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UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

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Memorandum to Chief Designing Engineer

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SECOND PROGRESS REPORT
ON
THE DESIGN AND MODEL STUDY OF THE TRANSITION AND EXIT OF THE 102-INCH RIVER OUTLETS
FRIANT DAM - CENTRAL VALLEY PROJECT - CALIFORNIA

by

H. G. DEWEY, JR., ASSISTANT ENGINEER

- - -

Denver, Colorado,
December 28, 1939

UNITED STATES

DEPARTMENT OF THE INTERIOR

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MEMORANDUM TO CHIEF DESIGNING ENGINEER
(H. G. Dewey, Jr.)

Subject: Second progress report - The design and model study of the transition and exit of the 102-inch river outlets - Friant Dam, Central Valley project, California.

1. Introduction. The preliminary design for regulating the flow of water in the San Joaquin River, and also for diverting the river flow during construction, included four 102-inch outlets to be installed in the Friant Dam (figure 1). The outlets would be placed in two groups and would discharge onto an apron and into the main stilling pool at the toe of the dam. The discharge through the outlets would be regulated by tube valves at the upstream end of the 102-inch conduits; the maximum discharge being 4,100 second-feet through each outlet, under a head of 220 feet.

It is desirable that a jet issuing from an outlet of this type should spread quickly on an apron so as to form a hydraulic jump in the pool immediately below. If the flow is not spread, but is allowed to enter the stilling pool as a solid jet, severe eddies may form and these, together with excessive surface and bottom velocities, may cause undesirable flow conditions in the pool. This disturbance may even continue far enough downstream to cause scouring beyond the apron.

Previous model studies of the Grand Coulee and Shasta Dams did not include any outlets from which the issuing jet had to be spread immediately. The outlets in these studies were located higher on the face of the dam than those of the Friant Dam, thus enabling the jets to spread of their own accord as they flowed down the face of the dam into the stilling pool. The first problem requiring immediate spreading of a jet issuing from an outlet occurred during the model studies of the outlets for the Marshall Ford Dam. From this study it was found that to spread a jet quickly and efficiently a transition should be placed in the outlet proper, followed by an exit trough or "beaver-tail" cut in the face of the dam.

The transition designed for the 102-inch river outlets of the Friant Dam (figure 2) embodied the principles of the Marshall Ford Dam outlet studies, but instead of maintaining constant area throughout the length of the transition, the area has been gradually

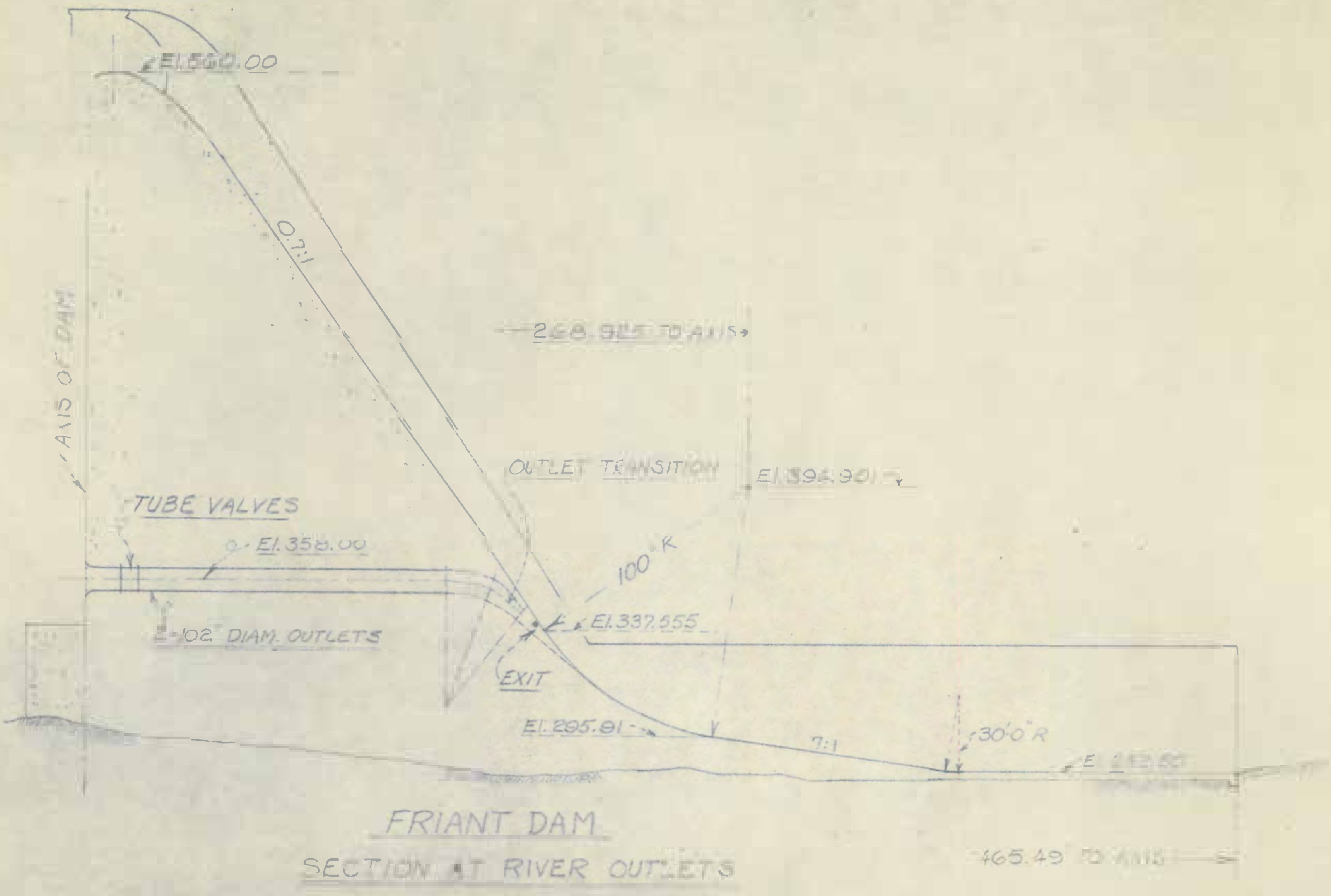
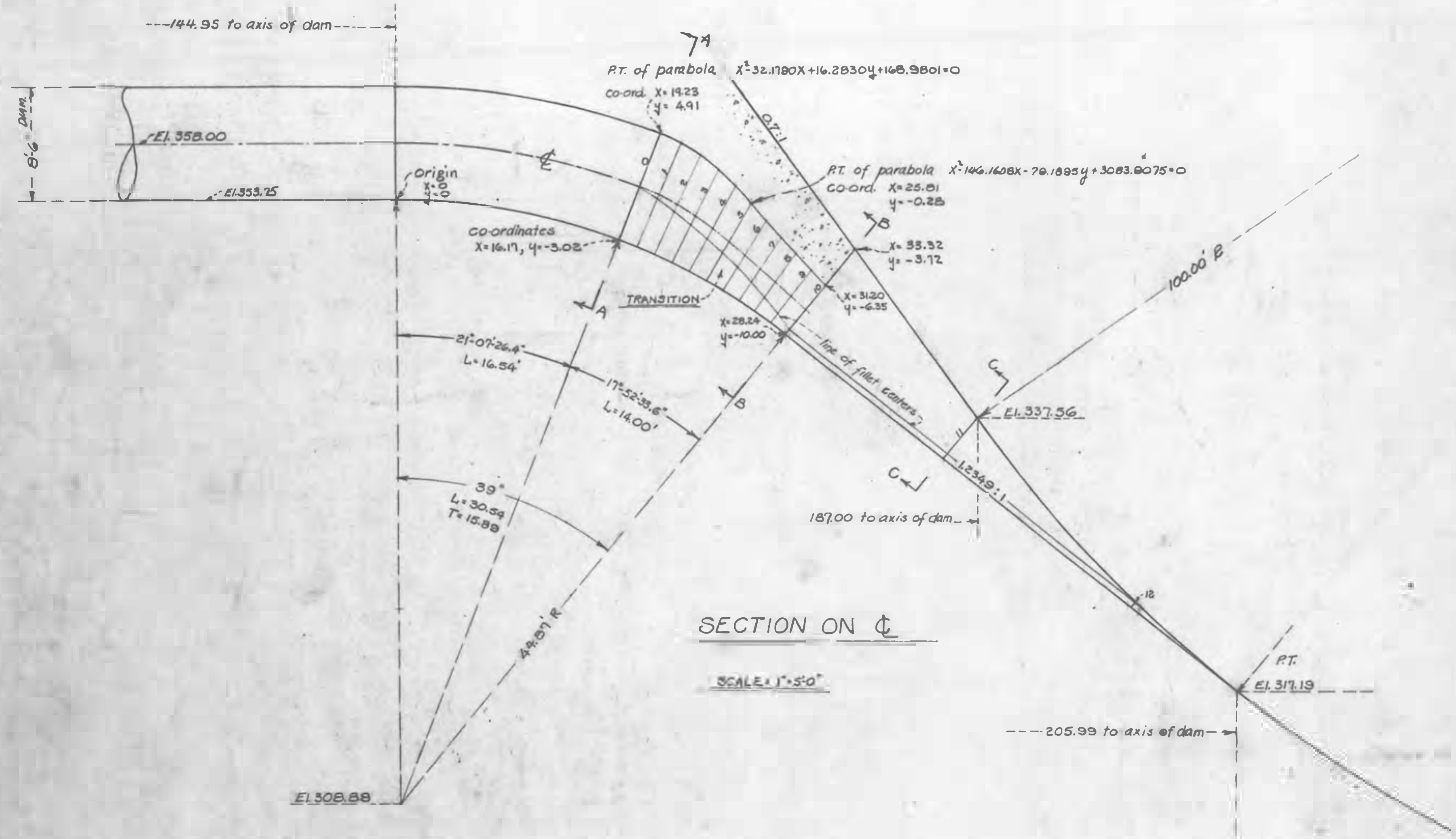
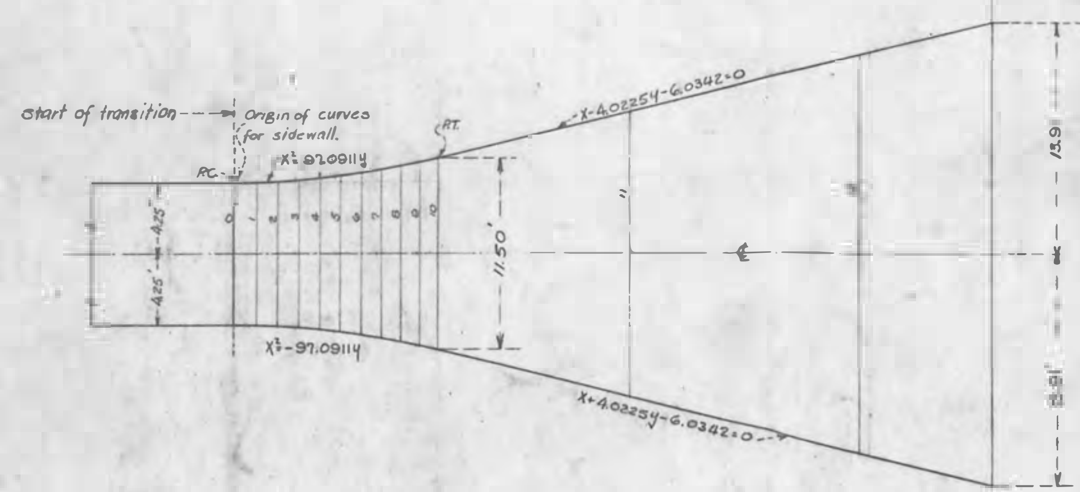

El. 560.00
0.7:1
AXIS OF DAM
TUBE VALVES
El. 358.00
2-102" DIAM. OUTLETS
298.925 TO AXIS
OUTLET TRANSITION
El. 394.901
100' R
El. 337.555
EXIT
El. 295.91
7:1
30'0" R
465.49 TO AXIS
FRIANT DAM
SECTION AT RIVER OUTLETS

FIGURE 1



SECTION ON ℄

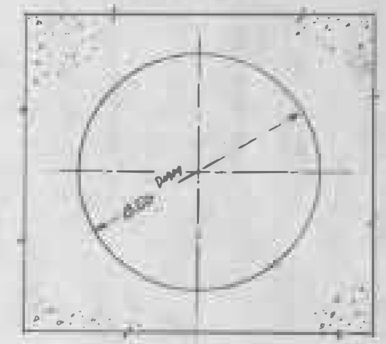
SCALE: 1"=5'-0"



