UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

HYDRAULICS OF STRATIFIED FLOW
SECOND PROGRESS REPORT
SELECTIVE WITHDRAWAL FROM RESERVOIRS

Report No. Hyd-595

HYDRAULICS BRANCH
DIVISION OF RESEARCH

OFFICE OF CHIEF ENGINEER
DENVER, COLORADO

SEPTEMBER 1969

Where approximate or nominal English units are used to express a value or range of values the converted metric units in parentheses are also approximate or nominal. Where precise English units are used the converted metric units are expressed as equally significant values. A table, conversion factors—British to metric units of measurement, is provided at the end of this report.

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by
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ABSTRACT

Selective outlet works for reservoirs provide an important control for maintaining water quality in the downstream river. The use of multilevel outlets is growing; however, design guidelines are seriously lacking. A study of previous theoretical and experimental work in selective withdrawal from reservoirs shows that a basis exists for development of practical design and operating criteria. Beginning with the theory for selective withdrawal from a continuously stratified fluid, a method is developed for the more general density profile. The method is verified with experimental data. Limitations of this method and the need for further experimental verification with model and prototype data are discussed. Has 24 references.

DESCRIPTORS–/ research and development/ reservoirs/ reviews/ *hydraulic models/ density currents/ hydraulics/ water quality/ temperature/ Froude number/ *stratification/ velocity distribution/ *thermal stratification/ theory/ bibliographies/ design criteria/ *stratified flow
IDENTIFIERS–/ *multilevel outlets/ *selective level releases/ progress reports

PURPOSE

The purpose of this report is to summarize the results of past research on selective withdrawal from reservoirs and to present a tentative theory for aiding in the solution of problems of design and operation within the Bureau of Reclamation.

CONCLUSIONS

1. Examination of previous theoretical and experimental studies indicates that enough work has been done to form a basis for development of practical design and operating criteria.

2. Application of these criteria will generally involve errors which cannot be accurately defined until verification data from models and prototypes become available.

3. A tentative theory and a computer program for its solution are presented in this report, along with experimental data.

APPLICATION

The material in this report is intended primarily for the use of Bureau of Reclamation designers in designing facilities for selective withdrawal from reservoirs. The contents should also be of interest to other researchers in this field. Emphasis is placed on the hydraulic engineering aspects of selective withdrawal, with limited discussion of other aspects of water quality.

INTRODUCTION

Attention is being focused on the control of water quality in streams and reservoirs. Bureau of Reclamation impoundments are used primarily for irrigation supplies; however, municipal and industrial uses have increased in recent years. Regardless of the use to which the stored water is put, the quality of the water is an important factor. Even if the water is released primarily for the purpose of generating electric power, the quality of the releases must be kept at a high level for maintenance of fish life, assimilation of wastes, or recreation in the downstream river.

The parameters most often controlled by selective withdrawal are temperature and dissolved oxygen. Many other physical or chemical properties are also considered, such as turbidity, taste and odor caused by reservoir plant growths, fertilizer and pesticide residues, and others.

Several Bureau of Reclamation projects include selective outlets with varying degrees of complexity. In most cases, the structures do not include multiple outlets but consist of a single outlet strategically placed, to exclude algal growth for example. A minor degree of selectivity is possible with simultaneous operation of the outlet works and powerplant where both facilities are available and when power and water requirements permit simultaneous operation.

At least 12 outlet works have been designed for selective withdrawals. Examples are Arrowrock, Shasta, Martinez, Grand Coulee, Yellowtail, Elephant Butte, Whiskeytown, and Arbuckle which include two or three outlets with vertical spacing from 10 to 100 feet. Casitas Dam includes multiple outlets in a sloping intake on the upstream face of the earth embankment. Cachuma Dam and Cheney Dam have multiple outlets in a vertical tower; similar facilities are included at Sanford Dam. Folsom Dam was successfully modified to allow selective withdrawal by placing shutter gates on the penstock trashracks. Bureau of Reclamation experience with selective withdrawal facilities at four of these dams has been described in a paper by Austin, Gray, and Swain.[1]

The use of multiple outlets is growing; however, design guidelines are seriously lacking. This report attempts to summarize previous and present theoretical and experimental work on the subject and to formulate tentative design criteria.

STRATIFICATION IN RESERVOIRS

Temperature stratification in a reservoir depends primarily upon the latitude and elevation of the reservoir and season. Other factors, such as wind, reservoir depth and shape, and reservoir operation also determine whether the reservoir is isothermal, has a continuous density gradient, or contains three or more distinct thermal layers. Most large reservoirs of the Bureau of Reclamation fall in the latter category. This simple stratification is characterized by a cool, more dense layer (hypolimnion) on the bottom, a warmer, less dense layer (epilimnion) at the surface, and a thermal gradient layer (thermocline) between the upper and lower layers. This condition typically occurs in the late spring and summer. In the fall or early winter, the surface of the reservoir cools and the water body may reach an isothermal condition through wind-induced mixing during the "fall turnover." If the

[1] Numbers in parentheses refer to references on page 15.

surface temperature falls to 4° C (maximum density of fresh water) the surface waters may sink into the reservoir and cause an exchange of water between the upper and lower portions of the reservoir. The reduced difference in density between the upper and lower layers during cooler weather in the spring reduces the stability of the reservoir and results in the "spring turnover." Well defined stratification reappears in the summer. The cycle is normally repeated annually. In shallow reservoirs the cycle may even occur daily.

Water quality is affected by reservoirs partly because of the increased travel time through a reach of the stream. Temperature or chemical stratification has additional effects, either beneficial or detrimental, on the quality of the stored water. Possible benefits of impoundments as enumerated by a Department of Interior task force are: sediment and turbidity reduction, hardness reduction through algal action, organic oxidation with biochemical oxygen demand (BOD) reduction and color reduction, coliform reduction, smoothing of incoming substances, methane fermentation in bottom muds, and increased assimilative capacity. Possible detrimental effects were listed as: less mixing which allows waste to hug the shore, lower reaeration, upstream movement of pollutants, algae blooms with undesirable esthetics and taste and odor problems, and thermal stratification. Possible detriments of thermal stratification in particular are: lower dissolved oxygen (DO) in the bottom layer, hydrogen sulfide formation, dissolution of iron or manganese, persistence of organic materials, pH reduction, and carbon dioxide increase.

Reservoirs also affect the quality of water in the river downstream from the dam. The quantity of DO in the stream governs the growth of fish and the capacity of the stream to assimilate wastes. Releases can be increased during periods of high waste transport for dilution. On the other hand, high velocities associated with large discharges may decrease contact time and reduce reaeration rates. The discharge temperature affects fish life and irrigation efficiency, and the quality of the stream determines recreational uses.

Water quality in a reservoir might be improved through artificial destratification by mechanical mixing with pumps or air curtains. The resistance to mixing is termed the "stability" of the reservoir and is defined as the energy required to lift the entire reservoir volume the distance between the center of gravity of a corresponding isothermal reservoir and the center of gravity of the stratified reservoir. Artificial destratification has been used successfully in relatively small impoundments but is believed to be economically impractical for most large reservoirs. However, the principle might be justified in limited zones near an outlet.

Reservoir water quality in selected zones has been improved by reaeration with various air-water contact devices. The effects of hydraulic equipment such as hollow-jet valves, cone valves, and turbine vents on aeration of discharges are being evaluated. Structures such as stilling basins might also have a noticeable beneficial effect on the quality of releases.

Proper design and operation of multiple outlets depend on a thorough knowledge of reservoir mechanics. The stratification pattern in a given reservoir will determine the optimum outlet arrangement for that particular case. The effects of reservoir bays and arms may allow selective zoning for boat dump areas, swimming facilities, or location of intake towers.

SELECTIVE WITHDRAWAL

Design of facilities for selective withdrawal can be divided into two categories: (1) improvement of existing facilities and (2) design of new facilities. Existing facilities can sometimes be improved by modification of outlets or by the creation of artificial barriers such as skimmer walls or submerged weirs.

The design of either new or modified outlets depends on reasonably accurate prediction of stratification patterns and the movement of layers in the reservoir. This prediction depends in turn on the reservoir size, shape, and location (latitude and altitude); the season during which the selective withdrawal occurs; and prevailing weather patterns. Optimum outlet operation can be achieved if reservoir conditions are continually monitored. Location of sampling stations and determination of sampling time intervals also require a basic understanding of reservoir mechanics.

Review of Previous Investigations

Theories for Prediction of Stratification.—As explained previously, the form and extent of reservoir stratification depends on several parameters which include latitude, elevation, wind, and reservoir geometry. Dake and Harleman (2) have conducted a comprehensive study of the mechanisms of thermal stratification in lakes and ponds. The parameters of latitude and elevation were further resolved into the more basic parameters of the rate of incoming solar radiation, the radiation absorbing capacity of the water, and the cooling of the reservoir water surface by evaporation and back radiation.

The Coriolis effect (earth's rotation) is added to thermal stratification and wind in analyses of water movement in reservoirs. The epilimnion in large reservoirs is circulated because the wind piles up the water on the downwind, right side of the reservoir (in the Northern Hemisphere). A case is cited (2) in which a difference in water surface elevation of only 0.1 foot from one side of the reservoir to the other can cause the thermocline to tilt as much as 50 feet. The difference in specific gravity between the epilimnion and hypolimnion was about 0.002.

Theoretical bases for determining the effects of wind induced circulation on thermal stratification are presently lacking. Hydraulic models can supply the required information if the rules for dynamic similitude can be determined.

Dake and Harleman describe the existing theories of Munk and Anderson, Ertel, Kraus and Rooth, Pivovarov, Weinberger, and Dutton and Bryson. Their review indicates that a reliable method for the prediction of thermal stratification in reservoirs remains to be developed; such a development forms the basis for their report. They contend that direct absorption of solar energy and transfer of energy by molecular diffusion form the theoretical foundation for prediction of stratification.

Dake and Harleman argue that the convection terms are important only in a limited zone near the surface because strong winds generally do not blow for long periods of time. They point out that even unprotected reservoirs develop well-defined thermal stratification.

The methods described by Dake and Harleman allow a detailed prediction of thermal stratification. Laboratory experiments performed with mercury vapor and/or infrared lamps as the radiation source showed good agreement with theory except in the bottom portion of the model. Measurements in two prototype lakes also support the theoretical prediction.

Wunderlich and Elder (3) have developed a graphical procedure for predicting the pattern of thermal stratification in reservoirs. Using records of inflow and outflow volume and inflow temperature, and computation of surface temperature by heat budget methods, the variation of outflow temperature with time is determined. Certain simplifying assumptions are made concerning the movement of water masses within the reservoir. The procedure allows estimation of the thermal regime of a reservoir and effects of outlet location and reservoir operation.

Orlob (4) has developed a detailed mathematical model for prediction of temperatures in streams and reservoirs. The analysis is performed by digital computer and considers both temporal and spatial variations of temperature within a stream, reservoir, or system of streams and reservoirs. Hydrologic and energy balances are performed on finite control volumes or "slices." One limitation lies in determining the "effective diffusivity" of heat within the reservoir. Prototype data are being analyzed to more adequately define this mechanism. Other refinements have recently been made; particularly the inclusion of selective withdrawal theory to replace the assumption that outflow occurs only from the slice at the elevation of the outlet.

This review indicates that the conditions of thermal stratification in a reservoir can be predicted through the use of mathematical models with sufficient accuracy to allow design of multiple outlets for selective withdrawal. Further refinement of the models will occur through model and prototype studies.

Theoretical and Experimental Studies in Selective Withdrawal.—Many studies have been conducted on selective withdrawal from point or line sinks at various locations in a laboratory flume. This discussion will be separated according to the type of sink and its location, the type of stratification (two- or three-layered or continuous density gradient), and whether the study was theoretical or experimental.

Withdrawal From a Line Sink at the Bottom of the Dam (Sluice).—Both theoretical and experimental investigations have been conducted of the flow of either a layered fluid or a continuous density gradient fluid through a line sink located at the bottom of a dam.

Huber and Reid (5) discuss withdrawal from a two-layered system.[2] This study was primarily experimental for purposes of verifying an earlier theoretical analysis by Huber (6). The theoretical analysis was concerned primarily with the shape of the interface at the point of incipient drawdown (the point at which the upper layer begins to flow through the sink; normal withdrawal is from the bottom layer). Relaxation techniques were used to define the interfacial shape. The investigation was extended beyond the point of incipient drawdown to determine the relative contribution of each layer for various withdrawal rates. The attempt by Huber and Reid to experimentally verify the theory was only partly successful because of viscosity effects in the small model. However, theoretical trends were verified by the experiments. Application of these results allows computation of withdrawal rates for sediment sluicing

[2] The two-layered system is a simplification of the three-layered system in nature. The thermocline is neglected.

without removing clear water from the higher regions of the reservoir, for example.

Harleman and Elder (7) discuss another application for withdrawal from a line sink. Theoretical and experimental considerations are applied to the design of a skimmer wall to exclude warm surface water from cooling water intakes of steam powerplants. Primary interest was in defining the condition of incipient drawdown. The theory is based on the premise that drawdown of the interface will occur when the withdrawal rate of the lower layer exceeds the critical discharge (in open channel homogeneous flow nomenclature). Both plane and radial skimmer walls were tested and the results substantiated the use of theoretical relationships in prototype designs.

C. S. Yih (8) formed the basis for later studies in the flow of a fluid with a continuous density gradient toward a line sink. The study has application, for example, to reservoirs which do not have a well-defined three-layered stratification. This condition often occurs during the late winter or early spring. Yih uses potential theory to show that withdrawal from discrete layers will occur below certain critical rates of discharge. The theory is limited to inviscid, nondiffusive flow and is generally limited to the region near the sink.

The Froude number of withdrawal is defined as

$$F = \frac{A}{d}\sqrt{g\beta} \qquad (1)$$

in which

A	=	$\sqrt{\rho U^2}$ where U is the velocity far upstream, and ρ is the average density far upstream
d	=	total depth of flow
g	=	acceleration of gravity
β	=	$\frac{\rho_o - \rho_1}{d}$, the density gradient.

Yih's theoretical solution gives the critical value of the Froude number as $1/\pi$ (0.318). Selective withdrawal can be accomplished only for Froude numbers below the critical value. Discharges with higher Froude numbers will draw from the entire depth of fluid.

Debler (9) based an experimental study on the results of Yih's theoretical work. His study yielded a critical Froude number value of 0.28 plus or minus 0.04. The lower critical value was believed to be caused by viscosity effects in the model, which were neglected in the theory.

Kao (10) presents a theoretical solution for the interface position and streamlines for a continuous density gradient with Froude number values below $1/\pi$.

Withdrawal From a Line Sink Above the Bottom.—Several theoretical and experimental investigations have also been conducted for this configuration. A. Craya (11) performed a theoretical analysis for withdrawal from a two-layered system. With dimensional analysis and two-dimensional potential theory, relationships were developed for the maximum withdrawal rate from a layer without withdrawal from the adjoining layer, the influence of the outlet size, and the maximum height of the outlet for selective withdrawal at a given discharge rate.

P. Gariel (12) verified Craya's work experimentally and extended the studies to the three-layered case and finally to the case of a continuous density variation.

Harleman (13) summarizes the work of Craya and Gariel and presents the equation:

$$\frac{Vc}{\sqrt{g'Zo}} = 1.52\left(\frac{Zo}{D}\right) \qquad (2)$$

for the maximum permissible velocity, Vc, through the slot to maintain selectivity, where

g'	=	densimetric acceleration of gravity $= \frac{\Delta\rho}{\rho o} g$
Zo	=	distance of outlet above interface
D	=	slot width (vertical dimension)
$\Delta\rho$	=	density difference between layers
ρo	=	density in layer being withdrawn

Koh (14) deals with the continuous density gradient case. Theoretical analyses were made for both two-dimensional flow toward a line sink and axisymmetric flow toward a point sink. Only the former case was investigated experimentally. The growth of the thickness of the withdrawal layer and the velocity distribution were presented with reference to the distance from the sink. Good agreement was obtained between theory and experiment. Koh's work was concerned with very slow motion of stratified fluids and included the effects of both viscosity and diffusion. Brooks and Koh (15) conclude that Koh's theory and experiments are applicable to large and moderate distances from the sink, respectively.

Gelhar and Mascolo (16) verify their contention that the mathematical model can be simplified by neglecting diffusive effects. Their investigation of withdrawal of a viscous, continuously stratified fluid between two homogeneous layers is extended to the linearly stratified case by a superposition method. Comparison with Koh's analytical results and with experimental data shows good agreement in both cases.

Withdrawal From a Point Sink Above the Bottom.—This configuration probably has the most direct influence on the design of selective withdrawal structures for large reservoirs. Koh's work for the axisymmetric case was introduced in the previous section and will be discussed in more detail later.

R. R. Long (17) investigated the flow of a linearly stratified fluid toward a point sink and discussed applications including meteorology and oceanography.

The internal Froude number is defined by Long as

$$F_i = \frac{q}{(g\frac{\Delta\rho}{\rho o} H^3)^{1/2}} \qquad (3)$$

where

q = discharge per unit width of channel
H = channel depth
$\Delta\rho$ = density difference from top to bottom
ρo = mean density

Substitution of

$$\frac{q}{H} = U,$$

$$g\frac{\Delta\rho}{\rho o} = g',$$

and d = H results in the form of Equation (1).

Long presented a solution for the distance between a line of constant density and the equilibrium position of that line and verified the theory with experiments in a salt water channel.

Harleman (13) presents the work of Craya (11) and Gariel (12) in the equation:

$$\frac{Vc}{\sqrt{g'Zo}} = 3.25\left(\frac{Zo}{D}\right)^2 \qquad (4)$$

In this case, D is the orifice diameter.

Bohan and Grace (18) describe model studies conducted with selective withdrawal from a reservoir with a general stratification pattern. Withdrawal was from square, circular or rectangular orifices. The tests included determination of the effects of orifice size, orifice location, density profile and velocity distribution in the withdrawal layer. The experimental data were found to fit the equation:

$$Vo = \left(\frac{Z^2}{Ao}\right)\sqrt{\frac{\Delta\rho'}{\rho o} gZ} \qquad (5)$$

where

Vo = average velocity through the orifice;
Z = vertical distance from centerline of orifice to upper or lower boundary of withdrawal layer;
Ao = area of orifice;
$\Delta\rho'$ = density differential across Z;
ρo = density at orifice centerline;
g = acceleration due to gravity.

The tests were conducted in a long, narrow, and relatively shallow channel. No attempt was made to determine the effect of the relative width of the flume and orifice. A sample design application was described. This study provided some basis for the development of general equations for selective withdrawal from square or circular orifices. This development is described later in this report.

