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HYDRAULIC LABORATORY REPORT NO. 40

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SECOND PROGRESS REPORT ON STUDIES OF THE FLOW OF WATER IN OPEN CHANNELS WITH HIGH GRADIENTS

By

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- - -

Denver, Colorado
October 13, 1938

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UNITED STATES
DEPARTMENT OF THE INTERIOR
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Branch of Design and Construction
Engineering and Geological Control
and Research Division
Denver, Colorado
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Laboratory Report No. 40
Hydraulic Laboratory
Compiled by: V. L. Streeter

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Subject: Second progress report on studies of the flow of water in open channels with high gradients.

1. Introduction. This report deals with evaluation of data given in the first progress report on the same subject by C. W. Thomas, July 27, 1938 (memorandum to the Chief Designing Engineer). In the design of steep chutes and spillways, the average velocity, depth, and velocity distribution should be known for any cross section of the chute or spillway. If the air content at a section is known it is necessary to obtain the average velocity in that section and the discharge per foot of width before the depth can be determined. The average velocity in any section is required before freeboard can be determined. In this report the assumptions upon which the design method is based are:

(1) Air in and above the water causes an additional resistance which is directly proportional to the square of the average velocity, and inversely proportional to some power of the hydraulic radius.
(2) The hydraulic radius is based on a depth equal to the discharge per foot of width divided by the average velocity in the section, or the hydraulic radius, assuming no air in the water.
(3) The value of "n" in the Manning formula is constant for the particular type of material of which the chute or spillway is composed.

With these assumptions, a method of design may be formulated. In this report, rectangular channels of constant width are assumed. The results may be extended to trapexoidal sections and varying widths. As yet no definite method of predicting air content in a given flow has been determined. Attempts have been made to obtain an exponential

formula containg air content, average velocity, hydraulic radius, and angle of chute with horizontal, but no relationship has been found which can be confirmed within satisfactory limits by the experimental data.

In this report the following topics will be discussed:

(1) Data available on high velocity flow.
(2) Applicability of existing open-channel formulas to aerated flow.
(3) Nature of air resistance.
(4) Analytical form of air resistance correction.
(5) Evaluation of coefficients.
(6) Application to design of a typical steep chute.
(7) Application to design of Shasta stilling pool.
(8) Present status of freeboard computations.

2. Data available on high velocity flow. All available data on high velocity flow are recorded in tables 2, 3, and 4 of the first progress report on this subject. In determining velocities from the oscillograms for Kittitas, the time distance required for the salt cloud to pass from one set of electrodes to another was obtained by averaging the time distances between the beginning points on the curves, and the time distances between other characteristic points on the curves. By using as time distance, that distance between beginning points on the curves, a velocity approximating that of maximum was obtained. Using an average of this time distance and the time distances between characteristic points should result in a velocity somewhat less than maximum. If the time distance is taken from the centers of gravity of the areas under the two curves, the resulting velocity should be an average for the section. As the velocities recorded in tables 2 and 3 of the first progress report were computed by the first method described above, it appears that they are greater

than the average velocities. To prove this point, several of the clearest oscillograms were selected and the time distances to the centers of gravity of the curves computed. The velocities from beginning points on the curves were also computed. The ratio of the velocities from the beginning points divided by the velocities from the centers of gravity for sixteen sets of curves averaged 1.40. The ratio of velocities as recorded in table 3 of the first progress report, obtained as described at the beginning of this paragraph, divided by the velocities from the centers of gravity averaged 1.24. As the oscillograms were in many cases indistinct, and in some cases the curves were not completely recorded, it is impossible to re-analyze all of them to obtain the average velocity. Instead of this the velocities in tables 2 and 3 have been reduced by dividing each velocity by 1.24, the average ratio of velocity as recorded in the report to the velocity as determined by using the time distance between centers of gravity. Tables 2 and 3 of the first progress report are reproduced in this report as tables 6 and 7, respectively, with the velocities corrected. With the new values of average velocity, the air content and value of Q/VD have also been recomputed. These changes have been made in tables 6 and 7.

3. <u>Applicability of existing open-channel formulas to aerated flow.</u> The Manning and Chezy formulas were intended for use on rivers and canals with small gradients, and comparatively small velocities. It is highly improbable that either formula

TABLE 6

OBSERVED VELOCITY

SLOPE 10° - 12'

Distance from electrode no. 1	0	10'	100'	110'	200'	210'	300'	310'	400'	410'
Number of electrode	1	2	3	4	5	6	7	8	9	10
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.									
89	1	(28.6)	3 3	(27.4) (27.4)	5 5					
193	1	(30.5)	3	(29.8)	5 5	(31.6) (30.5)	9 7			
362	1 1	(37.2) (43.1)	3 3	(40.5) (44.1)	5 5	(41.4) (38.3)	8 8	(49.2) (49.8)	9 9	
401	1 1	(37.7) (37.4)	3 3	(42.8) (41.4)	5 5					
491	1	(42.1)	3 3 3	(39.2) (45.1) (41.4)	5 5 5	(43.8) (45.5) (46.2)	8 8 9			
587	1	(44.5)	3 3	(44.1) (47.0)	5 5 5	(46.4) (49.2) (46.2)	7 7 7	(50.6) (53.1) (51.6)	9 9 9	
719	1 1	(46.6) (46.5)	3 3	(46.5) (45.3)	5 5 5	(41.3) (50.0) (56.5)	7 7 7	(56.0) (54.3) (51.3)	9 9 9	
777	1	(52.1)	3 3	(50.1) (51.7)	5 5	(50.9) -	7 7	(50.8) (51.9)	9 9	
922	1 1	(58.6) (49.2)	3 3 3	(60.1) (52.3) (49.0)	5 5 5	(53.3) (48.6) -	7 7 7	(54.8) (55.4) (52.2)	9 9 9	
1,005	1 1	(58.2) (59.8)	3 3	(51.1) (57.5)	5 5 5	(58.3) (55.5) (53.8)	7 7 7	(54.0) (51.2) (54.1)	9 9 9	

