UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

ANALOG AND ANALYTICAL STUDIES OF FLOW CONDITIONS IN THE CHANNEL BELOW OAKVILLE DAM--LOWER NUECES RIVER POWER PROJECT

Hydraulic Laboratory Report No. Hyd-333

ENGINEERING LABORATORIES

DEPARTMENT OF THE INTERIOR

OFFICE OF THE ASSISTANT COMMISSIONER AND CHIEF ENGINEER
DENVER, COLORADO

November 9, 1951

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Memorandum

Denver, Colorado
November 9, 1951

Chief Designing Engineer

Through: Chief, Engineering Laboratories Branch

R. E. Glover

Analytical check of flow conditions in the proposed channel below Oakville Dam--Lower Nueces River Project

1. Purpose

These computations were made to provide an independent check upon the analog data previously obtained. Another purpose was to estimate maximum and minimum water depths in the channel since the idealization required to adapt the data to analog treatment made it possible to obtain values for the fluctuations of level but did not give corresponding values for the water depths. Details of these computations and a summary of results are given in the following paragraphs.

2. Analytical Check

In order to bring the problem within reach of analytical treatment, it is necessary to assimilate the actual channel to one of uniform width and to assume for it a constant top width. It is also necessary to select a representative flow condition as a basis for estimating the friction losses. In addition it is expedient to make the friction losses proportional to the velocity of flow to avoid complicating non-linearities.

It will be noted that this idealization is very similar to that used as a basis for setting up the analog because the choice of representative flows and a corresponding linearized resistance is present in both cases. The analog has the advantage that a channel of varying cross sections can be represented while the analytical procedure is limited to treatment of a uniform channel.

Because most of the storage occurs in the dredged channel the idealized uniform channel is chosen to represent this reach as nearly as possible. The effect of the spillway channel storage is neglected. The computation is made in two parts. The first part is concerned only with the effect of variation about the mean flow while the second part is concerned with the effect of the mean flow. The methods used will now be described.

For the oscillating case let:

A represent the area of a channel cross section
C represent Chezy's constant
H the channel depth at the center
F the representative flow

g the acceleration of gravity
i the surface gradient due to F flowing at the depth H
Q the flow in the channel
R the hydraulic radius
t time
u a variable of integration
V velocity
W surface width
Y the elevation of the water surface above a horizontal datum

$K = \frac{F}{iW}$

The condition of continuity is expressed by the relation:

$$\frac{\partial Q}{\partial X} + W\frac{\partial y}{\partial t} = 0 \tag{1}$$

The flow relation is

$$Q = -\frac{F}{i}\frac{\partial y}{\partial x} \tag{2}$$

A substitution of Equation (2) into Equation (1) gives, after rearrangement:

$$\frac{\partial y}{\partial t} = \frac{F}{iW}\frac{\partial^2 y}{\partial x^2} \tag{3}$$

A solution of Equation (3) which satisfies the conditions:

$Q = Q_o$ when $x = 0$ for $t > 0$

$Y = 0$ for $x > 0$ when $t = 0$

is

$$Y = +\frac{Q_o x i}{F}\frac{1}{\sqrt{\pi}}\int_{\frac{x}{\sqrt{4Kt}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du \tag{4}$$

or

$$Y = +\frac{Q_o x i}{F}\,I \tag{5}$$

where

$$I = \frac{1}{\sqrt{\pi}} \int_{\frac{x}{\sqrt{4Kt}}}^{\infty} \frac{e^{-u^2}}{u^2} du \qquad (6)$$

This integral can be evaluated in terms or tabulated functions in the form:

$$I = \frac{1}{\sqrt{\pi}} \frac{e^{-\frac{x^2}{4Kt}}}{\left(\frac{x}{\sqrt{4Kt}}\right)} - \frac{2}{\sqrt{\pi}} \int_{\frac{x}{\sqrt{4Kt}}}^{\infty} e^{-u^2} du \qquad (7)$$

The second term in the right-hand member of this equation is expressible in terms of the "probability integral" which has been extensively tabulated.

If x approaches zero while t is greater than zero a limiting form which gives the rise of the water level at $x = 0$ is found to be:

$$Y_o = Q_o \sqrt{\frac{4it}{\pi FW}} \qquad (8)$$

In order to evaluate the water level fluctuations at the power plant we proceed as follows:

We choose

$F = 2,400$ ft^3/sec
$H = 7.0$ ft
Average bottom width 125 feet
Side slopes 2 horizontal on 1 vertical
Average top width $W = 153$ feet
Area of cross section 973 ft^2
Velocity corresponding to $F = 2,400$ is 2.467 ft/sec
The hydraulic radius is $R = 6.23$ feet
For a Kutters "n" of 0.030 the Chezy C value would be about 71.

then

$$i = \frac{V^2}{C^2 R} = \frac{(2.467)^2}{(71)^2 (6.23)} = 0.000194$$

also

$$\sqrt{\frac{4i}{\pi\ FW}} = \sqrt{\frac{0.000776}{1153596}} = \sqrt{(672.68)\ (10)^{-12}} = (25.93)\ (10)^{-6}$$

In order to isolate the fluctuations of level we will superimpose two flow patterns. The first pattern will be a flow of 4800 ft^3/sec which continues during the first 4 hours of each 24-hour period. The second pattern is a flow of -800 ft^3/sec which is continuous. These solutions obey the law of superposition. In the making of the computation, it will therefore be assumed that once a flow is begun it persists forever. The necessary changes will be obtained by adding or subtracting new flows as required. The first pattern for example will be obtained by imposing a flow of +4,800 ft^3/sec at the beginning of each 24 hours and superimposing a flow of -4,800 ft^3/sec 4 hours thereafter. The detail of the computation is shown in Table 1 below.

Table 1

COMPUTATION OF WATER LEVEL VARIATIONS AT THE OAKVILLE POWER PLANT BASED ON F = 2,400 ft^3/sec H = 7.0 FEET

Time hours	Time seconds	$\sqrt{t}$	F	F_1	Y_o feet	Variation of level feet
0.	0	0	+4, 300	-800	0	
4.	14400	120.000	-4, 800		+12.446	
20.	72000	268.328				
24.	86400	293.939	+4, 800		-2.910	15.356
28.	100800	317.490	-4, 800		+11.281	
44.	158400	397.994				
48.	172800	415.692	+4, 800		-3.232	14.514
52.	187200	432.666	-4, 800		+11.004	
68.	244800	494.772				
72.	259200	509.116	+4, 800		-3.385	14.389

$$(4,800)(25.93)(10)^{-6} = 0.124464$$
$$(800)(25.93)(10)^{-6} = 0.020744$$

Some idea of the difference which another choice of F and H could have can be obtained by comparing these results with the choice

$$F = 3,600 \text{ ft}^3/\text{sec}$$
$$H = 10 \text{ ft}$$

then

$$A = 1450 \text{ ft}^2$$
$$W = 165 \text{ ft}$$
$$V = 2.482 \text{ ft/sec}$$
$$R = 8.543$$
$$C = 76.7 \text{ for "n"} = 0.030$$

and

$$i = \frac{V^2}{C^2R} = 0.00012258$$

$$\sqrt{\frac{4i}{\pi FW}} = \sqrt{\frac{.00049032}{1866104}} = \sqrt{(262.75)(10)^{-12}} = (16.209)(10)^{-6}$$

With this choice the variations of level would be reduced to 9.600, 9.073 and 8.994 feet, respectively. The last of these figures would most nearly represent the fluctuations in level after a number of cycles of operation. These estimates of level fluctuations are probably on the high side because the surface area of the spillway channel was neglected and because the widest part of the proposed channel is adjacent to the power plant.

The second part of the computation, namely, that relating to the level about which the fluctuations take place, can now be made. The depth at which a steady flow of 800 cubic feet per second will flow in the improved channel below the dredged section will first be made. It will next be assumed that this average depth exists at the outlet of the dredged section and the depth at the power plant will then be computed by use of Bresse's tables, if necessary. For the dredged section the writer obtained a depth of 4.3 feet and for the channel downstream of the dredged section 4.4 feet. Using a different formula Mr. E. J. Rusho obtained 4.05 feet for the depth at which 800 cubic feet per second would flow in the 170-foot bottom width section at the plant if the water surface were parallel to the bottom. As an average for the dredged section he obtained 4.80 feet and for the channel downstream of the dredged section 4.36 feet. These values seemed close enough together to make a computation of backwater curves unnecessary.

The flow variation at the lower end of the dredged channel may be computed from the idealized relation:

$$Q = \frac{-F}{i} \frac{\partial y}{\partial x} \qquad (2)$$

with

$$\frac{\partial y}{\partial x} = -i \frac{2}{\sqrt{\pi}} \int_{\frac{x}{\sqrt{4Kt}}}^{\infty} e^{-u^2}\, du \qquad (9)$$

and

$$K = \frac{F}{iW} = \frac{2400}{(.000194)(153)} = \frac{2400}{0.029682} = 80860$$

$$\sqrt{4K} = \sqrt{323440} = 568.717$$

$$\frac{x}{\sqrt{4K}} = \frac{51744}{568.717} = 90.983$$

These computations are shown in the following table:

Table 2

COMPUTATION OF FLOW AT MILE 87.0
APPROXIMATELY 10 MILES DOWNSTREAM FROM THE POWER PLANT

Time hour	$\sqrt{t}$ seconds	$\frac{x}{\sqrt{4Kt}}$	$\frac{2}{\sqrt{\pi}} \int_{\frac{x}{\sqrt{4Kt}}}^{\infty} e^{-u^2} du$	F	Q ft³/sec	Remarks
0	0	D	0.0000	+4,800	0	
4	120.00	0.758	0.2837	-4,800	1362	
8	169.70	0.536	0.4484		790	
12	207.85	0.438	0.5356		418	
16	240.00	0.379	0.5920		271	
20	268.33	0.339	0.6316		190	
24	293.94	0.310	0.6611	+4,800	142	
28	317.50	0.286	0.6859	-4,800	1481	
32	339.41	0.268	0.7047		880	
36	360.00	0.253	0.7205		494	
40	379.48	0.240	0.7343		337	
44	397.99	0.229	0.7460		246	
48	415.69	0.219	0.7568	+4,800	194	
52	432.67	0.210	0.7665	-4,800	1528	
56	449.00	0.203	0.7740		917	Cycle
60	464.76	0.196	0.7816		531	Average
64	480.00	0.190	0.7882		369	640 cu. ft.
68	494.77	0.184	0.7947		277	per second
72	509.11	0.179	0.8002	+4,800	220	

3. Summary of Results.

A summary of analog and computed results are shown in Table 3 below:

Table 3

SUMMARY OF ANALOG AND COMPUTED DATA

Quantity	Analog	Computation
Fluctuations of water level at the power plant (mile 96.8)	9.7 feet	14.4 1/ and 9.0 2/ feet
Minimum and maximum water depths at the power plant		+0.9 to 15.3 1/ +2.2 to 11.2 2/
Mean depths with a flow of 800 cubic feet per second. In dredged channel (mile 87.0 to 96.8)		4.3 3/ feet
In improved river channel (downstream of mile 87.0)		4.3 feet
Flow variations at the downstream end of the dredged section (mile 87.0)	1040 ft^3/sec	1308 ft^3/sec
Flow variation 20 miles downstream from plant (mile 76.8)	365 ft^3/sec	
Flow at the downstream end of the dredged section (mile 87.0)		380 to 1688 4/ ft^3/sec

1/ With representative flow of 2,400 ft^3/sec and 7.0 feet depth.

2/ With representative flow at 3,600 ft^3/sec and 10.0 feet depth..

3/ Mr. Rusho estimates 4.05' depth at plant with 170-foot bottom width, 4.80 feet for the average dredged channel section and 4.36 feet depth in the improved channel downstream of mile 87.

4/ Computed values increased 160 ft^3/sec to bring average to 800 ft^3/sec. Flow cycle assumed at the Power Plant is 4800 ft^3/sec. for 4 hours and 0 for 20 hours in both cases.

4. Check computations:

These computations have been checked by Messrs. E. J. Rusho and Q. L. Florey.

II

Memorandum
Chief Designing Engineer
K. O. Vartia
R. E. Glover and C. R. Daum

Denver, Colorado
October 23, 1951
Through: Acting Head, Engineering Laboratories Branch

Study of flow conditions in the proposed channel below Oakville Dam--Lower Nueces River Project

Introduction

1. Present studies contemplate the use of the power plant at the Oakville Dam as a peaking plant. The mean flow of the river will be about 800 second feet during the summer months, but power plant operation may release flows of as much as 4,800 cubic feet per second into the channel. The present studies are being made to determine the fluctuations of level caused by such releases and to arrive at maximum flow rates in the lower channel. These flow rates are of interest because, if excessive, they may lead to erosion of the channel.

2. Following an exchange of correspondence between the Regional Director and the Chief Engineer, Mr. Vartia came to Denver on October 15. Upon his arrival consideration was given to the question of ways and means for obtaining the needed data in the most expeditious manner. Computation procedures, analog studies, and model studies were discussed. Computation procedures appeared undesirable because of the complications introduced by wave motion and friction. Model studies would probably provide the best answer in a technical sense but would be expensive and would consume a good deal of time. Analog studies were chosen because they promised a quick evaluation with an accuracy sufficient for present planning purposes.

Analog Design

3. Reference is made to the paper on "Application of an Analog Computer to the Hydraulic Problems of the Sacramento-San Joaquin Delta in California" of which a copy is attached. The methods there described were applied to the approximately 20 miles of channel below the dam. Of this length the first 10 miles of channel were assumed to be dredged. The dredged portion was assumed to have a base width of 170 feet at the power plant and to taper to a base width of 80 feet at the downstream end. Side slopes of two horizontal to one vertical were assumed. A Kutter's "n" of 0.030 was used for the dredged channel and the improved stream channel. Elsewhere an "n" of 0.050 was used. The channel was divided into twenty 1-mile reaches and the spillway channel was also included.

TABLE I

COMPUTATION OF ANALOG QUANTITIES

Main Channel

Mile	Effective flow ft^3/sec	Effective water depth ft	Average width ft	Area ft^2	Effective velocity ft/sec	Wetted perimeter ft	Hydraulic radius ft	Chezy's "C"	Resistance ohms	Top width ft	Capacitance micro-farads
1	3000	7.0	184	1288	2.329	201	6.41	72	116	198	1.21
2	2820	7.0	174	1218	2.315	191	6.38	72	123	188	1.15
3	2640	7.0	164	1148	2.300	181	6.34	72	130	178	1.09
4	2460	7.0	154	1078	2.282	171	6.30	72	139	168	1.025
5	2280	7.0	144	1008	2.261	161	6.26	72	148	158	0.965
6	2100	7.0	134	938	2.238	151	6.21	72	157	148	0.905
7	1920	7.0	124	868	2.211	141	6.16	72	171	138	0.84
8	1740	7.0	114	798	2.180	131	6.09	71	185	128	0.78
9	1560	7.0	104	728	2.142	121	6.02	71	212	118	0.72
10	1380	7.0	94	658	2.097	111	5.93	71	229	108	0.66
11	1200	7.0	94	658	1.823	111	5.93	71	199	107	0.655
12	1100	6.75	93.5	631	1.743	110	5.74	45	508	106	0.65
13	1000	6.50	93.0	604	1.655	109	5.55	44	547	105	0.64
14	900	6.25	92.5	578	1.557	108	5.35	43	581	104	0.635
15	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635
16	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635
17	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635
18	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635
19	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635
20	800	6.00	92.0	552	1.449	107	5.16	43	587	104	0.635

Spillway Channel

Channel	--	6.00	112	672	2.00	127	5.3	68	318	124	0.76
Pool	--	6.00	562	--	--	--	--	--	--	574	0.93

4. The results obtained from the analog are shown in the following table. These are based on a plant operation which causes 4,800 cubic feet per second to be released into the channel in a 4 hour period with no flow for the remainder of the 24 hours in each day. This is the worst condition that may be expected to materialize.

TABLE II

RESULTS OF ANALOG OPERATION

Mile	Water level fluctuation ft	Flow variation ft^3/sec	Total average head loss to end of 20-mile section ft
1	9.7	4250	12.75
2	8.6	4125	12.6
3	7.9	2950	12.4
4	7.8	2230	12.2
5	6.6	2020	12.0
6	6.2	1780	11.75
7	5.8	1630	11.5
8	5.35	1360	11.2
9	5.1	1250	10.8
10	4.7	1040	10.5
11	4.4	865	9.5
12	4.1	705	9.1
13	3.5	600	8.1
14	3.1	495	7.2
15	2.55	440	6.2
16	2.15	405	5.2
17	1.70	375	4.3
18	1.35	375	3.3
19	.90	365	2.2
20	.50	365	1.0
Spillway			
Channel		(600 inflow) (250 outflow)	12.3
Pool	7.9		

The correlation equations chosen are:

$$Y = 1.0\,E$$

$$Q = 500,000\,I$$

$$X = 10,000\,\xi$$

$$t = 1,728,000\,\eta$$

These are appropriate for an analog in which 1/20 second represents 24 hours of prototype time. Inertia effects were found to be of minor importance and were neglected. It is for this reason that no inductances appear in the analog. In the prototype hydraulic friction losses will vary with the square of the velocity and the depth. Since the hydraulic losses in the prototype must be represented by an electrical resistance in the analog, it was necessary to select some effective flow rates as a basis for proportioning the resistance. The rates shown were chosen by Mr. Vartia. It was contemplated that if the first choice proved to be unsuitable an improved choice could be made when the first analog run was completed. This first choice, however, proved to be reasonably satisfactory. Data for the first runs were read from an oscilloscope trace. Later runs are to be recorded on an oscillograph. The analog computations and quantities are shown below. Since an oscilloscope is not adaptable for measuring direct current quantities or constant voltages the minimum values of water level and of flow in the channel could not be obtained by this means. Oscillograph recordings are therefore being made and these will be forwarded as soon as possible.

Conclusions

1. The following conclusions can be drawn from the analog studies thus far made:

a. Water level fluctuations at the plant will be about 10 feet.

b. Flows in the lower channel will vary about 360 second feet from minimum to maximum or from approximately 620 to 980 cubic feet per second.

II

APPLICATION OF AN ANALOG COMPUTER TO THE HYDRAULIC PROBLEMS OF THE SACRAMENTO-SAN JOAQUIN DELTA IN CALIFORNIA

by

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A paper for presentation
at the Meeting of the
Hydraulic Division,
American Society of Civil
Engineers, Jackson, Miss-
issippi, Nov. 1-3, 1950

APPLICATION OF AN ANALOG COMPUTER TO THE HYDRAULIC PROBLEMS
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R. E. Glover, D. J. Hebert, and C. R. Daum

The Delta area in California is a roughly triangular tract of land lying just to the east of Suisun Bay. This area, which extends for a distance of about 50 miles north and south and has a maximum width of about 25 miles, was originally a marsh with a network of channels threading through it. At the present time, this area is agricultural land which has been reclaimed by constructing dikes along the old channels to inclose areas which can be pumped out and farmed.

The Delta is traversed by the Sacramento River which enters it from the north, by the San Joaquin which comes into it from the south, and by the North and South Forks of the Mokelumne River which come in from the east. The old network of channels, which has been effectively preserved and stabilized by the process of reclamation, still carries the flow of these streams through the Delta.

Tides coming into San Francisco Bay from the Pacific Ocean propagate themselves through Suisun Bay and into the Delta channels. Since the tidal currents generally exceed the currents due to stream flow, the direction of flow in the channels are periodically reversed and a mechanism is provided for propagation of ocean salinity into them. The salinity encroachment is held in check by stream flow which tends to flush the salinity out of the channels. In times of flood the salinity is driven back but in times of low stream flow the tidal ebb and flow succeeds in carrying some salt into the channels. The presence of salinity is a matter of concern to the farmers of the Delta lands because the ground surface of these reclaimed lands or "islands" is commonly below sea level so that the gradients are such as to carry water from the channels into the islands. Construction of the Shasta Reservoir on the upper Sacramento River has made a water supply available which is desired for use on some of the lands in the San Joaquin Valley across the Delta. To supply this demand the Tracy Pumping Plant will lift water out of the Delta channels at the south end of the Delta and the water to supply these pumps must be brought across the Delta through it channels.

The problem to be solved is then how to bring the Sacramento water across the Delta to the San Joaquin side without upsetting the balance of forces which now holds the salinity in check.

Reasons for Use of an Analog

One of the first methods of attacking the problem was by means of an hydraulic model. The channels of this model were reproduced to a scale of 1:4800 horizontal and 1:100 vertical. A tide generating apparatus and means for introducing stream flow were provided. Provision was also made for extractions to represent diversions for use on the Delta lands. Dyes were introduced to represent salinity. Flow patterns in the Delta were extensively studied with this model both for historic conditions and for the anticipated conditions as altered by pumping. It also provided a means for testing analytical procedures for estimating salinity propagation. After these results were obtained, the model had served its purposes and was dismantled.

