SOME FACTORS AFFECTING THE STABILITY OF CANALS
CONSTRUCTED IN COARSE GRANULAR MATERIALS

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SYNOPSIS

During the past 3 years, personnel of the Bureau of Reclamation have carried on studies to perfect methods for designing canals in coarse, noncohesive materials. A number of stable canals were located in the San Luis Valley of southern Colorado that had been in use since the late 1800's. Hydraulic measurements were made on test sections selected from the San Luis Valley canals, and other observations and measurements were made on the bed and natural bank materials which formed the test sections. The canals studied were constructed in the alluvial cone deposited by the Rio Grande River on the floor of the San Luis Valley.

The equations of Strickler and Keulegan relating median size of bed material to Manning's "n" value give values of "n" that are slightly lower than the average value determined for the San Luis Valley test sections. Irmay's equation relating maximum diameter and Manning "n" gives a value for "n" slightly lower than the value for San Luis Valley test sections when the bed material size of which there is 10 percent larger by weight is plotted against Manning's "n." The San Luis Valley tests have shown that Manning's "n" decreases as depth and velocity increases for a given channel and boundary conditions.

The use of limiting tractive force in designing canals in coarse, noncohesive material results in definite advantages in that it makes possible more economic designs. The force necessary to cause movement of noncohesive material on a sloping side of a channel is less than that required to cause motion of the same material on a level bed. The ratio of the force necessary to cause impending motion on a sloping side to that required on the level bed is shown to be a function only of the slope of the side and the angle of repose of the material.

INTRODUCTION

To perfect the methods used in the design of canals constructed in earth materials by the Bureau of Reclamation, a comprehensive program of studies was undertaken. One part of this study dealt with the design of canals in coarse, noncohesive material. Most of the work in this field carried on to date consisted of a study of observations taken on canals in the San Luis Valley of Colorado. The following paper presents the results of these observations, and the conclusions regarding the stability of canals.

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SCOPE OF THE STUDIES

In these studies, the major emphasis was on the resistance of coarse, noncohesive material to scour by flowing water and the limiting tractive forces which could be used in the design of such channels. Material was also collected on the hydraulic roughness of the canals. An analytical solution of the resistance to scour of the side slopes in such material was also developed.

THE SAN LUIS VALLEY

The San Luis Valley is a large open valley formed between the Sangre de Cristo and the San Juan Mountains in south-central Colorado. It has a length of about 80 miles and a width of about 40 miles. The bottom is saucer shaped and is formed of sediments brought down from the surrounding mountains and deposited on the bottom by water. Near the sides of the valley the slopes are rather steep, reaching 30 feet per mile, but they flatten as the distance from the sides increases; in the center the slopes are practically level. Where the larger streams enter the valley, they have formed alluvial cones with an apex where the streams emerge from the hills. The canals on which the observations were made were located on the alluvial cone formed by the Rio Grande River. The location of the river, the canals, and the confining hills is shown on Figure 1. Due to the confinement of the hills, the deposit has formed a section of a cone, with an angle between the sides of about 90° and a slope of about 15 feet per mile (2.84 m/km). Near the apex of the cone the subsoil consists of sand, gravel, and cobbles, the size of the cobbles decreasing with the distance from the apex. The location of the sections of the canals observed is shown on Figure 1. The Rio Grande Canal was constructed in 1879, and the Farmers Union and Prairie Canals were constructed in 1887. Most of canals and laterals are very stable. They cover a wide range of conditions of discharge, slope, and bed material size, as shown in Table I, and therefore presented an unusually favorable condition for studies of stable channels.