SLOPE 33° - 10'

Distance from electrode no. 11	0	10'	100'	110'	200'	210'
Number of electrode	11	12	13	14	15	16
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.					
193		12 12 12	(44.5) (41.5) (56.1)	14 14 14	(53.3) (54.0)	16 15
362	11 (63.5) 11 (54.7)	14 14 12	(56.4) (56.7) (57.3)	15 15 14	(52.7)	16
401	11 (60.1) 11 (60.0)	13 13 13	(65.3) (63.5) (58.5)	16 16 16		
491	11 (63.8)	12 13 12	(59.0) (64.3) (62.1)	13 15 14	(67.5) (61.5)	15 16
587	11 (67.5) 11 (63.7) 11 (60.4)	13 13 13 13	(64.0) (66.7) (67.5) (69.5)	15 15 16 15		
719	11 (77.2) 11 (71.1)	13 13 13	(75.2) (65.1) (66.3)	16 16 15		
777	11 (65.9)	13 13	(94.0) (85.6)	16 15		
922		12 12 12	(68.7) (77.6) (70.5)	13 13 14	(75.2) (74.5) (68.6)	15 15 16
1,005		12 12	(91.1) (75.1)	15 15		
				13 13 14	(81.6) (82.6) (74.1)	15 15 16

SLOPE 0° - 04'

Distance from electrode no. 17	0	10'	40'	50'
Number of electrode	17	18	19	20
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.			
362		18 18 18	(34.0) (35.5) (34.0)	20 20 20
491		17 17 18	(59.5) (48.1) (45.4)	20 20 20
587		17 17 17	(33.7) (45.3) (60.4)	19 20 20
719		17 17 17	(67.6) (60.0) (57.5)	20 20 19
777		17 17	(57.5) (50.6)	19 19
922		17 18	(58.6) (56.0)	20 20
1,005		17 18	(62.3) (55.4)	20 20

TABLE 7

COMPUTED ENTRAINED AIR

SLOPE 10° - 12'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A ft.^{2}	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	$\frac{Q}{AV}$	Air content percent
89	(1-3)	0.49	3.88	0.44	28.6	0.79	21
	(3-5)	0.50	3.96	0.45	27.4	0.82	18
193	(1-3)	0.80	6.35	0.68	30.5	0.99	1
	(3-5)	0.81	6.44	0.68	29.8	1.00	0
	(5-7)	0.83	6.55	0.70	30.5	0.97	3
	(5-9)	0.82	6.48	0.69	31.6	0.94	6
362	(1-3)	1.25	9.90	0.96	40.2	0.91	9
	(3-5)	1.25	9.95	0.96	42.4	0.86	14
	(5-8)	1.24	9.85	0.96	39.8	0.92	8
	(8-9)	1.24	9.84	0.96	49.5	0.74	26
401	(1-3)	1.26	10.06	0.98	37.6	1.00	0
	(3-5)	1.26	9.98	0.97	42.1	0.95	5
491	(1-3)	1.54	12.37	1.14	42.1	0.94	6
	(3-5)	1.55	12.30	1.13	42.0	0.96	4
	(5-8)	1.56	12.36	1.14	44.6	0.89	11
	(5-9)	1.56	12.43	1.15	46.2	0.86	14
587	(1-3)	1.80	14.32	1.26	44.5	0.92	8
	(3-5)	1.80	14.33	1.26	45.6	0.91	9
	(5-7)	1.79	14.28	1.26	47.3	0.87	13
	(7-9)	1.80	14.36	1.26	51.8	0.93	7
719	(1-3)	2.02	16.11	1.36	46.6	0.96	4
	(3-5)	2.03	16.12	1.37	46.0	0.97	3
	(5-7)	2.05	16.33	1.38	49.4	0.89	11
	(7-9)	2.07	16.49	1.39	53.8	0.81	19
777	(1-3)	2.24	17.87	1.45	52.1	0.83	17
	(3-5)	2.22	17.68	1.45	51.0	0.87	13
	(5-7)	2.21	17.58	1.44	50.9	0.87	13
	(7-9)	2.26	17.97	1.46	51.4	0.84	16
922	(1-3)	2.52	20.02	1.56	53.8	0.86	14
	(3-5)	2.49	19.79	1.55	53.8	0.87	13
	(5-7)	2.50	19.84	1.55	51.0	0.92	8
	(7-9)	2.51	19.96	1.56	54.1	0.86	14
1,005	(1-3)	2.70	21.50	1.63	59.0	0.79	21
	(3-5)	2.65	21.11	1.62	54.4	0.88	12
	(5-7)	2.66	21.15	1.62	55.8	0.86	14
	(7-9)	2.66	21.19	1.62	53.1	0.84	16

