2814 May, 1961 HY 3

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

AERATED FLOW IN OPEN CHANNELS

Progress Report
Task Committee on Air Entrainment in Open Channels
Committee on Hydromechanics

FOREWORD

The purpose of the Task Force on Aerated Flow in Open Channels is to investigate the status of present (1961) knowledge of the phenomena of aerated flow in open channels and to prepare a report on the subject. The personnel of the Task Force and a brief record of its activities are presented under the heading "Acknowledgments."

The reason for the interest in the phenomena of aerated flow as a mechanical mixture is involved with the proper design of steep channels or chutes. As air is entrained in the flow, the mixture increases in volume of "bulks." The bulking of aerated flow requires higher side walls than non-aerated flow. Thus, the phenomenon has important economic aspects in the design of chute spillways for earth dams, chutes for wasteways of irrigation canals, and high overflow spillways.

The process of air entrainment in chutes is inextricably involved in the mechanism of the generation of turbulence. The actual presence of large quantities of air in the form of bubbles, no doubt, affects the decay of turbulence. The complete problem is related to the problem of suspended sediment, except that solid particles tend to settle toward the bottom where turbulence is generated and air bubbles rise to the surface.

Although much has been learned of the mechanism of the phenomenon by experimentation, a rational solution has not been accomplished, in the knowledge of Task Force members. The objective of this Report is to include empirical relationships which should be helpful to engineers in preparing an economical design of a channel to carry aerated flow. Reliable empirical

Note.—Discussion open until October 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 3, May, 1961.

design criteria are not known to have appeared in the technical literature published in the United States.

HISTORICAL RESUME

A full account of the history of development of the knowledge of aerated flow is not within the scope of this Report, nor would it serve any useful purpose except for historians. A brief historical resume, however, is of value to afford the engineer a perspective of the problem, as well as to recognize outstanding contributions to the advance of the general information on the subject. A short list of references appears as Appendix I, followed by a more extended bibliography in Appendix II.

Three significant events have come to the attention of the Task Force members. The first important event concerns pioneer research in Germany, about 1925. The next two events involve the initiation of important research in the United States and a published symposium on entrainment of air in flowing water. The occurrence of the last two events were stimulated in no small measure by the interest of the original Committee on Hydraulic Research which preceded the organization of the Hydraulics Division of ASCE.

The Committee on Research of the Hydraulics Division inherited the functions of the old committee, upon organization of the Division. In 1957, the name of the Committee on Research was changed to the Committee on Hydromechanics and a new Committee on Research was formed. It is indeed appropriate, therefore, that the sponsoring committee of this task force should maintain a continuing interest in the difficult problem of bulking caused by aerated flow.

Instruments to measure the amount of air mixed with the water form an important aspect in the advance of the technical knowledge. R. Ehrenberger (1)[1] described a simple, yet effective, device for measuring the concentration of air entrained in the flow of chutes constructed in the laboratory at Versuchsanstalt fur Wasserbau, Vienna, Austria. His apparatus was simply a tube, pointing into the flow, and communicating through an elbow to a vertical glass tube of much larger diameter. The Pitot-like tube was rated in unaerated flow with a known velocity by measuring the time to fill a known volume of the vertical tube. The operation was repeated in aerated flow where the air bubbles rose to the surface in the vertical tube. By the increased time to fill the tube in aerated flow, Ehrenberger calculated the air concentration in the air-water mixture.

The Ehrenberger apparatus produced results which have been questioned because of the lack of similitude of flow into the Pitot-like tube in aerated and in unaerated flow. The apparatus was definitely crude compared with the modern electronic method, but ingenious, nonetheless.

Ehrenberger published empirical relationships defining air concentration as follows:

$$P_w = 0.42\ R^{-0.05}\ (\sin \alpha)^{-0.26} \quad \text{for } \sin \alpha < 0.476$$

$$P_w = 0.30\ R^{-0.05}\ (\sin \alpha)^{-0.74} \quad \text{for } 0.476 < \sin \alpha < 0.707$$

[1] Numerals in parentheses refer to corresponding references in Appendix I.

in which P_W is the percentage of water in the toal air-water mixture, R denotes the hydraulic radius, and α is the angle of the channel slope with the horizontal. Ehrenberger considers his formulas valid for $R < 0.30$ (0.98 ft). The width of his wood chute was 0.25 (0.82 ft). No doubt a substantial portion of the air was entrained by turbulence generated at the side walls. Therefore, the formulation of his results would not be applicable to a chute of different width.

