UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

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HYDRAULIC LABORATORY REPORT NO. 74

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FLOW OF WATER IN SUPERELEVATED CHANNELS AT VELOCITIES BOTH ABOVE AND BELOW CRITICAL

By

THE PERSONNEL OF THE SPILLWAY DESIGN SECTION
AND THE HYDRAULIC STRUCTURES LABORATORY

Compiled by

T. G. OWEN

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Denver, Colorado
January 24, 1940

PREFACE

The idea of applying the theory of superelevation to curved spillways was suggested by Raymond A. Hill, Consulting Engineer, Quinton, Code and Hill - Leeds and Barnard, Los Angeles, California. This idea was suggested by him while retained by the Salt River Valley Water Users Association of Arizona in the capacity of Consulting Engineer on designs made by the Bureau of Reclamation.

To date, six designs of curved and superelevated spillway channels have been prepared in the spillway section, division of dams, of the Bureau of Reclamation under the direction of D. C. McConaughy, Senior Engineer, and under the supervision of K. B. Keener, Senior Engineer. Particular credit is due D. A. Dedel, Engineer, D. V. C. Birrell, Associate Engineer, C. J. Hoffman, Associate Engineer, and A. W. Garnell, Junior Engineer, who severally contributed much to the development of the design theory.

The models of the Bartlett and Vallecito spillways described in this memorandum were designed and tested, and the data analyzed in the hydraulic laboratory of the Bureau of Reclamation by the writer under the direction of J. E. Warnock, Engineer in charge of the laboratory. All laboratories are under the supervision of Arthur Ruettgers, Senior Engineer, and R. F. Blanks, Engineer. All design work is under the supervision of J. L. Savage, Chief Designing Engineer, and all work of the Bureau of Reclamation is directed by R. F. Walter, Chief Engineer. The activities of the Bureau of Reclamation are directed by John C. Page, Commissioner.

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Branch of Design and Construction
Engineering and Geological Control
and Research Division
Denver, Colorado
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Hydraulic Laboratory
Compiled by: T. G. Owen
Reviewed by:

Subject: The flow of water in curved and superelevated channels at both subcritical and supercritical velocities.

1. General Theory and Development of Formulas. This discussion of the flow in open channels, curved in plan and superelevated in cross section, is divided into three parts. The first deals with flow at subcritical velocities in channels with parallel sides; the second concerns flow at supercritical velocities in channels of parallel sides; and the third concerns flow in tapering channels at supercritical velocities.

In these developments, consideration of the effect of friction will be first omitted, then later added.

The assumptions made are:

1. The energy gradient across any section normal to the center line is horizontal.

2. The discharge per foot of channel width is constant.

The symbols used in this discussion are defined as follows:

v = velocity.
h = velocity head.
r = radius measured from center of curvature to the point considered.
p = the hydrostatic pressure.
q = the discharge per foot of channel width.
z = elevation of any particle above the datum plane.
w = the weight per unit volume of the fluid.
a = the angle that the tangent to the water surface makes with the horizontal.
g = the acceleration of gravity.
C, C_1, C_2, and C_3 are constants.

From the first assumption it is obvious that

$$\frac{v^2}{2g} + \frac{p}{w} + z = \text{a constant.} \quad \text{(See Figure 1)}$$

In any horizontal plane z is constant, so

$$\frac{v^2}{2g} + \frac{p}{w} = \text{a constant.}$$

Differentiating with respect to r,

$$\frac{1}{2g} 2v \frac{dv}{dr} + \frac{1}{w} \frac{dp}{dr} = 0$$

$$\frac{v}{g} \frac{dv}{dr} + \frac{1}{w} \frac{dp}{dr} = 0$$

But the change in pressure across a stream tube is

$$\frac{dp}{dr} = \frac{wv^2}{gr}$$

Then,

$$\frac{v}{g} \frac{dv}{dr} + \frac{1}{w} \frac{wv^2}{gr} = 0$$

$$\frac{dv}{dr} + \frac{v}{r} = 0$$

or

$$\frac{dv}{v} + \frac{dr}{r} = 0$$

Integrating,

$$\log v + \log r = \log C$$

$vr = C$ ....(1)

or

$$v^2 r^2 = C^2$$

but

$$v^2 = 2\ gh$$

Then,

$$hr^2 = C_1 \quad \text{..} (2)$$

Taking the log of both sides,

$$\log h + 2 \log r = \log C_1$$

Differentiating,

$$\frac{dh}{h} + \frac{2\,dr}{r} = 0$$

$$\frac{dh}{dr} = -\frac{2h}{r}$$

But

$$\frac{dh}{dr} = \tan \alpha$$

Therefore,

$$\tan \alpha = -\frac{2h}{r} \quad \text{..} (3)$$

From the second assumption it follows that

$$q = av = (1 \cdot d)v = d \cdot v = C_2$$

But from equation (1), $vr = C$

$$\frac{d \cdot v}{vr} = \frac{C_2}{C} = C_3$$

or

$$\frac{d}{r} = C_3 \quad \text{..} (4)$$

Equation (3) may be derived also from a free-body analysis of a particle; and then equations (1) and (2) may be obtained from equation (3).

2. <u>Subcritical Flow.</u> The application of the above theory and equations to the design of a channel in which the flow is below critical velocity may be illustrated in an example.

Specifications of channel:

Width of channel	50 feet.
Radius of center line	150 feet.
Depth of water on center line at section considered	5 feet.
Velocity at center line of section considered	4 ft./sec.
Energy gradient at section considered	100 feet.

Critical depth under these conditions is 2.32 feet. The elements of the computations are arranged in tabular form and appear in table I. In this example, computations are made at five points, including the center line, but in wider channels it might be advisable to include more points.

TABLE I

Superelevation Computations for Subcritical Flow

No.	Item	Inside Wall	Inside Quarter-Point	Center Line	Outside Quarter-Point	Outside Wall
1	Radius	125.00	137.50	150.00	162.50	175.00
2	R_c/R_x	1.200	1.091	1.00	0.923	0.857
3	Velocity	4.80	4.36	4.00	3.69	3.43
4	Depth	4.17	4.58	5.00	5.42	5.83
5	Velocity head	0.36	0.30	0.25	0.21	0.18
6	Energy gradient	100.00	100.00	100.00	100.00	100.00
7	W. S. elev.	99.64	99.70	99.75	99.79	99.82
8	Bottom elev.	95.74	95.12	94.75	94.37	93.99
9	Superelevation	+0.72	+0.37	0.00	-0.38	-0.76

Explanation of computations:

Item No. 3 is obtained from equation (1).

$$vr = C$$

or

$$v_c r_c = v_x r_x$$

$$v_x = v_c \frac{r_c}{r_x}$$

where

v_c = velocity at center line,
r_c = radius at center line,
d_c = depth at center line.

Item No. 4 is obtained from equation (4).

$$\frac{d}{r} = C$$

or

$$\frac{d_c}{r_c} = \frac{d_x}{r_x}$$

and

$$d_x = \frac{d_c}{r_c} \cdot r_x = \frac{d_c}{r_c/r_x}$$

In item No. 9, the superelevation is the distance that any point on the channel floor is above (plus) or below (minus) the floor elevation at the center line.

Figure 2 shows the water surface and channel floor plotted in correct relation to each other. It is interesting to note that the superelevation occurs on the inside of the curve, which is the reverse of the condition obtained, as will be shown, when the flow occurs at a velocity above that of critical. The theory in the above design has not been verified in the laboratory. In the experimental investigation on the approach canal of the Green Mountain Dam spillway, some superelevation, as indicated by the above theory, was tried in the model. Theory indicated a total of eight feet of superelevation; the equivalent of four feet was tried in the model and found to disturb flow conditions down the chute even more than occurred with no superelevation. However, as will be seen by inspection of the plan of the approach channel (figure 4), this could not be considered a fair test of the theory. Figure 5 gives a comparison of the flow conditions with and without superelevation.

3. Supercritical Flow Considering No Friction. In this example, to facilitate comparison the channel specifications are identical with those in example 1 except that the velocity on the center line is above critical.

Specifications of channel:

Width of channel	50.0 feet.
Radius of center line	150.0 feet.
Depth of water at center line	5.0 feet.
Velocity at center line	30.0 ft./sec.
Energy gradient across section	100.0 feet.

Critical depth under these conditions would occur at 8.88 feet. As in the previous example, the elements of the computations are arranged in tabular form and appear in table II.

TABLE II

Superelevation Computations for Supercritical Flow

No.	Item	Inside Wall	Inside Quarter-Point	Center Line	Outside Quarter-Point	Outside Wall
1	Radius	125.00	137.50	150.00	162.50	175.00
2	R_a/R_x	1.20	1.091	1.00	0.923	0.857
3	Velocity	36.00	32.73	30.00	27.69	25.71
4	Depth	4.17	4.58	5.00	5.42	5.83
5	Velocity head	20.15	16.66	13.99	11.92	10.28
6	Energy gradient	100.00	100.00	100.00	100.00	100.00
7	W. S. elev.	79.85	83.34	86.01	88.08	89.72
8	Bottom elev.	75.68	78.76	81.03	82.66	83.89
9	Superelevation	-5.33	-2.25	0.00	+1.63	+2.88

The results of the computations shown in table II are plotted in figure 3 as solid lines. Superimposed for easy comparison are the water surface and channel floor curves for the subcritical flow condition.

4. Supercritical Flow Considering Friction. To recapitulate, in an ideal fluid with no friction present the velocity, depth, etc. on the center line are computed as if the channel were straight. With a given longitudinal bottom profile on the center line, the velocity at any station can be computed by a simultaneous solution of Bernoulli's equation and the relation $v = q/d$, where q is the discharge per foot of width and is assumed constant across any cross section. Then, having determined the elements of flow for the center line at any cross section, the elements of flow can be determined for as many other points in the cross section as desired by using the formulas vr = a constant and d/r = a constant, since the energy gradient is a horizontal plane for an ideal fluid with no friction.

However, when friction is introduced into the computations, two hydraulic conditions must be satisfied simultaneously: the relation $\tan \alpha = \frac{2H}{R}$ (equation 3) must hold true; and the equation of steady flow between successive stations must also be satisfied. This is probably most easily accomplished in the following manner. With the plan of the channel and the longitudinal profile of the floor at the center line given, the velocity, velocity head, depth, friction head, and water surface elevation at the center line can be computed as if the channel were not curved, by making use of the relations $d = q/v$ and, for instance, Manning's formula $s = (nv)^2/2.2082\ r^{4/3}$. The latter equation expresses the loss of head per foot of reach, (s), in terms of the coefficient of roughness, (n); the velocity, (v); and the hydraulic radius, (r). It is apparent that the loss per foot will be higher on the inside of the channel, where the velocity is higher and the hydraulic radius smaller, than on the outside of the channel. The energy gradient across the section is therefore no longer horizontal. To satisfy the two above-mentioned hydraulic conditions in one equation involves unmanageable integrals; so the procedure outlined below was followed.

(a) First, assuming the energy gradient to be horizontal across the cross section, the water surface and bottom elevations are computed at the sides and the quarter points by the method used in table II. Then the transverse slope of the water surface is computed at each of these points by the formula $\tan \alpha = 2H/R$, where R is the radius to the point and H is the velocity head at the point from the above computations.

(b) Next, the energy gradients at the sides and the quarter points are computed in a manner similar to the center-line computations. Having, from these computations, a new velocity head for each point, a new value of $\tan \alpha = 2H/R$ was then computed for the sides and the quarter points. With channel width as abscissas and values of $\tan \alpha$ found in subparagraph (a) as ordinates, a smooth curve was plotted. In the same manner, a smooth curve was plotted through the points of $\tan \alpha$ found in subparagraph (b). The area bounded by the center-line ordinate, the ordinate of any element - either curve, and the X-axis is equivalent to the difference in the water surface elevation between the center line and that element. The difference between the areas under the two curves represents the amount by which the water surface elevation must be adjusted in order that the two hydraulic conditions may be more nearly satisfied. The bottom elevations are then adjusted according to the indicated corrections and the computations repeated until as close agreement as desired is obtained. After the bottom elevations at the first station are determined, the entire process is repeated for the next station. When the bottom elevations for the entire spillway have been computed, it is necessary to plot the longitudinal profiles through the points at which computations were made for each cross section; that is, left side, left quarter point, right quarter point, right side, and others, if other points were used in the computations. If all the resulting longitudinal profiles do not show a continuous and progressive decrease in elevation from upstream to downstream, or, if any profile indicates a radius of curvature in the vertical plane small enough to result (due to centrifugal force) in an appreciable change in the vertical force on a prism of water from hydrostatic to either greater or less than hydrostatic, depending on whether the curvature is concave or convex, respectively, then the profile on the center line must be revised until these conditions are eliminated or their effect minimized. With the revised center line, the entire computations must be repeated.

The above outline indicates the procedure followed in the so-called "exact" method of design. In this "exact" method, however, the design assumes three conditions to be true: first, that the velocity distribution entering the curved channel is uniform; second, that the average velocity of an element in a vertical plane can be taken to represent the true velocity of that element; and third, that the discharge per foot of width is constant across the section. These assumptions only approximate the true condition. Furthermore, a curved channel can be designed for only one discharge; when operating at any other discharge, there will be either more or less superelevation than theory indicates. Because of these facts and because the computations in the so-called "exact" method are somewhat laborious, an "approximate" method of design was proposed in which the simplifying assumption was made that the energy gradient was horizontal in any cross section. This assumption greatly decreases the labor involved in computing the bottom elevations and does not result in elevations varying greatly from those indicated by the "exact" method. Table III shows this comparison for one case studied in which the channel width was 265 feet, the discharge 150,000 second-feet, and the maximum velocity approximately 55 feet per second.

TABLE III

Differences in Feet of Floor Elevations Obtained by Two Methods; "Approximate" Floor Minus "Exact" Floor Equals Difference

Station	Difference in Floor Elevations			
	Inside Wall	Inside Quarter-Point	Outside Quarter-Point	Outside Wall
0+50	0	0	0	0
1+00	-0.03	+0.01	-0.09	-0.19
1+50	-0.10	-0.01	-0.14	-0.34
2+00	-0.09	+0.02	-0.21	-0.50
2+50	-0.12	+0.03	-0.24	-0.59
3+00	-0.12	+0.01	-0.29	-0.64
3+50	-0.10	-0.02	-0.32	-0.68
4+00	-0.02	+0.04	-0.34	-0.69
4+50	+0.04	+0.04	-0.34	-0.72

Models whose prototypes have been designed by both methods have been built and tested in this laboratory. An example of the "exact" method is the Bartlett spillway; an example of the second method is the Vallecito spillway. General plans and sections of both spillways are given in the appendix, together with photographs of the models, water surface profiles, and velocities determined from the models.