Withdrawal of the Lower Layer From a Point Sink in the Bottom.—This configuration probably has a relatively rare application to design of selective outlets. Harleman, et al, (19) conducted a theoretical and experimental investigation of withdrawal from the lower layer through a vertical circular orifice in the bottom.

The relationship:

$$\frac{Vc}{\sqrt{g'Zo}} = 2.05\left(\frac{Zo}{D}\right)^2 \qquad (6)$$

was determined where the terms are as defined before, except that Zo is the depth of the lower layer.

Withdrawal of the Upper Layer From a Point Sink in the Upper Layer.—Harleman (13) describes an exploratory study by Rouse (22) of a vertical circular pipe withdrawing from the upper layer in a two-layered system of theoretically unlimited extent. The critical withdrawal rate is expressed by:

$$\frac{Vc}{\sqrt{g'Zo}} = 5.70\left(\frac{Zo}{D}\right)^{3/2} \qquad (7)$$

Zo is the distance of the pipe entrance above the interface.

This configuration also has only limited application to design of selective outlets.

Withdrawal Over a Weir.—Schuster (20) investigated experimentally the flow of both a two-layered system and a system with continuous density variation over a weir. A salt water model was used. The investigation was partly aimed at determining the feasibility of withdrawing the warm upper layer to reduce evaporation losses from the surface of the reservoir. The limited tests showed that, for practical discharge rates, selectivity could not be accomplished.

W. E. Knight (21) described the application of a submerged weir in the control of temperature and dissolved oxygen in the flow through a hydroelectric station. The weir was successful in improving the quality of downstream flows.

The Corps of Engineers report (18), cited earlier, also contains a description of limited tests on a submerged weir.

Prototype Studies.—The submerged weir study of Knight described in the preceding section was conducted in the prototype. The symposium proceedings referred to contain numerous discussions of effects of impoundments on water quality. However, studies which could be compared to model experiments or theory are lacking.

R. T. Jaske (22) presents data from Lake Roosevelt (Grand Coulee) which was obtained as part of a program of cooling Columbia River water for use at the Hanford atomic plant. The discussion regards both the temperature and hydraulic regimes of the reservoir. Comparisons are made with model results and with Debler's (9) theoretical work.

TVA, Bureau of Reclamation, and the Corps of Engineers have made velocity measurements in reservoirs. TVA is continuing comprehensive prototype measurements.

The Bureau of Reclamation has for several years monitored the water chemistry of Lake Mead. Region 2, with headquarters in Sacramento, California, is engaged in an extensive water quality program. The program includes evaluation of selective withdrawal for control of water temperature.

Measurements of temperature and water chemistry are being made in Cheney Reservoir, Kansas. Releases from multiple outlets pass through a treatment plant where discharge rates and quality parameters can be closely monitored. Measurements of temperature and conductivity are being made in Foss Reservoir, Oklahoma.

The Federal Water Pollution Control Administration is conducting and sponsoring measurements in several western reservoirs for purposes of verifying and refining mathematical models for temperature prediction.

It seems that an adequate volume of prototype data will eventually become available for comparison with hydraulic model study results and theoretical predictions.

APPLICATION OF PREVIOUS WORK TO USBR PROBLEMS

Prediction of Stratification

All methods described earlier are adequate for approximate prediction of reservoir stratification, but only under idealized conditions. The Water Resources Engineers prediction model appears to be the most useful at the present time.

A desirable alternative to mathematical prediction would be to monitor conditions in a similar nearby reservoir. However, this is seldom possible in the western states. If existing structures are being modified, existing conditions can be determined. A full 1-year cycle of data would be a minimal study. For purposes of this discussion, it will be assumed that one of these methods has been used to determine the stratification pattern for a given reservoir at various times of the year. Typical patterns are shown in Figure 1.

Three basic conditions may exist:

1. Isothermal (constant temperature)

2. Linear density variation

3. Three-layered stratification

Isothermal Condition

This condition requires no special configuration of withdrawal structure. Withdrawal at any level will draw from the entire depth of the reservoir. The isothermal condition may occur in the spring or fall when winds cause complete mixing of a reservoir with weak stratification. The winter pattern is often essentially isothermal, with only a shallow layer of less dense water near the surface.

Linear Density Variation

This condition is the most difficult to consider in the design of selective outlets. It is important to note that even a small variation in temperature from water surface to reservoir bottom can allow selective withdrawal. Yih, Debler, Kao, Gariel, Koh, Gelhar and Mascolo, Brooks and Koh, Long, and Schuster have investigated this condition, as discussed earlier in this report.

Brooks and Koh (15) suggest that Debler's experimental results be used until the theoretical solutions are clarified.

The recommended relationship is:

$$F' = \frac{V}{\sqrt{g'd}} = 0.28 \pm 0.04 \qquad (8)$$

where

F' = densimetric Froude number
V = average velocity in withdrawal layer
g' = $g\frac{\Delta\rho}{\rho}$
d = thickness of withdrawal layer

In Debler's experiments, the withdrawal layer thickness was measured above the slot centerline, located in the downstream bottom corner of the test flume. For the more general case of withdrawal with a linear density gradient, the withdrawal layer would be symmetrical above and below the slot when the slot is located somewhere above the floor. This would not be true, of course, if a boundary of the layer intersected the bottom of the reservoir or the water surface.

Equation (8) is applicable only to the region near the sink, because of the assumption of negligible viscosity. Brooks and Koh suggest that the relationship be presented in the form:

$$\frac{\delta}{a} = 2.7 \pm 0.2 \qquad (9)$$

where

δ = $2d$
a = $(q/\sqrt{g\epsilon})^{1/2}$
q = discharge per unit reservoir width
ϵ = $-\frac{1}{\rho o}\frac{d\rho}{dy}$
ρo = density at level of outlet
$\frac{d\rho}{dy}$ = vertical density gradient

Brooks and Koh further recommend that Koh's experimental and theoretical work be used for defining withdrawal layer thickness at moderate and large distances, respectively. Modification of Koh's work to a turbulent flow condition gives for moderate distances:

$$\frac{\delta}{a} = 8.4\left(k_2\frac{x}{a}\right)^{1/4} \qquad (10)$$

$$\text{for } 2.7 < \frac{\delta}{a} < 13.7$$

and for large distances:

$$\frac{\delta}{a} = 7.14\left(k_2\frac{x}{a}\right)^{1/4} \qquad (11)$$

$$\text{for } \frac{\delta}{a} > 13.7$$

In these equations, x is the distance from the dam and k_2 is a function of vertical eddy diffusivity which must be determined in the field. Brooks and Koh state that k_2 is on the order of 10^{-3}.

Note that equations (10) and (11) predict a growth of the withdrawal layer proportional to the one-fourth power of the distance from the dam.

Development of Equations for Design

Equations for withdrawal from a fluid with a linear density gradient can be generalized to allow approximate solution for nonlinear gradients. Two-dimensional theories are assumed to apply to the three-dimensional case and point sinks located at any depth are considered. An idealized reservoir geometry, with constant rectangular cross section is assumed.

The formula for the densimetric Froude number

$$F' = \frac{V}{\sqrt{g'd}}$$

forms the basis for the development. As discussed earlier, Debler found that F' = 0.28 ± 0.04 for withdrawal through a slot from a linear density gradient. For this discussion, the value 0.28 will be used.

V is the average velocity in the withdrawal layer, q/δ or $Q/W\delta$, where W is the reservoir width.

Q is equal to VoAo, where Vo is the velocity through the withdrawal orifice and Ao is the orifice area.

Ao, for a slot, is WD, where D is the vertical dimension of the slot. Therefore,

$$VW\delta = VoWD$$

$$V = \frac{VoD}{\delta}$$

$$\delta = 2d$$

$$V = \frac{VoD}{2d}$$

Substituting in Equation (8) for F' = 0.28:

$$F' = \frac{Vo}{\sqrt{g'd}} = 0.56\,\frac{d}{D} \qquad (12)$$

where d is the withdrawal layer thickness above or below the orifice centerline.

For a square orifice:

$$Ao = D^2$$

$$2VWd = VoD^2$$

$$V = \frac{VoD^2}{2Wd}$$

$$F' = \frac{Vo}{\sqrt{g'd}} = 0.56\,\frac{Wd}{D^2} \qquad (13)$$

For a circular orifice:

$$Ao = \frac{\pi D^2}{4}$$

$$2VWd = \frac{Vo\pi D^2}{4}$$

$$V = \frac{VoD^2}{2.54\,Wd}$$

$$F' = \frac{Vo}{\sqrt{g'd}} = 0.71\,\frac{Wd}{D^2} \qquad (14)$$

Similar forms can be developed for other orifice shapes. It should be noted that for a given flume width, equal orifice areas will result in identical relationships between F' and d, regardless of the orifice shapes.

As mentioned earlier in this report, the Waterways Experiment Station, Corps of Engineers, found that data for withdrawal through a square orifice from a 1-foot-wide flume fit the expression:

$$\frac{Vo}{\sqrt{g'd}} = \left(\frac{d}{D}\right)^2 = \frac{d^2}{Ao} \qquad (15)$$

(also Eq. (5))

Equations (13) and (14) were modified to allow comparison with the WES data.

The lines representing these relationships, plotted on Figure 2, suggest that the WES data might also fit Equations (13) and (14). Both Debler's tests and the WES experiments used dissolved salts to induce the density stratification, and both were conducted in relatively narrow flumes.

Computer Solution of Design Equations

Equations (13) and (14) were modified for solution by digital computer.

The equations in general form are:

$$\frac{Vo}{\sqrt{g'd}} = K\,\frac{Wd}{D^2} \qquad (16)$$

where K is a constant depending upon the shape of the orifice.

Squaring both sides of the equation:

$$\frac{Vo^2}{g'd} = K^2\,\frac{W^2d^2}{D^4}$$

As defined earlier:

$$g' = \frac{\Delta\rho}{\rho o}\,g$$

Therefore:

$$\frac{Vo^2}{\frac{\Delta\rho}{\rho o} gd} = K^2 \frac{W^2 d^2}{D^4}$$

Rearranging terms for convenience gives:

$$\frac{D^4 \rho o Vo^2}{g} = \Delta\rho K^2 d^3 W^2 \qquad (17)$$

For a given density profile and flow rate, the withdrawal layer thickness (above or below the orifice), d, is determined when the equality of Equation (17) is satisfied.

For a circular orifice, K = 0.505; for a square orifice, 0.314, when F' = 0.28.

A listing and detailed description of the computer program are given in the Appendix of this report.

Special conditions arise when the predicted upper boundary of the withdrawal layer intersects the water surface or when the lower boundary intersects the reservoir bottom.

The basic assumption in the solution of Equation (17) is that the total discharge is equally divided between the upper portion and the lower portion of the withdrawal layer. When intersection with the reservoir bottom or water surface occurs, this assumption is no longer valid. d is now less than the value required to satisfy Equation (17). The discharge above or below the orifice is then adjusted by multiplying the discharge by the ratio of the right side to the left side of Equation (17). The resulting excess discharge is applied to the other portion of the withdrawal layer. For example, assume a total discharge of 1,000 cubic feet per second. Under normal conditions, flow in the reservoir would approach the outlet equally from both above and below the outlet. Assume that under these conditions, the predicted upper boundary of the withdrawal layer intersects the water surface. d is then equal to the distance between the outlet and the water surface. The adjustment is performed as stated above, resulting in a lower discharge, say 400 cubic feet per second, from the upper part of the layer. The corresponding discharge from the lower portion must now be 600 cubic feet per second. The left-hand term in Equation (17) is adjusted accordingly, and solution for the lower boundary of withdrawal proceeds.

EXPERIMENTAL VERIFICATION

Description of Test Facility

Figure 3 shows the flume used for the laboratory tests. The flume is divided into a mixing compartment and a test compartment, with the refrigeration system located in the mixing compartment. The cold water flows through a selective-level bulkhead and sinks to the floor of the test compartment. After several hours, a well-defined thermal stratification exists in the test compartment. Outflow occurs through a small orifice with adjustable elevation and is recirculated to the mixing compartment or discharged to waste. Thermistor probes are placed in desired locations in the test compartment. The thermistors are connected through a scanning device to a digital thermometer and a printer where temperatures are displayed and recorded with an accuracy of 0.02° C. Two very accurate quartz probes, with a digital thermometer were used for calibration and checking of the thermistor probes.

Verification of Design Equations

A 5/8-inch-diameter circular tube or a 0.1-foot square tube was placed at the midpoint of the flume width, approximately 1 foot from the downstream wall. A thermistor was placed inside the tube for measurement of outflow temperature. The temperature profile in the 3-foot-deep flume was measured with 28 thermistors imbedded in a lucite rod (rather than an array as shown in Figure 3), placed 26 inches from the downstream wall and 8 inches from the left wall of the flume. The 30th thermistor was used to monitor the temperature in the mixing compartment, when necessary. The flow path from the selective bulkhead to the outflow tube was approximately 9 feet long.

Withdrawal layer thicknesses were observed for several temperature profiles, discharges, and outflow tube elevations. Most tests were performed for a 3-foot flume width; several were performed for a 2-foot width.

Figure 4 is a photograph of a typical test. Recording of temperatures began simultaneously with dropping of potassium permanganate crystals to form vertical dye streaks. The withdrawal layer boundaries were determined by deformation of the streaks as indicated by a scale attached to the plexiglass wall of the flume. It is estimated that the boundaries could be determined within ±0.02 foot. Discharge was measured with a 3/8-inch circular orifice in a 2-inch line. The orifice was calibrated in place using a weighing tank. Because of fluctuations in the mercury manometer and other reading errors, errors of approximately ±3 percent could be expected in measurement of discharge. The calibration remained applicable when checked after about 8 months of operation.

Figures 5-14 show temperature profiles, outlet locations, observed locations of the maximum velocity, and the theoretical and observed withdrawal layer boundaries for the test runs. Symbols are defined on Figure 5. The theory is based on $F' = 0.10$. These figures and Table 1 show excellent agreement between theory and observations for some runs and poor agreement for others. Generally, the runs performed with a falling head gave better results that those with recirculating flow. The inflow, especially when of a slightly different temperature than the outflow, generated currents which tended to obscure the withdrawal pattern.

Table 1 indicates that consistent results were obtained by Observer A and Observer B. Also, the distance from the outlet at which the layer was observed had no apparent influence.

Figures 5, 12, and 13 include static conditions (no inflow or outflow) and allow comparison with flowing conditions to determine the effect of withdrawal on the temperature profile. Figures 5A and B show a negligible short-term effect of withdrawal on the temperature profile in the zone of withdrawal. Figure 5 shows that, with zero inflow, the primary effect is in depletion of water within the withdrawal layer. This is visible as an apparent lowering of the inflection point, which was originally at elevation 1.67 feet. Twenty-five minutes from the start of the test this point was at 1.40 feet. Outlet elevation and discharge remained constant during the test. The lowering of the inflection point is also reflected in the increasing outflow temperature.

This test also shows a decrease in the withdrawal layer thickness and a lowering of the elevation of maximum velocity as the test proceeds. The layer was observed 4 feet upstream from the outlet. This unsteady condition apparently represents the adjustment period immediately following initiation of withdrawal. During this period, the outflow temperature remains nearly equal to the temperature at that level within the reservoir. Figure 5B shows that 5 minutes after initiation of withdrawal, a downstream movement occurs approximately 0.2 foot above the outlet level. Other, less dominant currents, both upstream and downstream occurred in the emptying flume but were not noted. As the system approaches the steady state, the dominant downstream current moves closer to the level of the outlet.

Figures 6A and 6B show the results of a test with equal inflow and outflow. The withdrawal pattern in Figure 6A is not being influenced by the inflow. In Figure 6B, however, the inflow, which is higher in temperature than the outflow, is obscuring the withdrawal pattern. Also note that the layer thickness is reduced, causing a slightly lower outflow temperature. The rather poor agreement between the theoretical and observed upper layer boundary in Figures 6A and 6B, shows that the author's method of treating the special condition of intersection with the bottom or water surface needs further refinement. Figure 6C, which is for a falling head, gives the same result.

Figures 6C and 7 show the effects of changing outlet elevation on essentially identical temperature profiles. Agreement between theory and observation is quite good. Agreement for the surface condition in Figure 7D is surprisingly perfect, perhaps accidentally.

Figures 8 and 9 describe a similar series of tests. Agreement for the surface condition in Figure 8A is very poor, which is true of several tests in the series. Note that the surface temperature was estimated. This could account for some error. The test series described in Figures 10 and 11 showed good agreement between theory and observation for the first two tests and poor agreement for the last three tests. Figures 10A and 10B show that agreement is good where the temperature profile is nearly linear (the basic assumption in development of the theory). Figure 10C suggests that, where the temperature gradient is decreasing with increasing elevation, the observed layer is larger than the theoretical layer. In Figures 10D and 11, where the temperature gradient is increasing, the observed layer is smaller than the theoretical layer. With a few exceptions, these observations are generally applicable to all the tests.

Figures 12 and 13 describe two series of tests conducted with a 0.1-foot square orifice. Results are very similar to those obtained with a circular orifice. Note in Table 1 that two different observers recorded the observations. Figure 12C shows that two cusps were observed in the velocity profile. The reason for this phenomenon has not been determined.

Figure 14 is for a test series with the square orifice and with the flume width reduced from 3 feet to 2 feet. Agreement between theory and observation was quite good.

The possible application of Equation (10), (Koh's experimental results for defining the layer thickness at moderate distances from the sink), was checked for test 4-16-69-1459, which does not have a perfectly linear density gradient:

$$\epsilon = -\frac{1}{\rho o}\frac{d\rho}{dy} = -\frac{1}{.999}\left(\frac{-.00021}{.35}\right) = .00601$$

$$a = \left[\frac{q}{\sqrt{g\epsilon}}\right]^{1/2} = \left[\frac{.00082}{.139}\right]^{1/2} = .0768$$

$$\delta = 8.4a\left(k_2\frac{x}{a}\right)^{1/4},\ k_2 = 10^{-3} =$$

$$8.4\left(.0768\right)\left(\frac{2}{.0768} \times 10^3\right)^{1/4} =$$

$$(.65)(2.26)(.178) = .261 \text{ ft}$$

Observed thickness = .35 foot

$$\frac{\delta}{a} = \frac{.261}{.0773} = 3.38$$

Therefore, the limitation that $2.7 < \frac{\delta}{a} < 13.7$ is met, though it might be considered to be in a transition zone.