Experimental work was initiated in the United States about 1939. During that year J. C. Stevens, F. ASCE, was chairman of the original Committee on Hydraulic Research. Upon the recommendation of ASCE, the Engineering Foundation made a small grant to Lorenz G. Straub, F. ASCE, Director, St. Anthony Falls Hydraulic Laboratory. ASCE had intended that this would be "seed" money, to stimulate other sponsorship, which indeed it proved to be. Later the funds were supplemented by support from the University of Minnesota, the United States Navy, and other sponsors.

The early adjustable metal flume at St. Anthony Falls was 1 ft wide. In an effort to achieve true two-dimensional flow, with no sidewall effect on the air entrainment, the flume was rebuilt to a width of 18 in. in 1950.

The most important advance in the science of aerated flow research at St. Anthony Falls Hydraulic Laboratory was the development of better methods of measuring the concentration of entrained air. The early method of measuring the entrained air volumetrically was to give way to an electronic method in 1950. This apparatus was described in a paper by O. P. Lamb and J. M. Killen (2). Essentially it involved a pair of electrodes set in the line of the flow direction. The electrical resistance of the laboratory water without entrained air was measured, in effect. The concentration of air in aerated flow could then be calculated by measuring the increase of resistance across the electrodes.

Two series of experiments were conducted with the wide flume and the electrical resistance apparatus. Results of a series with the smooth steel wall were in an unpublished report by Straub and A. G. Anderson (3), for the United States Navy in 1955. The results of the test series on walls roughened with granular particles were reported by Straub and Anderson (4) in 1958. Significantly, Straub was awarded a Research Prize by ASCE in 1958 for his work on aerated flow.

A Symposium on the Entrainment of Air in Flowing Water was published in the Transactions of ASCE in 1943. The paper by L. Standish Hall (5) reported on field tests of three chutes operated by the Pacific Gas and Electric Company. He also included field test data on the Kittitas chute by the Bureau of Reclamation, United States Department of the Interior (USBR). For reasons of large width and a mimimum amount of vertical and horizontal curvature, the Kittitas results proved the most desirable for later analysis.

The tests on the Kittitas chute were made by C. W. Thomas and reported to the USBR in 1938 (6). Thomas measured the water-surface profiles as well as the mean velocity by the salt-velocity method. With a known water discharge, the quantity of entrained air could be estimated.

PRESENT STATUS OF KNOWLEDGE

Narrow Chutes.—For purposes of this Report, a narrow chute is defined as one which has a width less than five times the depth. Air entrainment can ordinarily be considered to be a relatively simple two-dimensional problem for wide chutes. Three boundaries, the bottom and two sides, contribute to the generation of turbulence which causes air entrainment in narrow chutes. It is probale that the effect from one side wall reaches nearly to the opposite wall, thus compounding the air-entrainment effect in narrow chutes. Fig. 1 demonstrates development of the side-wall effect of the Uncompaghre Chute very clearly. Much of the data reported by Hall (5) fall in the category of narrow chutes. The discussions of this paper embraced attempts to generalize the results for the calculation of air concentration and the resultant bulking of the flow.

FIG. 1.—AIR ENTRAINMENT FROM SIDE-GENERATED TURBULENCE

FIG. 2.—AIR ENTRAINMENT CAUSED BY SIDE-WALL EFFECT IN NARROW CHUTES

In later years, other attempts at generalization of the air-entrainment phenomena were based on the Hall paper. Although these empirical conclusions were roughly applicable to narrow chutes, they lead to over-design of spillway chute side walls when applied to wide chutes. Fig. 2 shows the side-wall effect of the USBR Kittitas Chute.

Wide Chutes.—Maintaining the definition for a narrow chute expressed previously, a wide chute is considered to be one which has a width greater than five times the depth. An example of a wide chute may be seen in Fig. 3, which shows the Ft. Peck spillway. Close scrutiny of the water surface during operation led to the conclusion that there was a negligible amount of bulking from air concentration. Such an observation was impressive when contrasted with a high air concentration indicated by previous empirical criteria based on narrow chute operation.

