UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

- - -

# HYDRAULIC LABORATORY REPORT NO. 5

- - -

# CIRCULAR MEASURING WEIRS

By

FRANCESCO RAMPONI

TRANSLATED FROM
THE ITALIAN

By

F. L. PANUZIO

- - -

Denver, Colorado
September, 1936

HYD-5

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MEMORANDUM TO THE CHIEF DESIGNING ENGINEER

Translated from Italian

SUBJECT: "CIRCULAR MEASURING WEIRS"

By

Dr. Eng. Francesco Ramponi

(Assistant at the Istituto di Idraulica della Universita di Padova)

- - -

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# CIRCULAR MEASURING WEIRS

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- - -

1. In hydraulic laboratories and practice, the most commonly used type of weirs are: the rectangular weir with and without end contractions (Bazin), the $90^{\circ}$ triangular weir with the vertex at the bottom (Thomson), the trapezoidal weir (Cipolletti), and the circular weir of various diameters.

Other types of weirs have been proposed by other authors, as: the proportional weir (Rother) whose theoretical discharge is proportional to the head on the crest; the parabolic weir; and weirs whose sections are determined from a combination of the circular and the rectangular, of the rectangular and the triangular weir, etc. This last type has for obvious reasons very few applications.

Although the literature on rectangular and triangular weirs is large, the studies that consider the circular weirs are few. The use of the circular weir for flow measurements is comparatively recent. Until a few years ago the theoretical discharge equation was ill defined and was presented in a form both complicated and inconvenient for use.

In this article there is derived, for the circular weir, another equation which is more expedient for application and which gives approximate results that are acceptable in practice.

With the aid of the so-called "curve of errors for discharges" it will be shown that the circular weir, compared to the triangular and rectangular weirs, lends itself equally well to the measurement of large or small discharges. Other advantages of the circular weir over the other types of weirs as to construction and installation will be pointed out.

2. The theoretical discharge equation for the circular weir is found in an article of A. Staus and K. V. Sanden (A. Staus and K. V. Sanden: Der kreisrunde Ueberfall und Seine Abarten. Das Gas und Wasserfach, 1926; Helft 27-30).

The formula proposed by them, for a coefficient of discharge "m" equal to unity, is:

$$Q = q_i \, d^{5/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where "Q" is the discharge in liters per second and "d" is the diameter expressed in decimeters (dm.). The said $q_i$ is determined from the expression:

$$q_i = \frac{4}{15}\sqrt{2g} \cdot \left[2\left(1 - \frac{h}{d} + \frac{h^2}{d^2}\right) E - \left(2 - 3\frac{h}{d} + \frac{h^2}{d^2}\right) K\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where "h" is the head of water on the crest (lowest point) in feet and E and K represent two elliptical integrals of the first and second order. The lengths are all expressed in dm.

(For conversion from the metric units to the English units, the following table is supplied:

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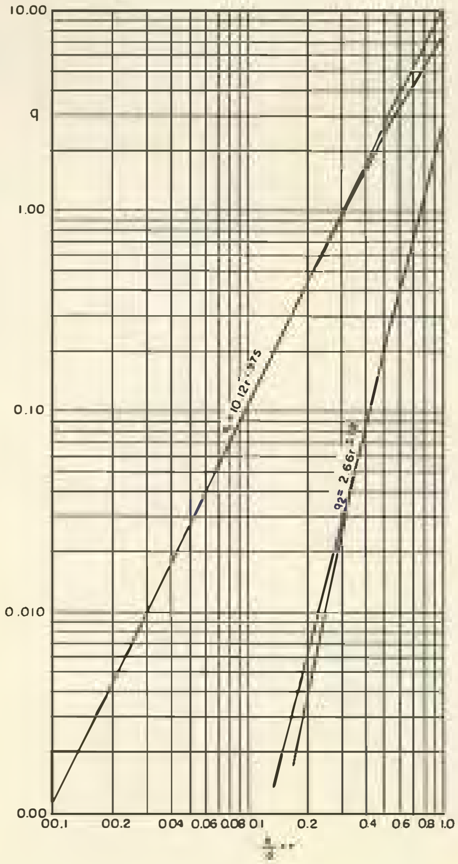


