Denver, Colorado, February 20, 1939.

MEMORANDUM TO ENGINEER C. P. VETTER
(H. G. Dewey, Jr.)

Subject: Comparison of model discharge coefficients with theoretical discharge coefficients for flow over roller gates.

1. During the 1 to 48 scale model studies of the Roza Diversion Dam, the left roller gate was calibrated for flow over the top of the gate. The data, converted to prototype, are shown in table 1.

2. In Technical Memorandum No. 562, "Design of Roller Gates," by C. P. Vetter, an analysis has been made of the pressure, velocity, discharge, and discharge coefficients for flow over roller gates by applying hydrodynamic principles. This analysis is reviewed here in part in order to present the experimental data more clearly.

3. The function that expresses the flow between two concentric boundaries is,

$$W = f(Z) = f(x + iy) = iC \log_e Z, \text{ where C is a}$$

real constant.

Now $\quad W = iC \log_e Z = iC \log_e (re^{i\theta}) = iC \log_e r + iC(i\theta)$

or $\quad W = -C\theta + iC \log_e r = \phi + i\psi$

hence

$$\phi = -C\theta \text{ .... (1)}$$

$$\psi = C \log_e r \text{ .... (2)}$$

The streamlines are given by, $\psi = C \log_e r$, where ψ is taken as any real arbitrary constant; hence, when C log r = constant, the streamlines are circles about the point of origin. One boundary or streamline is evidently the periphery of the roller gate. It is assumed that the free water surface is the other boundary having a radius $r = r_2$. This is not correct, however, but gives satisfactory results if the ratio of the radius of the roller gate to the depth of flow is not too small. The effect on the pressures along the periphery of the roller gate is not very great.

Assuming the streamlines are circles, the velocity will be tangential at all points, so that

$$V = V_\theta = \frac{\partial \phi}{r \partial \theta} = \frac{\partial(-C\theta)}{r\partial\theta} = -\frac{C}{r}$$

The constant, C, is considered to be positive for flow in a clockwise direction with respect to the origin and the observer, hence

$$V_\theta = \frac{C}{r} \qquad (3)$$

To determine C, integrate between r_1 and r_2, where r_1 is the radius of the roller gate, the lower boundary, and r_2 is the radius to the free surface from the origin.

Now $\qquad dq = V_\theta \cdot dr$, per foot

Then $\qquad q = \int_{r_1}^{r_2} V_\theta \, dr = \int_{r_1}^{r_2} \frac{C}{r} \cdot dr = C(\log_e r_2 - \log_e r_1)$

hence $\qquad q = C \log_e \frac{r_2}{r_1}$

or $$C = \frac{q}{\log_e \frac{r_2}{r_1}} \qquad (4)$$

Since $V_\Theta = \frac{C}{r}$, then we have that

$$V_\Theta = \frac{q}{r \log_e \frac{r_2}{r_1}} \qquad (5)$$

The velocity at the surface of the gate is, therefore,

$$V_{r1} = \frac{q}{r_1 \log_e \frac{r_2}{r_1}} \qquad (6)$$

and the velocity at the free surface is,

$$V_{r2} = \frac{q}{r_2 \log_e \frac{r_2}{r_1}} \qquad (7)$$

If it is assumed that at the free surface, $r = r_2$, that all available energy has been transformed to velocity, then if the total available head at some point on the water surface is h'(figure 1):

$$\frac{(Vr_2)^2}{2g} = \frac{q^2}{2g\, r_2^{\,2} \left(\log_e \frac{r_2}{r_1}\right)^2} = h' \qquad (8)$$

If q is known, this equation can be solved for r_2.

In order to get consistent results, q, the discharge per linear foot, should be determined by a method analogous to the one used for determining the velocities. For this purpose the highest point of the gate (having horizontal tangent) is assumed to act as control. This assumption is not correct, but the approximation is close enough for the purpose. A control section may be defined as a section which carries the maximum amount of water with a minimum amount of energy. Mathematically the depth at the control is obtained by differentiating the discharge with respect to the depth and equating the differential to zero. From equation (8),

$$q^2 = 2gh' \, r_2^{\,2} \left(\log_e \frac{r_2}{r_1}\right)^2$$

therefore, $q = \sqrt{2gh'} \cdot r_2 \log_e \frac{r_2}{r_1}$, but $r_2 = r_1 + d - h'$ (figure 1)

hence, $$q = \sqrt{2gh'}\,(r_1 + d - h') \log_e \left(\frac{r_1 + d - h'}{r_1}\right) \quad \text{.......... (9)}$$

$$\frac{dq}{d(h')} = \sqrt{2g}\left(\frac{r_1 + d - h'}{2\sqrt{h'}} - \sqrt{h'}\right)\left(\log_e \left(\frac{r_1 + d - h'}{r_1}\right)\right) - \sqrt{2gh'}$$