FIG. 3.—LOW AIR ENTRAINMENT FROM BOTTOM-GENERATED TURBULENCE IN A WIDE SPILLWAY CHUTE

Hydraulic design engineers have been concerned that the use of design criteria based on unselected field data for narrow chutes had influenced the tendency toward costly over-design of side walls. In recent years, engineers of the United States Army Waterways Experiment Station, Vicksburg, Miss., (WES), analyzed the smooth-channel data obtained at St. Anthony Falls (3) in combination with selected Kittitas field observations. The Kittitas data were selected on the basis of the observations with depths less than one-fifth of the width.

Straub and Anderson had found the air concentration, c, in two-dimensional smooth channel to be a function of $S/q^{2/3}$, in which S is the sine of the slope and q is the unit discharge.

The WES issued a hydraulic design criteria chart in June, 1957, based on the foregoing analysis of the Straub and Anderson smooth-channel results. The curve of best fit was

$$\bar{c} = 0.38 \log_{10}\left(\frac{S}{q}\right)^{2/3} + 0.77 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

This chart presented a much more economical basis for the design of wide chute spillway walls than did previous criteria from unselected narrow chute results.

As mentioned previously, Straub and Anderson (4), presented the results of tests on rough channels in December, 1958. They concluded that for an artificially roughened channel, air concentration was a function of $S/q^{1/5}$. This function of slope and unit discharge was based on an empirical relationship between $\bar{c}$ and V^*/d_t in which V^* is the shear velocity and d_t is a so-called transition depth as defined therein. The new Straub and Anderson data on rough channels were also analyzed in comparison with selected Kittitas field results. The large unit discharge of the prototype chute at Kittitas produced small values of $S/q^{1/5}$, and consequently small concentrations of air. The curve of best fit for the combined laboratory and field data was determined by use of a digital computer and is shown in Fig. 4. The standard error for air concentration was $\sigma_c = 0.061$. The Task Force believes that this relationship offers a realistic basis for the design of wide chute spillways within the limitations of the present status of knowledge.

Overflow Spillways.—The relationship represented by the curve in Fig. 4 is believed applicable to wide ungated overflow spillways, although no prototype observations are yet available for verification. A laboratory study by W. J. Bauer (7) made at the State University of Iowa, Ames, Iowa, covered an investigation of the development of the turbulent boundary layer on an overflow spillway. His observations were not carried beyond the point on the profile where the turbulent boundary layer reached the surface. It is at this part where the phenomenon of air entrainment begins.

The Task Force is not aware of any research which deals with the effect of spillway crest piers on air entrainment. Examination of the operation of overflow spillways in the field indicate that the turbulence generated at the pier noses can cause substantial air entrainment. Indeed, it is believed that for modern spillways with a large design head compared with the width of the gate bay, the pier disturbance has a greater effect on air entrainment and its resultant bulking, than does the bottom turbulence alone. The effect of piers on air entrainment at the Chief Joseph Dam on the Columbia River may be seen in Fig. 5.

NEED FOR FUTURE RESEARCH

This Report has presented an empirical criterion as a guide to the judgment of the engineers in the design of wide steep channels. Nevertheless, much remains to be learned of the basic mechanics of the generation of turbulence, its energy as the water surface is approached, and of the actual phenomenon of air entrainment.