The nature of the canals is shown on Figure 2. The canals were constructed by slip scrapers drawn by horses. The original dimensions are not available, and it is not known to what extent their shape has been modified by the flowing water or by cleaning operations since they were constructed. There is little doubt that in the upstream canals much of the finer material in the soil through which the canals were constructed has been moved downstream by the flowing water. During most of the years of operation of the canals, some coarse material entered through the head gate; but in the Rio Grande Canal a trap had been recently installed just below the head gate by means of which most of the moving material above sand sizes seemed to be ejected from the canal.
### Table I

**DATA ON SAN LUIS VALLEY CANAL TEST SECTIONS**

<table>
<thead>
<tr>
<th>Test section</th>
<th>Median size bed material inches</th>
<th>1950 measurements</th>
<th>1952 measurements</th>
<th>Maximum sustained discharge cfs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q cfs</td>
<td>Mean velocity ft/sec</td>
<td>Slope of energy gradient</td>
<td>Manning's &quot;n&quot;</td>
</tr>
<tr>
<td>1</td>
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<td>944</td>
<td>4.80</td>
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</tr>
<tr>
<td>2</td>
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<tr>
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<td>456</td>
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<td>1.34</td>
<td>0.00319</td>
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<tr>
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<td>7.6</td>
<td>1.52</td>
<td>0.00973</td>
</tr>
<tr>
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<td>124.5</td>
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</tr>
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<tr>
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<td>0.00202</td>
</tr>
<tr>
<td>17</td>
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</tr>
<tr>
<td>18</td>
<td>0.83</td>
<td>62.5</td>
<td>2.21</td>
<td>0.00083</td>
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</table>
OBSERVATIONS IN CANALS AND ON CANAL BED MATERIAL

A reconnaissance was made of the canals, and reaches were selected for preliminary study. Cross sections at 100-foot intervals were taken of these reaches when the canals were dry. The data were plotted, and the most regular portions of the reaches were selected for use in making the hydraulic measurements. The length of the reaches ranged from 600 to 2,200 feet and were very straight and regular in section. The water surface elevations were determined at 200-foot intervals by a specially made instrument shown in Figure 3. During an observation, the instrument was placed on an iron pin which had been driven in the canal near the bank. The top elevation of the pin was determined by accurate leveling. The instrument was a stilling well and hook gage, the well being connected to the water through a horizontal tube directed upstream with holes in the sides of the tube. From the elevation thus determined and the plotted cross sections, the areas of flow were determined. In most cases, the discharges were determined from rating curves of various control sections in the canals which had been calibrated by numerous current meter discharge measurements. In other cases, individual current meter discharge measurements were made. From the areas and discharges, the mean velocity and the velocity head at each section were computed. The sum of the velocity head and water surface elevation at the point gave the elevation of the energy gradient. The slope of this gradient between observation points was computed, and the values averaged to obtain the slope used in computing the hydraulic roughness and tractive force. The cross-sectional areas and hydraulic radii and depth of the sections were averaged to obtain the mean values in the computations.

The canals had sustained without scour, in some cases, flows considerably above that observed. The largest flow carried over a period of 22 years was determined for each section from records of the Colorado State Engineer and the irrigation companies. This discharge was called the maximum sustained flow. The elevation of the water surface for the maximum sustained flow was estimated, from the cross-section dimensions based on the observed water levels and slope and the hydraulic roughness computed from the measurements. Based on the maximum sustained discharge and the observed slope, the tractive force which they would cause on the bottom of the canal was determined to obtain a safe value for use in design.

Samples were taken of the material forming the surface layer of the bottom of the canal over 1 square yard of area in each section where the material composition represented, as nearly as could be determined from visual observation, average conditions for the section. Mechanical analyses of the bed samples are shown in Table II. Samples of the material in which the canal was constructed were obtained by excavating into the bank of each section. Mechanical analyses of the natural bank material were made with the results shown in Table III.
DETERMINATION OF HYDRAULIC ROUGHNESS

The most universally used discharge formula for open channels in the United States is that of Manning:

\[ Q = \frac{1.486}{n} A R^{2/3} S^{1/2} \]

where \( n \) is the boundary roughness coefficient, \( Q \) the discharge in cfs, \( A \) the cross section area in square feet, \( R \) the hydraulic radius in feet, and \( S \) the slope of the energy gradient. In computing the hydraulic roughness of the test sections, Manning's "\( n \)" value and equation were used in all computations. Where other formulas are shown the roughness value is shown as a function of Manning's "\( n \)."

Hydraulic measurements, including discharge and water surface slope, were made on 15 test sections in the San Luis Valley canals in 1950 and 14 test sections in 1952. By use of Manning's formula, the average roughness coefficient was determined for each test section. See Table I.