SLOPE 33° - 10'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A ft.^{2}	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	$\frac{Q}{AV}$	Air content percent
193	(12-14)	1.02	8.07	0.83	44.0	0.55	45
	(14-16)	1.14	9.04	0.90	53.4	0.40	60
	(14-15)	1.14	9.04	0.90	54.0	0.40	60
362	(11-14)	1.50	11.83	1.11	59.1	0.52	48
	(12-14)	1.50	11.83	1.11	57.2	0.53	47
	(14-15)	1.63	12.88	1.17	56.5	0.50	50
	(14-16)	1.63	12.88	1.17	52.7	0.53	47
401	(11-13)	1.49	11.75	1.11	60.0	0.57	43
	(13-16)	1.53	12.08	1.12	62.4	0.52	48
491	(12-13)	1.94	15.32	1.32	59.0	0.55	45
	(11-13)	1.94	15.32	1.32	63.8	0.51	49
	(12-14)	1.94	15.32	1.32	62.1	0.55	45
	(13-15)	1.90	15.02	1.30	65.8	0.50	50
	(14-16)	1.90	15.02	1.30	61.5	0.53	47
587	(11-13)	2.05*	16.23*	1.37	64.0	0.57	43
	(13-15)	2.05	16.16	1.37	66.7	0.55	45
	(13-16)	2.05	16.16	1.37	67.5	0.53	47
719	(11-13)	2.18*	17.26*	1.43	74.1	0.56	44
	(13-15)	2.20	17.42	1.43	66.3	0.62	38
	(13-16)	2.20	17.42	1.43	70.2	0.58	42
777	(11-13)	2.35*	18.61*	1.50	65.9	0.63	37
	(13-16)	2.40	19.00	1.51	89.9	0.46	54
922	(12-13)	2.63*	20.83*	1.60	73.1	0.61	39
	(12-14)	2.63*	20.83*	1.60	70.5	0.63	37
	(13-15)	2.68	21.28	1.61	75.0	0.58	42
	(14-16)	2.68	21.28	1.61	68.6	0.63	37
1,005	(12-15)	-	-	-	83.1	-	-
	(13-15)	-	-	-	82.2	-	-
	(14-16)	2.94*	23.20*	1.71	74.1	.58	42

* Point gage readings made only at the end of the section on account of spray.

SLOPE 0° - 04'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A ft.^{2}	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	$\frac{Q}{AV}$	Air content percent
362	(18-20)	1.52	12.03		34.5	0.87	13
491	(17-20)	1.82	14.41		53.8	0.63	37
	(18-20)	1.82	14.41		45.4	0.74	26
587	(17-19)	2.14	16.95		33.7	1.00	0
	(17-20)	2.14	16.95		48.9	0.71	29
719	(17-19)	2.36	18.69		57.5	0.67	33
	(17-20)	2.36	18.69		63.8	0.61	39
777	(17-19)	2.61	20.76		54.1	0.70	30
922	(17-20)	2.91	22.93		58.6	0.68	32
	(18-20)	2.91	22.93		56.0	0.71	29
1,005	(17-20)	2.90	22.98		62.2	0.71	29
	(18-20)	2.90	22.98		55.3	0.80	20

applies directly to the solution of high velocity flow. In the Chezy formula

$$V = C \sqrt{R\,S} \qquad (1)$$

the velocity is seen to vary as the square root of the hydraulic radius. According to this formula, an increase in air content results in a larger hydraulic radius, and hence a greater velocity. This result is quite the opposite one would expect in aerated flow. A derivation of the Chezy formula will be undertaken to show that it is incorrect in high velocity flow to use the hydraulic radius of the "insufflated" section, without changing the value of the coefficient C.

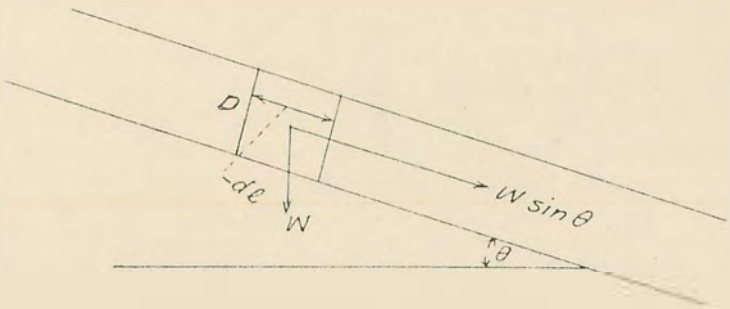

Figure 1

Let $100\,\mu$ = percent of air in water-air mixture by volume
b = width of channel (rectangular)
w = unit weight of water
τ = shear per unit area at surface of contact between water and channel

Writing the equilibrium conditions for volume $Db\,d\ell$

$$Db\,d\ell\,(1-\mu)\,w \sin\theta = \tau(2D+b)\,d\ell$$

$$\text{or } \tau = \frac{Db}{2D+b}\,(1-\mu)\,w \sin\theta$$

$$\text{Let } q = k V^2$$

Assuming k to be the same in "insufflated" and not "insufflated" water.

$$R = \frac{Db}{2D+b}$$

$$\text{and } S = \sin \theta$$

$$\text{then } k V^2 = R(1-\mu)\, w\, S$$

$$V = (1-\mu)^{\frac{1}{2}} \left(\frac{w}{\tau}\right)^{\frac{1}{2}} \sqrt{R\, S}$$

$$\left(\frac{w}{\tau}\right)^{\frac{1}{2}} = \text{Chezy } C$$

$$\text{and } V = (1-\mu)^{\frac{1}{2}} C \sqrt{R\, S} \qquad (2)$$

In other words, the coefficient C in the Chezy formula must be multiplied by $(1-\mu)^{\frac{1}{2}}$ when applied to water and air mixtures.

