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MEMORANDUM TO CHIEF DESIGNING ENGINEER  
SUBJECT: FORCES EXERTED BY SEEPAGE WATER

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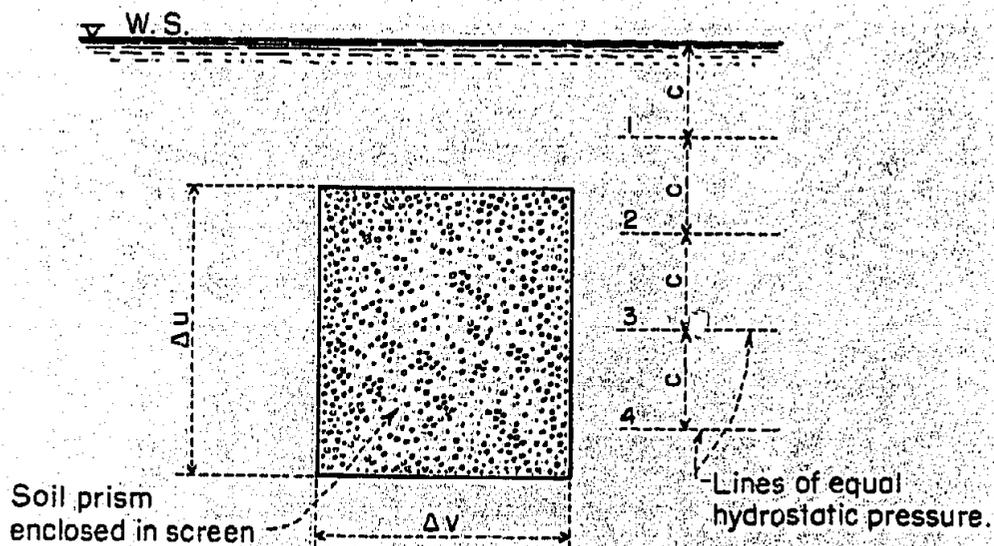
## FORCES EXERTED BY SEEPAGE WATER

The purpose of the following discussion is to develop the relations describing the pressures transmitted to soil particles by water flowing through them, and to present these relations in such a form that they will be readily applicable to a semi-graphic calculation of the forces tending to produce sliding in embankments.

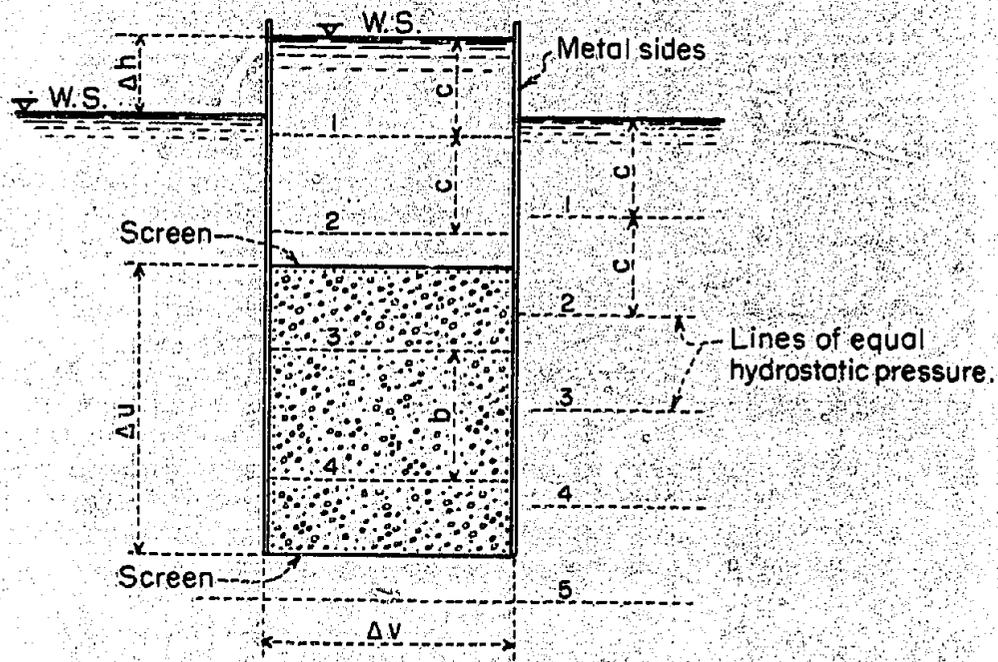
The analysis is not concerned with the determination of piezometric pressures within the water in any sense, but is confined to a study of the effects of such pressures upon the forces acting between soil particles. It is therefore assumed that the locations of lines of equal hydrostatic pressure within the soil mass are known either from theoretical considerations or from experimental data.