Letting $\frac{dq}{d(h')} = 0$, we have

$$\log_e \left(\frac{r_1 + d - h'}{r_1}\right) = \frac{2h'}{r_1 + d - 2h'} \quad \text{........................ (10)}$$

From this equation h' may be determined for any values of r_1 and d. When h' has been determined, q is found from equation (9);

with $$q = (r_1 + d - h') \log_e \left(\frac{r_1 + d - h'}{r_1}\right) \sqrt{2gh'}$$

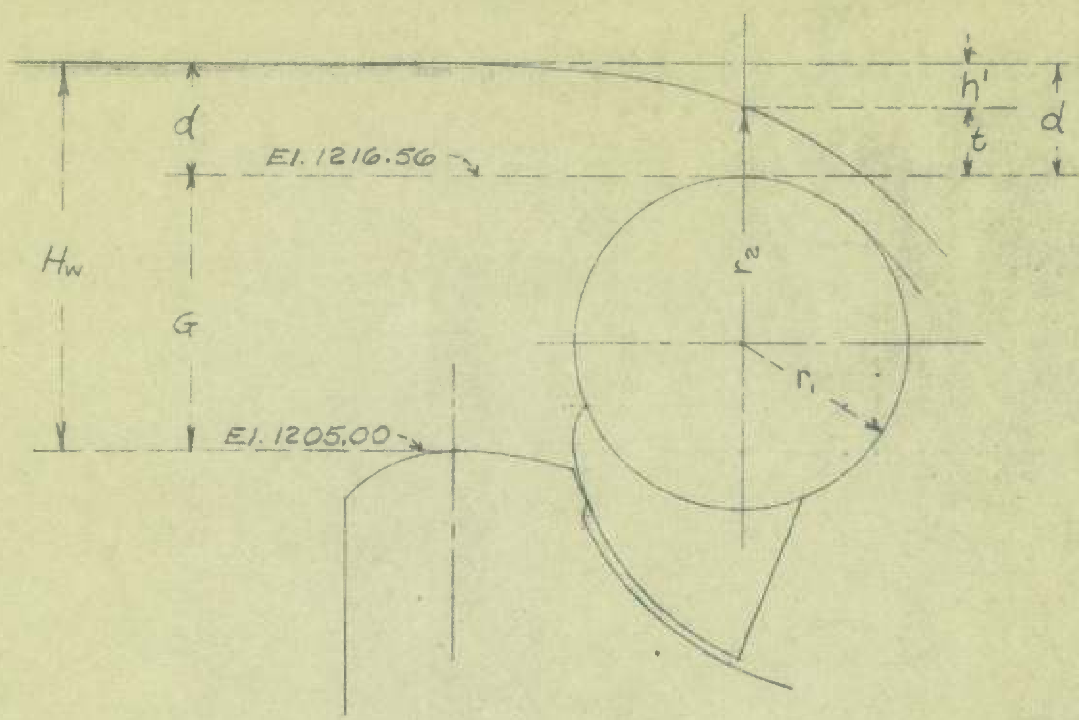
or $$q = C\, d^{3/2} \quad \text{.. (11)}$$

then $C = (r_1 + d - h')\sqrt{\frac{2gh'}{d^3}}\log_e\left(\frac{r_1 + d - h'}{r_1}\right)$ .... (12)

4. In table 1, the values of C were obtained from the relation $C = \frac{q}{d^{3/2}}$, equation (11). The discharge and depth were measured on the model. Figure 2 shows these experimental values of C plotted against d. Figure 3 shows the experimental plot of q vs. d. The values of C were also obtained (table 2) using equation (12) and inserting experimental values of d and h'. The value of r_1 was constant, $r_1 = 7.0$ feet (1.75-inch model). These coefficients are plotted on figure 2. The values of q (table 3) were obtained from the relation, $q = Cd^{3/2}$, where C was taken from table 2, and d from the measured values on the model. These data are plotted on figure 3. The theoretical discharge coefficients were obtained by first solving for h' in equation (10), then substituting in equation (12). These data (table 4) are plotted on figure 2. The theoretical discharge was obtained from the relation, $q = Cd^{3/2}$, where the values of C and d were taken from table 4. These data (table 5) are plotted on figure 3.

