USE OF AN ELECTRONIC COMPUTER TO ANALYZE DATA FROM STUDIES OF CRITICAL TRACTIVE FORCES FOR COHESIVE SOILS

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To help improve design criteria for earth-lined and unlined canals, an establiched program for an IBM 650 electronic digital computer was used to develop multiple linear empirical correlations of critical tractive forces versus standard soil properties. Data were collected by testing in a tractive force apparatus hand-out, undisturbed 8-inch (0.203 meter) cohesive soil samples, taken from operating irrigation canals in the western United States. The tractive force apparatus was developed and calibrated in the laboratories of the Bureau of Reclamation.[1])

A number of correlations were attempted. Linear equatitons and several coefficients, indicating the confidence which can be placed in the equations, were obtained for the range of soils tested. The equatinos were obtained more rapidly, and at considerably less cost, than could have been attaned by manual methods.

Pour améliorer les critères employés pour projeter des canaux revêtus en terre et ceux sans revêtement, un programme existant, qui emploie un calculateur électronique digital „IBM 650", fut utilisé pour développer des multiples corrélations linéaries empiriques des forces tractives critiques avec les propriétés étalons des sols. Ces données étaient rassemblées au moyen des essais dans un appareil pour mésurer la force tractive sur les échantillons de 0.203 mètre coupés a la main, sans dérangement, des sols cohesifs. Ces échantillons furent prises des canaux d'irrigation en usage dans l'ouest des États-Unis. Ce appareil-ci était développé et calibré dans les laboratoires du Bureau of Reclamation.

Plusieurs de corrélations en étaient essayées. Des équations linéaires, et quelques coefficients indiquants la confiance qu'on peut avoir en ces équations, ont étés obtenues pour la portée des sols sur lesquels on faisait les essais. Avec le calculateur les équations sont derivés plus rapidement et considérablement plus économiquement que possible par les méthodes manuelles.

INTRODUCTION

Tractive force or boundary shear may be defined as the force per unit area exerted by a fluid flowing past a surface or boundary. The force, acting on the surface of the boundary in the direction of flow, is dependent on flow conditions and the characteristics of the boundary material. In an earth channel, tractive force is a primary agent tending to cause erosion. For every earth material, there is a tractive force, or range of tractive forces, above which erosion will occur and below which the material will remain essentially stable. This tractive force is thought of as critical when determining the stability of soils.

Although a number of studies have been conducted to determine the tractive forces necessary to move non-cohesive materials, little

[1]) Bureau of Reclamation Progress Report No. 3, „Canal Erosion and Tractive Force Study (Correlation of Laboratory Test Data)" — Lowercost Canal Lining Program, General Report No. GEN—26, October 1960

Fig. 1

information is available regarding forces necessary to erode cohesive soils. As a resulf of this lack of information, and to establish better design criteria for cohesive soils, the Bureau of Reclamation established an experimental program intended to exted over a period of several years, and to include the study of various soils and climatic and hydrarlic conditions. This paper presents information on a portion of this program. Hand-cut, undisturbed *8-inch (0.203 meter)* cohesive soil samples were collected from *46 canal* and lateral reaches throughout the western United States, Figure 1. All of these soils fell in a limited range on the plasticity index-liquid-limit chart, Figure 2, and no large deviation in mean particle size was present between soils. No reaches containing large quantities of sand and gravel were tested.

To provide controlled tractive forces acting on the *8-inch (0.203 meter)* soil samples, a tractive-force testing apparatus was developed, Figure 3. The apparatus consists of a covered cylindrical tank *3* feet *(0.914 meter)* in diameter, a *3-bladed* impeller powered with a