FIG. I

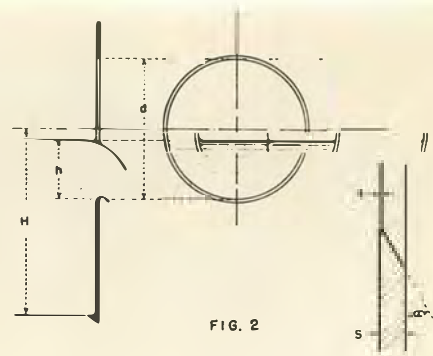


FIG. 2

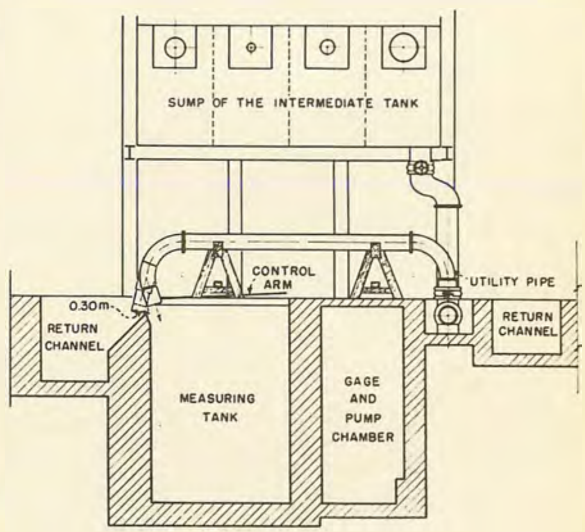


FIG. 4

0 0.50 1.00 1.50m

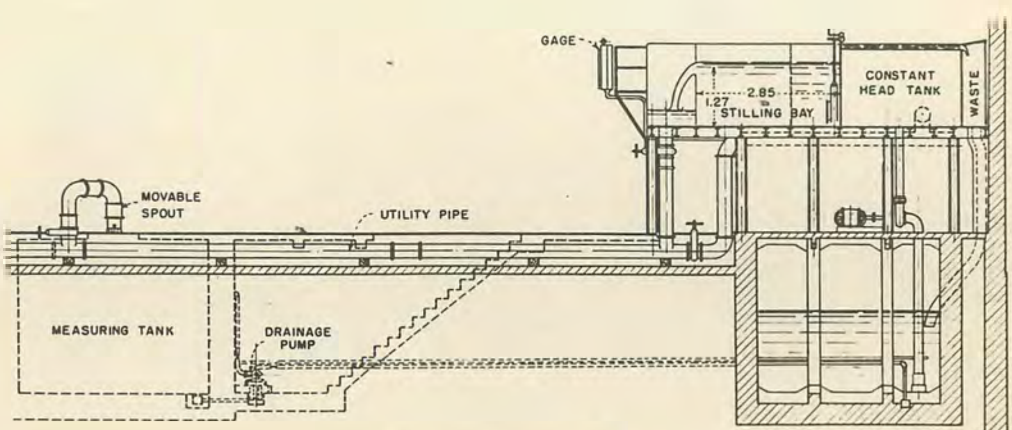


FIG. 3

0 1 2 3 4 5m

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1 liter = 0.0353 cu. ft.

28.31 liters = 1.0 cu. ft.

1 meter = 39.37 inches

1 dm. = 3.937 inches

1 cm. = 0.3937 inches

1 mm. = 0.3937 inches

The application of formula (2) is, however, so impractical that the same Staus in a more recent article (A. Staus: Zur Berechnung kreisrunder Ueberfalle. Wasserkraft und Wasserwirtschaft, 1930; Heft 11) found it necessary to furnish a table, the values of $q_i$ for values of $\frac{h}{d}$, increasing by 0.01 increments between 0 to 1.0, inclusive.

For the determination of $q_i$ a graphical solution has been proposed which, however, does not simplify the problem very much. (Von S. Gradstein e A. Walther: Zeichnerische Behandlung des kreisrunden Ueberfalls. Das Gas u Wasserfach, 1931; Heft 10).

A formula for $q_i$ was sought that would be simple in form and that would give values approximately equal to the theoretical values. This formula can be expressed as:

$$q_i = k\, r^{a} - k_1 r_i^{a_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where "r" represents the ratio $\frac{h}{d}$ (h and d have the significance dimensions as above); k, $k_1$ and $a$, $a_1$ are some constants whose evaluation can be made with the values that are furnished from the tables of Staus.

Having obtained such values, the curve $(r, q_i)$ was drawn on logarithmic paper (figure 1). A curve is obtained which for small values of "r", not greater than 0.5, can be practically assimulated in a straight line; while for values "r" greater than 0.5, such a line tends progressively to bend towards the axis of "r".

If a tangent is constructed to the curve for a small value of "r", for example, "r" = 0.1, the said tangent can be represented by the following expression:

$$q_i = 10.12 \, r^{1.975}$$

which may be used to determine the discharge for rather small values of "r".

Now, the difference $q_2 = q_1 - q_i$ is calculated for a certain number of values of "r". This new series of values as a function of the term "r" can be represented by

$$q_2 = 2.66 \, r^{3.78}$$

and then by definition

$$q_i = q_1 - q_2 = 10.12 \, r^{1.975} - 2.66 \, r^{3.78} \quad \ldots\ldots\ldots\ldots\ldots \quad (3')$$

The lengths are always expressed in dm. and the discharge in liters per second (l/s).

In order to decide on the accuracy of the proposed formula, each value of $q_i$, that was determined by equation (3'), is compared in table I with the corresponding values given by the table of Staus. As it can be observed, the discrepancy between the two values do not ever exceed 5 percent. Therefore, the claim is made that the new formula can be rightly used in general practice. However, the point is not neglected that the constants $k$, $k_1$, and $\alpha$, $\alpha_1$ of equation (3), if determined by a procedure different from that used here, can assume values slightly different from those that are shown in equation (3').

TABLE I

| $r$ | $q_1$ | $q_i$ Staus | $q_2 = q_1 - q_i$ | $2.66\ r^{3.78}$ | $q_i$ new formula (3') | $\Delta$ | $\Delta$ % |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.1072 | 0.1072 | 0.0000 | 0.0004 | 0.1068 | -0.0004 | -3.7 |
| 0.2 | 0.4214 | 0.4173 | 0.0041 | 0.0061 | 0.4153 | -0.0020 | -4.8 |
| 0.3 | 0.9386 | 0.9119 | 0.0267 | 0.0281 | 0.9106 | -0.0013 | -1.4 |
| 0.4 | 1.6567 | 1.5713 | 0.0854 | 0.0833 | 1.5734 | +0.0021 | +1.3 |
| 0.5 | 2.5742 | 2.3734 | 0.2008 | 0.1936 | 2.3806 | +0.0072 | +3.0 |
| 0.6 | 3.6899 | 3.2939 | 0.3960 | 0.3857 | 3.3042 | +0.0103 | +3.1 |
| 0.7 | 5.0033 | 4.3047 | 0.6986 | 0.6908 | 4.3125 | +0.0078 | +1.8 |
| 0.8 | 6.5130 | 5.3718 | 1.1412 | 1.1443 | 5.3687 | -0.0031 | -0.6 |
| 0.9 | 8.2187 | 6.4111 | 1.8076 | 1.7862 | 6.4325 | +0.0214 | +3.3 |
| 1.0 | 10.1200 | 7.4705 | 2.6495 | 2.6600 | 7.4600 | -0.0105 | +1.3 |

The effective discharge will then be:

$$Q = mq_i d^{5/2} = md^{5/2}\ (10.12\ r^{1.975} - 2.66\ r^{3.78}) \quad \ldots\ldots\ldots. (4)$$

where $m$ is the coefficient of discharge whose value depends on several elements, and principally: on the height of head ($h$); on the diameter of the weir ($d$); on the thickness and shape of the circumference of the weir; on the form and dimensions of the channel of approach, or in other words on the "velocity of approach"; on the ruggedness of the channel; on the temperature of the water; etc. This, therefore, is a value to be determined experimentally.