5. From the results obtained, it can be readily seen that:

(1) The experimental discharge coefficients obtained from the relation $q = Cd^{3/2}$, in which q and d are measured, are less than the theoretical discharge coefficients obtained from equating (12). See figure 2.



$$Q = Cd^{3/2} \text{ per foot, where}$$

$$C = \left[r_1 + d - h'\right] \sqrt{\frac{2gh'}{d^3}} \cdot \log_e \left[\frac{r_1 + d - h'}{r_1}\right]$$

DISCHARGE OVER ROLLER GATE

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TABLE 1 - EXPERIMENTAL DATA.

RUN No.	$Q_{ft.}$ C.F.S	H_W	H_G	d	t	h'	r_1	$d^{3/2}$	$C = \frac{Q}{d^{3/2}}$
1	9.147	13.608	11.56	2.050	1.376	0.674	7.00	2.935	3.117
2	13.632	14.208	11.56	2.650	1.728	0.922	7.00	4.314	3.160
3	18.323	14.736	11.56	3.178	2.080	1.098	7.00	5.666	3.234
4	22.838	15.168	11.56	3.610	2.400	1.210	7.00	6.859	3.330
5	27.733	15.648	11.56	4.090	2.702	1.388	7.00	8.272	3.353
6	32.087	16.032	11.56	4.474	2.976	1.498	7.00	9.463	3.391
7	36.616	16.404	11.56	4.846	3.200	1.646	7.00	10.668	3.432

TABLE 2 - EXPERIMENTAL DATA IN THEORETICAL FORMULA

RUN No.	r_1	d	h'	r_1+d-h'	$\sqrt{\frac{2gh'}{d^3}}$	$\log_e\left[\frac{r_1+d-h'}{r_1}\right]$	$C=\left[r_1+d-h'\right]\sqrt{\frac{2gh'}{d^3}}\cdot\log_e\left[\frac{r_1+d-h'}{r_1}\right]$
1	7.00	2.050	0.674	8.376	2.245	0.17948	3.374
2	7.00	2.650	0.922	8.728	1.786	0.22073	3.441
3	7.00	3.178	1.098	9.080	1.484	0.25905	3.491
4	7.00	3.610	1.210	9.400	1.287	0.29490	3.567
5	7.00	4.090	1.388	9.702	1.143	0.32642	3.620
6	7.00	4.474	1.498	9.976	1.038	0.35417	3.667
7	7.00	4.846	1.646	10.200	0.965	0.37637	3.705

TABLE 3 - EXPERIMENTAL "Q" FROM THEORY.

d	$d^{3/2}$	C From Table 2	$Q_{ft.}$	% Greater than meas. Q - Table 1
2.050	2.935	3.374	9.903	8.26
2.650	4.314	3.441	14.844	8.89
3.178	5.666	3.491	19.780	7.95
3.610	6.859	3.567	24.466	7.13
4.090	8.272	3.620	29.945	7.98
4.474	9.463	3.667	34.701	8.15
4.846	10.668	3.705	39.525	7.94

TABLE 5 - THEORETICAL "C"

d	$d^{3/2}$	C From Table 4	Q_{ft}
1.0	1.000	3.232	3.232
2.0	2.828	3.369	9.528
3.0	5.196	3.499	18.181
4.0	8.000	3.624	28.992
5.0	11.180	3.743	41.847
6.0	14.696	3.856	56.668

TABLE 4 - THEORETICAL DATA.

r_1	d	h'	r_1+d-h'	$\sqrt{\frac{2gh'}{d^3}}$	$\log_e\left[\frac{r_1+d-h'}{r_1}\right]$	$C=\left[r_1+d-h'\right]\sqrt{\frac{2gh'}{d^3}}\cdot\log_e\left[\frac{r_1+d-h'}{r_1}\right]$
7.00	1.00	0.325	7.675	4.575	0.09204	3.232
7.00	2.00	0.633	8.367	2.257	0.17838	3.369
7.00	3.00	0.932	9.068	1.491	0.25883	3.499
7.00	4.00	1.224	9.776	1.110	0.33401	3.624
7.00	5.00	1.510	10.490	0.882	0.40451	3.743
7.00	6.00	1.792	11.208	0.731	0.47072	3.856