Size analysis of the average top layer of bed material for each test section was determined by sieving, the results being as shown in Table III. Following the usual procedure, the median size of bed material for each test section was plotted against Manning's roughness value "\( n \)" as shown in Figure 4. Two points for Test Sections 8 and 10 plotted for the 1950 data fall considerably to the right of all other points. The depths on the 1950 measurements for these two points were extremely low and consequently high "\( n \)" values were obtained.

Strickler 3/ arrived at a formula based on empirical studies made in Switzerland in 1923, which relates Manning's roughness coefficient "\( n \)" and median size of bed material \( K_{50} \) in inches. His formula \( n = \frac{K_{50}^{1/6}}{44.4} \) is plotted on Figure 4 showing its relationship to the plotted data obtained from the San Luis Valley test sections.

Keulegan 4/ proposed a formula similar to Strickler's relating Manning's roughness value to the equivalent median grain roughness of bed material \( K_{50} \) in inches. Keulegan's formula \( n = \frac{K_{50}^{1/6}}{46.9} \) is also plotted on Figure 4. Both Keulegan's and Strickler's formulas are dimensionless, assuming \( n \) has the dimensions of \( L^{1/6} \) as pointed out by


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<tr>
<th>Canal test section</th>
<th>Percent passing by weight</th>
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<tr>
<td>8</td>
<td>87.6</td>
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<td>9</td>
<td>99.4</td>
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<td>61.6</td>
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<td>11</td>
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<td>14</td>
<td>100</td>
</tr>
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<td>15</td>
<td>83.9</td>
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<td>16</td>
<td>80.4</td>
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<tr>
<td>18</td>
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### Table III

**SAN LUIS VALLEY CANAL TEST SECTIONS**

**NATURAL MATERIAL IN BANKS**

**MECHANICAL ANALYSIS**

**U. S. Standard Sieve Size**

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<thead>
<tr>
<th>Canal section</th>
<th>3&quot;</th>
<th>1-1/2&quot;</th>
<th>3/4&quot;</th>
<th>3/8&quot;</th>
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<th>#8</th>
<th>#16</th>
<th>#30</th>
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</tr>
</tbody>
</table>
Rouse 5/ and give slopes of 6 when plotted on log-paper. This slope follows quite closely the average slope of points obtained from the San Luis Valley data.

Irmay 6/ has used a somewhat different approach in finding a relationship between Manning's roughness coefficient and size of bed material. Instead of using the median diameter, he uses what he calls the maximum diameter and relates this to Manning's "n." His relation is \[ n = \frac{K_m}{49.0} \] where \( K_m \) is the maximum diameter in inches of particles in the bed. Where the bed is made up of mixtures of sizes, the maximum size of bed material may be that size of only one rock in a given area. To show a relationship between Irmay's formula and the data obtained on the San Luis Valley canals, the bed material size of which 10 percent was larger was plotted against Manning's "n" value for each test section. Figure 5 shows a plot of Irmay's formula together with the San Luis Valley data. Although Irmay's equation gives slightly lower values of "n" than an average line through the plotted points, there is quite good agreement.

It is true that the largest particles or projections from a boundary have the greatest influence on determining the roughness coefficient for flow in open channels. Just what percentage of the boundary area of an open channel must be covered by a given size of bed material or projection to make this size predominant in determining the roughness coefficient has not been determined.

In designing canals in coarse, granular material, it is necessary to choose a value of Manning's roughness coefficient based on a knowledge of the natural bank material. If the bank material has a considerable percentage of large particles, then it is advantageous to design the canal so the smaller material will be scoured out and the larger particles will remain in place, making the canal stable. Figure 6 is a plot showing the relationship between Manning's "n" value for the stable canal test section and the size of natural bank material of which there is 25 percent larger. An average line for the plotted points drawn to a slope of 6 gives the relationship of \[ n = \frac{K_{25}^{1/6}}{39} \] where \( K_{25} \) is the size in inches of which 25 percent is larger. This relationship should hold for other alluvial deposited materials having similar gradations. As the canals from which this relation was determined were unusually straight and uniform in section, if this relation is used for purposes of canal design, a factor of safety should be added.