Defining R in a new way

$$R_{net} = \frac{bD(1-\mu)}{2D+b} = \frac{Q}{V(2D+b)}$$

in which Q = total discharge of the channel

and V = average velocity in a section

the Chezy formula remains

$$V = C \sqrt{R_{net}\, S} \qquad (3)$$

In a similar manner it may be shown that the Manning formula should use the R_{net} as defined above when water contains air.

Both the Chezy and Manning formula coefficients were obtained empirically for low velocities. Under these conditions there is undoubtedly very little resistance to flow from the water-air surface. For high velocity flow, a great deal of air is set in motion by the water with additional losses which would not be included in the Chezy or Manning formulas. In order to use either of these

formulas it will be necessary to correct them in some manner for the additional losses.

4. Nature of air resistance. Little is known of the mechanics of air resistance. It has been frequently observed that high air velocities occur over channels carrying water at high velocity. The force necessary to keep this air in motion, which is an added resistance to flow, comes from the water. This force on the air would be practically independent of the depth of water but the retarding force on a pound of water would be inversely proportional to the depth. As air resistance is usually proportional to the square of the velocity for high velocities, it is reasonable to assume this to be the case until further data can be obtained.

5. Analytical form of air resistance correction. The Manning formula is

$$S = \frac{n^2 V^2}{2.2082 R^{4/3}} \tag{4}$$

to this may be added a term to account for the air resistance, so that the formula becomes

$$S = \frac{n^2 V^2}{2.2082 R_{net}^{4/3}} + \frac{K V^2}{R_{net}^{p}} \tag{5}$$

n = Kutter's roughness coefficient.

As R_{net} cannot be determined until μ is known, a slightly different definition of R will be used, which is

$$R_{\mu=o} = \frac{Q/V}{b + 2Q/Vb} \tag{6}$$

This is the hydraulic radius based on $\mu=o$. The $R_{\mu=o}$ will be slightly less than R_{net}. As the values of "K" and "p" are obtained

experimentally, the values will be adjusted to compensate for the slight change in hydraulic radius. The formula for steady flow then becomes

$$S = \frac{n^2 V^2}{2.2082 R_{\mu=o}^{4/3}} + \frac{K V^2}{R_{\mu=o}^{p}} \qquad (7)$$

S = sine of angle between energy gradient and horizontal
b = width of the rectangular channel
V = average velocity in section

$$R_{\mu=o} = \frac{Q}{bV + 2Q/b}$$

Q = total discharge
K, p, = constants determined from experimental data by method of least squares.

For steady flow conditions, the available energy is represented by S. This energy is divided into two parts; the first part of the equation represents the loss due to internal friction and friction between water and solid surface. The second part, which represents the loss due to air resistance, is directly proportional to the square of the velocity and inversely proportional to some power of the depth or hydraulic radius.

In order to make this equation applicable to the design of steep channels and spillways it is combined with Bernoulli's equation as follows:

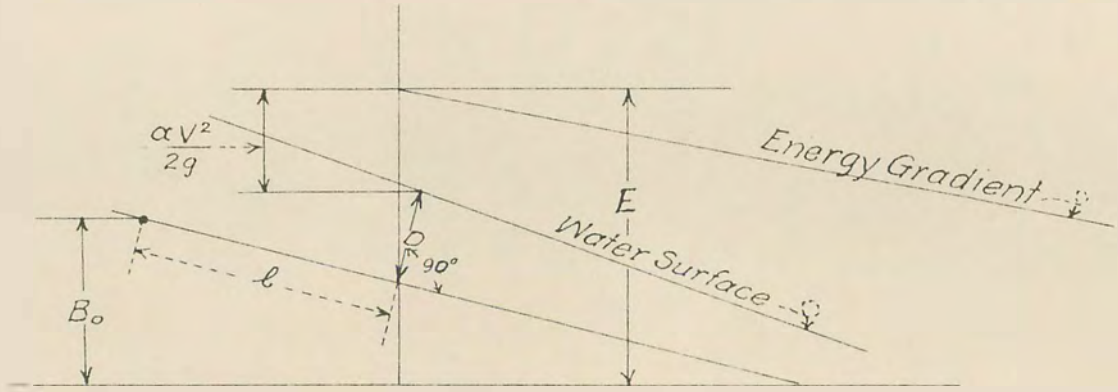

Figure 2

From Bernoulli's equation (figure 2):

$$E = B_o - \ell \sin \theta + D \cos \theta + \frac{\alpha V^2}{2g} \qquad (8)$$

in which $$\alpha = \frac{\int V_i^3 \, dA}{V^3 A}$$

where
A = cross-sectional area
V_i = velocity at elemental area, dA
V = average velocity in the section

With α defined as above, $\frac{\alpha V^2}{2g}$ is the average kinetic energy per pound of water for the section. Taking the derivative of (8) with respect to ℓ

$$\frac{dE}{d\ell} = -\sin \theta + \frac{dD}{d\ell} \cos \theta + \frac{\alpha V}{g} \frac{dV}{d\ell} \qquad (9)$$

with $Q_o = VD$ = discharge per foot of width

$$dD = \frac{-D\,dV}{V} = \frac{-Q_o}{V^2} dV \qquad (10)$$

Combining (9) and (10):

$$\frac{dE}{d\ell} = -\sin \theta \frac{-Q_o}{V^2} \cos \theta \frac{dV}{d\ell} + \frac{\alpha V}{g} \frac{dV}{d\ell} \qquad (11)$$