### NOMENCLATURE

- $\alpha$  = angle between an equal pressure line and the horizontal  
 $b$  = the spacing of equal pressure lines in percolating water  
 $c$  = an arbitrary constant = the spacing of equal pressure lines in static water  
 $\gamma$  = unit weight of water  
 $\gamma'$  = unit weight of a solid soil mass (no voids)  
 $\gamma_1$  = unit weight of an imaginary static liquid, equivalent in buoyancy to percolating water  
 $e$  = void ratio =  $\frac{\gamma'}{\gamma} - 1$   
 $h$  = piezometric head  
 $p$  = unit hydrostatic (piezometric) pressure =  $\gamma h$   
 $p'$  = pressure between soil particles (contact pressure) across a unit of gross area including voids plus solids  
 $\Delta p$  = difference between the average unit fluid pressures on the ends of the small, finite soil prism  
 $P$  = total hydrostatic pressure on a given area  
 $P'$  = total pressure between soil particles (contact pressure) across a given area  
 $p'_u$  = component of soil pressure in the  $u$  direction  
 $p'_v$  = component of soil pressure in the  $v$  direction  
 $s'$  = shearing or tangential stress between soil particles on a unit of gross area including voids plus solids  
 $u$  = the dimension or direction perpendicular to lines of equal hydrostatic pressure  
 $v$  = the dimension or direction parallel to lines of equal hydrostatic pressure  
 $\Delta u, \Delta v, \text{ and } l$  = the dimensions of the small finite soil prism  
 $\Delta V$  = the gross volume of the small finite soil prism  
 $w$  = wet unit weight of soil, voids filled with water  
 $w'$  = dry unit weight of the soil based on the unit of the gross volume including voids



**FIG. 1 - SOIL PRISM IN STATIC WATER**



**FIG. 2 - SOIL PRISM SUBJECTED TO VERTICAL FLOW**

To provide a readily visualized approach to the more general case of percolation through an earth dam, the special cases of no flow and of vertical flow through a soil prism immersed in water are considered (figure 1). The method was suggested by that of L. F. Harza in an article relating to the same subject<sup>1</sup>. The soil is first considered as if confined by a pervious container such as a screen.

It is clear that the force required to suspend the soil prism,  $\Delta V$  ( $= l \times u \times \Delta v$ ), in static water is equal to the wet weight of that prism diminished by the difference between the pressures acting upon the full areas of the ends including voids. If the unit pressure difference is  $\Delta p$ , then the force required to suspend the soil prism in water is

$$P' = w \Delta V - \Delta p \Delta v. \quad (1)$$

This is fully consistent with a concept which treats of the buoyant effects on individual particles. According to the principle of Archimedes, the force required to suspend a solid in a liquid is equal to the dry weight of the solid diminished by the weight of the displaced liquid. But  $w' \Delta V$  is the dry weight of the soil, and  $\frac{w'}{\gamma'} \gamma \Delta V$  is the weight of the volume of water equal to the volume occupied by the soil particles so that

$$P' = w' \Delta V - \frac{w'}{\gamma'} \gamma \Delta V \quad (2)$$

However,

$$w' \Delta V - \frac{w'}{\gamma'} \gamma \Delta V = w \Delta V - \gamma \Delta u \Delta v; \quad (3)$$

and since in this case  $\Delta p = \gamma \Delta u$ , it is apparent that equation (1) is identical with equation (2).