In connection with studies of the spillways by use of models, it should be noted that the assumption was made that if a model could be designed by application of the theory and if the model operated satisfactorily, then the prototype could be designed also by the same theory and it, too, would operate satisfactorily. To obtain a model which is both geometrically and dynamically similar to the prototype, the model must be a geometrical reduction of the prototype and also have corresponding velocities. To satisfy the latter condition, the coefficients of roughness in the two structures must bear the following relationship to each other:

$$n_m \cdot N^{1/6} = n_p$$

where

n_m = the model coefficient of roughness,

n_p = the prototype coefficient of roughness,

N = the scale ratio = $\frac{\text{Length prototype}}{\text{Length model}}$.

Table IV gives the model scale which will give the correct model velocity corresponding to the prototype velocity for various model and prototype roughness values.

TABLE IV

Scales of Models Requiring No Friction Correction
for Various Roughness Values

Prototype n	Values of Model n		
	0.009	0.010	0.011
0.011	3.334	1.772	1.000
0.012	5.619	2.986	1.686
0.013	9.083	4.827	2.725
0.014	14.168	7.530	4.250
0.015	21.433	11.391	6.430

From this table it is evident that even with the lowest model roughness factor and a comparatively high value of the prototype factor, the scale ratio is rather low.

In the case of the Bartlett spillway, a roughness factor of 0.014 was assumed for the prototype and 0.010 for the model. This would require a scale ratio of 1 to 7.53. Because the designed discharge for the prototype was 175,000 second-feet, the required model discharge for the indicated scale ratio would be 1,125 second-feet. With this value of the scale ratio, the available laboratory capacity was exceeded many times over, both as to discharge and necessary floor space. The scale chosen was 1 to 100 and required 1.75 second-feet for the designed discharge and 2.50 second-feet for the maximum flood condition. It was decided to design a prototype spillway which would have a value of roughness corresponding to a model value of 0.010 so that a model could be built which would be both geometrically and dynamically similar to the prototype. If the model of this prototype structure (which has a roughness factor corresponding to the model value of 0.010) operated satisfactorily, it was assumed that the other prototype structure, designed by the same theory, would operate equally well. Since the two prototype structures were to be identical in every respect except for the roughness coefficient, then the only change to be made in designing the second structure would be a relocation of the energy gradient and adjustment of the bottom elevations. This amounts to correcting the model for friction. Sample computations for the Bartlett spillway follow.

It will be seen in figure 6 of the appendix that the curved part of the spillway starts at station 2+50 with an A.R.E.A. spiral on the center line. This spiral continues to station 4+00. Beyond this point there are 25 feet of 600-foot radius curve and from there on to the end, the radius of the center line is 500 feet. To simplify computations, the walls of the spillway were assumed vertical throughout; and in the spiral part of the channel, the radius at any point in a cross section was taken as the radius of the center line plus or minus the distance from the center line. The radius of the spiral at any point of the center line was computed by the formula

$$R_x = \frac{R_c \cdot L_c}{L_x}$$

where

R_x = radius of spiral at any point,

L_x = length from start to point x,

R_c = terminal radius of spiral,

L_c = total length of spiral.

In the Bartlett spillway, R_c = 600 feet; L_c = 150 feet. The radius of the center line at station 3+00 is then

$$R_{3+00} = \frac{600 \cdot 150}{50} = 1{,}800 \text{ feet.}$$

The sides and the quarter-points were chosen as the points at which to make computations. Sections at 50-foot intervals on the center line were investigated. Within the reach of the spiral, the lengths of elements along the sides and quarter-points were determined by the equations:

$$\Delta L = L_{x_2} - L_{x_1}$$

and

$$L_x = L_c \left(1 \pm \frac{a}{2 R_c}\right)$$

where

L_{x_2} = the total length from the start of the spiral to the station under consideration,

L_{x_1} = the total length from the start of the spiral to the previous stations,

L_c = the total length of the spiral center line to the station investigated,

a = the distance from the center line to the element under consideration,

R_c = the radius of the center line at the station considered,

ΔL = the length of the element between stations.

The computations of the lengths of elements between stations 2+50 and 3+00 are arranged in table V.

TABLE V

Computations for Longitudinal Length of Elements

Item	Inside Wall	Inside Quarter-Point	Center Line	Outside Quarter-Point	Outside Wall
R_x	1715.00	1757.50	1800.00	1842.50	1885.00
a	85.00	42.50	0	42.50	85.00
$a/2\,R_c$	0.02361	0.01180	0	0.01180	0.02361
$1 \pm a/2\,R_c$	0.97639	0.98820	1.00000	1.01180	1.02361
$L_{x_2} = L_c(1 \pm \frac{a}{2\,R_c})$	48.82	49.41	50.00	50.59	51.18
L_{x_1}	0	0	0	0	0
$\Delta L = L_{x_2} - L_{x_1}$	48.82	49.41	50.00	50.59	51.18

Computations were next made along the center line as if the channel were straight. Elevations of the bottom were taken from the proposed longitudinal profile along the center line, as shown in figure 6 of the appendix. Computations having already been carried down to station 2+50, the following quantities at that station were known:

Bottom elevation at 2+50, $E_b = 1737.33$

Velocity, $V = 49.129$

Velocity head, $H_v = 37.526$

Depth, $d = 20.945$

Hydraulic radius, $r = 16.804$

Energy gradient, E.G. = 1795.80

Width of channel, $W = 170$ feet

Discharge for design, $Q = 175,000$ second-feet

Discharge per foot, $q = 1,029$ second-feet

Prototype roughness factor, $n = 0.014$

The computations for station 3+00 are given in table VI, which represents the solution by trial of Manning's formula. An alternative solution is given by a combination of Bernoulli's and Manning's formulas.

$$\frac{q}{V} + \frac{V^2}{2g} + \frac{n^2}{8.8328}\left(qV + \frac{2}{W}\right)^{4/3}(V + V_o)^2$$

$$+ E_b + H_{f_o} - 1795.80 = 0.$$

In addition to the above defined symbols, V is the velocity at the station under consideration; V_o is the velocity at the adjacent upstream station; and H_{f_o} is the summation of head losses from station 2+50 down through the adjacent upstream station. The above equation can, of course, be solved by Newton's method, but the trial solution, as represented by table VI, is more simple and faster. The procedure followed is to assume a depth at station x, compute the corresponding energy gradient, and then readjust the assumed depth until the computed energy gradient at station 2+50 (which equals the energy gradient at station x plus ΣH_f from station 2+50 down through station x) approximates the actual energy gradient at station 2+50. Computations are referred to the energy gradient at 2+50 so that there will be no cumulative error.

TABLE VI

Hydraulic Computations Along Center Line of Channel
Station 3+00

No.	Item	Trial No. 1	Trial No. 2	Trial No. 3
1	Assumed depth, d	19.39	19.40	19.41
2	Corresponding velocity, V	53.069	53.041	53.014
3	Velocity head, H_v	43.786	43.740	43.695
4	Velocity at previous station, V_o	49.129	49.129	49.129
5	Average velocity throughout reach, V_a	51.099	51.085	51.072
6	Hydraulic radius, r	15.788	15.795	15.802
7	Hydraulic radius at previous station, r_o	16.804	16.804	16.804
8	Average hydraulic radius throughout reach, r_a	16.296	16.300	16.303
9	$1/2.2082\ r_a^{4/3} = k$	0.010964	0.010960	0.010957
10	$k(n\ V_a)^2 = s$	0.005611	0.005606	0.005602
11	$s \cdot$ length of reach $= \Delta H_f$	0.2806	0.2803	0.2801
12	$\Delta H_f + \Delta H_{f_o} + \dots = \Sigma H_f$	0.2806	0.2803	0.2801
13	Bottom elevation, E_b	1732.40	1732.40	1732.40
14	$\Sigma(E_b + d + H_v + \Sigma H_f) =$ computed E.G. at sta. 2+50	1795.857	1795.820	1795.785
15	Actual energy gradient at sta. 2+50	1795.80	1795.80	1795.80
16	Difference in E.G.'s	+0.57	+0.20	-0.015

The next step in the computations is to assume the energy gradient across a section to be horizontal and compute the water surface and bottom elevations at the sides and quarter points, and then the transverse water slope at those points. Table VII illustrates the computations at station 3+00.

TABLE VII

Floor Elevation Computations, Assuming a Horizontal Energy Gradient
Station 3+00

No.	Item	Inside Wall	Inside Quarter-Point	Center Line	Outside Quarter-Point	Outside Wall
1	R_x	1715.00	1757.50	1800.00	1842.50	1885.00
2	$(R_c/R_x)^2$	1.1016	1.0489	1.000	0.95439	0.91185
3	H_{vx}	48.134	45.832	43.695	41.702	39.843
4	$H_{vx} + \Sigma H_{f_c}$	48.414	46.112	43.975	41.982	40.123
5	W. S. elev.	1747.386	1749.688	1751.825	1753.818	1755.677
6	$\sqrt{H_{vx}}$	6.9379	6.7699	6.6102	6.4577	6.3121
7	$d_x = \dfrac{128.30395}{\sqrt{H_{vx}}}$	18.493	18.952	19.410	19.868	20.327
8	Bottom elev.	1728.893	1730.736	1732.40	1733.950	1735.350
9	Tan $\alpha = 2\, H_{vx}/R_x$	0.05613	0.05216	0.04855	0.04527	0.04227

In table VII, item 3 is computed from equation (2) and becomes $H_{vx} = H_{vo} \left(\frac{R_o}{R_x}\right)^2$ where H_{vc} is 43.695 (value from table VI), and the last term is item 2 in table VII. Item 4 in table VII is taken from table VI (item 12). Item 5 in table VII is found by subtracting values of item 4 from the energy gradient at station 2+50 (1795.80). Item 7, table VII, is derived by combining equations (2) and (4), the general form of which is $d_x = d_c\,(H_c/H_x)^{1/2}$. Item 9, table VII, is equation (3).

Computations are next made between stations along longitudinal elements located at the sides and quarter-points. The velocity heads, energy gradients, etc., are computed, and then new values of tan α at each point, based on the new values of the velocity head, are found. The value of tan α at the center line does not change. Using that value and the new values of tan α at the quarter-points and sides, the water surface elevations are computed by the method outlined below. Referring to figure 7, it is evident that ΔH is the difference in the water surface elevations between points 1 and 2 and also equal to the difference in the values of the velocity heads at the two points. Furthermore,

$$\Delta H = a_1 \tan \alpha_1 + a_2 \tan \alpha_2 = H_1 - H_2$$

$$a_1 + a_2 = R_2 - R_1$$

Assuming that the values of $\tan \alpha_1$ and $\tan \alpha_2$ are known, we then have two equations involving three unknowns. To solve for the value of ΔH we must assume one condition. There are two choices available: first, to assume the water surface to be composed of parabolic curves between the points under investigation, in which case $a_1 = a_2$; second, to assume the energy gradient horizontal, in which case equations (2) and (3) will hold, and an expression for the ratio of a_1 to a_2 can be found in terms of R_1 and R_2. The latter assumption is used in the derivation that follows.

From equation (3):

$$\tan \alpha = \frac{2H}{R}$$

$$\Delta H = H_1 - H_2 = a_1 \frac{2H_1}{R_1} + a_2 \frac{2H_2}{R_2}$$

From equation (2):

$$H_1 R_1^2 = C_1 \quad \text{or} \quad H_1 = \frac{C_1}{R_1^2}$$

and

$$H_2 R_2^2 = C_1 \quad \text{or} \quad H_2 = \frac{C_1}{R_2^2}$$

Therefore,

$$\Delta H = \frac{C_1}{R_1^2} - \frac{C_1}{R_2^2} = \frac{2a_1}{R_1} \cdot \frac{C_1}{R_1^2} + \frac{2a_2}{R_2} \cdot \frac{C_1}{R_2^2}$$

$$\frac{C_1}{R_1^2} - \frac{C_1}{R_2^2} = \frac{2a_1 C_1}{R_1^3} + \frac{2a_2 C_1}{R_2^3}$$

$$\frac{1}{R_1^2} - \frac{1}{R_2^2} = \frac{2a_1}{R_1^3} + \frac{2a_2}{R_2^3}$$

$$\frac{2a_1 R_2^3 + 2a_2 R_1^3}{R_1^3 R_2^3} = \frac{R_2^2 - R_1^2}{R_1^2 R_2^2}$$

$$2a_1 R_2^3 + 2a_2 R_1^3 = R_1 R_2^3 - R_1^3 R_2 \ldots\ldots\ldots\ldots\ldots (A)$$

From the figure:

$$a_1 + a_2 = R_2 - R_1$$

$$a_1 = R_2 - R_1 - a_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

$$a_2 = R_2 - R_1 - a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C)$$

(A)

$$2a_1 R_2^3 + 2R_1^3 (R_2 - R_1 - a_1) = R_1 R_2^3 - R_1^3 R_2$$

$$2a_1 R_2^3 + 2R_1^3 R_2 - 2R_1^4 - 2a_1 R_1^3 = R_1 R_2^3 - R_1^3 R_2$$

Solving for a_1:

$$2a_1 (R_2^3 - R_1^3) = R_1 R_2^3 - R_1^3 R_2 - 2R_1^3 R_2 + 2R_1^4$$

$$a_1 = \frac{1}{2} \cdot \frac{R_1 R_2^3 - 3R_1^3 R_2 + 2R_1^4}{R_2^3 - R_1^3}$$

Substituting (B) in (A):