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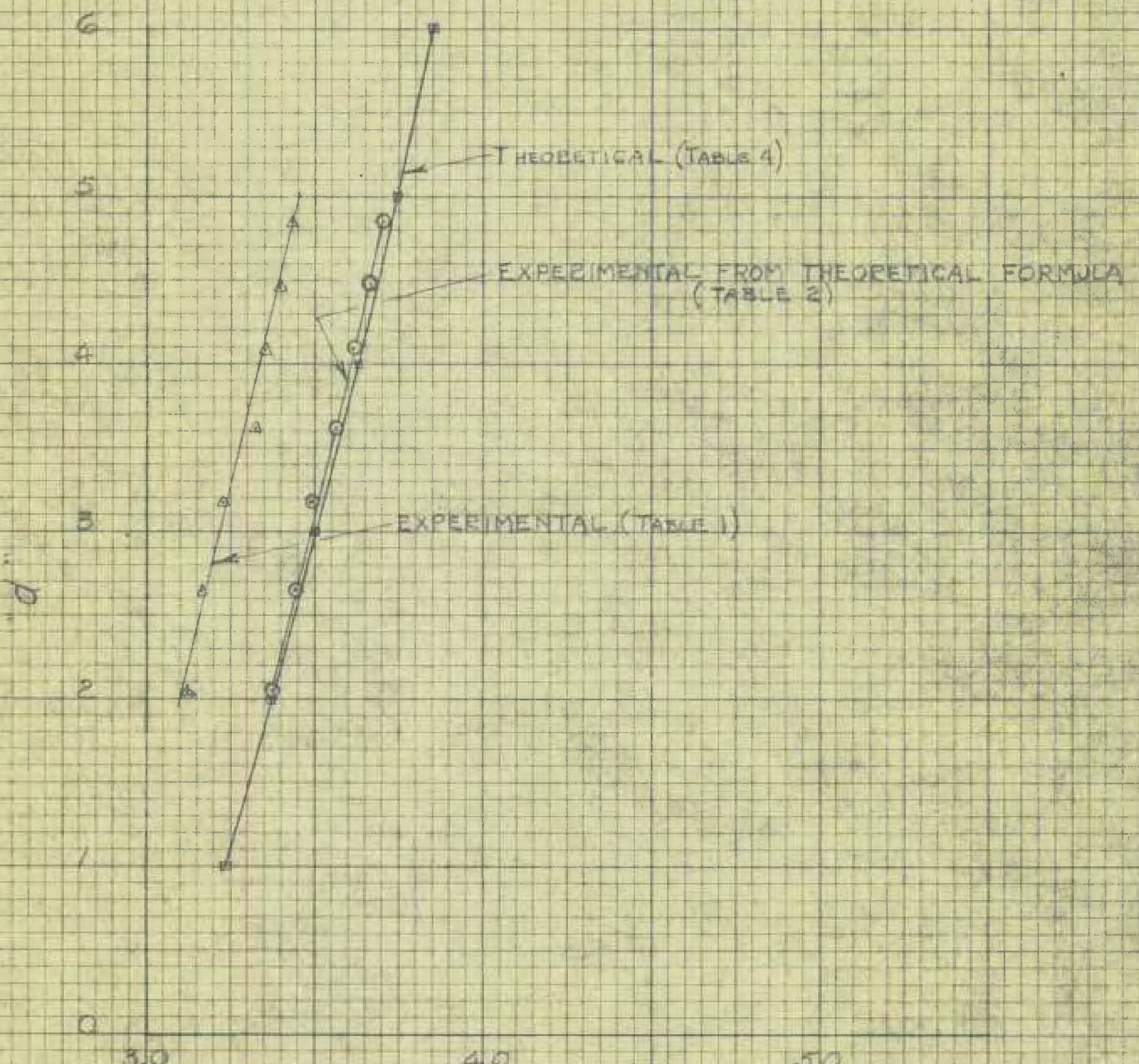
FIGURE 2



COMPARISON OF EXPERIMENTAL AND THEORETICAL DISCHARGE COEFFICIENTS
LEFT ROLLER GATE
ROZA DIVERSION DAM
- HYDRAULIC MODEL STUDIES -