3. We have applied the proposed formula in the calibration of four circular weirs of the diameters of 100, 200, 300, and 400 mm. (The exact measure of the diameters were 100.1; 200.0; 299.8; 400.0 mm., respectively.)

The weirs were made from thin sheets of brass, 0.75 m. square and 5 mm. thick. The circumference of the orifice was beveled as in figure 2, and the thickness (S = 1 mm.) was maintained constant for the four diameters. The said weirs were installed in the lesser width of four, rectangular, metal "stilling channels", which had the same dimensions (1.09 m. wide, 2.85 m. long, and 1.27 m. deep from the bottom of tank to the center of the weir). The water supply for the weirs came from a constant head tank through gate-controlled openings in the thin upstream wall of the approach channel. At the midlength of the approach channel, a vertical system of stilling grills was placed.

It was attempted, moreover, to place the four weirs under identical conditions of operation. In order to insure this similarity, some check calibration experiments were made with the four weirs which were placed each time in the same stilling channel.

Quantity measurements of most discharges were determined by means of an accurately calibrated tank, having the bottom dimensions of 2 m. by 4 m. and a height of 3 m. On the other hand, the smaller discharges of the 100-mm. diameter weir were determined by weight. The maximum total discharge was about 100 l/sec.

The figures 3 and 4 give a diagrammatical illustration of the apparatus (for further information see E. Scimemi "Il Laboratorio di Idraulica del R. Istituto Superiore di Ingegneria di Padova e le sue ricerche." "L'Energia Elettrica", Settembre 1935, Vol. XII).

The calibration curves for the four weirs are shown in figures 5 and 6; and the discharge coefficients "m", determined by means of the equation (4), are shown as a function of "r" in figure 7.

The coefficient "m", as previously stated, is determined by several elements. However, curves a, b, c, and d of figure 7 show how "m" varies with a variation of the term "r" and how "m" varies with a variation of the diameter for a constant value "r". In fact, in this case (curve e figure 7) the coefficient "m" tends to decrease slightly with an increase of the diameter.

It appears from a preliminary inspection that weirs of different sizes should have a fixed coefficient of discharge for they are, among themselves, similar in form and dimensions (the named hypothesis may not even be qualified).

On the other hand, these experimental weirs, besides varying in diameter, have a constant thickness (S = 1 mm.) at the circumference of the weir. The said thickness influences the coefficient of contraction of the jet and consequently the coefficient "m". That is, a small increase in "S" diminishes the contraction of the jet and increases the discharge coefficient.

The experimental research from these weirs for such an argument is not conclusive because the variation of the coefficient "m" for variations of the thickness "S" is small. However, a calibration was made for a weir with a diameter of 100 mm. and a thickness (S) of 0.03 mm. This weir was placed under the same conditions as the weir of 100 mm. diameter that was previously tested, figure 8. With this reduction of thickness (S), the new discharge coefficient, with an equality of other conditions, should almost be equal to that of 300 mm. diameter weir. The results of the experiment worked out very favorably, figure 7, curve f.

The discharge coefficient also depends, in a certain way, on a so-called "velocity of approach" that forms upstream from the weir. This velocity can be basically fixed by the ratio of the liquid area in the approach channel to the liquid area at the weir face. Thus in this case, knowing that the weirs were installed in channels of the same dimensions, there should result a different coefficient of the velocity of approach for each diameter of the weir.

To better define the influence that the velocity of approach has on the coefficient of discharge, a comparison is made between the values of "m" from our calculations with those determined from two distinct series from the experiments of Staus. Naturally, such a comparison is only indicative since it is not

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possible to guarantee the conformity of the method and of the installation in different groups of tests.

The size and layout of our apparatus has already been described, figure 3. Now, for comparison, the apparatus of Staus will be described. The first series of experiments (1926) were carried out in a canal of 600-mm. width and of 500-mm. height from the bottom of the canal to the center of the weir. The weirs, which were constructed from sheets of 4-mm. thickness, were made with conical square corner diverging towards the outside and with diameters of 150 and 300 mm., inclusive, increasing by 25 mm. increments. On the other hand, the second series of experiments (1931) were carried out in a canal of a width equal to 3d (diameters) of a height equal to 1.5d (from the bottom of the canal to the center of the weir), and of a length equal to or greater than 3d. The weirs had an opening of 200, 225, 250, 275, and 300 mm. and a diverging conical square edge.

A comparison was only possible for discharge coefficients for weirs of 200 and 300 mm. diameter, figure 9a.

It is quickly noted from the curve that the discrepancies were greatest for the large values of the factor "r". Nevertheless, it is apparent that the discrepancies were almost exclusively due to the different velocities of approach.

The expression for the coefficient of the velocity of approach was assumed to be that given by Bazin for rectangular weirs and later modified by Hégly for semicircular Weirs (M. Hégly:

Experiences sur des deversoirs a nappė libre avec contraction laterale. Annales des Ponts et Chaussées. Year 1921; page 290-389). The expression is:

$$m_1 = 1 + k \left(\frac{\omega}{\Omega}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where "$\omega$" is the flowing liquid area at the face of the weir, "$\Omega$" is the liquid area in the approach channel, and "k" is the constant that was assumed by Hégly equal to unity.

In table II the increases of the discharge coefficients in thousandths, for these experiments and for two series of experiments of Staus, were calculated by means of equation (5) for weirs of 200 and 300 mm. diameter.