With the Tracy pumps in operation it will be necessary to increase the transfer of water from the Sacramento to the San Joaquin channels in order to replenish the water supply of the southern part of the Delta and thereby maintain a proper balance of flow. Studies of the possibilities of artificial channels connecting the Sacramento and Mokelumne channels to increase the transfer were carried out analytically using the Hardy Cross procedure for determining the division of flow and the methods previously established with the aid of the model for estimating salinity intrusion. A tidal phase difference existed at one of the sites which could be utilized to increase the transfer. Since gates would be necessary in any case for protection during floods, it would be possible to open the gates when the tidal currents were favorable and to close them when they were adverse. An analytical approach to this problem based on wave propagation formulas proved to be very difficult, and while some of these computations were actually made, the process proved to be so laborious as to make it desirable to search for some other method of solution.

The electronic analog computer built to expedite these computations not only was successful for this purpose but gave also a more rapid means of studying flow distribution in the Delta and a means for evaluating the effect of tidal currents on the effective flow resistance of the Delta channels. The appearance of the completed analog is shown in Figure 1.

Analog Requirements

In order to solve the Delta problem, it is required that the analog be able to reproduce the square law relation between friction and velocity which is characteristic of fluid flow. In addition, it is required to represent the wave motion associated with the tides. To do this, the factors of inertia and of storage due to water level changes must be accounted for. The factors employed in this analog to represent the hydraulic factors are shown in Table 1.

Table 1

CORRESPONDING HYDRAULIC AND ANALOG QUANTITIES

Hydraulic	Electrical
Quantity of flow	Current
Water surface elevations	Voltage
Inertia	Inductance
Storage	Capacity
Frictional drag	Resistance
Time	Time

Description of the Analog

The analog is designed on the basis of circuits of the type shown in Figure 2.

The inductances are air-cored coils which are either of commercial types or were wound as required. The condensers are commercial units of the paper or mica type. In the large channels, having very low frictional resistances, linear resistors, having appropriate average values for the currents flowing, were used. In some of the smaller channels, however, it was necessary to use some type of square-law resistor. This was obtained by taking advantage of certain vacuum tube characteristics which have approximately the the required form of variation. These were used with resistors in parallel and in series to obtain the desired characteristic. A biasing voltage was also required in this adjustment. The circuit used in such cases is shown in Figure 3.

A tube with two elements is used to permit current to flow in either direction. This type of resistor is not wholly satisfactory since the tubes show differences which make it necessary to adjust each one separately. The current carrying capacity is restricted within narrow limits, and it is necessary, therefore, to design the analog around these elements. Net current flows were read on d-c milliammeters. Tidal amplitudes and phase differences are read on a cathode-ray oscilloscope. The gate keeper was represented by a rectifier circuit using a 6N7 type tube. This also had some short comings near the zero point which introduced an effect analogous to gate leakage. In spite of these minor difficulties, the analog operates in a very satisfactory manner. Some idea of the speed with which the analog works may be obtained from the fact that the analog runs through about 500 days of actual tidal changes in each second of operating time.

Basic Equations

In setting up the correlation equations, the electrical circuits were assumed to have their inductance and capacity uniformly distributed along their length. In practice, these elements and the square law resistance were lumped. The inertia and storage factors were considered together, and the resistances were considered separately. For purposes of explanation, the following notation will be used:

In the hydraulic channel let:

g represent the acceleration of gravity
H the depth of the stream
L the length of a channel
M a constant applying to a channel specifying its flow resistance
Q the flow
t time
W the width of the stream at the surface
x distance along a stream
y the surface elevation above sea level
ρ weight of water per unit of volume

A longitudinal section of a stream channel is shown in Figure 4. The shaded element represents a lamina of width W, depth H, and length dx. For analytical purposes the actual channel is assimilated to a uniform rectangular channel which has the same top width and cross sectional area as the actual channel. As stated previously, frictional forces are not introduced into the dynamical equations, but are treated separately. Since x represents a distance measured along the stream from some fixed point on the bank, the planes defined by x and $x + dx$ do not change position with time. It is assumed that y is small compared to H.

The continuity condition requires that if the quantities of water flowing through the planes x and $x + dx$ differ, then the surface elevation must rise or fall as required to accommodate the changes of volume. If small quantities are neglected, this requirement is expressed by

$$W dx \frac{\partial y}{\partial t} = +Q - \left(Q + \frac{\partial Q}{\partial x} dx\right)$$

If a surface gradient $\frac{\partial y}{\partial x}$ is present, the water depth on one side of the lamina will be greater than on the other by the amount $\frac{\partial y}{\partial x} dx$ and the additional pressure due to this head differential will cause the water within the lamina to be accelerated. Thus the requirements of Newton's law are expressed to a first order of approximation by:

$$\frac{\rho WH}{g} dx \frac{\partial}{\partial t} \left(\frac{Q}{WH}\right) = -\rho WH \frac{\partial y}{\partial x} dx$$

These two equations can be simplified by cancelling common terms and collecting. Then the equation of continuity is

$$\frac{\partial Q}{\partial x} + \frac{W \partial y}{\partial t} = 0 \qquad (1)$$

and Newton's law takes the form

$$\frac{\partial y}{\partial x} + \frac{1}{gHW} \frac{\partial Q}{\partial t} = 0 \qquad (2)$$

It is of interest to note that if Q is eliminated from the two equations above, one obtains the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{gH} \frac{\partial^2 y}{\partial t^2} \qquad (3)$$

The relation between flow and gradient for the hydraulic channel can be expressed in the form

$$Q = M\sqrt{\frac{\partial y}{\partial x}} \qquad (4)$$

which may be recognized as a form of the Chezy formula.

In the electrical circuits let

C represent the capacity per unit length of circuit
E the potential with respect to ground
I current
K a constant applying to a circuit
r resistance per unit length of circuit
η time in the analog
λ inductance per unit length of circuit
ξ distance along a circuit

Then the equations for the idealized electrical circuits* which correspond to equations (1) and (2) for the hydraulic channels are:

$$\frac{\partial I}{\partial \xi} + c\,\frac{\partial E}{\partial \eta} = 0 \qquad (5)$$

$$\frac{\partial E}{\partial \xi} + \lambda\,\frac{\partial I}{\partial \eta} = 0 \qquad (6)$$

from which there is obtained on elimination of I

$$\frac{\partial^2 E}{\partial \xi^2} = \lambda c\,\frac{\partial^2 E}{\partial \eta^2} \qquad (7)$$

For the circuits provided with an electronic resistor for representation of hydraulic resistances of the type expressed by equation (4)

$$I = K\sqrt{\frac{\partial E}{\partial \xi}} \qquad (8)$$

or if the circuit has a linear resistance

$$I = \frac{1}{r}\,\frac{\partial E}{\partial \xi} \qquad (9)$$

Correlation Equations

The electronic analog operates at a frequency of 1,000 cycles per second. The sinusoidal variations imposed on the analog approximately represent tidal oscillations having a frequency of

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*See, for example, "The Theory of Sound" by Lord Rayleigh, Volume 1, Paragraph 235x, page 467. The equations (5) and (6) can be obtained from Rayleigh's equation 1 by letting R = 0, K = 0. In this form they are a simplified version of Heaviside's equations for a long line.

about two cycles per day. The correlation equations which were found suitable for use with the available electrical components are:

$$
\begin{aligned}
y &= 0.1 \; E \\
Q &= 10{,}000{,}000 \; I \\
x &= 10{,}000 \xi \\
t &= 45{,}000{,}000 \eta
\end{aligned}
$$

Other applications would, of course, require other constants. An analogous electrical quantity is obtained by substituting the above relations into the hydraulic equations: for example, equation (4) is

$$Q = M\sqrt{\frac{\partial y}{\partial x}}$$

which on substitution becomes

$$10{,}000{,}000 \; I = M\sqrt{\frac{0.1 \; \partial E}{10{,}000 \partial \xi}}$$

or

$$I = \frac{M}{3.2 \times 10^9}\sqrt{\frac{\partial E}{\partial \xi}}$$

Then the quantity $\frac{M}{3.2 \times 10^9}$ is the K value to use in the equation (8)

$$I = K\sqrt{\frac{\partial E}{\partial \xi}}$$

By this choice of constants the electrical circuit is given resistance characteristics which are analogous to the hydraulic friction in the corresponding actual channel. The other relations are treated in a similar way.

Boundary Conditions

To account for the stream flow it was necessary to introduce direct currents of specified amounts at certain points in the analog and to take them out at certain other points. In general the currents fed into the network represent river flows entering the Delta area, while currents leaving the network represent the draft of the Tracy pumps and the flow from the Delta area into Suisun Bay. To introduce these currents, voltages of controllable magnitude were introduced between the network and the ground wire (see Figure 2). Control of the currents was obtained by variable resistors located at the points where the currents enter and leave the network.

The tides were represented by alternating voltages of specified magnitude applied between the network and the ground wire at the point on the analog representing the entrance to Suisun Bay. A blocking condenser was used here to prevent the flow of direct current. The actual tides occurring at this point vary somewhat from day to day due to varying phase relations between the lunar and solar components. In the analog these tidal variations were replaced by a single sinusoidal variation of average amplitude. The connections arranged for introducing the direct currents representing stream flow would permit the alternating currents representing the tides to pass into the ground wire at other points than that representing the entrance to Suisun Bay. Since this would introduce errors inductive blocking impedences were introduced into the direct current circuit wherever necessary to confine the alternating currents to the proper network circuits. Where stream channels continued beyond the area represented by the analog, lumped impedences were introduced to represent those portions beyond the analog area. In most cases these were determined by trial so that known tidal behavior would be properly represented.

In order to protect the direct current meters where necessary from loss of field due to the alternating-current components, they were shunted by condensers having impedences which were low compared to the resistance of the meter.

<u>Results</u>

The analog has assisted materially in the solution of the problem of flow transfer through the Delta. The results obtained check well with those obtained by other means.

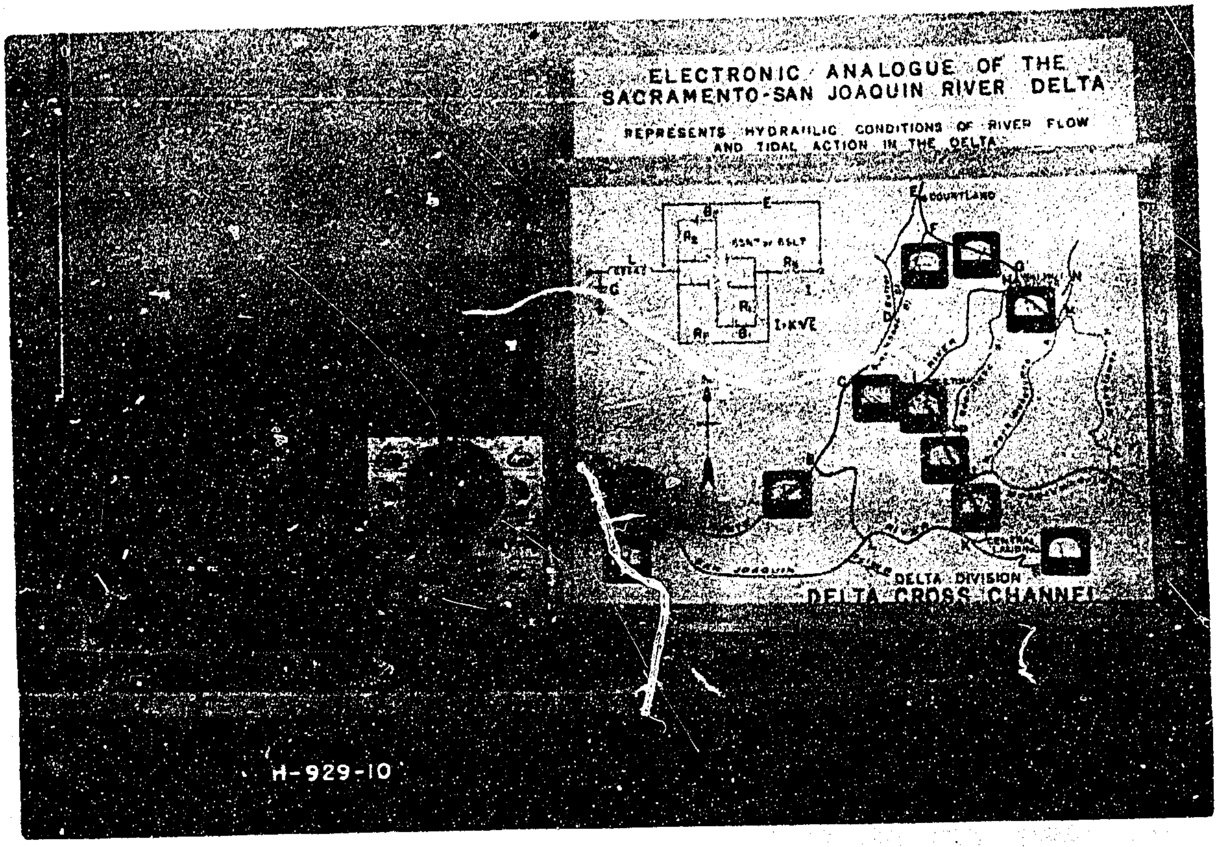


Figure 1. External appearance of the Electronic Analog.

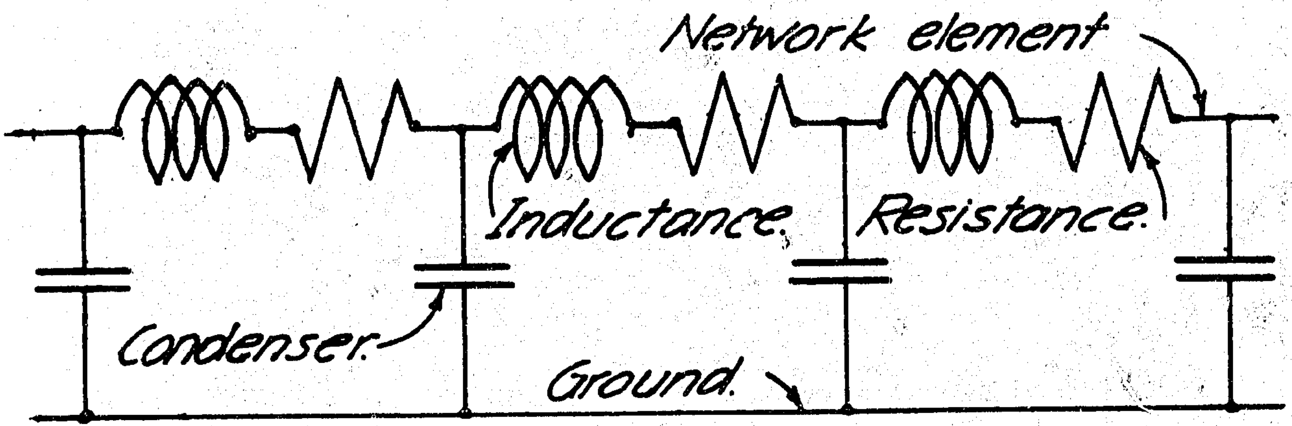


Fig 2. Basic Analog Circuit.

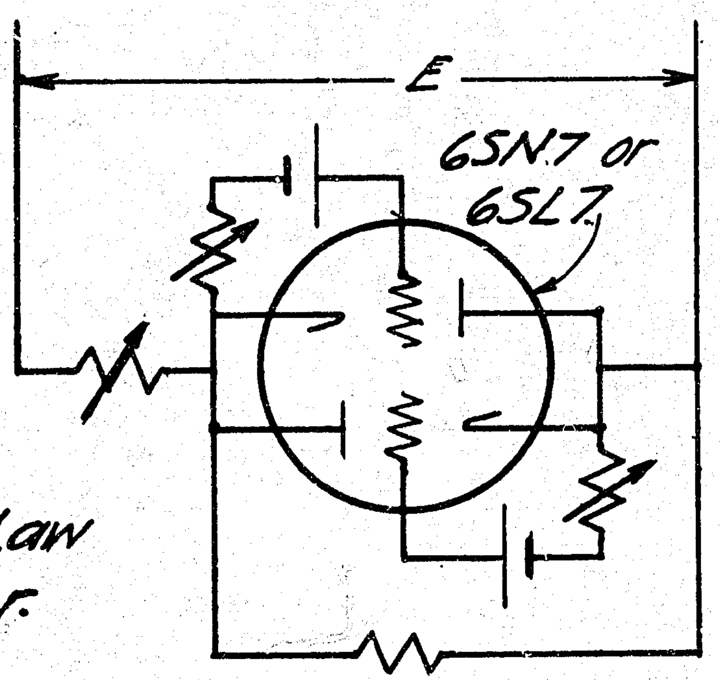


Fig 3.
Square Law
Resistor.

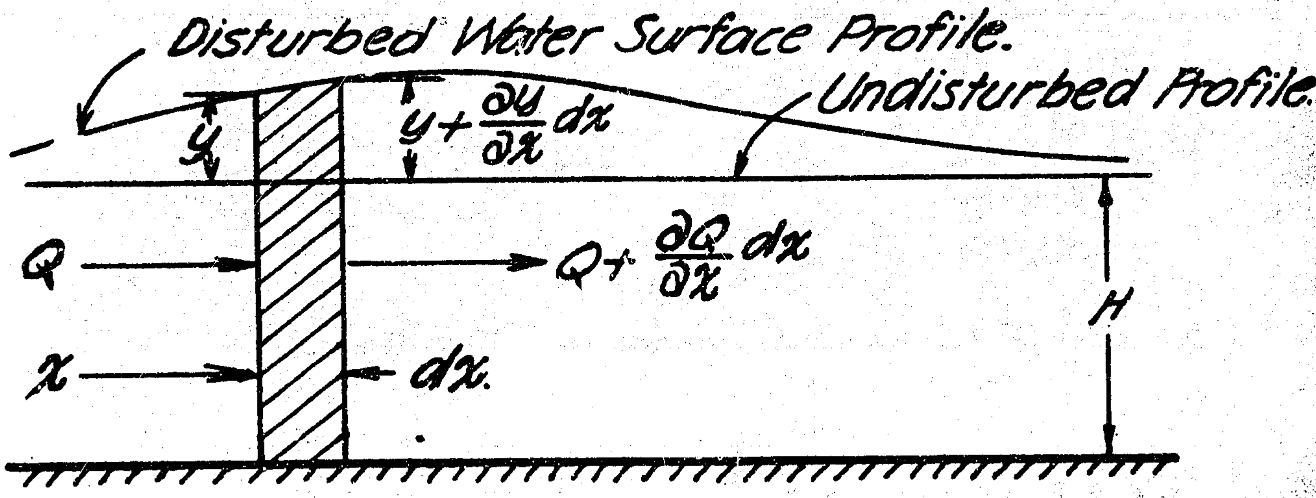


Fig 4. Longitudinal Section of a Channel.

IV

OSCILLOGRAPH RECORDS
OAKVILLE DAM
LOWER NUECES POWER PROJECT
TEXAS

Made for Mr. K. O. Vartia of the
Austin Area Planning Office
Region 5
October 31, 1951

Records made by
C. R. Daum
and
L. O. Timblin Jr.