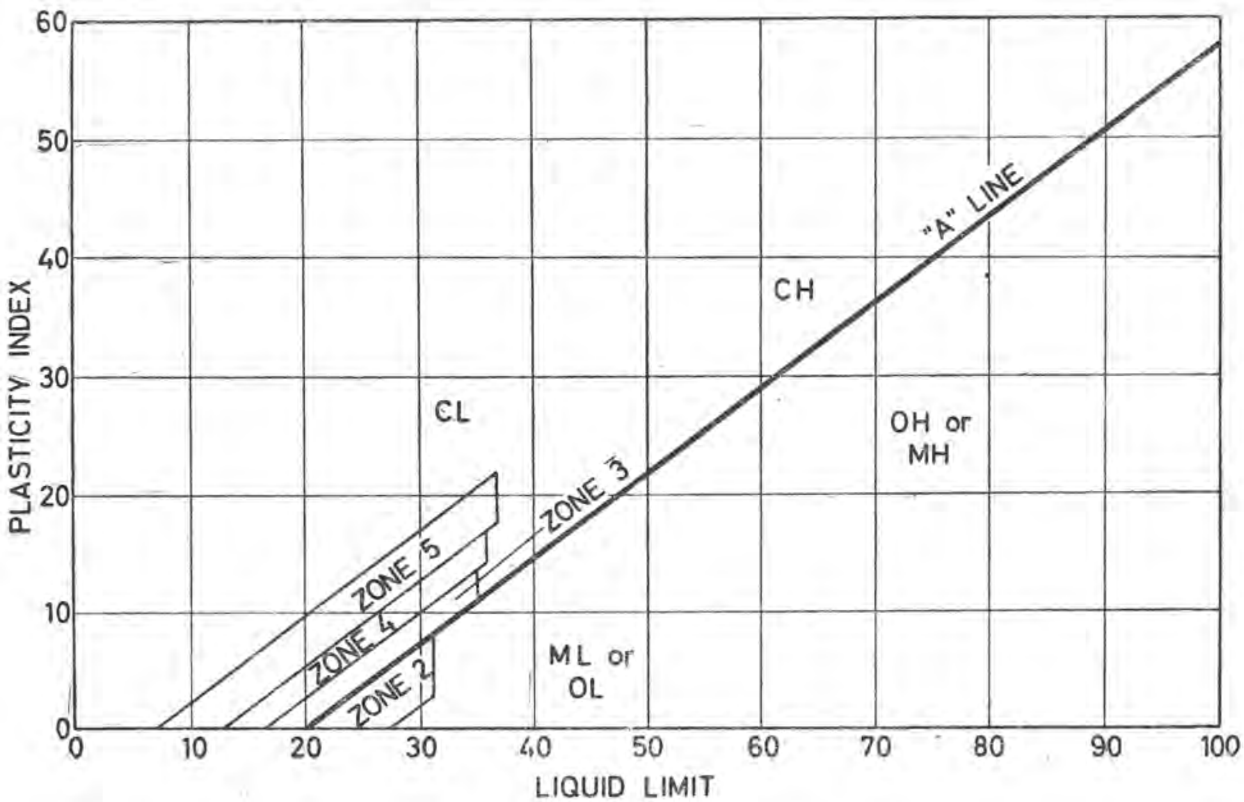

Fig. 2

RANGE OF SOILS TESTED

motor driven by compressed air, a recess for the sample, a pressure gage and a pressure regulator. The *8-inch (0.203 meter)* sample is saturated and placed in the apparatus so that the soil surface is in the same plane as the bottom of the cylindrical tank, and covered with *12 inches (0.305 meter)* of water. The impeller is rotated slowly at first. The rotation forces water to flow across the soil surface, thus creanting small tractive forces on the sample. The velocity of the impeller is gradually increased until erosion of the sample surface occurs. Velocity of the impeller at the time of erosion is related to the tractive force acting on the sample by a combination of experimental and theoretical analysis. When critical rotational velocity is attained, erosion usually starts quite suddenly and progresses rapidly, leaving little question that the critical condition has been reached. Because the tractive force varies across the sample, the position of the eroded area is important for establishing the critical tractive force which started erosion.

CREATING

After the erosion tests were completed, vane shear values of the soils were obtained by using a vane shear apparatus having a *4-bladed* vane *4 inches (0.1016 meter)* high and *2 inches (0.0508 meter)* in diameter. Other stan-

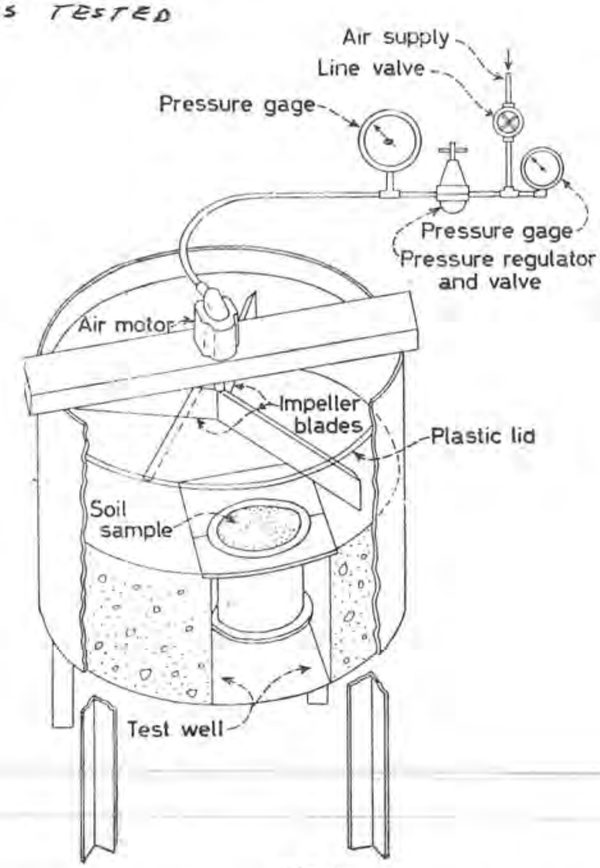

Fig. 3

TRACTIVE FORCE TEST TANK.

dard laboratory test values obtained on the soils included, the plasticity index, liqued limit, shrinkage limit, density of the soil tested, percent maximum density, and gradation factors. A ~~long~~ log -probability method[2]) was used to explain the soil gradation. By this method, the mean size of soil particle, scatter of particles about the mean, and a function of the amount of very fine material in the soil is used to explain the mechanical analysis of the soil.

RESULTS OBTAINED

Table 1, summarizes the results of laboratory test performed on the soil samples. The first column of this table identifies the location, in the western United States, from which the soil was obtained, and the number of the test section in the area. For example ~~2–7~~ 7-2 refers to geographical ~~erea~~ area No. 7 and test section No. 2 in that area. Column 1, gives the number assigned to the sample at the time it was obtained for later ~~identificion~~ identification. Column *2* gives the tractive force, ~~im~~ in pounds, per square foot, at which ~~is~~ it was determined that erosion started. The plasticity index of the sample is given in Column *3*. Column *4* shows the density, in pounds per cubic foot, of the sample when tested. Columns *5*, *6*, *7* and *8* refer to the gradation of the soil. Column *7* is a measure of the geometric mean diameter. The geometric mean diameter may be determined by applying the equation.

$$M_\Phi = \text{Log } 2\, \xi_{(m)}$$

subscripts

where: M_Φ = value shown in Column 7

$\xi_{(m)}$ = geometric mean diameter

Column *6* is an indication of the scatter of particles about the geometric mean. The σ_Φ value furnishes a measure of the distribution of particles. As the value σ_Φ in Column *6* becomes greater, the range of distribution of particles about the mean increases. The k_Φ value in Column *5* furnishes a measure of the amount of very fine particle sizes in the material. As the value of k_Φ, becomes greater, the percentage of fine material increases. Column *8* is the product of Columns *5*, *6* and *7*. Column *9* shows the shrinkage limit of the soil tested, and Column *10* show the ~~cane~~ VANE shear values converted to pounds per square foot.

Column *11* show the percent maximum density of the soil when tested and Column *12* show the liquid limit of the soil as determined from laboratory tests.

ANALYSIS

Initial analysis was made using a graphical method of deviations described by M. Ezekiel[3]). However, this method proved slow, and it appeared impractical to make all the correlations desired. To make analysis of many factors more practical, a multiple linear correlation program, which had been written for an *IBM 650* electronic digital computer was used. The data shown in Table 1 were punched on cards which were used in the computer.