(A)

$$2R_2^3 (R_2 - R_1 - a_2) + 2a_2 R_1^3 = R_1 R_2^3 - R_1^3 R_2$$

$$2R_2^4 - 2R_1 R_2^3 - 2a_2 R_2^3 + 2a_2 R_1^3 = R_1 R_2^3 - R_1^3 R_2$$

Solving for a_2:

$$2a_2 (R_1^3 - R_2^3) = R_1 R_2^3 - R_1^3 R_2 + 2R_1 R_2^3 - 2R_2^4$$

$$a_2 = \frac{1}{2} \cdot \frac{3R_1 R_2^3 - R_1^3 R_2 - 2R_2^4}{R_1^3 - R_2^3}$$

$$\frac{a_1}{a_2} = \frac{R_1 R_2^3 - 3R_1^3 R_2 + 2R_1^4}{2 (R_2^3 - R_1^3)} \cdot \frac{2 (R_1^3 - R_2^3)}{3R_1 R_2^3 - R_1^3 R_2 - 2R_2^4}$$

$$\frac{a_1}{a_2} = \frac{R_1 R_2^3 - 3R_1^3 R_2 + 2R_1^4}{R_1^3 R_2 - 3R_1 R_2^3 + 2R_2^4} = \frac{R_1 (R_2^3 - 3R_1^2 R_2 + 2R_1^3)}{R_2 (R_1^3 - 3R_1 R_2^2 + 2R_2^3)}$$

Let

$$R_2 = KR_1 \quad \text{or} \quad K = \frac{R_2}{R_1}$$

Then,

$$\frac{a_1}{a_2} = \frac{R_1 (K^3R_1^3 - 3R_1^2 KR_1 + 2R_1^3)}{KR_1 (R_1^3 - 3R_1 K^2R_1^2 + 2K^3 R_1^3)}$$

$$= \frac{R_1^4 (K^3 - 3K + 2)}{KR_1^4 (1 - 3K^2 + 2K^3)} = \frac{K^3 - 3K + 2}{2K^4 - 3K^3 + K}$$

Table VIII gives values of $\frac{a_1}{a_2}$ corresponding to values of K. Values of a_1 and a_2 are then found from the value of the ratio $\frac{a_1}{a_2}$ and $a_1 + a_2 = R_2 - R_1$. The value of ΔH is then found from the equation:

$$\Delta H = a_1 \cdot \tan \alpha_1 + a_2 \cdot \tan \alpha_2$$

The water surface elevations at the sides and quarter-points are then found by subtracting the sum of $\Delta H + H_{v_e} + \Sigma \Delta H_{f_c}$ from the hydraulic gradient at station 2+50. The bottom elevations at these points are found by subtracting the depths at the quarter-

TABLE VIII

Values of $\frac{a_1}{a_2}$ Corresponding to K

Values of K	Values of $\frac{a_1}{a_2}$
1.01	0.98689
1.02	0.97419
1.03	0.96123
1.04	0.94889
1.05	0.93708
1.06	0.92525
1.07	0.91375
1.08	0.90248
1.09	0.89144
1.10	0.88068

points and sides from the water surface elevations at those points. If these elevations do not agree with the bottom elevations used in the longitudinal element computations, the latter may be revised in three ways: first, by trial; that is, assume a new bottom elevation; second, plot smooth curves through the two values of tan α (at sides, quarter points, and center line) using tan α as ordinates and channel width as abscissa, taking the difference in areas under the curves as the correction factors; third, take as the correction factor the difference between the computed values of ΔH for sides and quarter points, using the values of tan α found from table VII and the values of ΔH found from values of tan α from the longitudinal element computations. With the revised bottom elevation, the calculations are repeated until as close agreement as desired is obtained.

The above procedure is illustrated by sample computations for the Bartlett spillway. In addition to sample computations for the prototype structure, the correction for "excess friction" in the model is also shown. To correct for "excess model friction," a prototype structure was designed, using a value of roughness factor that would scale down to the model value. Then prototype dimensions were reduced in the usual way to obtain corresponding model dimensions. Actually, in the case of the Bartlett spillway, the prototype was designed using a roughness factor of 0.014, and in the model design a value of "excess friction loss" was included. This value of "excess friction loss" was computed in the following manner:

$$n_p' = n_m \cdot N^{1/6} = 0.010 \cdot 2.15443 = 0.02154$$

and

$$H_{f_p}'/H_{f_p} = (n_p'/n_p)^2 = (0.02154/0.014)^2 = 2.368$$

or

$$H_{f_p}' = 2.368\ H_{f_p}$$

where

n_p' = the prototype roughness factor that will scale down to the model value of roughness,

n_m = the model roughness factor,

n_p = the actual prototype roughness factor,

N = the model scale ratio = $\dfrac{\text{prototype length}}{\text{model length}}$ = 100,

H_{f_p}' = the head loss due to friction in the hypothetical prototype structure,

H_{f_p} = the head loss due to friction in the actual prototype structure.

Expressed in words, the equation $H_{f_p}' = 2.368\ H_{f_p}$ means that in two prototype structures which are identical in every respect except for the roughness factors, the ratio of the friction losses varies directly as the square of the ratio of the roughness factors. The "excess friction loss" in the hypothetical prototype structure is computed and shown in table IX.

TABLE IX

"Excess Friction" Computations

Station	ΣH_{f_p}	$\Sigma H'_{f_p} = 2.368 \Sigma H_{f_p}$	$\Sigma \Delta H_f = \Sigma H'_{f_p} - \Sigma H_{f_p}$ = "Excess Friction"
2+50	0.00	0.00	0.00
3+00	0.28	0.66	0.38
3+50	0.64	1.52	0.88
Etc.			

In computing table IX the values of ΣH_{f_p} are taken from table VI, item 12.

The next step in the computations is to compute the water surface and bottom elevations at the various cross sections, as outlined above. These are arranged in table X.

TABLE X

Floor Elevation Computations
Energy Gradient Not Assumed Horizontal
Station 3+00

Trial No. 1

Pertinent Data: $n_p' = 0.02154$; $r_c/d_c = 0.814$; $s_o = 0.01192$
$H_{vc} = 43.695$; $H_{f_{pc}}' = 0.66$

No.	Item	Inside Wall R = 1715	Inside Quarter-Point R = 1757.5	Outside Quarter-Point R = 1842.5	Outside Wall R = 1885
1	Actual proto. bot. el.	1728.89	1730.74	1733.95	1735.35
2	Excess friction, $\Sigma\ \Delta H_f$	0.38	0.38	0.38	0.38
3	Hypothetical bot. el.	1728.51	1730.36	1733.57	1734.97
4	Velocity head, H_{vx}	48.07	45.79	41.72	39.88
5	$\sqrt{2g\ H_{vx}} = V_x$	55.605	54.270	51.802	50.647
6	$q/V_x = d_x$	18.506	18.961	19.864	20.317
7	$\frac{r_c}{d_c}\ d_x = 0.814\ d_x = r_x$	15.064	15.434	16.169	16.538
8	$\frac{1}{2.2082\ r^{4/3}} = k_x$	0.01217	0.01179	0.01108	0.01075
9	$k(n_p' V_x)^2 = s$	0.01746	0.01611	0.013795	0.01279
10	$\frac{s_o + s}{2} = \text{Av. } s_a$	0.01469	0.014015	0.01286	0.012355
11	Element length = ΔL	48.82	49.41	50.59	51.18
12	$s_a \cdot \Delta L = \Delta H_{f_x}$	0.717	0.692	0.651	0.632
13	$\Sigma\ \Delta H_{f_x} = H_{f_x}$	0.717	0.692	0.651	0.632
14	$d_x + H_{vx} + H_{f_x}$	67.293	65.443	62.235	60.829
15	Hyd. grad. at 2+50	1795.803	1795.803	1795.805	1795.799
16	$\frac{2H_{vx}}{R_x} = \tan\alpha$	0.05606	0.05211	0.04529	0.04231
17	ΔH	4.435	2.138	-1.993	-3.854
18	$\Delta H + H_{vc} + \Sigma\ H_{f_{pc}}'$	48.790	46.493	42.362	40.507
19	W. S. el.	1747.010	1749.307	1753.438	1755.293
20	d_x	18.506	18.961	19.864	20.317
21	Bottom elevation	1728.504	1730.346	1733.574	1734.976

TABLE X (Continued)

Floor Elevation Computations
Energy Gradient Not Assumed Horizontal
Station 3+00

Trial No. 2

Pertinent Data: $n_p' = 0.02154$; $r_c/d_c = 0.814$; $s_o = 0.01192$
$H_{vc} = 43.695$; $H_{f_{pc}}' = 0.66$

No.	Item	Inside Wall R = 1715	Inside Quarter-Point R = 1757.5	Outside Quarter-Point R = 1842.5	Outside Wall R = 1885
3	Hypothetical bot. el.		1730.35		
4	Velocity head, H_{vx}		45.80		
5	$\sqrt{2g\ H_{vx}} = V_x$		54.276		
6	$q/V_x = d_x$		18.959		
7	$\frac{r_c}{d_c}\ d_x = 0.814\ d_x = r_x$		15.433		
8	$\frac{1}{2.2082\ r^{4/3}} = k_x$		0.01179		
9	$k(n_p'V_x)^2 = s$		0.01611		
10	$\frac{s_o + s}{2} = \text{Av. } s_a$		0.014015		
11	Element length = ΔL		49.41		
12	$s_a \cdot \Delta L = \Delta H_{f_x}$		0.692		
13	$\Sigma\ \Delta H_{f_x} = H_{f_x}$		0.692		
14	$d_x + H_{vx} + H_{f_x}$		65.451		
15	Hyd. grad. at 2+50		1795.801		
16	$\frac{2H_{vx}}{R_x} = \tan\alpha$		0.05212		
17	ΔH		2.138		
18	$\Delta H + H_{vc} + \Sigma\ H_{f_{pc}}'$		46.493		
19	W. S. el.		1749.307		
20	d_x		18.959		
21	Bottom elevation		1730.348		

In table X, item 1 is taken from table VII. Item 2 comes from table IX. The hypothetical bottom elevation (item 3) is the corresponding elevation in the second prototype structure which has the same plan and corresponding velocities as the first prototype structure, but it has a value of roughness factor that will reduce to the model value according to the similitude relation. The value of this hypothetical bottom elevation is found by subtracting item 2 from item 1. The next step is to assume a velocity head (item 4) and run through the ensuing computations in much the same manner as was done in table VI for the hydraulic computations down the center line of the channel. If the correct velocity head were assumed, the computed hydraulic gradient at station 2+50 (item 15) will check the actual (1795.80); if it does not check, a new velocity head is assumed and the computations are repeated until as close an agreement as desired is obtained before proceeding with the rest of the computations. In item 7 it is assumed that the ratio of the hydraulic radius at any point to the hydraulic radius at the center line varies directly as the ratio of the depths at the corresponding points. Item 8 is interpolated from a table[1] of values of k corresponding to values

[1] Such a table is given in "Handbook of Hydraulics" by H. W. King, table 107, p. 310, third edition, McGraw-Hill, 1939.

of r. Item 10 is the average friction loss per foot throughout the reach, s being taken from item 9, and s_o from the adjacent upstream station at the corresponding point in the cross section. In this particular example the adjacent upstream station is 2+50, at which the floor is level in cross section and the friction loss per foot is constant across the section. Item 11 is taken from table V. Item 13 is the summation of the friction losses from station 2+50 down to and including the station under investigation. After the desired agreement between the computed and the actual hydraulic gradient at station 2+50 (item 15) has been obtained, items 16 to 21, inclusive, are then calculated. In item 16 the velocity head found in item 4 is used. The computations for item 17, ΔH, are shown in table XI. Values of ΔH_x on the right side of the center line appear as negative quantities because ΔH is defined as the difference between the value of the velocity head at the point investigated and that at the center line. The water surface elevation (item 19) is found by subtracting item 18 from the energy gradient at 2+50 (1795.80). Item 20 is the same d_x found in item 6. The bottom elevation (item 21) is, of course, the water surface elevation minus the depth. If this bottom elevation does not agree closely enough with the bottom elevation given in item 3, a revision of the bottom elevation is necessary. Such a revision was indicated after the first trial computations for the inside quarter-point only. In these computations, the revision of the bottom elevation was made by trial instead of by making corrections as indicated by plotting curves of $\tan \alpha$ found in tables VII and X, or computing new values of ΔH from the values of $\tan \alpha$ from table VII. It is evident that the order of computations must be from the center line progressively outward since the ΔH for the outside points includes the ΔH for the quarter-points. (See table XI)

The foregoing illustrates the manner in which the bottom elevations at each section were computed for the Bartlett spillway and also how the computations were made for the hypothetical prototype structure, the values of which reduce directly to a 1:100 scale model. As previously pointed out, the above computations must be repeated for each section, working progressively downstream. Also, when all computations have been completed, it is necessary to investigate the longitudinal profiles through each of the cross-sectional points determined. If any profile does not show a progressive decrease in elevation from upstream to downstream, or if any profile indicates a radius of curvature in the vertical plane small enough to result in an appreciable change in force on a prism of water from hydrostatic to greater or less than hydrostatic, due to centrifugal force, it will be necessary to revise the center-line profile and repeat the entire set of computations until a satisfactory design is obtained.

Figure 8 in the appendix gives a comparison between the actual water surfaces as determined by a 1:100 scale model and the theoretical water surfaces; also, the velocities as determined from the model, and as indicated by theory. The sections shown in figure 8 are for the original model before it was decided to incorporate a bucket at the end of the lined section, as shown in figure 6 of the appendix.

Figures 9 to 11, inclusive, show the model in operation under various conditions of discharge. Figure 12 shows the flow pattern on the bottom and sides for the designed discharge. The flow pattern indicates cross flow on the bottom toward the inside of the curve. This condition is to be expected because the channel was designed for the mean velocity in a vertical plane and because the normal velocity distribution in the vertical plane indicates that the velocity on the floor is considerably below the mean value. Could a similar pattern have been taken along a surface passing through points of maximum velocity in the vertical plane, cross flow to the outside of the curve would no doubt have been indicated. These points are brought out to illustrate the inherent and insurmountable objectionable property of curved spillways; namely, that the spillway will operate completely satisfactorily only when the actual velocity at any point agrees with the velocity used in designing the spillway at that point.