CURRENT RECORDS

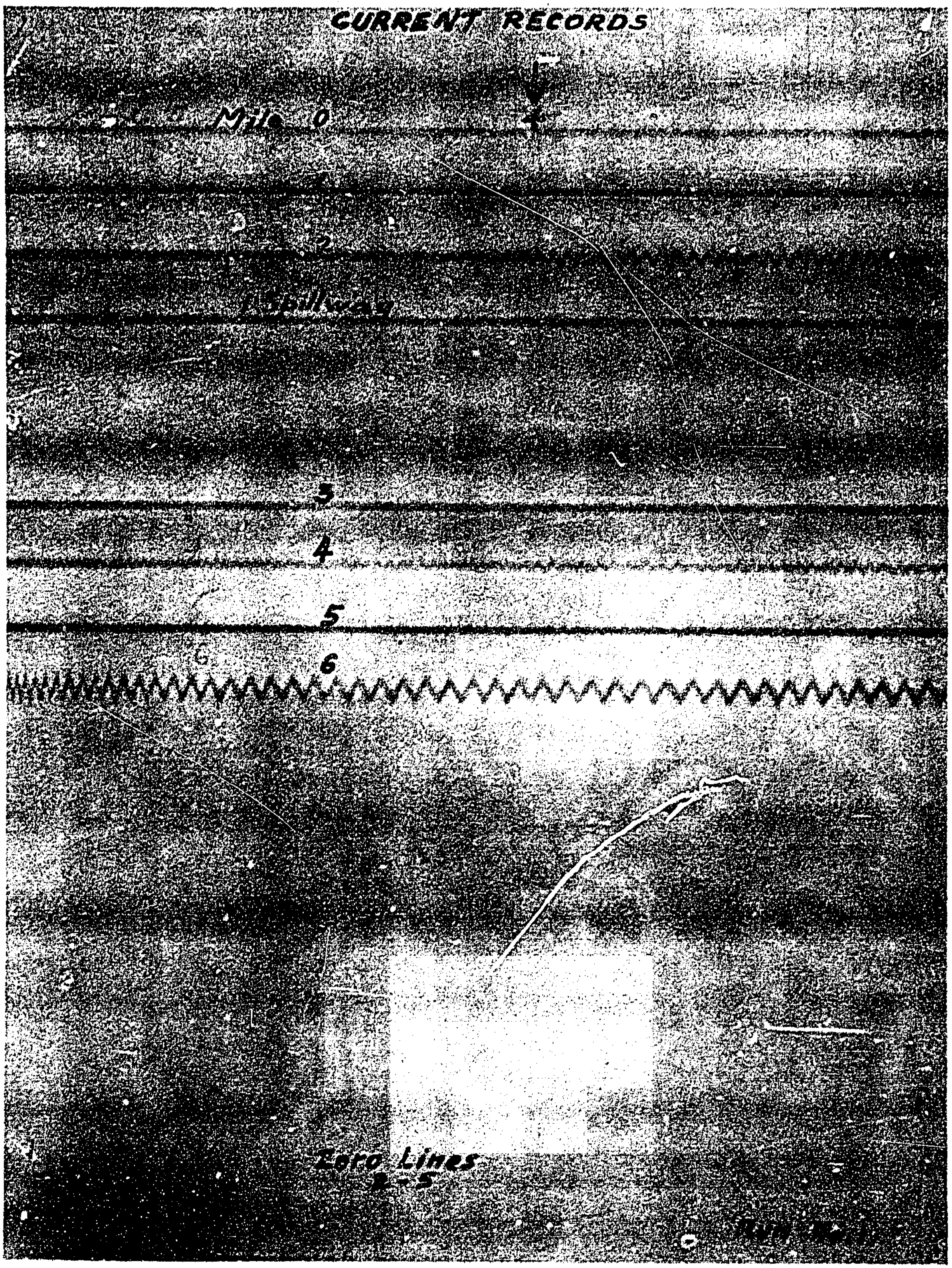
These records show the wave shape, magnitude and phase shift of the currents in the mile sections of the analog. One milliampere of current in the analog represents 500 cubic feet per second of flow in the prototype. The galvanometers were adjusted to give a deflection of 1 inch per 0.5 milliampere; therefore, 1-inch deflection represents 1000 cubic feet per second.

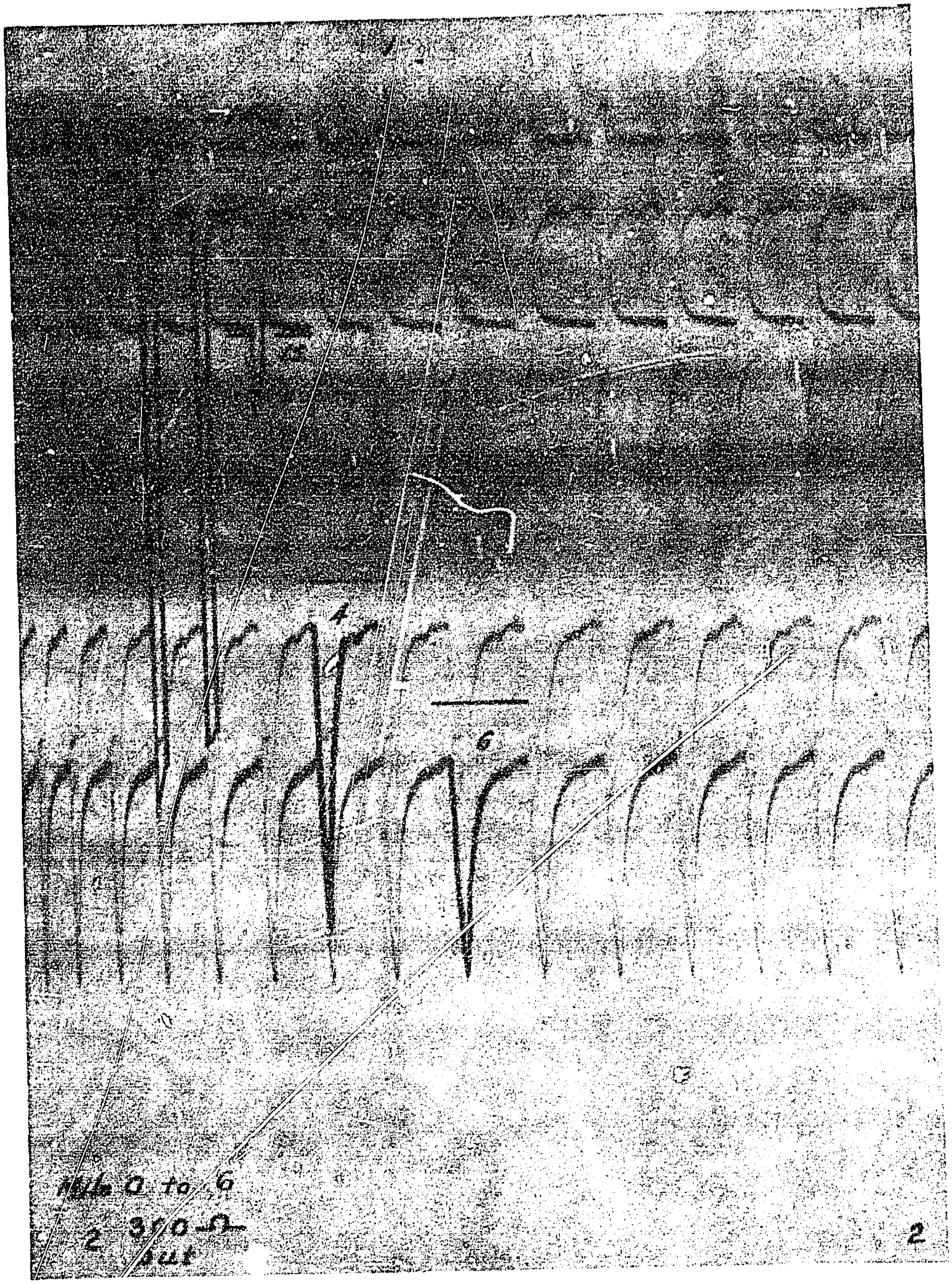
DATA FOR CURRENT RECORDS

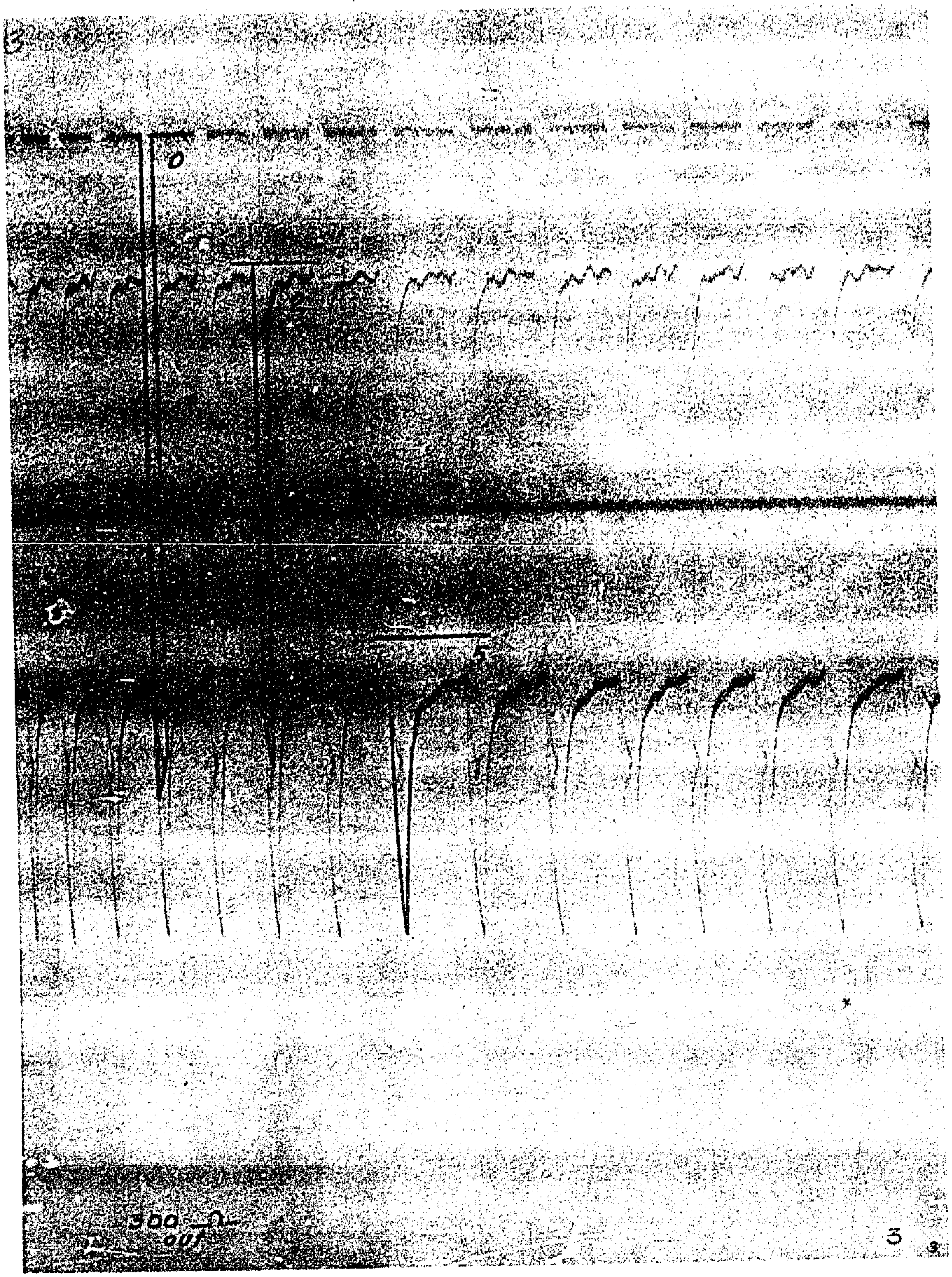
Run	Mile	Trace	300 ohm	Remarks
1	0	1	—	Zero line
	1	2	—	Zero line
	2	3	—	Zero line
	1S	4	—	Zero line
	3	5	—	Zero line
	4	6	—	Zero line
	5	7	—	Zero line
	6	8	—	Zero line
2	0	1	Out	10 ma input
	1	2	Out	10 ma input
	1S	4	Out	10 ma input
	4	6	Out	10 ma input
	6	8	Out	10 ma input
3	0	1	Out	10 ma input
	2	3	Out	10 ma input
	3	5	Out	10 ma input
	5	7	Out	10 ma input
4	(Same as run 3)		In	10 ma input
5	(Same as run 2)		In	10 ma input
6	(Same as run 1)		In	10 ma input
7	(Same as run 1)		—	Zero line
8	(Same as run 2)		In	10 ma input
9	(Same as run 3)		In	10 ma input
10	(Same as run 3)		Out	10 ma input
11	(Same as run 2)		Out	10 ma input
12	0	1	—	Zero line
	7	2		Zero line
	8	3		Zero line
	9	4		Zero line
	10	5		Zero line
	11	6		Zero line
13	(Same as 12)		Out	10 ma input
14	(Same as 12)		In	10 ma input
15	(Same as 12)		Out	10 ma input
16	(Same as 12)		In	10 ma input
17	0	1	—	Zero line
	12	2	—	Zero line
	13	3	—	Zero line
	14	4	—	Zero line
	15	5	—	Zero line
	16	6	—	Zero line
18	(Same as 17)		In	10 ma input

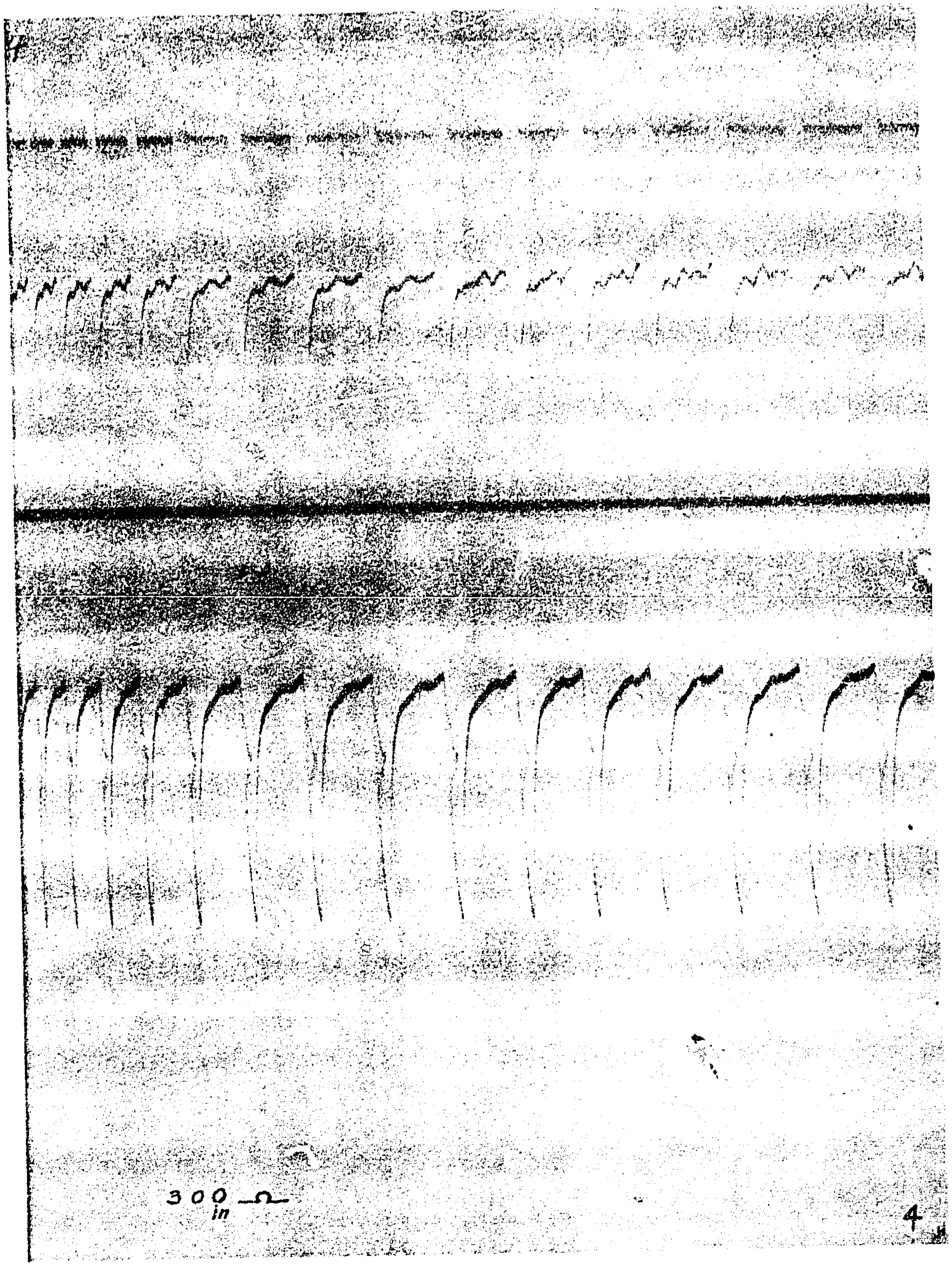
DATA FOR CURRENT RECORDS—Continued

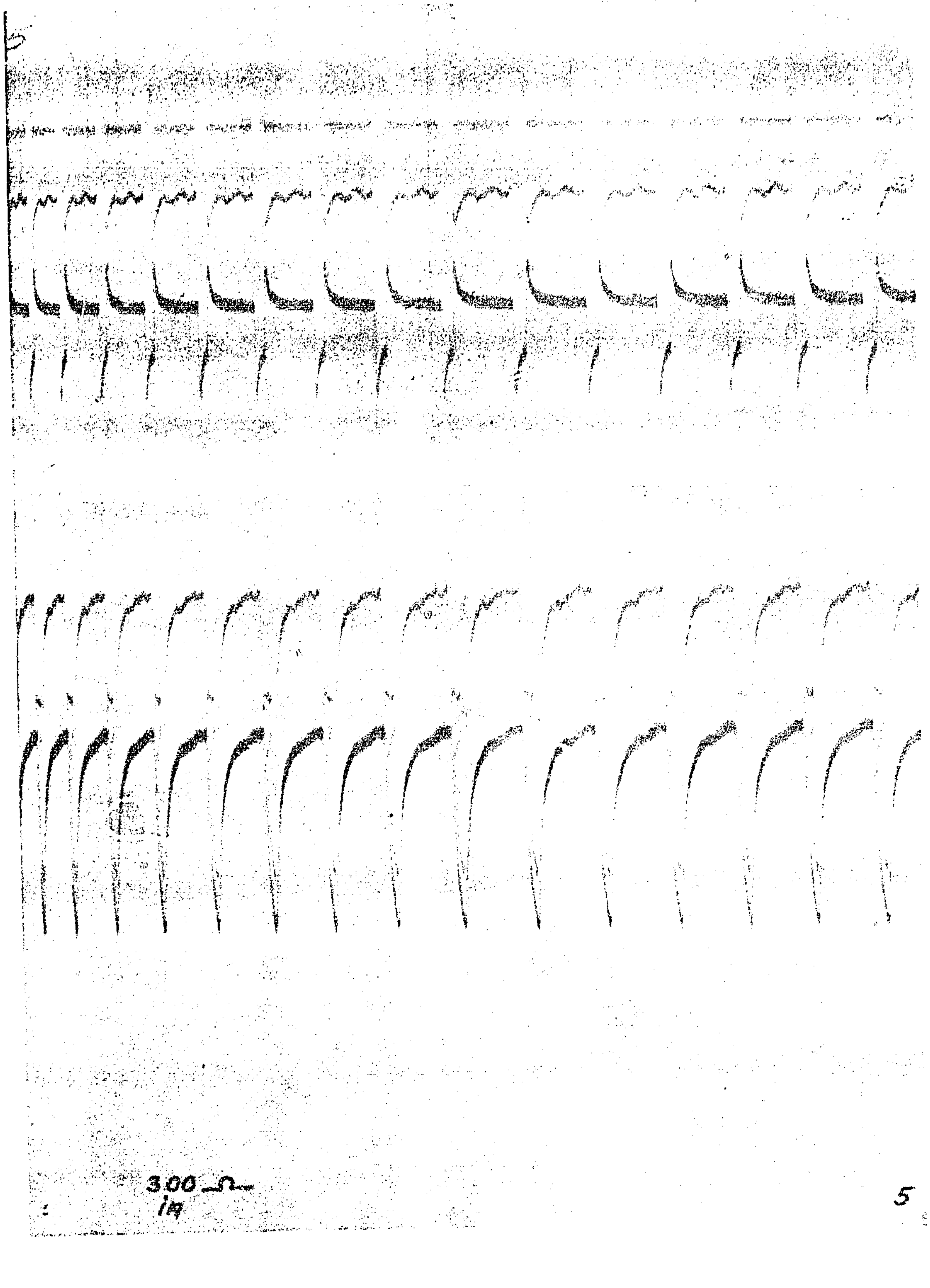
19	(Same as 17)		Out	10 ma input
20	0	1	—	Zero line
	17	2	—	Zero line
	18	3	—	Zero line
	19	5	—	Zero line
	20	6	—	Zero line
21	(Same as 20)		Out	10 ma input
22	(Same as 20)		In	10 ma input
23	0	1	In	10 ma input

