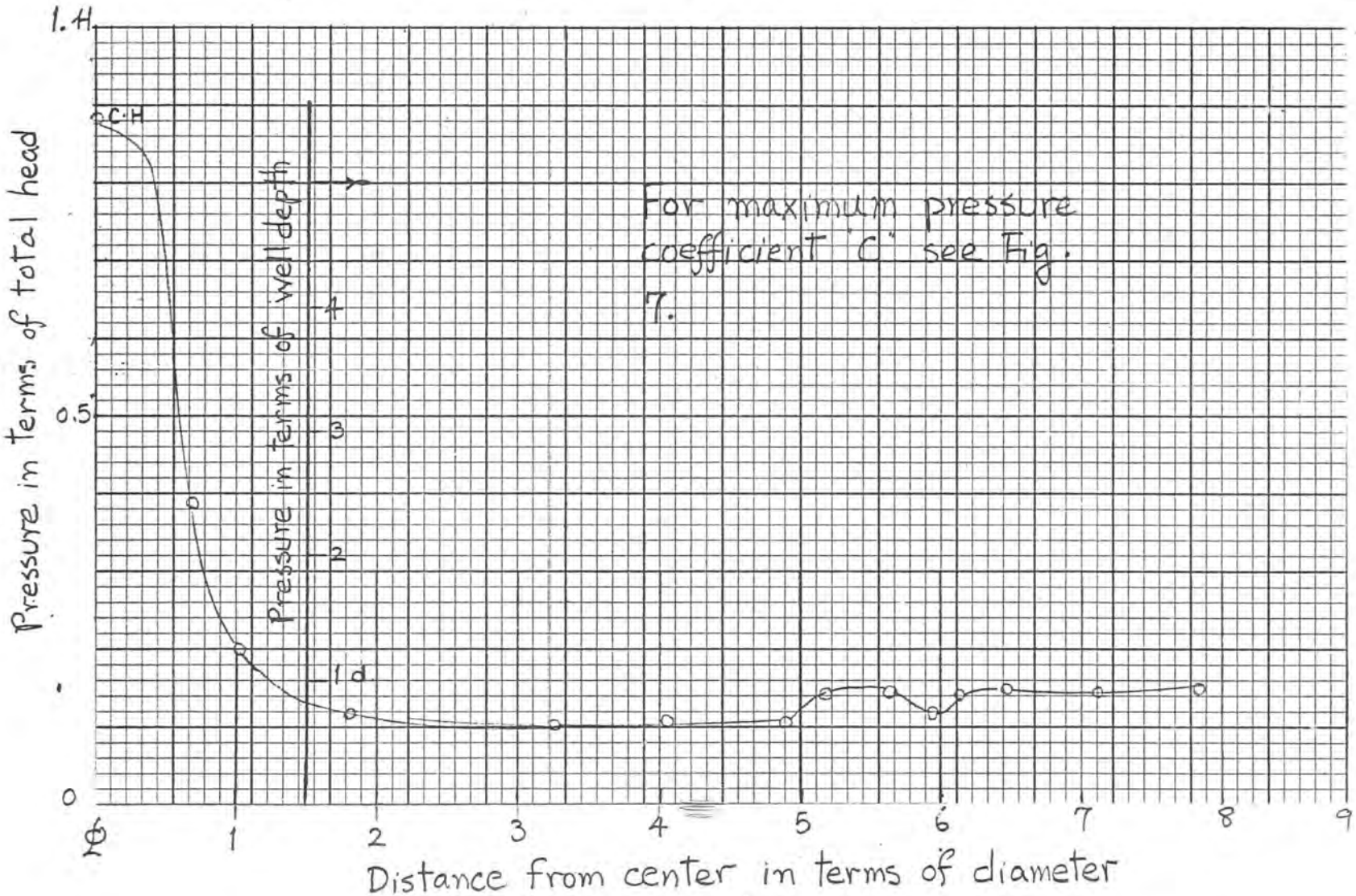


FILLET INSTALLATION



PRESSURE-DISTRIBUTION COEFFICIENT