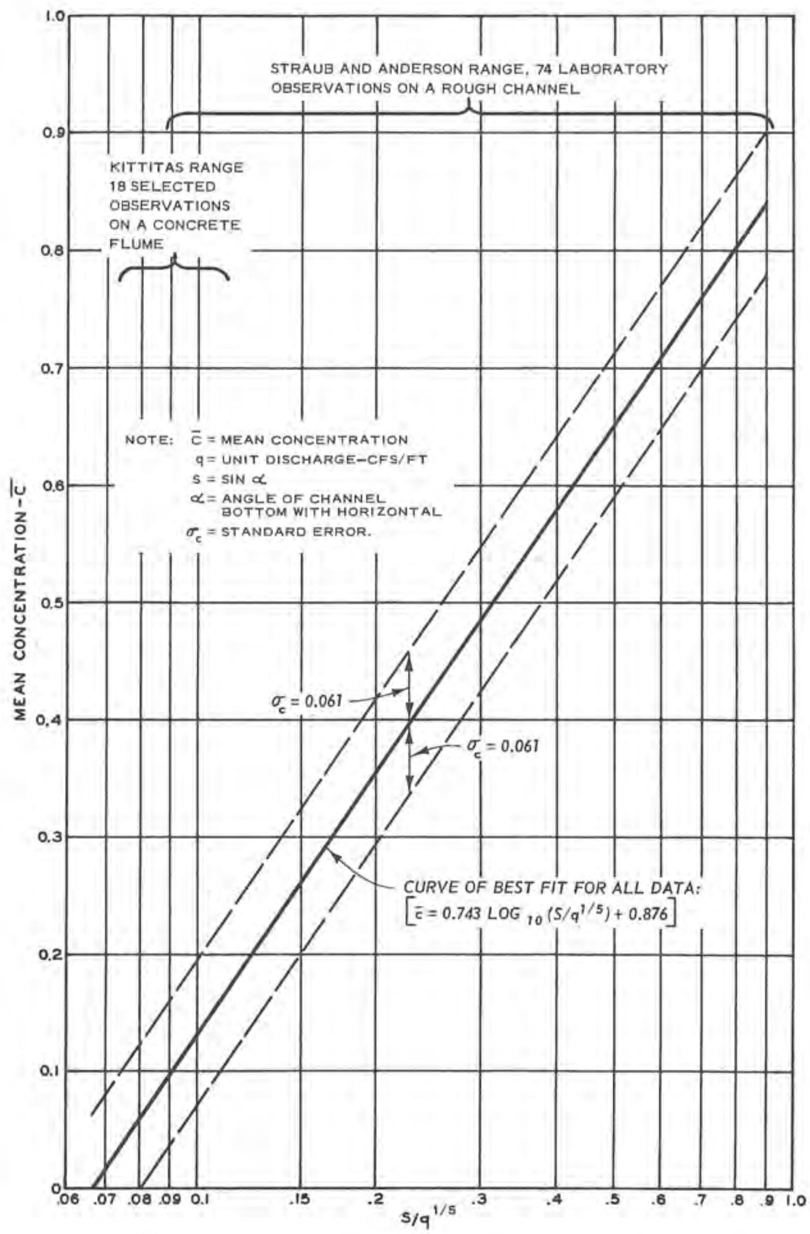

FIG. 4.—AIR ENTRAINMENT BASED ON TESTS OF A ROUGH LABORATORY CHANNEL AND FIELD TESTS

Theory.—The problem is related to the more fundamental one of the generation of turbulence at the boundary, its diffusion upward through the liquid, and the action of the vertical components of turbulence at the water surface. More needs to be known of the reflection or dissipation of the vertical components at the surface with various degrees of turbulence and the actual piercing of the surface by turbulence cells. It is undoubtedly the latter phenomenon which gives rise to air entrainment.

The present (1961) knowledge of turbulence is based primarily on the great mass of observations with the flow of air. The work in aeronautics is not

FIG. 5.—EFFECT OF SPILLWAY PIERS ON AIR ENTRAINMENT

concerned with the phenomenon of turbulence in the vicinity of a liquid with a free water surface. Unfortunately, the hot wire anemometer, which is the principal measuring device in wind tunnel turbulence studies, is not readily adaptable to use with air-water mixtures. Nevertheless, theoretical studies of turbulence in open channels should start with a study of the theory and test results of aerodynamics.

Laboratory Equipment.—Although the combined effect of both side walls and bottom are important in certain structures, the more simple two-dimensional problem of a wide channel should be approached first. A disadvantage of studying wide channels with steep slopes is that large discharges are needed to obtain substantial depth. The fundamental problem of turbulence in open channels can be studied with fairly light slopes. Practically uniform flow can be obtained with a reasonable channel length, if the bottom is artificially roughened. A fairly wide channel with a rough bottom and smooth side walls would then have very little side-wall effect.

More laboratory work is needed to perfect instruments which will measure, at least, the longitudinal and vertical components of turbulence velocities in water. The Hubbard-Ling hot-film anemometer (8)(9) is one approach and several laboratories have experimented with pressure transducers mounted on stream-lined supports.