It seems likely that viscosity accounted for the larger-than-predicted withdrawal layer thickness. Koh's original experimental equation for viscous, diffusive withdrawal was checked for application to this observation. The equation is:

$$\delta = 8.4\, D^{0.056}\nu^{0.189}q^{0.133}(g\epsilon)^{-0.189}x^{0.245} \qquad (18)$$

where

D = molecular diffusion coefficient for heat, ft^2/sec

ν = kinematic viscosity, ft^2/sec

q = unit discharge, cfs/ft

ϵ = $-\frac{1}{\rho o}\frac{d\rho}{dy}$, ft^{-1}

x = distance from orifice, ft

Assuming $D = 3.1 \times 10^{-5}$ ft^2/sec and $\nu = 1.58 \times 10^{-4}$ ft/sec ($\nu/D = 5$), δ was calculated to be 1.102 ft for test 4-16-69-1459. This is more than three times the observed thickness of 0.359 ft.

Calculation of the parameter

$$\frac{q}{D\alpha_o x^{2/3}}\left(\alpha_o = \left[\frac{\epsilon g}{D\nu}\right]^{1/6}\right)$$

gives a value of 1.33, which lies near the lower limit of $0.3 \leq \frac{q}{D\alpha_o x^{2/3}} < 10^3$, for application of Equation (18).

The results of these calculations are inconclusive.

Velocity Distribution

The WES experiments showed that "when the orifice CL (centerline) fell below the center of the withdrawal zone the maximum velocity was found to occur below the orifice CL and similarly when the orifice CL fell above the center of the withdrawal zone, the maximum velocity occurred above the orifice CL." This observation was not verified by the present tests. No particular trend was noted; the position of maximum velocity was more or less random.

Figure 15 shows that 80 percent of the observed points lie within or very near to a ± 5 percent variation between observed and theoretical withdrawal layer boundaries.

In 3 out of 30 cases, the outflow temperature was within 0.05° C of the reservoir temperature at the level of the outlet. In 14 cases, the difference was within 0.10° C, and in 18 cases, 0.15° C. In 9 cases, the difference was greater than 0.20° C. The largest differences occurred with the square orifice and 2-foot-wide flume (Figure 14).

These differences represent the skewness of the velocity distribution; a distribution symmetrical about the outlet would result in zero difference.

The data also show that for all tests with the circular orifice the outflow temperature was greater than the corresponding reservoir temperature, while the opposite was true for the tests with the square orifice. There is no apparent reason why the orifice shape should influence the velocity distribution in this way.

The Waterways Experiment Station recommends computation of the velocity distribution according to the equation:

$$\frac{v}{V_{max}} = \left(1 - \frac{y}{Y}\frac{\Delta\rho}{\Delta\rho_{max}}\right)^2 \qquad (19)$$

where

v	=	velocity at point y;
V_{max}	=	maximum velocity;
Y	=	distance above or below point of maximum velocity to boundary of withdrawal layer;
$\Delta\rho$	=	density differential from point of maximum velocity to y;
$\Delta\rho_{max}$	=	density differential from point of maximum velocity to boundary of withdrawal layer.

The relationship between the average velocity (q/δ) and the required maximum velocity is determined through an integration procedure. It is assumed that the velocity is constant across the width of the reservoir. Elder and Wunderlich (24) measured a non-uniform distribution in Fontana Reservoir, although the variation was not large.

Knowledge of the velocity distribution and cross-sectional shape of the reservoir allows calculation of the discharge from any specific level of the reservoir. Thus, the withdrawal of other materials such as dissolved oxygen can be determined, if their distribution in the reservoir is known.

Comparison of Test Results with Prototype Data

Limited field measurements (24) in TVA's Fontana and Cherokee Reservoirs indicated a simple relationship:

$$\delta = 5a \qquad (20)$$

throughout the length of the reservoir.

A densimetric Froude number of 0.10, which much of the present model data fit, corresponds to:

$$\delta = 6.32a \qquad (21)$$

Equation (20) corresponds to F' = 0.16. The TVA data actually varied through an approximate range $4.2 < \frac{\delta}{a} < 6.1$, or $0.11 < F' < .23$.

As explained earlier, the WES data might be considered to conform to F' = 0.28. The general agreement of the WES data with Debler's findings and the approximate conformity of the present data to TVA prototype measurements are very interesting. Both the WES and Debler used salt models while TVA's measurements and the present study involved only temperature stratification. The molecular diffusion coefficient for heat is on the order of 500 times greater than that for sodium chloride. This would tend to increase the withdrawal layer thickness in thermal heat models and thus decrease the apparent critical value of F'.

The author agrees with the TVA suggestion that a modified form of the inviscid, nondiffusive theory can be applied for a reasonable prediction of withdrawal layer thickness. Until more verification data become available, Equation (20) should be used.

The reasonable agreement between model and prototype data suggests that hydraulic models can be used for estimating withdrawal layer thicknesses and for studying flow patterns in stratified reservoirs.

The model data illustrate that the assumed relationship is most deficient when the withdrawal layer intersects the reservoir bottom or water surface. Future work will consist of including velocity distribution in the computer analysis, in place of assumed distribution of discharge. Measurement of velocity distribution in the test flume will allow a more precise comparison with predicted results and with the findings of other experimenters, and will allow inclusion of variable reservoir width.

The present study used a "re-entrant" type of outlet configuration. The author assumes that the small orifice size, with respect to the flume width and depth, precludes any effect of orifice configuration. This will be investigated further in future tests.

SAMPLE PROBLEM

Temperature profile:

Elevation 4750 (bottom)	T = 11.00° C
Elevation 4770	12.00
Elevation 4780	13.00
Elevation 4810	23.00
Elevation 4820 (water surface)	25.00

Outlet size and shape: 2.0-foot-diameter, circular
Outlet elevation = 4810.63
Discharge = 16 cfs
Reservoir width = 150 feet

Determine limits of withdrawal layer for F' = 0.16 ($\frac{\delta}{a} = 5$). Assume that no suspended or dissolved materials are present to alter the density profile. It is further assumed that the reservoir width remains constant in the region of the withdrawal layer.

The computer program shown in the appendix is used for the solution. Slight modifications, necessary when changing from model to prototype calculations, are described in the program description.

The input data sheet and results are shown in Figures 16 and 17, respectively.

DEFINITION OF TERMS

a	–	defined as $(q/\sqrt{g\epsilon})^{1/2}$, ft
A	–	defined as $\sqrt{\rho U^2}$, units depend on units of ρ ($g^{1/2}$ - ft/$ml^{1/2}$ sec, for ρ in mg/1)
Ao	–	area of outlet, ft^2
d	–	withdrawal layer thickness, above or below outlet, ft
D	–	outlet size (vertical width, diameter) ft; also molecular diffusion coefficient, ft^2/sec
F	–	Froude number, dimensionless
Fi, F'	–	internal or densimetric Froude number, dimensionless
g	–	acceleration of gravity, ft/sec^2
g'	–	densimetric acceleration of gravity, ft/sec^2
H	–	channel depth, ft
K	–	constant, dependent upon outlet shape
k_2	–	function of vertical eddy diffusivity
q	–	discharge per unit of channel width, ft^3/sec - ft
Q	–	total discharge, ft^3/sec
T	–	temperature, C^o
U, V	–	average velocity, ft/sec
υ	–	local velocity, ft/sec
Vc	–	maximum permissible withdrawal velocity to maintain selectivity, ft/sec
V_{max}	–	maximum velocity, ft/sec
Vo	–	average velocity through outlet, ft/sec
W	–	width of reservoir or channel, ft
x	–	horizontal, upstream distance from outlet, ft
y	–	distance above or below point of maximum velocity, ft
Y	–	distance from point of maximum velocity to boundary of withdrawal layer, ft
Z	–	distance from centerline of outlet to boundary of withdrawal layer, ft
Zo	–	distance from outlet to interface, ft
αo	–	defined as $(\epsilon q/D\nu)^{1/2}$, $sec^{1/2}/ft^{3/2}$
β, $d\rho/dy$	–	vertical density gradient (β is positive in sign), g/ml - ft
δ	–	total withdrawal layer thickness, ft
ϵ	–	defined as $-1/\rho o\ d\rho/dy$, ft^{-1}
ν	–	kinematic viscosity, ft^2/sec
ρ	–	density, g/ml
ρo	–	mean density, or density at level of outlet, g/ml
$\Delta\rho$, $\Delta\rho'$	–	density difference across a given distance, g/ml
$\Delta\rho_{max}$	–	density difference from point of maximum velocity to boundary of withdrawal layer, g/ml

REFERENCES

(1) Austin, G. H., Gray, D. A., Swain, D. G., "Report on Multilevel Outlet Works at Four Existing Reservoirs," ASCE Hydraulics Division Conference, Cambridge, Massachusetts, August 21-23, 1968.

(2) Dake, J.M.K., and Harleman, D. R. F., "An Analytical and Experimental Investigation of Thermal Stratification in Lakes and Ponds," *M.I.T. Hydrodynamics Laboratory Report No. 99,* September 1966.

(3) Wunderlich, W. O., and Elder, R. A., "The Mechanics of Stratified Flow in Reservoirs," 1967 Symposium of the American Fisheries Society, Athens, Georgia.

(4) Orlob, G. T., "Prediction of Thermal Energy Distribution in Streams and Reservoirs," Report prepared by Water Resources Engineers, Inc., Walnut Creek, California, for California State Department of Fish and Game, June 30, 1967.

(5) Huber, D. Grant, and Reid, Thomas L., "Experimental Study of Two-Layered Flow Through a Sink," *Journal of the Hydraulics Division, ASCE,* January 1966.

(6) Huber, D. Grant, "Irrotational Motion of Two Fluid Strata Towards a Line Sink," *Journal of the Engineering Mechanics Division, ASCE,* August 1960.

(7) Harleman, Donald R. F., and Elder, Rex A., "Withdrawal from Two-Layered Stratified Flows," *Journal of the Hydraulics Division, ASCE,* July 1965.

(8) Yih, Chia-Shun, "On the Flow of a Stratified Fluid," *3rd U.S. National Congress on Applied Mechanics,* 1958.

(9) Debler, W. R., "Stratified Flow Into a Line Sink," *Journal of the Engineering Mechanics Division, ASCE,* July 1959.

(10) Kao, T. W., "A Free-Streamline Solution for Stratified Flow Into a Line Sink," *Journal of Fluid Mechanics,* Volume 21, March 1965.

(11) Craya, A., "Theoretical Research on the Flow of Nonhomogeneous Fluids," La Houille Blanche, January-February 1949, in French.

(12) Gariel, P., "Experimental Research on the Flow of Nonhomogeneous Fluids," La Houille Blanche, January-February 1949, in French.

(13) Harleman, D.R.F., "Stratified Flow," Section 26 of *Handbook of Fluid Dynamics,* edited by V. L. Streeter, McGraw-Hill, New York, 1961.

(14) Koh, R.C.Y., "Viscous Stratified Flow Toward a Sink," *Journal of Fluid Mechanics,* No. 24, Part 3, 1966.

(15) Brooks, N. H., and Koh, R.C.Y., "Selective Withdrawal from Density-Stratified Reservoirs," Proceedings of ASCE Specialty Research Conference on "Current Research into the Effects of Reservoirs on Water Quality," Portland, Oregon, January 22-24, 1968.

(16) Gelhar, L. W., and Mascolo, D. M., "Non-diffusive Characteristics of Slow Viscous Stratified Flow Towards a Line Sink," *M.I.T. Hydrodynamics Laboratory Report No. 88,* February 1966.

(17) Long, R. R., "Velocity Concentrations in Stratified Fluids," *Journal of the Hydraulics Division, ASCE,* January 1962.

(18) Bohan, J. P., and Grace, J. L., Jr., "Mechanics of Flow from Stratified Reservoirs in the Interest of Water Quality," Technical Report H-69-10, U.S. Army Engineer Waterways Experiment Station, July 1969.

(19) Harleman, D.R.F., Morgan, R. L., and Purple, R. A., "Selective Withdrawal from a Vertically Stratified Fluid," *International Association for Hydraulic Research, 8th Congress,* August 1959.

(20) Schuster, J. C., "Hydraulic Model Study of Stratified Flow over a Weir," *U.S. Bureau of Reclamation Report No. Hyd-425,* November 1956.

(21) Knight, W. E., "Improvement of the Quality of Reservoir Discharges through Control of Discharge Elevation," *Proceedings of Symposium on Streamflow Regulation for Quality Control,* Taft Sanitary Engineering Center, June 1965.

(22) Jaske, R. T., "An Evaluation of the Use of Selective Discharges from Lake Roosevelt to Cool the Columbia River," Report BNWL-208, Pacific Northwest Laboratory, Richland, Washington, February 1966.

(23) Rouse, H., "Seven Exploratory Studies in Hydraulics," *Journal of the Hydraulics Division, ASCE,* August 1956.

(24) Elder, R. A., and Wunderlich, W. O., "The Prediction of Withdrawal Layer Thickness in Density Stratified Reservoirs," TVA Water Resources Research Laboratory Report No. 4, March 1969.

Table 1—Comparison of Theoretical and Observed Withdrawal Layers, F′ = 0.10

Test number	Orifice shape	Orifice size	Orifice elevation	Flume width	Theoretical upper limit	Observed upper limit	Theoretical lower limit	Observed lower limit	Discharge	Observer	Remarks
3-25-69-1050	Circular	.052 ft.	1.67 ft.	3.0 ft.	1.88 ft.	2.00 ft.	1.36 ft.	1.44 ft.	.00345 cfs	A	Unless otherwise noted all tests are with falling head, no inflow.
3-25-69-1100			1.67		1.87	1.83	1.39	1.47	.00345	A	
3-25-69-1110			1.67		1.87	1.83	1.42	1.48	.00345	A	Observed 4 ft. from outlet.
4-16-69-1311			0.20		0.72	0.85	0.00	0.00	.00345	B	Recirculating, inflow = outflow.
4-16-69-1323			0.20		0.71	0.75	0.00	0.00	.00345	B	Recirculating, inflow = outflow.
4-16-69-1405			0.20		0.69	0.85	0.00	0.00	.00245	B	
4-16-69-1423			1.70		1.92	1.90	1.49	1.45	.00245	B	
4-16-69-1443			2.00		2.20	2.20	1.79	1.80	.00245	B	
4-16-69-1459			2.30		2.46	2.45	2.12	2.10	.00245	B	
4-16-69-1513			2.50		2.60	2.60	2.30	2.30	.00245	A	Upper boundary of layer intersected w.s.
4-18-69-0922			2.60		2.77	2.96	2.44	2.40	.00245	B	Upper boundary of layer intersected w.s.
4-18-69-0934			2.20		2.35	2.35	2.04	1.95	.00245	B	
4-18-69-0948			1.70		1.89	1.95	1.48	1.35	.00245	B	
4-18-69-1002			1.20		1.41	1.40	1.01	1.05	.00245	B	
4-18-69-1010			0.70		0.92	0.95	0.43	0.60	.00245	B	
4-18-69-1021			0.20		0.43	0.40	0.00	0.00	.00245	B	
5-16-69-0816			2.20		2.42	2.40	1.97	1.93	.00486	B	
5-16-69-0824			1.70		1.94	1.95	1.45	1.43	.00486	B	
5-16-69-0832			1.10		1.34	1.38	0.87	0.72	.00486	B	
5-16-69-0840			0.60		0.82	0.73	0.24	0.40	.00486	B	
5-16-69-0854			0.10		0.57	0.46	0.00	0.00	.00486	B	Observed 2 ft. from outlet

Test number	Orifice shape	Orifice size	Orifice elevation	Flume width	Theoretical upper limit	Observed upper limit	Theoretical lower limit	Observed lower limit	Discharge	Observer	Remarks
6-26-69-1250	Square	0.1	1.25	↓	1.47	1.80	1.04	0.90	.00345	A	Observed 1.67 ft. from outlet
6-26-69-1305	↓	↓	1.50	↓	1.73	1.85	1.27	1.25	.00345	A	↓
6-26-69-1355	↓	↓	1.75	↓	1.97	2.10	1.52	1.50	.00345	A	↓
6-27-69-1303	↓	↓	2.00	↓	2.27	2.25	1.71	1.65	.00345	B	↓
6-27-69-1317	↓	↓	2.30	↓	2.56	2.61	2.04	2.06	.00345	B	
6-27-69-1338	↓	↓	2.60	↓	2.82	2.83	2.36	2.43	.00345	B	
7- 2-69-1050	↓	↓	1.25	2.0	1.51	1.56	0.99	1.01	.00345	B	
7- 2-69-1107	↓	↓	1.90	↓	2.16	2.12	1.63	1.66	.00345	B	
7- 2-69-1123	↓	↓	2.40	↓	2.63	2.71	2.16	2.21	.00345	B	