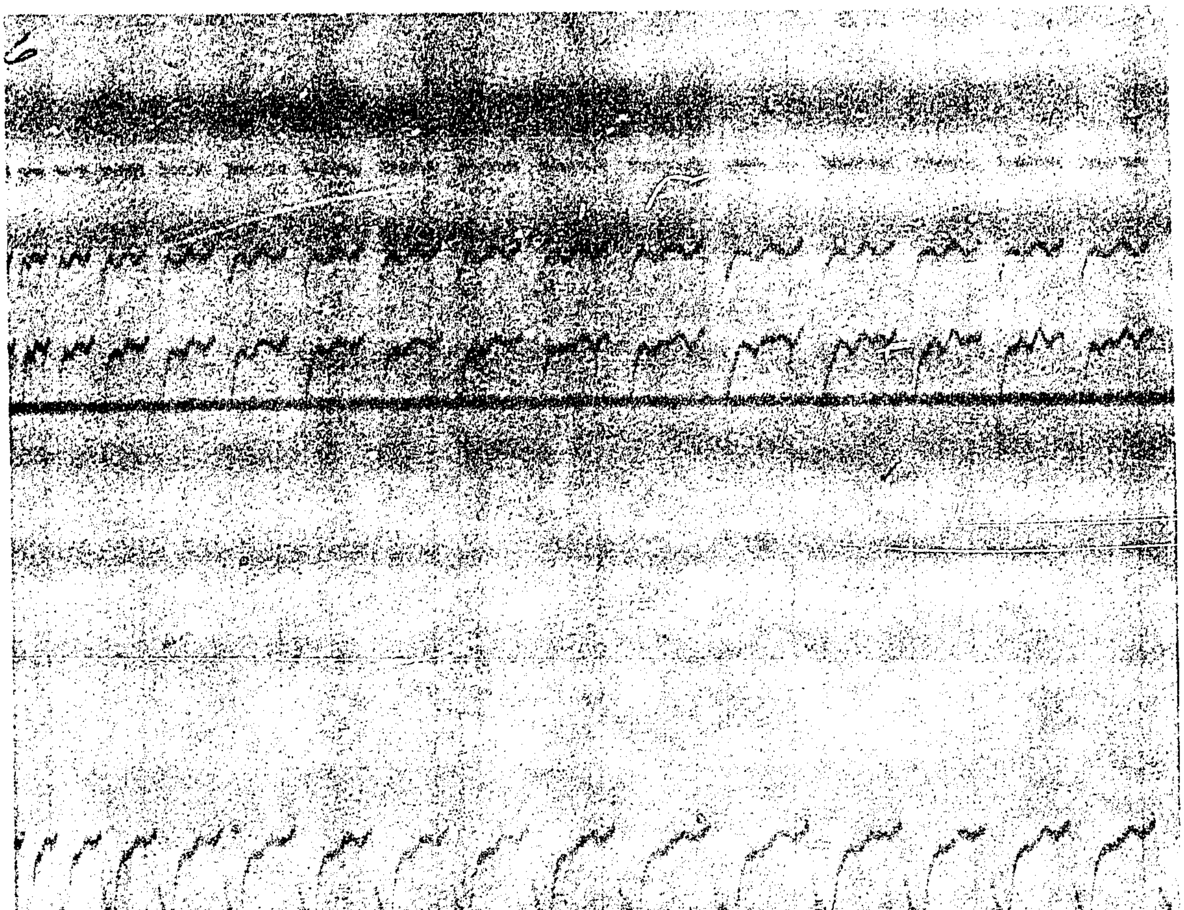

CURRENT RECORDS
Mile 0
2
Spillway
3
4
5
6
Zero Lines
2-5


a to 6
2 350 Ω
out
2

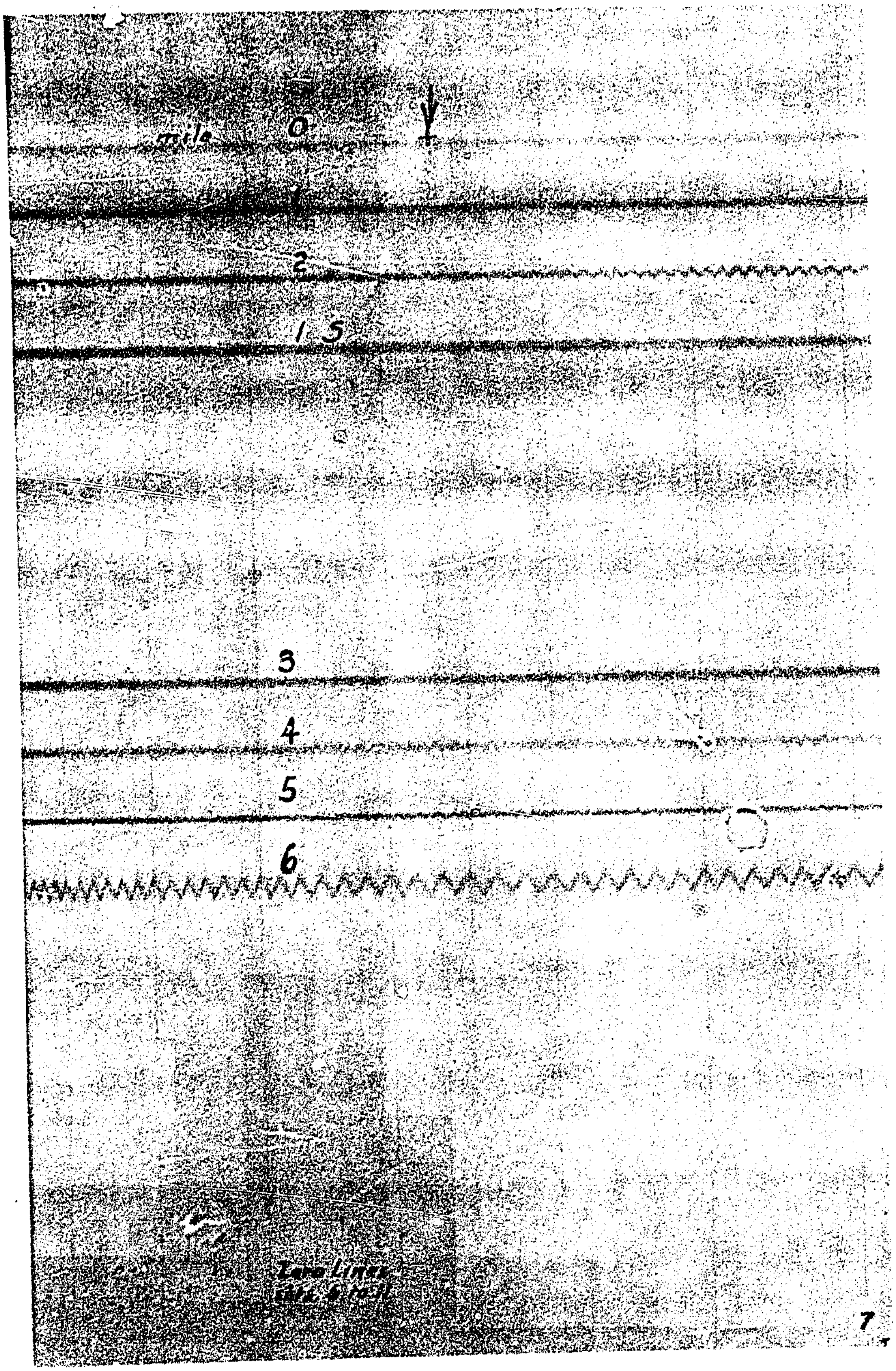

3
0
5
300 Ω out
3

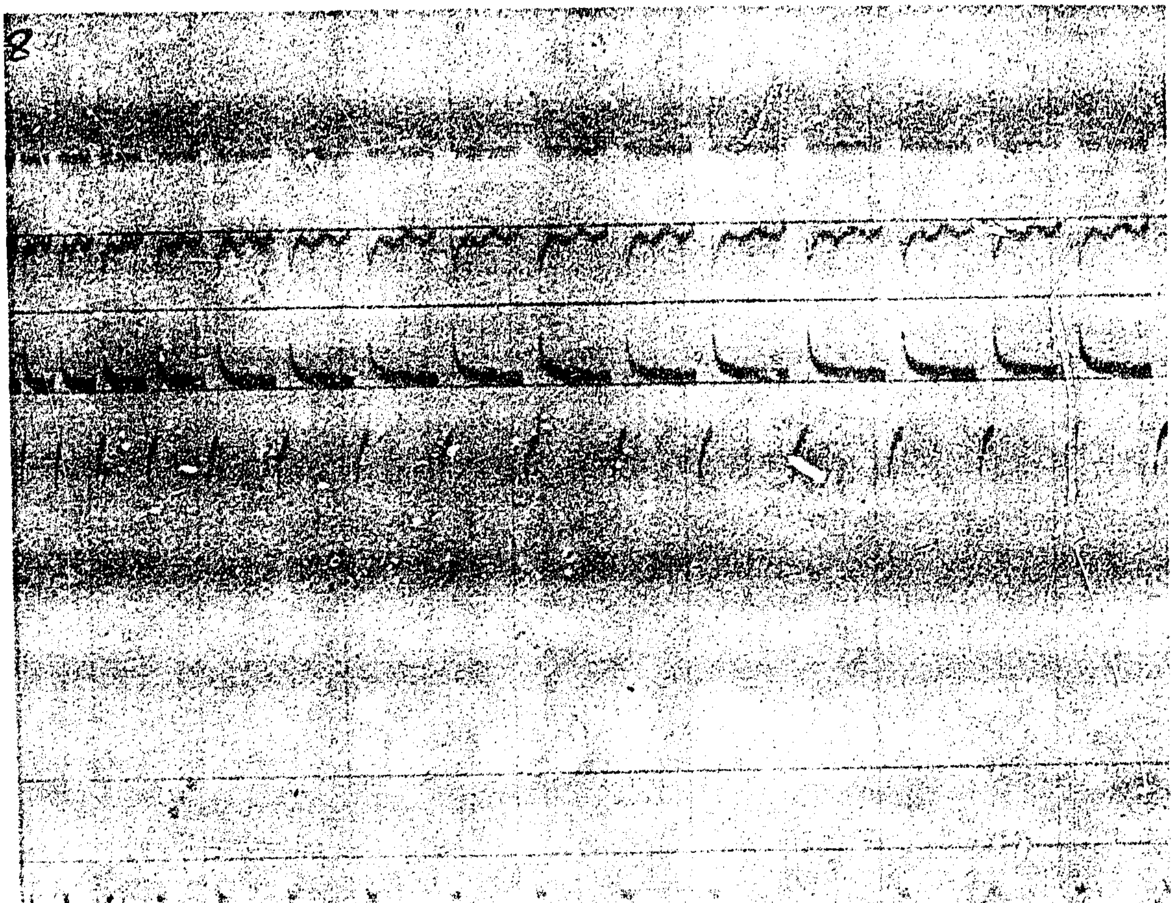

300 in Ω

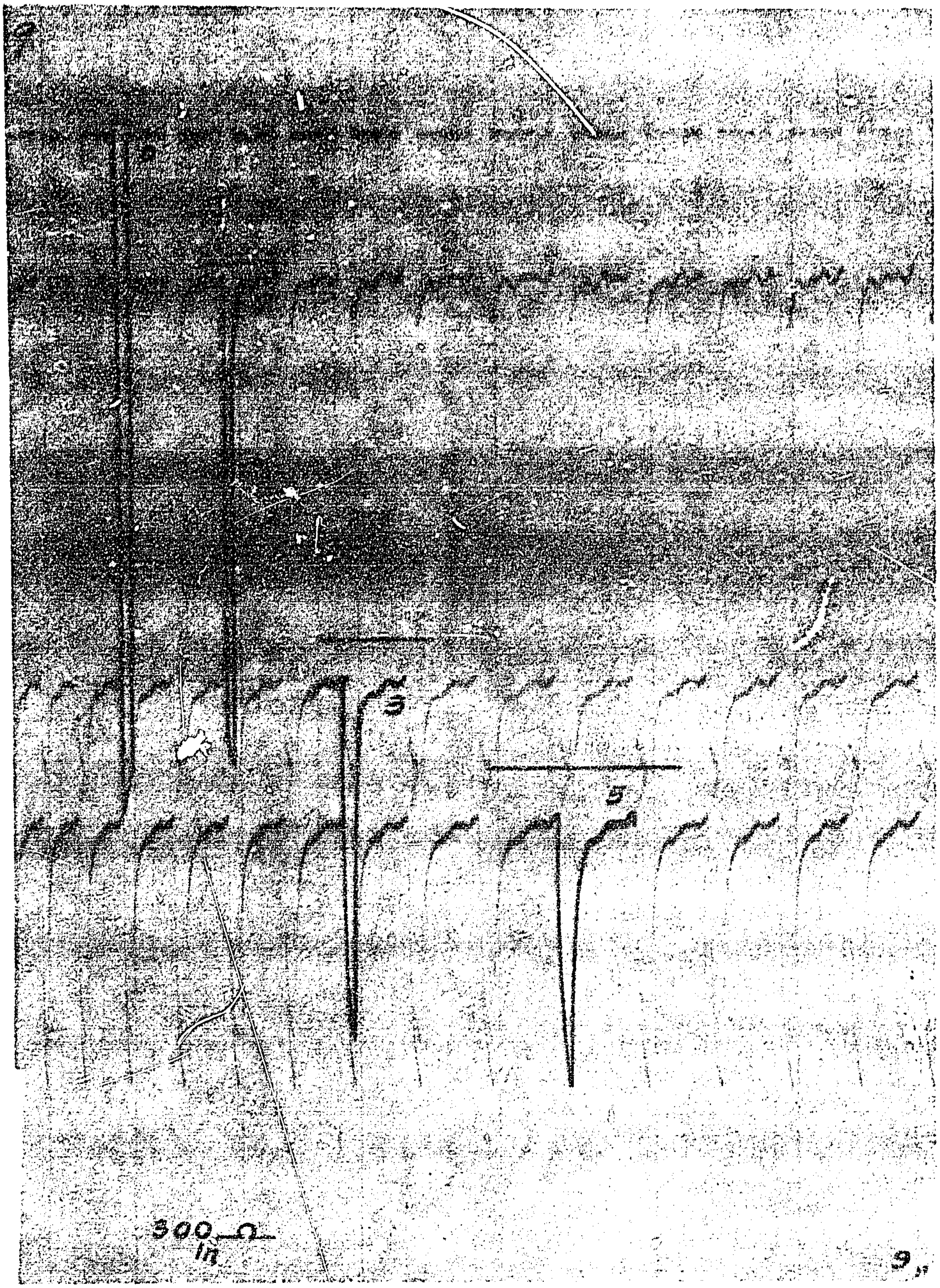

300 Ω
1A



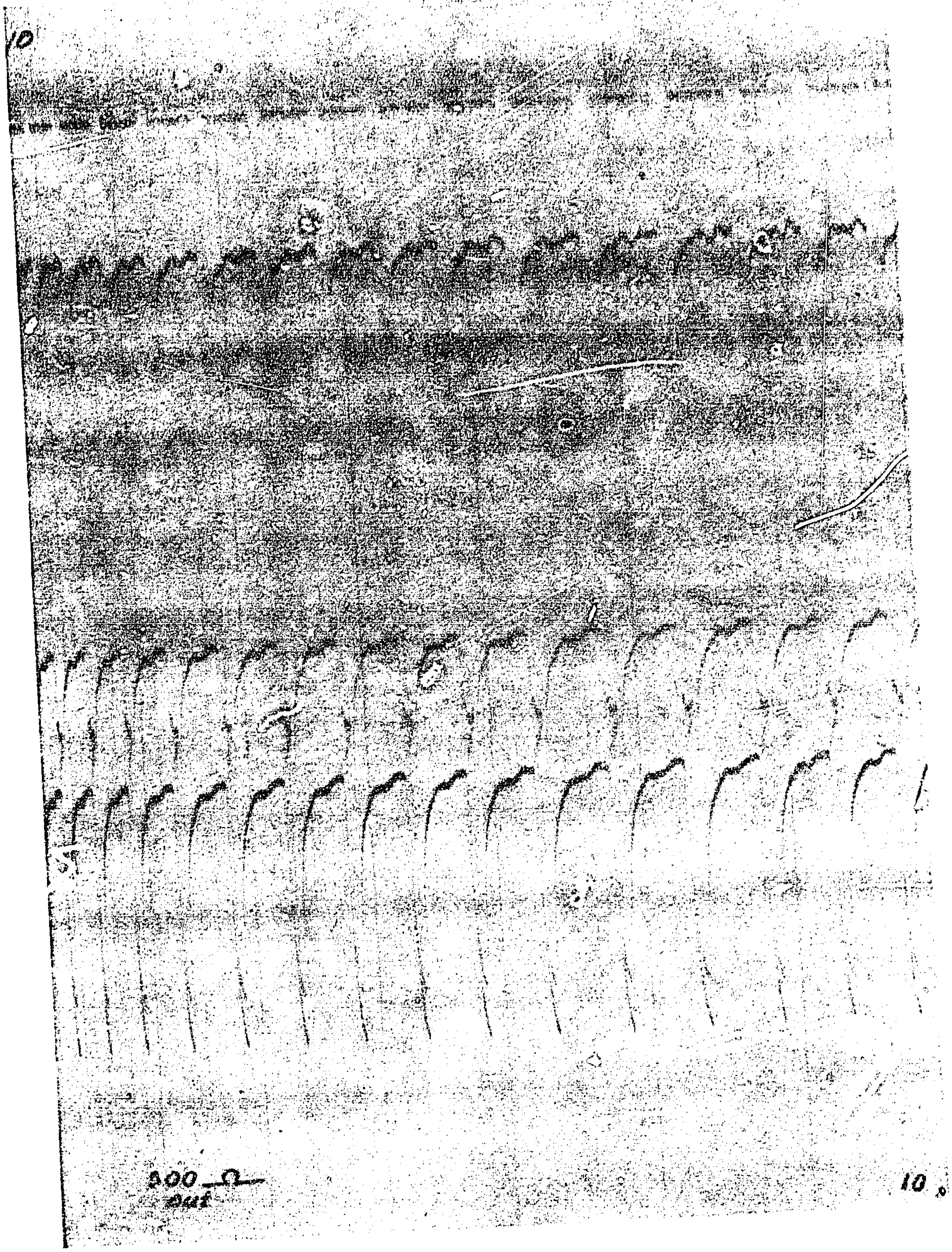
6
300 Ω
1m
6
6

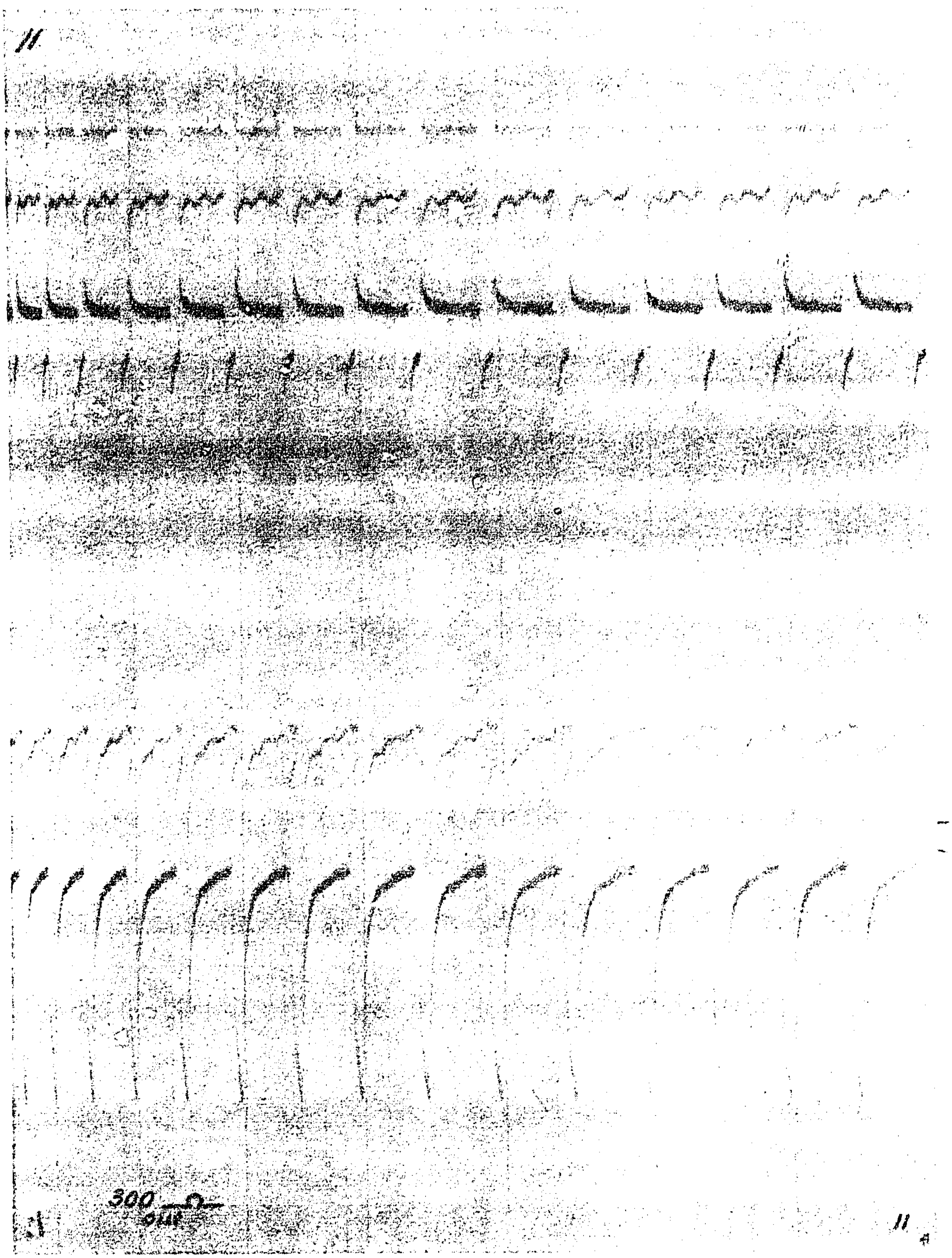

mile
0
1
2
1 5
3
4
5
6
Zero Lines

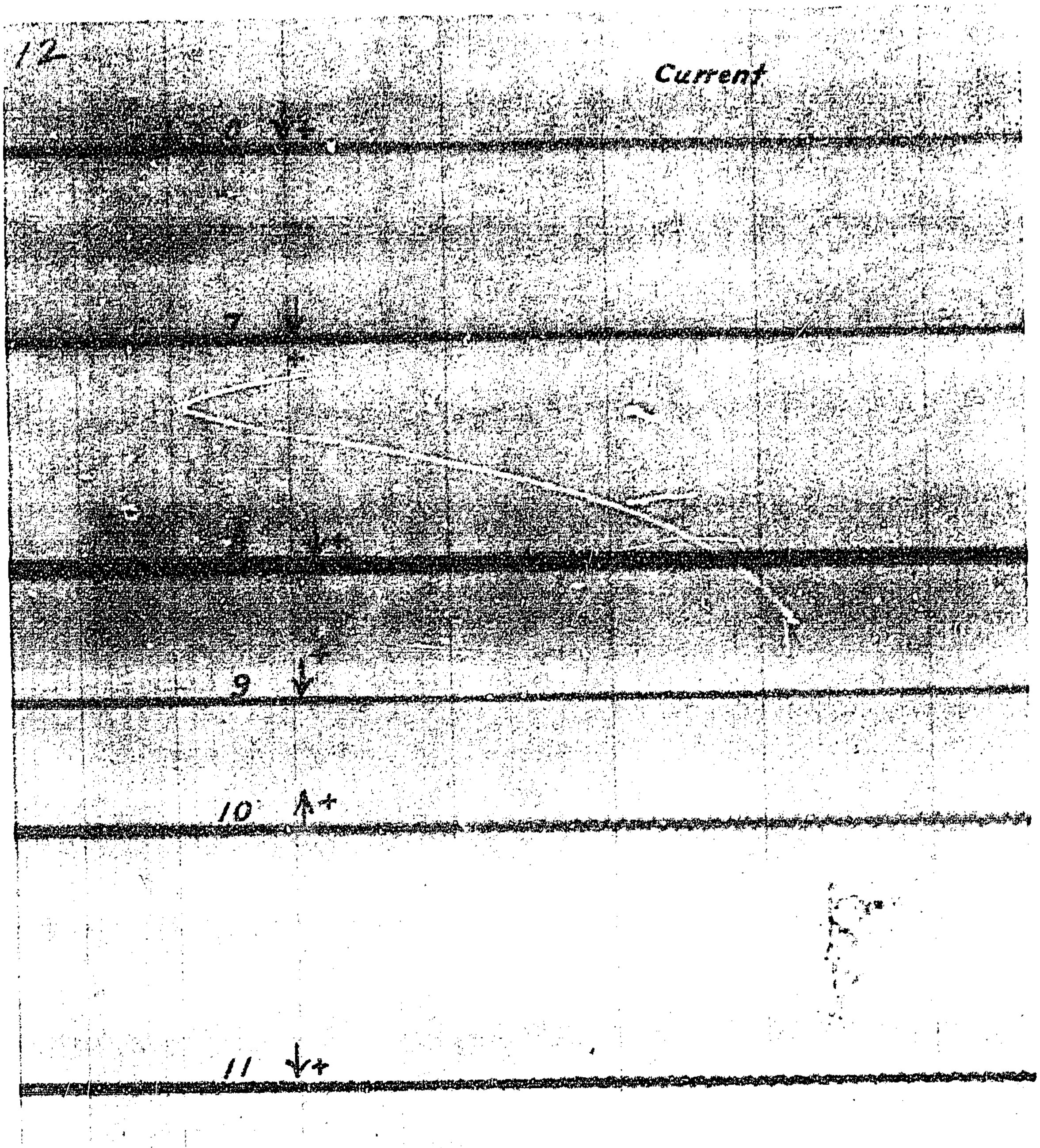

8
300
In
8



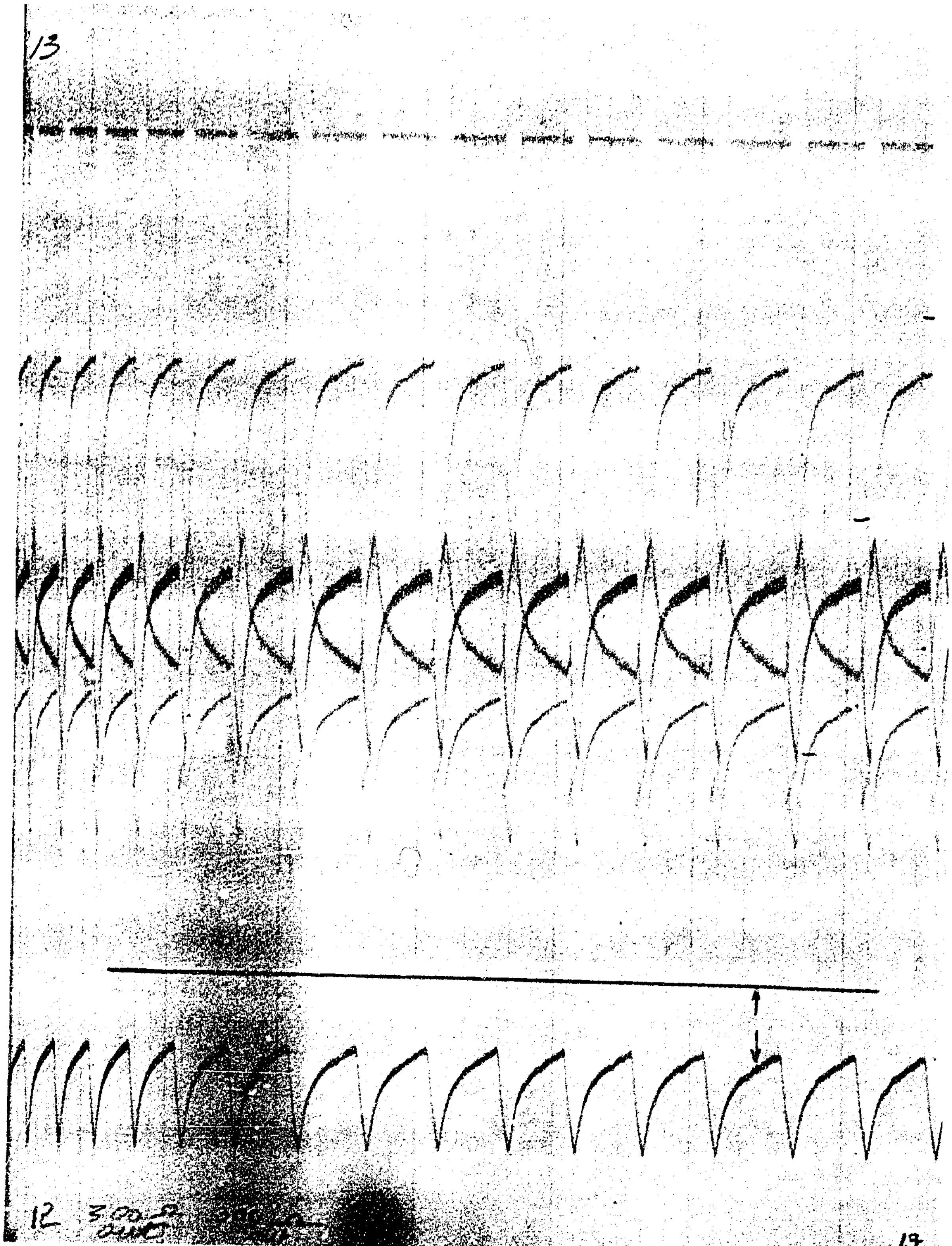
5
5
500 Ω
in.