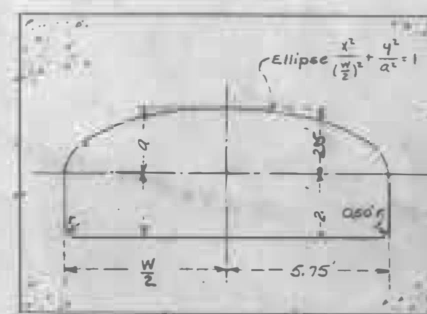
PROJECTION OF INVERT ON HORIZONTAL PLANE

TABLE OF TRANSITION DIMENSIONS

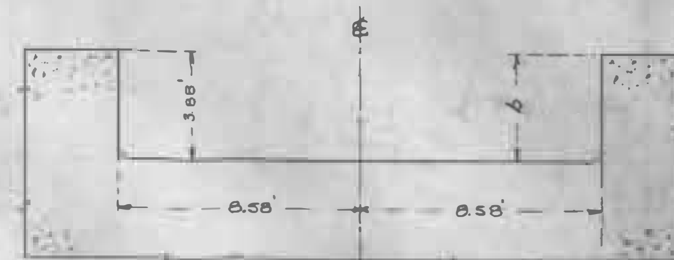
SECTION	Co-ord. INVERT		Co-ord. ROOF		a	b	r	W/2
	x	y	x	y				
0	16.17	-3.02	19.23	+4.91	4.25	4.25	4.25	4.26
1	17.47	-3.54	20.74	+4.19	4.20	4.20	3.88	4.27
2	18.75	-4.11	22.14	+3.27	4.06	4.06	3.50	4.32
3	20.01	-4.71	23.45	+2.19	3.86	3.86	3.13	4.40
4	21.26	-5.35	24.67	+1.00	3.61	3.61	2.75	4.52
5	22.48	-6.04	25.81	-0.28	3.32	3.32	2.38	4.66
6	23.68	-6.76	26.90	-1.57	3.05	3.05	2.00	4.83
7	24.86	-7.51	27.98	-2.82	2.82	2.82	1.63	5.03
8	26.01	-8.31	29.06	-4.02	2.63	2.63	1.25	5.25
9	27.14	-9.14	30.13	-5.20	2.47	2.47	0.88	5.49
10	28.24	-10.00	31.20	-6.35	2.35	2.35	0.50	5.75
11	39.61	-19.21				3.88	0.50	8.58
12	53.28	-30.28				0.50	0.50	11.97



SEC A-A



SEC B-B



SEC. C-C

FRIANT DAM
PROPOSED DESIGN FOR THE TRANSITION AND EXIT OF THE 102" RIVER OUTLETS

H.G.D.JR. 9-13-38

reduced until the area of the extreme end section of the transition is 85 percent of the area of the 102-inch outlet. The method of designing the transition for the Friant Dam outlets and the basic assumptions made are described in full in the Appendix.

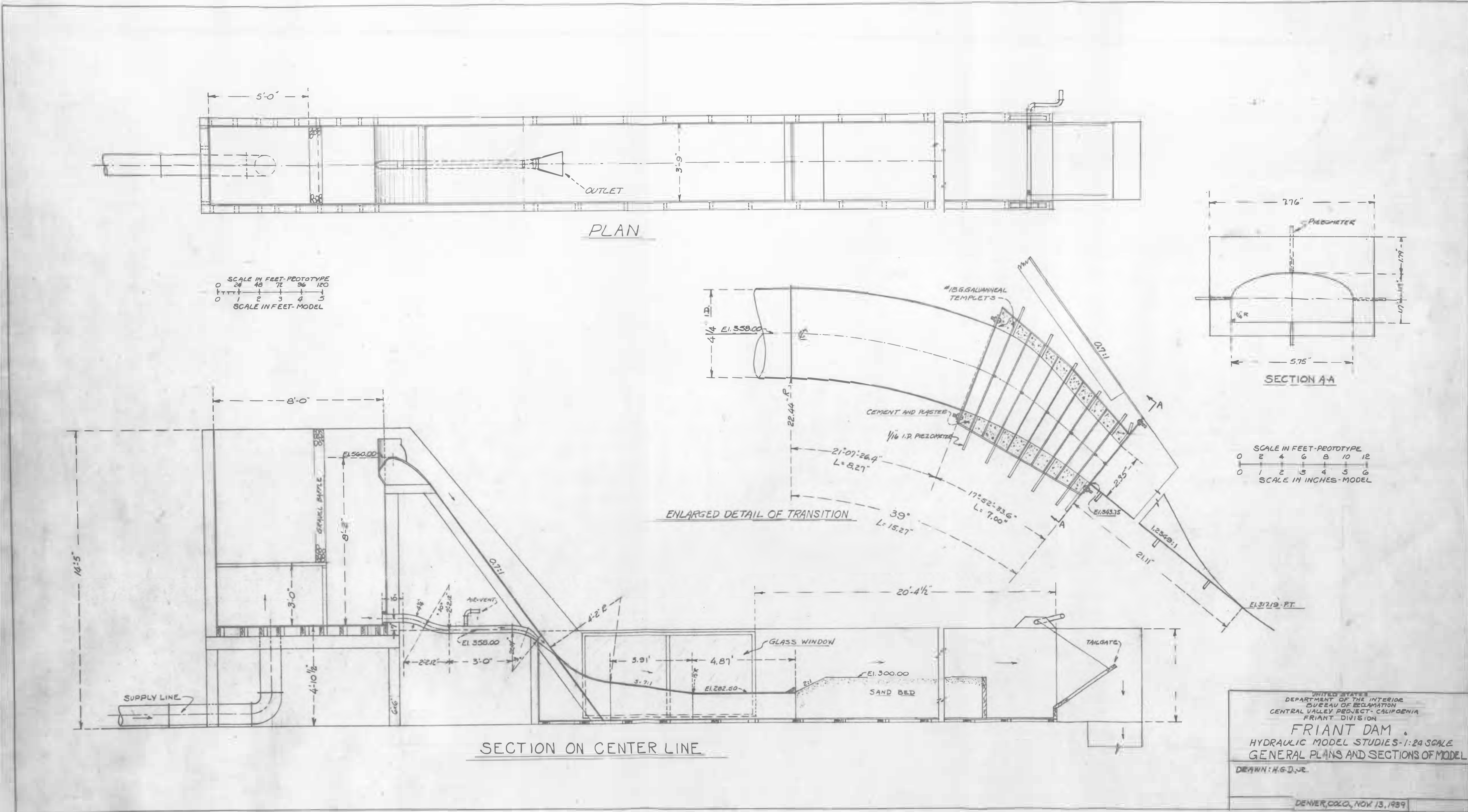
Model studies were incorporated with the design of the Friant Dam outlet transition and exit to determine: (1) The pressures within the transition under different heads; (2) the spread of the issuing jet on the apron, and other flow conditions; (3) the velocity distribution in the flow at the extreme end section of the transition, at the end of the exit, and on the horizontal part of the apron; and (4) the calibration curves for the transition, treating it as an orifice.

The first progress report, October 12, 1939, included drawings of the design of the outlet transition and exit (figure 2); and of the pressures within the transition (figures 4A, 4B, and 4C), as determined from a study of a 1 to 24 scale sectional model (see paragraph 2). The tests on this model have now been completed and are discussed in this report.

Additional tests were made on a complete 1 to 60 scale model (as compared to tests on a 1 to 24 scale sectional model described in this report) to determine a correct horizontal spacing of the outlets on the face of the dam, and to locate training walls in the stilling pool for improving the hydraulic jumps formed during operation of the outlets. This study, however, has been abandoned, as the preliminary design of the outlets has been changed.

Because of the difficulty of regulating the flow in the outlets by tube valves at the upstream end, the outlets have been removed from the spillway and have been placed to the left of the main stilling pool. The tube valves, moreover, have been placed at the downstream end of the outlets, eliminating the outlet transition and exit. This design requires a separate stilling pool below the tube valves, since it is impractical to place the tube valves at the face of the main spillway. Accordingly, a 1 to 34.375 scale model study is now in progress to evolve a satisfactory design of this separate stilling pool. Upon completion of these tests, a report will be submitted covering the details of the design.

2. The model. Preliminary tests on the Shasta Dam spillway were made on a 1 to 40 scale sectional model. Upon completion of these tests, the same model was adapted for tests of the Friant Dam spillway, but on a scale of 1 to 24 (figure 3). One of the four river outlets was installed on this model, but lack


5'-0"
3'-9"
OUTLET
PLAN
SCALE IN FEET-PROTOTYPE
0 24 48 72 96 120
0 1 2 3 4 5
SCALE IN FEET-MODEL
8'-0"
14'-5"
El. 560.00
GRAVEL BAFFLE
8'-2"
3'-0"
4'-10 1/2"
0.7:1
4'-2" R
AIR-VENT
El. 358.00
2'-2.12"
3'-0"
SUPPLY LINE
SECTION ON CENTER LINE
20'-4 1/2"
GLASS WINDOW
3.91'
4.87'
El. 282.50
El. 300.00
SAND BED
TAILGATE
ENLARGED DETAIL OF TRANSITION
El. 358.00
22.44" R
#18 G. GALVANNEAL TEMPLETS
CEMENT AND PLASTER
1/16 I.D. PIEZOMETER
21°-07'-26.4"
L= 8.27"
17°-52'-33.6"
L= 7.00"
39°
L= 15.27"
2.35"
El. 343.75
1.2549:1
21.11"
El. 317.19 - P.T.
7.76"
PIEZOMETER
SECTION A-A
5.75"
SCALE IN FEET-PROTOTYPE
0 2 4 6 8 10 12
0 1 2 3 4 5 6
SCALE IN INCHES-MODEL
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FRIANT DIVISION
FRIANT DAM
HYDRAULIC MODEL STUDIES - 1:24 SCALE
GENERAL PLANS AND SECTIONS OF MODEL
DRAWN: H.G.D. JR.
DENVER, COLO., NOV. 13, 1939

of sufficient depth of the head box made it necessary to place a reversed curve at the upstream end of the outlet (Section on Center Line, figure 3).

The model transition was made of metal templets, which were cut to correct shape and placed at equal intervals along the transition (Enlarged Detail of Transition, figure 3). Piezometers were attached to each templet and were made flush with the inner surface of the transition, formed by placing cement-plaster mortar between the templets and screeding to a smooth surface. Additional piezometers were placed along the floor of the exit.

3. Pressures in transition. Pressures within the transition were recorded for heads of 220, 150, 50, and 25 feet above elevation 358.00 (center line of 102-inch outlet). Since there was no upstream control in the model outlet, the discharges for these heads were greater than for the actual conditions. A study of figures 4A, 4B, and 4C reveals that the pressures within the transition are satisfactory. The greatest negative pressure occurs at piezometer 18 on the crown, but it is only slightly greater than -10 feet of water, prototype. It is believed that the pressures within the transition would be satisfactory for all operating conditions, even though the tests were made for a greater discharge than will exist in the prototype, and the alinement was not exact.