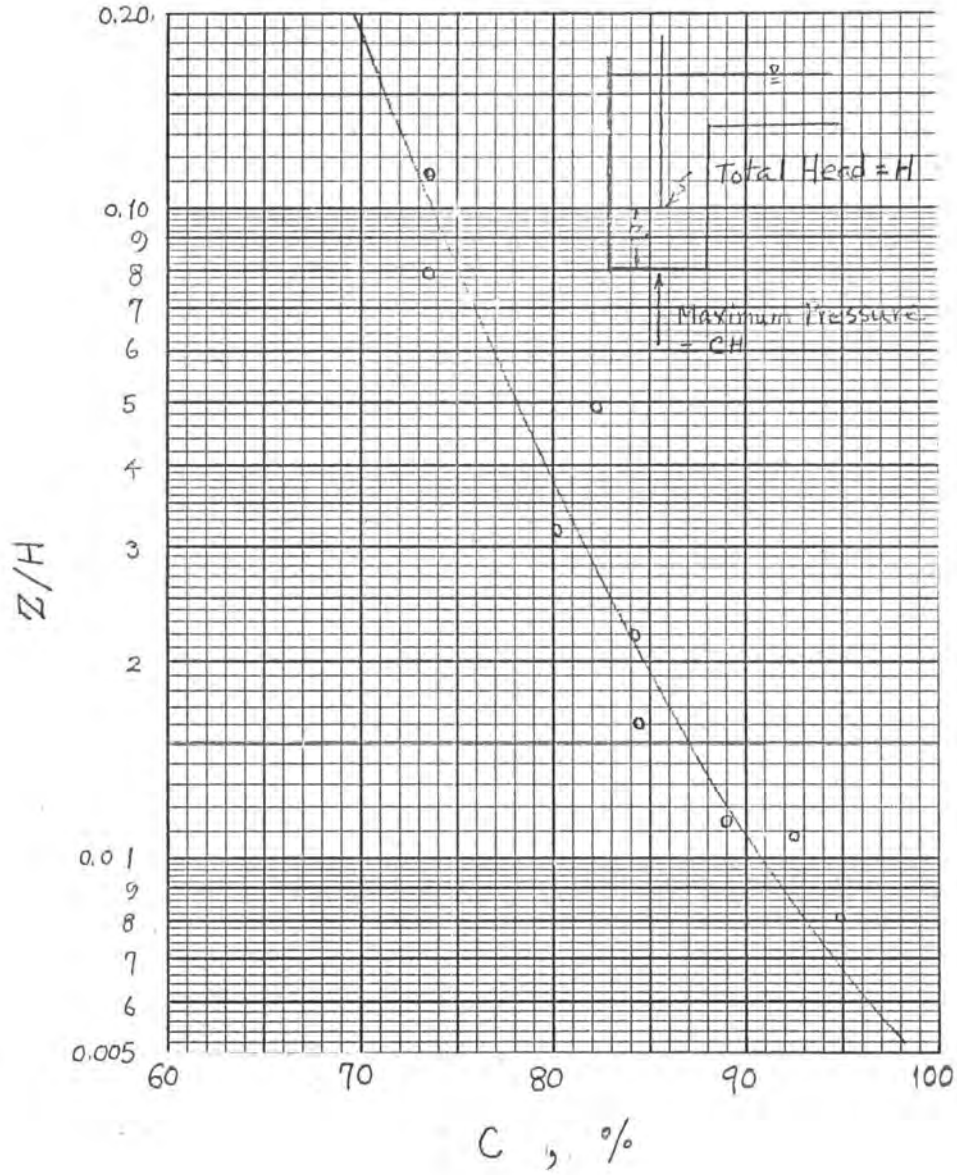
FIGURE 5



DIMENSIONLESS PRESSURE PROFILE ON WELL FLOOR.