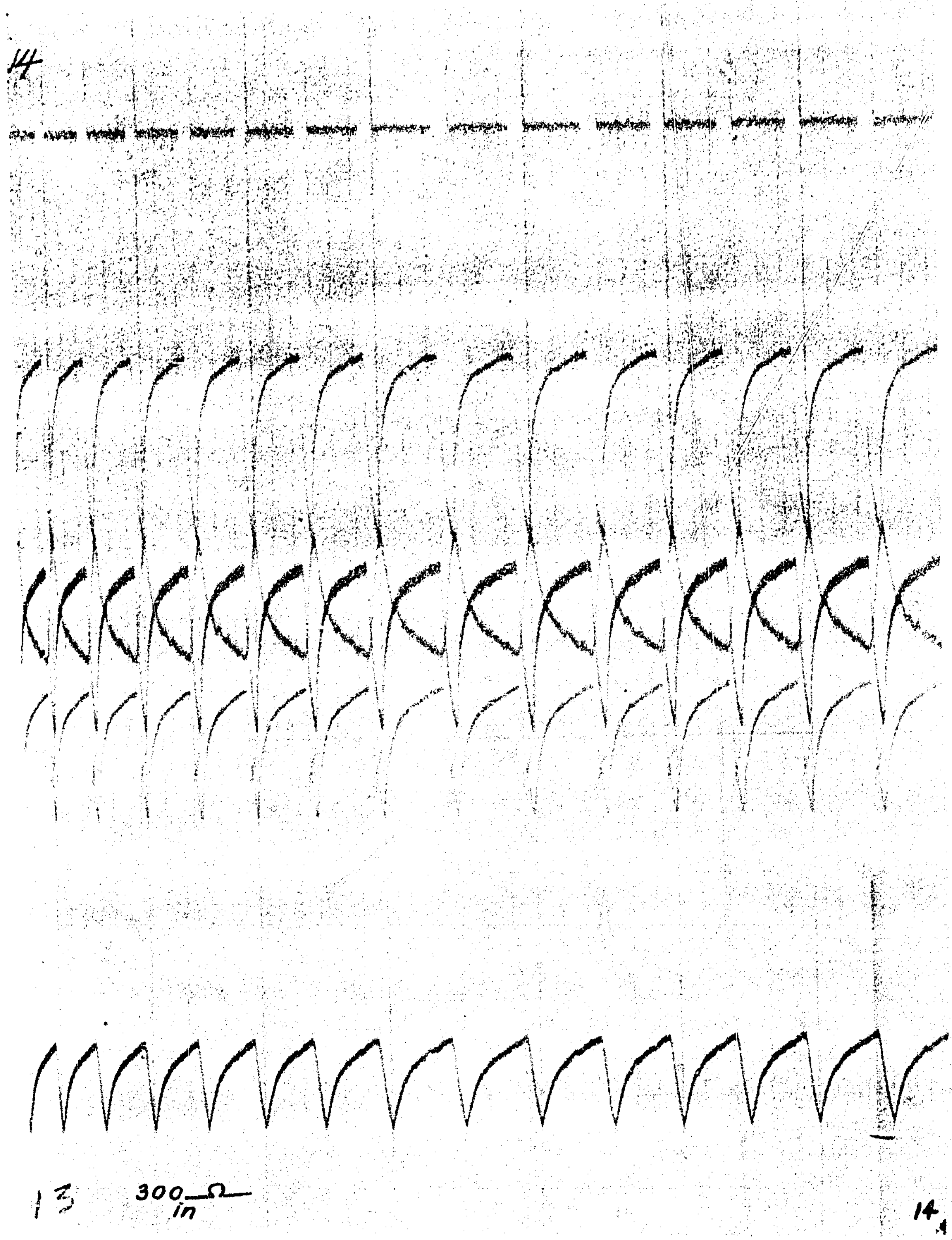

10
500 Ω
10

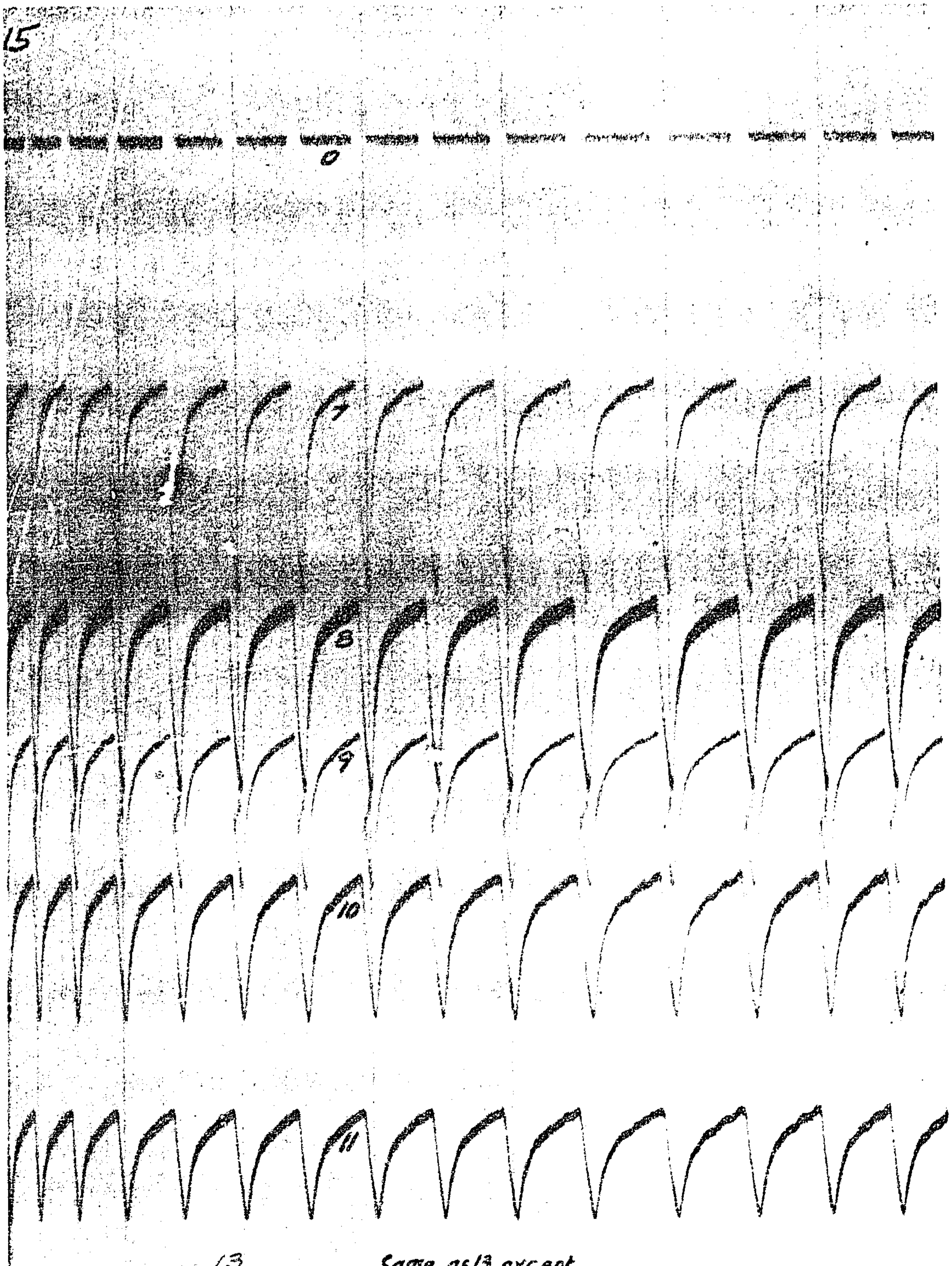

II
300



Zero Lines
shts. 12 & 13

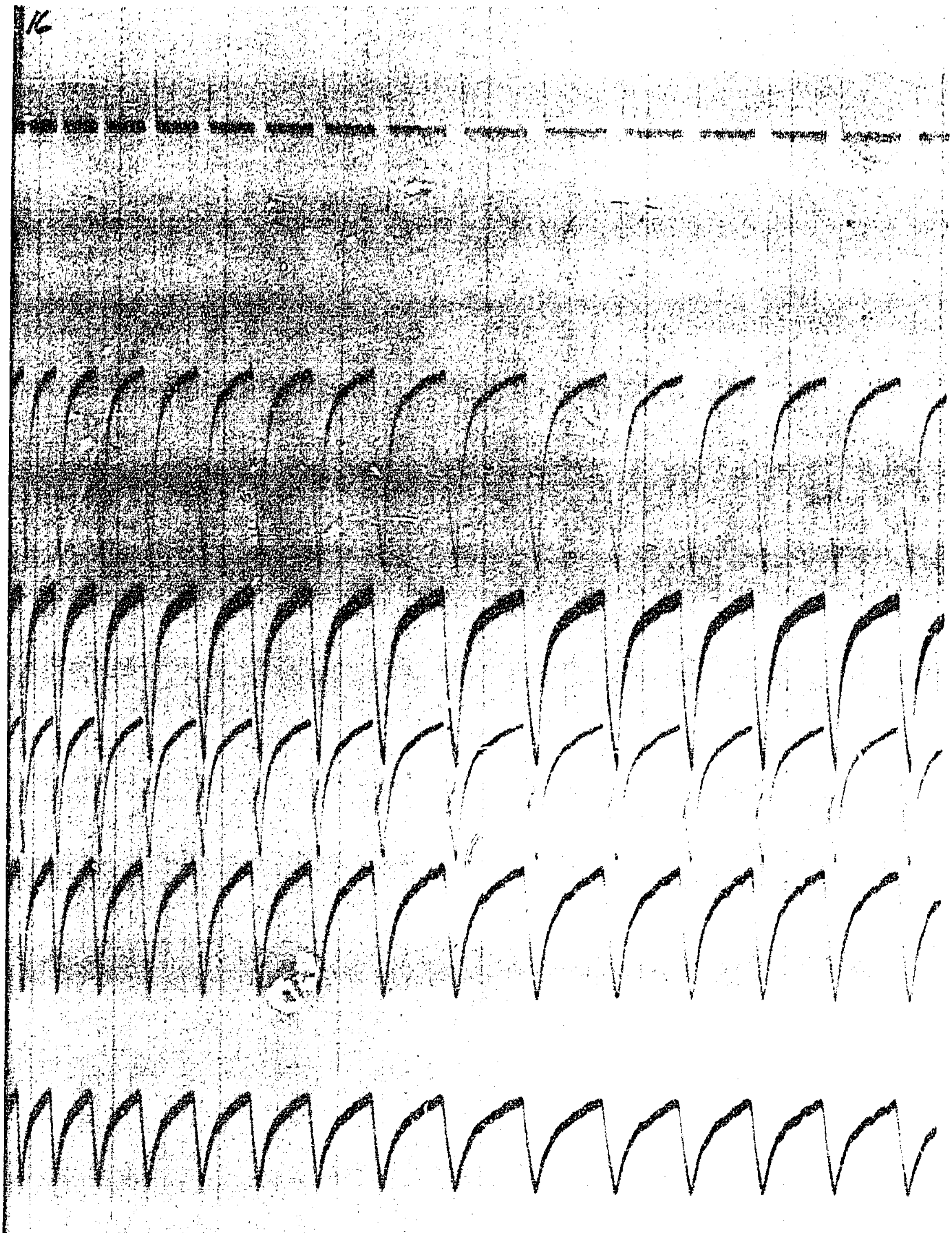

13
12


14
13
300 Ω in
14

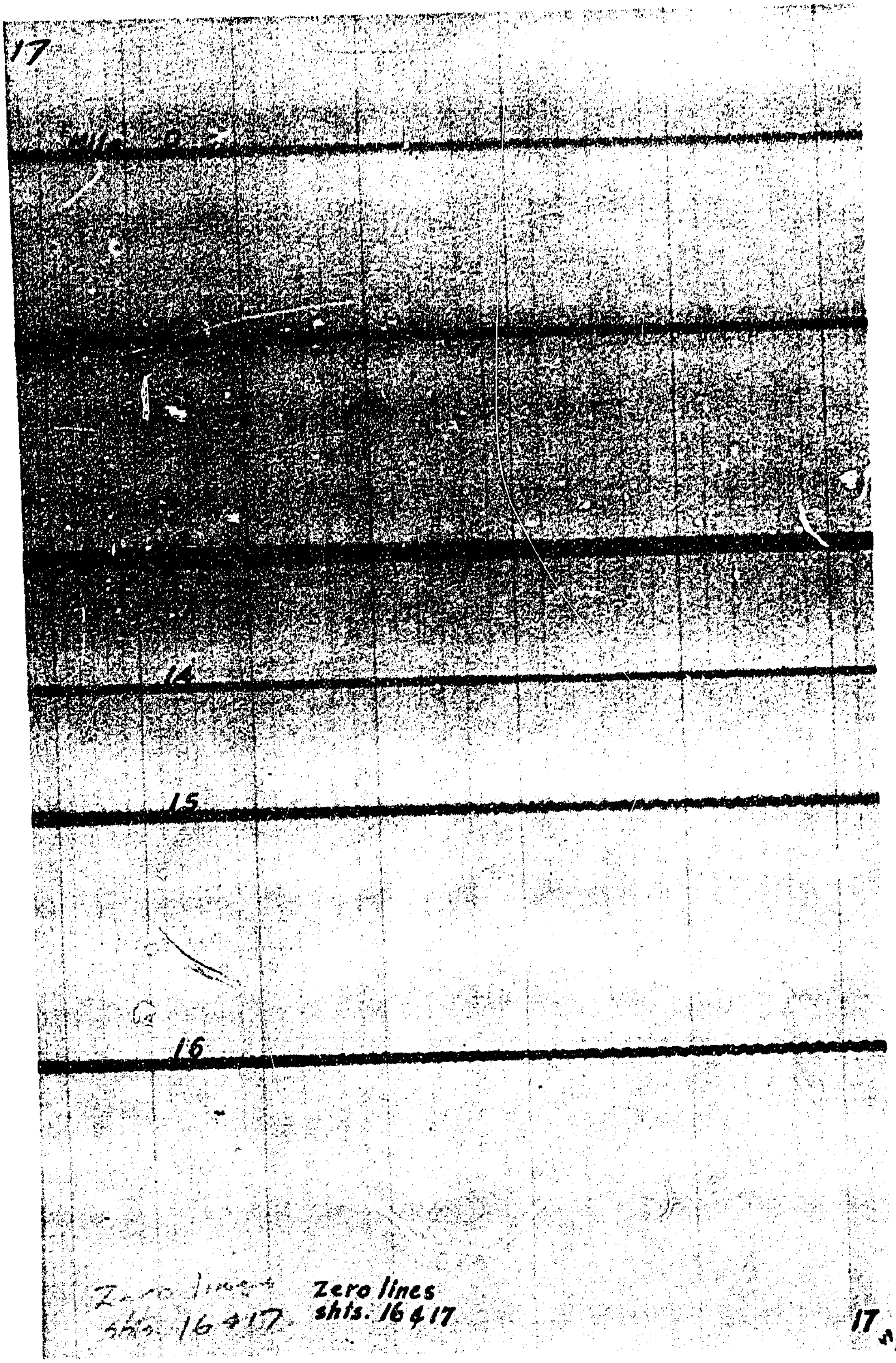