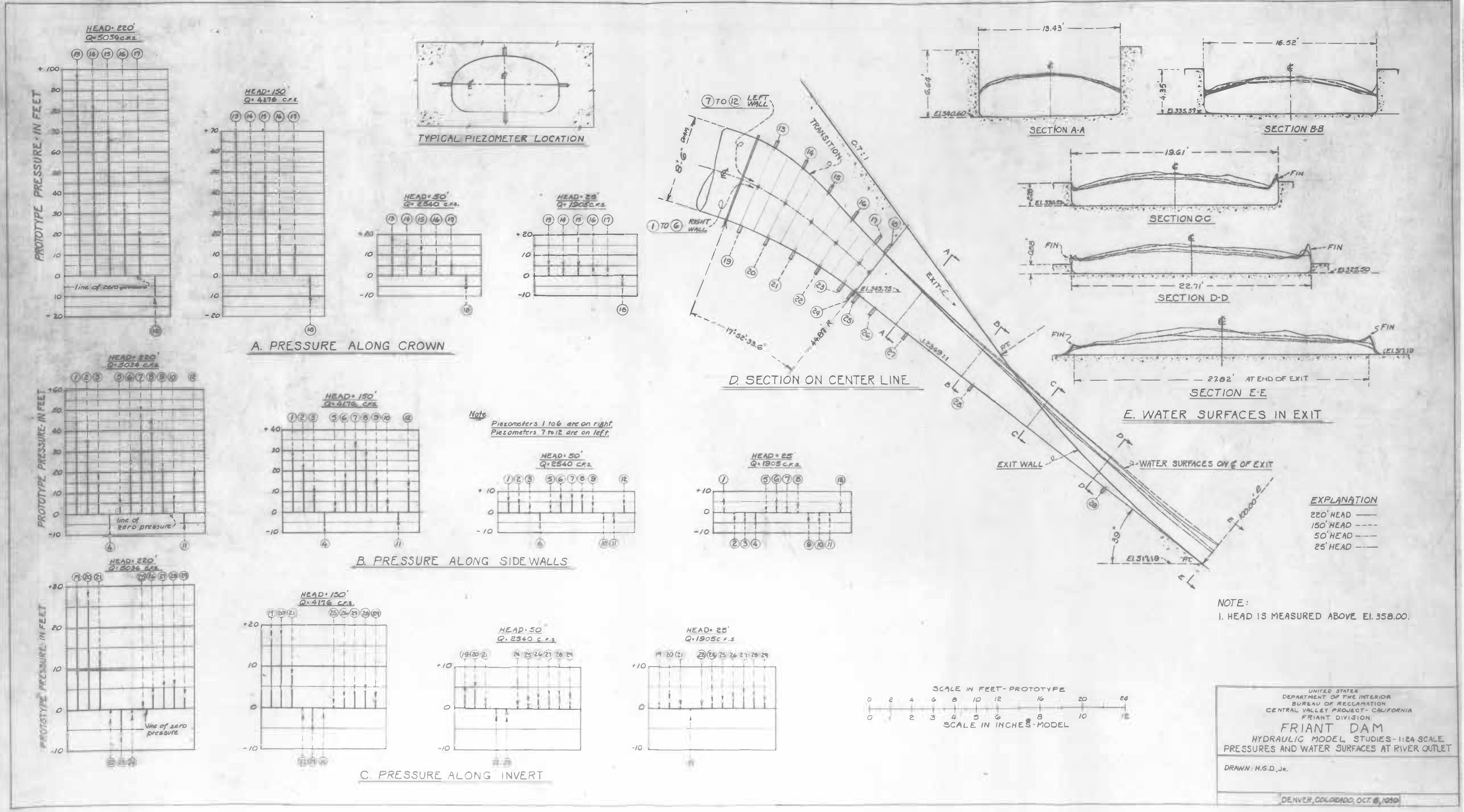
4. Spread of issuing jet, and other flow conditions. Hydraulic literature contains little data relative to the spreading of a jet on a horizontal or sloping surface. Moreover, a mathematical analysis is particularly difficult because flow of this type is three-dimensional. One of the most recent studies of the spread of a jet on a horizontal surface was made at the University of Iowa. This study was quite elementary, but served to indicate the nature of the problem.[1]

[1] Spreading of a Water Jet on a Flat Floor (at Super-critical Velocities); a thesis by Enver Muratzade; August, 1939; University of Iowa. Copy on file in U.S.B.R. technical library.

No attempt was made during the study of the Friant river outlets to make a mathematical analysis of the spread of the jet, but data were taken to record the amount of spread. It is planned, at some later date, to make an attempt at a mathematical analysis of the spreading of the jet.

It was previously noted that to spread a jet effectively a transition should be placed in the outlet proper. This enables the flow to diverge under pressure. The addition of an exit cut in the face of the dam is required because of the necessity to embed the transition in the concrete of the dam. Accordingly, the spreading of the jet is still confined in the exit and does not

FIGURE 4



TYPICAL PIEZOMETER LOCATION
A. PRESSURE ALONG CROWN
B. PRESSURE ALONG SIDEWALLS
C. PRESSURE ALONG INVERT
D. SECTION ON CENTER LINE
E. WATER SURFACES IN EXIT
PROTOTYPE PRESSURE - IN FEET
HEAD = 220' Q = 5034 c.f.s.
HEAD = 150' Q = 4176 c.f.s.
HEAD = 50' Q = 2540 c.f.s.
HEAD = 25' Q = 1905 c.f.s.
line of zero pressure
Note
Piezometers 1 to 6 are on right.
Piezometers 7 to 12 are on left.
7 TO 12 LEFT WALL
1 TO 6 RIGHT WALL
TRANSITION
0.7:1
8'-6" DIAM.
EXIT-℄
EXIT WALL
WATER SURFACES ON ℄ OF EXIT
El. 343.75
El. 317.19
100.00'-R
SECTION A-A
SECTION B-B
SECTION C-C
SECTION D-D
SECTION E-E
13.43'
16.52'
19.61'
22.71'
27.82' AT END OF EXIT
FIN
EXPLANATION
220' HEAD
150' HEAD
50' HEAD
25' HEAD
NOTE:
1. HEAD IS MEASURED ABOVE El. 358.00.
SCALE IN FEET - PROTOTYPE
SCALE IN INCHES - MODEL
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HYDRAULIC MODEL STUDIES - 1:24 SCALE
PRESSURES AND WATER SURFACES AT RIVER OUTLET
DRAWN: H.G.D., Jr.
DENVER, COLORADO, OCT. 6, 1950

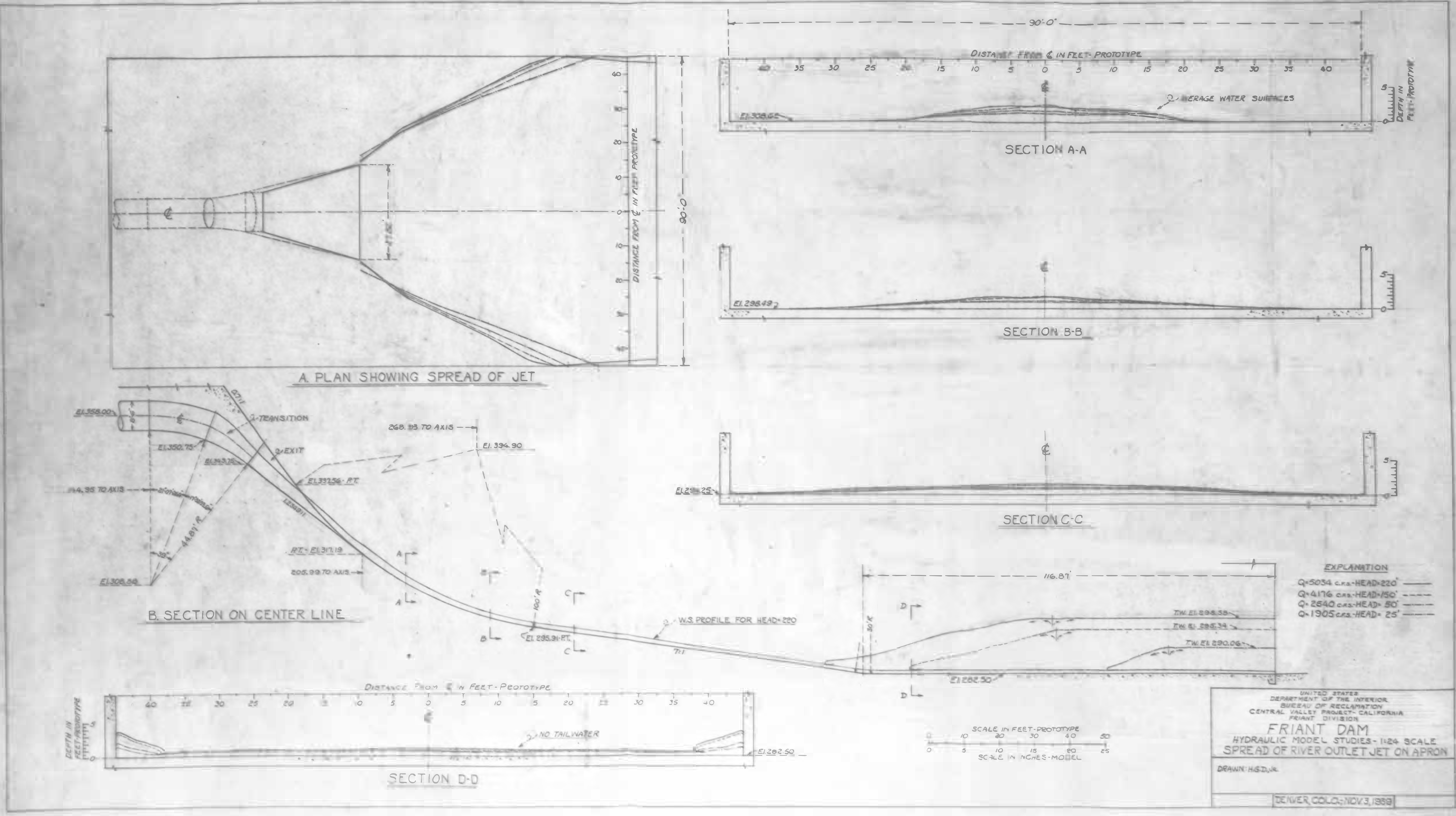
spread freely until the flow reaches the apron. Figure 4E shows cross sections of the flow at various points in the exit. It can be seen that a fin forms along the side walls of the exit (figures 4E and 6A). These fins were not excessive and could not be eliminated, because, when a high-velocity jet is allowed to expand or spread, fins will form along the walls which confine the jet. Any further convergence of the confining walls will only aggravate this condition. It is important, therefore, to make proper allowance for divergence in the exit below the transition.

Figures 5 and 6B show the spread of the issuing jet on the apron proper. Under normal prototype conditions, the tailwater will be considerably greater than that used during these tests. However, a problem may develop in the future which will require a jet issuing from an outlet to be spread over an apron and enter a stilling pool uniformly enough to allow a hydraulic jump to form. Accordingly, the spread of the jet was recorded assuming free flow on the apron. By studying figure 5, it may be seen that the jet spreads uniformly, and on the horizontal part of the apron (Section D-D, figure 5), the jet is of nearly uniform depth, except for the side fins caused by the intersection of the spreading jet against the side walls of the pool (figure 5A).

Observations, only, were made of the flow on the apron for normal tailwater conditions. These revealed that as the spreading jet plunged into the tailwater, return flow occurred along the side walls. This condition was also prevalent in the 1 to 60 scale model, but it was eliminated by training walls placed in the stilling pool. These tests, however, were not completed.

Unfavorable flow conditions have been found to exist when flow in a spillway passes over an outlet exit. Such conditions were found during the model studies of the Shasta Dam. In these studies, after the outlets had been spaced properly, it was necessary to add a streamlined deflector or "eyebrow" on the face of the dam immediately above the exit. By doing this, the flow passing over the exit was deflected upward, thus eliminating excessive splash and spray, normally caused by the spillway flow plunging into the exit. It was also found with no flow through the outlet, but with flow over the spillway, that the flow passing over the exit evacuated the air from the outlet. With the pressure in the outlet less than atmospheric, the spillway flow was pulled down into the exit, causing excessive spray. This condition, however, was readily eliminated by aerating the outlets.