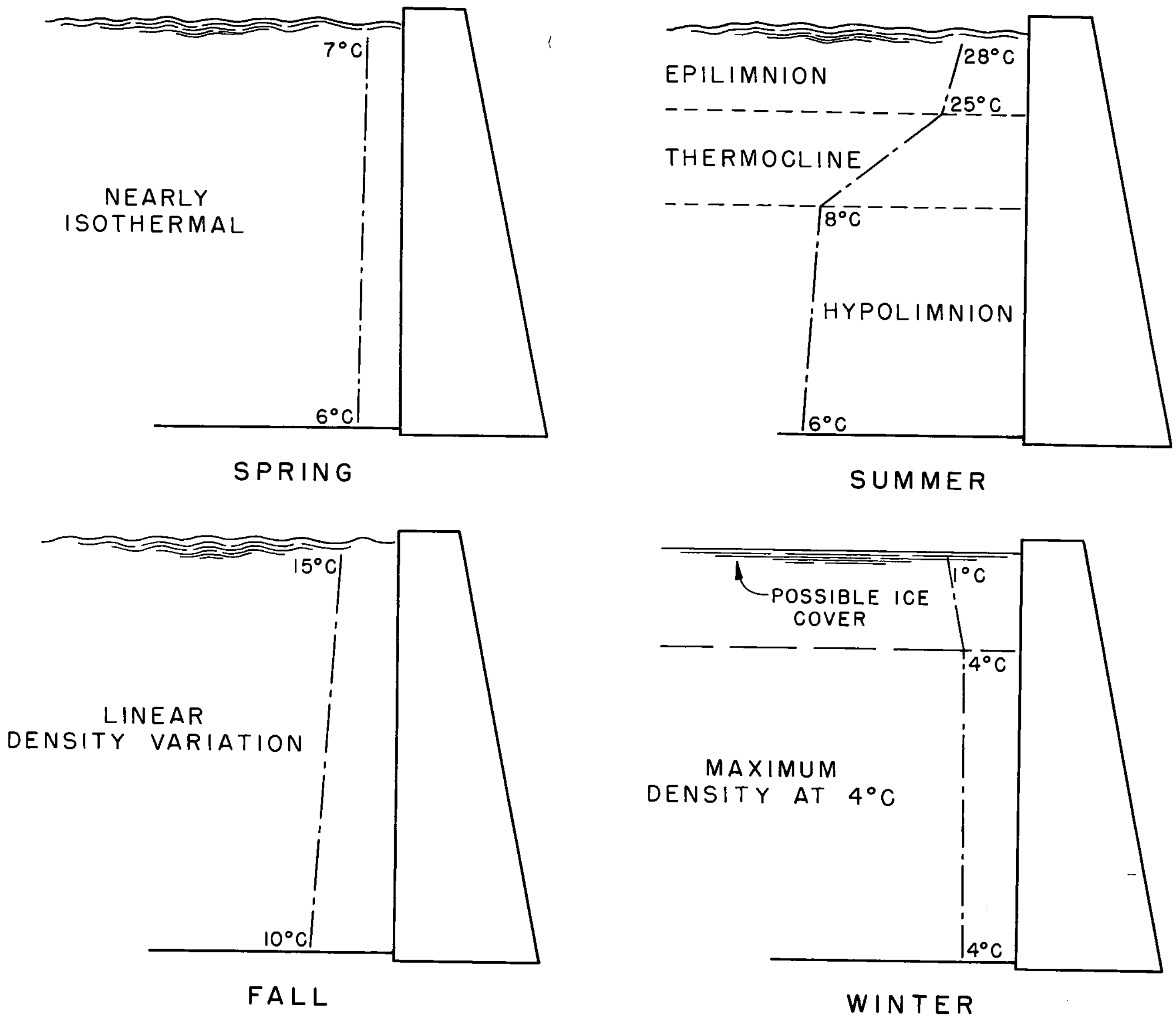
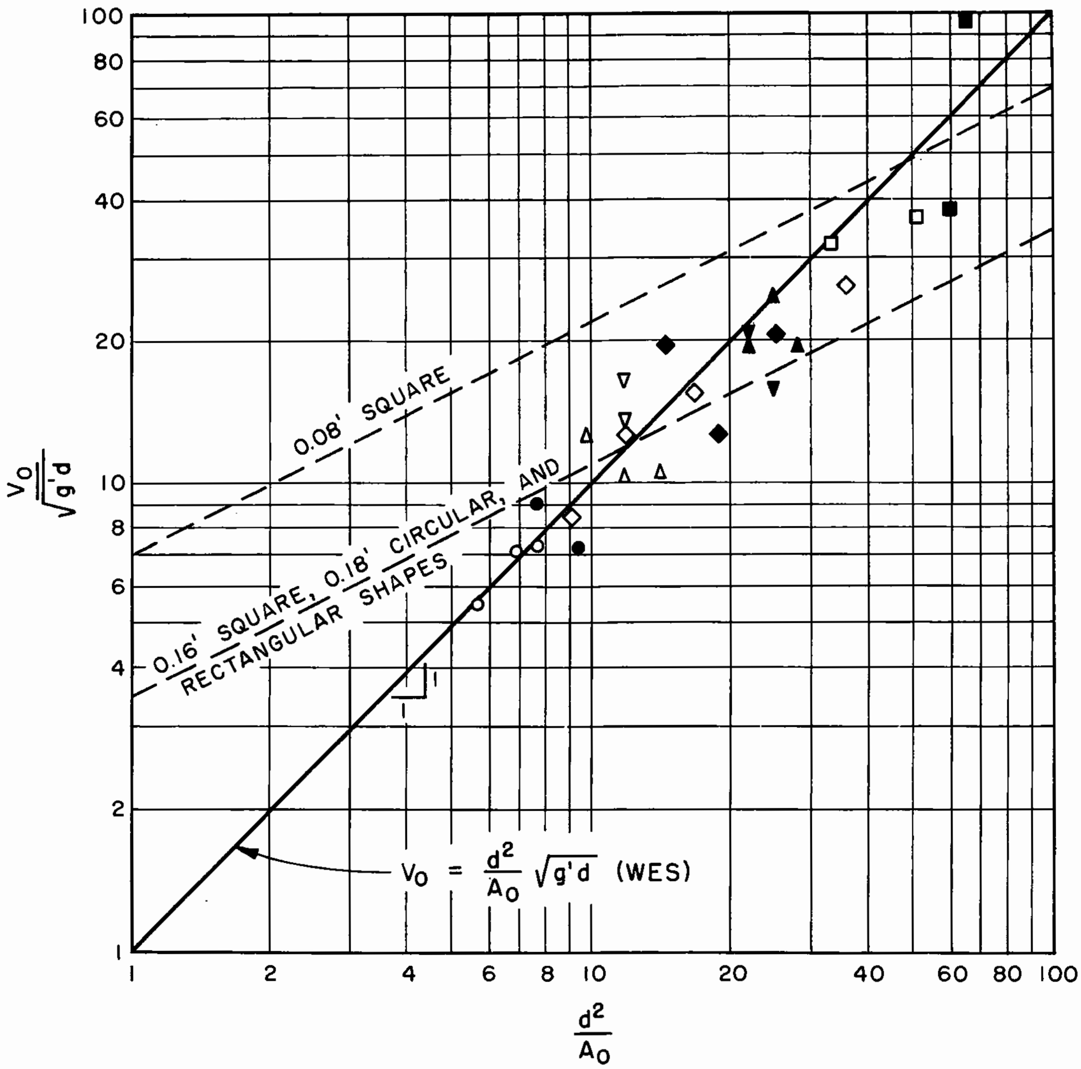


Figure 1
Report HYD-595



LEGEND

0.08-FT. SQUARE ORIFICE

□ LOWER } WITHDRAWAL CHARACTERISTICS
■ UPPER

0.16-FT. SQUARE ORIFICE

○ LOWER } WITHDRAWAL CHARACTERISTICS
● UPPER

0.11- BY 0.23-FT. RECTANGULAR ORIFICE (LONG AXIS VERT)

▽ LOWER } WITHDRAWAL CHARACTERISTICS
▼ UPPER

0.11- BY 0.23-FT. RECTANGULAR ORIFICE (LONG AXIS HORIZ)

△ LOWER } WITHDRAWAL CHARACTERISTICS
▲ UPPER

0.18-FT. DIAM ORIFICE

◇ LOWER } WITHDRAWAL CHARACTERISTICS
◆ UPPER

----- EQ'S (13) AND (14)

Partially reproduced from Reference (18)

SELECTIVE WITHDRAWAL FROM RESERVOIRS

Comparison of Eq's (13) and (14) with WES Data for F' = 0.28

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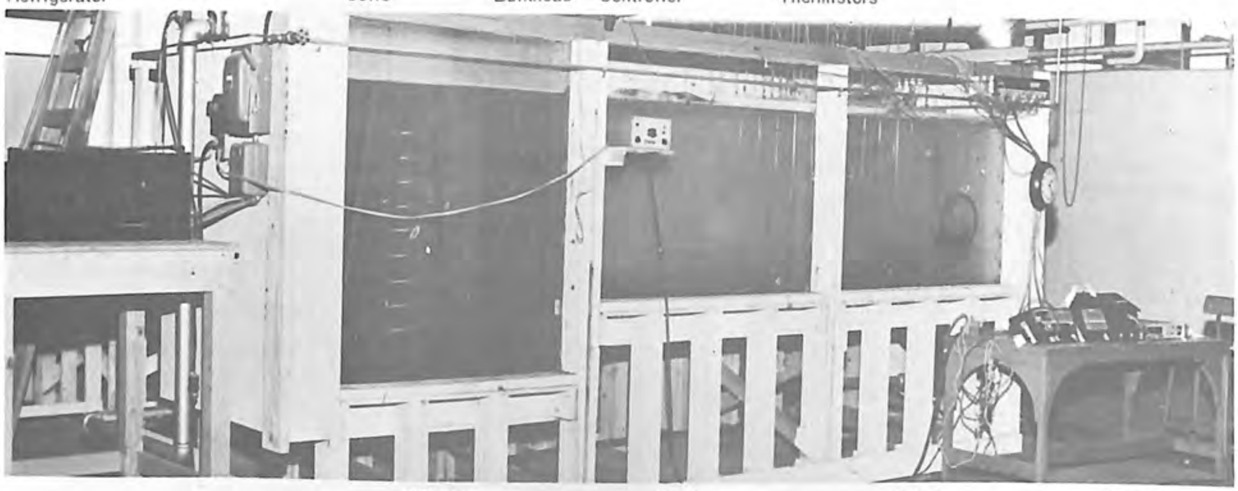

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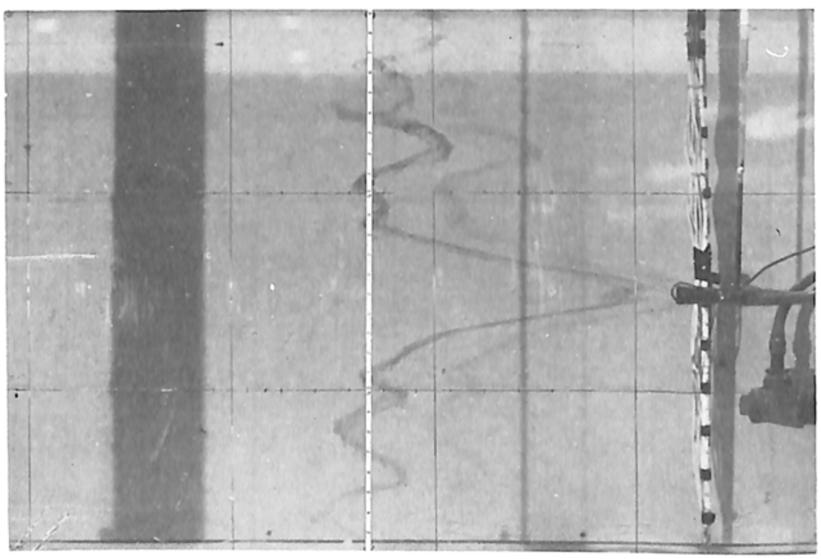
SELECTIVE WITHDRAWAL FROM RESERVOIRS

Test Facility and Instrumentation



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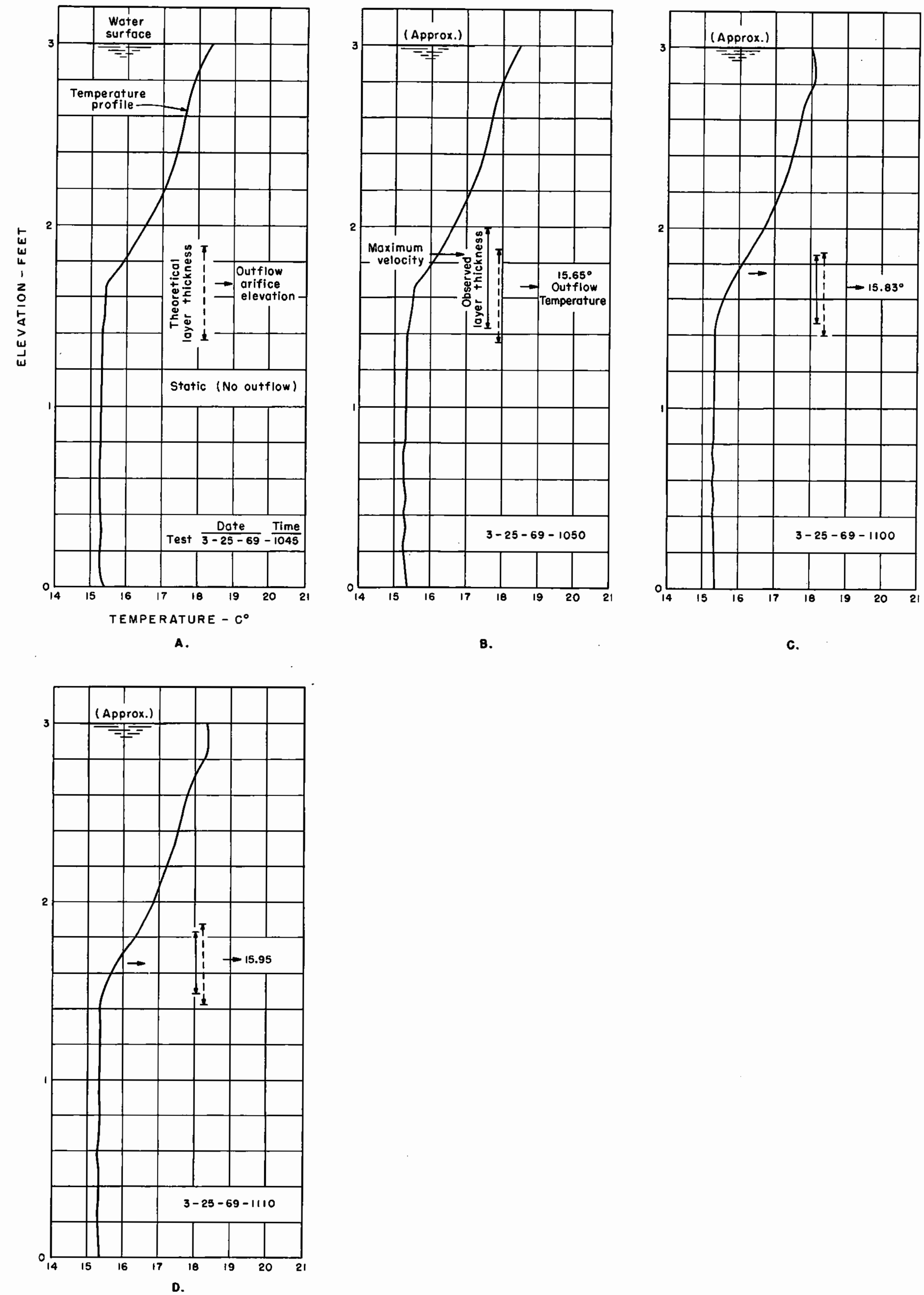
SELECTIVE WITHDRAWAL FROM RESERVOIRS

Typical Test



SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

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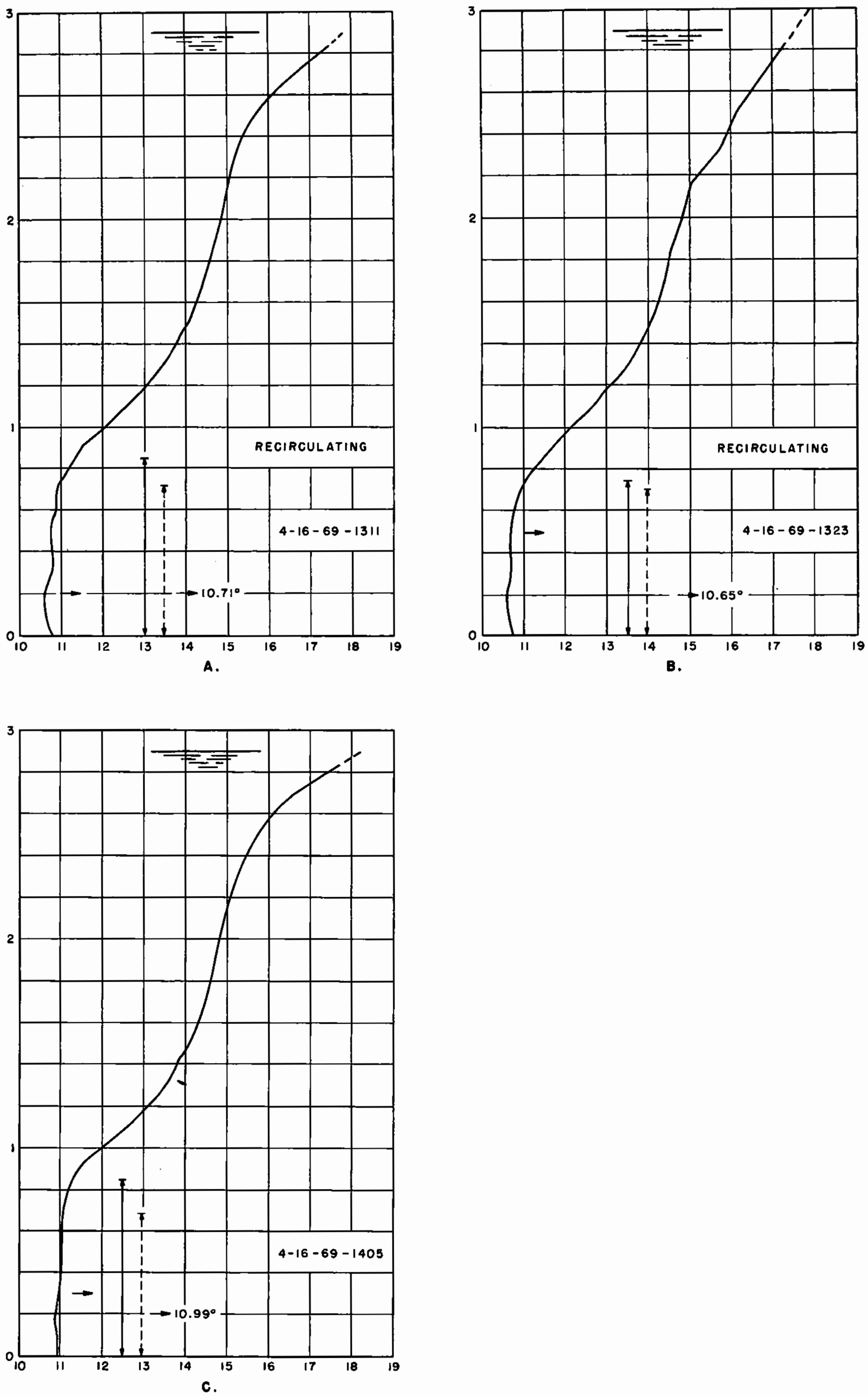

SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

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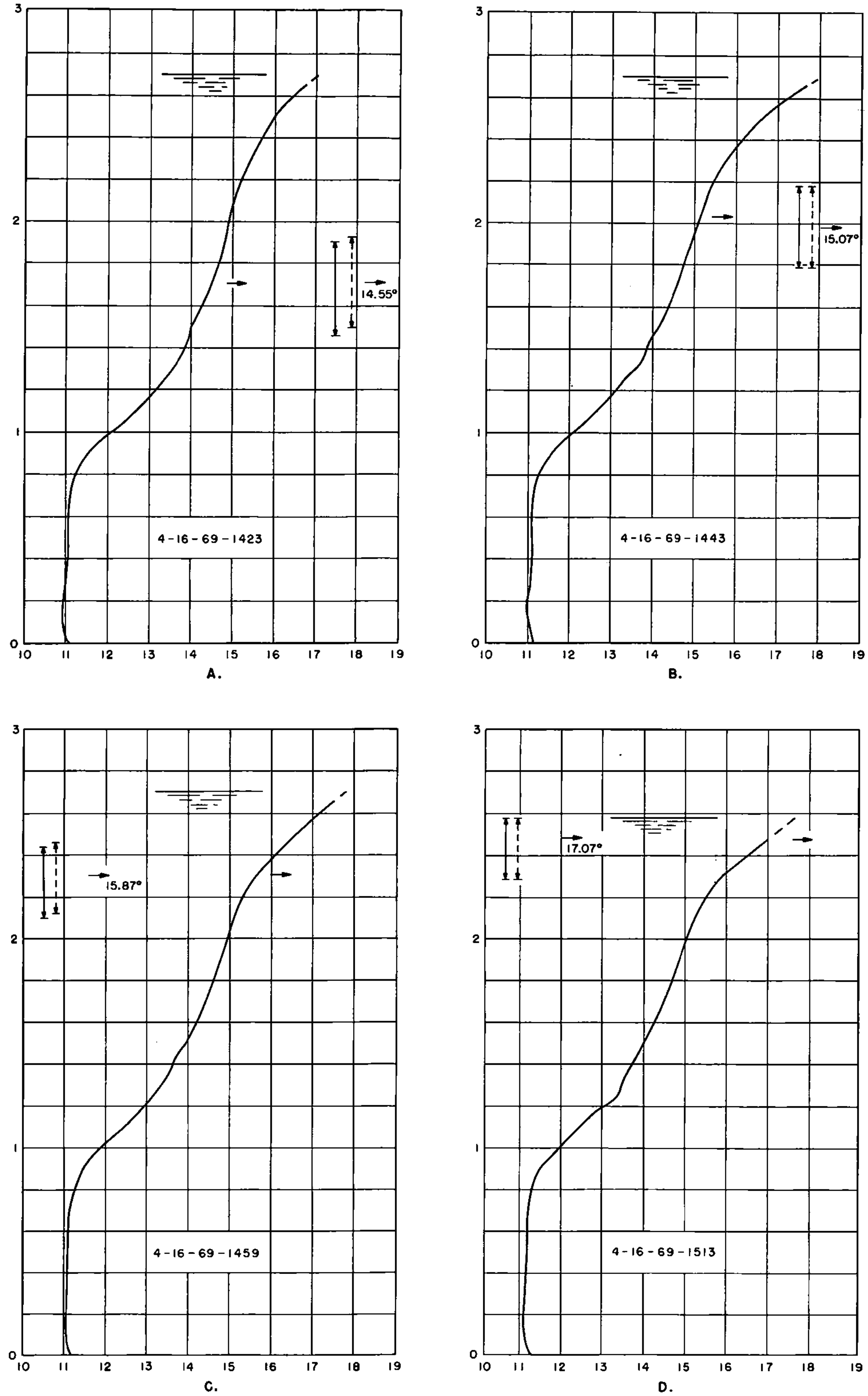

SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

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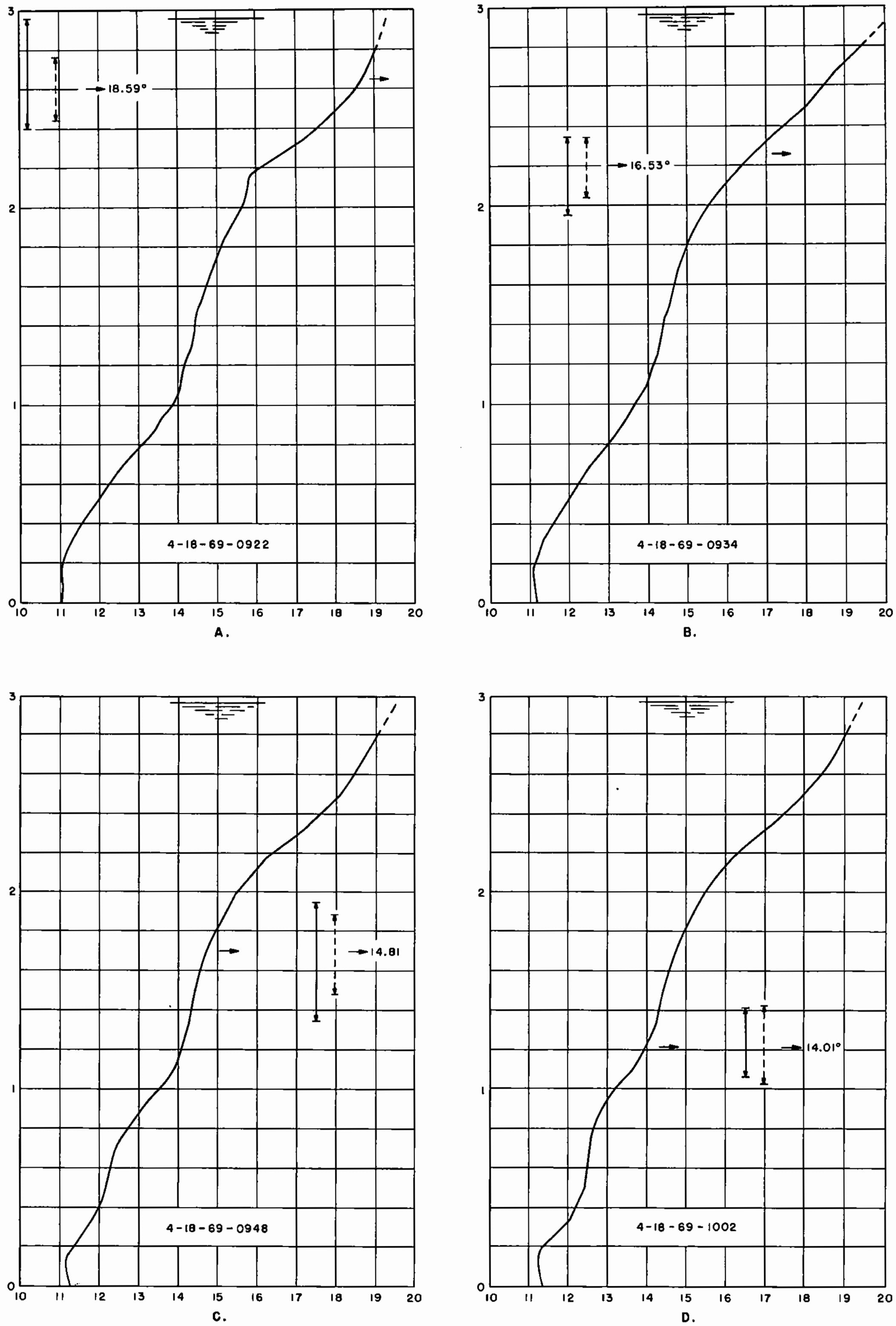
Figure 8
Report HYD-595



SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

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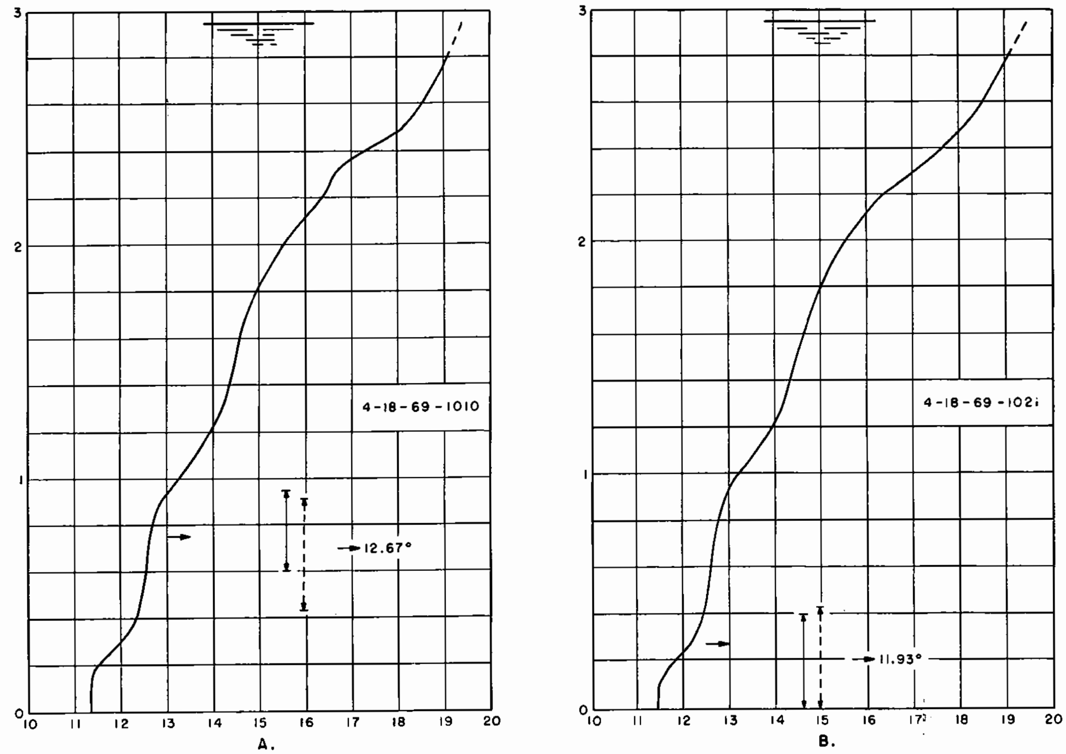

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WITHDRAWAL TESTS

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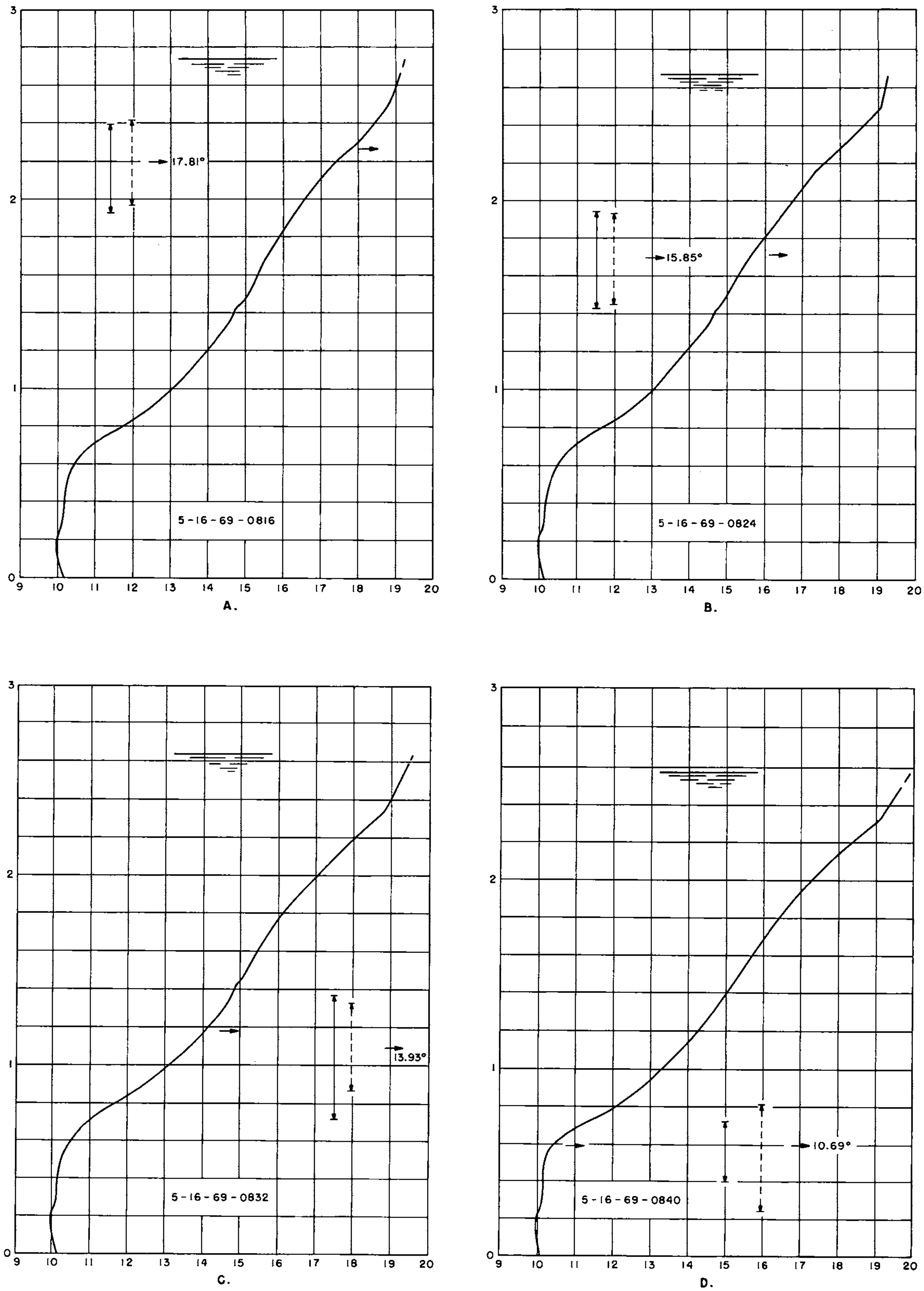
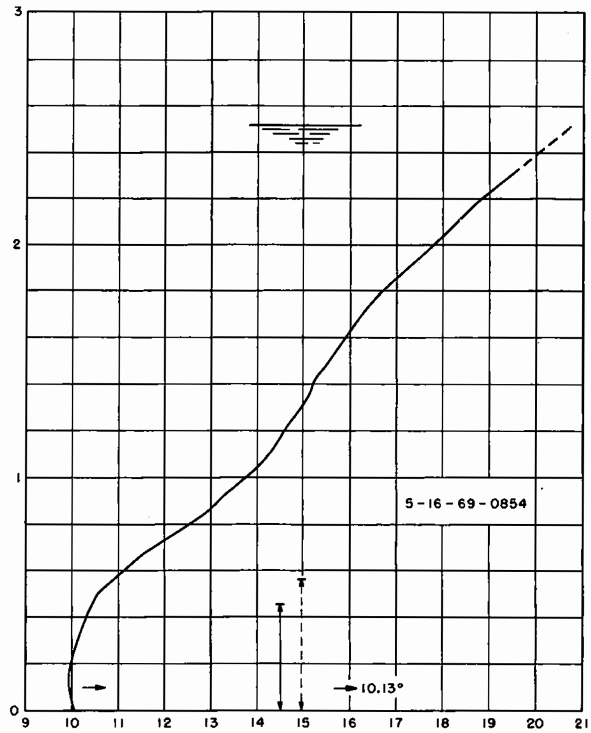


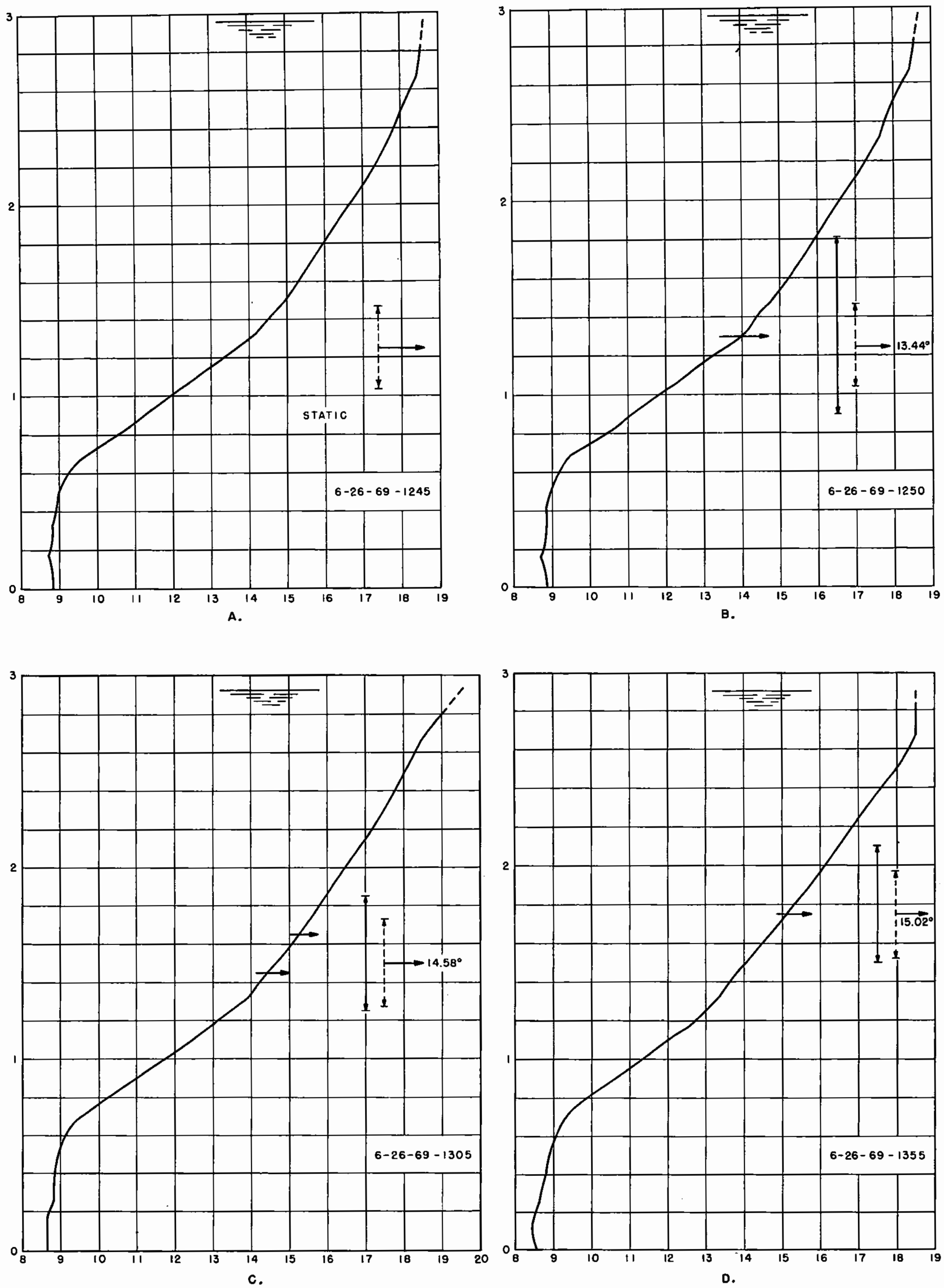
SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