TABLE II

| | Our experiments | | Staus (1926) experiments | | Staus (1931) experiments |
|---|---|---|---|---|---|
| d / r | 200 | 300 | 200 | 300 | 200 and 300 |
| 0.2 | | | 0.3 | 1.8 | 2.3 |
| 0.3 | | | 0.6 | 4.5 | 5.8 |
| 0.4 | | | 0.9 | 8.7 | 10.8 |
| 0.5 | 0.13 | 0.65 | 2.8 | 13.8 | 17.0 |
| 0.6 | 0.20 | 0.95 | 4.0 | 19.3 | 23.7 |
| 0.7 | 0.27 | 1.33 | 5.3 | 24.8 | 30.0 |
| 0.8 | 0.34 | 1.67 | 6.4 | 29.3 | 35.0 |
| 0.9 | 0.41 | 1.90 | 8.6 | 32.6 | 38.4 |
| 1.0 | 0.43 | 2.07 | 8.7 | 32.8 | 38.5 |

For these experiments the effect of the velocity of approach can be neglected, because the correction amounts to a maximum increase of 2 percent for the weir of 300 mm. under a full head ($h = d$). On the other hand, for the first series of experiments of Staus a maximum increase of 33 percent was found for a weir of 300 mm. diameter under a full head. For the second series of experiments, the maximum increase of the discharge coefficient for weirs of any diameter is 38 percent for full head, since the dimensions of the approach channel were proportional to the diameter of the weir.

If now the discharge coefficient "u", reduced by the coefficient of velocity of approach of equation (5) is computed for these three series of experiments, three curves, that are practically superimposed are obtained for each weir diameter, figure 9b. This result approximately verifies the equation (5) for the coefficient of the velocity of approach.

It should not be difficult to find an analytical expression, of close approximation, for the discharge coefficient "m" as a function of the ratio $h/d = r$. For example Staus, for the second series of experiments (1931), gives:

$$m = 0.555 + \frac{1}{110\ h/d} + 0.041\ \frac{h}{d}$$

A study was made on the application of an equation of the type

$$m = cr^{\gamma} \left[1 + k \left(\frac{\omega}{\Omega}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The said equation, used in cases where the velocity of approach can be neglected, reduces to only the first exponential term and therefore is very convenient to use. So in such cases, the effective discharge represented by equation (4) is always expressed according to the exponential binomial-equation of the type of equation (3).

Equation (6), if applied to these experiments, after making the term $\left[1 + k\left(\frac{\omega}{\Omega}\right)^2\right]$ equal to unity and by taking values for $c$ and $\gamma$ from the following tabulation:

| d | c | $\gamma$ |
|---|---|---|
| 100 | 0.681 | -0.03 |
| 200 | 0.572 | " |
| 300 | 0.569 | " |
| 400 | 0.570 | " |

gives good results for values of "r" less than 0.9. Above values of 0.9, errors of the order of 1 percent are perpetuated in the valuation of the discharge, table III.

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TABLE III

| r | h | Q new formula (4) | Q experimentally | Δ | Δ % | h | Q new formula (4) | Q experimentally | Δ | Δ % |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Weir of 100 mm. | | | | | Weir of 200 mm. | | |
| 0.1 | 0.1 | 0.0666 | | | | 0.2 | 0.3701 | | | |
| 0.2 | 0.2 | 0.2547 | | | | 0.4 | 1.4145 | | | |
| 0.3 | 0.3 | 0.5499 | | | | 0.6 | 3.0545 | 3.05 | +0.004 | +1.31 |
| 0.4 | 0.4 | 0.9419 | 0.94 | +0.002 | +2.13 | 0.8 | 5.2330 | 5.22 | +0.013 | +2.49 |
| 0.5 | 0.5 | 1.4157 | 1.41 | +0.006 | +4.25 | 1.0 | 7.8648 | 7.85 | +0.015 | +1.91 |
| 0.6 | 0.6 | 1.9542 | 1.95 | +0.004 | +2.05 | 1.2 | 10.8568 | 10.85 | +0.007 | +0.64 |
| 0.7 | 0.7 | 2.5387 | 2.53 | +0.009 | +3.56 | 1.4 | 14.1037 | 14.10 | +0.004 | +0.28 |
| 0.8 | 0.8 | 3.1478 | 3.15 | -0.002 | -0.63 | 1.6 | 17.4878 | 17.50 | -0.012 | -0.68 |
| 0.9 | 0.9 | 3.7584 | 3.78 | -0.022 | -5.82 | 1.8 | 20.8796 | 21.00 | -0.120 | -5.71 |
| 1.0 | 1.0 | 4.3450 | 4.40 | -0.055 | -12.50 | 2.0 | 24.1380 | 24.55 | -0.312 | -12.71 |
| | | | Weir of 300 mm. | | | | | Weir of 400 mm. | | |
| 0.1 | 0.3 | 1.0127 | | | | 0.4 | 2.0864 | | | |
| 0.2 | 0.6 | 3.8707 | | | | 0.8 | 7.9744 | 8.00 | -0.026 | -3.25 |
| 0.3 | 0.9 | 8.3577 | 8.35 | +0.008 | +0.96 | 1.2 | 17.2194 | 17.30 | -0.081 | -4.68 |
| 0.4 | 1.2 | 14.3180 | 14.30 | +0.018 | +1.26 | 1.6 | 29.4991 | 29.50 | -0.001 | -0.03 |
| 0.5 | 1.5 | 21.5190 | 21.40 | +0.119 | +5.56 | 2.0 | 44.3352 | 44.20 | +0.135 | +3.05 |
| 0.6 | 1.8 | 29.7055 | 29.60 | +0.105 | +3.55 | 2.4 | 61.2017 | 61.30 | -0.102 | -1.66 |
| 0.7 | 2.1 | 38.5895 | 38.30 | +0.289 | +7.54 | 2.8 | 79.5050 | 79.60 | -0.105 | -1,32 |
| 0.8 | 2.4 | 47.8489 | 47.80 | +0.049 | +1.02 | 3.2 | 98.5819 | | | |
| 0.9 | 2.7 | 57.1296 | 57.20 | -0.070 | -1.22 | 3.6 | 117.7024 | | | |
| 1.0 | 3.0 | 66.0470 | 66.60 | -0.553 | -8.33 | 4.0 | 136.0700 | | | |

4. Whenever it is possible to restrict the field of application of the calibration curve of a circular weir to a limited path (i.e. $h \leqq d/2$), it is easy to find for this curve an analytical expression both very simple and sufficiently accurate.