TABLE XI

Computations for ΔH
Station 3+00

Item	Inside Wall	Inside Quarter-Point	Outside Quarter-Point	Outside Wall
		TRIAL NO. 1		
R_2	$= R_{iq}$ = 1757.5	$= R_c$ = 1800.0	$= R_{oq}$ = 1842.5	$= R_o$ = 1885.0
R_1	$= R_i$ = 1715.0	$= R_{iq}$ = 1757.5	$= R_c$ = 1800.0	$= R_{oq}$ = 1842.5
$\frac{R_2}{R_1} = K$	1.02478	1.02418	1.02361	1.02307
$\frac{a_1}{a_2}$	0.96800	0.96877	0.96951	0.97021
$42.5/(1 + a_1/a_2) = a_2$	21.5955	21.587	21.579	21.571
$42.5 - a_2 = a_1$	20.9045	20.913	20.921	20.929
$\tan \alpha_2$	$= \tan \alpha_{iq}$ = 0.05212	$= \tan \alpha_c$ = 0.04855	$= \tan \alpha_{oq}$ = 0.04529	$= \tan \alpha_o$ = 0.04231
$\tan \alpha_1$	$= \tan \alpha_i$ = 0.05606	$= \tan \alpha_{iq}$ = 0.05211	$= \tan \alpha_c$ = 0.04855	$= \tan \alpha_{oq}$ = 0.04529
$a_1 \tan \alpha_1 + a_2 \tan \alpha_2 = \Delta H_x$	$= \Delta H_{iq} + \Delta H_i$ = 4.43549	$= \Delta H_{iq}$ = 2.13782	$= \Delta H_{oq}$ = -1.99302	$= \Delta H_{oq} + \Delta H_o$ = -3.85356
		TRIAL NO. 2		
R_2		$= R_c$ = 1800.0		
R_1		$= R_{iq}$ = 1757.5		
$\frac{R_2}{R_1} = K$		1.02418		
$\frac{a_1}{a_2}$		0.96877		
$42.5/(1 + a_1/a_2) = a_2$		21.587		
$42.5 - a_2 = a_1$		20.913		
$\tan \alpha_2$		$= \tan \alpha_c$ = 0.04855		
$\tan \alpha_1$		$= \tan \alpha_{iq}$ = 0.05212		
$a_1 \tan \alpha_1 + a_2 \tan \alpha_2 = \Delta H_x$		$= \Delta H_{iq}$ = 2.13803		

5. Flow at Supercritical Velocities in Channels Tapered in Plan. It will be recalled that the Bartlett spillway was an example of the curved spillway with parallel walls designed by the so-called "exact" method. The discussion that follows deals with curved spillways which are tapered in plan and designed by the so-called "approximate" method in which the simplifying assumption is made that the energy gradient across a section is horizontal. The Vallecito spillway will be used as an example of this type. In this case it was necessary to use two spirals on the center line, one from the start of the curvature and extending down to the midpoint and the second from the midpoint down to the tangent. After the desired location of the gate structure and the downstream tangent part of the spillway channel had been located, the twin spirals that would best fit the topography were located on the center line. Then, by a cut-and-try process, twin spirals which would approximate the side walls and quarter-points, taking into account the desired taper in width, were evolved. Making the assumption that the discharge per foot of width is constant across any section, the energy gradients on the center line are computed for all stations as if the channel were straight. These computations are similar to those of table VI. Then, at each station, the radii of curvature for each desired point in the cross section are computed from the formula $R = \dfrac{R_s \cdot L_s}{L}$, where R is the radius of curvature at the point, R_s is the terminal radius of the spiral, L_s is the total length of the spiral, and L is the length from the start of the spiral to the point considered. Having these data, the water surface and bottom elevations for the quarter points and the side walls are computed by the formula whose development follows.

The development of the formula for computing the water surface depends upon two assumptions: first, that the energy gradient across a section is horizontal; and second, that the water surface between verticals may be considered parabolic. From the latter assumption it follows that the tangents to the water surface at verticals 1 and 2 (figure 13) will intersect at the midpoint of the interval between the verticals, or at b/2 from either one.

From the figure

$$H_2 = K + Z$$

and

$$K = H_1 + \frac{b}{2} \tan \alpha_1$$

$$Z = \frac{b}{2} \tan \alpha_2$$

but

$$\tan \alpha_1 = \frac{2H_1}{R_1}$$

and

$$\tan \alpha_2 = \frac{2H_2}{R_2}$$

$$K = H_1 + \frac{b}{2} \cdot \frac{2H_1}{R_1} = H_1 + \frac{bH_1}{R_1}$$

$$\tan \alpha_2 = \frac{2H_2}{R_2} = \frac{2(K + Z)}{R_2} = \frac{Z}{\frac{b}{2}}$$

$$Z = \frac{b}{2} \cdot \frac{2(K + Z)}{R_2} = \frac{bK}{R_2} + \frac{bZ}{R_2}$$

$$Z(1 - \frac{b}{R_2}) = \frac{bK}{R_2}$$

$$Z = \frac{bK}{R_2} \cdot \frac{R_2}{R_2 - b} = \frac{K}{(\frac{R_2}{b} - 1)}$$

Therefore,

$$H_2 = K + \frac{K}{(\frac{R_2}{b} - 1)}$$

This formula was used in computing the values of the velocity head at the various points in each cross section of the Vallecito spillway. The procedure is to work from the center line progressively to either side. When working to the left of the center line, the value of H_2 is the unknown; when working to the right, H_1 is the unknown. When the various values of the velocity heads have been computed, the water surface elevations can be computed by subtracting the velocity head from the energy gradient. The velocity is found by the formula $V = \sqrt{2\ gH}$; and the depth is found from $d = q/V$ where q equals the discharge per foot of width. The bottom elevation equals the water surface elevation minus the depth.

This method of computing the water surface and the bottom elevations at the cross sections was adopted by the designers in preference to using the formula $H_2 = H_1 \left(\frac{R_1}{R_2}\right)^2$ for the following reason. In laying out the plan for a tapered twin spiral spillway, it is practically impossible, if not theoretically so, to have the start of the spirals representing the walls, quarter-points, and center line occur at the same station as referred to the center line. Thus the start of the spiral for the outside wall for Vallecito occurs at station 5+97.78; while the center-line spiral starts at station 6+16.23, and the inside wall spiral starts at station 6+81.41. At station 7+00, the radius of the inside wall spiral is about 3,760 feet; for the center line it is 1,020 feet; and for the outside wall it is 819 feet. In an ordinary curve, the radius of curvature at the inside is less than that at the outside. Here the reverse is true. It is evident that under this condition the velocity heads at various points in a cross section, figured by the equation HR^2 = constant, would be in error. It was for this reason that the method used in computing the various elevations for Vallecito was developed.

Figure 14 shows the general plan and sections of the Vallecito spillway. A model of the spillway was built and revised twice to diminish the effect of centrifugal force due to curvature in the vertical plane. Figure 15 shows cross-sectional water surface profiles as determined from the model compared with the theoretical water surfaces at the various sections. The final bottom elevations at each section are shown in solid lines; the original and first alterations are shown dotted. Deviation from the theoretical water surface is due in part, at least, to the disturbance caused by the piers at the crest; and this effect is magnified by the converging walls. Where practical, piers should be eliminated and the rate of convergence made as small as possible. In the appendix in figures 16 to 20 are shown pictures of transition No. 3 with no flow, one-quarter, one-half, three-quarters, and full designed discharge.

The hydraulic features of the design of curved channels, with parallel walls for both subcritical and supercritical velocities, have been herein treated in considerable detail; and the design of curved spillways, tapered in plan and having supercritical velocities, has been outlined.