FIGURE 5


90'-0"
DISTANCE FROM ℄ IN FEET-PROTOTYPE
DEPTH IN FEET-PROTOTYPE
AVERAGE WATER SURFACES
El. 308.62
SECTION A-A
El. 298.49
SECTION B-B
El. 291.25
SECTION C-C
A. PLAN SHOWING SPREAD OF JET
DISTANCE FROM ℄ IN FEET-PROTOTYPE
El. 358.00
℄-TRANSITION
℄-EXIT
El. 350.75
El. 349.25
268.93 TO AXIS
El. 394.90
El. 337.56-P.T.
114.35 TO AXIS
44.61' R.
El. 308.56
P.T.-El. 317.19
205.99 TO AXIS
B. SECTION ON CENTER LINE
100' R
El. 285.91-P.T.
W.S. PROFILE FOR HEAD=220
7:1
116.87'
T.W. El. 296.38
T.W. El. 295.34
T.W. El. 290.06
El. 282.50
EXPLANATION
Q=5034 c.f.s.-HEAD=220'
Q=4176 c.f.s.-HEAD=150'
Q=2640 c.f.s.-HEAD=50'
Q=1905 c.f.s.-HEAD=25'
NO TAILWATER
El. 282.50
SECTION D-D
SCALE IN FEET-PROTOTYPE
SCALE IN INCHES-MODEL
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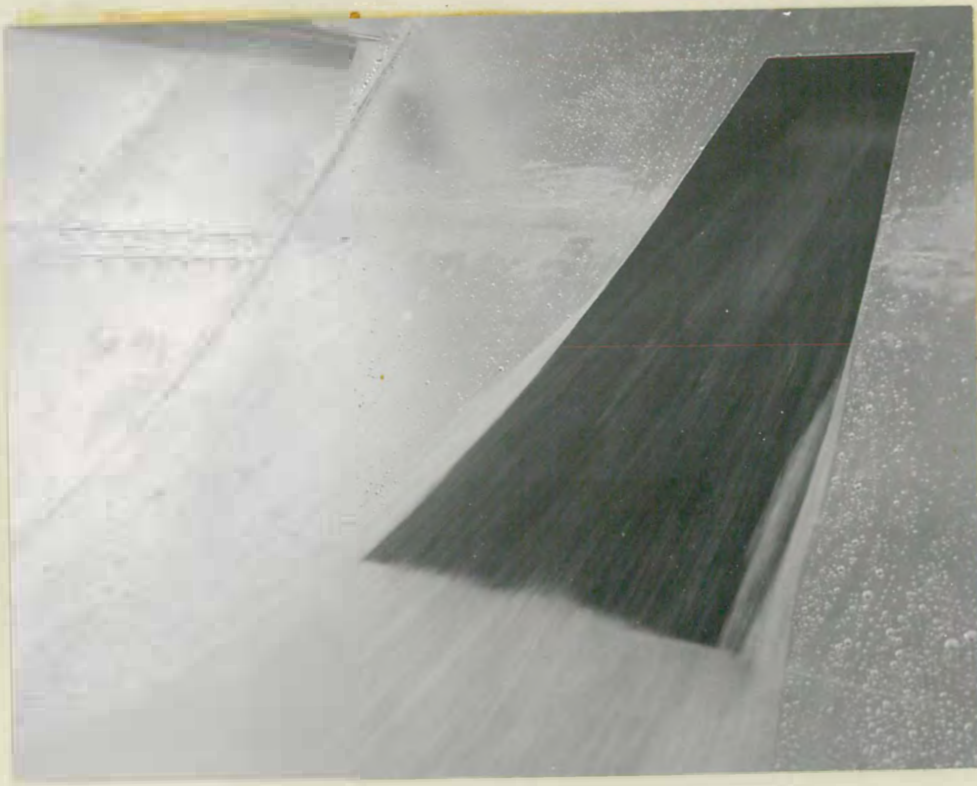
HYDRAULIC MODEL STUDIES - 1:24 SCALE
SPREAD OF RIVER OUTLET JET ON APRON
DRAWN: H.S.D.,JR.
DENVER, COLO., NOV. 3, 1938

A. OUTLET DISCHARGE 4,700 SECOND-FEET
HEAD= 190 FEET ON ELEV. 358.00



B. OUTLET DISCHARGE 4,700 SECOND-FEET
HEAD= 190 FEET ON ELEV. 358.00

RECOMMENDED DESIGN

A. SPILLWAY DISCHARGE 90,000 SECOND-FEET
NO FLOW IN OUTLET. AIR VENT OPEN.

B. SPILLWAY DISCHARGE 90,000 SECOND-FEET
OUTLET DISCHARGE 4,700 SECOND-FEET
HEAD= 190 FEET ON OUTLET(GATE ELEV. 35[illegible].00)

RECOMMENDED DESIGN

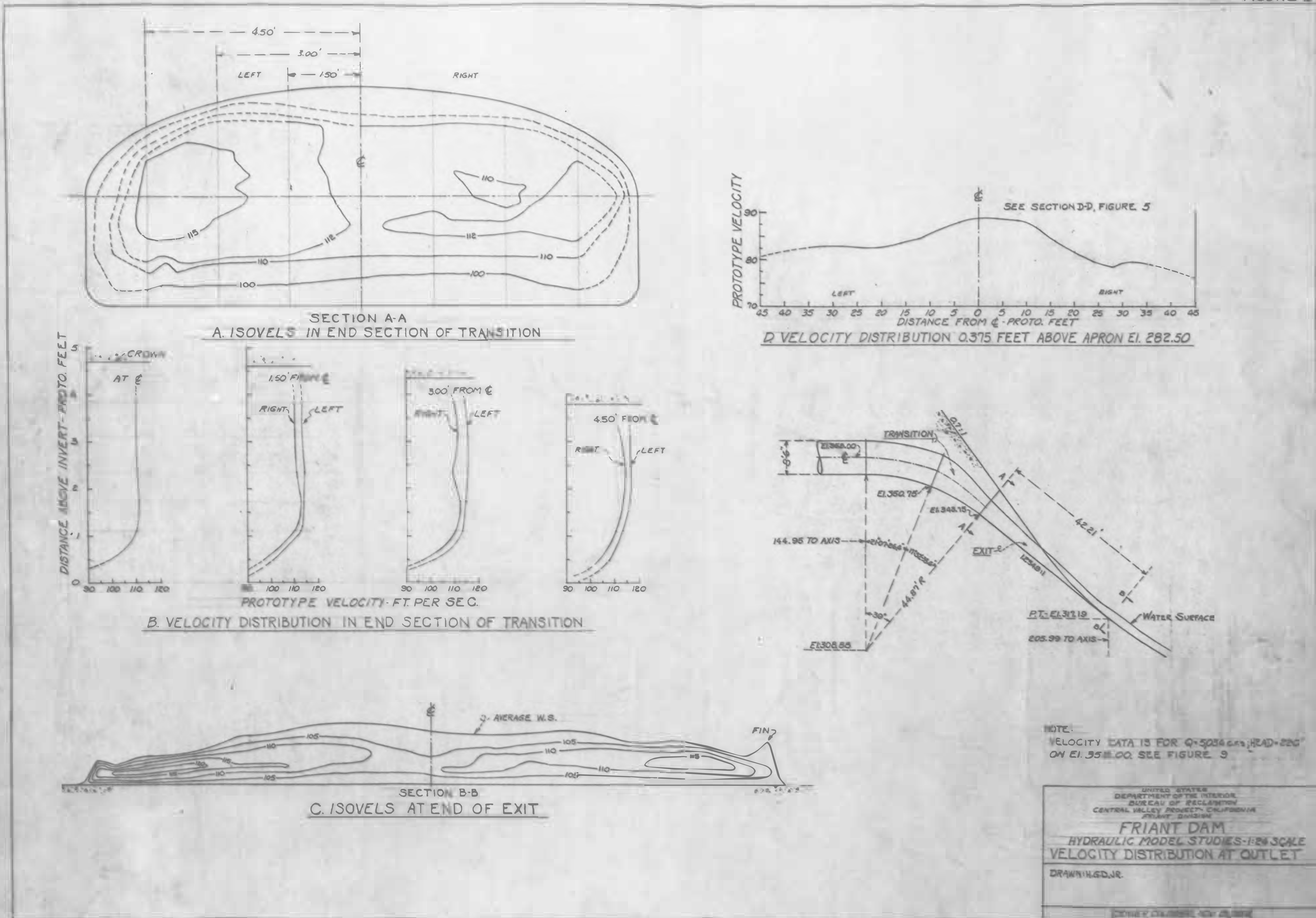
A similar study was made of the Friant Dam river outlets to determine if any of the unfavorable conditions were present which had existed on the Shasta Dam for flow over the spillway. Fortunately, the Friant Dam outlets were much nearer the stilling pool than those on the Shasta Dam. As the spillway flow increased (no flow in the outlet), the water plunging into the exit was deflected upward at the floor of the exit until its trajectory was nearly parallel with the slope of the apron. The flow then entered the tailwater immediately without creating any undesirable disturbance. It was necessary, moreover, to supply air to the outlet when the spillway flow approached its maximum (figure 7A). Without air, the flow plunging into the exit was badly disintegrated causing considerable spray.

The flow conditions were also found to be satisfactory for flow both in the spillway and in the outlet. No streamlined deflectors or "eyebrows" were required as in the case of the Shasta Dam. The intersection of the spillway flow with the outlet flow caused the former to be deflected upward; but the two flows appeared to blend together and entered the tailwater without any excessive disturbance (figure 7B). The presence of spillway flow has practically no effect on the spreading of the jet issuing from the outlet.

It can be concluded from a comparison of the studies made on the Shasta Dam and Friant Dam outlets that, in any event, the spillway flow which is deflected from an exit has a flat trajectory and arches away from the face of a dam. Accordingly, the higher the outlets are on the face of a dam, the more pronounced is the disturbance caused by the deflected water before it enters a stilling pool.

5. Velocity distribution. Figure 8 shows the velocity distribution, in terms of the prototype, at the downstream end of the transition, at the end of the exit, and on the horizontal part of the apron. The velocities are greater on the left side of the model, which is probably due to the reversed curve in the model outlet (figure 3), and may be due to a slight error in model construction. In any event, the velocity distribution is quite uniform for flow through this type of transition.

6. Calibration of transition. The transition was calibrated to determine a head-discharge curve and a head-coefficient of discharge curve (figure 9). The head, H, was taken to the center line of the extreme end section of the transition, and the area in the relation $Q = CA\sqrt{2gH}$ was that of the same section. The coefficient of discharge, C, varied from 0.80 to 0.85 with an increase in head from 35 to 230 feet.


4.50'
3.00'
1.50'
LEFT
RIGHT
115
112
110
100
SECTION A-A
A. ISOVELS IN END SECTION OF TRANSITION
PROTOTYPE VELOCITY
SEE SECTION D-D, FIGURE 5
LEFT
RIGHT
DISTANCE FROM ℄ - PROTO. FEET
D. VELOCITY DISTRIBUTION 0.375 FEET ABOVE APRON El. 282.50
DISTANCE ABOVE INVERT - PROTO. FEET
CROWN
AT ℄
1.50' FROM ℄
3.00' FROM ℄
4.50' FROM ℄
RIGHT
LEFT
PROTOTYPE VELOCITY - FT. PER SEC.
B. VELOCITY DISTRIBUTION IN END SECTION OF TRANSITION
TRANSITION
El. 350.75
El. 348.15
144.95 TO AXIS
EXIT ℄
42.21'
44.87' R
30°
El. 308.85
P.T. - El. 317.19
205.99 TO AXIS
WATER SURFACE
℄
AVERAGE W.S.
FIN
105
110
115
SECTION B-B
C. ISOVELS AT END OF EXIT
NOTE:
VELOCITY DATA IS FOR Q=5034 c.f.s.; HEAD=220'
ON El. 35 .00. SEE FIGURE 9
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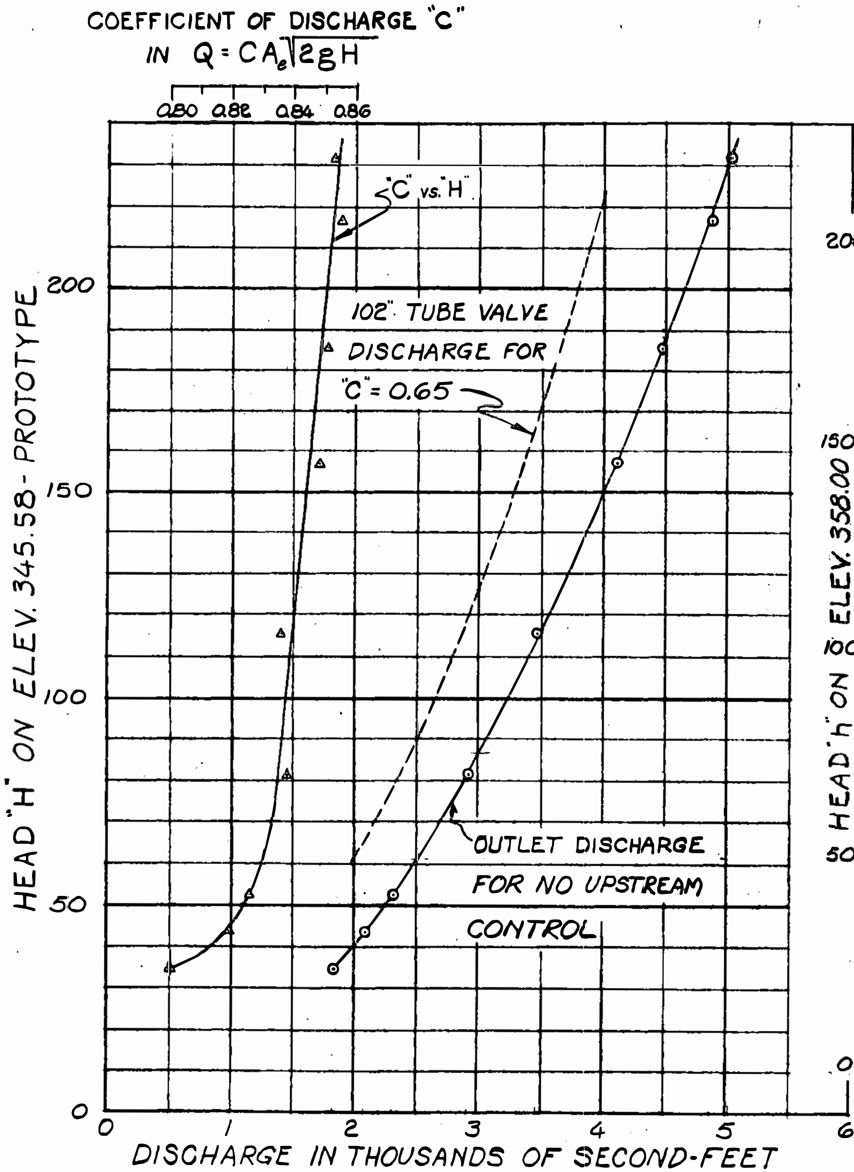
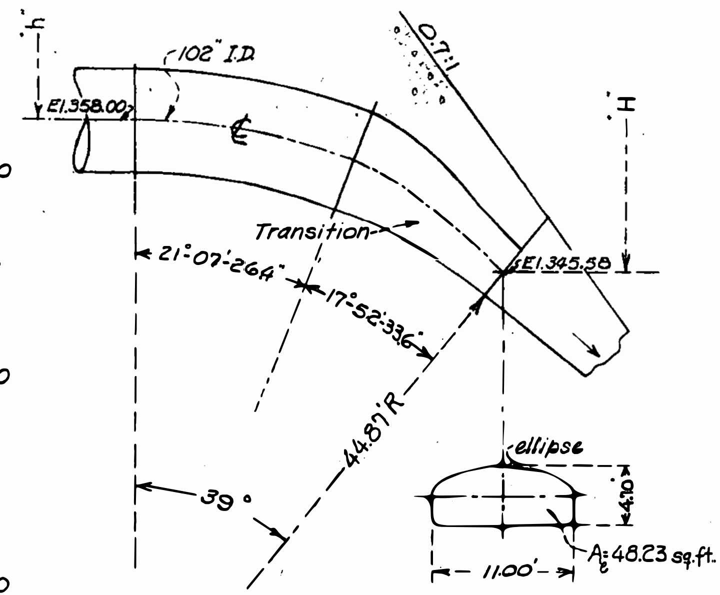
HYDRAULIC MODEL STUDIES - 1:24 SCALE
VELOCITY DISTRIBUTION AT OUTLET
DRAWN: H.G.D., JR.