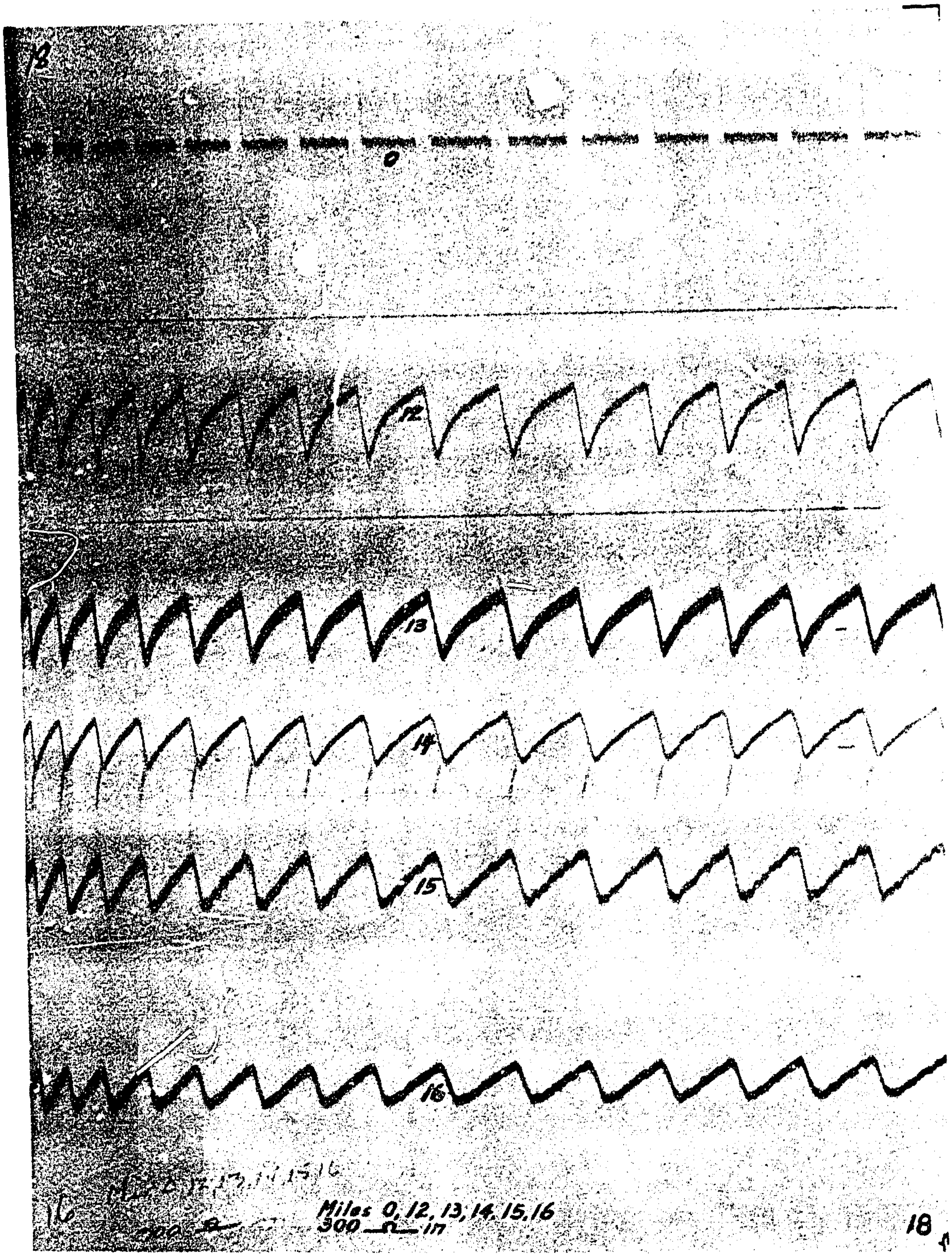
Same as 13 except
10 right end up

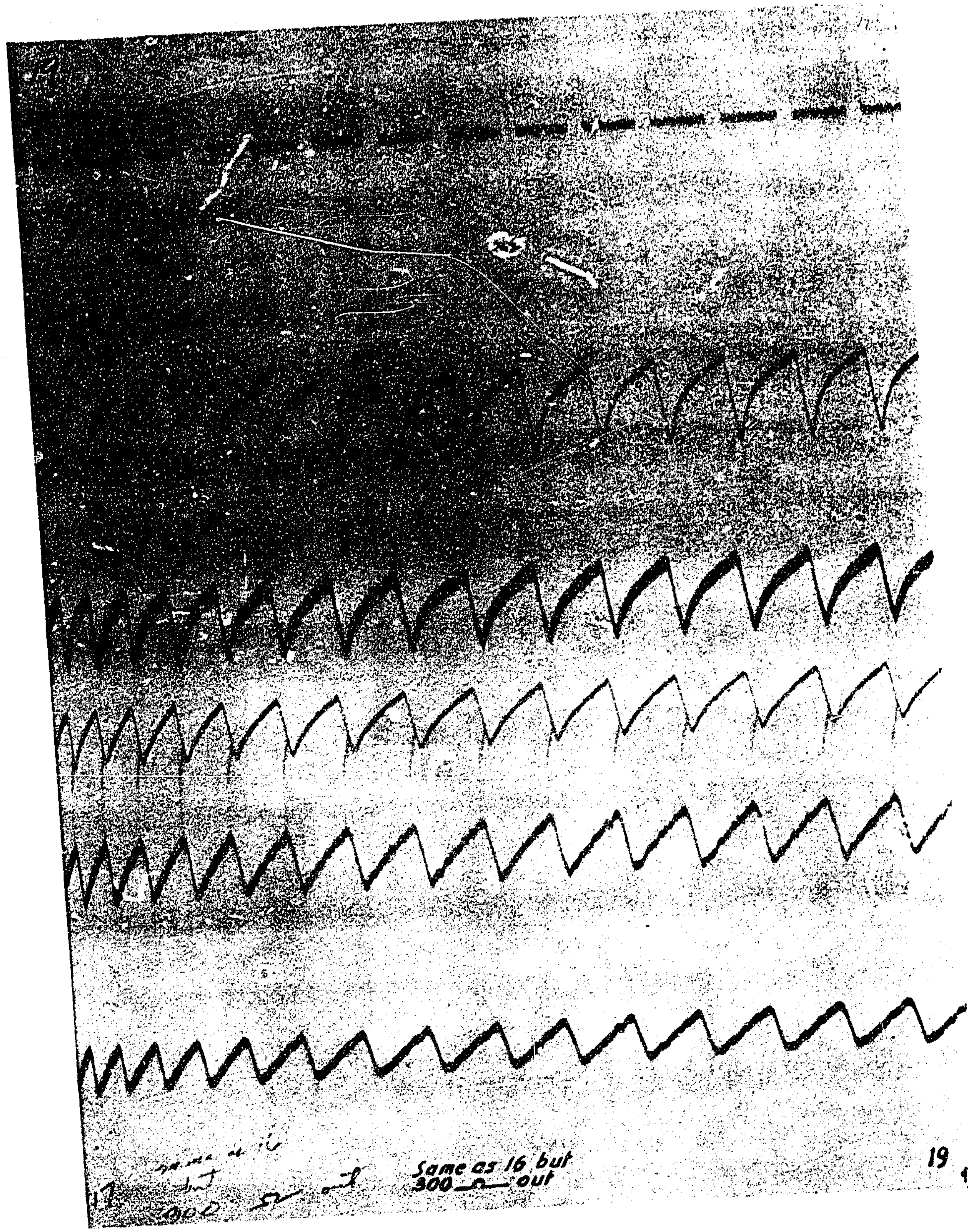
15

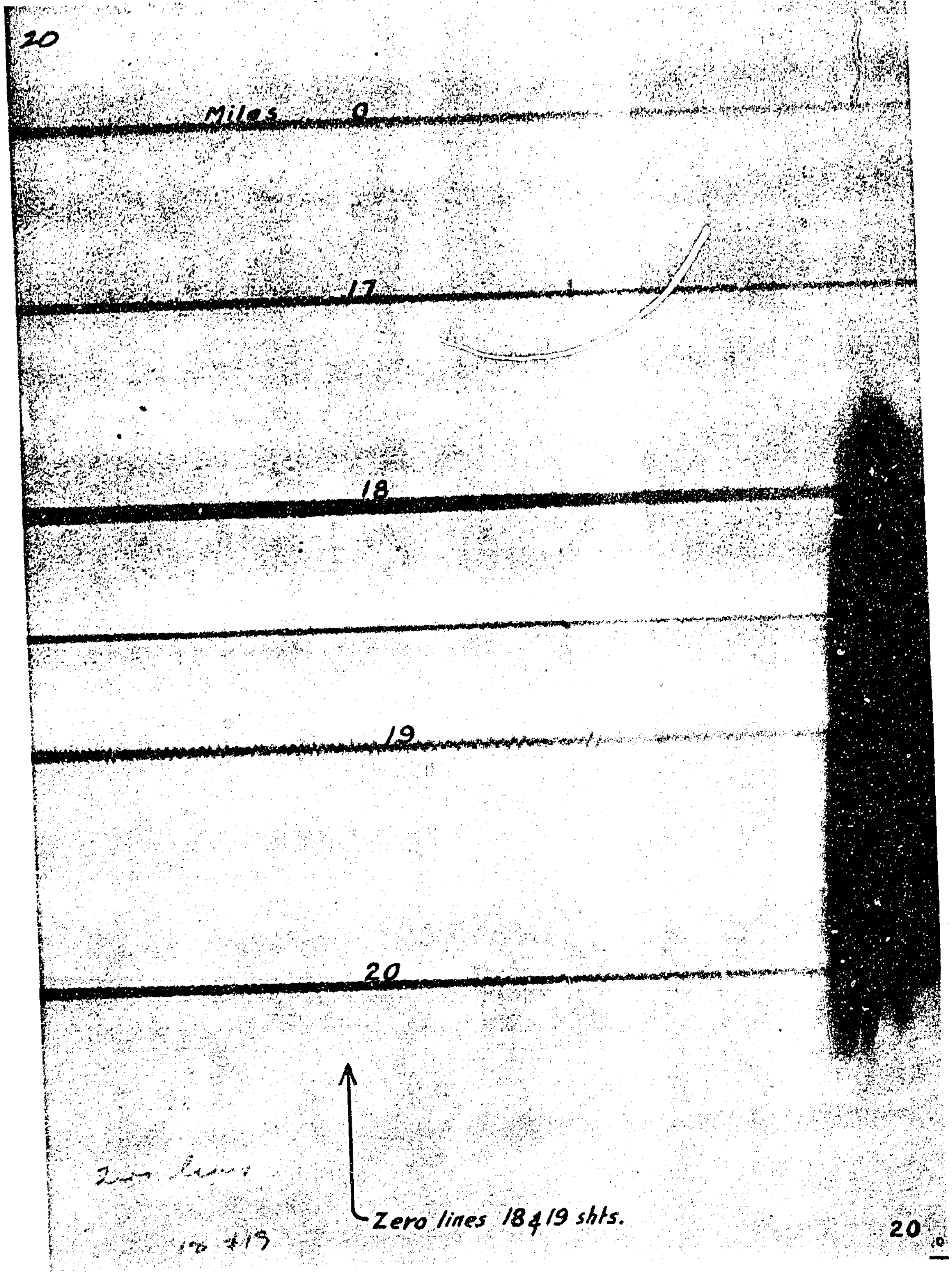


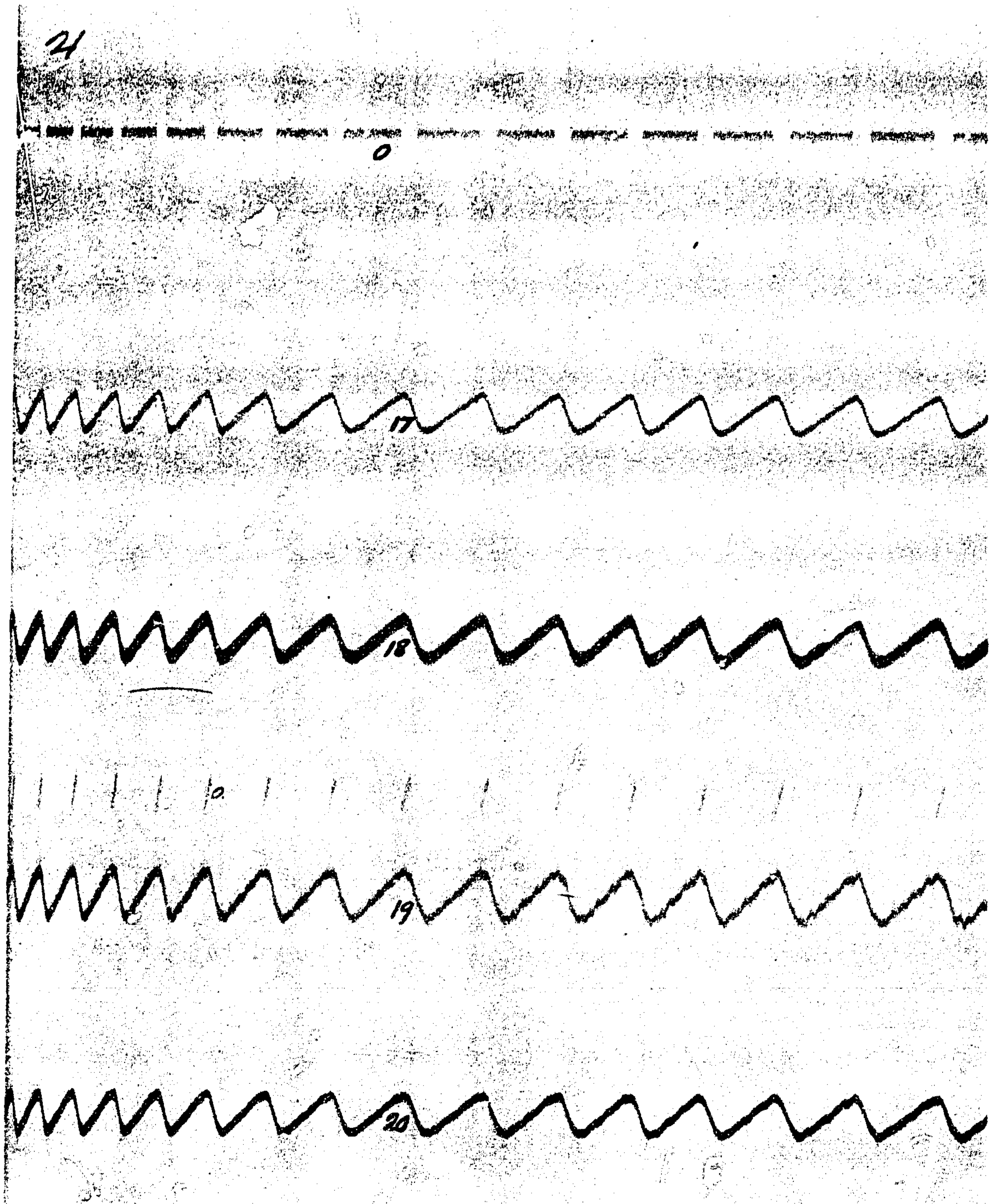
Same as 14 except 10 righted.