- - -

T. G. Owen.

APPENDIX

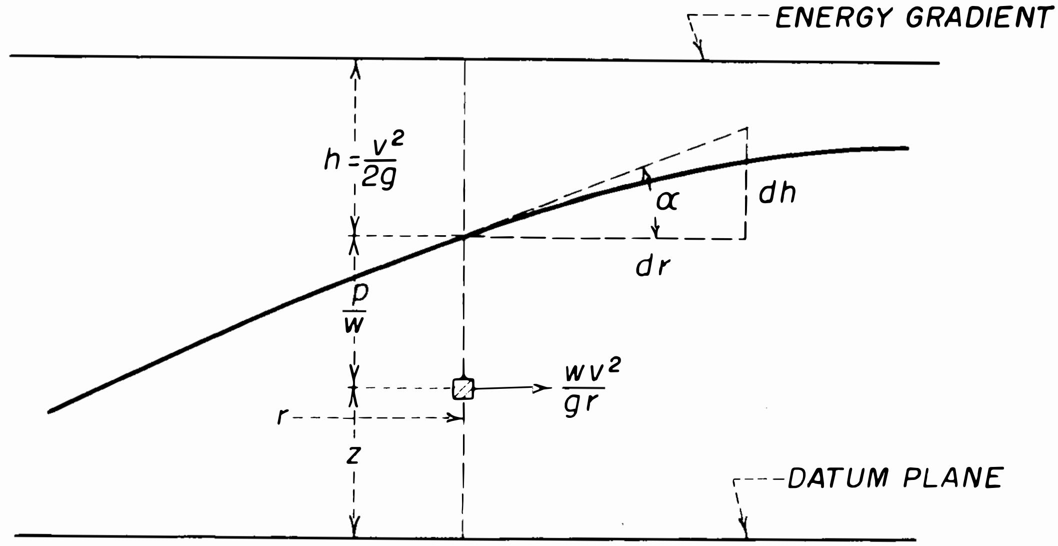


FIGURE 1

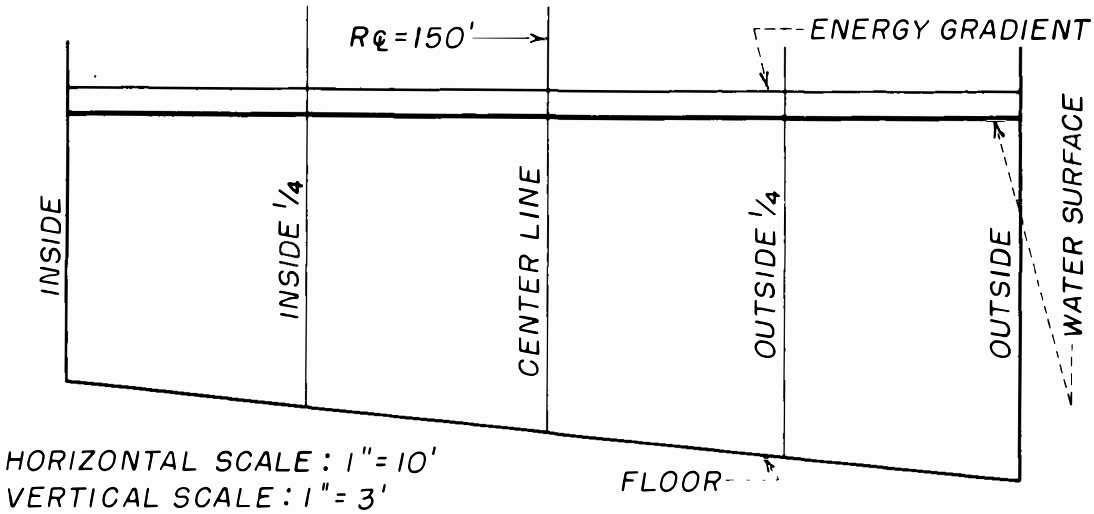


FIGURE 2

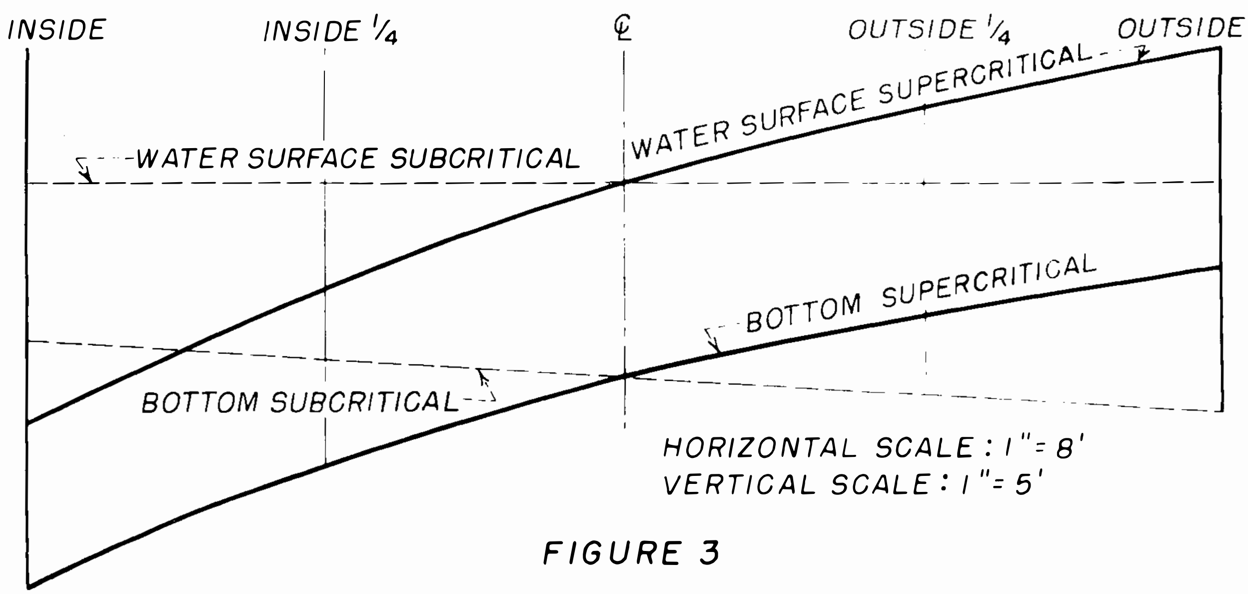
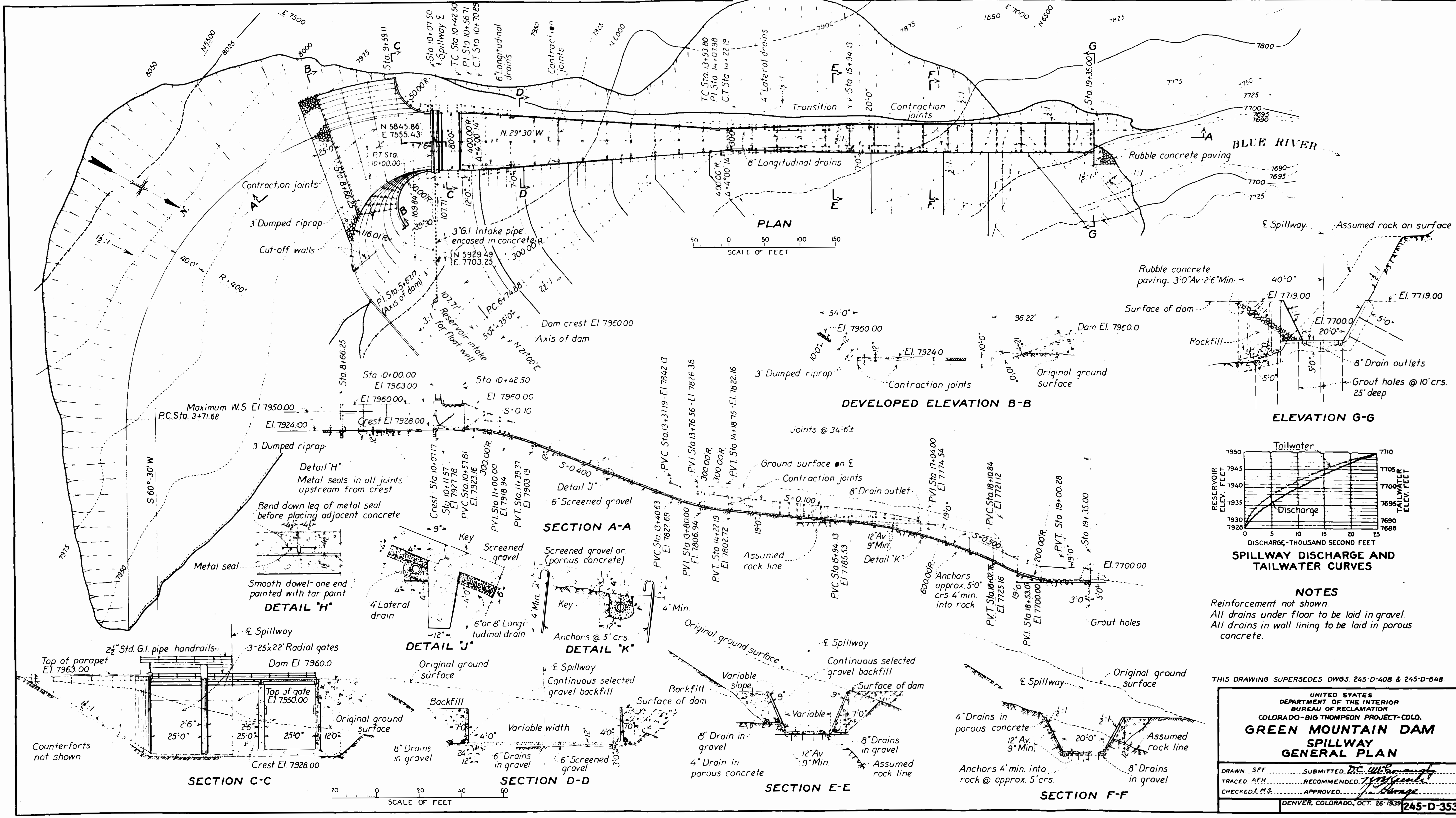
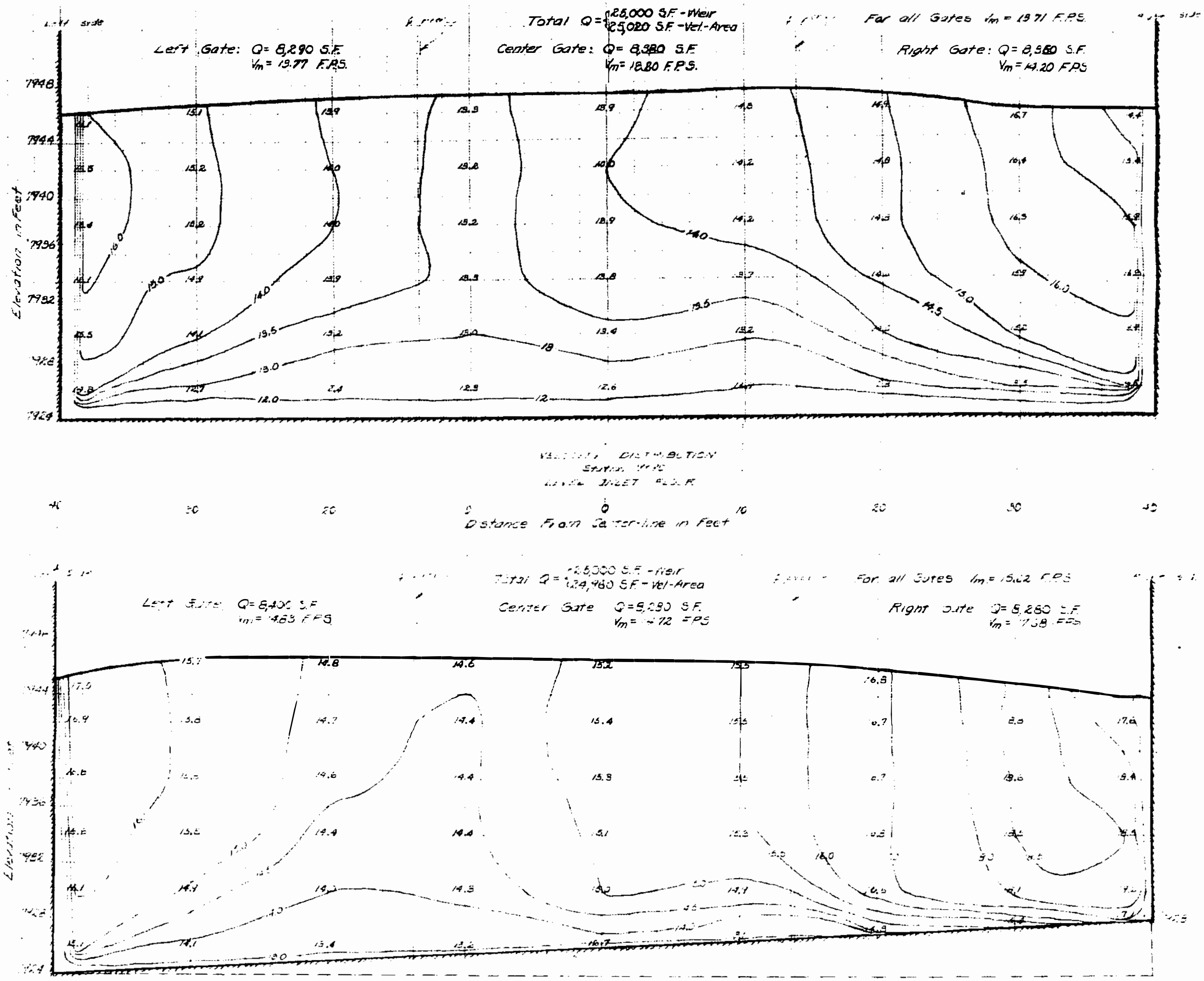


FIGURE 3

FIGURE 4

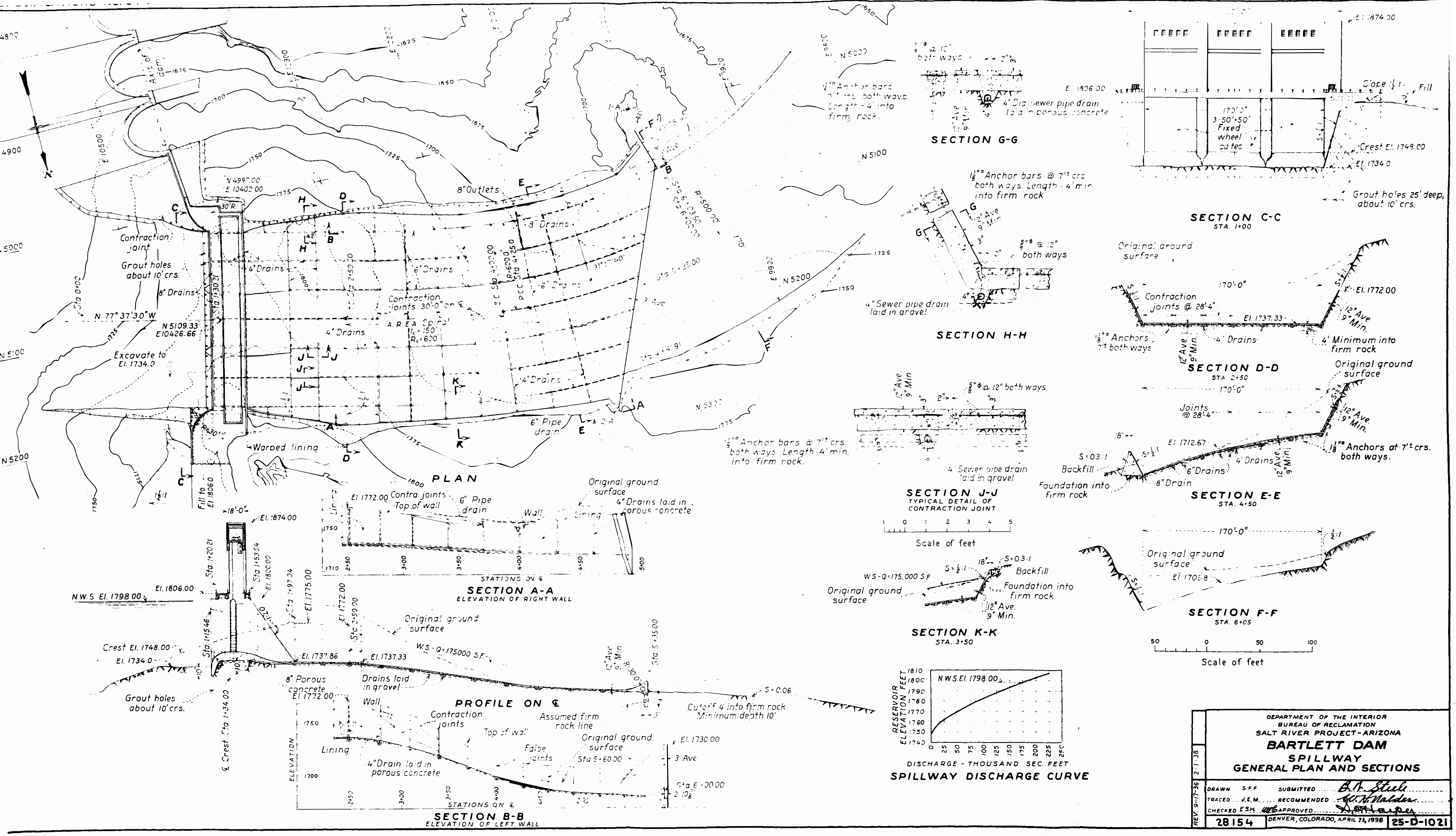

PLAN
SCALE OF FEET
BLUE RIVER
Transition
Contraction joints
Rubble concrete paving
8" Longitudinal drains
6" Longitudinal drains
4" Lateral drains
3" Dumped riprap
Cut-off walls
Contraction joints
3" G.I. Intake pipe encased in concrete
Reservoir intake for float well
Dam crest El 7960.00
Axis of dam
N 29°30' W.
N 5845.86
E 7555.43
N 5929.49
E 7703.25
SECTION A-A
Maximum W.S. El 7950.00
P.C.Sta. 3+71.68
El. 7924.00
Crest El 7928.00
Sta 10+00.00
El 7963.00
Sta 10+42.50
El 7960.00
S=0.10
S=0.400
S=0.100
S=0.500
Ground surface on ℄
Contraction joints
8" Drain outlet
Assumed rock line
Anchors approx. 5'0" crs 4' min. into rock
El. 7700.00
Grout holes
Joints @ 34'6"±
S 60°30' W
Detail "H"
Metal seals in all joints upstream from crest
Bend down leg of metal seal before placing adjacent concrete
Metal seal
Smooth dowel- one end painted with tar paint
DETAIL "H"
Detail "J"
6" Screened gravel
4" Lateral drain
6" or 8" Longitudinal drain
Key
Screened gravel
DETAIL "J"
Screened gravel or (porous concrete)
Anchors @ 5' crs.
DETAIL "K"
DEVELOPED ELEVATION B-B
El. 7960.00
El. 7924.0
Dam El. 7960.0
3' Dumped riprap
Contraction joints
Original ground surface
ELEVATION G-G
℄ Spillway
Assumed rock on surface
Rubble concrete paving. 3'0" Av 2'6" Min.
Surface of dam
Rockfill
El. 7719.00
El. 7700.0
8" Drain outlets
Grout holes @ 10' crs. 25' deep
SPILLWAY DISCHARGE AND TAILWATER CURVES
Tailwater
Discharge
RESERVOIR ELEV. FEET
TAILWATER ELEV. FEET
DISCHARGE-THOUSAND SECOND FEET
NOTES
Reinforcement not shown.
All drains under floor to be laid in gravel.
All drains in wall lining to be laid in porous concrete.
SECTION C-C
℄ Spillway
2½" Std. G.I. pipe handrails
3-25'x22' Radial gates
Top of parapet El 7963.00
Dam El. 7960.0
Top of gate El 7950.00
Original ground surface
Counterforts not shown
Crest El. 7928.00
SECTION D-D
Original ground surface
Backfill
Continuous selected gravel backfill
Surface of dam
Variable width
8" Drains in gravel
6" Drains in gravel
6" Screened gravel
SECTION E-E
Original ground surface
Continuous selected gravel backfill
Variable slope
Variable
Surface of dam
8" Drain in gravel
4" Drain in porous concrete
8" Drains in gravel
Assumed rock line
SECTION F-F
Original ground surface
4" Drains in porous concrete
Anchors 4' min. into rock @ approx. 5' crs.
Assumed rock line
8" Drains in gravel
THIS DRAWING SUPERSEDES DWGS. 245-D-408 & 245-D-648.
UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
COLORADO-BIG THOMPSON PROJECT-COLO.
GREEN MOUNTAIN DAM
SPILLWAY
GENERAL PLAN
DRAWN SFF
TRACED AFH
CHECKED LMS
SUBMITTED
RECOMMENDED
APPROVED
DENVER, COLORADO, OCT. 26, 1939
245-D-353



COLORADO-BIG THOMPSON PROJECT
GREEN MOUNTAIN DAM SPILLWAY
COMPARISON OF VELOCITY DISTRIBUTIONS
AT STA 9+40 FOR LEVEL AND SLOPING
INLET FLOORS