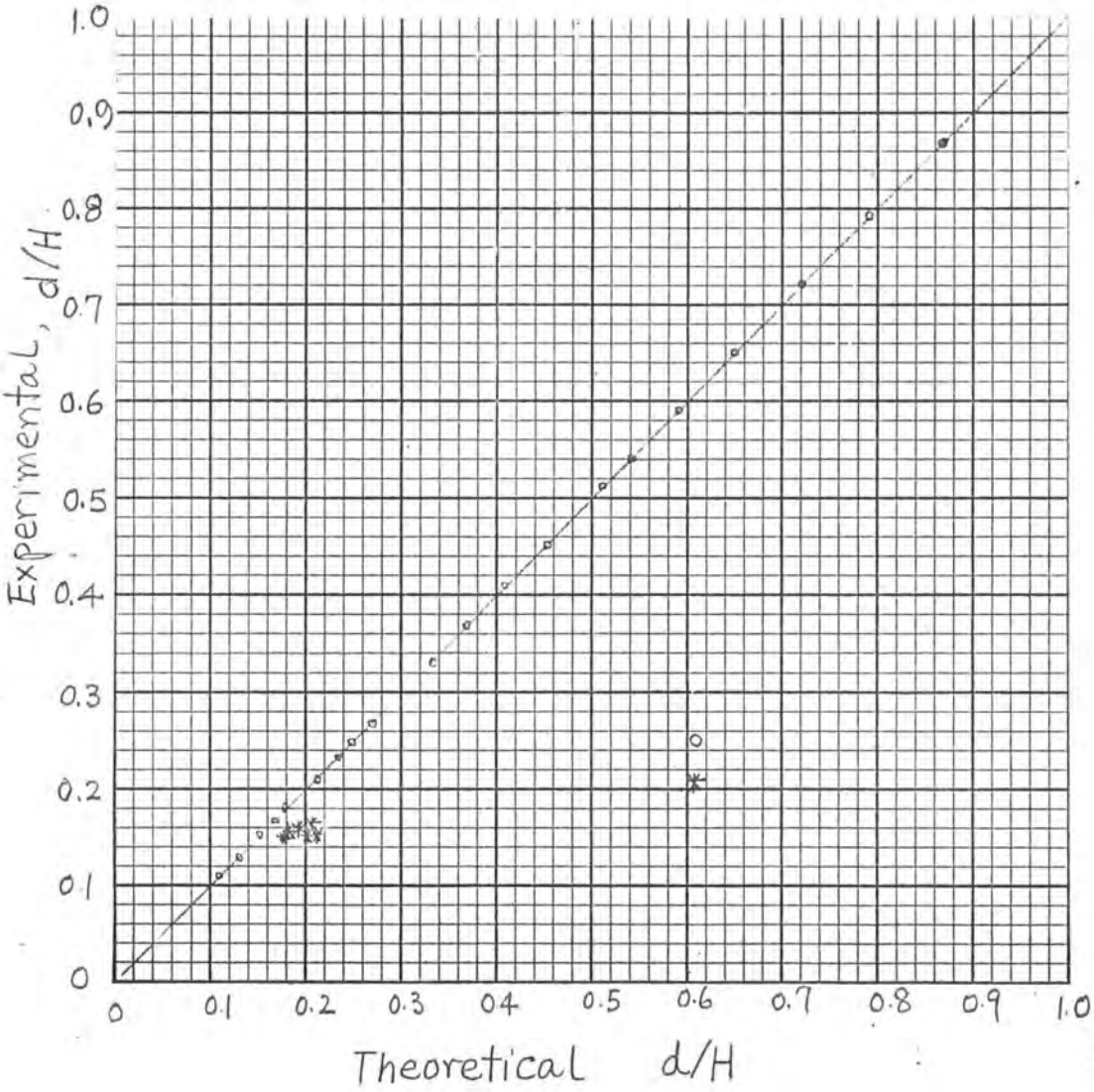
FIGURE 6

FIGURE 7.