17
14
15
16
Zero lines
shts. 16 & 17
Zero lines
shts. 16 & 17
17

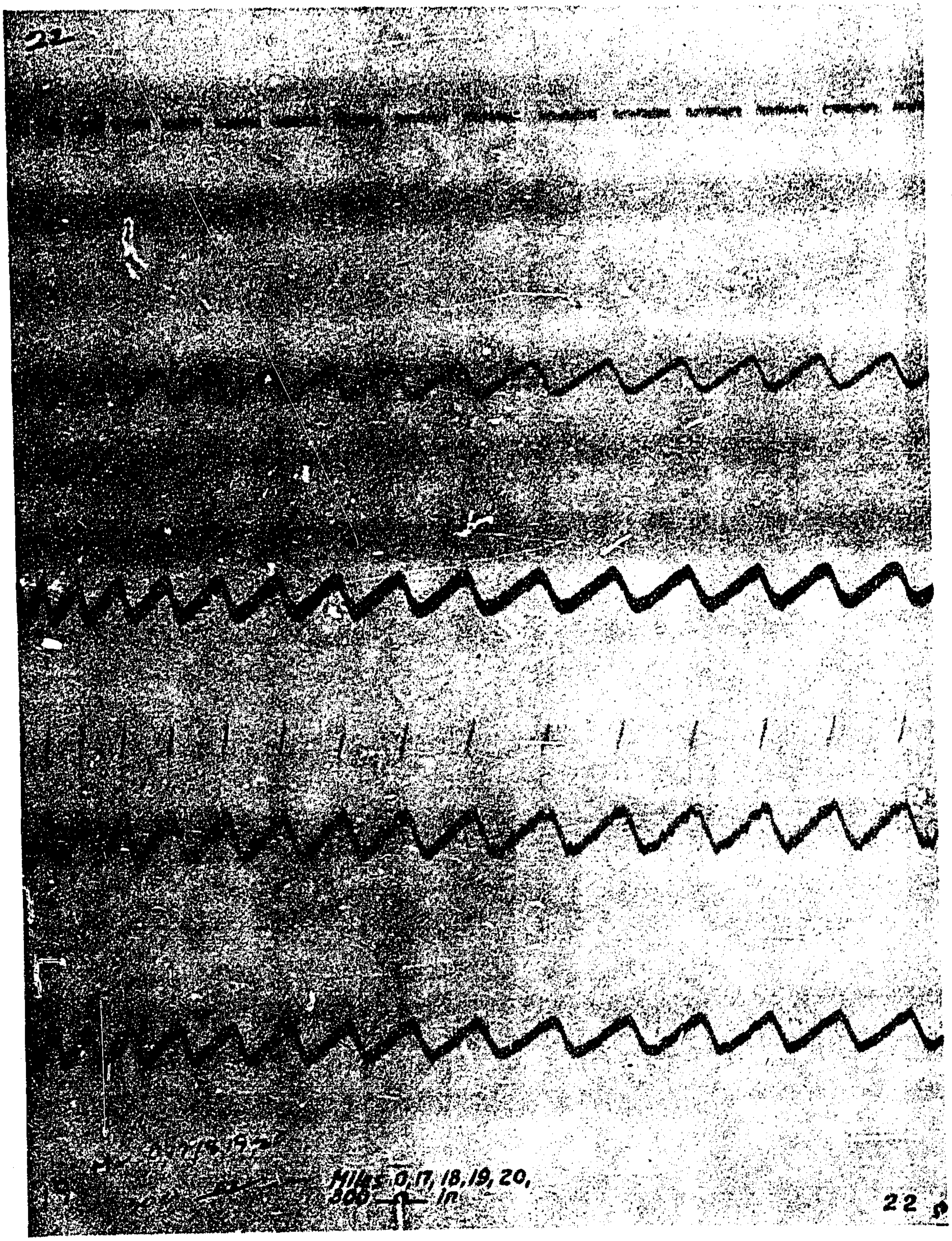

0
12
13
14
15
16
Miles 0, 12, 13, 14, 15, 16
300 Ω 117
18

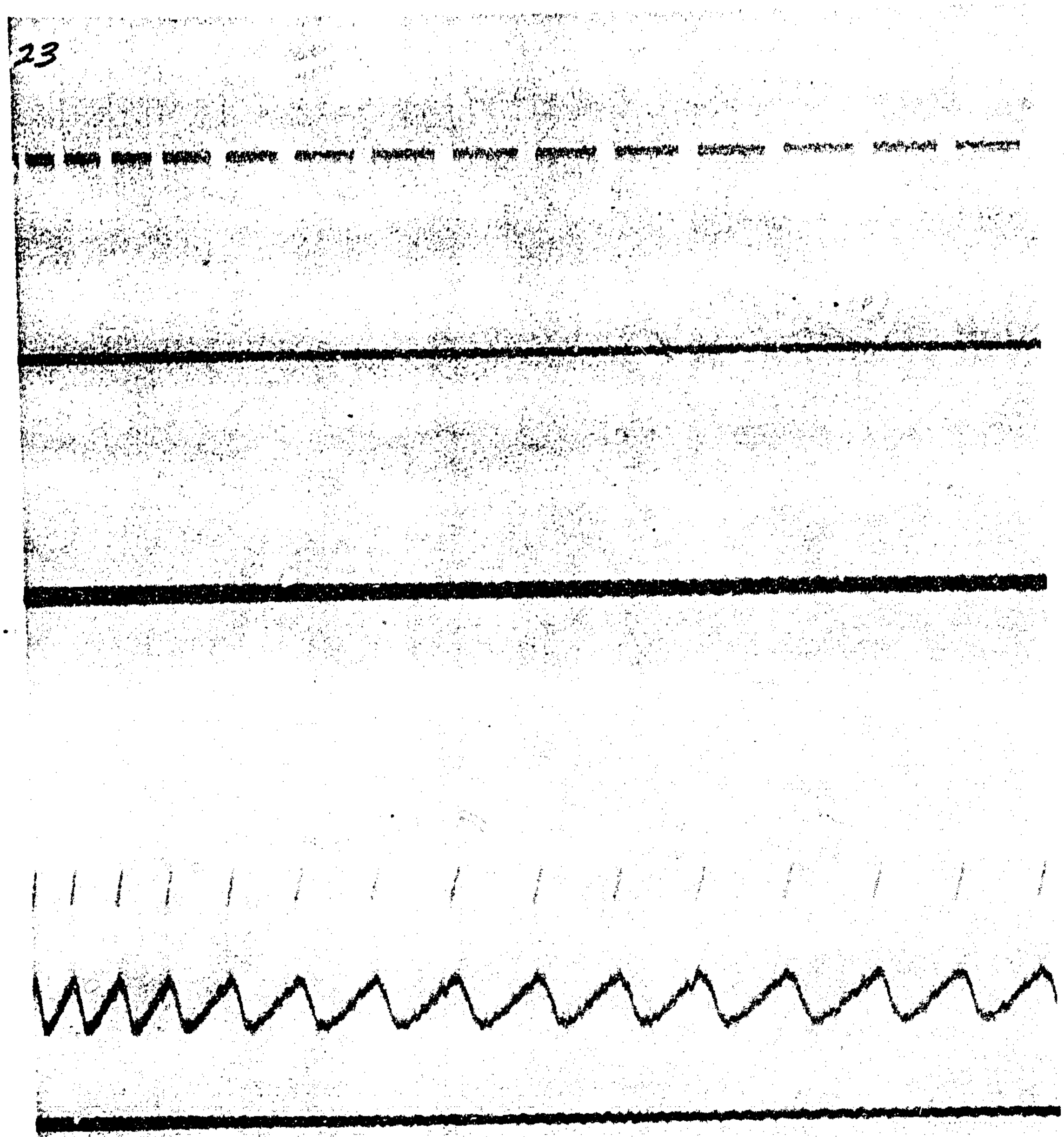

Same as 16 but
300 Ω out
19


20
Miles 0
17
18
19
20
Zero lines 18 & 19 shts.
20



Miles 0, 17, 18, 19, 20, 300 Ω out


22
Hills 0,17, 18,19, 20,
300 Ω In
22


23

IV B

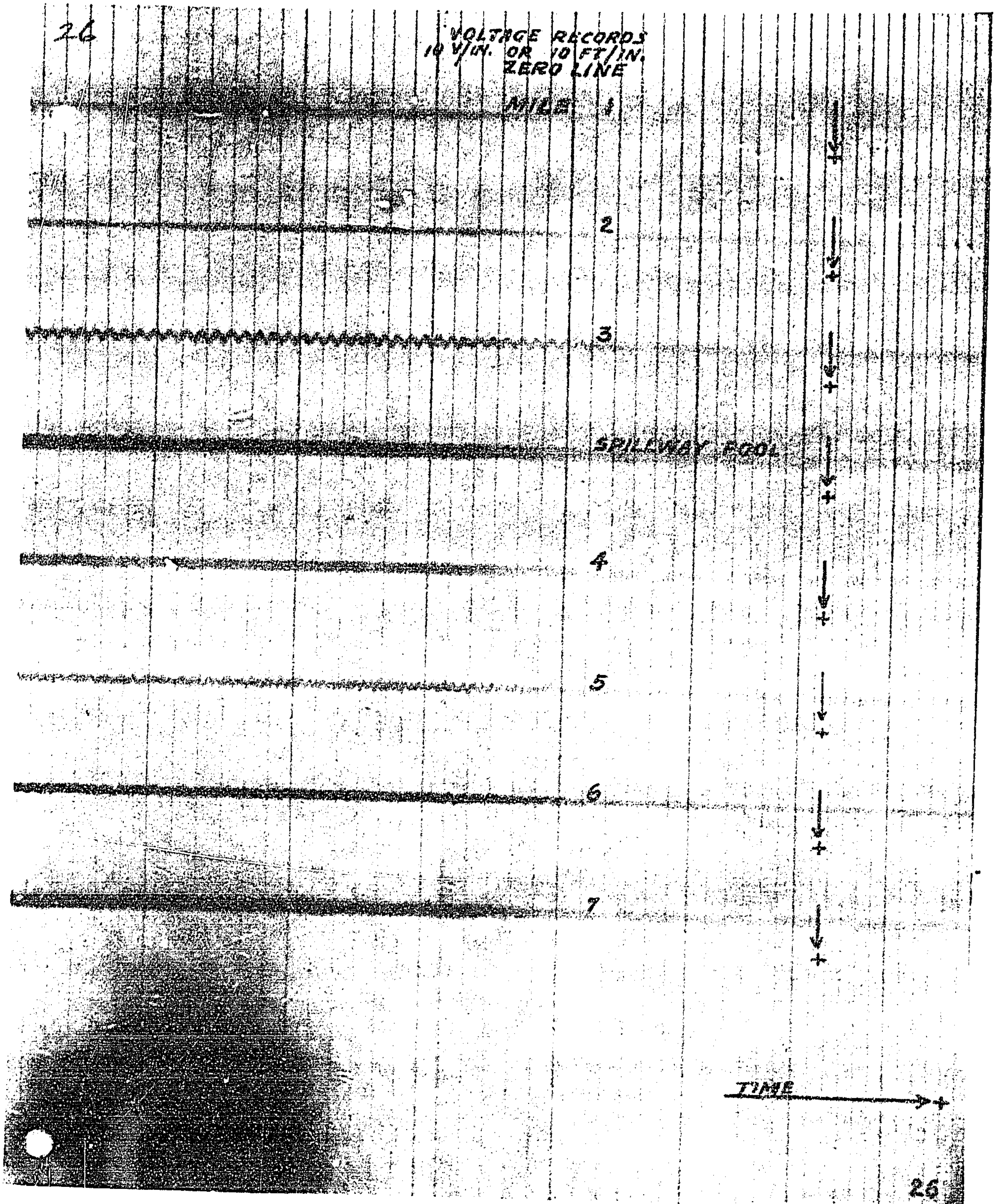
VOLTAGE RECORDS

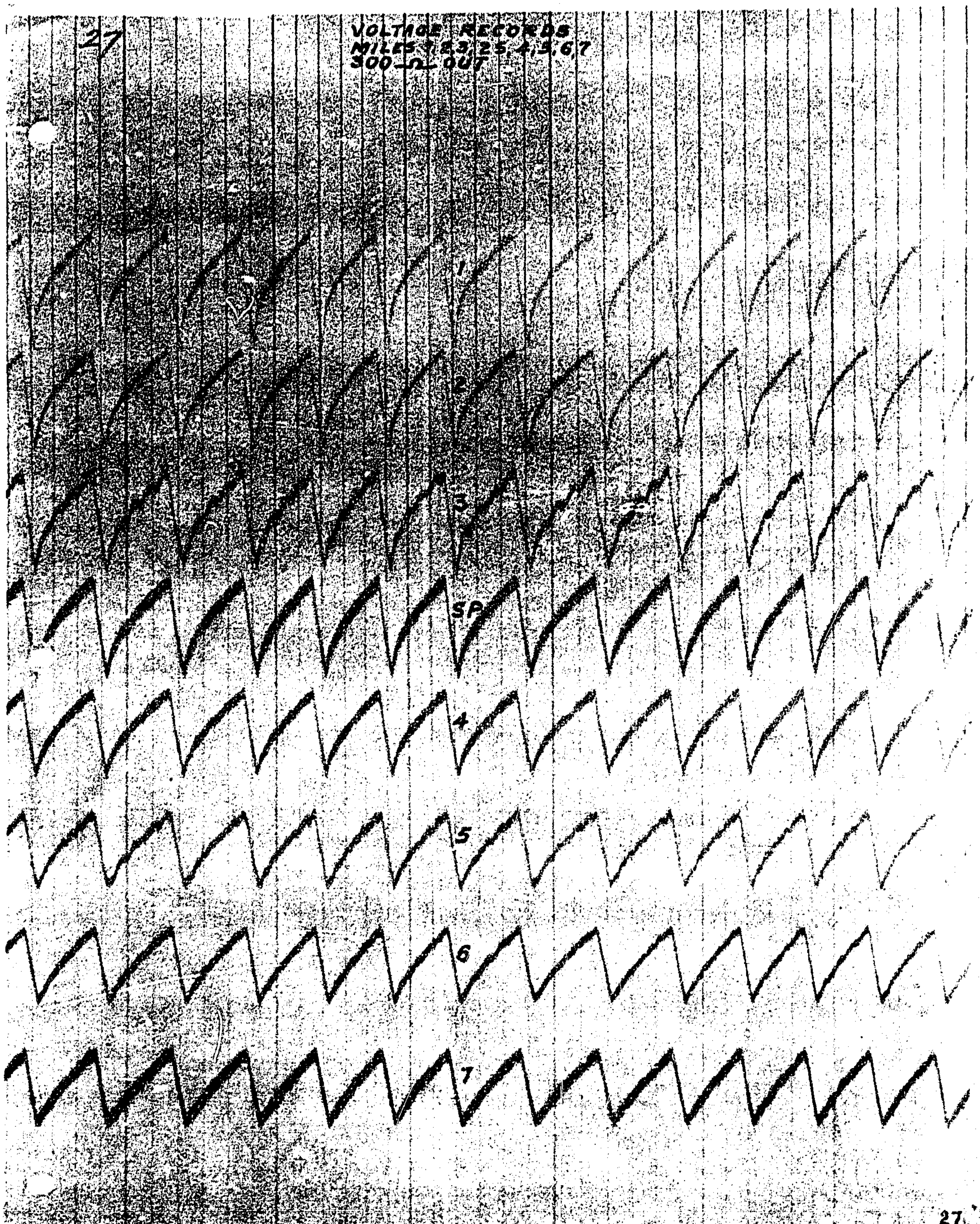
These records show the wave shape, magnitude and phase shift of the voltages with respect to ground for the points at the beginning of the mile sections of the analog. The input current was 10 milliampere.

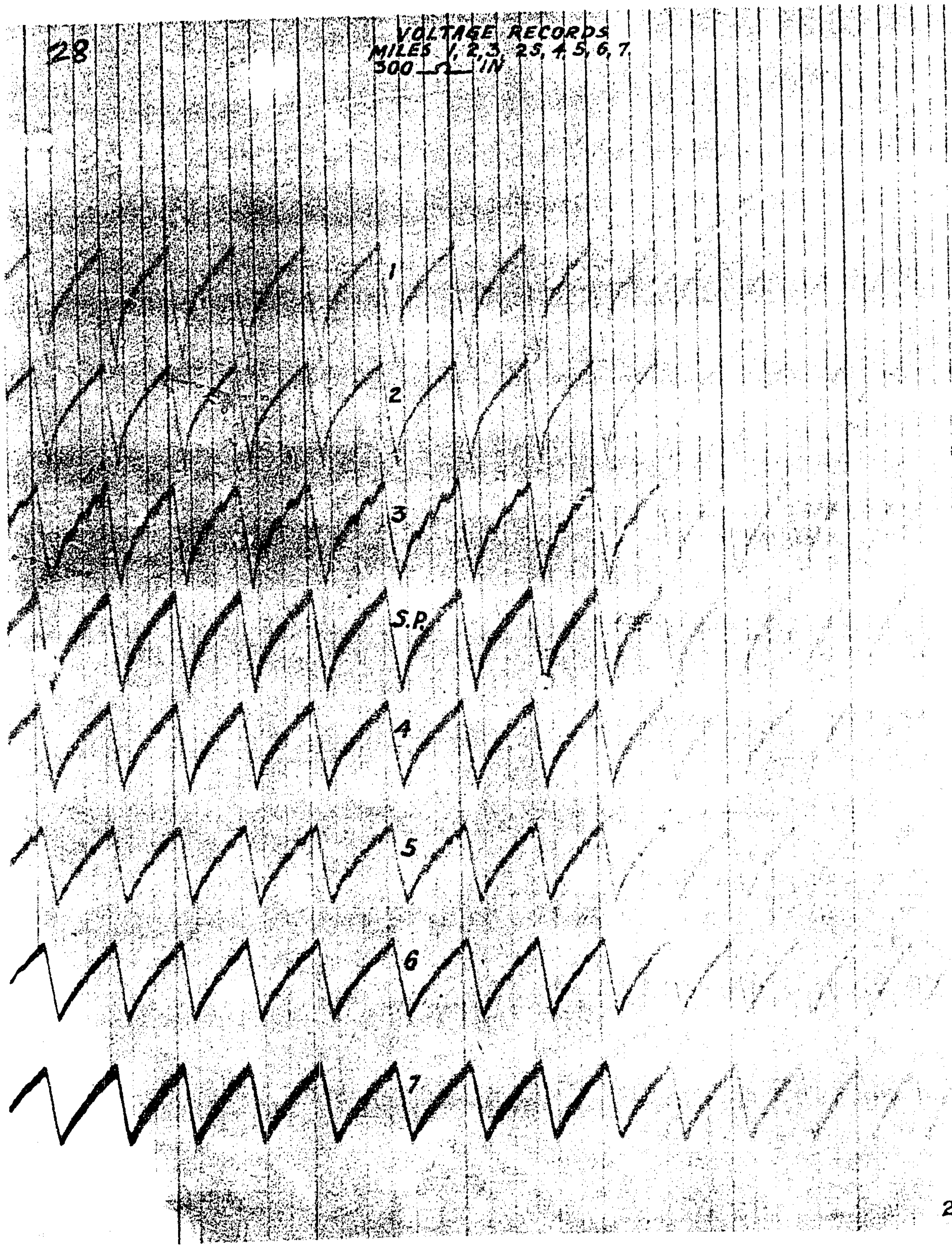
One volt in the analog represents 1 foot in the prototype. The voltage amplifiers were adjusted so that 10 volts would result in a deflection of 1 inch; therefore, 1-inch deflection represents 10 feet.

