MEMORANDUM TO CHIEF DESIGNING ENGINEER
(V. L. Streeter)

Subject: Use of Manning's formula in the design of short steep chutes.

1. Manning's formula has been applied to steep channels with drops of 10 to 15 feet. The computed depths have been checked experimentally in the hydraulic laboratory for slopes of 3:1 and 0.8:1.

2. The average energy of the water per pound at a section is taken as $1.1 \frac{v^2}{2g} + D \cos \theta + y$ where $v$ is the average velocity; $D$ is the depth measured normal to the slope; $\theta$ the angle between the channel bottom and the horizontal; and $y$ the elevation of the bottom section. The coefficient of 1.1 of the $\frac{v^2}{2g}$ term was first assumed and then verified by measurements made by G. C. Wright, under the direction of E. W. Lane, on the South Canal of the Uncompahgre Project in the summer of 1931.

3. $E = 1.1 \frac{v^2}{2g} + D \cos \theta + B-L \sin \theta$ where $E$ is the total energy per pound above datum $O-O$. Then
\[
\frac{dE}{dL} = \frac{1.1}{g} v \frac{dv}{dL} + \frac{dD}{dL} \cos \theta - \sin \theta
\]

Manning's formula is

\[
v = \frac{1.486}{n} R^{2/3} S^{1/2}
\]

\( R \) = hydraulic radius
\( n \) = Kutter's roughness coefficient
\( S = \frac{dE}{dL} \) = sine of angle between energy gradient and horizontal

Transforming,

\[
\frac{dE}{dL} = -\frac{n^2 v^2}{2.2082 R^{4/3}}
\]

The negative sign is required as \( E \) decreases with an increase in \( L \).

Equating the two expressions for \( \frac{dE}{dL} \) above:

\[
-\frac{n^2 v^2}{2.2082 R^{4/3}} = \frac{1.1}{g} v \frac{dv}{dL} + \frac{dD}{dL} \cos \theta - \sin \theta
\]

Let \( Q = Q \) per foot of width; then \( vD = Q \) = constant.

\[
v = \frac{Q}{D}
\]

\( D \frac{dv}{dL} + v \frac{dD}{dL} = 0 \)

\[
\frac{dv}{dL} = -\frac{v}{D} \frac{dD}{dL}
\]

Substituting above

\[
-\frac{n^2 Q^2}{2.2082 R^{4/3} D^2} = -\frac{1.1}{g} \frac{Q^2}{D^3} \frac{dD}{dL} + \frac{dD}{dL} \cos \theta - \sin \theta
\]

Solving for \( \frac{dL}{dD} \),

\[
\frac{dL}{dD} = \frac{\cos \theta - \frac{1.1}{g} \frac{Q^2}{D^3}}{\sin \theta - \frac{n^2 Q^2}{2.2082 R^{4/3} D^2}}
\]

which can be integrated graphically.
4. This method of computation has been verified in the laboratory on two slopes, 3:1 and 0.8:1 as shown in the following table:

<table>
<thead>
<tr>
<th>Slope</th>
<th>Horizontal Length</th>
<th>Width</th>
<th>Roughness</th>
<th>Q per foot</th>
<th>Depth</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>to Vertical Feet</td>
<td>Feet</td>
<td></td>
<td>Width</td>
<td>Depth</td>
<td>Observed to Computed</td>
</tr>
<tr>
<td>3:1</td>
<td>28.50</td>
<td>1.25</td>
<td>0.009</td>
<td>3.52</td>
<td>0.180</td>
<td>0.171 -5.0</td>
</tr>
<tr>
<td>3:1</td>
<td>28.50</td>
<td>1.25</td>
<td>0.009</td>
<td>6.00</td>
<td>0.278</td>
<td>0.278 0.0</td>
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<tr>
<td>0.8:1</td>
<td>11.40</td>
<td>3.76</td>
<td>0.009</td>
<td>1.197</td>
<td>0.0625</td>
<td>0.0598 -4.3</td>
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<tr>
<td>0.8:1</td>
<td>11.40</td>
<td>3.76</td>
<td>0.009</td>
<td>1.994</td>
<td>0.0937</td>
<td>0.0928 -1.0</td>
</tr>
</tbody>
</table>