An expression of this type is that furnished by W. Greve (W. Greve: "Semicircular weirs calibrated at Purdue University" "Eng. N. R." July 1934, p. 182).

Greve performed the calibration of the semicircular weirs with openings of: 6-3/16, 12, 24, and 29-15/16 inches, and he established the

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equation expressed in English units

$$Q = mh^{n} \qquad (9)$$

The values of "m" and "n", transferred into the decimal-metric system (Q in mc/sec. and h in meters), are tabulated below:

| Diam. mm. | 157.2 | 304.9 | 457.2 | 609.6 | 760.4 |
|---|---|---|---|---|---|
| n | 1.86 | 1.87 | 1.88 | 1.88 | 1.89 |
| m | 0.483 | 0.765 | 0.994 | 1.174 | 1.391 |
| for n = 1.88 constant | | | | | |
| m | 0.510 | 0.785 | 0.994 | 1.174 | 1.369 |

The discharges calculated with the formula of Greve agree very well with the results from our formula (4) for heads not greater than the radius. The differences between the two formulae are less than 1 percent.

The author, moreover, admits that the coefficient "m" can be considered as a function, exponential type, of the diameter so that the general equation of discharge would assume the form $Q = cd^{s}h^{n}$.

Another formula, well known for semicircular weirs, is that of Hégly

$$Q = m\,\omega\sqrt{2gh} \qquad (10)$$

Q and h are expressed in mc/sec. and meters; $\omega$ is the area of the weir below the level of the head and m is the coefficient of discharge:

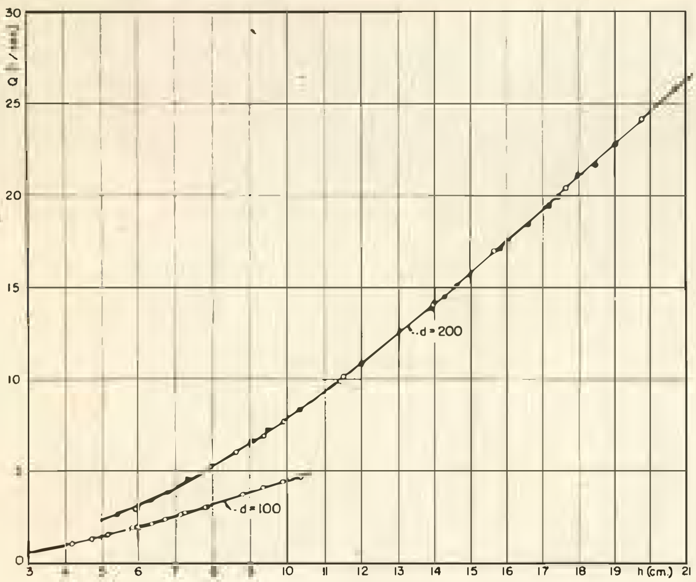


FIG. 5

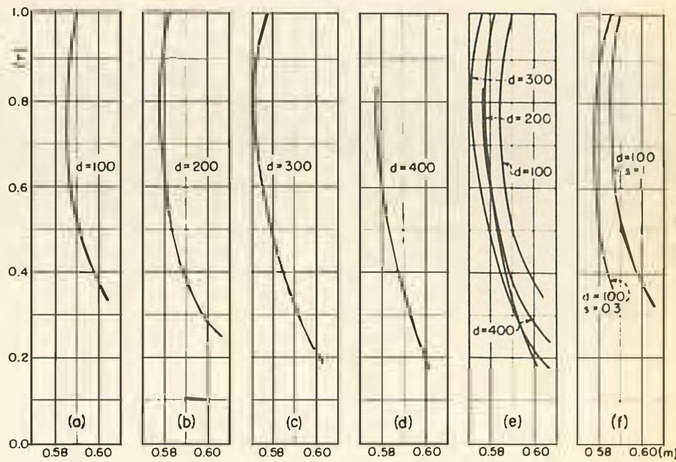


FIG. 7

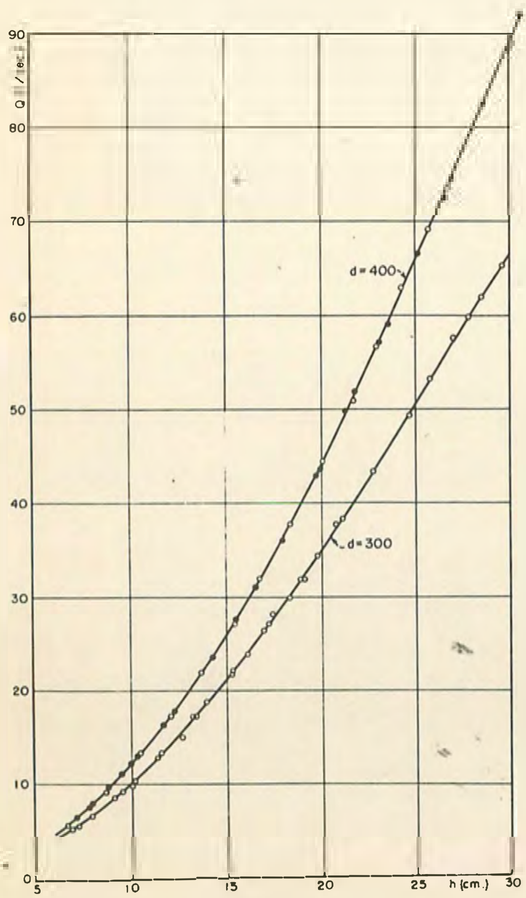


FIG. 6

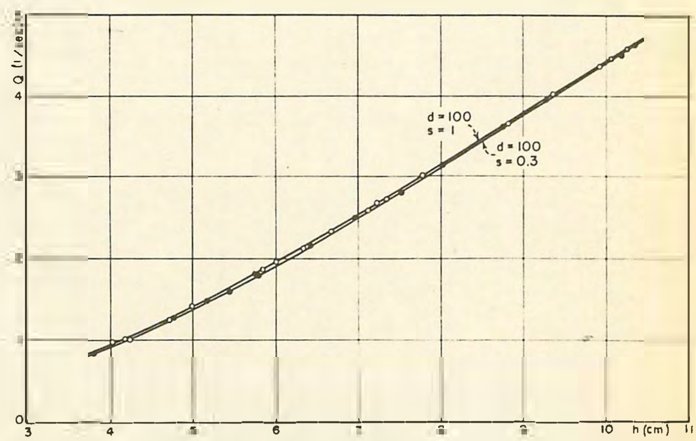


FIG. 8

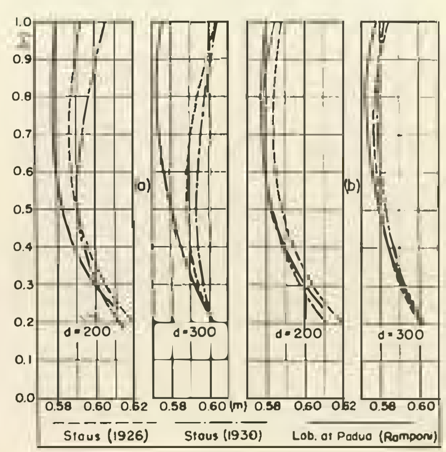


FIG. 9

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$$m = \left(0.350 + \frac{0.002}{h}\right) \left[1 + \left(\frac{\omega}{\Omega}\right)^2\right] \text{ .................... (11)}$$

where and have the same meaning as given in equation (5). This formula is valid between the limits of the field examined by the author. This field is for weirs of about 1 meter in diameter for a height of head not greater than a radius.

However, the same formula can be generalized for weirs of any diameter, giving:

$$M = \left(0.350 + 0.002 \frac{d}{h}\right) \left[1 + \left(\frac{\omega}{\Omega}\right)^2\right] \text{ .................... (11')}$$

TABLE IV

| d: | 1000 | | 400 | | 300 | | 200 | | 100 | |
|---|---|---|---|---|---|---|---|---|---|---|
| r | A | B | A | B | A | B | A | B | A | B |
| 0.1 | 20.993 | 20.611 | 2.124 | 2.086 | 1.034 | 1.014 | 0.375 | 0.370 | 0.066 | 0.066 |
| 0.2 | 80.230 | 78.822 | 8.119 | 7.974 | 3.955 | 3.877 | 1.435 | 1.414 | 0.253 | 0.255 |
| 0.3 | 171.051 | 170.177 | 17.305 | 17.219 | 8.433 | 8.373 | 3.060 | 3.054 | 0.541 | 0.550 |
| 0.4 | 291.842 | 291.490 | 29.541 | 29.499 | 14.385 | 14.344 | 5.222 | 5.233 | 0.923 | 0.942 |
| 0.5 | 435.453 | 438.117 | 44.068 | 44.355 | 21.460 | 21.558 | 7.791 | 7.865 | 1.376 | 1.416 |
| 0.6 | 597.267 | 604.766 | 60.440 | 61.202 | 29.438 | 29.759 | 10.068 | 10.857 | 1.888 | 1.954 |

A = Modified formula of Hegly

B = New formula (4)

In table IV for the simpler case of no velocity of approach, there are assembled some comparisons for different diameters between the modified formula of Hegly and new formula (4). (In equation (4), $m = cr^{\gamma}$ was substituted with c and $\gamma$ as given by the tabulation under topic 3. For weirs of 1 meter diameter, $m = 0.570\ r^{-0.03}$ was used.)

From the author's results two valid equations are proposed: one for heads less than 0.6 d, and the other for heads equal to or greater than 0.6 d.

The first expression is

$$q_i = 8.90\ r^{1.91} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

($q_i$ in liters per second and lengths in dm) and gives the theoretical discharge for $d = 1$ dm and $m = 1$. Such an equation can be used with good results as shown in the table that follows:

| r | $q_i$ form.(7) | $q_i$ form.(3') | Δ |
|---|---|---|---|
| 0.1 | 0.109 | 0.107 | +0.002 |
| 0.2 | 0.412 | 0.417 | -0.005 |
| 0.3 | 0.893 | 0.912 | -0.019 |
| 0.4 | 1.547 | 1.571 | -0.024 |
| 0.5 | 2.368 | 2.373 | -0.005 |
| 0.6 | 3.368 | 3.294 | +0.074 |

The effective discharge, for a weir of diameter d, would be

$$Q = md^{5/2} \times 8.90\ r^{1.91} = m\ 8.90\ d^{0.59} h^{1.91} \quad \ldots\ldots\ldots\ldots (7')$$

where the coefficient of discharge (m) can be expressed by an exponential function ($m = cr$ ) which does not in any way modify the structure of the discharge equation (7').

For values of "r" greater than 0.6, observing that for this interval the calibration curve has a path almost linear (see figure 5), the following equation is derived:

$$q_i = 10.63\ r - 3.13 \quad \dots\dots \quad (8)$$

for the theoretical discharge ($d = 1$ dm; $m = 1$) which gives the following results:

| r | $q_i$ form.(8) | $q_i$ form.(3') | Δ |
|---|---|---|---|
| 0.60 | 3.248 | 3.294 | -0.046 |
| 0.65 | 3.779 | 3.790 | -0.011 |
| 0.70 | 4.311 | 4.305 | +0.006 |
| 0.75 | 4.842 | 4.834 | +0.008 |
| 0.80 | 5.374 | 5.372 | +0.002 |
| 0.85 | 5.905 | 5.913 | -0.008 |
| 0.90 | 6.437 | 6.451 | -0.014 |
| 0.95 | 6.968 | 6.976 | -0.008 |
| 1.00 | 7.500 | 7.470 | +0.030 |

For the true discharges:

$$Q = md^{5/2}\ (10.63\ r - 3.13) \quad \dots\dots \quad (8')$$

with m, the discharge coefficient, that stays within the interval of validity of the formula and that is taken constant for each diameter, or that is independent of the term "r".

5. An element of great importance for bulkhead weirs concerns the error in the discharge measurement caused by an inaccurate evaluation of the head of water on the weir.