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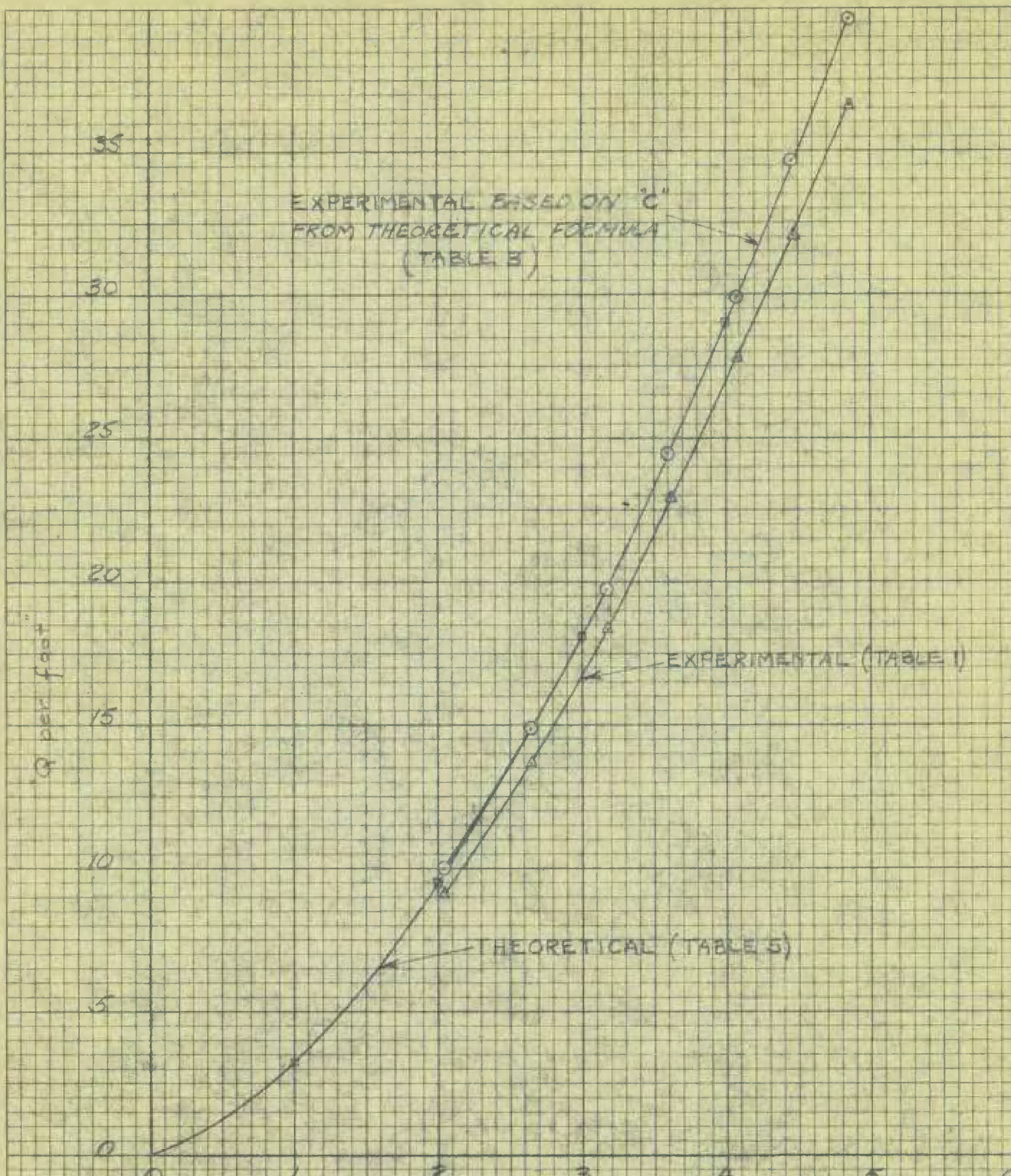
FIGURE 3



COMPARISON OF EXPERIMENTAL
AND THEORETICAL DISCHARGE
LEFT ROLLER GATE
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- HYDRAULIC MODEL STUDIES -

(2) When the experimental values of d and h' obtained from the model are substituted in equation (12), the coefficients obtained (table 2), which are independent of the measured discharge, agree very nearly with the theoretical coefficients (figure 2). The discharge, therefore, obtained by using these coefficients (table 3) agrees very nearly with the theoretical discharge (figure 3).

(3) The experimental discharge (table 1) is about 8 percent less than the theoretical discharge (table 3 and table 5). See figure 3.

6. Although more experimental data are needed, it appears that the theoretical discharge coefficients give a discharge that is too large. This may be due, in part, to the assumption that the control occurs at the highest point on the surface of the gate through the vertical. If critical depth is computed and compared to the values of t (figure 1), it is seen that the values of t are less than critical depth. Although the difference is slight in model dimensions, it indicates that critical depth occurs upstream from the vertical. If equations (9), (10), and (12) could be obtained assuming the control section to be rotated an angle θ upstream from the vertical, the equations so obtained may give values in closer agreement with the experimental data. It is not entirely correct to assume that the model and theoretical coefficients should agree exactly, especially when the effect of roughness and viscosity is considered. But since the model data are

quite consistent even for very small discharges, it appears that there is a tendency for the theoretical discharge coefficients to be too large.

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