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FRIANT DIVISION

FRIANT DAM

HYDRAULIC MODEL STUDIES - 1:24 SCALE

CALIBRATION OF RIVER OUTLET TRANSITION

DRAWN: H.G.D.JR.

DENVER, COLO., NOV. 6, 1939

FIGURE 9

Although there was no upstream control in the model - the prototype flow will be controlled by 102-inch tube valves - this calibration is particularly valuable inasmuch as there are little or no data available on the coefficient of discharge for this particular type of transition.

7. Conclusions.

(1) When it is necessary to spread a jet issuing from an outlet quickly and efficiently, a transition similar to the one discussed should be placed in the outlet proper. This will produce, under pressure, a divergence of the flow which will continue to spread laterally in the open beyond the transition.

(2) To prevent eddies and high-velocity flow in stilling pools during operation of outlets, it is desirable to obtain a hydraulic jump as the issuing jet enters the tailwater. If the jet enters a wide pool, it is necessary to confine the hydraulic jump, otherwise return flow will drown the jump, and eddies will form.

(3) Outlets should be placed as near as possible to the bottom of a spillway or dam. Submergence of the outlets, however, is not recommended.

APPENDIX

DESIGN OF OUTLET TRANSITION

1. Introduction. The purpose of a transition placed in an outlet and its performance for various flow conditions has been discussed in detail in the preceding pages. The procedure used in the design of the transition for the Friant Dam river outlets is discussed in this appendix to facilitate any future design of this type. For convenience, a summary of the design procedure is given in paragraph 11.

2. Basic assumptions. Before the transition can be designed, it is necessary to make the following assumptions concerning the transition:

(a) Length along invert.
(b) Reduction in area from beginning to end of transition.
(c) Total drop from invert of outlet to end of transition.
(d) Shape.
(e) Amount of concrete between crown of transition (at downstream end) and face of dam.
(f) Fillet radius.

(a) Length along invert. The length of transition is determined by inspection and its choice is described more fully in paragraph 6. However, the length should be kept at a minimum, especially if the transition is to be made as a casting. Abruptness should be avoided, since a short diverging transition may develop high negative pressures along its inner surface.

(b) Reduction in area from beginning to end of transition. The reduction in area from the beginning to the end of the transition may be taken as 15 percent; that is, the area of the extreme end section of the transition will be 85 percent of the area of the circular outlet. This area reduction is uniform, but is not necessarily a straight-line variation between sections equally spaced along the invert. The value of reduction, 15 percent, was determined from studies made on the outlets of the Grand Coulee Dam. In any event, some reduction is required to prevent excessive negative pressures from occurring upstream in the outlet. However, if the flow through the outlet will be controlled farther upstream by a valve, the amount of reduction used must not cause the transition to act as a control (figure 9).

(c) Total drop from invert of outlet to end of transition. The total drop in invert is arbitrary and is dependent on the circumstances of any particular problem. Any considerable drop may create negative pressures in the transition unless the area of the end section of the transition is reduced correspondingly. In most cases the radius of the invert will be large enough to conform to the necessary tangent requirements at the face of the dam, thus preventing any excess drop in invert or a too rapid downstream curvature.

(d) Shape. The shape of the transition must cause the issuing jet to spread. It is evident that a jet of circular cross-section impinging tangent to an apron will not spread quickly. Accordingly, it has been found that a transition should have a section which is elliptical above the center line and rectangular below the center line with fillets in the corner of the rectangular part (Section B-B, figure 10). This shape tends to flatten the jet and to accelerate spreading.

The longitudinal shape may consist of a constant-radius invert with a parabolic crown; or a combination of an arc at the upstream end of the crown tangent to a straight line extending to the end of the transition.

The amount of divergence, in plan, may be between five and eight degrees; an excessive divergence may cause the flow to separate from the walls and create negative pressures. Since the exit cut in the face of the dam is made tangent to the walls of the transition at the start of the exit, the divergence will also be governed by the distance between outlets. The exits must not intersect since the jets would cause excessive fins at their point of intersection.

(e) Amount of concrete between crown of transition (at downstream end) and face of dam. This amount must not be less than three feet, unless a deflector or "eyebrow" is placed above the transition on the dam. In this case, the three feet would be measured from the face of the deflector to the crown.

(f) Fillet radius. The change of fillet radius from section to section is a straight-line variation. The fillet radius at the start of the transition will be the radius of the circular outlet, and the fillet radius at the end of the transition may be taken as three or six inches.

3. Equations used in design. The design of a transition requires certain equations and well known relations from analytical geometry:

(a) $\frac{y - y_1}{x - x_1} = m$, the equation for a straight line, where the point P (x_1, y_1) is known, and m is the slope of the line.

(b) $(x - h)^2 = a(y - k)$, the equation of a parabola with center at(h, k) and its axis parallel to the y-axis; "a" is a constant and if positive, parabola opens upward, if negative, parabola opens downward.

(c) If two lines are normal to each other their slopes are negative reciprocals.

DESIGN PROCEDURE

4. Known data and assumptions. The preliminary arrangement is shown on figure 1. It is required to design the downstream end of the outlet (shown dotted, figure 1). The elevation of the center line of the outlets, slope of dam, and radius into pool are known. The following assumptions are made:

(a) Length of transition = 14 feet.
(b) Reduction of area = 15 percent.
(c) Drop in invert = 10 feet.
(d) Shape of transition = Circular to ellipse plus rectangle. Divergence about 6 degrees.
(e) Amount of concrete between crown of end section and face of dam = 3 feet minimum.
(f) Fillet radius straight-line variation. Radius in end section = 0.50 foot.

5. Computations for end section of transition. The area of the 102-inch outlet is known:

$$A_o = \frac{\pi}{4} (8.5)^2 = 56.745 \text{ sq. ft.}$$

Then area of end section by 4(b) is:

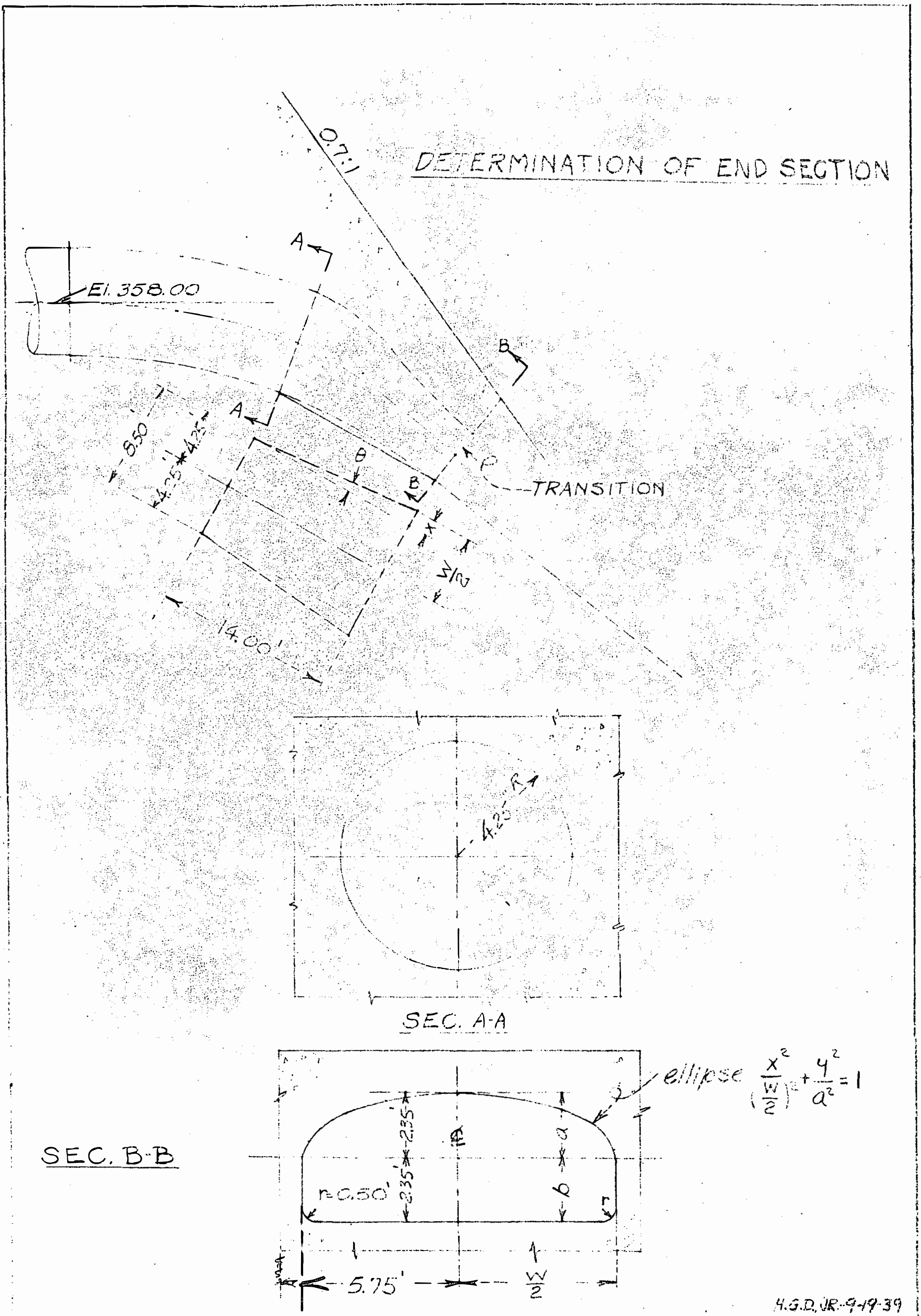
$$A_e = 0.85 \times 56.745 = 48.233 \text{ sq. ft.}$$

Referring to figure 10, the width of the end section may be determined. Project the invert on the chord, as shown, and assume chord length is equal to length of transition, or 14 feet. This is necessary since the invert radius and the central angle are not known. Assuming, as in 4(d), that the divergence is 6 degrees, then referring to figure 10:

$$x = 14 \tan 6^o$$

or $$x = 14 \times 0.1051 = 1.47$$

then $w/2 = 1.47 + 4.25 = 5.72$ feet. Use $w/2 = 5.75$ feet.