FIGURE 5

FIGURE 8 DRAWING No. 11



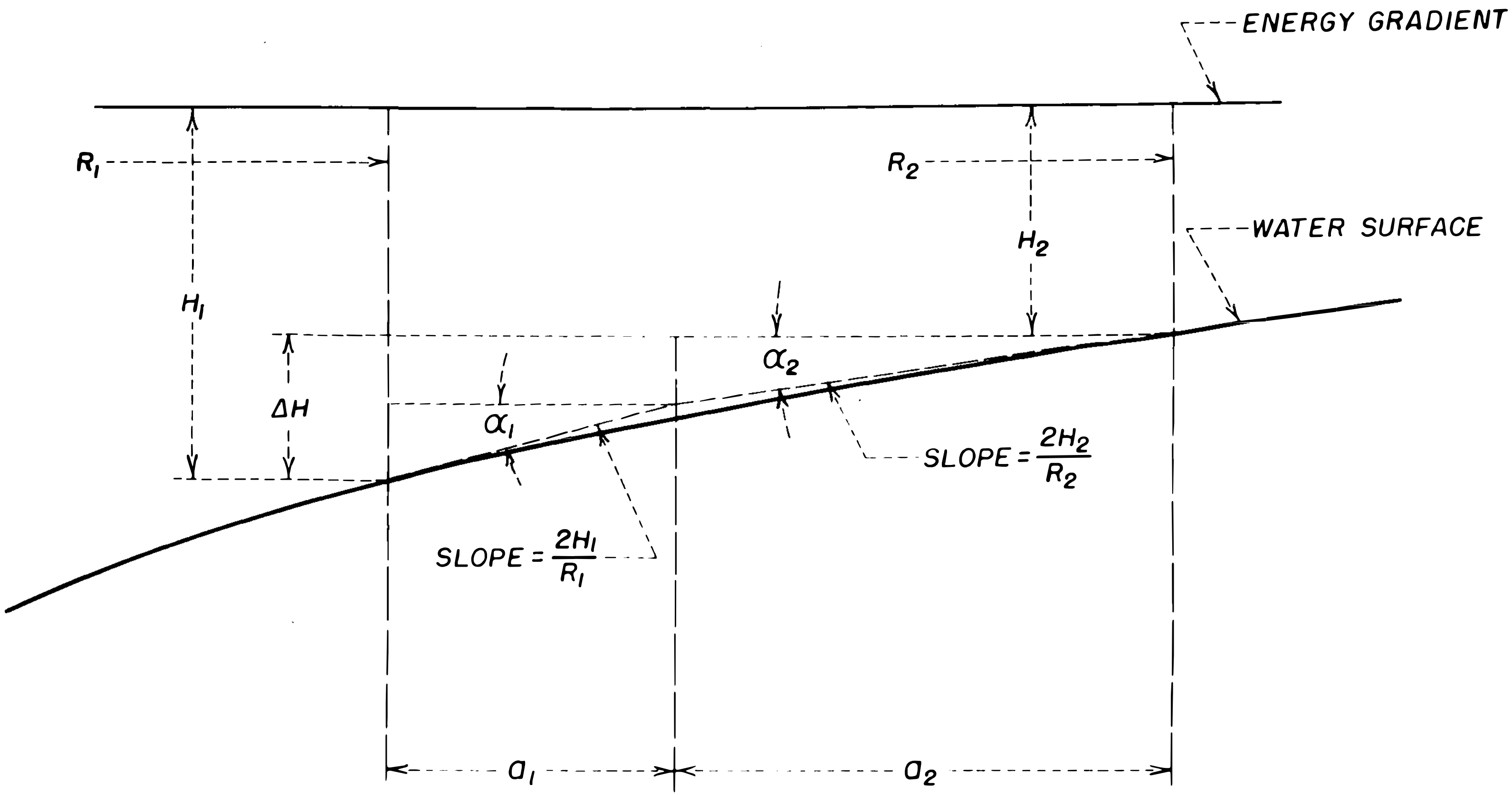
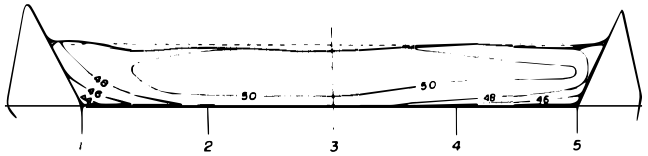
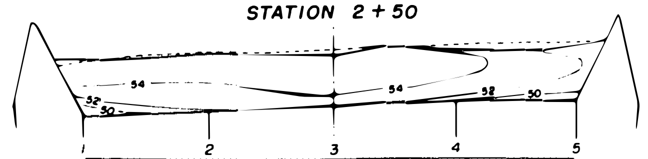

ENERGY GRADIENT
R_1
R_2
WATER SURFACE
H_1
H_2
ΔH
α_1
α_2
SLOPE = $\frac{2H_1}{R_1}$
SLOPE = $\frac{2H_2}{R_2}$
a_1
a_2

FIGURE 7



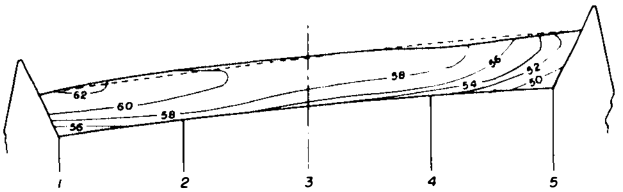
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	49.3	49.3	49.3	49.3	49.3	49.30
AV. ACTUAL VEL.	47.3	50.6	50.8	49.7	48.5	49.38

STATION 2 + 50



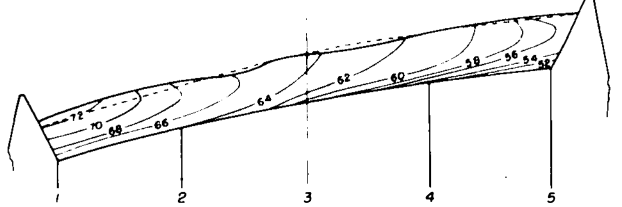
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	55.6	54.3	53.0	51.8	50.7	53.08
AV. ACTUAL VEL.	52.9	53.6	54.4	53.6	51.3	53.16

STATION 3 + 00



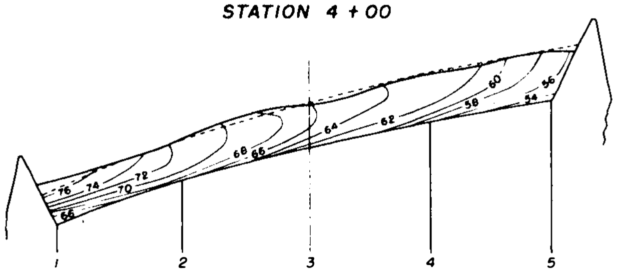
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	63.6	60.5	57.7	55.2	52.8	57.96
AV. ACTUAL VEL.	58.6	59.7	58.2	57.3	51.2	57.06

STATION 3 + 50



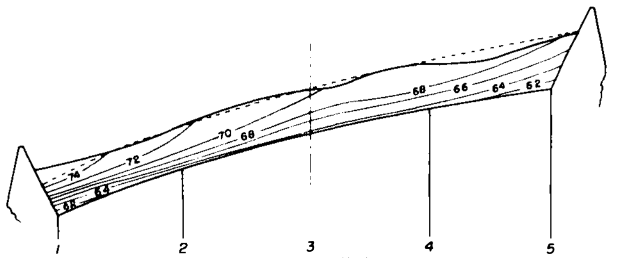
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	72.6	67.2	62.6	58.6	55.0	65.20
AV. ACTUAL VEL.	67.2	67.0	63.1	60.1	54.9	62.46

STATION 4 + 00



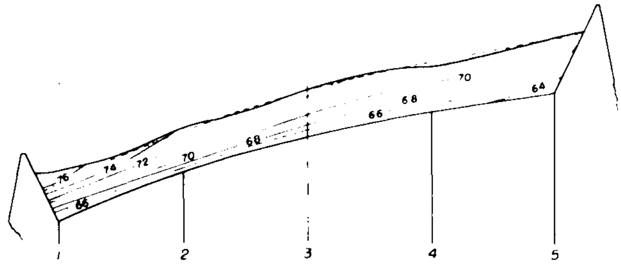
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	77.3	70.6	65.0	60.1	56.1	65.82
AV. ACTUAL VEL.	70.9	71.0	65.5	62.1	56.8	65.26

STATION 4 + 50



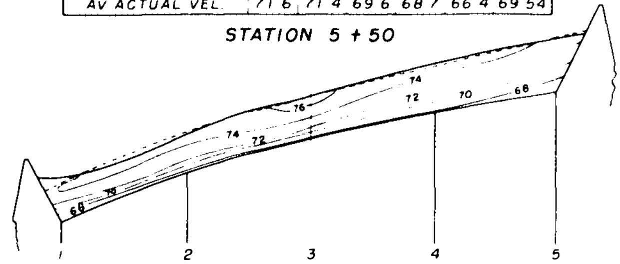
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	78.4	71.6	65.8	61.0	56.9	66.74
AV. ACTUAL VEL.	70.0	69.7	67.6	67.1	64.7	67.82

STATION 5 + 00



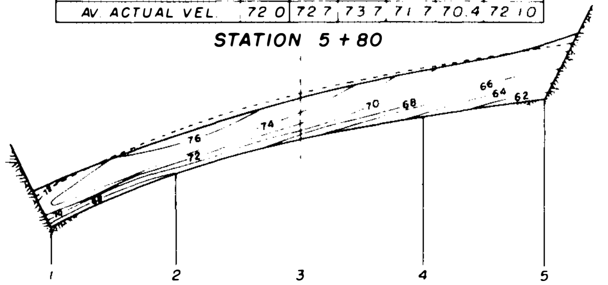
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	79.0	72.2	66.2	61.8	57.7	67.38
AV. ACTUAL VEL.	71.6	71.4	69.6	68.7	66.4	69.54

STATION 5 + 50



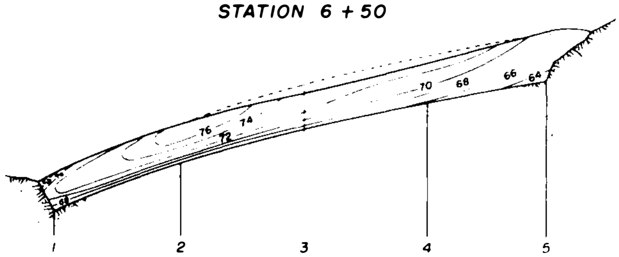
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	79.3	72.6	67.0	62.3	58.2	67.88
AV. ACTUAL VEL.	72.0	72.7	73.7	71.7	70.4	72.10

STATION 5 + 80



SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	79.1	72.8	67.0	62.9	59.0	68.16
AV. ACTUAL VEL.	72.3	74.6	73.1	67.7	65.7	70.68

STATION 6 + 50



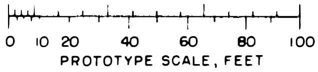
SECTION	1	2	3	4	5	MEAN
AV. THEORETICAL VEL.	79.0	72.9	67.0	63.4	59.5	68.36
AV. ACTUAL VEL.	71.0	73.5	71.7	70.1	66.0	70.46

STATION 7 + 00

EXPLANATION

------- Theoretical water surface

——— Actual water surface



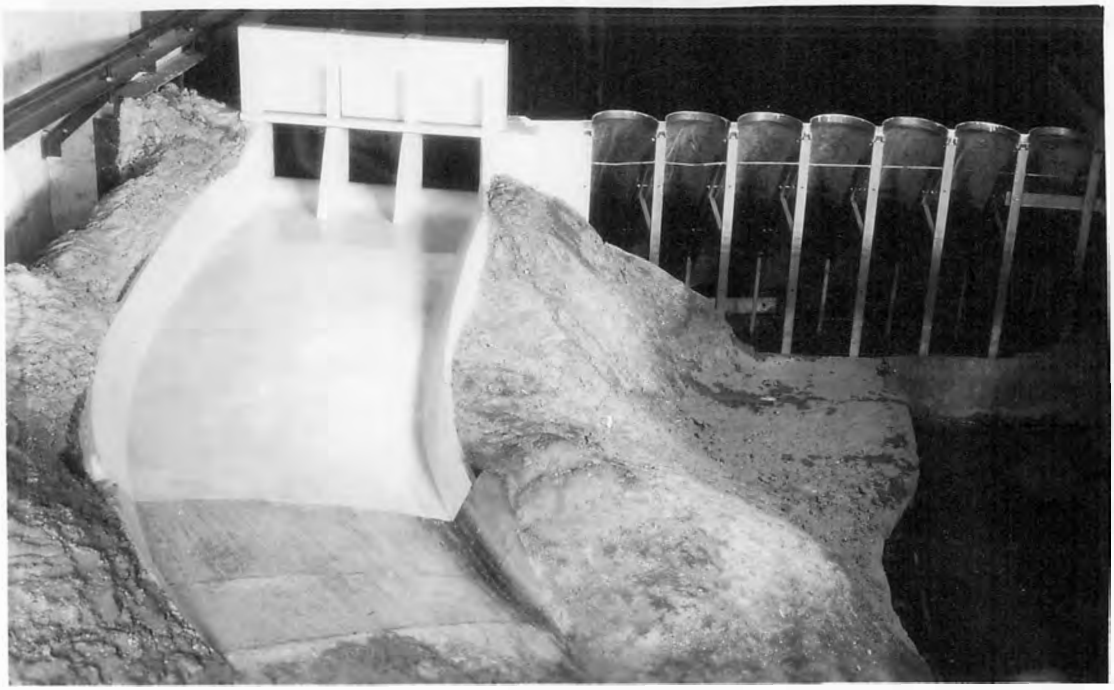
UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
SALT RIVER PROJECT - ARIZONA

BARTLETT DAM

HYDRAULIC MODEL STUDIES OF SPILLWAY - SCALE RATIO 1:100

COMPARISON OF THEORETICAL AND ACTUAL WATER SURFACE PROFILES AND VELOCITY TRAVERSES AT 175,000 SECOND-FEET DISCHARGE

DRAWN T.G.O. D.C.W.	SUBMITTED
TRACED M.E.K.	RECOMMENDED
CHECKED	APPROVED
DENVER, COLORADO — JAN. 13, 1940	



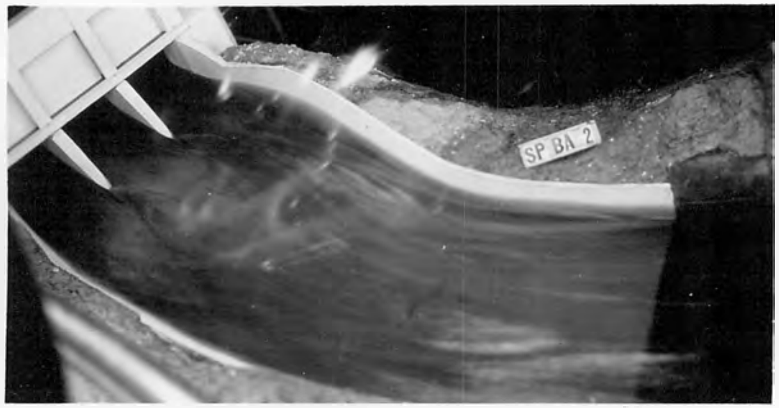
View from Downstream

View from Downstream

View from Upstream

Figure 9. Various Views of the Model, No Flow.