710



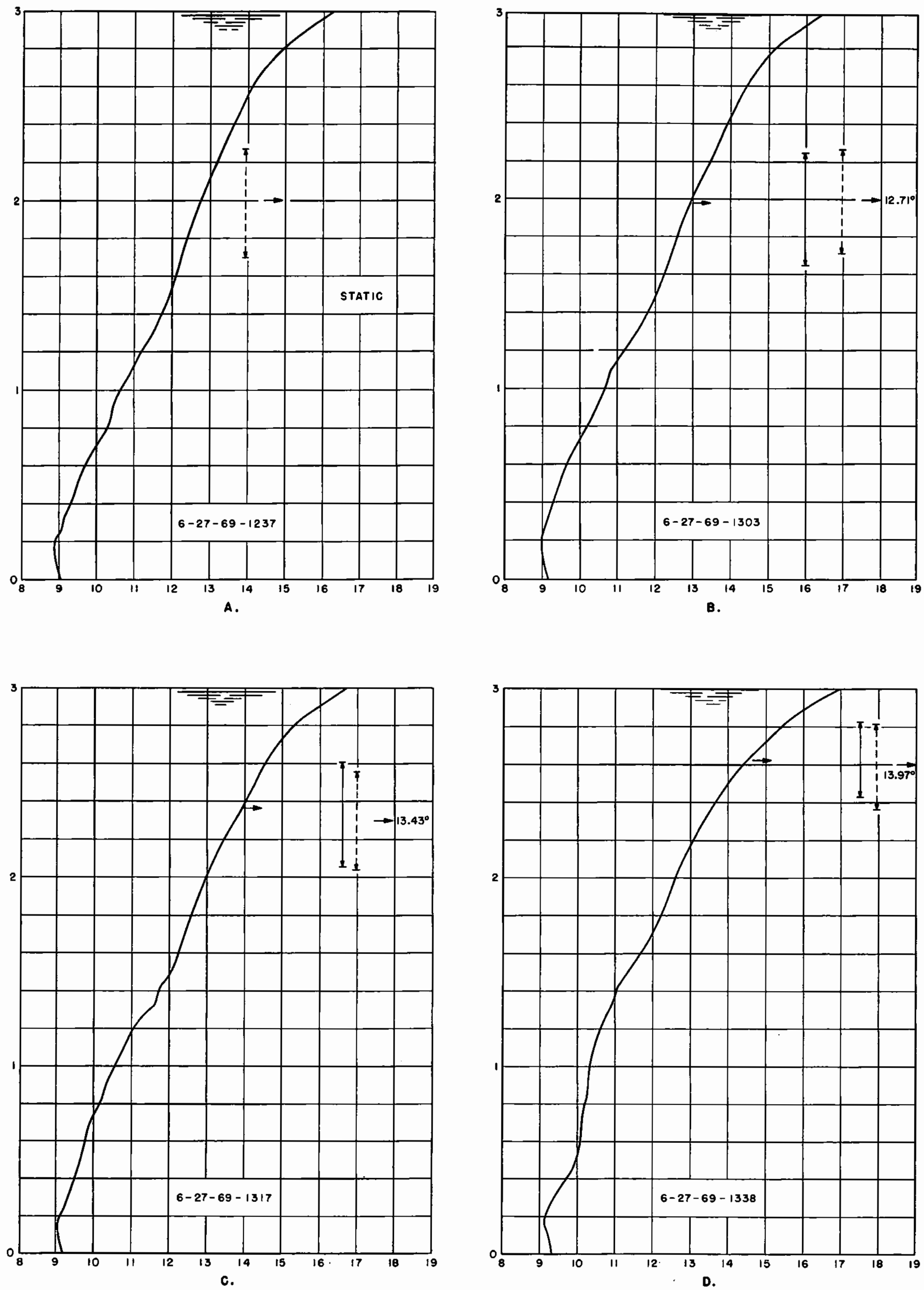
SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

710



SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

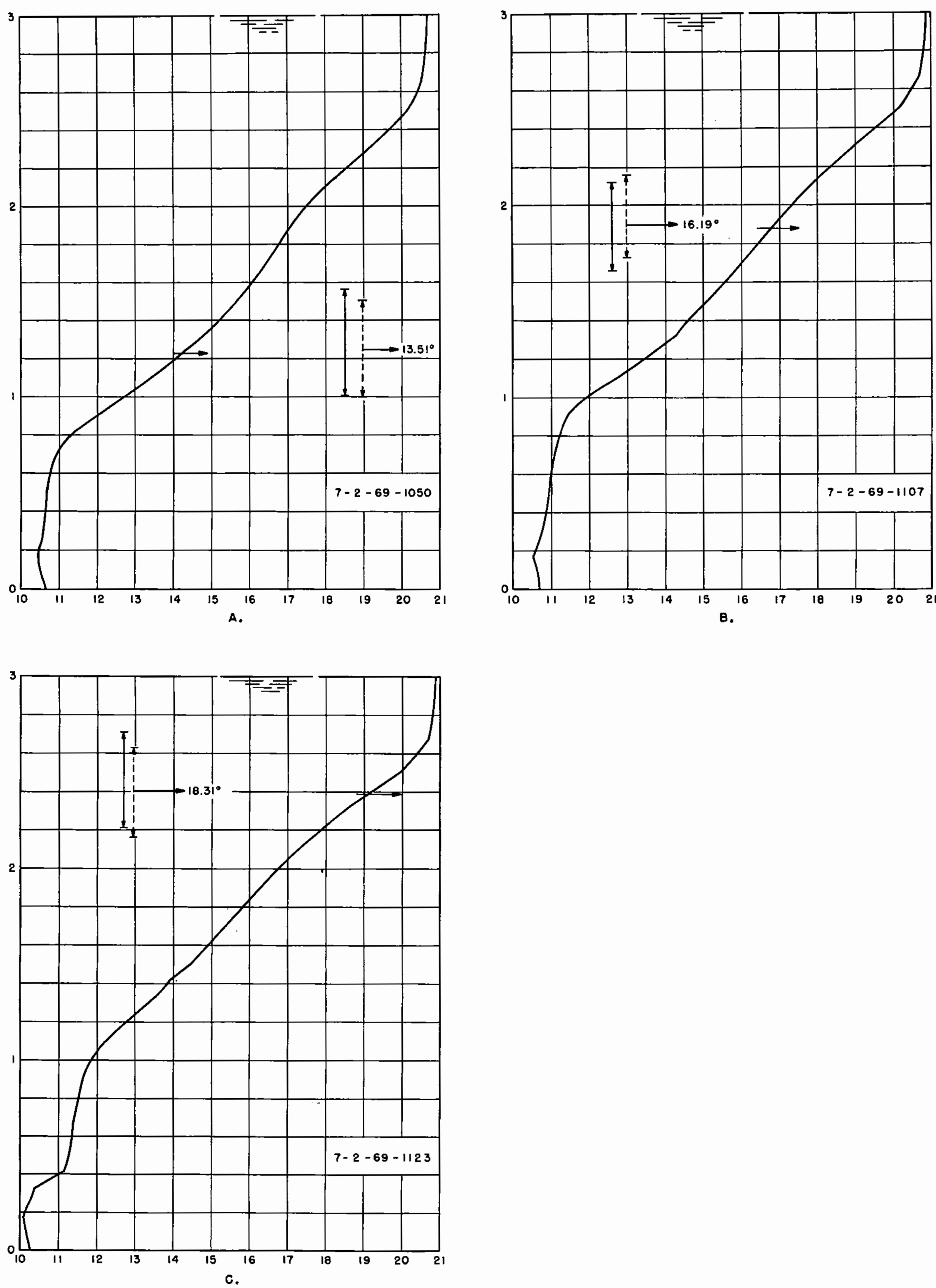
710



SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

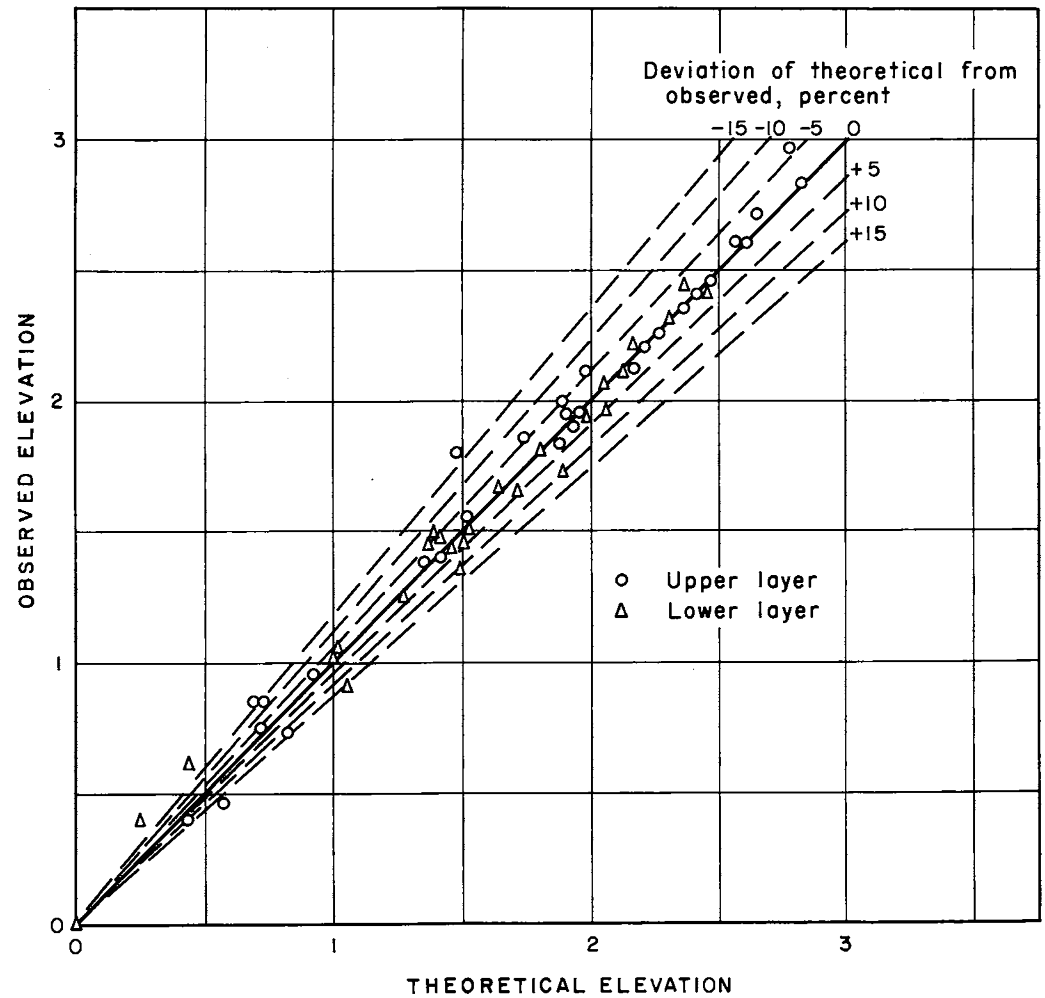
710

Figure 14
Report HYD-595



SELECTIVE WITHDRAWAL FROM RESERVOIRS
WITHDRAWAL TESTS

710



SELECTIVE WITHDRAWAL FROM RESERVOIRS

Comparison of Theoretical and Observed Withdrawal Layers, F' = 0.10

710

BUREAU OF RECLAMATION

LABORATORY PUNCH CARD DATA

USE *BUFF* COLORED CARDS	PROBLEM	JOB NO.
	DETAIL INPUT DATA FOR SAMPLE PROBLEM	#
	FEATURE	RETURN TO
	PROJECT	ROOM BLDG. PHONE

1–8	9–16	17–24	25–32	33–40	41–48	49–56	57–64	65–72	73–80
150.	2.	4820.	4750.	.166	5				
4750.	11.								
4770.	12.								
4780.	13.								
4810.	23.								
4820.	25.								
4810.63	16.								

REMARKS

BY | DATE | CHECKED | SHEET OF

ELEVATION	TEMP	DENSITY
4750.00	11.00	.99963
4770.00	12.00	.99952
4780.00	13.00	.99940
4810.00	23.00	.99757
4820.00	25.00	.99707

CIRCULAR OUTLET

OUTLET SIZE= 2.000 DISCHARGE= 16.00
OUTLET ELEV= 4810.63 UPPER LIMIT= 4812.70 LOWER LIMIT= 4807.60

STRATIFIED FLOW IN RESERVOIRS
Results of Sample Problem

APPENDIX

UNITED STATES
DEPARTMENT OF THE INTERIOR
Bureau of Reclamation

ELECTRONIC COMPUTER PROGRAM ABSTRACT

HEADER CARD

DESCRIPTIVE NAME OF PROGRAM: SELECTIVE WITHDRAWAL LAYER THICKNESS (47)

AUTHOR: (48) KING DL (72) — 73: 0 80: 0

CARD 1

PROGRAM STATUS: (1) P — DATE: (2) 010169 (7) — APPLICATION CODE: (8) 701 (10) — COMPUTER: (11) HØN 800 (26)

LANGUAGE: (27) AUTØMATH 1800 (38) — STORAGE REQUIRED: (39) 1500 (45) — TYPE: (46) W

ORIGINATING ORGANIZATION: (47) RECLAMATIØN-ØCE (62) — DOCUMENTATION: (63) PART (66) — MAILING CODE: (67) D-1190 (72) — 73: 1 80: 1

CARD 2

PROGRAM ID: (1) STRAT (8) — CUSTODIAN'S NAME: (9) D. L. KING (24)

CUSTODIAN'S MAIL CODE (DENVER OFFICE): (25) 293A (28) — ABSTRACT STATUS: (29) (33) — 73: 2 80: 1

PURPOSE — CARDS 3 THRU 7

COMPUTES WITHDRAWAL LAYER THICKNESS IN STRATIFIED RESERVØIR FRØM MØDIFIED FØRM ØF DEBLERS EQUATIØN, FØR SPECIFIED VALUE ØF THE DENSIMETRIC FRØUDE NUMBER. CØMPUTES DENSITY PRØFILE FRØM TEMPERATURE DATA.

(73–80: 3 1, 3 2, 3 3, 3 4, 3 5)

METHODS — CARDS 8 THRU 11

SØLUTIØN ØF AN ALGEBRAIC EQUATIØN USING AN ITERATIVE TECHNIQUE AND TABLE LØØK-UP PRØCEDURE.

(73–80: 4 1, 4 2, 4 3, 4 4)

LIMITATIONS — CARD 12

ZERØ TØ 30 DEGREES C. SQUARE AND CIRCULAR ØUTLET. ØNE ØUTLET ØPERATING.

(73–80: 5 1)

DUPLICATE THE FOLLOWING COLUMNS IN ALL CARDS. FILE NO. (74) H-175 (79)
SEE REVERSE SIDE FOR INSTRUCTIONS FOR FILLING OUT THE ABSTRACT.

NARRATIVE DESCRIPTION OF ELECTRONIC DIGITAL COMPUTER PROGRAM TO DETERMINE SELECTIVE WITHDRAWAL LAYER THICKNESSES

The subject computer program was developed primarily for determining the theoretical withdrawal layer thicknesses for stratified reservoirs. This program was initially developed to aid with model studies, but it is equally applicable to investigation of prototype structures.

The program is written in the Fortran IV language for a Honeywell 800 Computer (see Program Listing). It can be used with either square or round withdrawal outlet shapes, with any outlet elevation, reservoir width, reservoir depth, and outflow discharge. The program as it now exists can be used with water temperatures between 0^{o} and 30^{o} C. Also, no considerations are given to other causes of density variations (such as salinity and turbidity).

The withdrawal layer thicknesses are computed using equation (17) from the text of this report. The term $D^4 \rho o Vo^2/g$ is first evaluated for the particular outlet conditions. The program then searches for the two depths at which $\Delta\rho K^2 d^3 W^2$ is equal to the above term. These two depths are the upper and lower limits of the withdrawal layer.

The computation is carried forward in a series of steps as shown in the accompanying flow chart, beginning with the correction of the temperature readings for the various levels (this can be omitted by removing three statements from the program). With the correct temperatures the densities are then computed. The $D^4 \rho o Vo^2/g$ term is then evaluated for the particular outlet conditions. The program then evaluates the $\Delta\rho K^2 d^3 W^2$ term at each temperature level starting from the highest. These values are then compared to $D^4 \rho o Vo^2/g$ until the point of equality is passed. That interval is then broken into 100 increments and again the $\Delta\rho K^2 d^3 W^2$ terms are computed and compared to $D^4 \rho o Vo^2/g$ until the point of equality is again passed. The position of the upper limit of withdrawal is thus obtained. A similar procedure is then executed to obtain the lower boundary. The program will compensate for cases in which either the water surface or the bottom is located in what would otherwise be the computed withdrawal layer. Also, it will solve cases in which the upper and lower boundaries are both between the same temperature levels.

Printed output consists of:

1. Listing of the elevation above the bottom, the corrected temperature at that level, and the density at that level.
2. Shape of the outlet (square or circular).
3. The outlet size.
4. The outlet elevation.
5. The discharge.
6. The upper limit of withdrawal.
7. The lower limit of withdrawal.

An example of the printed output for analysis of model data is included in this appendix. When prototype data are used, the format for printing the discharge must be modified (for example, from F8.5 to F8.0).

LIST OF SYMBOLS FOR PROGRAM TO COMPUTE SELECTIVE WITHDRAWAL LAYER THICKNESS

C(I)	– Temperature correction factors, C^o
W	– Width of reservoir, ft
D	– Diameter of outlet, ft
WS	– Elevation of water surface, ft
BOT	– Bottom elevation, ft
SHAPE	– Coefficient considering discharge and outlet shape
N	– Number of data points
H(I)	– Elevations of temperature readings, ft
T(I)	– Temperature readings and corrected readings, C^o
R(I)	– Water densities, g/ml
ELØRIF	– Elevation of outlet, ft
Q	– Outflow discharge, cfs
ZUP	– Distance from outlet to upper boundary of withdrawal, ft
ZLØ	– Distance from outlet to lower boundary of withdrawal, ft
GRAD	– Density gradient between temperature levels, g/ml per ft
RØRIF	– Density at level of outflow, g/ml
AØRIF	– Cross sectional area of outlet, ft^2
VØRIF	– Velocity in outlet, fps
FACTOR	– $D^4 \rho o V o^2 / g$ term from Equation (17)
K	– Indicates if ZLØ has been evaluated
DELRU(I)	– Differences in densities between outlet and higher points, g/ml
DELRL(I)	– Differences in densities between outlet and lower points, g/ml
K1	– Indicates if ZUP has been evaluated
M	– N+l for a DØ LØØP limit
PRØD	– $\Delta \rho K^2 d^3 W^2$ term from Equation (17)
RHØ	– Densities at various elevations, g/ml
DELR	– Difference between outlet density and densities at specific points, g/ml
ELUP	– Elevation of upper withdrawal layer boundary, ft
ELLØ	– Elevation of lower withdrawal layer boundary, ft

COMPUTER REQUIRED

The program conforms to USASI specifications for FORTRAN IV and is compatible with most computers using FORTRAN IV compilers. The program as written has been run on a Honeywell H-800 computer.