In the multiple linear correlation it was assumed that the critical tractive force was a linear function of several variables, and that no single variable was closely enough related to it to result in a satisfactory estimate. The linear relation takes the following form:

$$T_c = a + b_1 X_1 + b_2 \cancel{X_3}^{\,X_2} + \ldots + b_k X_k$$

T_c = the critical tractive force

a = a constant

$b_1, b_2 \ldots b_k$ = constants

$X_1, X_2 \ldots X_k$ = independent variables such as plasticity, index, density of soil, etc.

To find the equation which best fits the data shown in Table *1*, the standard method of least squares was used. By this method

$$\sum_{i=1}^{i=n} (T_i - T_i')^2$$

where: T'_i is a point on the oot curve corresponding to the data point T_i are made to be a minimum, by substituting the desired equation for T_i and using the calculus to obtain the desired set of equations. The number of equations necessary to solve and the difficulty of solution increases with the number of independent variables used. For large numbers of equations their solution becomes in-

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[2]) Otto G. H., A Modified Logarithmic Probability Graph for Interpretation of Mechanical Analyses of Sediments, Journal of Sedimentary Petrology, Volume 9, No. 2, pp. 62—76, August 1939.

[3]) Ezekiel M., Methods of Correlation Analysis, Second Edition, Wiley, New York, 1941

Table 1

DATA OBTAINED FROM LABORATORY TEST SAMPLES

Zone 2

	1	2	3	4	5	6	7	8	9	10	11	12
Region and reach	Sample No. (18F-)	Most prob T.F.	P.I.	Density	k'_ϕ	σ_ϕ	M_ϕ	$k'_\phi \sigma_\phi M_\phi$	S.L.	V.S.	% Max. density	LL
7-2	361-1	0.053	7.2	93.4	1.60	2.88	7.54	34.8	20.72	2.20	85.0	30.7
4-4	537	0.027	0.2	91.7	0.07	1.16	5.12	0.416	22.97	1.33	84.6	22.2
4-4	539	0.038	4.9	95.7	1.00	2.72	4.48	12.2	20.32	1.15	88.4	26.7
4-6	571	0.038	4.1	96.6	0.50	2.16	4.10	4.43	24.51	1.07	85.2	29.3
4-6	572	0.042	6.7	89.3	1.20	2.14	4.20	10.8	24.53	1.48	78.7	29.3
1-6	673	0.023	0	86.6	0.80	1.26	5.48	5.52	22.21	1.10	80.7	21.3
1-8	691	0.028	0.4	95.3	0.10	1.59	5.24	0.833	25.11	0.66	91.6	23.1
2-3	445	0.030	0	94.5	1.04	2.22	3.84	8.87	22.85	0.82	90.4	23.2
2-4	450	0.020	0	75.3	2.00	1.41	4.25	11.99	26.70	0.90	71.5	21.8
2-7	468	0.033	0	100.1	1.40	1.93	3.90	10.54	24.5	0.66	95.3	20.1
4-3	525	0.018	0	91.4	1.55	1.64	3.44	8.87	28.31	0.98	83.6	27.2
4-3	526	0.027	0	88.9	2.00	1.15	3.72	8.56	24.36	0.90	81.2	23.1
1-2	637	0.029	0	91.8	2.00	1.30	5.20	13.52	24.64	0.66	100	25.4
1-2	638	0.017	0	78.3	0.80	1.34	5.34	5.72	28.16	0.57	85.7	26.8
1-3	647	0.024	0	78.6	2.00	1.20	5.20	12 5	27.08	0.84	73.8	24.5
1-4	656-1	0.032	0	93.8	2.00	1.54	5.50	16.94	22.71	1.48	92.0	23.5
1-4	657-1	0.045	0	96.6	1.50	1.45	5.10	11.09	22.18	1.93	94.8	23.8
1-5	664	0.030	0	92.5	1.50	1.68	4.94	12.45	21.19	0.49	86.5	21.5
1-5	665	0.030	0	93.9	2.00	1.10	4.40	9.68	21.43	0.90	87.7	22.3
1-5	666	0.017	0	74.0	1.40	1.08	4.20	6.35	24.71	0.66	69.2	22.0
1-6	673-2	0.028	0	88.2	1.50	1.39	5.40	11.26	22.21	1.10	81.9	21.5
1-6	674	0.029	0	86.6	1.10	1.36	5.25	7.85	21.59	0.66	80.6	22.0
1-7	682	0.024	0	87.6	0.50	2.05	5.79	5.93	25.87	1.31	82.8	25.5
1-7	683	0.020	0	80.7	0.69	1.88	5.68	7.38	23.26	0.98	78.7	23.0
1-7	684	0.025	0	86.8	0.81	1.48	5.79	6.95	29.59	1.39	79.4	24.0
1-8	692	0.022	0	84.8	0.28	1.76	5.51	2.72	22.69	0.66	81.5	24.2
1-8	693	0.025	0	89.5	0.70	1.56	5.15	5.61	25.9	0.98	86.0	25.1
1-9	700	0.021	0	85.3	-0.25	1.92	6.10	-2.93	27.9	1.64	86.4	28.0
*	709-1	0.018	0	84.5	1.00	1.38	5.60	7.73	26.36	1.18	81.1	24.5
*	710-1	0.026	0	85.0	1.40	1.39	5.48	10.7	25.88	0.90	82.3	25.8
*	710-2	0.028	0	90.0	1.30	1.24	5.40	8.69	25.88	1.31	86.3	25.5
*	711	0.022	0	82.2	0.85	1.52	5.75	7.42	26.32	1.31	78.8	27.4
*	711-2	0.025	0	85.8	1.30	1.28	5.42	9.00	26.32	1.23	82.3	20.8

NOTES

NOTS: T. F. = most probable tractive force (pound per square foot) determined from tractive force machine

P. I. = plasticity index

k_ϕ = phi skewness

σ_ϕ = phi standard devation

M_ϕ = phi arithmetical mean diameter.

S. L. = shrinkage limit

V. S. = vane shear — pound per square foot

*) Field data not obtained because of construction.