Using the Manning formula with a correction for air resistance, as given in equation (7):

$$S = \frac{dE}{d\ell} = \frac{-n^2 V^2}{2.2082 R_{\mu=o}^{4/3}} - \frac{KV^2}{R_{\mu=o}^{p}} \qquad (12)$$

The negative sign is used in the right side of the equation to show a decrease in E for an increase in ℓ. Equating (11) and (12) and

solving for $\frac{d\ell}{dV}$

$$\frac{d\ell}{dV} = \frac{a\frac{V}{g} - \frac{Q_o}{V^2}\cos\theta}{\sin\theta - \frac{n^2V^2}{2.2082\,R_{\mu=o}^{4/3}} - \frac{KV^2}{R_{\mu=o}^{p}}} \qquad (13)$$

in which K and p are to be determined from experimental data. Although equation (13) appears complicated the right side may be expressed as a function of V for any particular canal with constant width, provided Q_o, the roughness and the slope are known. Under such conditions $R_{\mu=o} = \frac{bQ_o/V}{b + 2\,Q_o/V}$

where b = width.

Equation (13) may be written $\frac{d\ell}{dV} = f(V)$.

Plotting $\frac{d\ell}{dV} = f(V)$ as ordinate against V as abscissa, as shown in figure 3, the cross-hatched area is the length to that point

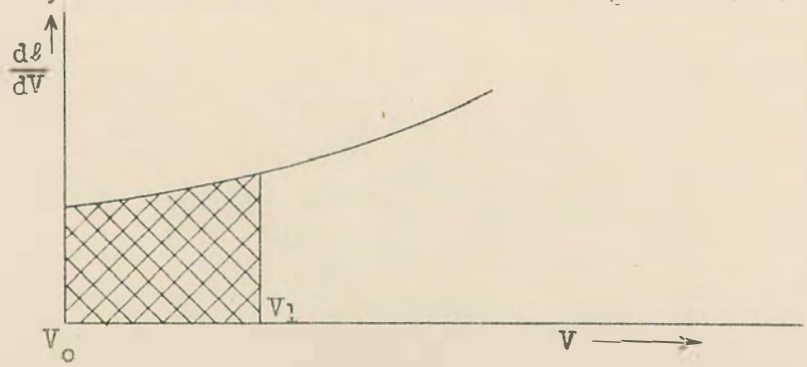

Figure 3

in the chute where the velocity is V_1. The velocity at the beginning of slope, or any other point in the chute at which the velocity is known is V_o.

6. Evaluation of coefficients. For all practical purposes the velocities contained in table (6) for the Kittitas are terminal velocities. Equation (7) may be used to determine the constants. It will be handled as follows:

$$S - \frac{n^2V^2}{2.2082\ R^{4/3}_{\mu=o}} = \frac{K\ V^2}{R^p_{\mu=o}}$$

$$\log\left(S - \frac{n^2V^2}{2.2082\ R^{4/3}_{\mu=o}}\right) = \log K + 2 \log V - p \log R_{\mu=o}.$$

Let $$\log\left(S - \frac{n^2V^2}{2.2082\ R^{4/3}_{\mu=o}}\right) = X$$

$$\log K = A$$
$$\log V = Z$$

and $$\log R_{\mu=o} = Y$$

then $$X = A + 2Z - pY$$

and $$X - 2Z = A - pY.$$

By least squares, the two condition equations for determination of A and p are:

$$\text{I} \qquad \Sigma\ (X - 2Z) = \Sigma A - p\Sigma Y$$

$$\text{II} \qquad \Sigma Y(X - 2Z) = A\Sigma Y - p\Sigma Y^2.$$

These two equations in two unknowns, result in K = 0.0000364 and p = 1.46.

7. Application to the design of a typical steep chute. To illustrate the use of equation (13) with the constants as determined above, a typical steep chute will be considered.

TYPICAL CHUTE DESIGN

$$\frac{d\ell}{dv} = \frac{0.03607v - \frac{141.42}{v^2}}{0.7071 - 0.000000055755v^2(v+50)^{4/3} - 0.000000015908v^2(v+50)^{1.46}}$$