RUNNING TIME

With the Honeywell H-800, about 16 seconds of central processor time are required for compilation. Execution time depends on the number of temperature profiles examined. A single temperature profile should require only a few seconds for execution. 1417 words of core memory are required.

PREPARATION OF INPUT DATA

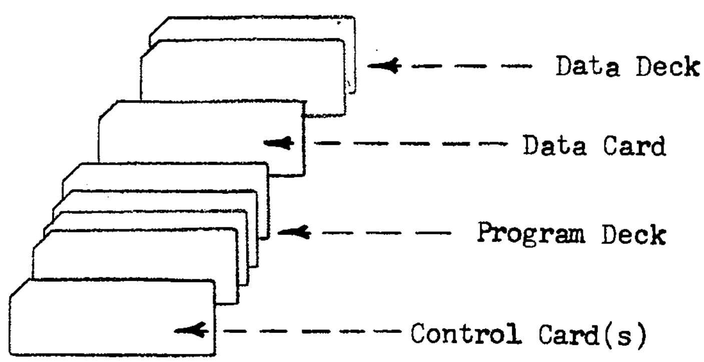
The first 31 data cards contain corresponding values of temperature and density for temperatures from 0 to 30° C, in one degree increments. These data are placed in columns 1-16 in an 8-column format, with the position of the decimal point unspecified (8F.O).

The program as written for analysis of model data specifies 28 cards to follow, which contain correction values for the thermistor readings. This correction is deleted by eliminating statements 0006, 0007, and 0014 from the program (see Program Listing).

The next data card contains the variables of reservoir width, outlet size, water surface elevation, bottom elevation, and outlet shape in columns 1-40, with an 8F.0 format. Columns 41 and 42 contain the value of the number of elevation-temperature cards to follow, in an I2 integer format.

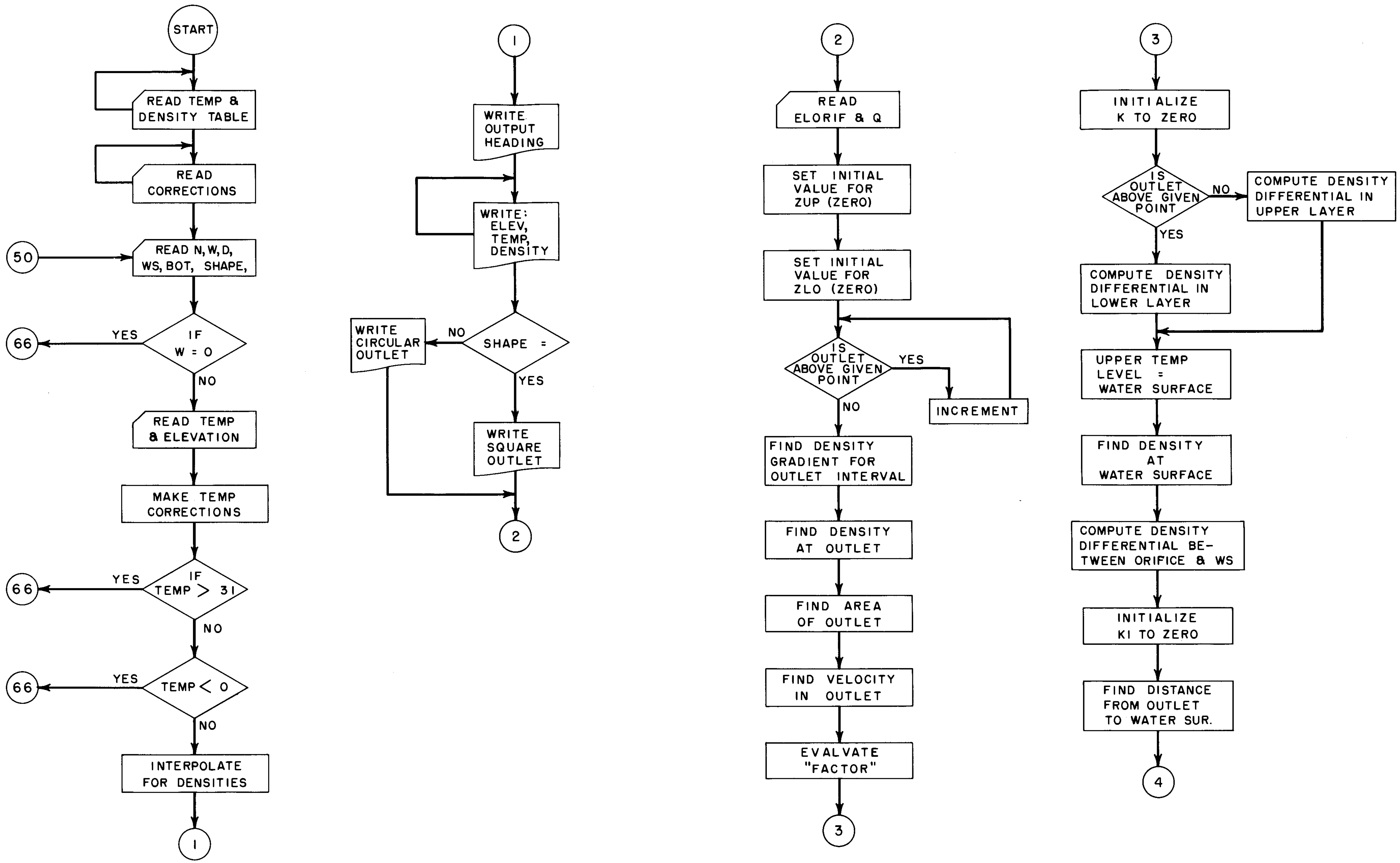
The remaining cards contain corresponding values of elevation and temperature. The last data card should have 0.0 entered in Columns 1-8.

The deck should be stacked according to the diagram below. No subroutines are used in this program.

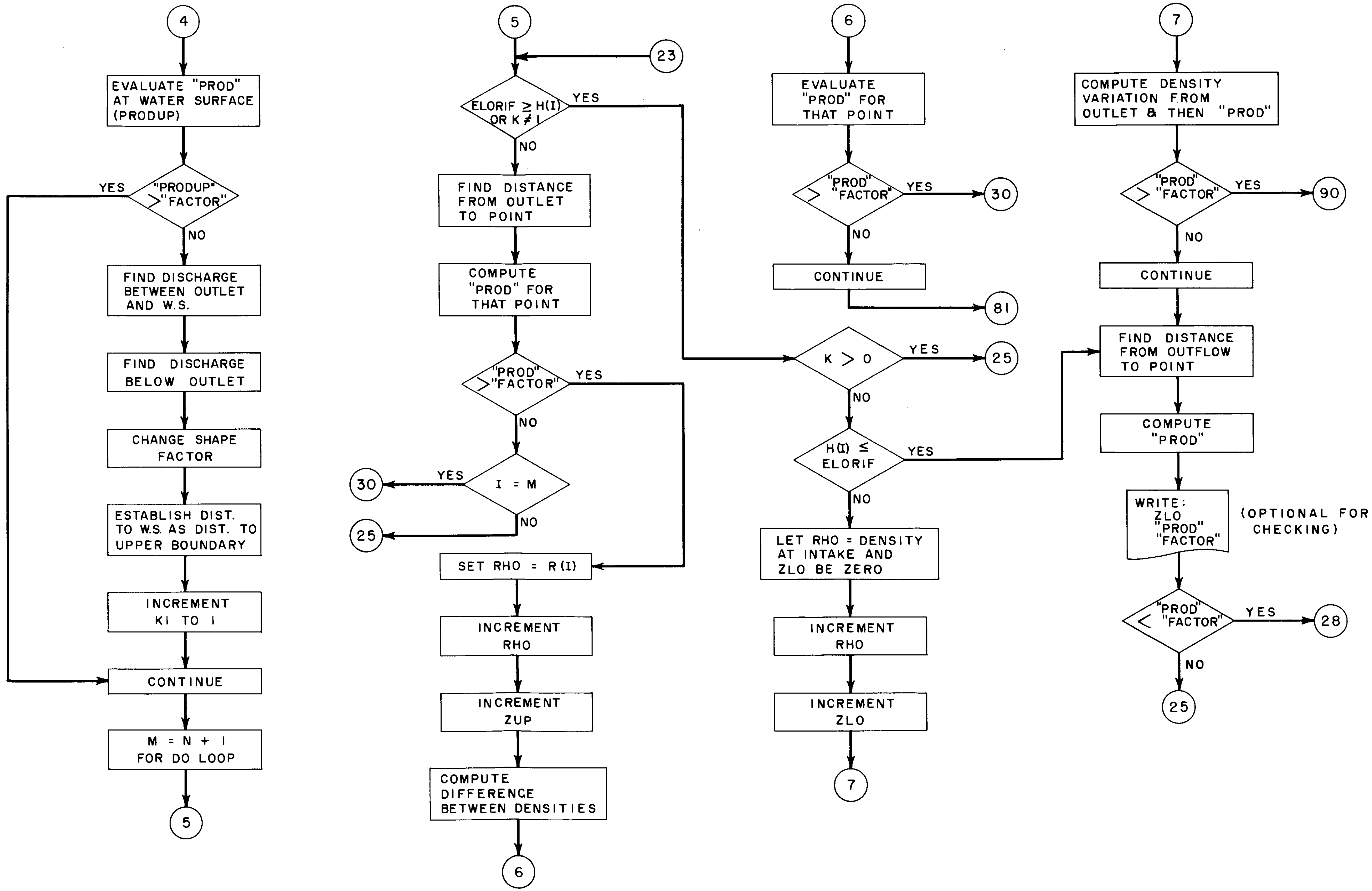


GPO 855-234

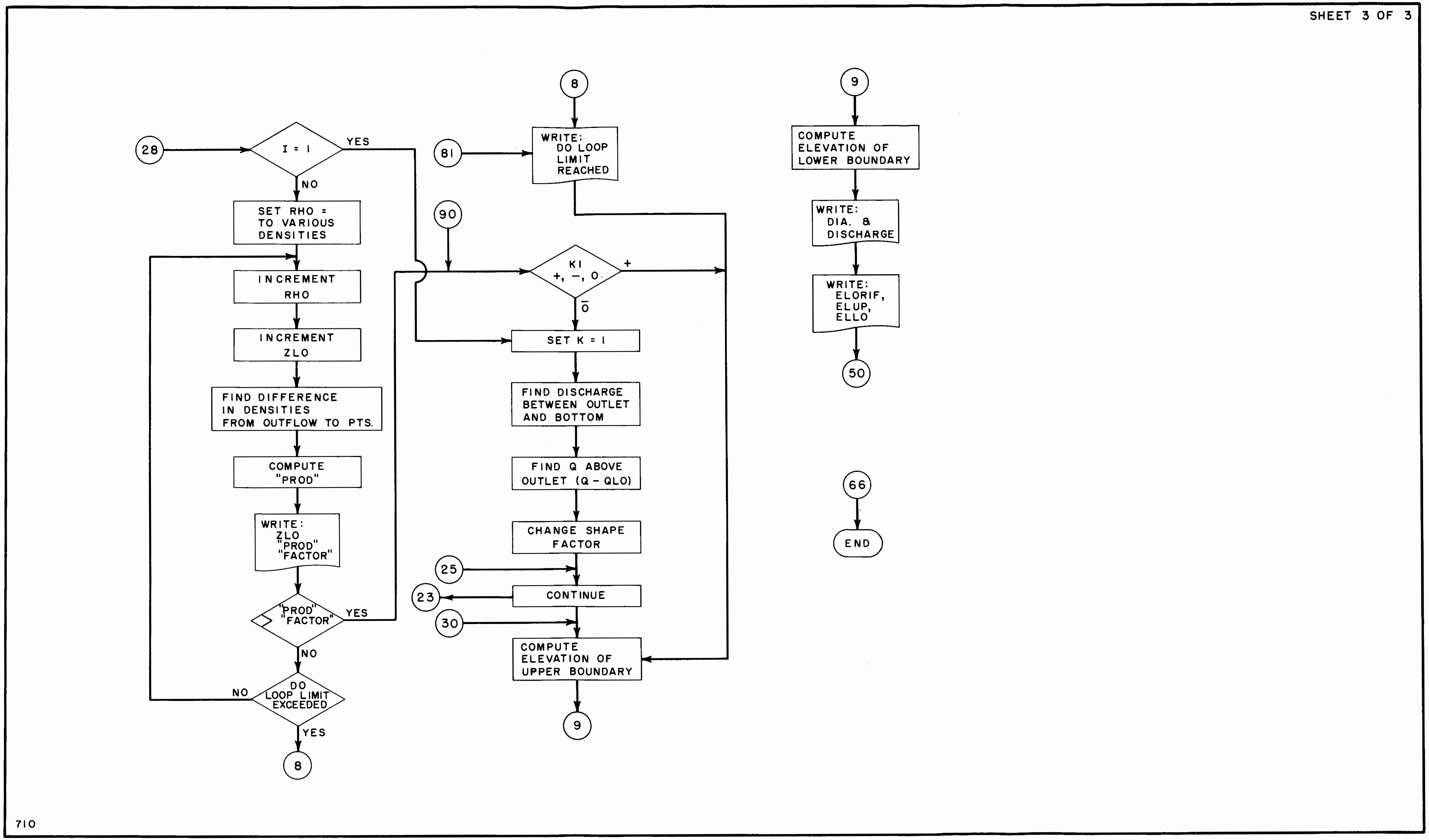
FLOW DIAGRAM


START
READ TEMP & DENSITY TABLE
READ CORRECTIONS
50
READ N, W, D, WS, BOT, SHAPE,
IF W = 0
YES
66
NO
READ TEMP & ELEVATION
MAKE TEMP CORRECTIONS
IF TEMP > 31
YES
66
NO
TEMP < 0
YES
66
NO
INTERPOLATE FOR DENSITIES
1
1
WRITE OUTPUT HEADING
WRITE: ELEV, TEMP, DENSITY
SHAPE =
NO
WRITE CIRCULAR OUTLET
YES
WRITE SQUARE OUTLET
2
2
READ ELORIF & Q
SET INITIAL VALUE FOR ZUP (ZERO)
SET INITIAL VALUE FOR ZLO (ZERO)
IS OUTLET ABOVE GIVEN POINT
YES
INCREMENT
NO
FIND DENSITY GRADIENT FOR OUTLET INTERVAL
FIND DENSITY AT OUTLET
FIND AREA OF OUTLET
FIND VELOCITY IN OUTLET
EVALVATE "FACTOR"
3
3
INITIALIZE K TO ZERO
IS OUTLET ABOVE GIVEN POINT
NO
COMPUTE DENSITY DIFFERENTIAL IN UPPER LAYER
YES
COMPUTE DENSITY DIFFERENTIAL IN LOWER LAYER
UPPER TEMP LEVEL = WATER SURFACE
FIND DENSITY AT WATER SURFACE
COMPUTE DENSITY DIFFERENTIAL BE-TWEEN ORIFICE & WS
INITIALIZE KI TO ZERO
FIND DISTANCE FROM OUTLET TO WATER SUR.
4

710


4
EVALUATE "PROD" AT WATER SURFACE (PRODUP)
"PRODUP" > "FACTOR"
YES
NO
FIND DISCHARGE BETWEEN OUTLET AND W.S.
FIND DISCHARGE BELOW OUTLET
CHANGE SHAPE FACTOR
ESTABLISH DIST. TO W.S. AS DIST. TO UPPER BOUNDARY
INCREMENT KI TO I
CONTINUE
M = N + I FOR DO LOOP
5
5
23
ELORIF ≥ H(I) OR K ≠ I
YES
NO
FIND DISTANCE FROM OUTLET TO POINT
COMPUTE "PROD" FOR THAT POINT
"PROD" > "FACTOR"
YES
NO
I = M
YES
30
NO
25
SET RHO = R(I)
INCREMENT RHO
INCREMENT ZUP
COMPUTE DIFFERENCE BETWEEN DENSITIES
6
6
EVALUATE "PROD" FOR THAT POINT
"PROD" > "FACTOR"
YES
30
NO
CONTINUE
81
K > 0
YES
25
NO
H(I) ≤ ELORIF
YES
NO
LET RHO = DENSITY AT INTAKE AND ZLO BE ZERO
INCREMENT RHO
INCREMENT ZLO
7
7
COMPUTE DENSITY VARIATION FROM OUTLET & THEN "PROD"
"PROD" > "FACTOR"
YES
90
NO
CONTINUE
FIND DISTANCE FROM OUTFLOW TO POINT
COMPUTE "PROD"
WRITE: ZLO "PROD" "FACTOR"
(OPTIONAL FOR CHECKING)
"PROD" < "FACTOR"
YES
28
NO
25

710


28
I = I
YES
NO
SET RHO = TO VARIOUS DENSITIES
INCREMENT RHO
INCREMENT ZLO
FIND DIFFERENCE IN DENSITIES FROM OUTFLOW TO PTS.
COMPUTE "PROD"
WRITE: ZLO "PROD" "FACTOR"
"PROD" "FACTOR"
YES
NO
DO LOOP LIMIT EXCEEDED
NO
YES
8
8
81
WRITE: DO LOOP LIMIT REACHED
90
KI +, −, O.
+
0
SET K = I
FIND DISCHARGE BETWEEN OUTLET AND BOTTOM
FIND Q ABOVE OUTLET (Q − QLO)
CHANGE SHAPE FACTOR
25
23
CONTINUE
30
COMPUTE ELEVATION OF UPPER BOUNDARY
9
9
COMPUTE ELEVATION OF LOWER BOUNDARY
WRITE: DIA. & DISCHARGE
WRITE: ELORIF, ELUP, ELLO
50
66
END
710