Zone 3

	1	2	3	4	5	6	7	8	9	10	11	12
Region and reach	Sample No. (18F-)	Most prob T.F.	P.I.	Density	k'_ϕ	σ_ϕ	M_ϕ	$k'_\phi \sigma_\phi M_\phi$	S.L.	V.S.	% max. density	LL
7-5	352-2	0.036	10.8	76.0	1.20	1.90	6.46	14.73	23.92	1.80	67.9	31.5
7-5	359	0.043	11.3	76.2	2.10	1.66	6.14	21.40	27.12	2.20	70.1	34.7
7-6	370-1	0.025	0.8	85.7	1.55	2.24	5.19	18.02	15.45	1.45	82.3	19.7
7-1	382	0.033	6.4	72.1	1.50	2.18	5.50	17.99	17.71	1.80	69.8	26.0
7-2	391-1	0.042	10.2	82.9	1.50	1.91	6.15	17.62	16.15	1.80	79.1	30.8
7-8	400-1	0.043	8.6	81.9	2.00	2.21	6.60	29.17	19.60	1.50	74.1	31.0
7-3	407-1	0.039	5.4	92.5	2.00	1.88	6.21	23.35	18.04	1.90	88.3	26.7
7-3	407-2	0.040	5.6	84.2	2.00	2.07	6.49	26.87	18.04	1.90	80.2	27.5
4-1	514	0.037	9.7	73.4	1.57	2.76	5.88	25.48	19.31	0.41	69.0	31.8
4-3	527	0.040	8.5	91.6	2.00	2.46	4.54	22.34	20.98	0.90	83.8	29.5
4-8	582	0.049	4.8	99.3	1.40	2.41	3.98	13.43	18.3	1.64	84.5	24.3
1-1	629	0.033	5.3	85.3	1.00	1.81	6.08	11.0	19.62	1.43	81.3	24.4
1-1	630	0.041	3.3	78.8	1.8	2.22	6.68	26.69	20.15	1.43	75.2	22.6
1-3	646	0.028	5.6	78.7	0.90	2.81	4.31	10.9	19.67	0.74	74.0	25.5
1-3	648	0.040	3.5	91.6	1.10	1.58	5.70	9.91	22.01	0.98	85.8	24.2
2-4	451-1	0.023	0	76.5	1.90	1.31	3.29	8.19	22.64	0.57	73.2	18.8
2-7	467	0.029	0	99.3	0.60	2.20	3.60	4.75	24.5	0.98	94.1	19.5

Zone 4

	1	2	3	4	5	6	7	8	9	10	11	12
Region and reach	Sample No. (18F-)	Most prob T.F.	P.I.	Density	k'_ϕ	σ_ϕ	M_ϕ	$k'_\phi \sigma_\phi M_\phi$	S.L.	V.S.	% max. density	LL
7-5	351-2	0.045	11.3	99.5	1.75	1.78	6.27	19.6	19.6	1.80	88.8	31.3
7-7	375-1	0.042	6.7	102.0	2.00	2.00	4.55	18.2	21.2	1.45	94.3	22.1
7-2	389-1	0.050	10.8	112.9	1.90	2.40	6.25	28.5	25.1	1.80	100	29.5
7-2	389-2	0.042	10.9	72.7	2.00	2.36	6.60	31.2	25.1	1.80	69.6	29.1
7-8	398-1	0.046	10.5	86.1	2.00	2.75	6.98	38.4	19.7	1.50	83.1	28.1
7-8	398-2	0.042	9.9	84.4	1.56	2.05	6.30	20.2	19.7	1.50	81.8	29.2
7-8	399-1	0.053	16.0	73.7	1.70	3.00	7.38	37.6	18.1	1.50	70.6	35.2
7-3	406-2	0.057	15.4	81.6	1.70	3.10	7.53	39.7	15.2	1.90	79.5	35.2
7-3	408-1	0.046	16.8	76.2	1.10	2.02	6.56	14.6	21.1	1.90	72.9	36.2
2-1	435	0.016	9.9	50.8	0.08	2.34	6.44	1.21	19.2	0.61	48.1	29.3
2-8	472	0.030	5.4	93.2	0.50	3.26	4.55	7.42	18.4	0.57	86.3	21.7
2-9	478	0.037	4.4	110.1	0.60	3.01	3.67	6.64	17.7	0.57	98.4	19.5
4-1	513	0.045	12.0	91.0	0.60	3.07	5.70	10.5	17.1	0.90	85.3	32.7
4-4	538	0.050	7.9	103.8	2.00	3.08	4.76	29.3	16.5	1.33	95.7	23.9
4-4	546	0.049	15.1	68.2	1.45	3.08	5.71	25.5	18.7	0.57	65.0	35.1
4-4	547	0.032	6.5	98.7	1.10	2.85	5.85	18.4	15.8	0.82	93.8	25.4
4-9	600	0.044	13.3	112.2	-0.6	4.5	5.1	-13.8	20.2	2.05	99.8	33.1
4-9	601	0.040	16.9	81.9	-0.28	3.14	6.44	-5.66	19.8	0.98	74.6	36.5
4-10	610	0.042	13.6	96.2	0.4	2.40	6.80	6.53	20.2	2.30	89.9	31.7
1-1	628	0.030	4.4	82.8	1.40	1.84	6.10	15.7	14.4	1.15	79.0	22.4
5-5	836-1	0.015	2.5	99.0	-0.9	2.70	4.61	-11.2	15.5	1.32	89.3	17.9
5-5	838-2	0.025	1.3	104.8	1.4	2.60	5.01	18.2	14.2	1.32	89.0	17.6
7-7	377-2	0.027	0	101.1	2.00	1.71	3.57	12.2	13.0	1.45	84.0	14.0