These results may be interpreted to mean that the water in the voids is completely supported by its contact with the water external to the prism and that, in addition, the water in the voids completely surrounds each particle and exerts an upward force against it proportional to the volume of the particle. The unbalanced portion of the weight of each soil particle is transmitted downward through successive layers of particles across surfaces of mutual contact (neglecting vertical friction along the sides of the container), with the result that the pressures between particles accumulate until at the bottom the force between the particles and the supporting screen aggregates  $P'$ .

It might be argued that if the particles are in contact, they cannot be completely surrounded by water and that as a consequence the

<sup>1</sup>"Uplift and Seepage under Dams," Proc., A.S.C.E., September, 1934.

full buoyant effects of the water are not attained. Terzaghi<sup>2</sup>, however, cites experimental evidence to show that the hydrostatic pressures against solid walls are diminished only in the order of 2% by the presence of clay bearing against such walls, and that for other granular materials the effects of the areas of contact are practically zero. The reduction in the buoyant effects on particles caused by the exclusion of water pressure from the areas of contact between them may therefore be reasonably neglected.

Suppose now, that the soil prism is enclosed in metal side walls with screens at top and bottom and that in addition a flow is maintained through the sand by the expedient of holding the water surface over the upper end at a higher elevation than that over the lower end (figure 2).

Neglecting the weight of the container, the force  $P'$  required to suspend the soil prism is now obviously the total weight of the soil and water in the container less the total water pressure across the bottom of the container. Since the weight of the water above the soil prism is simply the total pressure across the container at the top screen, the suspending force,  $P'$ , may again be said to equal the saturated weight of the soil prism less the difference between the total pressures acting on the two end screens, or

$$P' = w \Delta V - \Delta p \Delta v \quad (1)$$

In this case, however,  $\Delta p$  is no longer equal to  $\gamma \Delta u$  as in equation (3), but may have any value,  $\gamma (\Delta u - \Delta h)$ . Archimedes' principle is therefore no longer applicable.

In order to introduce the concept of friction forces, the foregoing condition may be analyzed from the point of view of energy loss. In passing from the top to the bottom of the soil prism, a slug of water of volume  $\epsilon \frac{w'}{\gamma'} \Delta V$  loses energy of position equal in amount to  $\epsilon \frac{w'}{\gamma'} \Delta V \gamma \Delta u$  and gains energy of pressure equal to  $\epsilon \frac{w'}{\gamma'} \Delta V \Delta p$ . If the net loss of energy is set equal to the work done by the friction forces  $F$ , tending to resist flow,

$$F \Delta u = \epsilon \frac{w'}{\gamma'} \Delta V \gamma \Delta u - \epsilon \frac{w'}{\gamma'} \Delta V \Delta p,$$

or

$$F = \frac{w'}{\gamma'} \gamma \Delta V - \epsilon \frac{w'}{\gamma'} \Delta V \Delta p. \quad (4)$$

The force of friction transmitted to the particles and thus to the screen at the bottom is seen from equation (4) to be equivalent to the weight of the water in the voids diminished by the difference between the pressures on the void areas at the two ends, the void area being

<sup>2</sup>"Large Retaining Wall Tests", Engr. News-Rec; Feb. 1, 22, Mar. 8, 29, and Apr. 19, 1934. Specifically, Part III, "Action of Water on Fine-Grained Soils," Mar. 8. pp. 316-317.