BARTLETT SPILLWAY, MODEL SCALE 1:100

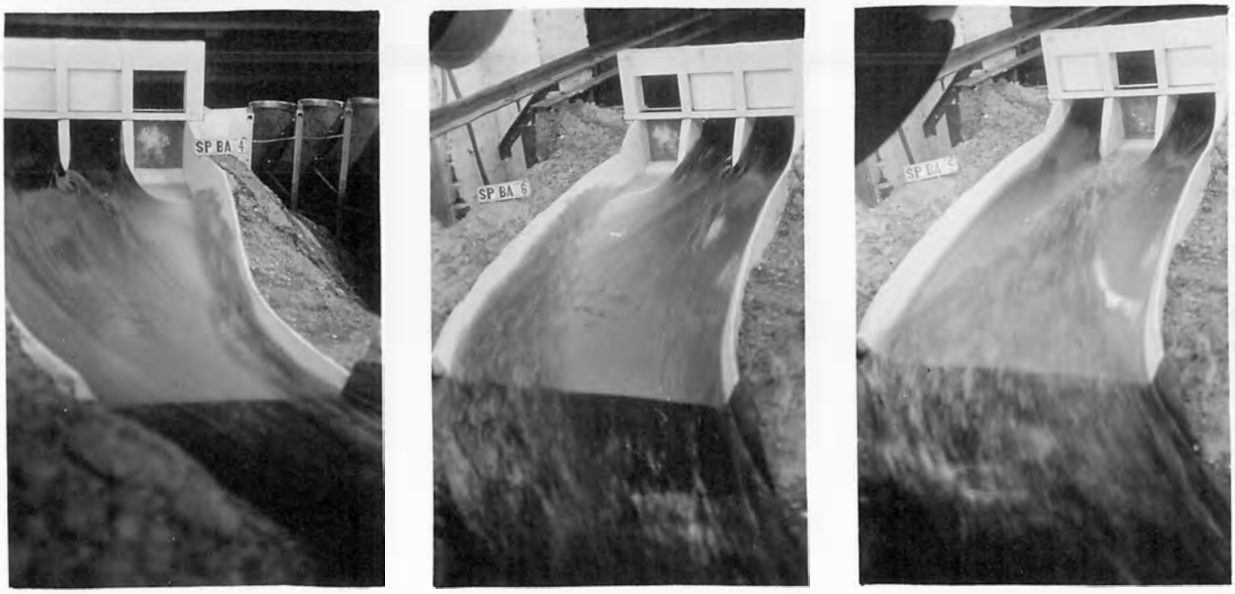


View of Left Wall

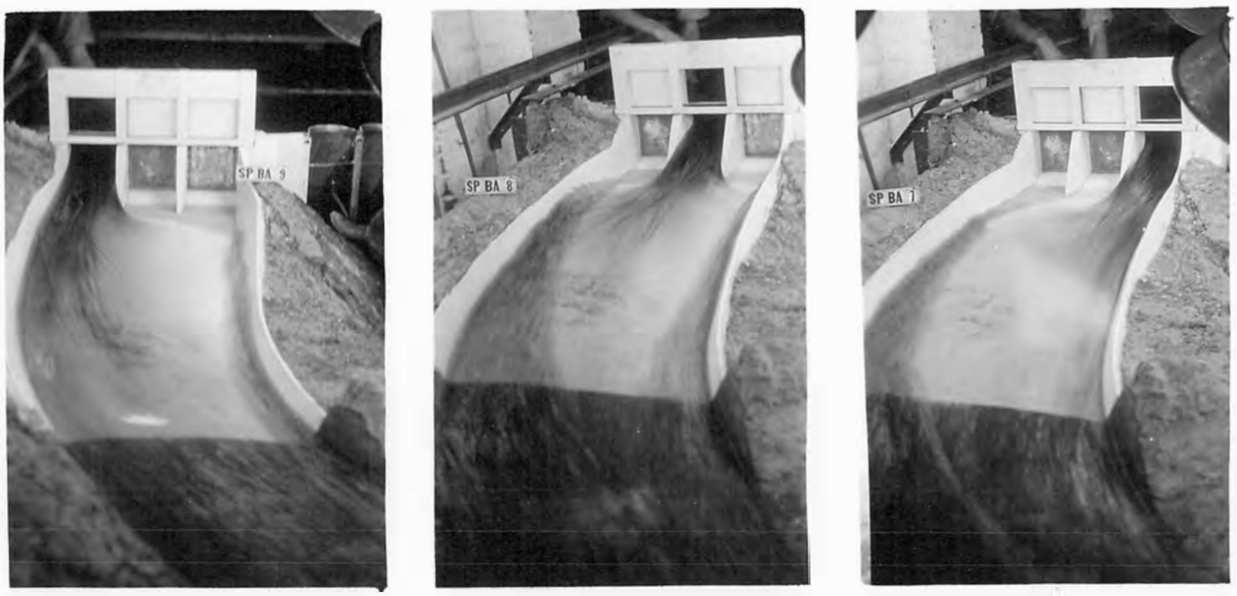
View of Right Wall

Figure 10. Various Views of the Model Discharging 175,000 Second-Feet.

BARTLETT SPILLWAY, MODEL SCALE 1:100



Views of Various Combinations Using Two Gates Open



Views of Operation with One Gate Open

Figure 11. Operation with Various Gate Combinations.

BARTLETT SPILLWAY, MODEL SCALE 1:100

View Looking Upstream

View of Floor and Left Wall

Figure 12. Flow Patterns on Channel Floor after Flow of 175,000 Second-Feet.

BARTLETT SPILLWAY, MODEL SCALE 1:100

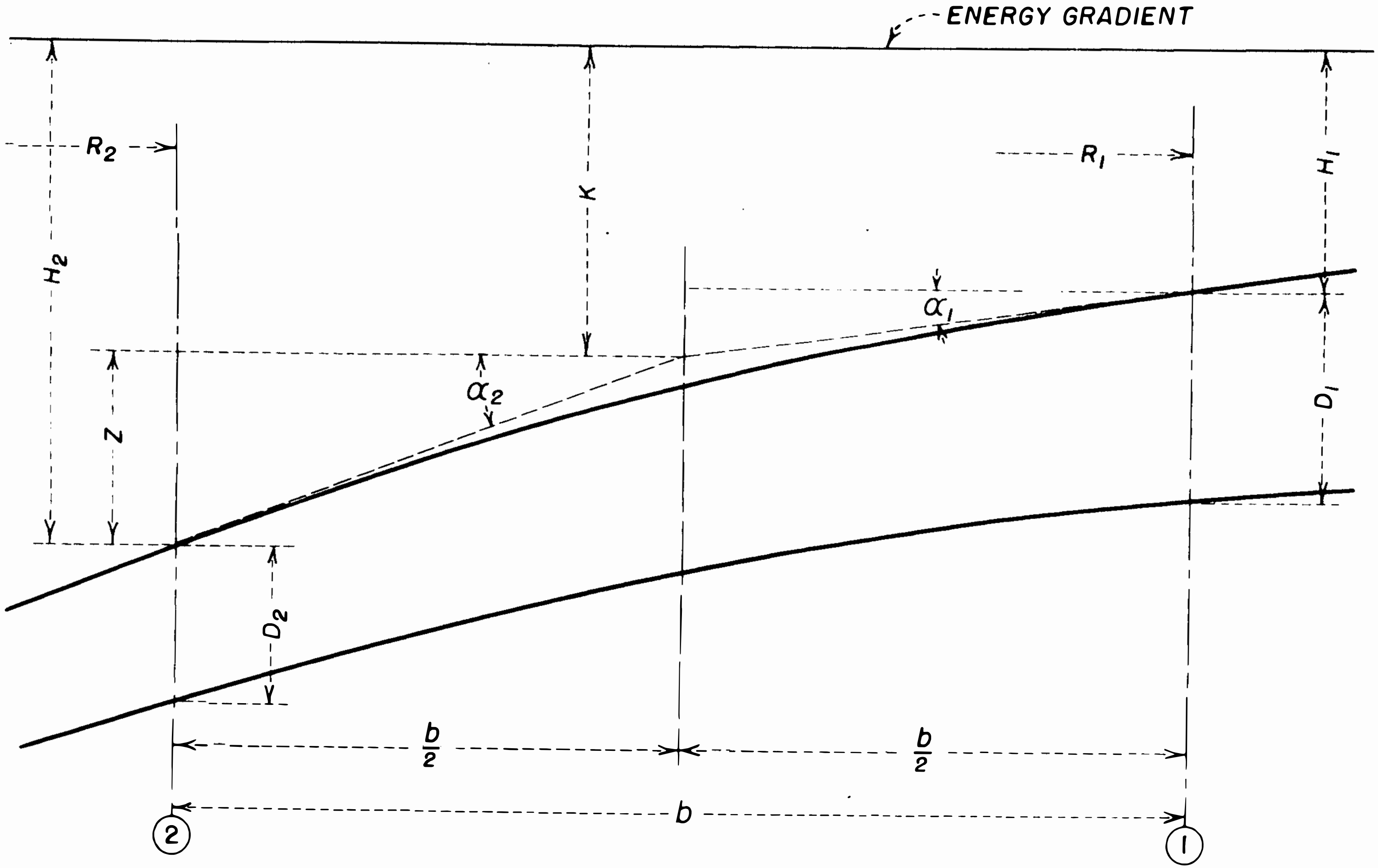
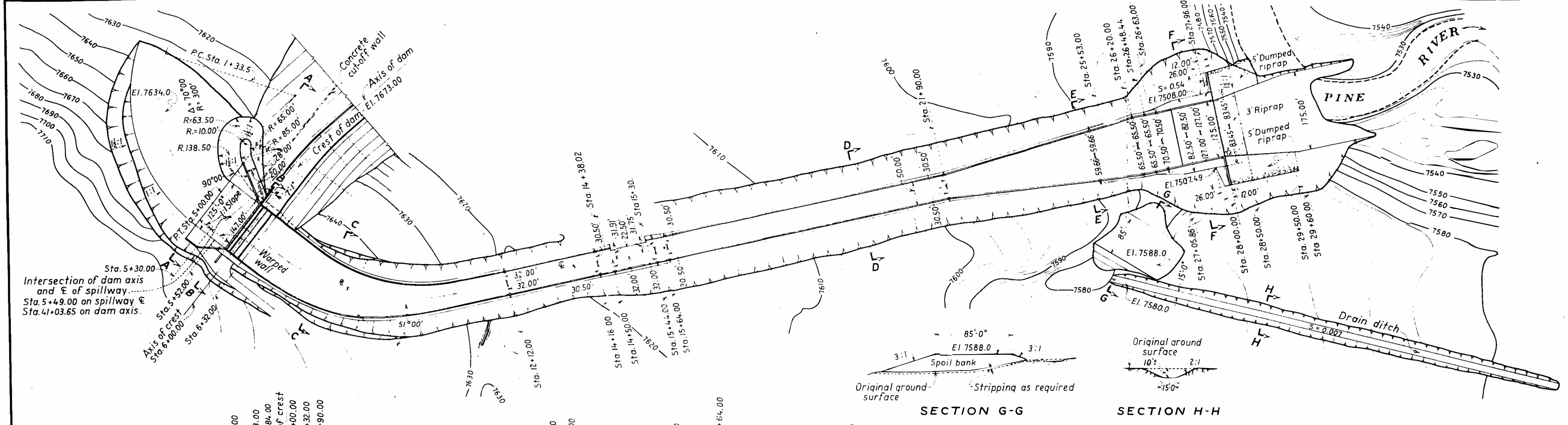
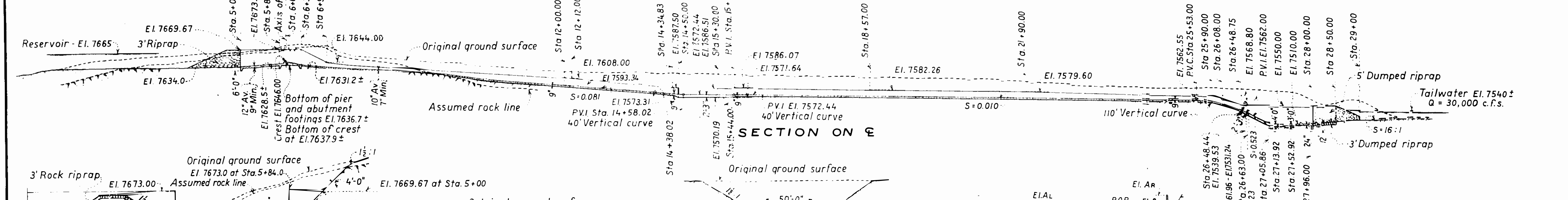
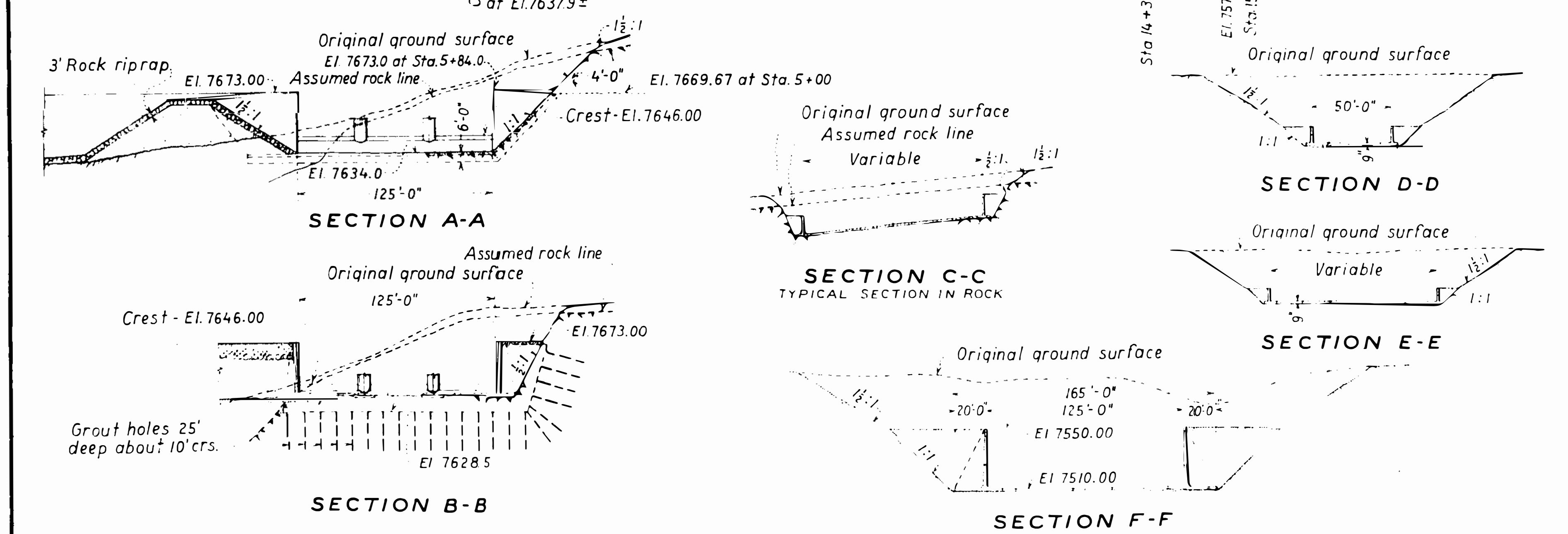