	1	2	3	4	5	6	7	8	9	10	11	12
Region and reach	Sample No. (18F-)	Most prob T.F.	P.I.	Density	k'_ϕ	σ_ϕ	M_ϕ	$k'_\phi \sigma_\phi M_\phi$	S.L.	V.S.	% max. density	LL
7-5	351-1	0.032	13.6	87.4	1.70	1.90	6.26	20.22	19.6	1.80	78.0	31.1
7-5	350-1	0.046	14.0	88.1	2.50	2.78	7.22	50.2	20.8	1.80	79.0	30.8
7-6	368-2	0.034	7.5	98.0	2.0	3.20	4.21	26.94	13.7	1.45	92.0	21.6
7-6	368-3	0.040	10.1	93.9	2.0	3.75	5.40	40.5	13.7	1.45	85.5	24.4
7-6	369-1	0.028	9.6	97.4	0.09	2.78	6.22	1.56	17.8	1.45	85.7	25.3
7-3	406-1	0.041	17.3	74.0	2.00	3.30	7.62	50.3	15.2	1.90	73.0	33.9
2-5	457	0.051	21.0	81.2	1.4	3.68	4.68	24.1	16.4	0.41	79.0	36.4
2-6	461	0.036	10.0	106.1	0.80	3.03	4.75	11.5	16.5	0.82	94.0	25.5
2-6	462	0.034	10.0	103.6	0.50	3.06	4.76	7.28	16.5	0.61	92.0	25.5
2-8	473	0.029	10.2	97.1	0	3.02	3.22	0	21.9	0.66	93.0	25.8
4-6	559	0.027	10.2	85.6	0.62	3.98	6.40	15.8	18.4	0.82	82.2	25.1
4-6	560	0.039	13.0	95.5	1.2	3.60	5.49	23.7	18.4	1.10	91.7	27.9
4-7	570	0.031	13.5	85.2	1.50	2.50	3.98	14.9	23.3	2.05	75.2	28.6
4-11	618	0.039	12.5	95.9	1.1	2.38	6.24	17.82	15.7	0.82	92.4	29.3
4-11	620	0.053	16.5	118.3	0.60	2.21	6.90	9.15	17.7	1.97	100	33.6
5-6	775-1	0.030	24.8	73.6	0.80	4.15	8.18	27.2	8.5	0.90	71.3	42.9
5-6	777-1	0.042	22.8	97.7	0.66	3.94	7.54	19.6	11.4	0.82	94.6	40.6
5-7	785-1	0.042	14.2	89.8	0.86	3.61	7.61	23.6	16.3	0.39	83.0	29.4
5-7	785-2	0.039	14.9	89.8	0.74	3.61	7.35	19.6	16.3	0.41	83.0	30.1
5-7	786-1	0.031	14.8	82.2	1.4	2.84	6.40	25.4	16.1	0.39	75.9	29.0
5-8	793-1	0.032	12.7	109.2	0.89	3.89	6.70	23.2	17.8	0.82	91.2	25.2
5-1	799-1	0.042	6.1	102.9	1.30	6.30	8.68	71.1	15.0	0.41	83.4	17.1
5-2	817-1	0.040	8.2	105.6	1.8	3.85	6.15	42.6	14.2	0.84	91.0	22.0
5-2	817-2	0.034	9.4	103.7	1.1	4.00	6.32	27.81	14.2	0.84	88.9	22.0
5-2	818	0.039	5.3	103.5	1.8	3.30	5.61	33.3	14.5	1.05	88.5	18.3
5-4	826-1	0.034	11.4	86.9	1.8	3.68	6.18	40.9	18.3	0.74	74.6	25.0
5-4	827-2	0.027	7.5	88.7	2.0	3.20	7.08	45.3	16.1	1.25	76.3	19.4
5-4	828-1	0.021	10.0	80.9	0.9	3.10	5.68	15.85	13.9	1.25	69.5	23.1
5-5	837-1	0.033	15.1	88.9	0.7	3.00	5.50	11.6	15.0	1.25	80.0	30.7
5-5	837-2	0.030	8.2	106.4	1.5	3.00	5.36	24.1	15.0	1.32	95.7	20.3
5-5	837-4	0.022	7.5	92.8	1.8	3.10	5.48	30.6	15.0	1.32	83.6	20.1
5-5	838-1	0.039	9.2	91.7	1.50	3.30	5.62	27.8	14.2	1.32	82.7	22.4
5-5	838-3	0.028	9.6	88.6	1.8	2.85	5.20	26.7	14.2	1.32	79.9	22.8
7-7	377-1	0.029	0	117.2	2.00	1.68	3.54	11.9	13.0	1.45	98.0	13.0

volved and time consuming, if the correlations are conducted without the use of a computer.

As the cost of conducting a correlation by the computer was nominal, numerous correlations were tried. Data for the limited range of soils tested were divided into zones parallel to the A line on the plasticity index-liquid limit chart, Figure 2. Since correlations could be conducted rapidly with the computer, it was possible to conduct correlations for each zone, and the same correlations for all data combined.

out

Results obtained from the computer are shown in Table 2. The correlations are numbered from *1* to *21*. For each correlation, results, for *4* zones and the combined data were obtained, resulting in over *100* correlations being conducted. The final data include the best fit linear equation, a standard deviation for each equation, a correlation coefficient for each equation, and a standard deviation for each variable used. For each correlation ,the critical tractive force (tractive force at time erosion started) was used for the dependent variable, and only linear correlations were attempted.

Table 2

MULTIPLE LINEAR CORRELATION VALUES USING ELECTRONIC COMPUTER

Values in General Equation: $TF = a + b_1\,PI + b_2\,D + b_3\,k'_\Phi + b_4\,\sigma_\Phi + b_5\,M_\Phi + b_6\,k'_\Phi\sigma_\Phi M_\Phi + b_7\,SL + b_8\,VS + b_9\,D^{0}/_{0}$