The three-dimensional problem could well be investigated by first studying the side-wall effect, alone. A wide laboratory flume is again indicated to minimize reflection from the opposite wall. The experimenter would probably choose to roughen only one wall and maintain the bottom and opposite wall smooth.

The behavior of the turbulence from discrete wall protuberances, whether knobs or vertical strips, as the disturbance reaches the surface needs to be studied. Little is known of the effect of joint offsets in channel walls. The effect of piers on overflow spillways belongs to this class of problems. The Task Force believes that the fundamental research could best be performed in the laboratory, where the geometrical variables may be changed with little expense.

Field Tests.—More observations on existing prototype structures should be made and publicized, if only to alert the profession to the important design problems involved. The problem of holding measuring instruments in high velocity flows of the prototype is severe in itself. Provisions for measurement of such a rudimentary variable as velocity are badly needed in the prototype. Velocity observations would yield information for friction-loss studies as well as for basic information on high velocity turbulent flow. The provision for high-velocity measurement must normally be done during construction, by embedded metal which can hold a support at the bottom or a span for attaching the support from above. Embedded metal has been installed for the attachment of instrument piers to the bottom of the Ft. Randall chute spillway. Observations at this installation must await the availability of test flows, which is a common problem with all field tests.

Air entrainment caused by the disturbances of crest piers on overflow spillways may be a critical condition for the design of side walls of this type of spillway. General field observations of the pier effect may well serve as a useful guide in the design of future spillways, until more basic work can be accomplished in the laboratory.

ACKNOWLEDGMENTS

The interest of the Committee on Hydromechanics (the Hydraulics Division's "old" Committee on Research), subsequent to World War II, began at the Minnesota International Hydraulics Convention at Minneapolis on September 3, 1953. At that convention, J. M. Robertson, a committee member, presided at an open meeting on the subject of "Research Needs on Aerated Flow."

At the following meeting of the Committee in 1954, a proposal to establish a Task Force on Aerated Flow in Open Channels was discussed. However, it was not until October 27, 1955, that the Task Force was officially established.

The original task force membership was:

W. J. Bauer

D. Colgate

J. P. Lawrence

W. W. DeLapp, Chairman

F. B. Campbell, Contact Member from Committee on Research.

J. P. Lawrence subsequently resigned from the task force and was replaced by A. G. Anderson.

The task force arranged a technical session for the Jackson, Miss., ASCE Convention on February 18, 1957. W. W. DeLapp presented a progress report on the activities of the Task Force and F. B. Campbell made some formal remarks on the design problems involving air entrainment. The following papers were presented:

"Experiments on Self-Aerated Flow in Open Channels," by L. G. Straub and A. G. Anderson.

"Entrainment of Air by Flowing Water in Circular Conduits with a Downgrade Slope," by J. C. Kent.

The paper by Straub and Anderson was subsequently published in the Journal of the Hydraulics Division, Vol. 84, No. HY7, December, 1958, Paper 1890.

The Task Force met at Minneapolis on May 28, 1957. The members enjoyed the privilege of counsel with L. G. Straub, Director, St. Anthony Falls Hydraulic Laboratory, during the meeting. A bibliography was prepared soon thereafter.

On account of the urgency of other professional business, W. W. DeLapp and W. J. Bauer each requested to be relieved of their duties on the Task Force.

The sponsoring Committee on Hydromechanics met at Iowa City, Iowa, on June 14, 1958. The committee invited retiring member F. B. Campbell to assume chairmanship of the Task Force on Aerated Flow in Open Channels. The reconstituted Task Force met in Kansas City, Mo., on April 25, 1959, and agreed on an outline for the foregoing report.

Respectfully submitted,

W. Douglas Baines
Frederick R. Brown
Emmett M. Laursen
E. Roy Tinney
Norman H. Brooks, Chairman,
Committee on Hydromechanics

A. G. Anderson
D. Colgate
F. B. Campbell, Chairman, Task
Force on Air Entrainment in
Open Channels

APPENDIX I.—LIST OF REFERENCES

1. "Wasserbewegung in steilen Rinnen mit besonder Berucksichtigung der Selbstbeluftung" ("Flow in Steep Chutes, with Special Reference to Self-Aeration"), by R. Ehrenberger, Oesterreichischer Ingenieur-und Architektverein No. 15/16 and 17/18, 1926. Translated by E. F. Wilsey, U.S. Bur. of Reclamation.