Average v	v^2	$(v+50)^{4/3}$	$(v+50)^{1.46}$	$0.03607v$	$\frac{141.42}{v^2}$	$5.5755\times10^{-8}\,v^2(v+50)^{4/3}$	$1.5908\times10^{-8}\,v^2(v+50)^{1.46}$	Air resistance $\frac{d\ell}{dv}$	No air resistance $\frac{d\ell}{dv}$	Air resistance ℓ	No air resistance ℓ
18.6	345.96	280.81	479.77	0.6709	0.40878	0.0054	0.0026	0.3749	0.3735	0.0	0.0
20	400.0	288.49	494.15	0.7214	0.35355	0.0076	0.0031	0.5281	0.52581	0.63	0.63
30	900.0	344.71	600.50	1.0821	0.15713	0.0173	0.0086	1.3579	1.34104		
40	1,600.0	403.33	713.17	1.4428	0.08839	0.0360	0.0182	2.0745	2.01822	27.41	26.99
50	2,500.0	464.16	831.76	1.8035	0.05657	0.0647	0.0332	2.8671	2.71934		
60	3,600.0	527.06	955.94	2.1642	0.03928	0.1058	0.0547	3.8875	3.53392	85.51	86.14
70	4,900.0	591.89	1,085.4	2.5249	0.02886	0.1617	0.0846	5.4166	4.57654		
80	6,400.0	658.55	1,220.0	2.8856	0.02210	0.2350	0.1242	8.2308	6.06552	198.13	174.77
90	8,100.0	726.95	1,359.4	3.2463	0.01746	0.3283	0.1752	15.8567	8.52394		
100	10,000.0	796.99	1,503.45	3.607	0.01414	0.4444	0.2392	152.8876	13.67671	945.64	354.23
101								∞			
105	11,025.0	832.61	1,577.2	3.7874	0.01283	0.5118	0.2766		19.3270		
110	12,100.0	868.61	1,652.0	3.9677	0.01159	0.5860			32.6673		560.32
112.5	12,656.25	886.8	1,689.8	4.0579	0.01117	0.6258			49.7753		
115	13,225.0	904.99	1,728.0	4.1480	0.01069	0.6673			103.9526		840.09
116.1	13,479.2	913.04		4.1877	0.01049	0.6862			199.8605		1,007.19
120	14,400.0	941.74	1,805.1	4.3284	0.00982	0.7561					
117.3											
125	15,625.0	978.85	1,883.0	4.5088	0.00905						

SHASTA SPILLWAY COMPUTATIONS

$$\frac{d\ell}{dv} = \frac{\frac{\alpha v}{g} - \frac{Q_o}{v^2}\cos\theta}{\sin\theta - \frac{n^2 v^2}{2.208R^{4/3}} - \frac{0.0000364v^2}{R^{1.46}}}$$

$$\frac{d\ell}{dv} = \frac{0.03327v - \frac{311.78}{v^2}}{0.7808 - 0.000000013846\,v^{\frac{10}{3}} - 0.000000004187\,v^{3.46}}$$

$\theta = 51^\circ\ 20'$
$\cos\theta = 0.6248$
$\sin\theta = 0.7808$
$\alpha = 1.07$
$g = 32.16$
$n = 0.011$
$Q_o = 499$ c.f.s.
$L = 502'$

v	v^2	$v^{\frac{10}{3}}$	$v^{3.46}$	$0.03327v$	$\frac{311.78}{v^2}$	$1.3846\times10^{-8}\,v^{\frac{10}{3}}$	$4.187\times10^{-9}\,v^{3.46}$	Air resistance $\frac{d\ell}{dv}$	No air resistance $\frac{d\ell}{dv}$	Air resistance ℓ	No air resistance ℓ
44.55	1,984.7	313,266	507,010	1.4822	0.1571	0.0043	0.0021	1.7111	1.707	0.0	0.0
50.0	2,500	460,500	755,860	1.6635	0.1247	0.0064	0.0032	1.9953	1.987	10.10	10.07
60.0	3,600	845,610	1,420,400	1.9962	0.0866	0.0117	0.0059	2.5021	2.483		32.43
70.0	4,900	1,413,600	2,421,300	2.3289	0.0636	0.0196	0.0101	3.0160	2.976	60.17	59.73
80.0	6,400	2,206,100	3,843,200	2.6616	0.0487	0.0305	0.0161	3.5588	3.482		92.0
90.0	8,100	3,266,900	5,776,600	2.9943	0.0385	0.0452	0.0242	4.1549	4.018	131.52	129.5
100.0	10,000	4,642,000	8,317,600	3.327	0.0312	0.0643	0.0348	4.8347	4.600		172.5
110.0	12,100	6,377,680	11,567,000	3.6597	0.0258	0.0883	0.0484	5.6418	5.248	228.64	221.7
120.0	14,400	8,522,500	15,630,000	3.9924	0.0216	0.1180	0.0654	6.6468	5.991		277.8
130.0	16,900	11,130,000	20,617,000	4.3251	0.0184	0.1541	0.0863	7.9695	6.872	362.63	342
135.0	18,225	12,620,000	23,493,000	4.4914	0.0171	0.1747	0.0984	8.8129	7.382		
140.0	19,600	14,247,000	26,645,000	4.6578	0.0159	0.1973	0.1116	9.8366	7.955	451.06	415.9
144.9										502	
145.0	21,025	16,019,000	30,084,000	4.8242	0.0148	0.2218	0.1260	11.1072	8.604	503.42	
150.0	22,500	17,931,000		4.9905	0.0138	0.2483			9.346		502

$$\frac{d\ell}{dv} = \frac{0.03327v - \frac{417.41}{v^2}}{0.5481 - 1.3846\times10^{-8}\,v^{\frac{10}{3}} - 0.4187\times10^{-8}\,v^{3.46}}$$