σ_n = standard deviation of variable

Correlation	PI		Density		k'_ϕ		σ_ϕ		M_ϕ		$k'_\phi\sigma_\phi M_\phi$		SL		VS		% maximum density		a	Standard deviation	Correlation coefficient
	b_1	σ_1	b_2	σ_2	b_3	σ_3	b_4	σ_4	b_5	σ_5	b_6	σ_6	b_7	σ_7	b_8	σ_8	b_9	σ_9			
1. Zone 2	0.00246	0.00046	0.00074	0.00010	0.00337	0.00106	-0.00122	0.00223	0.00223	0.00077									-0.05296	0.00336	0.91
Zone 3	0.00111	0.00041	0.00052	0.00015	0.00412	0.00280	0.00077	0.00308	0.00190	0.00133									-0.03268	0.00460	0.76
Zone 4	0.00202	0.00020	0.00025	0.00004	0.00802	0.00079	0.00366	0.00125	-0.00106	0.00096									-0.01559	0.00282	0.97
Zone 5	0.00125	0.00027	0.00046	0.00011	0.00530	0.00178	0.00182	0.00143	0.00007	0.00098									-0.03654	0.00552	0.66
All data	0.00099	0.00015	0.00029	0.00006	0.00432	0.00096	-0.00138	0.00102	0.00129	0.00070									-0.00880	0.00658	0.72
2. Zone 2	0.00311	0.00061			0.00306	0.00135	-0.00053	0.00282	0.00104	0.00094							0.00055	0.00012	-0.02937	0.00422	0.86
Zone 3	0.00127	0.00054			0.00404	0.00339	0.00110	0.00371	0.00124	0.00159							0.00048	0.00022	-0.02450	0.00556	0.62
Zone 4	0.00203	0.00018			0.00782	0.00069	0.00324	0.00109	-0.00142	0.00081							0.00031	0.00004	-0.01597	0.00244	0.98
Zone 5	0.00094	0.00024			0.00533	0.00180	0.00162	0.00143	0.00079	0.00099							0.00056	0.00013	-0.04102	0.00555	0.66
All data	0.00099	0.00015			0.00461	0.00096	-0.00074	0.00096	0.00116	0.00068							0.00036	0.00007	-0.01399	0.00652	0.73
3. Zone 2			0.00062	0.00015	0.00425	0.00143	0.00669	0.00212	0.00218	0.00105			-0.00066	0.00039					-0.03788	0.00455	0.83
Zone 3			0.00046	0.00018	0.00578	0.00341	0.00489	0.00392	0.00397	0.00145			0.00056	0.00051					-0.05478	0.00560	0.61
Zone 4			0.00016	0.00011	0.00799	0.00191	0.00923	0.00268	0.00454	0.00175			0.00076	0.00050					-0.04971	0.00677	0.80
Zone 5			0.00019	0.00011	0.00192	0.00209	0.00083	0.00185	0.00210	0.00117			0.00046	0.00046					-0.00883	0.00716	0.23
All data			0.00022	0.00007	0.00478	0.00118	0.00302	0.00122	0.00283	0.00078			0.00018	0.00023					-0.01877	0.00779	0.57
4. Zone 2					0.00399	0.00168	0.00888	0.00237	0.00114	0.00119			-0.00108	0.00043			0.00031	0.00015	0.00330	0.00534	0.76
Zone 3					0.00556	0.00404	0.00535	0.00467	0.00347	0.00168			0.00059	0.00061			0.00033	0.00024	-0.04028	0.00655	0.38
Zone 4					0.00783	0.00189	0.00891	0.00267	0.00438	0.00161			0.00075	0.00049			0.00022	0.00013	-0.05067	0.00667	0.80
Zone 5					0.00294	0.00200	0.00100	0.00173	0.00257	0.00111			0.00058	0.00043			0.00042	0.00015	-0.03302	0.00667	0.42
All data											No correlation										
5. Zone 2	0.00190	0.00077			0.00353	0.00168	0.00379	0.00333	-0.00032	0.00137					0.00589	0.00312			0.01151	0.00529	0.76
Zone 3	0.00069	0.00056			0.00257	0.00386	0.00260	0.00445	0.00038	0.00199					0.00412	0.00389			0.01526	0.00636	0.44
Zone 4	0.00179	0.00028			0.00856	0.00108	0.00589	0.00168	-0.00312	0.00126					0.00436	0.00181			0.00903	0.00390	0.94
Zone 5	0.00061	0.00029			0.00286	0.00232	0.00097	0.00217	0.00035	0.00126					-0.00070	0.00340			0.01964	0.00704	0.29
All data	0.00073	0.00016			0.00363	0.00102	0.00161	0.00102	-0.00002	0.00071					0.00518	0.00153			0.01437	0.00684	0.69
6. Zone 2	0.00162	0.00042	0.00059	0.00018							0.00035	0.00011			0.00320	0.00175	0.00003	0.00017	-0.03593	0.00339	0.91
Zone 3	0.00072	0.00040	0.00111	0.00037							0.00037	0.00016			0.00232	0.00215	-0.00084	0.00046	-0.00512	0.00404	0.82
Zone 4	0.00188	0.00020	0.00027	0.00025							0.00041	0.00005			-0.00153	0.00175	0.00012	0.00029	-0.01935	0.00347	0.95
Zone 5	0.00104	0.00023	0.00016	0.00023							0.00025	0.00006			0.00061	0.00194	0.00039	0.00029	-0.03375	0.00526	0.70
All data	0.00077	0.00011	-0.00009	0.00013							0.00021	0.00005			0.00502	0.00130	0.00040	0.00017	-0.00631	0.00636	0.74