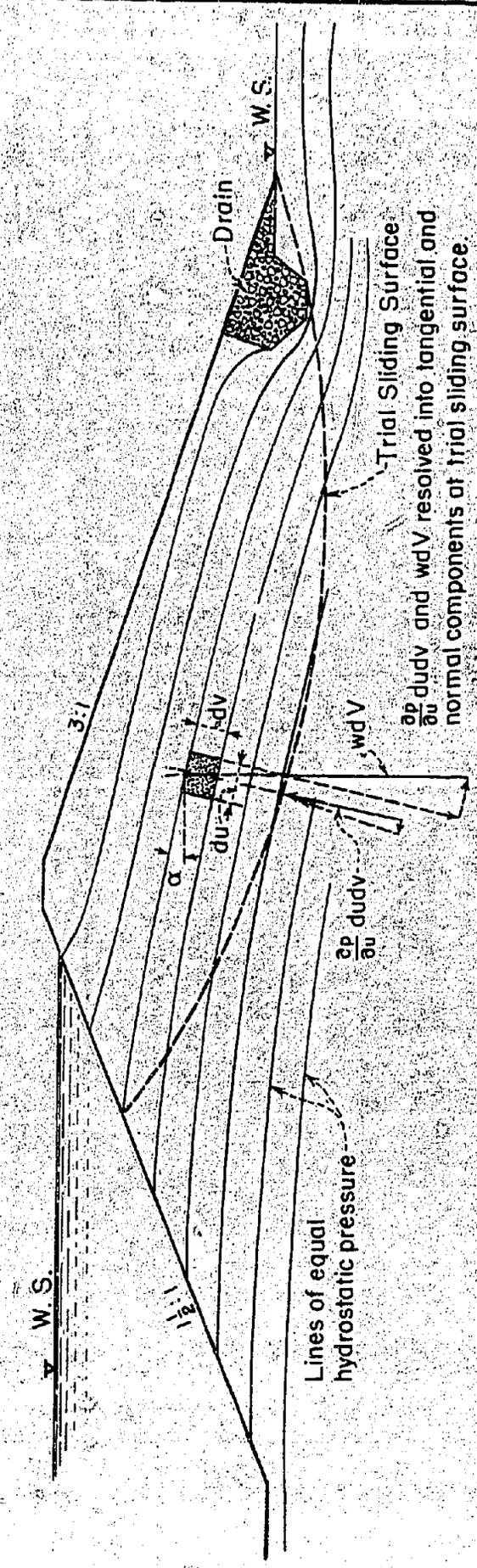
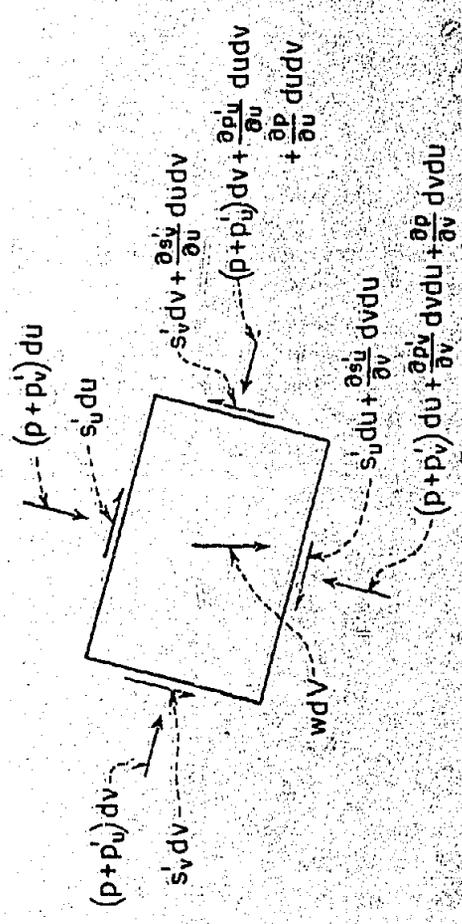


FIG. 3 - SEEPAGE FORCES IN AN EARTH DAM

defined as an area equivalent to that which would be occupied by the cross sections of the voids if the voids were all straight vertical tubes.

But the total weight,  $P'$ , supported by the screen at the bottom is

$$P' = w' \Delta V + \frac{w'}{\gamma} \gamma \Delta V - \frac{w'}{\gamma} \Delta V \Delta p - \frac{w'}{\gamma} \Delta V \Delta p. \quad (5)$$

This is simply equation (1) broken down into solid and liquid parts. Subtracting (4) from (5), we see that

$$P' - F = w' \Delta V - \frac{w'}{\gamma} \Delta V \Delta p, \quad (6)$$

or that, in addition to the friction forces tending to produce pressure between the grains, there must be taken into consideration the weight of the grains diminished by an upward force on the grains proportional to the pressure gradient.