2. "An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures," by O. P. Lamb and J. M. Killen, St. Anthony Falls Hydr. Lab., Tech. Paper No. 2, Series B, 1950.

3. "The Distribution of Air in Self-Aerated Flow in Smooth Open Channels," by L. G. Straub and A. G. Anderson, St. Anthony Falls Hydr. Lab. Project Report No. 48, July, 1955.

4. "Experiments on Self-Aerated Flow in Open Channels," by L. G. Straub and A. G. Anderson, Proceedings, ASCE, Vol. 84, No. HY7, December, 1958.

5. "Entrainment of Air in Flowing Water (Symposium): open Channel Flow at High Velocities," by L. S. Hall, Transactions, ASCE, Vol. 108, 1943, p. 1394.

6. "Progress Report on Studies of the Flow of Water in Open Channels with High Gradients," by C. W. Thomas, U. S. Bur. of Reclamation, Hydr. Lab. Report HYD 35, July 27, 1938.

7. "Turbulent Boundary Layer on Steep Slopes," by W. J. Bauer, Transactions, ASCE, Vol. 119, 1954, p. 1212.

8. "Constant-Temperature Hot-Wire Anemometer with Application to Measurements in Water," by P. G. Hubbard, Doctoral Dissertation, State Univ. of Iowa, Iowa City, Iowa, 1954.

9. "Measurement of Flow Characteristics by the Hot-Film Technique," by S. C. Ling, Doctoral Dissertation, State Univ. of Iowa, Iowa City, Iowa, 1955.

APPENDIX II.—BIBLIOGRAPHY ON AERATED FLOW IN OPEN CHANNELS[a]

201. "The High Velocity Flow of Water in a Small Rectangular Channel," by W. W. DeLapp, Ph.D. Dissertation, Univ. of Minnesota, Minneapolis, 1947.

[a] Filed in Engineering Societies Library, New York, October, 1959.

202. "The Flow of Water in Channels Under Steep Gradients," by W. F. Durand, Transactions, Amer. Soc. of Mech. Engrs., January, 1940, pp. 9-14.

203. "Wasserbewegung in steilen Rinnen (Schusstennen) mit besonder Beruecksichtigung der Selbsteluftung" ("Flow in Steep Chutes, with Special Reference to Self-Aeration"), by R. Ehrenberger, Osterreichischer Ingenieur-und Architektverein, No. 15/16 and 17/18, 1926. Translated by E. F. Wilsey, U. S. Bur. of Reclamation.

204. "Open Channel Flow of Water-Air Mixtures," by H. A. Einstein, Transactions, AGU, Vol. 35, No. 2, April, 1954, pp. 235-242.

205. "Head Loss and Air Entrainment by Flowing Water in Steep Chutes," by Ante Frankovic, Proceedings, Internatl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 467-476.

206. "An Experimental Study of Bubbles Moving in Liquids," by W. L. Haberman and R. K. Morton, Proceedings, ASCE, Vol. 80, Proceedings-Separate No. 387, January, 1954.

207. "The Measurement of Concentrations and of Velocities in a Current of Air and Water Mixed," by G. Halbronn, La Houille Blanche, May-June, 1951, pp. 394-405.

208. "Air Entrainment in Steeply Sloping Flumes," by G. Halbronn, R. Durand, de Lara, and G. Cohen, Proceedings, Internatl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 455-466.

209. "Entrainment of Air in Flowing Water (Symposium): Open Channel Flow at High Velocities," by L. S. Hall, Transactions, ASCE, Vol. 108, 1943, p. 1394.

210. "The Influence of Air Entrainment on Flow in Steep Chutes," by L. S. Hall, Univ. of Iowa Studies in Engrg., Bulletin 31, Iowa City, 1946, pp. 298-314.

211. "Air Entrainment on Spillway Faces," by G. H. Hickox, Civil Engineering, Vol. 15, No. 12, December, 1945, p. 562.