It is seen on Table I that the "n" values for the test sections are lower in every section except No. 15 for the 1952 measurements than for the 1950 measurements. It is also shown that the discharge and consequently the mean velocities and values of depth and hydraulic

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radii are higher in 1952 than 1950. These measurements support the general knowledge that Manning's roughness coefficient "n" becomes smaller for the same boundary material as the depth of flow increases. A plot showing the relationship between relative roughness $K_{35}/R$, the diameter of bed material of which 35 percent is larger over the hydraulic radius, and Manning's "n" value is shown in Figure 7. The points thus plotted for the 1950 and 1952 measurements did follow closer to an average line drawn through the points with a slope of 6 than the plot of average size of bed material versus Manning's "n." Although the relative roughness versus Manning's "n" plot (Figure 7) indicates that Manning's equation and "n" values are not constant for a given particle size throughout the range of hydraulic radii found in the San Luis Valley tests, it is felt that there are not enough data to justify the recommendation of a new velocity formula.

DETERMINATIONS OF LIMITING TRACTIVE FORCE

Because of the wide range of slopes and material sizes available in the San Luis Valley canals, the conditions were particularly favorable to the determination of the limiting velocities and boundary shears that could be safely used for the design of canals through coarse, noncohesive material. As shown in Table I, the median diameters of the material ranged from 0.79 to 3.23 inches, the slopes from 0.00080 to 0.00973, and the mean velocities from 1.34 to 6.53 feet per second.

Extensive studies carried on by the Bureau of Reclamation 7/ have emphasized the use of tractive force or shear as the best criterion for design of canals for stability against scour, and there is good reason to believe that this will displace the use of mean velocities. For this reason, the analysis of the results of these experiments has been presented on the basis of limiting shear allowable in canals, although the basic data are given from which one may obtain the velocity relations if desired.

Canals constructed through coarse, noncohesive material rarely, if ever, pass through a material of a narrow range of particle sizes. In practically every case the material covers a considerable range, usually extending from sand on up to gravel or cobbles. Unless the boundary shear values and velocities are very low, some material will be scoured out of such a canal when it is put into operation. Whether or not the results are satisfactory depends upon whether or not the amount moved out produces unsatisfactory conditions. In the San Luis Valley Canal sections on which measurements were made, the finer material had been removed from the top layer of the bed and a paving of coarser material was left. Between the larger particles, however, were found smaller ones; and even sand particles were present immediately under the top particles. It was hoped that substantially all of the material above a certain size would be found to have been removed from the bed so that this size could

be used as an index of design, but this was found not to be the case; and no satisfactory analysis of the data based on a specific size left in the bed was found.

It was decided, therefore, to base the criterion of design on the relation of the tractive forces produced by the maximum sustained flow in the canals to the nature of the natural material through which the canal was originally constructed. Mechanical analyses of samples of undisturbed material taken from the canal banks are given in Table III. An advantage of this approach is that samples of material through which a canal is proposed can be readily obtained before the canal is built, and mechanical analyses of them can be made.

In order to make the results of these studies available for design purposes as soon as possible, an arbitrary parameter was adopted to describe the nature of the natural material. This was taken to be the sieve size of which 25 percent of the weight of the material was larger. It was recognized that a better parameter can probably be obtained and studies along this line are planned; but as these studies would require considerable time, the approximate analysis was adopted so the results could be used until a better parameter could be developed.

By plotting the 25 percent larger size of the natural bank material against the tractive force for the maximum sustained flows in the various sections, as shown in Figure 8, it is possible to arrive at a relation between these variables which can be used in design. The line A represents a probable relation for the safe limiting value of tractive force for the various sizes; but to be conservative, a factor of safety has been added and the line B is recommended for design purposes. For ease in remembering this relation, it may be stated in English units as: the limiting tractive force in pounds per square foot recommended for design is equal to four-tenths of the size in inches of the sieve opening on which 25 percent of the weight of the natural bank material will be retained. In metric units, this is practically equivalent to the relation: tractive force in Kg per m² is equal to 0.8 the sieve opening in centimeters on which 25 percent of the material will be retained.