The foregoing discussion is confined to the special case in which flow is parallel to the direction of gravity. In ground water and earth dam problems this will be the exception rather than the rule. A typical flow net for the model of an earth dam on an impervious foundation is shown in figure 3. As may be seen, the directions of flow bear little relation to the directions of the lines of equal pressure.

To generalize the argument suggested above, consider the equilibrium of a soil prism of convenient length,  $du$ , and width  $dv$  (thickness unity), with sides parallel and perpendicular respectively to lines of equal pressure (see figure 3). The soil in the prism is restrained on all sides by the contacts between soil particles and is acted upon by the hydrostatic pressures and its own weight. The water in the prism is acted upon by hydrostatic pressures, by the resistance of the soil particles to change in hydrostatic pressure, and its own weight.

If we isolate the prism with its soil and water content intact and apply the general equations of static equilibrium, the forces transmitted from the water to the soil are readily determined. For static equilibrium, (i.e., assuming that no masses are being accelerated) the sum of the components in any direction of all the forces acting on the body must be zero. Adding the components in the  $u$  direction, therefore, yields

$$\frac{\partial P' u}{\partial u} du dv + \frac{\partial S' u}{\partial v} dv du + \frac{\partial P}{\partial u} du dv + w du dv \sin \alpha = 0,$$

or, since  $\frac{\partial P}{\partial u} = 0$ ,

$$\frac{\partial P' u}{\partial u} dv + \frac{\partial S' u}{\partial v} = -w \sin \alpha dv. \quad (7)$$

Similarly, considering the components in the v direction, there results

$$\frac{\partial p'_v}{\partial v} dv du + \frac{\partial s'_v}{\partial u} du dv + \frac{\partial p}{\partial v} dv du + w dv du \cos \alpha = 0,$$

or,

$$\frac{\partial p'_v}{\partial v} dv + \frac{\partial s'_v}{\partial u} dv = -\frac{\partial p}{\partial v} dv - w \cos \alpha dv. \quad (8)$$

Of these two equations, the right hand members represent the forces acting on the prism due to gravity and to changes in the hydrostatic pressure. Adding the two equations vectorially, there results

$$\left( \frac{\partial p'_u}{\partial u} dv + \frac{\partial s'_u}{\partial v} dv \right) + \left( \frac{\partial p'_v}{\partial v} dv + \frac{\partial s'_v}{\partial u} dv \right) = w \sin \alpha dv + \left( -\frac{\partial p}{\partial v} dv - w \cos \alpha dv \right) \quad (9)$$

The right and left hand members, however, need be concurrent only if

$$\frac{\partial s'_u}{\partial v} \text{ and } \frac{\partial s'_v}{\partial u} \text{ are zero, which is unlikely.}$$

In making a graphical analysis, it is nevertheless convenient to neglect  $\frac{\partial s'_u}{\partial v}$  and  $\frac{\partial s'_v}{\partial u}$ . The error of this procedure is

of the same character as that introduced by assuming in the absence of percolating water that the weight of the soil is transmitted vertically downward indefinitely in the same line of action. As Terzaghi has pointed out,<sup>3</sup> its effect is only to cause a slight redistribution of pressures without affecting their total. For practical purposes, therefore, the increment in soil pressure or contact pressures through the soil prism,  $du dv$ , is equal to and has the opposite direction to the vector sum comprising the right hand member of equation (9), and may be considered to act through the center of gravity of the prism.

To obtain the total pressures between soil particles along a typical trial sliding surface in an embankment it is therefore only necessary to divide the area intercepted by the sliding arc into convenient finite rectangular elements, to determine the forces due to hydrostatic pressure difference (corresponding to  $\frac{\partial p}{\partial v} dv$ ) from the equal pressure lines and the forces of gravity from the wet weights,

<sup>3</sup>"The Mechanics of Shear Failures on Clay Slopes and the Creep of Retaining Walls", Public Roads, Dec., 1929.

and to add their several effects vectorially. In fig. 3, the forces,  $\frac{\partial p}{\partial v} du dv = \frac{\partial p}{\partial v} dv$  and  $w dv$  are shown projected to the trial sliding surface and there resolved into their tangential and normal components. In practice, certain simplifications of this procedure are used, but results are easily shown to be identical.