212. "Entrainment of Air in Flowing Water and Technical Problems Connected with It," by V. Jevdjevich and L. Levin, Proceedings, Interntl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 439-454.

213. "An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures," by Owen P. Lamb and John M. Killen, SAF Tech. Paper No. 2, Series B, 1950.

214. "An Electronic Equipment for Measurement of Air Entrained in Flowing Water," by Owen P. Lamb and John M. Killen, Annual Report (Tech.), Research Publication No. 18, Central Water and Power Research Sta., Poona, 1952, pp. 75-76.

215. "An Experimental Channel for the Study of Air Entrainment in High-Velocity Flow," by Owen P. Lamb, SAF Project Report No. 34, November, 1952.

216. "Recent Studies on Flow Conditions in Steep Chutes," by E. A. Lane, Engineering News-Record, January 2, 1936, p. 5.

217. "Entrainment of Air in Swiftly Flowing Water, Observations of the Flow Over Spillways Yield Conclusions of Interest to Hydraulic Engineers," by E. A. Lane, Civil Engineering, February, 1939.

218. "Notes on the Flow Mechanics of Water-Air Mixtures," by Leon Levin, La Houille Blanche, Vol. 10, No. 4, August-September, 1955, pp. 555-557; Applied Mechanics Reviews, April, 1956, p. 160.

219. "Some Prototype Observations of Air-Entrained Flow," by V. Michels and M. Lovely, Proceedings, Internatl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 403-414.

220. "Field Tests on High Velocity Flow and its Air Content," by A. Okada and K. Fukuhara, Tech. Research Inst. of Electric Power Industry, Japan. Presented at Eighth Congress, Internatl. Assoc. for Hydr. Research, Montreal, Canada, August, 1959.

221. "Flow of Water Mixed with Air in Channels and on Spillways," by Dorin Pavel, Hydr. Proceedings, No. 2, Budapest, Hungary, 1951, pp. 168-171.

222. "Air Entrainment in Chute Spillways," by M. S. Priest, Tech. Annual Report, Govt. of India, Central Water and Power Research Sta., Poona, Research Publication No. 19, 1943, pp. 114-122.

223. "Air Entrainment by Water in Steep Open Channels," by M. S. Priest, Tech. Paper No. 3, Engrg. Inst. of Canada, 1951.

224. "The Distribution of Air in Self-Aerated Flow in a Smooth Open Channel," by L. G. Straub and A. G. Anderson, SAF Project Report No. 48, July, 1955.

225. "Experiments on Selff-Aerated Flow in Open Channels," by L. G. Straub and A. G. Anderson, Proceedings, ASCE, Vol. 84, No. HY7, December, 1958.

226. "Velocity Measurements of Air-Water Mixtures," by L. G. Straub, J. M. Killen, and Owen P. Lamb, Transactions, ASCE, Vol. 119, 1954, p. 207.

227. "Experimental Studies of Air Entrainment in Open Channel Flow," by L. G. Straub and Owen P. Lamb, Proceedings, Internatl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 425-438.

228. "Progress Report on Studies of the Flow of Water in Open Channels with High Gradients," by C. W. Thomas, USBR Hydr. Lab. Report HYD 35, July 27, 1938.

229. "Ricerca Sperimentale Sulle Correnti Rapide," by Michele Viparelli, Univ. of Naples, Instituto di Idraulica e Construzioni Idrauliche, Tech. Paper No. 64, 1951.

230. "The Flow in a Flume with 1:1 Slope," by Michele Viparelli, Proceedings, Internatl. Hydr. Convention, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1953, pp. 415-424.

231. "Study of Air Entrainment in Chute Spillways," by Michele Viparelli, Annual Report (Tech.), Research Publication No. 18, Central Water and Power Research Sta., Poona, India, 1952, pp. 71-74 (30° slope). (1954 Report, No. 20, pp. 284-293 for 45° slope.)

232. "Fast Water Flow in Steep Channels" ("Ecoulement d'eau rapide dans les canaux a forte pente"), by Michele Viparelli, AIRH Ville Congress, Lisbon, Portugal, Section D, 1957, La Houille Blanche, January-February, 1958, p. 95.

233. "Correnti Rapids," by Michele Viparelli, L'Energia Elettrica, No. 7, 1958.