Such an error depends, in a majority of cases, on the imperfections of our observations; and frequently on the small oscillations that manifest themselves in the quiet liquid of the stilling well. (Neglecting those certain errors that can arise from causes of physical nature, as, for example, strong variations of temperature that change the dimensions of the weir; and also neglecting certain instrumental errors as weirs not perfectly formed, etc.)

Assume that the reading of the head "h" was effected by an error $\pm \Delta h$, to which there will correspond an error $\pm \Delta Q$ for the discharge $Q$. If $\Delta h$ is sufficiently small, the relation can be written:

$$\Delta Q = \frac{dQ}{dh} \cdot \Delta h \qquad (12)$$

with this equation it is possible to determine the absolute error of the discharge for an error $\Delta h$ in the head. Yet in practice one is interested to learn the error relative to the discharge $Q$ or $R = \frac{\Delta Q}{Q}$. If $\Delta h$ is placed equal to 0.1 mm., "R" may be written:

$$R = \frac{\Delta Q}{Q} = 0.001 \frac{\frac{dQ}{dh}}{Q} = \frac{\frac{dQ}{dh}}{Q} \% \qquad (12')$$

for $Q$ in liters per second and $h$ in dm. This relation was used by Staus and Sanden in the construction of the curve of errors for weirs of every form as a function of the discharge, where the discharge coefficient "m" is equal to unity.

This relation will be used for a comparison between the curve of errors of the circular weir, of the right-angled triangular weir, and of a rectangular weir with end contractions. In order to

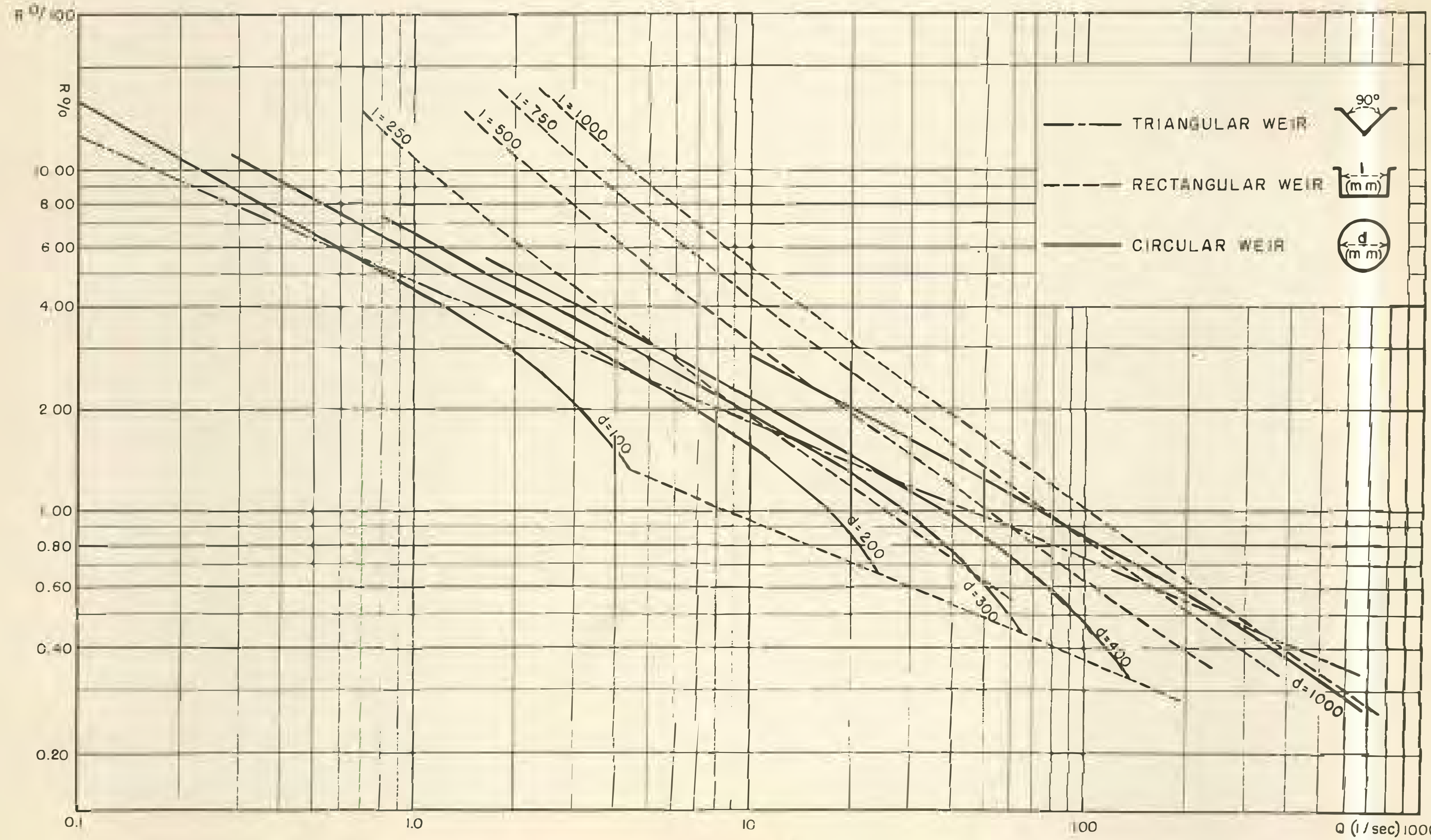
R %/100
R %
10.00
8.00
6.00
4.00
2.00
1.00
0.80
0.60
0.40
0.20
0.1
1.0
10
100
Q (l/sec) 1000
l = 250
l = 500
l = 750
l = 1000
d = 100
d = 200
d = 300
d = 400
d = 1000
TRIANGULAR WEIR
90°
RECTANGULAR WEIR
l (m m)
CIRCULAR WEIR
d (m m)

FIGURE 10

stay within the practical field, the errors are referred to their respective true discharge (rather than to that with $m = 1$). No velocity of approach is assumed.