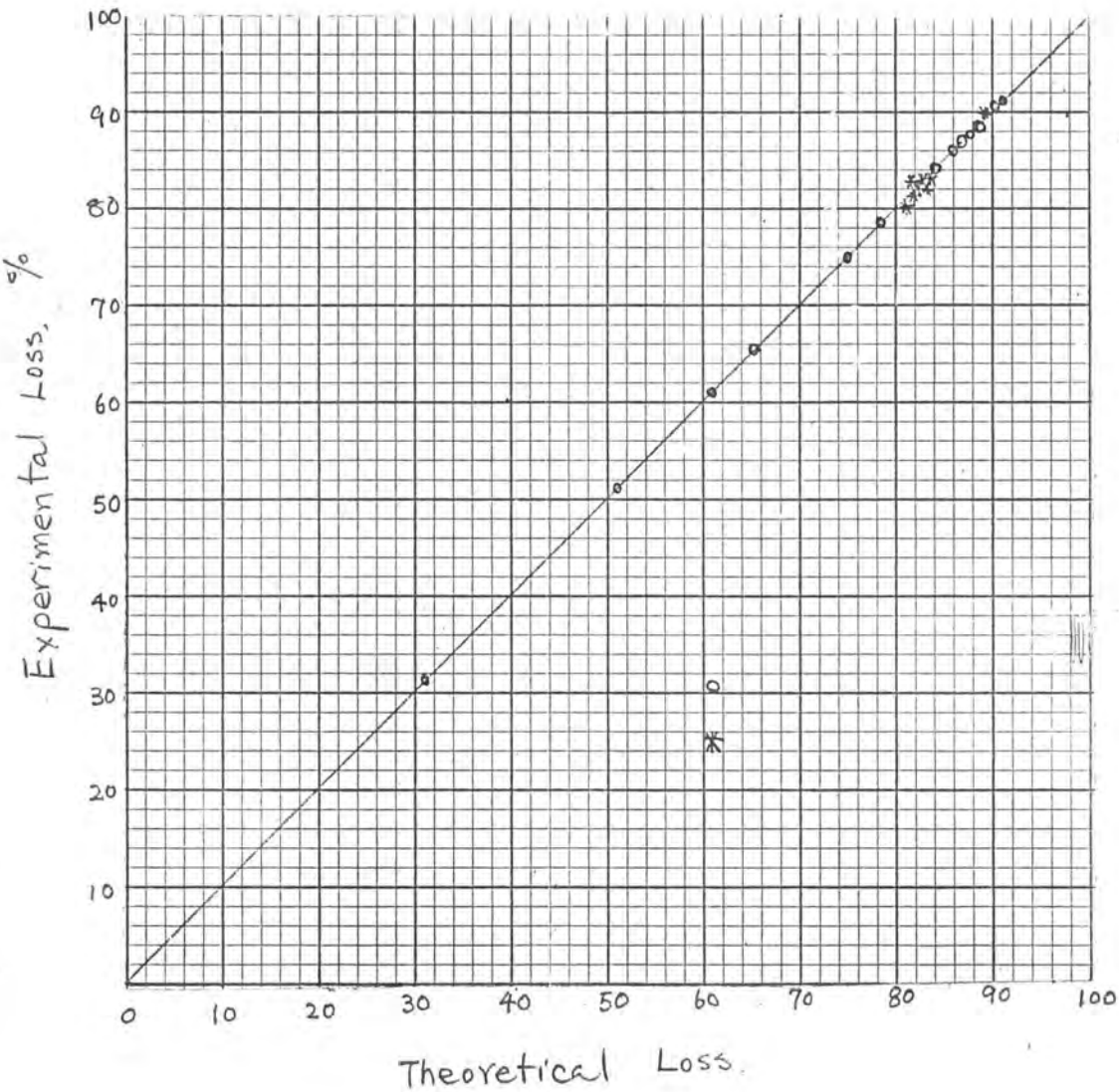
MAXIMUM PRESSURE IN TERMS OF TOTAL HEAD.

FIGURE 8



VERIFICATION OF WELL DEPTH EQUATION

FIGUR



VERIFICATION OF LOSS EQUATION