$\theta = 33^\circ 14'$
$\sin\theta = 0.5481$
$\cos\theta = 0.8365$
$L = 201.7'$

v	v^2	$v^{\frac{10}{3}}$	$v^{3.46}$	$0.03327v$	$\frac{417.41}{v^2}$	$1.3846\times10^{-8}\,v^{\frac{10}{3}}$	$0.4187\times10^{-8}\,v^{3.46}$	Air resistance $\frac{d\ell}{dv}$	No air resistance $\frac{d\ell}{dv}$	Air resistance ℓ	No air resistance ℓ
144.9	20,996	15,908,000	30,012,000	4.8208	0.0199	0.2213	0.1257	23.8732		0.0	
146.0	21,316	16,387,000	30,807,000	4.857	0.0196	0.2269	0.1290	25.1686		26.97	
148.0	21,904	17,147,800	32,292,000	4.924	0.0191	0.2374	0.1352	27.9481			
150.0	22,500	17,931,000	33,828,000	4.990	0.0185	0.2483	0.1416	31.485	16.584	139.23	0.0
152.0	23,104	18,741,800	35,413,000	5.057	0.0181	0.2595	0.1483	35.9152	17.460	206.57	
154.0	23,716	19,576,700		5.1236	0.0176	0.2711			18.433		69.90
156.0	24,336	20,436,000		5.1901	0.0172	0.2830			19.513		
158.0	24,964	21,323,300		5.2567	0.0167	0.2952			20.718		148.04
160.0	25,600	22,237,000		5.3232	0.0163	0.3079			22.094		
162.0	26,244	23,176,700		5.3897	0.0159	0.3209			23.652		236.53
151.86										201.7	
160.46											202

$$\frac{d\ell}{dV} = \frac{\dfrac{\alpha V}{g} - \dfrac{Q_o}{V^2}\cos\theta}{\sin\theta - \dfrac{n^2V^2}{2.2082\,R_{\mu=o}^{4/3}} - \dfrac{0.0000364\,V^2}{R_{\mu=o}^{1.46}}} \qquad (14)$$

Given: Concrete chute, rectangular cross section, 8 feet wide, angle of floor with horizontal 45 degrees, discharge 1,600 second-feet.

To find: Average velocity at any section including air resistance and average velocity if no air resistance is assumed.

Solution: Assume $n = 0.012$; $\alpha = 1.16$; water enters at critical depth, $D_c = 10.75$; $V_c = 18.6$ feet per second.

$$R_{\mu=o} = \frac{8D}{8 + 2D} = \frac{1,600}{V\left(8 + \dfrac{400}{V}\right)} = \frac{200}{V + 50}$$

$$\frac{d\ell}{dV} = \frac{0.036070\,V - \dfrac{141.42}{V^2}}{0.7071 - 0.000,000,055,755\,V^2(V + 50)^{4/3} - 0.000,000,015,908\,V^2(V + 50)^{1.46}}$$

The solution of this equation is shown in table (8). The value of ℓ is determined from $\frac{d\ell}{dV}$ either by the method outlined in section 5 or by Simpson's rule. The terminal velocities with the air resistance term and without the air resistance term are found to be 101 feet per second and 117.3 feet per second, respectively. The distances down the chute at which 99 percent of the terminal velocities are reached are 946.6 feet and 1,007.2 feet for the two cases with air resistance and without air resistance, respectively. The length may be plotted against

the velocity, or against $\frac{Q_o}{V} = D_{net}$. As the water contains air, the actual depth will be greater than D_{net}.

8. Application to the design of Shasta stilling pool. At maximum reservoir elevation, the Shasta spillway discharge per foot of width is 499 second-feet, and the difference in elevation between crest and pool floor 487.5 feet. The thirteen sluiceways discharge a total of about 63,000 second-feet at maximum reservoir elevation. Assuming the flow is uniform at the entrance to the pool, the discharge per foot of width is 667 second-feet. The water discharging from the sluices has less velocity than water at the same elevation discharging over the crest, due to the greater losses in the sluiceways than down the face of the spillway. The discharge per foot of width will be increased in the neighborhood of the sluiceway outlets. As this water has less momentum per pound of water than that arriving from the crest, the average velocity of water will be less than that of the water from the crest. The increase in depth due to the increase in discharge per foot of width downstream from the sluiceway outlets tends to increase the velocity over that which would exist due to flow over the crest alone. These two factors, decrease in momentum per pound of water, and increase in hydraulic radius will work against each other. Computation of momentum and of velocity due to increased hydraulic radius indicates that the velocity at the foot of the dam will actually be less when the sluiceways are discharging with the crest, than when the crest

alone is discharging. The exact amount of decrease in velocity is impossible to determine as the angle of spread of sluiceway jet and the manner in which it will mix with the crest jet are not known. In order to be on the safe side, it will be assumed that the water enters the pool at the velocity which would occur if the sluiceways were not operating. The computations for Shasta spillway are given in table (8). The net hydraulic radius, $R_{\mu=0}$, is taken equal to the net depth Q_0/V. The curved portion leading into the pool from elevation 659.98 to the upstream end of the pool floor is taken as a straight section at an angle of 33° - 14' with the horizontal. This assumed tangent is 201.7 feet long, whereas the length down the face of the dam to elevation 659.98 is 502.0 feet long. It was assumed that the face of the dam extends up to elevation 1025.22 as a straight line of angle 51° - 20' with the horizontal. Bernoulli's equation was written between a point on the reservoir water surface and elevation 1025.22 to determine the velocity at elevation 1025.22. No losses were assumed for this calculation. If the beginning velocity is somewhat in error, it will affect the velocity at the toe of the dam very little, as $\frac{d\ell}{dV}$ is small at elevation 1025.22. The method of finding the velocity at any length down the face of the dam is the same as for the chute.

In determining the necessary tail water to maintain a jump, the discharge per foot of width and the entering velocity are sufficient. The air content of the water does not increase its

momentum appreciably, but the air content increases the depth of the entering jet and hence the static pressure existing in the jet. The static pressure is of no practical importance as compared with the momentum term, so that even when the static pressure is doubled the conjugate depth remains the same within a fraction of one percent.