**CRITICAL TRACTIVE FORCE ON CHANNEL SIDE SLOPES**

Since the San Luis Valley canals were shallow and the side slopes very flat, the results obtained were considered to be applicable to the level bottoms of canals. There is good reason to believe, however, that a given material will move at a lower tractive force on the sloping side than on the level bottom of a canal; 8/ and where the side slopes are as large as those commonly used in canal design, limiting tractive forces on the sides are likely to be appreciably smaller than those on the canal bottom. This factor must therefore be considered in canal design. Since no experimental data were available to determine

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the magnitude of the limiting values for sides having various slopes, the following analytical analysis applicable to canals in coarse, non-cohesive material was devised. As the cohesion in the material is increased, the effect of the side slope on resistance to scour becomes less, and for highly cohesive material is probably of little or no importance. Up to the present, analysis has been developed only for the case of coarse, noncohesive material acted on by clear water.

**Forces Acting on a Particle on a Sloping Canal Side**

As is pointed out in the previously mentioned reference 7, a particle resting on the sloping side of a canal in which water is flowing is acted on by two forces (1) the force due to the action of the flowing water on the particle which acts in the direction of the flow, and (2) the force due to the weight of the particle which tends to cause the particle to roll down the side slope. Movement of the particle will begin when the resultant of these two forces is large enough to start it.

The force on the particle downward in the direction of the sloping side for particles below the water surface, as shown in Figure 9, is equal to $W_s \sin \phi$ where $W_s$ is the submerged weight of the particle and $\phi$ is the angle with the horizontal of the sloping side. The force on the particle due to the velocity of the water is $a T_s$, in which "a" is the effective area of the particle, and $T_s$ is the tractive force on the side of the canal. $T_s$ acts in the direction of flow and is expressed in units of weight per unit area. It would be difficult to determine the value of "a" for any particle, but for the purpose of this study it is not necessary as it is canceled out in the solution.

The resultant of these two forces which are at right angles to each other is $\sqrt{W_s^2 \sin^2 \phi + a^2 T_s^2}$, and when this force is large enough, the particle will move.

**Magnitude of the Critical Tractive Force on Sloping Side**

A determination of the conditions for impending motion can be made by analyzing the forces which act on particles on a surface sloping at the angle of repose for the material involved. Consider the condition of a particle on the side of a canal filled with water, in which there is no flow, when the side slope is equal to the angle of repose $\theta$. If $W_s$ is the submerged weight of the particle, the force acting on the particle downward in the direction of the surface slope is $W_s \sin \theta$ and that acting normal to the surface is $W_s \cos \theta$, and the ratio of these two components is $\tan \theta$. Since for a given material the angle of repose is nearly independent of the particle size, motion of the particles is independent of whether $W_s$ is large or small, but depends only on whether the ratio of the component of the force down the slope to that normal to the slope is greater than $\tan \theta$. 

11
Since motion is impending, the resistance to motion of the particle is equal to the force tending to cause motion, or to $W_s \sin \theta$, which is also equal to $W_s \cos \theta \tan \theta$. It is reasonable to suppose that resistance to motion of a particle on any other slope or on the level is also equal to the normal force times $\tan \theta$, and this hypothesis is therefore adopted. 9/

Applying the hypothesis to the case of a particle on the side of a canal in which water is flowing, the resultant of the two forces which act on the particle should cause impending motion when the ratio of this resultant force to the normal force acting on the particle is equal to $\tan \theta$. Then

$$\sqrt{\frac{W_s^2 \sin^2 \theta + a^2}{W_s \cos \theta}} = \tan \theta$$

and

$$T_s = \sqrt{\frac{W_s^2 \cos^2 \theta \tan^2 \theta - W_s^2 \sin^2 \theta}{a^2}}$$

Also, motion of a particle on a level surface is in a state of impending motion due to the tractive force when $a \# L/W_s = \tan \theta$, and $T_L = \frac{W_s \tan \theta}{a}$. In this case $a \# L$ is the tractive force which will cause impending motion of a single particle on the level surface. "a" is again the effective area of a single particle and $T_L$ is the average tractive force per unit area.