DETERMINATION OF END SECTION
0.7:1
El. 358.00
A
B
8.50
4.25
4.25
θ
B
X
W/2
14.00'
TRANSITION
4.25' R
SEC. A-A
ellipse $\frac{x^2}{(\frac{W}{2})^2}+\frac{y^2}{a^2}=1$
SEC. B-B
2.35'
2.35'
a
b
r
r=0.50'
5.75'
W/2
H.G.D. JR. 9-19-39

This is only approximate since the actual divergence will depend on the projection of the invert on a horizontal plane, but the length of this projection is not yet known. Although this length will be less than 14 feet, if the original angle of divergence assumed is not too great, then the actual divergence will be within 5 to 8 degrees, as is desired. This approximation will then suffice.

Now on Section B-B, figure 10, $w/2$ is known and hence w, and the fillet radius was assumed to be 0.50 foot as in 4(f). Letting $a = b$, and knowing the area of the end section $A_B = 48.233$ square feet, it is possible to find a and b, thus:

$A_B = 1.7854Wb - 0.4292r^2$, which is derived considering general terms of the end section, adding area of ellipse and rectangle with fillets.

Therefore $48.233 = 1.7854 \times (2 \times 5.75)b - 0.4292\ (0.50)^2$

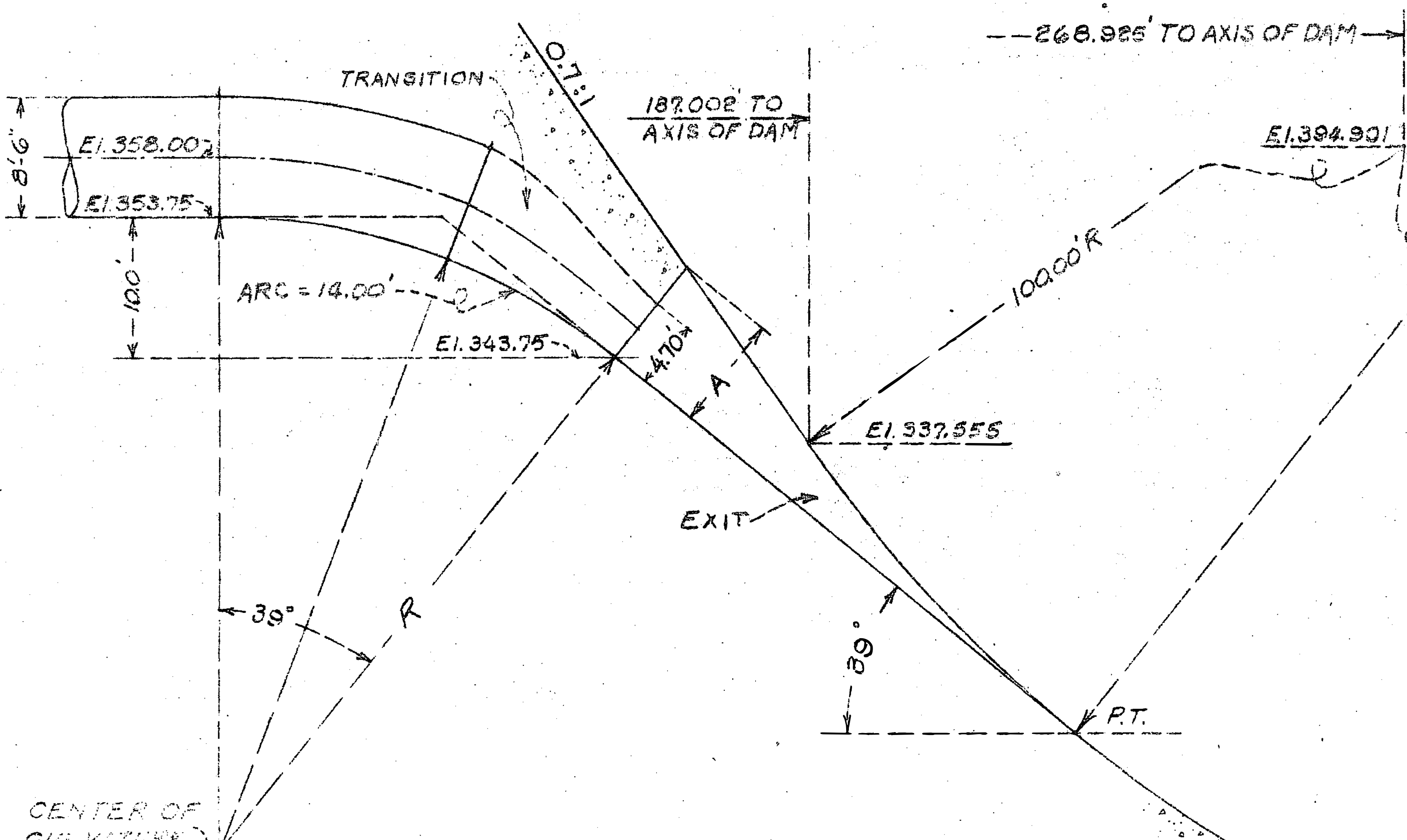
$\therefore\ b = a = 2.35$ feet.

Now the end section is completely determined as indicated on figure 10.

6. Locating invert and transition. A sketch must now be made to scale, locating the outlets and face of dam and radius in the pool, as shown on figure 11. A horizontal line is drawn 10 feet below invert of outlet (elev. 343.75) representing total drop of invert) as assumed in 4(c). Various lines are then drawn tangent to the 100-foot radius apron and made to intersect the line at elevation 343.75. The distance A is now known (figure 11), since the height of end section is a + b = 4.70 feet, and the amount of concrete needed above crown of end section is 3 feet minimum according to 4(e); hence, A = 7.70 feet minimum. By scaling from the point of intersection of various tangent lines with the line at elevation 343.75 to the 0.7 to 1 slope of dam, this value of A can be satisfied; however, for sake of simplicity, make slope of tangent line an even angle such as the 39-degree slope on figure 11. This will make A greater than 7.70, but it should not exceed the minimum value too much. When the correct tangent line is located, extend it to intersect elevation 353.75, the invert elevation of the outlet. Then find center of curvature so that invert arc will be tangent to the invert at elevation 353.75, and tangent to the 39-degree slope at the exit. This central angle will be the same value as the angle for slope of tangent line, or 39 degrees as on figure 11.

It is now possible to compute the radius at the invert and all other data as indicated on figure 12. These computations

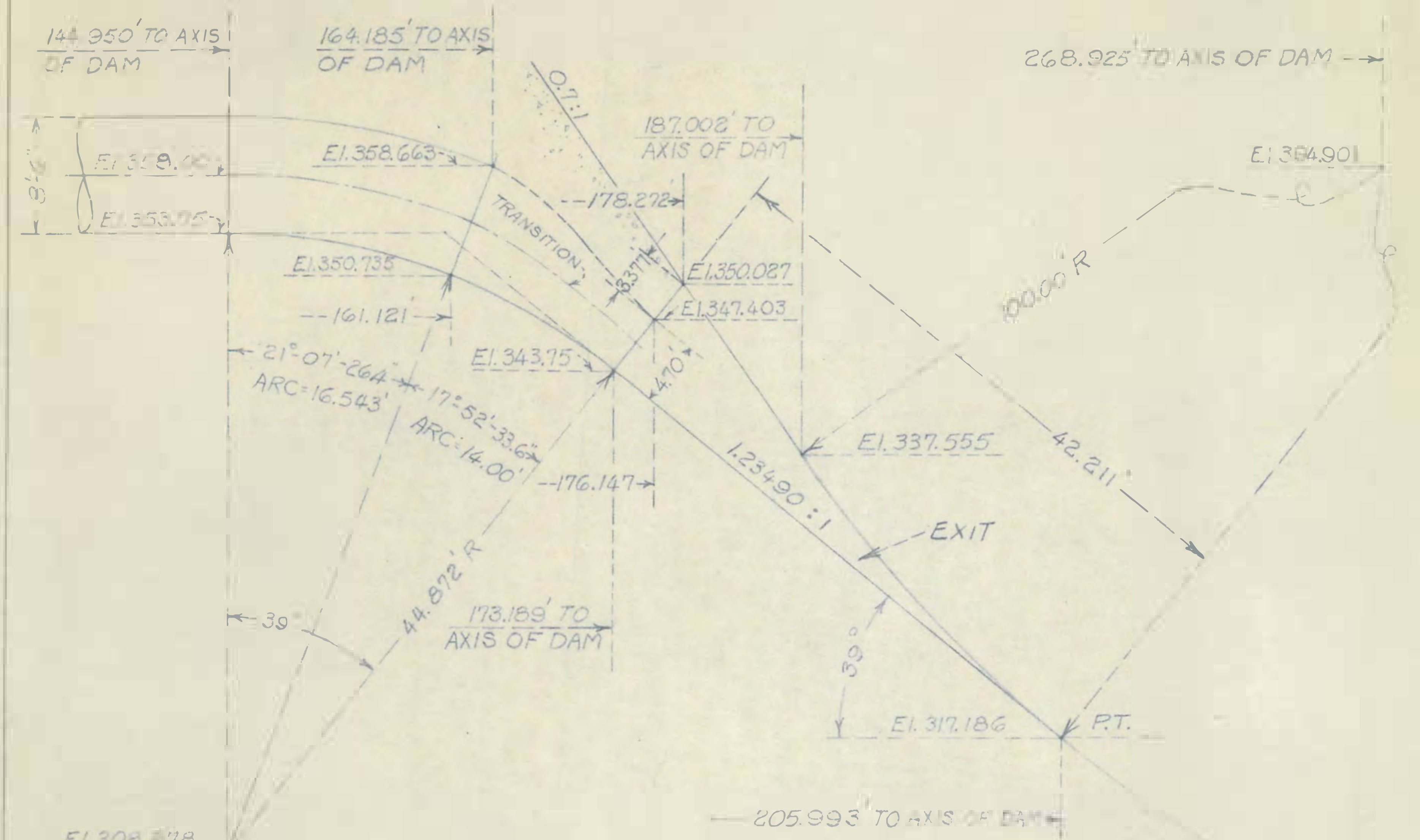
FIGURE 11



SECTION ON ℄ OF OUTLET

LAYOUT FOR LOCATING INVERT AND TRANSITION

H.G.D. JR.-3-18-39



SECTION ON ℄ OF OUTLET

LOCATION OF INVERT AND TRANSITION

FIGURE 12

are not given since they require a simple application of trigonometry. The point of intersection of the line through the end section with the 0.7 to 1 slope of the dam is best found by writing equations of each line, taking elevation 308.678 as the origin, and solving the equations for the point of intersection.

If the assumed length of transition appears to be too short, it can be made to suit by trial on a sketch similar to figures 11 or 12. It is probably desirable to make the length so as to have the central angle equal to about one-half the total central angle. If the length is changed then the divergence should be rechecked following the procedure as in (5) above. If the divergence is now too large, it should be reduced using the correct value of invert length projected on horizontal plane. Then the dimensions of the end section must be recomputed as in (5) above.

In this particular problem the actual horizontal length of invert is 12.068 feet, hence the angle of divergence θ may be checked:

$$\tan \theta = \frac{5.75 - 4.25}{12.068} = \theta = 7^{o}\text{-}5.3'.$$

In the original assumptions, using the method as on figure 10, the angle of divergence was $\theta = 6^{o}$. The actual divergence is therefore still within the limits of 5 to 8 degrees.

7. Establishing side-wall curves. At this point in the design, the position of the transition has been established as shown on figure 12. It is now necessary to determine the curves for the side walls. By projecting the invert on a horizontal plane, it is possible to write equations for the side walls, and at the same time to obtain true widths of the sections, since true width will show in this view (figure 13B).

The side-wall curves may be taken as parabolas with the origin as shown on figure 13B. The vertex of the parabola is at the origin and the parabola must go through the point P ($x = 12.068$, $y = 1.50$).