Correlation	PI		Density		k'_ϕ		σ_ϕ		M_ϕ		$k'_\phi \sigma_\phi M_\phi$		SL		VS		% maximum density		a	Standard deviation	Correlation coefficient
	b_1	σ_1	b_2	σ_2	b_3	σ_3	b_4	σ_4	b_5	σ_5	b_6	σ_6	b_7	σ_7	b_8	σ_8	b_9	σ_9			
7. Zone 2	0.00216	0.00044	0.00053	0.00020											0.00412	0.00197	0.00011	0.00019	-0.03520	0.00388	0.88
Zone 3	0.00104	0.00044	0.00096	0.00043											0.00336	0.00245	-0.00071	0.00053	0.00076	0.00471	0.75
Zone 4	0.00170	0.00039	-0.00040	0.00046											0.00157	0.00338	0.00085	0.00055	-0.01318	0.00692	0.79
Zone 5	0.00095	0.00028	0.00032	0.00027											0.00000	0.00234	0.00001	0.00033	-0.00871	0.00638	0.50
All data	0.00093	0.00011	0.00001	0.00014											0.00531	0.00140	0.00025	0.00018	-0.00121	0.00686	0.69
8. Zone 2			0.00090	0.00020							0.00054	0.00012			0.00506	0.00205	-0.00023	0.00019	-0.04341	0.00414	0.86
Zone 3			0.00136	0.00038							0.00047	0.00016			0.00339	0.00224	-0.00126	0.00043	0.00758	0.00439	0.78
Zone 4			-0.00085	0.00053							0.00034	0.00013			0.00620	0.00371	0.00114	0.00066	0.00718	0.00836	0.66
Zone 5			-0.00036	0.00025							0.00022	0.00008			0.00011	0.00248	0.00081	0.00035	-0.00529	0.00675	0.40
All data			-0.00001	0.00016							0.00034	0.00005			0.00603	0.00156	0.00025	0.00020	0.00119	0.00766	0.59
9. Zone 2	0.00183	0.00042	0.00062	0.00019							0.00039	0.00011					0.00002	0.00017	-0.03432	0.00353	0.90
Zone 3	0.00084	0.00039	0.00112	0.00038							0.00041	0.00016					-0.00081	0.00046	-0.00615	0.00407	0.82
Zone 4	0.00179	0.00017	0.00022	0.00024							0.00039	0.00005					0.00016	0.00029	-0.01850	0.00345	0.95
Zone 5	0.00104	0.00023	0.00017	0.00022							0.00025	0.00006					0.00038	0.00028	-0.03256	0.00518	0.71
All data	0.00081	0.00011	-0.00005	0.00014							0.00022	0.00005					0.00037	0.00018	-0.00169	0.00678	0.70
10. Zone 2	0.00249	0.00042									0.00035	0.00013					0.00051	0.00011	-0.02053	0.00408	0.87
Zone 3	0.00130	0.00045									0.00033	0.00020					0.00046	0.00020	-0.01361	0.00513	0.69
Zone 4	0.00172	0.00015									0.00038	0.00005					0.00042	0.00006	-0.01896	0.00343	0.95
Zone 5	0.00094	0.00019									0.00026	0.00006					0.00057	0.00012	-0.03192	0.00514	0.71
All data	0.00081	0.00011									0.00022	0.00005					0.00030	0.00007	-0.00124	0.00675	0.70
11. Zone 2			0.00067	0.00014							0.00059	0.00013	-0.00060	0.00039					-0.02229	0.00449	0.84
Zone 3			0.00038	0.00017							0.00069	0.00021	0.00051	0.00049					-0.01831	0.00571	0.59
Zone 4			0.00013	0.00011							0.00045	0.00012	0.00124	0.00058					-0.00298	0.00848	0.65
Zone 5			0.00016	0.00011							0.00016	0.00008	0.00028	0.00044					0.01126	0.00717	0.22
All data			0.00017	0.00007							0.00033	0.00006	-0.00007	0.00020					0.01386	0.00820	0.51
12. Zone 2											0.00060	0.00016	-0.00110	0.00044			0.00038	0.00015	0.01736	0.00546	0.74
Zone 3											0.00064	0.00024	0.00049	0.00056			0.00025	0.00022	-0.00476	0.00635	0.44
Zone 4											0.00044	0.00012	0.00123	0.00057			0.00020	0.00014	-0.00756	0.00831	0.67
Zone 5											0.00021	0.00008	0.00035	0.00042			0.00036	0.00015	-0.00707	0.00679	0.38
All data											No correlation										
13. Zone 2	0.00243	0.00035	0.00066	0.00010	0.00276	0.00107													-0.03570	0.00372	0.89
Zone 3	0.00140	0.00035	0.00049	0.00015	0.00473	0.00270													-0.02071	0.00461	0.76
Zone 4	0.00207	0.00016	0.00032	0.00005	0.00657	0.00083													-0.01749	0.00347	0.95
Zone 5	0.00126	0.00025	0.00045	0.00011	0.00505	0.00175													-0.02875	0.00555	0.66
All data	0.00098	0.00010	0.00023	0.00005	0.00455	0.00097													-0.00005	0.00666	0.71

Correlation	PI		Density		k'_ϕ		σ_ϕ		M_ϕ		$k'_\phi \sigma_\phi M_\phi$		SL		VS		% maximum density		a	Standard deviation	Correlation coefficient
	b_1	σ_1	b_2	σ_2	b_3	σ_3	b_4	σ_4	b_5	σ_5	b_6	σ_6	b_7	σ_7	b_8	σ_8	b_9	σ_9			
20. Zone 2			0.00068	0.00017									-0.00114	0.00046					-0.00465	0.00563	0.73
Zone 3			0.00019	0.00021									-0.00001	0.00060					0.02080	0.00736	Imaginary
Zone 4			0.00005	0.00014									0.00143	0.00074					0.00809	0.01076	0.27
Zone 5			0.00011	0.00011									0.00011	0.00045					0.02228	0.00746	Imaginary
All data			0.00011	0.00008									-0.00055	0.00020					0.03445	0.00909	0.29
21. Zone 2	0.00306	0.00050			0.00274	0.00159													0.02238	0.00552	0.74
Zone 3	0.00099	0.00043			0.00326	0.00349													0.02569	0.00603	0.52
Zone 4	0.00161	0.00024			0.00634	0.00140													0.01674	0.00586	0.85
Zone 5	0.00066	0.00025			0.00260	0.00202													0.02390	0.00681	0.38
All data	0.00094	0.00011			0.00417	0.00103													0.02216	0.00714	0.66