Ratio of Force on Sloping Side to that on Level Surface Necessary to Cause Impending Motion

For purposes of design it is desirable to know the ratio of the tractive force per unit area, $T_s$ which will cause impending motion on the canal side to $T_L$ which will cause impending motion on a level surface. If this ratio is $K$, then

$$K = \frac{T_s}{T_L} = \sqrt{\frac{\frac{W_s^2 \cos^2 \theta \tan^2 \theta - W_s^2 \sin^2 \theta}{a^2}}{\frac{W_s \tan \theta}{a}}}$$

9/ This hypothesis has also been used by A. A. Kalinske in "Movement of Sediment as Bed Load in Rivers," Transactions, AGU, Vol. 28, August 1947, p. 616.
It will be seen that this ratio is a function only of the inclination of the sloping side $\theta$ and the angle of repose of the material $\theta$. Figure 10 is a diagram giving the value of $K$ for various values of $\theta$ and $\theta$.

A solution for the value of $K$ was independently derived by C. H. Fan,¹⁰ which is

$$K = \cos \theta \sqrt{1 - \frac{\tan^2 \theta}{\tan^2 \theta}}$$

Although the two equations are not identical, the difference between the values of $K$ obtained by them is negligible.

In designing a canal in coarse, noncohesive material so that the material on the side slopes will not be moved, it is necessary to establish that the shear at any point on the sides is less than $K$ times the shear that will cause impending motion for that material on a level surface. As the shear on the sides and bottom of a channel is not uniformly distributed, it is necessary also to know this distribution. For trapezoidal cross sections, this distribution is given in a report previously mentioned.⁷ The magnitude of shears which are safe on a level surface for various sizes of coarse, noncohesive material and data on angles of repose are also given in this report. Observations on angles of repose failed to detect any material difference between values under water and those in the dry condition.

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FIGURE 1

SAN LUIS VALLEY PROJECT
CANAL TEST SECTIONS
STABLE CHANNEL STUDIES
Pictures showing nature of typical test sections.
Gage used for measuring water surface elevations accurately in rough flow.
FIGURE 4

KEULEGAN'S EQUATION
\[ n = \frac{K_{50}}{46.9} \]

STRICKLER'S EQUATION
\[ n = \frac{K_{50}}{44.4} \]

GROSS STABLE CHANNEL STUDIES
SAN LUIS VALLEY CANALS
MEDIAN BED MATERIAL SIZE IN INCHES

1950 DATA
1952 DATA
IRMAY'S EQUATION

\[ n = \frac{k_{m}^{6/3}}{49.0} \]

**FIGURE 5**

**STABLE CHANNEL STUDIES**
**SAN LUIS VALLEY CANALS**

**BED MATERIAL, SIZE IN INCHES**

10% IS LARGER BY WEIGHT
FIGURE 6

EQUATION

\[ n = \frac{K_{25}}{39} \]

STABLE CHANNEL STUDIES
SAN LUIS VALLEY CANALS

NATURAL BANK MATERIAL
SIZE IN INCHES
25% IS LARGER BY WEIGHT

© 1950 DATA
△ 1952 DATA
FIGURE 7

STABLE CHANNEL STUDIES
SAN LUIS VALLEY CANALS

MANNINGS n

RELATIVE ROUGHNESS

K_{35} = \frac{R}{R}

BED MATERIAL SIZE IN FEET - 35% LARGER BY WEIGHT

HYDRAULIC RADIUS - FEET

.01 .02 .03 .04 .05 .06 .07 .08 .09 .10

1950 DATA

1952 DATA
Line representing relations of tractive forces $\text{lb/ft}^2 = \frac{1}{2}$ diameter in inches

Tractive force $\text{KG/m}^2 = \text{diameter in centimeters (Approximately)}$

Tentatively recommended for design

Tractive force in $\text{lbs/ft}^2 = 0.4$ diameter in inches

RESULTS OF STUDIES ON SAN LUIS VALLEY CANALS

FIGURE 8
TRACTIVE AND GRAVITY FORCES
Based on equation $K = \cos \phi \sqrt{1 - \frac{\tan^2 \theta}{\tan^2 \phi}}$

$\theta$ - Angle of Repose

$\phi$ - Angle of side slope

For non-cohesive material

$K = \text{critical tractive force on sides in fraction of value for level bottom}$

Figure 10