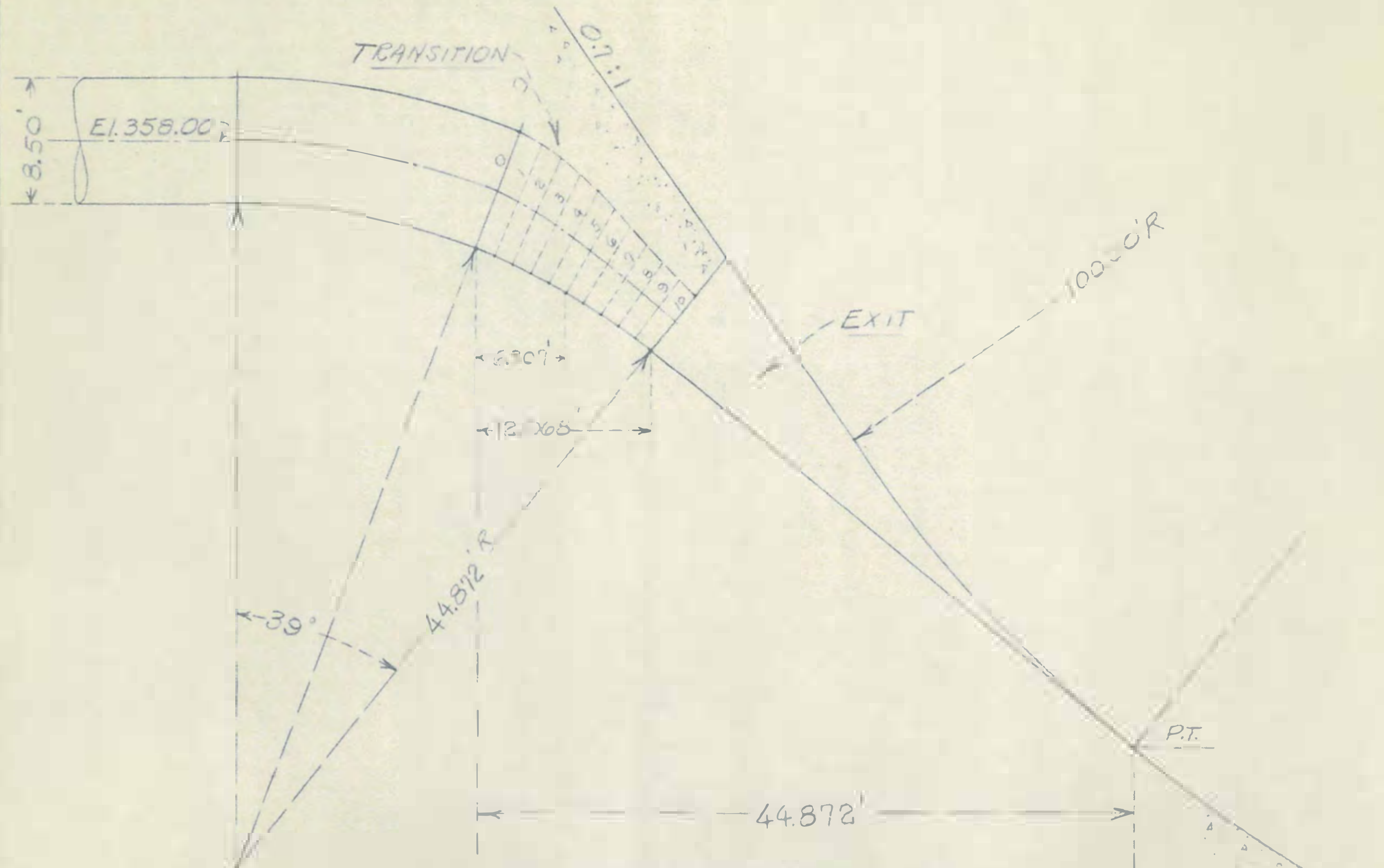
Hence $x^2 = K \cdot y$

or $$K = \frac{x^2}{y} = \frac{(12.068)^2}{1.50}$$

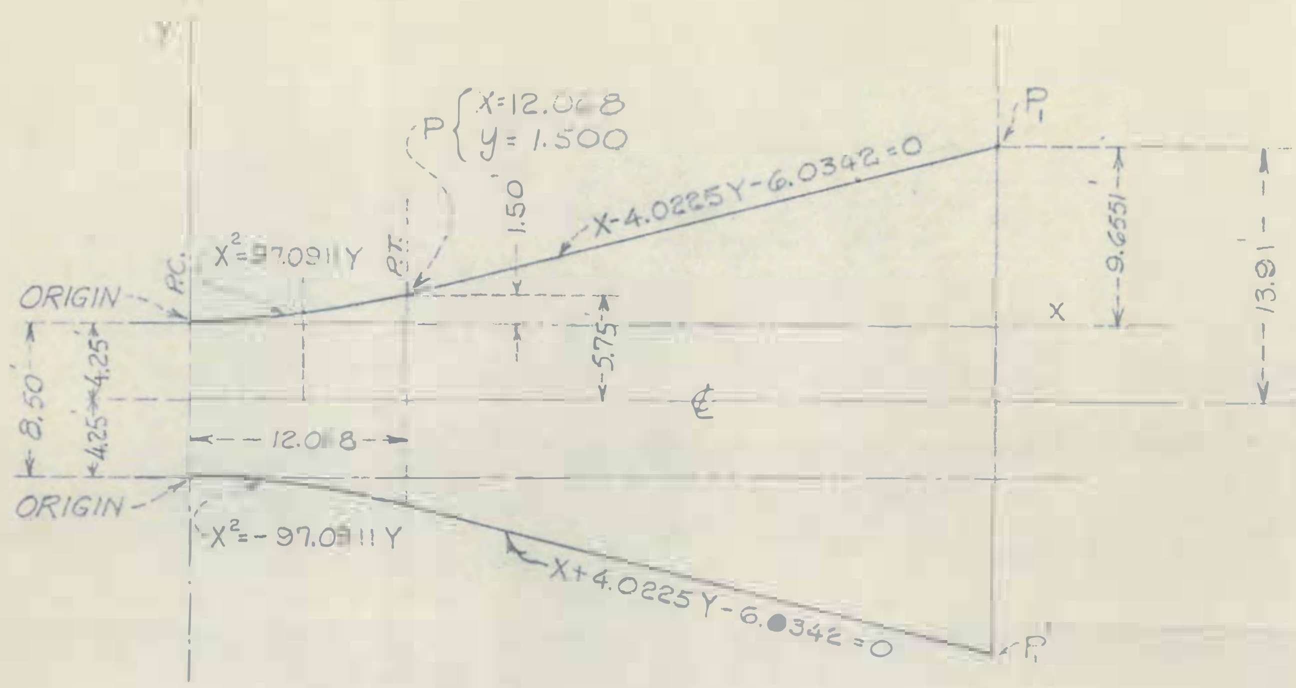
$\therefore$ $K = 97.0911$

and $x^2 = 97.0911\ y.$

FIGURE 13



A. SECTION ON ℄ OUTLET



B. PROJECTION OF INVERT ON HORIZONTAL PLANE ESTABLISHING SIDEWALL CURVES

H. 9-19-39

The walls of the exit must be tangent to the parabola at P(12.068, 1.50). Hence, it is necessary to find the equation of the straight line passing through P, and tangent to the parabola $x^2 = 97.0911y$ at P, and passing through point P_1, where the exit terminates on the apron (figure 13B).

Slope of parabola at P:

$$x^2 = 97.0911y$$

$$\frac{dy}{dx} = \frac{2x}{97.0911} \text{ and for } x = 12.068$$

then $\frac{dy}{dx} = 0.2486$

Equation of straight line is:

$$\frac{y - y_1}{x - x_1} = m$$

or $$\frac{y - 1.50}{x - 12.068} = 0.2486$$

hence $x - 4.0225y - 6.0342 = 0$, satisfies the conditions.

It is now necessary to find the width of the exit at its tangency to the apron at $P_1P'_1$ (figure 13B). At this point $x = 44.872$ feet, which is found from the data on figure 12.

Hence, using the equation just found:

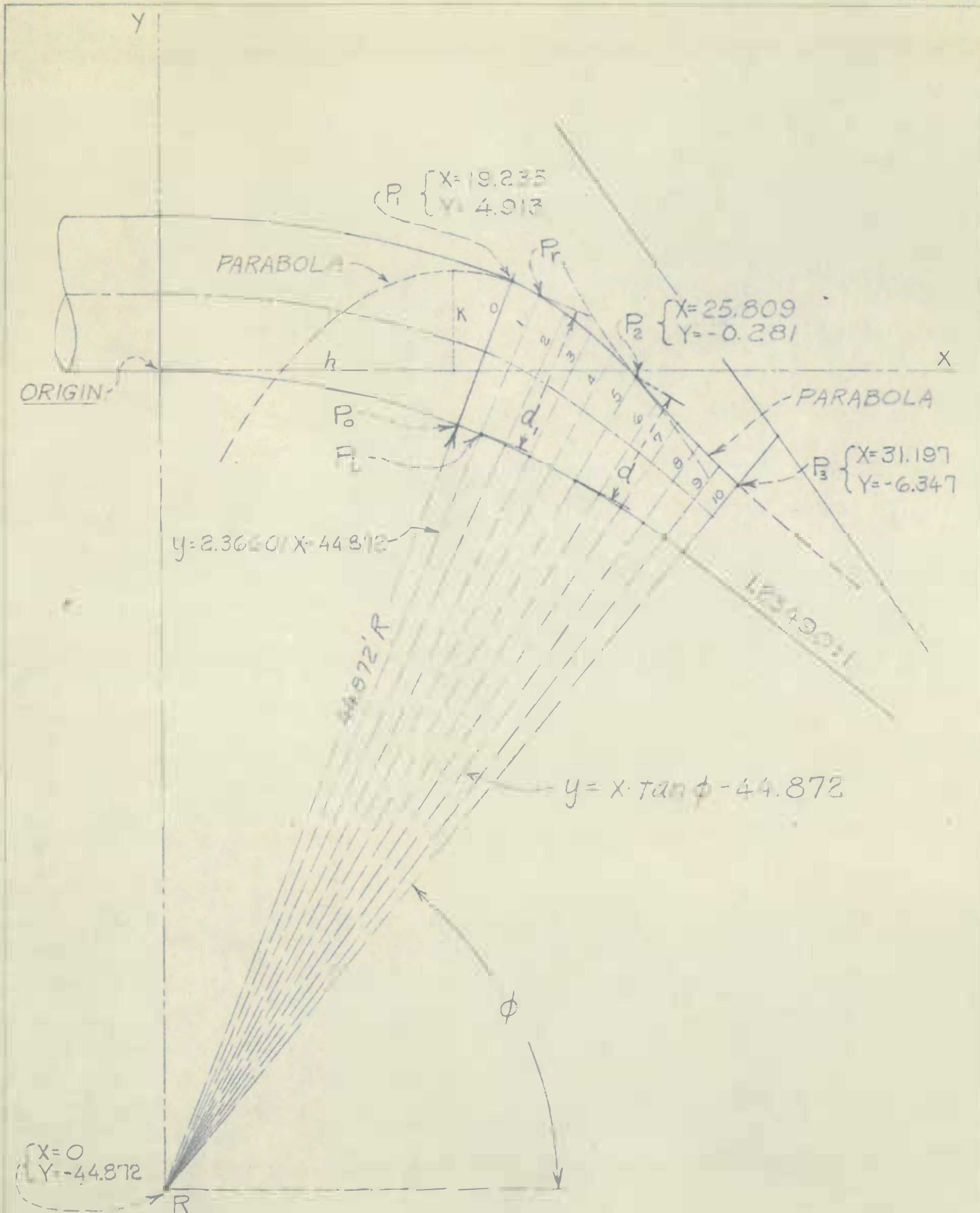
$$x - 4.0225y - 6.0342 = 0$$

and for $x = 44.872$, it is found that $y = 9.6551$. Hence, width from center line to P_1 or P'_1 is

$$w/2 = 9.6551 + 4.25$$

$$w/2 = 13.9051.$$

8. Determination of crown of transition. The crown of the transition may consist of two parabolas which are tangent to each other at a section halfway between the beginning and end of the transition. A convenient origin is chosen for these parabolas as shown on figure 14. The coordinates of points P_1 and P_3 are obtained from the data on figure 12, but referring these data to the new origin of figure 14. It remains to determine the coordinates of point P_2 where the parabolas will be tangent.



DETERMINATION OF ROOF OF TRANSITION

On figures 13A and 14, the transition has been divided into ten equal arc lengths along the invert, and each section is drawn radial. By using the data on figure 12, it is possible to compute the elevation and the distance from the axis of dam for the point on the invert at section 5, or for an arc length of 7 feet. It is found with reference to the origin on figure 13B that $x = 6.3068$ (figure 13A). To determine the width in plan at the midsection, we let $x = 6.3068$ in $x^2 = 97.0911y$ (figure 13B).

Hence $y = 0.4097$

Then width of midsection $= 8.50 + 2 \times 0.4097$

or $W = 9.3194$.