Correlation	PI		Density		k'_ϕ		σ_ϕ		M_ϕ		$k'_\phi \sigma_\phi M_\phi$		SL		VS		% maximum density		a	Standard deviation	Correlation coefficient
	b_1	σ_1	b_2	σ_2	b_3	σ_3	b_4	σ_4	b_5	σ_5	b_6	σ_6	b_7	σ_7	b_8	σ_8	b_9	σ_9			
14. Zone 2	0.00252	0.00042	0.00056	0.00022													0.00010	0.00020	-0.03297	0.00410	0.87
Zone 3	0.00128	0.00042	0.00095	0.00044													-0.00065	0.00054	0.00008	0.00486	0.73
Zone 4	0.00179	0.00034	-0.00037	0.00045													0.00083	0.00054	-0.01391	0.00678	0.80
Zone 5	0.00095	0.00027	0.00032	0.00027													0.00001	0.00032	-0.00871	0.00627	0.52
All data	0.00099	0.00011	0.00005	0.00015													0.00021	0.00019	0.00393	0.00729	0.64
15. Zone 2			0.00098	0.00025											0.00786	0.00244	-0.00025	0.00024	-0.04662	0.00521	0.77
Zone 3			0.00131	0.00047											0.00558	0.00263	-0.00134	0.00053	0.02422	0.00546	0.64
Zone 4			-0.00135	0.00057											0.00826	0.00415	0.00168	0.00071	0.01037	0.00957	0.52
Zone 5			-0.00018	0.00026											-0.00039	0.00271	0.00044	0.00036	0.01485	0.00739	Imaginary
All data			0.00020	0.00018											0.00693	0.00178	-0.00007	0.00023	0.01318	0.00882	0.37
16. Zone 2	0.00181	0.00037	0.00064	0.00010							0.00039	0.00011							-0.03422	0.00347	0.91
Zone 3	0.00120	0.00036	0.00049	0.00014							0.00037	0.00017							-0.01869	0.00438	0.78
Zone 4	0.00183	0.00016	0.00036	0.00005							0.00040	0.00004							-0.01753	0.00338	0.95
Zone 5	0.00114	0.00022	0.00044	0.00010							0.00022	0.00006							-0.02630	0.00525	0.70
All data	0.00078	0.00011	0.00021	0.00006							0.00020	0.00005							0.00577	0.00688	0.69
17. Zone 2	0.00242	0.00038	0.00066	0.00011															-0.03247	0.00405	0.87
Zone 3	0.00153	0.00037	0.00045	0.00015															-0.01061	0.00493	0.72
Zone 4	0.00199	0.00033	0.00030	0.00010															-0.00773	0.00701	0.78
Zone 5	0.00096	0.00025	0.00034	0.00011															-0.00853	0.00617	0.54
All data	0.00096	0.00011	0.00021	0.00006															0.00807	0.00730	0.64
18. Zone 2	0.00306	0.00040															0.00054	0.00012	-0.02054	0.00446	0.84
Zone 3	0.00160	0.00044															0.00044	0.00021	-0.00753	0.00544	0.64
Zone 4	0.00192	0.00030															0.00040	0.00012	-0.01271	0.00672	0.80
Zone 5	0.00076	0.00023															0.00037	0.00014	-0.00584	0.00632	0.51
All data	0.00100	0.00011															0.00028	0.00008	0.00356	0.00726	0.65
19. Zone 2													-0.00168	0.00049			0.00038	0.00018	0.03697	0.00645	0.61
Zone 3													-0.00003	0.00062			0.00002	0.00025	0.03524	0.00756	Imaginary
Zone 4													0.00144	0.00072			0.00014	0.00018	0.00050	0.01062	0.31
Zone 5													0.00012	0.00044			0.00023	0.00015	0.01346	0.00733	0.08
All data													-0.00060	0.00020			0.00013	0.00009	0.03427	0.00910	0.29

pounds subscripts

NOTES: PI = plasticity index. D = density, 16 pounds per square foot. k_ϕ = phi skewness. σ_ϕ = phi standard deviation M_ϕ = phi arithmetic mean diameter. SL = shrinkage limit. VS = vane shear, pounds per square foot. D% = percent maximum density.

Data in zones generally resulted in higher correlation coefficients than when all data were combined. However, when the general data are considered, including deviations and consistency of line slopes in zones, it appears that a sufficient relationship to explan the allowable critical tractive force may result from using the combined data. For instance, the equation.

$$T_c = -0.00124 + 0.00081\ PI + 0.00030\ D\% + 0.00022\ M_\Phi\ \sigma_\Phi\ k'_\Phi$$

subscripts

where:

T_c = critical tractive force in pounds per square foot

PI = plasticity index

D% = in-place percent maximum soil density

$M_\Phi\ \sigma_\Phi\ k'_\Phi$ = a mathematical description of soil gradation

subscripts

LL = liquid limit

and limits are:

$$0 < PI < 22$$

$$65 < D\% < 100$$

$$-12 < M\Phi\sigma\Phi k\Phi < 40$$

$$13 < LL < 42$$

is in indicated by the computer to fit the combined data with a correlation coefficient of *0.70* and a standard deviation of *0.00675*. There are several other correlations which also appear to have some significance.

It is emphasized that all correlations attempted were linear, all equations obtained are empirical, and each equation is bounded by limits. However, the general method may be used in future applications where closely controlled soils in a wider range of limits are manufactured, and curvilinear relationships of best fit are attempted.

CONCLUSIONS AND SUMMARY

A study was conducted to aid the development of a method for determining critical tractive forces of cohesive soils from standard soil properties. Soil samples were obtained from canals scattered throughout the western United States and soil and hydraulic properties were measured in the laboratory. The dependent variable, critical tractive force, was measured on a specially designed and calibrated erosion machine. Multiple linear correlations were made using various combinations of soil properties as independent variables. The correlations were made utilizing a program which was already available for a IMB IBM *650* electronic computer. Utilizing the computer and program made it possible to use many groupings of the variables, and obtain the best fit equations, their correlation coefficients, and their standard deviations. Table *2* lists the various correlations. The speed and flexibility of the computer made it possible to arrange data in zones for correlation purposes, as well as correlate all data together. Data were arranged in zones parallel to the *A* line on the plasticity index-liquid limit chart.

It is concluded that a digital computer of the type used for thi this study is satistactory for obtaining various multiple linear correlations. The total thime time consumed in solving the equations is small, even when a large number of independent variables are used. It permits a vast number of correlations in a limited time, and with very little increase in time furnishes additional data which indicated the amount of confidence which can be placed in the correlations.

In some areas of this study, there are insufficient data, and the data are of limited range. In the future, it is planned to manufacture closely controlled soil sample with a wider range of limits, and to extend analysis to curvilinear relations.

All relationships presented in Table 2 are not considered tentative and should be used for design purposes without further confirmation.

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