PROGRAM LISTING

```
            C     COMPUTATION OF WITHDRAWAL LAYER THICKNESS                        1
0001              ODIMENSION H(50),T(50),R(50),DELRU(50),DELRL(50),C(50),          2
                  1TEMP(50),DENS(50)                                               2A
0002            1 FORMAT (5F8.0,I2)                                                3
0003            2 FORMAT (2F8.0)                                                   4
0004              DO 200 J=1,31                                                    5
0005          200 READ (2,2) TEMP(J),DENS(J)                                       6
0006              DO 9  I=1,28                                                     6A
0007            9 READ (2,2) C(I)                                                  6B
0010           50 READ (2,1) W,D,WS,BOT,SHAPE,N                                    6
0011              IF (W .EQ. 0.0) GO TO 66                                         6AA
0012            7 DO 10 I=1,N                                                      7
0013              READ (2,2) H(I),T(I)                                             7A
0014              T(I)=T(I)+C(I)                                                   7AA
0015              IF (T(I).GT.31.) GO TO 66                                        7BB
0016              IF (T(I).LT.0.) GO TO 66                                         7CC
0017              DO 210 J=2,31                                                    7B
0020              TEM1=TEMP(J)                                                     7C
0021              TEM2=TEMP(J-1)                                                   7D
0022              DEN1=DENS(J-1)                                                   7E
0023              DEN2=DENS(J)                                                     7EE
0024              IF (T(I).GT.TEM1) GO TO 210                                      7FF
0025              IF (T(I).LT.4.) GO TO 215                                        7GG
0026              R(I)=DEN1-(T(I)-TEM2)*(DEN1-DEN2)                                7HH
0027              GO TO 10                                                         7G
0030          215 R(I)=DEN1+(T(I)-TEM2)*(DEN2-DEN1)                                7II
0031              GO TO 10                                                         7I
0032          210 CONTINUE                                                         7J
0033           10 CONTINUE                                                         8
0034              WRITE (3,52)                                                     8A
0035           52 FORMAT (1H1,1X,24H ELEVATION TEMP  DENSITY)                      8B
0036              DO 54 I=1,N                                                      8C
0037           54 WRITE (3,55) H(I),T(I),R(I)                                      8D
0040           55 FORMAT (F8.2,F8.2,F8.5)                                          8E
0041              IF (SHAPE .EQ. 0.040) GO TO 75                                   8F
0042              WRITE (3,70)                                                     8G
0043           70 FORMAT (1H0,16H CIRCULAR OUTLET)                                 8H
0044              GO TO 11                                                         8I
0045           75 WRITE (3,76)                                                     8J
0046           76 FORMAT (1H0,14H SQUARE OUTLET)                                   8K
0047           11 READ (2,2) ELORIF,Q                                              9
0050              ZUP=0.0                                                          10A
0051              ZLO=0.0                                                          10B
0052              DO 15 I=1,N                                                      11
0053              IF (ELORIF .GT. H(I)) GO TO 15                                   12
0054              GRAD=(R(I-1)-R(I))/(H(I)-H(I-1))                                 12A
0055              RORIF=R(I-1)-GRAD*(ELORIF-H(I-1))                                13
0056              AORIF=3.1416*D*D/4.                                              14
0057              VORIF=Q/AORIF                                                    15
0060              FACTOR=D**4*RORIF*VORIF**2/32.174                                16
0061              GO TO 16                                                         17
0062           15 CONTINUE                                                         18
            C     LIMITS OF WITHDRAWAL                                             19
0063           16 K=0                                                              20
0064              DO 20 I=1,N                                                      21
0065              IF (ELORIF .GT. H(I)) GO TO 21                                   22
```

```
      DELRU(I)=RORIF-R(I)                                              23
      GO TO 20                                                         24
   21 DELRL(I)=R(I)-RORIF                                              25
   20 CONTINUE                                                         26
      H(N+1)=WS
      R(N+1)=R(N)+(R(N)-R(N-1))*(WS-H(N))/(H(N)-H(N-1))
      DELRU(N+1)=RORIF-R(N+1)
      K1=0                                                             26A
      ZUPP=WS-ELORIF                                                  26B
      PRODUP=DELRU(N+1)*SHAPE*ZUPP**3*W**2                               2
      IF(PRODUP.GT.FACTOR)GOTO22                                       26D
      QUP=(Q*PRODUP/FACTOR)/2.0                                        26E
      QLO=Q-QUP                                                        26F
      SHAPE=SHAPE*(Q/(2.0*QLO))**2                                     26G
      ZUP=ZUPP                                                         26H
      K1=1                                                             26I
   22 CONTINUE                                                         26J
      M=N+1
      DO 25 I=1,M
      IF(ELORIF.GE.H(I).OR.K.NE.1)GOTO24                                28
      ZUP=H(I)-ELORIF                                                   29
      PROD=DELRU(I)*SHAPE*ZUP**3*W**2                                   30
      IF (PROD .GT. FACTOR) GO TO 26                                    31
      IF(I.EQ.M)GOTO30
      GO TO 25                                                          32
   26 RHO=R(I)                                                         32A
      DO 40 J=1,100                                                    32B
      RHO=RHO+ (R(I-1)-R(I))*0.01                                       33
      ZUP=ZUP-0.01*(H(I)-H(I-1))                                        34
      DELR=RORIF-RHO                                                    35
      PROD=DELR*SHAPE*ZUP**3*W**2                                       36
      IF (PROD .LT. FACTOR) GO TO 30                                    37
   40 CONTINUE                                                         37A
      IF (PROD .LT. FACTOR) GO TO 81                                   37B
      RHO=R(I-1)                                                       37C
      DO 41 J=1,100                                                    37D
      RHO=RHO+(R(I-2)-R(I-1))*0.01                                     37E
      ZUP=ZUP-0.01*(H(I-1)-H(I-2))                                     37F
      DELR=RORIF-RHO                                                   37G
      PROD=DELR*SHAPE*ZUP**3*W**2                                      37H
      IF (PROD .LT. FACTOR) GO TO 30                                   37I
   41 CONTINUE                                                         37J
   24 IF (K .GT. 0) GO TO 25                                            38
      IF(H(I).LE.ELORIF)GOTO27
      RHO=RORIF
      ZLO=0.
      DO95 J=1,100
      RHO=RHO+(R(I-1)-RORIF)*0.01
      ZLO=ZLO+0.01*(ELORIF-H(I-1))
      DELR=RHO-RORIF
      PROD=DELR*SHAPE*ZLO**3*W**2
      IF(PROD.GT.FACTOR)GOTO90
   95 CONTINUE
   27 ZLO=ELORIF-H(I)
      PROD=DELRL(I)*SHAPE*ZLO**3*W**2                                   40
      IF (PROD .LT. FACTOR) GO TO 28                                    41
```

```
0156           GO TO 25                                                               42
0157        28 IF (I .EQ. 1) GO TO 29                                                 42A
0160           RHO=R(I)                                                               42B
0161           DO 80 J=1,100                                                          42C
0162           RHO=RHO+ (R(I-1)-R(I))*0.01                                            43
0163           ZLO=ZLO+0.01*(H(I)-H(I-1))                                             44
0164           DELR=RHO-RORIF                                                         45
0165           PROD=DELR*SHAPE*ZLO**3*W**2                                            46
0166           IF(PROD  .GT.FACTOR)GOTO90                                             47
0167        80 CONTINUE                                                               48
0170        81 WRITE (3,82)                                                           48A
0171           GO TO 30                                                               48B
0172        82 FORMAT (22H DO LOOP LIMIT REACHED)                                     48C
0173        90 IF(K1)29,29,30                                                         48D
0174        29 K=1                                                                    49
0175           QLO=(Q*PROD/FACTOR)/2.0
0176           QUP=Q-QLO
0177           SHAPE=SHAPE*(Q/(2.0*QUP))**2                                           49C
0200        25 CONTINUE                                                               50
0201        30 ELUP=ELORIF+ZUP                                                        51
0202           ELLO=ELORIF-ZLO                                                        52
0203           WRITE (3,61) D,Q                                                       82A
0204        61 FORMAT (1H0,13H OUTLET SIZE=,F8.3,11H DISCHARGE=,F8.5)                 82B
0205           WRITE (3,60) ELORIF,ELUP,ELLO                                          83
0206        600FORMAT (13H OUTLET ELEV=,F8.2,13H UPPER LIMIT=,F8.2,13H LOWER LIMI     84
              1T=,F8.2)                                                               85
0207           GO TO 50                                                               87
0210        66 END                                                                    88
```

DATA CARD CONFIGURATION

BUREAU OF RECLAMATION

LABORATORY PUNCH CARD DATA

USE .... COLORED CARDS	PROBLEM	JOB NO.
	DETAIL DATA CARD CONFIGURATION	#
	FEATURE	RETURN TO
	PROJECT	ROOM BLDG. PHONE

Columns 1–8	Columns 9–16	Columns 17–24	Columns 25–32	Columns 33–40	Columns 41–48	Description
←TEMP→	←DENS→					FIRST 31 CARDS, TEMPERATURE AND DENSITY TABLE
←C→						NEXT 28 CARDS, THERMISTOR CORRECTIONS (OPTIONAL)
←W→	←D→	←WS→	←BOT→	←SHAPE→	N	RESERVOIR AND OUTLET GEOMETRY. N= NUMBER OF DATA POINTS.
←H→	←T→					ELEVATIONS AND CORRESPONDING TEMPERATURES, N CARDS
←ELORIF→	←Q→					OUTLET ELEVATION AND DISCHARGE

REMARKS

BY DATE CHECKED SHEET OF

GPO 833969

AMPLE PROGRAM OUTPUT

ELEVATION	TEMP	DENSITY
.00	15.25	.99909
.08	15.21	.99909
.17	15.21	.99909
.25	15.21	.99909
.33	15.21	.99909
.42	15.21	.99909
.50	15.21	.99909
.58	15.21	.99909
.67	15.21	.99909
.75	15.21	.99909
.83	15.21	.99909
.92	15.21	.99909
1.00	15.21	.99909
1.08	15.23	.99909
1.17	15.21	.99909
1.25	15.23	.99909
1.33	15.23	.99909
1.42	15.23	.99909
1.50	15.27	.99908
1.67	15.45	.99906
1.83	16.13	.99895
2.00	16.71	.99885
2.17	17.13	.99878
2.33	17.43	.99873
2.50	17.61	.99869
2.67	17.75	.99867
2.83	17.83	.99865
3.00	17.69	.99868

CIRCULAR OUTLET

OUTLET SIZE= .052 DISCHARGE= .00345
OUTLET ELEV= 1.31 UPPER LIMIT= 2.01 LOWER LIMIT= .00

7-1750
(2-67)
Bureau of Reclamation

CONVERSION FACTORS--BRITISH TO METRIC UNITS OF MEASUREMENT

The following conversion factors adopted by the Bureau of Reclamation are those published by the American Society for Testing and Materials (ASTM Metric Practice Guide, January 1964) except that additional factors (*) commonly used in the Bureau have been added. Further discussion of definitions of quantities and units is given on pages 10-11 of the ASTM Metric Practice Guide.

The metric units and conversion factors adopted by the ASTM are based on the "International System of Units" (designated SI for Systeme International d'Unites), fixed by the International Committee for Weights and Measures; this system is also known as the Giorgi or MKSA (meter-kilogram (mass)-second-ampere) system. This system has been adopted by the International Organization for Standardization in ISO Recommendation R-31.

The metric technical unit of force is the kilogram-force; this is the force which, when applied to a body having a mass of 1 kg, gives it an acceleration of 9.80665 m/sec/sec, the standard acceleration of free fall toward the earth's center for sea level at 45 deg latitude. The metric unit of force in SI units is the newton (N), which is defined as that force which, when applied to a body having a mass of 1 kg, gives it an acceleration of 1 m/sec/sec. These units must be distinguished from the (inconstant) local weight of a body having a mass of 1 kg; that is, the weight of a body is that force with which a body is attracted to the earth and is equal to the mass of a body multiplied by the acceleration due to gravity. However, because it is general practice to use "pound" rather than the technically correct term "pound-force," the term "kilogram" (or derived mass unit) has been used in this guide instead of "kilogram-force" in expressing the conversion factors for forces. The newton unit of force will find increasing use, and is essential in SI units.

Table I

QUANTITIES AND UNITS OF SPACE

Multiply	By	To obtain
	LENGTH	
Mil	25.4 (exactly)	Micron
Inches	25.4 (exactly)	Millimeters
	2.54 (exactly)*	Centimeters
Feet	30.48 (exactly)	Centimeters
	0.3048 (exactly)*	Meters
	0.0003048 (exactly)*	Kilometers
Yards	0.9144 (exactly)	Meters
Miles (statute)	1,609.344 (exactly)*	Meters
	1.609344 (exactly)	Kilometers
	AREA	
Square inches	6.4516 (exactly)	Square centimeters
Square feet	929.03*	Square centimeters
	0.092903	Square meters
Square yards	0.836127	Square meters
Acres	0.40469*	Hectares
	4,046.9*	Square meters
	0.0040469*	Square kilometers
Square miles	2.58999	Square kilometers
	VOLUME	
Cubic inches	16.3871	Cubic centimeters
Cubic feet	0.0283168	Cubic meters
Cubic yards	0.764555	Cubic meters
	CAPACITY	
Fluid ounces (U.S.)	29.5737	Cubic centimeters
	29.5729	Milliliters
Liquid pints (U.S.)	0.473179	Cubic decimeters
	0.473166	Liters
Quarts (U.S.)	946.358*	Cubic centimeters
	0.946331*	Liters
Gallons (U.S.)	3,785.43*	Cubic centimeters
	3.78543	Cubic decimeters
	3.78533	Liters
	0.00378543*	Cubic meters
Gallons (U.K.)	4.54609	Cubic decimeters
	4.54596	Liters
Cubic feet	28.3160	Liters
Cubic yards	764.55*	Liters
Acre-feet	1,233.5*	Cubic meters
	1,233,500*	Liters

Table II

QUANTITIES AND UNITS OF MECHANICS

Multiply	By	To obtain
	MASS	
Grains (1/7,000 lb)	64.79891 (exactly)	Milligrams
Troy ounces (480 grains)	31.1035	Grams
Ounces (avdp)	28.3495	Grams
Pounds (avdp)	0.45359237 (exactly)	Kilograms
Short tons (2,000 lb)	907.185	Kilograms
	0.907185	Metric tons
Long tons (2,240 lb)	1,016.05	Kilograms
	FORCE/AREA	
Pounds per square inch	0.070307	Kilograms per square centimeter
	0.689476	Newtons per square centimeter
Pounds per square foot	4.88243	Kilograms per square meter
	47.8803	Newtons per square meter
	MASS/VOLUME (DENSITY)	
Ounces per cubic inch	1.72999	Grams per cubic centimeter
Pounds per cubic foot	16.0185	Kilograms per cubic meter
	0.0160185	Grams per cubic centimeter
Tons (long) per cubic yard	1.32894	Grams per cubic centimeter
	MASS/CAPACITY	
Ounces per gallon (U.S.)	7.4893	Grams per liter
Ounces per gallon (U.K.)	6.2362	Grams per liter
Pounds per gallon (U.S.)	119.829	Grams per liter
Pounds per gallon (U.K.)	99.779	Grams per liter
	BENDING MOMENT OR TORQUE	
Inch-pounds	0.011521	Meter-kilograms
	1.12985×10^6	Centimeter-dynes
Foot-pounds	0.138255	Meter-kilograms
	1.35582×10^7	Centimeter-dynes
Foot-pounds per inch	5.4431	Centimeter-kilograms per centimeter
Ounce-inches	72.008	Gram-centimeters
	VELOCITY	
Feet per second	30.48 (exactly)	Centimeters per second
	0.3048 (exactly)*	Meters per second
Feet per year	0.965873×10^{-6}*	Centimeters per second
Miles per hour	1.609344 (exactly)	Kilometers per hour
	0.44704 (exactly)	Meters per second
	ACCELERATION*	
Feet per second^2	0.3048*	Meters per second^2
	FLOW	
Cubic feet per second (second-feet)	0.028317*	Cubic meters per second
Cubic feet per minute	0.4719	Liters per second
Gallons (U.S.) per minute	0.06309	Liters per second
	FORCE*	
Pounds	0.453592*	Kilograms
	4.4482*	Newtons
	4.4482×10^{-5}*	Dynes
	WORK AND ENERGY*	
British thermal units (Btu)	0.252*	Kilogram calories
	1,055.06	Joules
Btu per pound	2.326 (exactly)	Joules per gram
Foot-pounds	1.35582*	Joules
	POWER	
Horsepower	745.700	Watts
Btu per hour	0.293071	Watts
Foot-pounds per second	1.35582	Watts
	HEAT TRANSFER	
Btu in./hr ft^2 deg F (k, thermal conductivity)	1.442	Milliwatts/cm deg C
	0.1240	Kg cal/hr m deg C
Btu ft/hr ft^2 deg F	1.4880*	Kg cal m/hr m^2 deg C
Btu/hr ft^2 deg F (C, thermal conductance)	0.568	Milliwatts/cm^2 deg C
	4.882	Kg cal/hr m^2 deg C
Deg F hr ft^2/Btu (R, thermal resistance)	1.761	Deg C cm^2/milliwatt
Btu/lb deg F (c, heat capacity)	4.1868	J/g deg C
Btu/lb deg F	1.000*	Cal/gram deg C
Ft^2/hr (thermal diffusivity)	0.2581	Cm^2/sec
	0.09290*	M^2/hr
	WATER VAPOR TRANSMISSION	
Grains/hr ft^2 (water vapor transmission)	16.7	Grams/24 hr m^2
Perms (permeance)	0.659	Metric perms
Perm-inches (permeability)	1.67	Metric perm-centimeters

Table III

OTHER QUANTITIES AND UNITS

Multiply	By	To obtain
Cubic feet per square foot per day (seepage)	304.8*	Liters per square meter per day
Pound-seconds per square foot (viscosity)	4.8824*	Kilogram second per square meter
Square feet per second (viscosity)	0.092903*	Square meters per second
Fahrenheit degrees (change)*	5/9 exactly	Celsius or Kelvin degrees (change)*
Volts per mil	0.03937	Kilovolts per millimeter
Lumens per square foot (foot-candles)	10.764	Lumens per square meter
Ohm-circular mils per foot	0.001662	Ohm-square millimeters per meter
Millicuries per cubic foot	35.3147*	Millicuries per cubic meter
Milliamps per square foot	10.7639*	Milliamps per square meter
Gallons per square yard	4.527219*	Liters per square meter
Pounds per inch	0.17858*	Kilograms per centimeter

GPO 835-159

ABSTRACT

Selective outlet works for reservoirs provide an important control for maintaining water quality in the downstream river. The use of multilevel outlets is growing; however, design guidelines are seriously lacking. A study of previous theoretical and experimental work in selective withdrawal from reservoirs shows that a basis exists for development of practical design and operating criteria. Beginning with the theory for selective withdrawal from a continuously stratified fluid, a method is developed for the more general density profile. The method is verified with experimental data. Limitations of this method and the need for further experimental verification with model and prototype data are discussed. Has 24 references.

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HYD-595
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DESCRIPTORS–/ research and development/ reservoirs/ reviews/ *hydraulic models/ density currents/ hydraulics/ water quality/ temperature/ Froude number/ *stratification/ velocity distribution/ *thermal stratification/ theory/ bibliographies/ design criteria/ *stratified flow
IDENTIFIERS–/ *multilevel outlets/ *selective level releases/ progress reports

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