The equation (12') applied to equation (4) of circular weirs is carried, by simple operations, to the form

$$R = \frac{3.78}{h} \cdot \frac{1.988 - r^{1.805}}{3.804 - r^{1.805}} \%$$

For the right-angled triangular weir, using the form of Hégly

$$Q = \left[0.310 + \frac{0.002}{h}\right] \sqrt{2g}\; h^{5/2}$$

with Q and h expressed in mc/sec. and in meters, the expression for "R" is

$$R = \frac{2.5}{h} \%$$

in which in order to keep increments of h ($\Delta$ h) in terms of thousandths of dm (0.001 dm), "h" must be read in dm.

Analogously for the rectangular weirs with end contractions, using the formula of Hégly

$$Q = \left[0.405 - 0.03 \frac{L - l}{L} + \frac{0.0027}{h}\right] \sqrt{2g}\; lh^{3/2}$$

(Q in mc/sec. and h in m.) and assuming, for simplification, that the width "l" of the weir is 1/4 of the width of the canal, then with

$$Q = \left[0.3825 + \frac{0.0027}{h}\right] \sqrt{2g}\; lh^{3/2}$$

the equation (R) is found (with $h$ in dm and $\Delta h$ = 0.001 dm)

$$R = \frac{1.5}{h} \%$$

The curve of errors is constructed with the coordinates R and Q. One quickly observes from the formula that for the circular weirs the value of R, besides varying with the head, varies for the same discharge (Q) with the diameter of the orifice. Also, the error R of the rectangular weir, for the same discharge, varies dependently with the length "l" of the weir. For a better comparison, therefore, the curve of errors is determined for different diameters of the circular weir and for different lengths of the rectangular weir. These curves are presented in a logarithmic scale since, it is known that, in this scale they present lines with nearly rectilinear paths. It is also noted that the curve of errors for the triangular and rectangular weirs would be exactly straight lines in the logarithmic diagram, if the coefficient of discharge was independent of the head.

The curves (figure 10) point out how the triangular 90° weir, when considering the probable error relative to the discharge, is most suited for small discharges, while the rectangular is most suited for large discharges. The circular weir, for the large discharges, approaches the rectangular weir, while for the small discharges, it approaches the triangular weir. Therefore, selecting the proper diameter, the circular weir yields itself sufficiently well to measure either the small or the large discharges.

Furthermore, the discharge of 50 liters per second can be, for example, given from the triangular weir with an error of 0.95 percent; the same discharge can be given from a circular weir of 300 mm. of diameter with an error of 0.62 percent; and from a rectangular weir, length 25 cm., with an error of 0.64 percent.

The same three weirs can give the discharge of 5 l/sec. with an error of 2.4 percent for triangular, of 2.8 percent for circular of 300 mm., and of 3.2 percent for rectangular weir of 0.25 m. length.

For a very small discharge (about 1 liter/sec.), for which the relative error is more clearly displayed, there is an error of 4.7 percent with a triangular weir; of 6.6 percent with the circular weir of 300 mm. diameter (with a circular weir of 100 mm. the error would be 4.5 percent, smaller than the error given by a triangular weir) and of 11 percent with rectangular weir of 0.25 m. length. (It may not be necessary to mention that for too small heads on the crest the vena adheres to the outside wall of the weir and the discharge increases, as is known, in a very particular way.)

For much larger discharges, of the order of 300 liters/sec., the 90° triangular weir, the circular weir of 1 meter diameter, and the rectangular weir of 1 meter length, can give an error almost equal to about 0.4 percent.

Therefore, concerning the relative errors in the measurement of discharges, the circular weir gives errors either almost equal to or less than those given by other types of weirs which

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are more in use.

6. For comparison with other weirs, it must be noted, moreover, that the circular weir is in a very favorable position when considering its construction and its mounting.

With respect to its construction, the circular weir, which is made in a lathe, gives the maximum guarantee exactness. In addition to which, its dimensions are easily controllable. The triangular weir on the other hand demands extreme care in the construction of the vertex; this must be finished by hand, and in the determination of the angle of opening of the weir. Analogously, the same must be said for other types of weir.

Concerning the installation of the weir in the stilling channel, the circular weir, as a virtue of its form, eliminates the danger of lateral inclination of the weir. For a concrete illustration, the case of $90^{\circ}$ triangular weir can be given; by means of inaccurate mounting or through settling during the experiment, it may be displaced in a manner that its axis of symmetry, generally in a vertical position, is inclined laterally (left or right) by a certain angle $\alpha$. Under such conditions, for a height of water $h$, the flowing liquid section surpasses the value $h^2$ (which it should be if the weir were under normal conditions) by a larger value:

$$h^2 \; 1/2 \; \left[ tg\,(45 - \alpha) + tg\,(45 + \alpha) \right]$$

Consequently, the increase in the discharge is: for $\alpha = 1^{\circ}$, 0.6 percent; for $\alpha = 2^{\circ}$, 5 percent; and for $\alpha = 5^{\circ}$, 15 percent.

In an analogous manner, the rectangular weir, with the crest not horizontal, produces errors in the evaluation of the discharge. These errors can be a little sensitive especially when the reading of the head of water is referred to a point of the crest close to the walls.

Conclusions

Recapitulation of the conclusions of these studies concerning circular weirs shows:

(a) Equation (4) which was derived from the calibration curve.

$$Q = m\, q_i\, d^{5/2} = m d^{5/2}\,(10.12\, r^{1.975} - 2.66\, r^{3.78}) \quad ..... (4)$$

This equation can be used for good results as compared to the experiments and also to the formula of Staus and Sanden.

(b) Moreover, how equation (4) can be expressed in the exponential binomial form when one assumes $m = cr^{\gamma}$.

(c) With examples, how for each diameter of the circular weir, it is possible to find an equation always of the exponential binomial form that includes in itself the coefficient "m" and that does not give very large errors.

(d) Several equations of calibration which are valid only for a certain interval of head.

(e) Clearly the sensitiveness of the circular weir in measuring discharges. It was clearly brought out how the circular weir unites, in part, the characteristics of the triangular and of the rectangular weir.

(f) The advantages of the circular weir over other weirs as to construction and to installation for operations.

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