It is assumed that the area of the midsection will be the average of the area of the beginning and end sections, hence:

$A_M = 1/2\ (A_o + A_e$ and from paragraph 5

$A_M = 1/2\ (56.745 + 48.223)$

$\therefore\ A_M = 52.489$ sq. ft.

Since area of any section (except circular only) is from paragraph 5: $A = 1.7854Wb - 0.4292V^2$ we can find b. But r, fillet radius for midsection, is $1/2\ (4.25 + 0.50) = 2.375$ feet.

Hence $52.489 = 1.7854 \times 9.3194b - 0.4292\ (2.375)^2$

from which $b = a = 3.325$ feet.

And $d = 2 \times 3.325 = 6.65$ (figure 14).

From figure 14 it is now possible to compute the coordinates of point P_2. The angle ϕ to section 5 is $\phi = 59^\circ 56' 16.8''$, which may be obtained from the data on figure 12. From point R (figure 14) to point P_2, the distance is $44.872 + 3.325 \times 2 = 51.522$; then $x = 51.522 \cos (59^\circ 56' 16.8'') = 25.809$; and from R, $y = 51.522 \sin (59^\circ 56' 16.8'') = 44.591$, or referred to the origin (figure 14), $y = -0.281$; hence the coordinates of P_2 are P_2 (25.809, -0.281).

To review: It is required to pass a parabola (figure 14) through the point P_1 and tangent to the crown of the circular outlet at that point, and to pass through the point P_2; furthermore, another parabola is to pass through P_2, being tangent to the first parabola (connecting P_1 and P_2), and passing through P_3. All computations should be carried to at least four significant places past the decimal point. If this is not done, a check is difficult to make. Final tabulation, however, is only necessary to two significant places.

The coordinates of the known points are (figure 14):

P_1 (19.235, 4.913)

P_2 (25.809, -0.281)

P_3 (31.197, -6.347)

The equation of the parabola through P_1 and P_2 will be:

$$(x - h)^2 = a(y - K) \quad \ldots \ldots \ldots \ldots (1)$$

substituting:

$$(19.235 - h)^2 = a(4.913 - K) \quad \ldots \ldots \ldots \ldots (2)$$

and $$(25.809 - h)^2 = a(-0.281 - K) \quad \ldots \ldots \ldots \ldots (3)$$

in which h, K, and "a" are unknown. It has been specified that this parabola must be tangent to the outlet crown at P_1, hence the slope at this point will be the negative reciprocal of the slope of the line extending through R and section 0 (figure 14).

The slope of this line is, therefore:

$$m = \tan \varnothing = \frac{4.913 + 44.872}{19.235} = 2.58825.$$

Now slope of parabola at P_1:

$$m = -\frac{1}{2.58825} = -0.38636.$$

From the general equation of the parabola:

$(x - h)^2 = a(y - K)$ the slope at any point is:

$m = \frac{dy}{dx} = 2/a\ (x - h)$ where x is any point on the curve;

therefore, at P_1:

$$m = 2/a\ (x - h)$$

or $$-0.38636 = 2/a\ (19.235 - h), \text{ therefore}$$

$$a = 5.1765h - 99.5703 \quad \ldots \ldots \ldots \ldots (4)$$

Substituting this value of "a" in (2) and (3), we obtain:

$$(19.235 - h)^2 = (5.1765h - 99.5703)(4.913 - K) \quad . \; . \; . \; . \; . \quad (5)$$

$$(25.809 - h)^2 = (5.1765h - 99.5703)(-0.281 - K) \quad . \; . \; . \; . \quad (6)$$

If (5) and (6) are solved for h, it is found that

$$h = 16.0895$$

Now from (4): $a = 5.1765h - 99.5703$ and with $h = 16.0895$, then $a = -16.2830$.

From (2) or (3); but using (2):

$$(19.235 - h)^2 = a(4.913 - K)$$

and since $h = 16.0895$ and $a = -16.2830$,

$$(19.235 - 16.0895)^2 = -16.2830(4.913 - K)$$

$$\therefore K = 5.5206.$$

Hence, the equation of the parabola passing through P_1 and tangent to the outlet crown at P_1 and passing through P_2:

$$h = 16.0895$$

$$K = 5.5206$$

$$a = -16.2830$$

$$(x - h)^2 = a(y - K)$$

hence,

$$(x - 16.0895)^2 = -16.2830\,(y - 5.5206)$$

which reduces to:

$$x^2 - 32.1790x + 16.2830y + 168.9801 = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Now a parabola must be put through P_2 and be tangent to equation (7) at P_2, and also pass through P_3. Coordinates are:

P_2 (25.809, -0.281)

P_3 (31.197, -6.347)

hence:

$$(25.809 - h)^2 = a(-0.281 - K) \quad \ldots \ldots \ldots \ldots \quad (8)$$

$$(31.197 - h)^2 = a(-6.347 - K) \quad \ldots \ldots \ldots \ldots \quad (9)$$

$$\frac{dy}{dx} = \frac{2}{a}(x - h)$$

at P_2; $x = 25.809$

$a = -16.2830$

$K = 16.0895$

$\therefore dy/dx = m = 2/-16.2830\ (25.809 - 16.0895)$

or $m = -1.1939$ at P_2

For parabola through P_2 and P_3:

$$-1.1939 = 2/a\ (25.809 - h) \text{ at } P_2 \quad \ldots \ldots \ldots \ldots \quad (10)$$

In a manner similar to that for the parabola through P_1 and P_2, equations (8), (9), and (10) are solved obtaining:

$h = 73.0804$

$K = -28.4992$

$a = 79.1895$

Therefore $(x - 73.0804)^2 = 79.1895\ (y + 28.4992)$ which reduces to:

$$x^2 - 146.1608x - 79.1895y + 3083.9075 = 0 \quad \ldots \ldots \ldots \quad (11)$$

Now equation (7) is a parabola tangent to the crown of the outlet at P_2, and passing through P_2 and P_3; and equation (11) is a parabola tangent to equation (7) at P_2, and passing through P_2 and P_3.

9. <u>Height of each section.</u> The height of each section will evidently be the radial distance from the invert to the crown of the transition, d on figure 14. To determine these values of d, it is necessary to find the coordinates of the points of intersection of each section with the invert and the crown. With these coordinates, the values of d can be obtained as will be shown.

Each section or radial line through the point R (figure 14) may be expressed as:

$$y = mx + b \quad \ldots \ldots \quad (12)$$

where $m = \tan \phi$ to each line, and b = value of y for $x = 0$.

Since the invert has been divided into equal arc lengths, the value of ϕ to each line may be readily determined, using given angles on figure 12. The value of b is, $b = -44.872$ (figure 14). Hence, equation of any section line is:

$$y = x \cdot \tan \phi - 44.872 \quad \ldots \ldots \quad (13)$$

A sample computation will be made solving for the intersection of the line through section 1. A check on the coordinates of P_1, P_2, and P_3 should also be made by finding the intersection of lines through section 0, section 5, and section 10 (figure 14). In the case of section 5, both parabolas may be used since P_2 is a common point.

The angle to section 1 may be found as:

$\phi = 67^{\circ}$-05.31264' and $\tan \phi = 2.36601$.

Then the equation of the radial line through R and section 1 (figure 14) is:

$y = 2.36601x - 44.872$ and

from equation (7):

$x^2 - 32.1790x + 16.2830y + 168.9801 = 0.$

By solving these equations, it is found that the coordinates of the point of intersection, P_r (figure 14), are:

P_r (20.7377, 4.1936).

Now it is necessary to determine the coordinates of the point of intersection of the line through section 1 at the invert. This is best done by using sin and cos functions of ϕ to this line, as follows:

As just shown to section 1, $\phi = 67^{\circ}$-05.31264' for which

$\sin \phi = 0.92111$

$\cos \phi = 0.38931.$

Since the radius is 44.872 from R to the invert, then

$x = 44.872 \cos \phi = 17.4691$, and from R

$y = 44.872 \sin \phi = 41.3320$, but referred to the origin (figure 14), the coordinates are:

P_i (17.4691 , -3.5400)

Now these coordinates are known:

At crown, P_r (20.7377, 4.1936)

At invert, P_i (17.4691, -3.5400).

The slope of the line through these points is, as before, $\phi = 67^{\circ}$-05.31264'. It is possible to find d, therefore, since:

$$d_i = \frac{y_r - y_i}{\sin \phi}$$

or

$$d_i = \frac{x_r - x_i}{\cos \phi}$$

Hence

$$d_i = \frac{4.1936 + 3.5400}{0.92111} = 8.3959$$

or

$$d_i = \frac{20.7377 - 17.4691}{0.38931} = 8.3959$$

Now, as on figure 10, section B-B; $a = b = d/2 = 4.1979$.

Each section should be treated in this manner with a check on previous computations available at points P_1, P_2, P_3, at the crown; and corresponding points on the same lines at the invert. It is advisable to make a large-scale drawing of the transition and check each coordinate and "d" value as it is computed.

10. Width at each section. It remains, finally, to compute the width of each section. When the procedure in paragraph 9 is completed, the x-coordinate of each point of intersection of radial section line with the invert will be known. Now referring to figure 13B, reduce these values of x to the origin for the side-wall parabola, thus, in the case of section 1:

$x_i = 17.4691$

and from figure 14:

$x_o = 16.1714.$

The point P_o (figure 14) is also the origin for the parabolas of the side walls on figure 13B. Then the value of x for section 1 which is to be substituted in $x^2 = 97.0911y$ is:

$$x = x_1 - x_o = 1.2977$$

then

$$y = \frac{(1.2977)^2}{97.0911} = 0.0173$$

therefore:

$$w/2 = 4.2500 + 0.0173$$

$$w/2 = 4.2673$$

and $w = 8.5346.$

Every section is treated in this manner to obtain its width.

Finally, a tabulation may be made of each section in a manner similar to that following, referring values to the origin of figure 14:

Section number	Coordinate of invert		Coordinate of roof		Section data			
	x	y	x	y	a	b	r	w/2
0								
1	17.47	-3.54	20.74	+4.19	4.20	4.20	3.88	4.27
2								
etc.								

Figure 2 shows the details of the complete design.

11. Summary of design procedure. A brief summary of the numerous steps taken in the design of the transition is given and reference is made to the paragraph explaining the design in detail.

(a) Basic assumptions, paragraph 2.

(b) Equations used in design, paragraph 3.

(c) Known data and assumptions as applied to particular problem discussed, paragraph 4.

(d) Computations for end section of transition, paragraph 5. Determine area of circular outlet and obtain area of end section of transition according to assumption in paragraph 2 (b). Determine width of end section by projecting invert on chord (figure 10), and assuming chord length is same as arc length as taken in paragraph 2(a). Angle of divergence (angle θ, figure 10) assumed as in paragraph 2(d). Using width of end section and area, compute height of end section.

(e) Locating invert and transition, paragraph 6. Make a scaled drawing of apron, face of dam, and locate outlets (figure 11). Using assumed drop in invert from paragraph 2(c), depth of concrete from paragraph 2(e), and height of end section from paragraph 5, draw a line tangent to curved apron so as to intersect horizontal line representing elevation of drop in invert. Scale perpendicular distance from this point to face of dam. This distance (A, figure 11) must be at least equal to height of end section plus depth of concrete required above crown of end section. Extend tangent line correctly established to intersect invert of circular outlet. Determine center of curvature (figure 11). With data given, compute all information as shown on figure 12.

(f) Establishing side-wall curves, paragraph 7. Project invert established on horizontal plane (figure 13B) and using coordinates of invert at start and end of transition (figure 12), establish a parabola through these points. Compute equation of side walls of exit.

(g) Determination of crown of transition, paragraph 8. Divide transition into radial sections, taking equal arc lengths along invert (figures 13 and 14). Determine width and height of midsection assuming area of midsection to be average of first and last sections. Compute coordinates of crown at midsection. Knowing the coordinates of the crown at the start of the transition, at the midsection, and at the end section, determine two parabolas, (figure 14), which will satisfy these coordinates, and which will be tangent at the midsection.

(h) Height of each section, paragraph 9. Determine equation of each radial-section line (figure 14) and find their point of intersection with the parabolas of the crown. Also find the points of intersection of the radial section lines with the invert. With the coordinates of these points of intersection and the slope of each line, find the height of each section included between the invert and the crown along each of the radial lines.

(i) Width of each section, paragraph 10. Using the coordinates of each section at the invert, refer them to the origin of parabolas for the side walls (figure 13B), and compute the width of each section. Tabulate data of sections.

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