ENERGY GRADIENT
R_2
K
R_1
H_1
H_2
α_1
α_2
Z
D_1
D_2
$\frac{b}{2}$
$\frac{b}{2}$
b
2
1

FIGURE 13

FIGURE 14







NOTES

Trenches for cut-offs and drains not shown.
For alinement data of curved transition see Dwg. 191-D-61.

EXCAVATION DATA FOR CURVED TRANSITION

STA. ON ℄	DL	ELEVATIONS AL	BL	L.Q.P.	℄	R.Q.P.	BR	AR	DR
6+32	73.5	7673.0	7639.2	7639.2	7639.3	7639.6	7639.9	7673.0	73.5
+90	67.2	7646.0	7633.3	7633.8	7634.7	7635.6	7636.8	7647.0	65.8
7+00	66.8	7644.9	7632.3	7632.9	7633.8	7635.0	7636.3	7647.0	65.7
+50	64.3	7640.3	7627.0	7628.7	7630.8	7632.7	7634.8	7647.0	64.7
8+00	60.4	7636.0	7623.4	7626.3	7629.3	7631.8	7634.2	7647.0	62.2
+50	55.3	7632.8	7620.4	7624.3	7627.7	7630.7	7633.3	7647.0	58.1
9+00	49.3	7629.5	7616.9	7621.1	7624.6	7627.4	7629.9	7644.9	51.5
+50	42.4	7624.8	7612.6	7616.5	7619.4	7622.0	7623.6	7639.2	44.9
10+00	37.0	7619.5	7607.7	7610.8	7613.2	7614.9	7616.2	7633.7	40.3
+50	34.2	7617.3	7602.7	7605.0	7606.5	7607.7	7608.7	7626.1	35.3
11+00	32.9	7615.3	7599.3	7600.5	7601.5	7602.1	7602.5	7618.4	32.6
+50	32.2	7612.3	7596.4	7596.8	7597.3	7597.6	7597.5	7613.5	32.0
12+12	32.0	7608.0	7592.3	7592.3	7592.3	7592.3	7592.3	7608.0	32.0

Note: Elevations and distances shown to nearest tenth of foot. Detail dimensions and distances shown on concrete and reinf drawings.

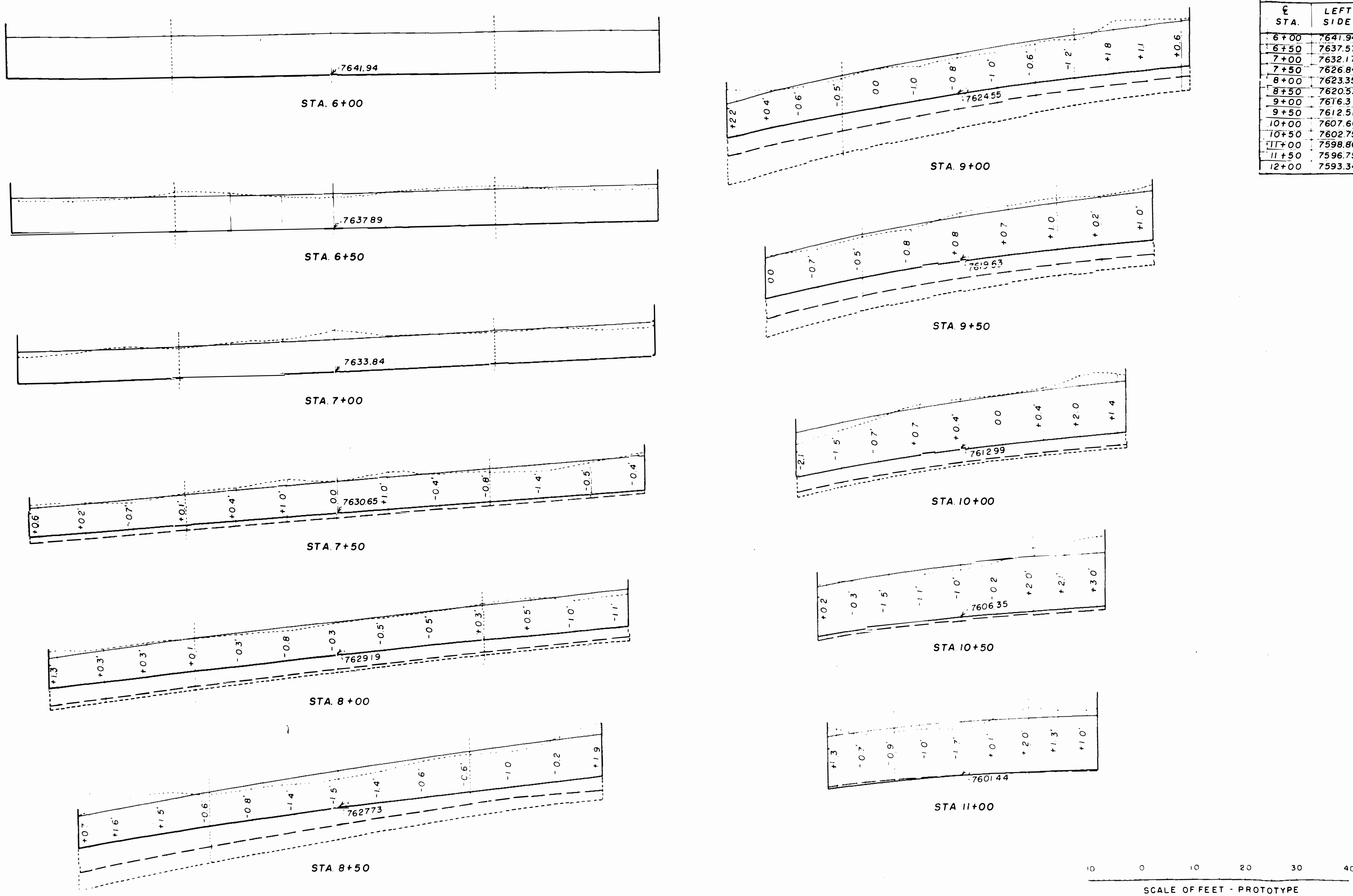
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
PINE RIVER PROJECT - COLORADO
VALLECITO DAM
SPILLWAY
EXCAVATION PLAN AND SECTIONS

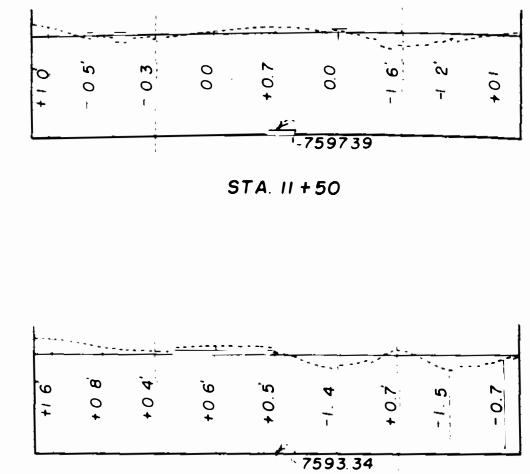
REV. 7-11-39 | REV. 9-30-39 | 4-15-39 | 5-19-39 | TRACED 6-7-38 | 6-22-38 | 7-30-38

DRAWN A.W.G. SUBMITTED: [signature]
TRACED R.E.B. RECOMMENDED: [signature]
CHECKED: R.V.C.B. APPROVED: W. H. Nalder

DENVER, COLORADO., MAY 11, 1938. 191-D-85

BOTTOM ELEVATIONS - SECTIONS AT RT. ∠s TO ℄						W. S. ELEVATIONS		ENERGY GRADIENT
℄ STA.	LEFT SIDE	LEFT QUARTER POINT	℄	RIGHT QUARTER POINT	RIGHT SIDE	LEFT SIDE	RIGHT SIDE	
6+00	7641.94	7641.94	7641.94	7641.94	7641.94	7649.9	7649.9	7664.06
6+50	7637.57	7637.58	7637.89	7638.22	7638.73	7644.4	7645.9	7663.76
7+00	7632.17	7632.81	7633.84	7635.03	7636.28	7638.3	7643.1	7663.17
7+50	7626.84	7628.70	7630.65	7632.67	7634.65	7632.6	7641.5	7662.48
8+00	7623.35	7626.34	7629.19	7631.82	7634.22	7629.3	7641.7	7661.76
8+50	7620.53	7624.34	7627.73	7630.70	7633.32	7626.8	7641.7	7661.09
9+00	7616.31	7620.80	7624.55	7627.48	7629.73	7623.2	7638.8	7660.46
9+50	7612.51	7616.65	7619.63	7621.99	7623.66	7620.5	7633.4	7659.83
10+00	7607.66	7610.72	7612.99	7614.75	7616.08	7616.7	7626.4	7659.16
10+50	7602.75	7604.78	7606.35	7607.58	7608.41	7612.6	7619.2	7658.41
11+00	7598.86	7600.47	7601.44	7602.08	7602.36	7609.2	7613.2	7657.58
11+50	7596.75	7597.22	7597.39	7597.51	7597.51	7607.5	7608.4	7656.68
12+00	7593.34	7593.34	7593.34	7593.34	7593.34	7603.8	7603.8	7655.68





EXPLANATION

Theoretical water surface
Actual water surface
Floor, taper transition No.3
Floor, taper transition No.2
Floor, taper transition No.1

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
PINE RIVER PROJECT-COLORADO

VALLECITO DAM

HYDRAULIC MODEL STUDIES OF SPILLWAY-MODEL SCALE 1:60
COMPARISON OF THEORETICAL AND ACTUAL WATER SURFACES
TWIN SPIRAL TAPER TRANSITION No.3

DRAWN L.L.A.	SUBMITTED
TRACED O.R.S.	RECOMMENDED
CHECKED	APPROVED

DENVER, COLORADO, JANUARY 27, 1938

View from Upstream

View from Downstream

View from Side

Figure 16. Views of the Model of Transition No. 3. No Flow.

VALLECITO SPILLWAY, MODEL SCALE 1:60

View from Upstream

View from Downstream

View from Side

Figure 17. Views of the Model of Transition No. 3. Designed Discharge of 30.000 Second-Feet. No Gates Used.

VALLECITO SPILLWAY, MODEL SCALE 1:60

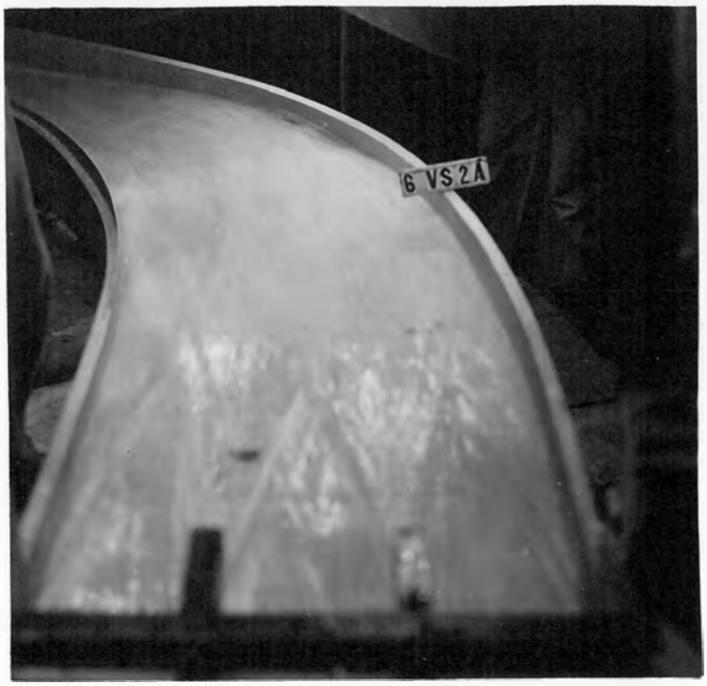
View from Upstream

View from Downstream

View from Side

Figure 18. Views of the Model of Transition No. 3. Three-Quarters Maximum Discharge, 22,500 Second-Feet. All Gates Equally Open.

VALLECITO SPILLWAY, MODEL SCALE 1:60



View from Upstream

View from Downstream

View from Side

Figure 19. Views of the Model of Transition No. 3. Half Maximum Discharge, 15,000 Second-Feet. All Gates Equally Open.

VALLECITO SPILLWAY, MODEL SCALE 1:60

View from Upstream

View from Downstream

View from Side

Figure 20. Views of the Model of Transition No. 3. One-Quarter Maximum Discharge, 7,500 Second-Feet. All Gates Equally Open.

VALLECITO SPILLWAY, MODEL SCALE 1:60