The hydraulic-jump formula will be expressed so that the necessary tail water depth over the apron, D_2, is a function of Q_o, the discharge per foot of width and V_1, the entering average velocity.

$$D_2 = -\frac{Q_o}{2V} + \sqrt{\frac{2Q_o \gamma V_1}{g} + \frac{Q_o^2}{4V_1^2}} \qquad (15)$$

where $$\gamma = \frac{\int V_i^2 \, dA}{V^2 A}$$

in which γ is a correction to the momentum term required because of the velocity distribution; V_i is the velocity at area dA and A is area of cross section. This formula is correct when air is present in large quantities in the water only when the entering momentum is large compared with the static force of the jet. The exact equation is:

$$D_2^3 - D_2\left[D_1^2 \ (1-\mu) + \frac{2V_1^2}{g}\gamma D_1(1-\mu)\right] + \frac{2V_1^2}{g}\gamma D_1^2(1-\mu)^2 = 0 \qquad (16)$$

in which 100μ is the percent of air contained in the entering jet; and $D_1 = \frac{Q_o}{(1-\mu)V_1}$ is the actual depth of the entering jet. The above equation (16) is a cubic in D_2 and may be solved easily by trial.

Using equation (15): D_2 is found for Shasta stilling pool for three cases:

(a) Assuming an air resistance:

$V_1 = 151.86$ Q per foot = 667 second-feet
$\gamma = 1.045$ $D_2 = 78.97$ feet
Floor elev. = Tail water elev. 632.0 - 79.0 = elev. 553.0.

(b) Assuming no air resistance:

$V_1 = 160.46$ Q per foot = 667 second-feet
$\gamma = 1.045$ $D_2 = 81.35$ feet
Floor elev. = Tail water elev. 632.0 - 81.4 = elev. 550.6.

(c) Assuming no losses down the face of the dam

$V_1 = 181.5$ Q per foot = 667 second-feet
$\gamma = 1.045$ $D_2 = 86.88$ feet
Floor elev. = Tail water elev. 632.00 - 86.9 = elev. 545.1.

The above tail water depths are based on the assumption that the sluiceway discharge has been uniformly distributed throughout the spillway width at entrance to the pool. This assumption will be investigated by model studies.

9. Present status of freeboard computations. At present little is known about the amount of air present in water flowing at high velocities. Attempts have been made to fit the data available with formulas as follows, by the method of least squares:

$$\mu = K\, R^{\alpha}_{\mu=0}\, V^{\beta}$$

$$\mu = K(R_{\mu=0} \cos \theta)^{\alpha}$$

$$\mu = K\, R^{\alpha}(\cos \theta)^{\beta}$$

in which α, β, and K are constants.

None of the above formulas would fit the data to a satisfactory degree. Either the data are not reliable enough, or the type of formula is incorrect.

In determining the amount of freeboard, vertical curves will usually be critical points. The maximum velocity and not the average velocity, should be used in computing the trajectory to design the shape of vertical curve. The maximum velocity might well be taken as 1.5 times the average velocity of the section as an additional safeguard. The highest air content found in the Kittitas data is 60 percent. It would seem reasonable that the air content would be directly proportional to some power of the velocity, and inversely proportional to some function of the hydraulic radius and slope of the bottom of the canal. This has not been verified, however, by the existing data.

10. Conclusions. It has been shown that the time distances between centers of gravity of the curves on the oscillogram may be used to compute the average velocities and the time distances between beginning points on the curves to obtain maximum velocities. The present oscillograph is not satisfactory for taking records of high velocity flow. It should be completely rebuilt, or a new oscillograph obtained which is more adaptable to high velocity measurements. With an improved oscillograph, the maximum and average velocities for a section should be obtained within reasonable limits. These two values would then give some picture of the velocity distribution.

In section 3, the importance of using a correct

interpretation of the hydraulic radius in connection with the Manning and Chezy formulas has been discussed. The hydraulic radius must be defined as the area of cross section, assuming no air in the water, divided by the total wetted perimeter including air. This permits the customary value of coefficient to be used. The Manning or Chezy formulas, however, have no means of expressing air resistance. The nature of air resistance is discussed and a correction term for air resistance in the Manning formula proposed. Coefficients for the new air resistance term are evaluated from the Kittitas data. Applications of the formula to the design of a typical steep chute and to Shasta stilling pool are given. The unimportance of air content in the entering jet in the hydraulic jump is emphasized. Shasta stilling pool floor elevations are computed for three cases:

(1) Flow down the spillway computed by the usual Manning formula.
(2) Flow using the Manning formula with an additional air resistance term.
(3) Assuming no losses down the face of the dam.

The relationship among the factors, velocity, hydraulic radius, and slope of channel or spillway to the air content has not as yet been determined.

Many additional experimental data are needed to further refine the coefficients obtained from the Kittitas investigation and to extend and confirm the application of the Kittitas data to flow down spillways. It is most important that further experimental investigations be carefully planned, particularly with regard to how the data are to be used. Tests should be made on the highest

available spillways and on other chutes.

11. Acknowledgements. The preparation of this memorandum was under the direct supervision of J. E. Warnock. Valuable suggestions and aid in computations were given by D. J. Hebert. The basis for the present method of treatment of high velocity flow was taken from a memorandum to the Chief Designing Engineer by D. J. Hebert and V. L. Streeter, August 30, 1938. C. W. Thomas assisted in the computation of average velocities from the Kittitas oscillograms. The data on which this report is based came from the first progress report on the same subject by C. W. Thomas.

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