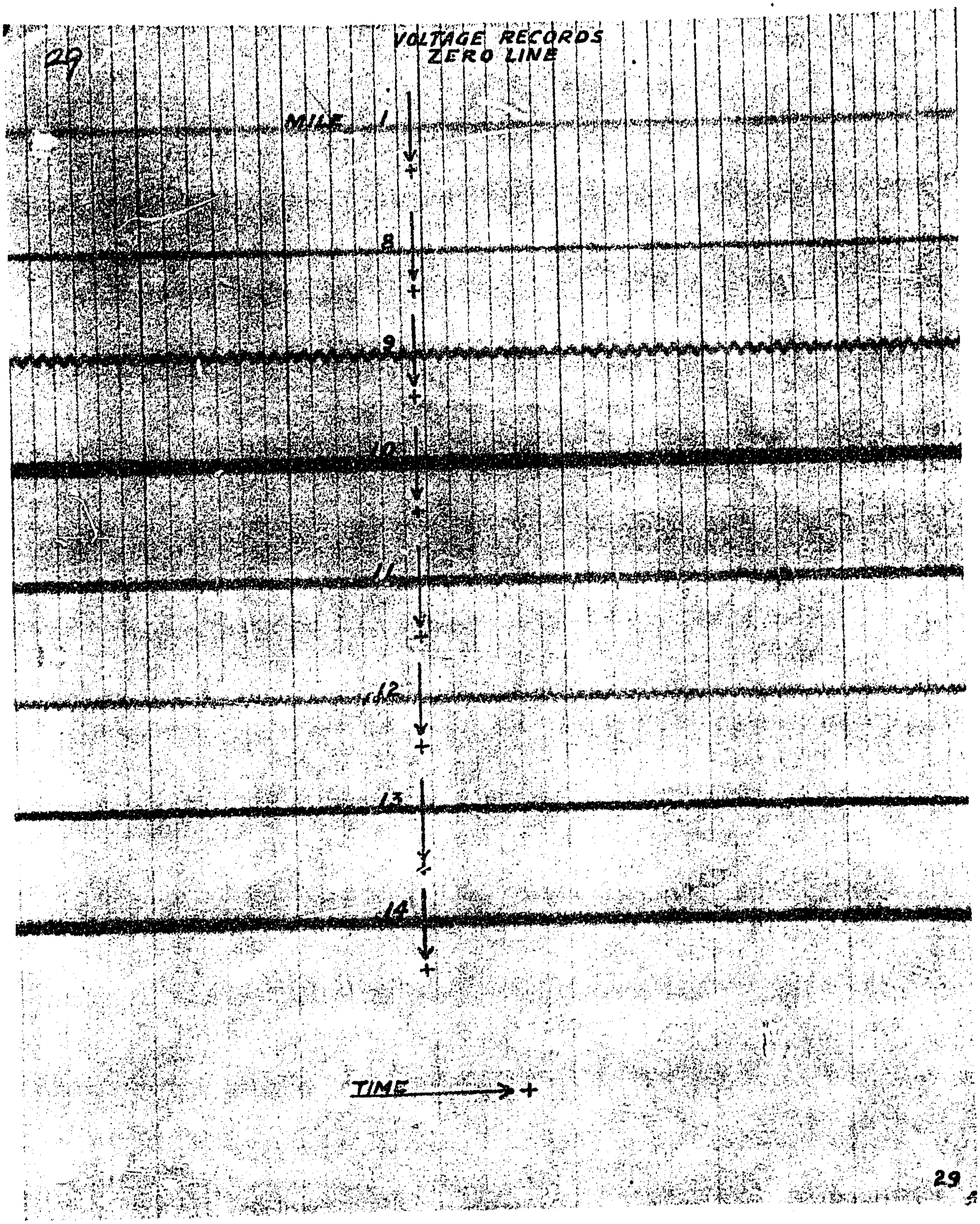
DATA FOR VOLTAGE RECORDS

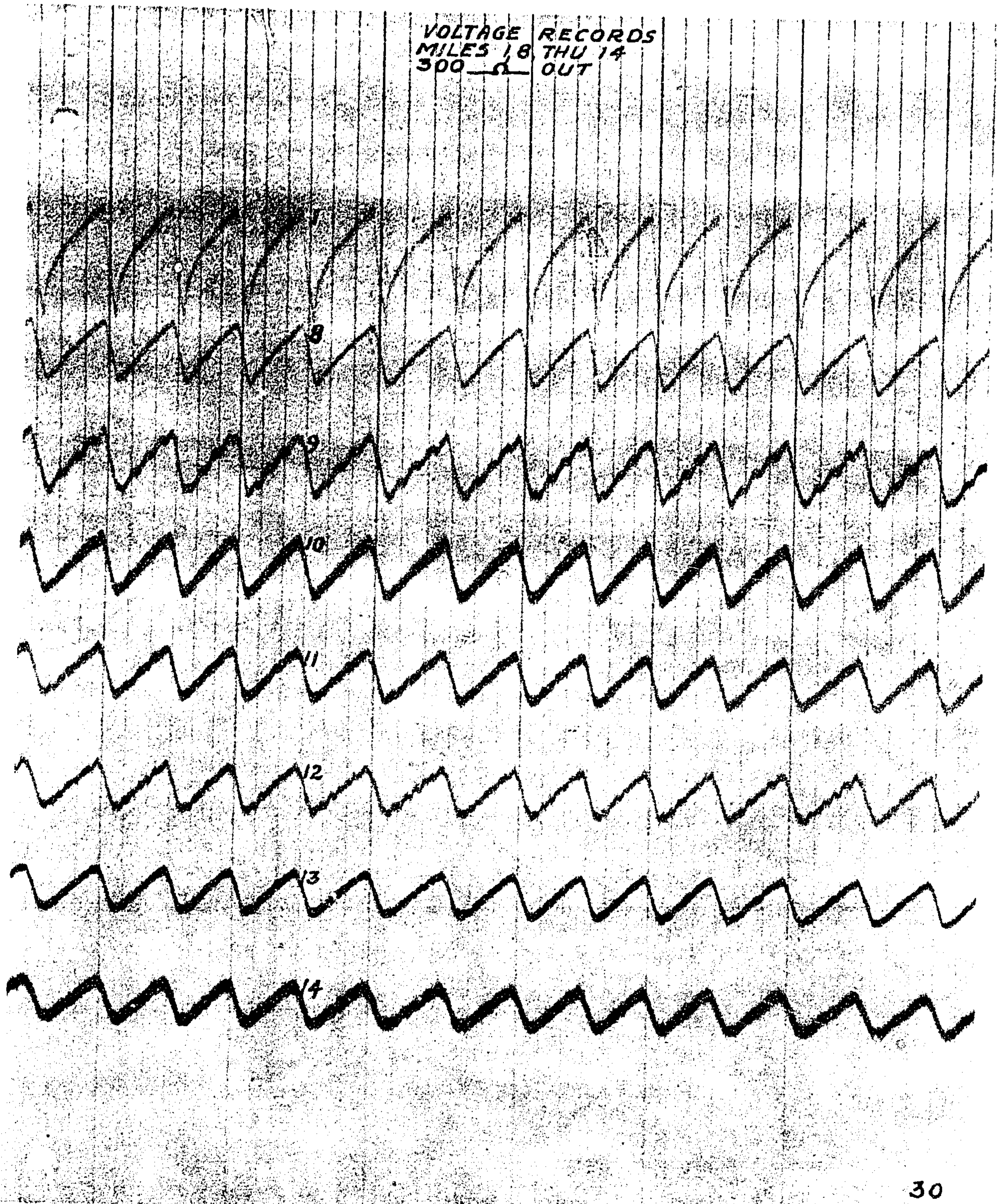
Run	Mile	Trace	300 ohm	Remarks
1	1	1	—	Zero line
	2	2	—	Zero line
	3	3	—	Zero line
	S.P.	4	—	Zero line
	4	5	—	Zero line
	5	6	—	Zero line
	6	7	—	Zero line
	7	8	—	Zero line
2	(Same as run 1)		Out	10 ma input
3	(Same as run 1)		In	10 ma input
4	1	1	—	Zero line
	8	2	—	Zero line
	9	3	—	Zero line
	10	4	—	Zero line
	11	5	—	Zero line
	12	6	—	Zero line
	13	7	—	Zero line
	14	8	—	Zero line
5	(Same as 4)		In	10 ma input
6	(Same as 4)		Out	10 ma input
7	1	1	—	Zero line
	15	2	—	Zero line
	16	3	—	Zero line
	17	4	—	Zero line
	18	5	—	Zero line
	19	6	—	Zero line
	20	7	—	Zero line
8	(Same as run 7)		Out	10 ma input
9	(Same as run 7)		In	10 ma input

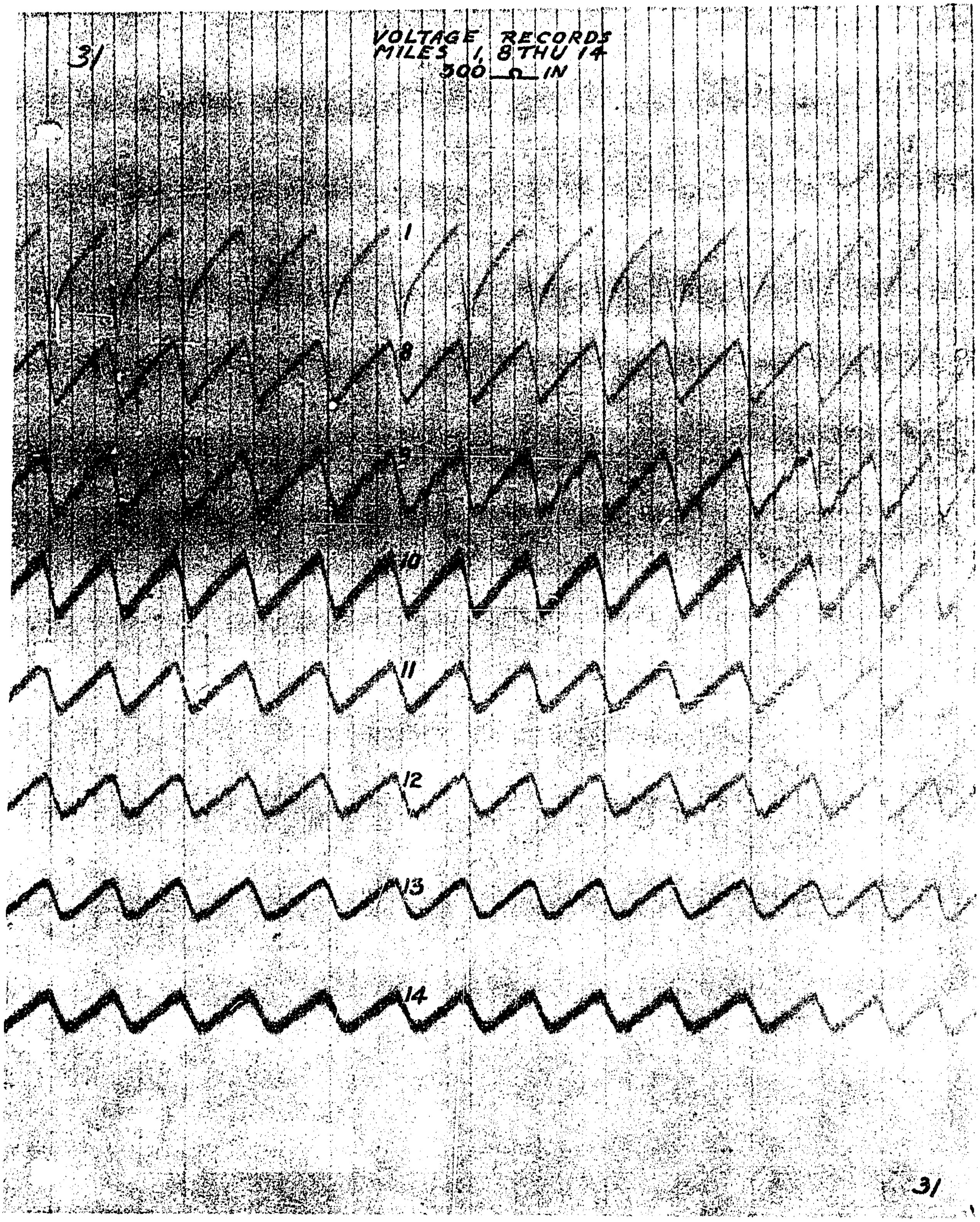

VOLTAGE RECORDS
10 V/IN. OR 10 FT/IN.
ZERO LINE
MILE 1
2
3
SPILLWAY POOL
4
5
6
7
TIME

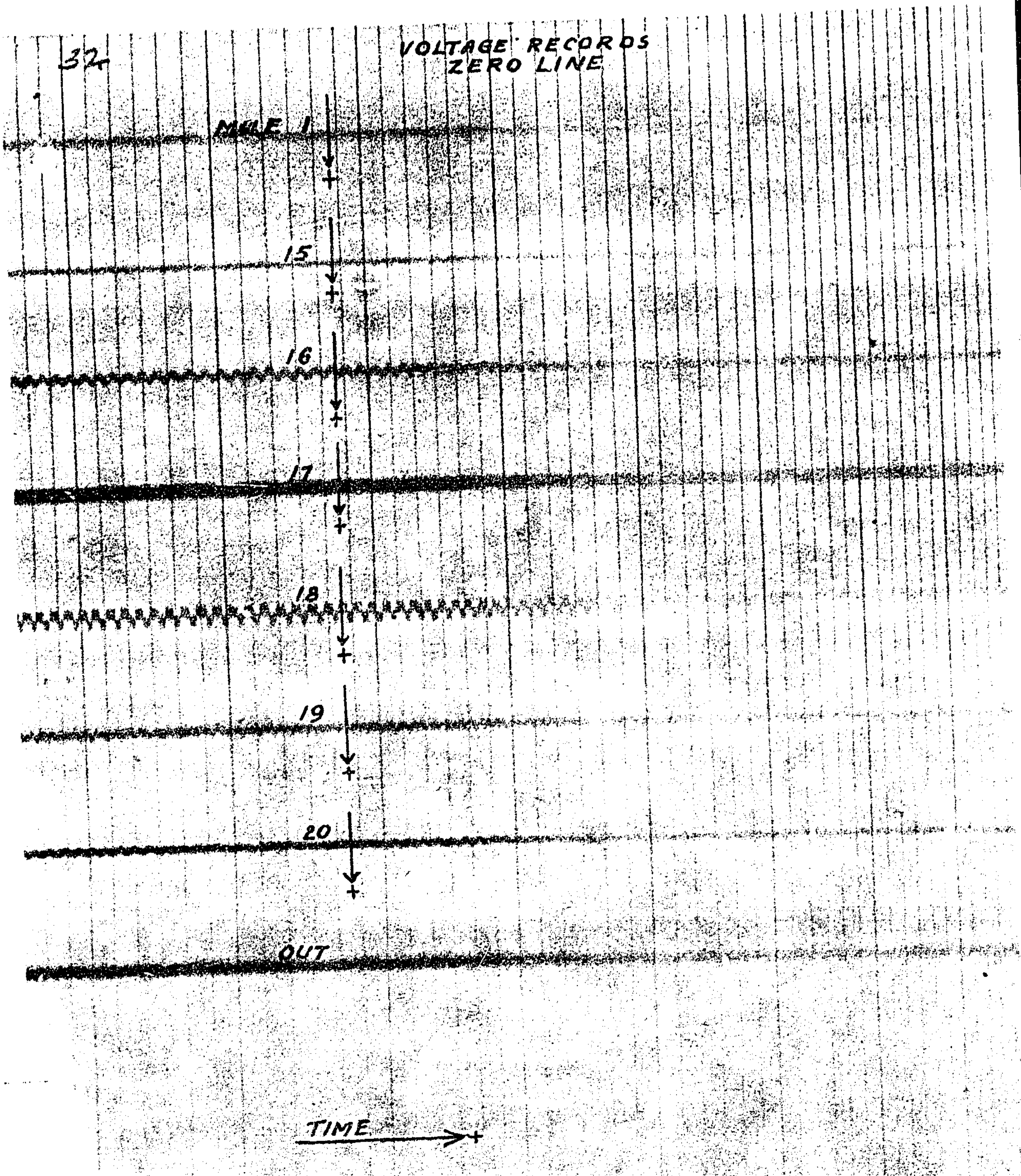

27
VOLTAGE RECORDS
MILES 1,2,3,25,4,5,6,7
300-Ω OUT
1
2
3
SP
4
5
6
7

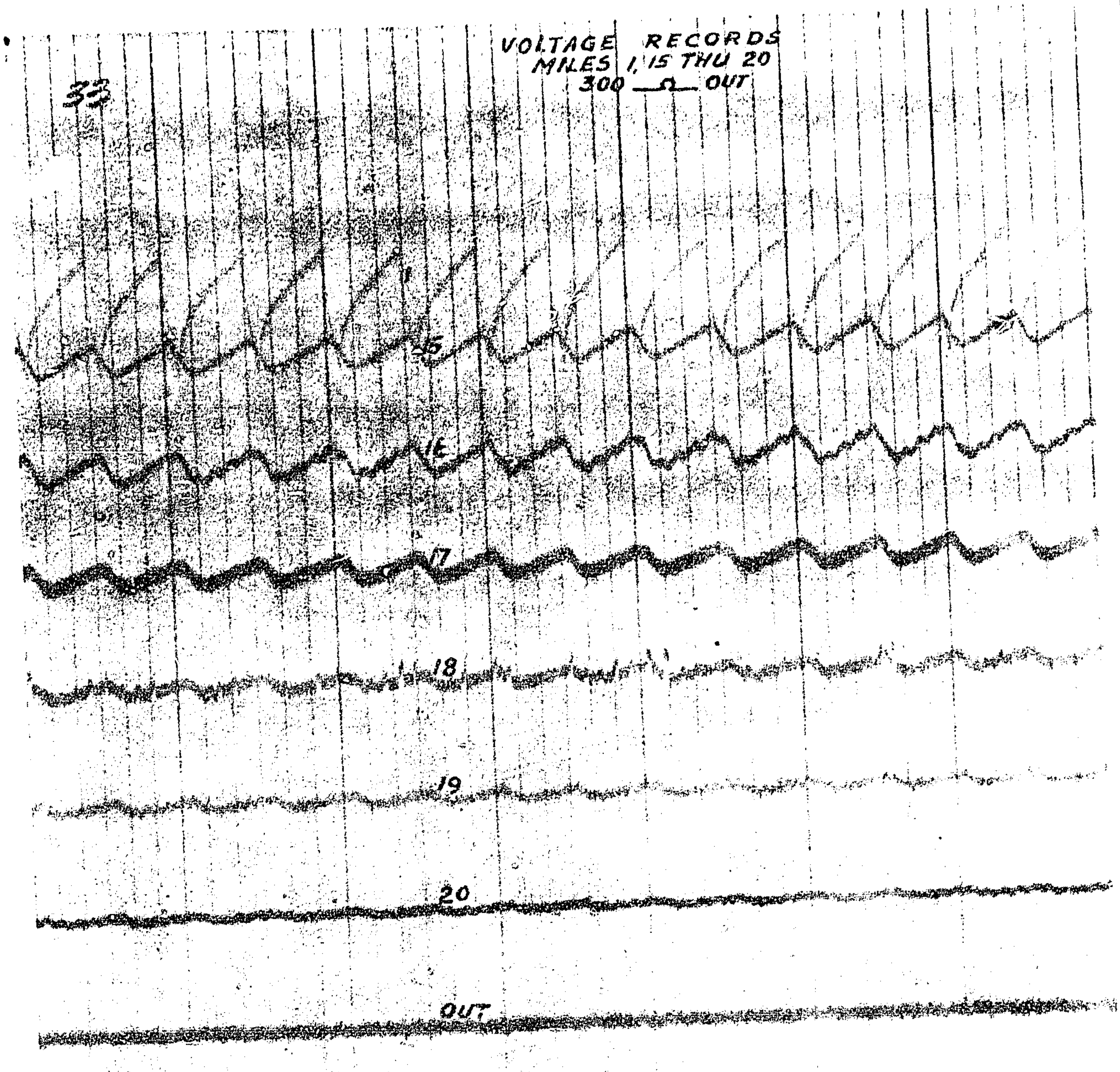

28
VOLTAGE RECORDS
MILES 1, 2, 3, 2S, 4, 5, 6, 7
500 Ω IN
1
2
3
S.P.
4
5
6
7

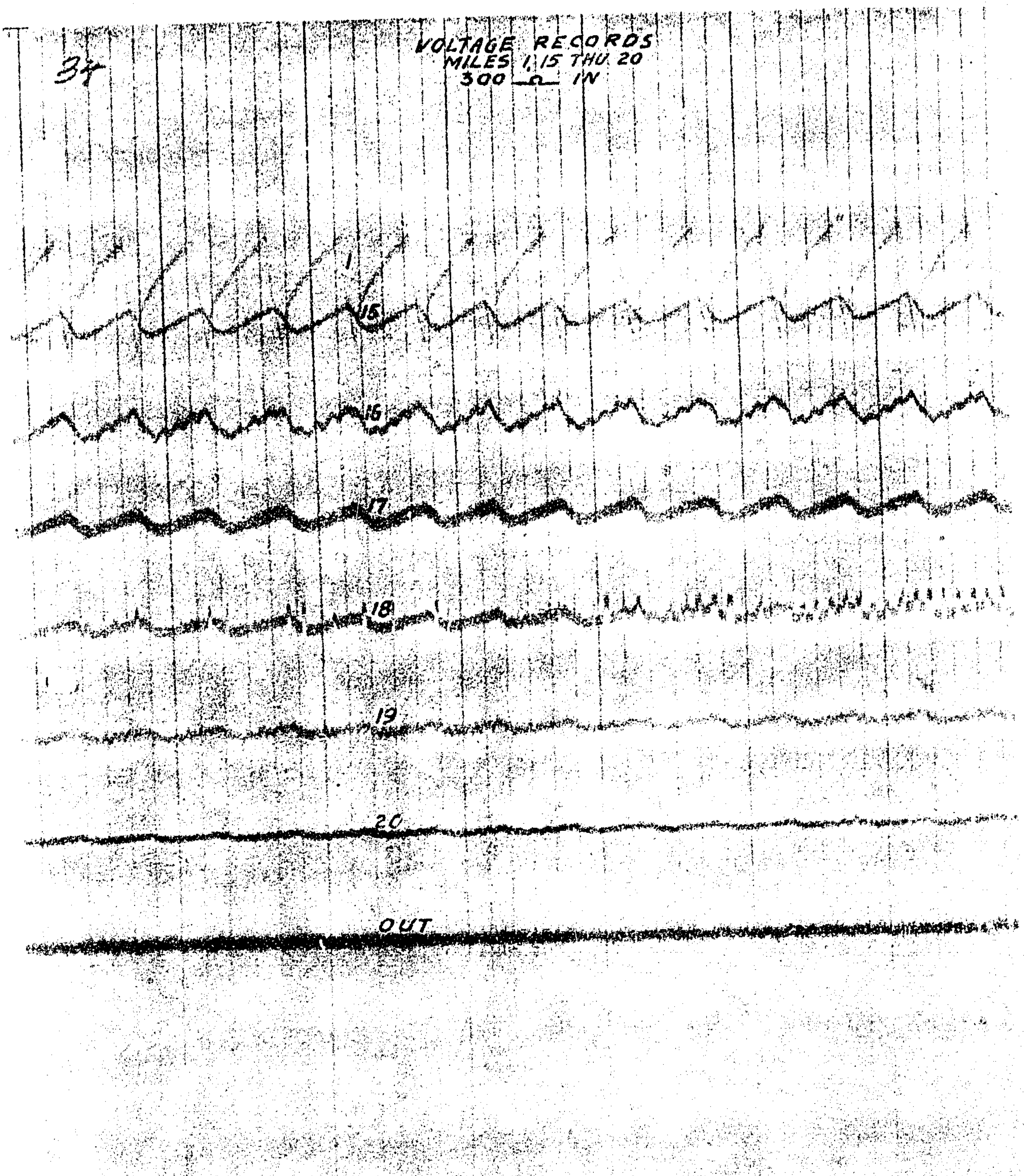

29
VOLTAGE RECORDS
ZERO LINE
MILE 1
8
9
10
11
12
13
14
TIME


VOLTAGE RECORDS
MILES 18 THU 14
300 Ω OUT
8
9
10
11
12
13
14


31
VOLTAGE RECORDS
MILES 1, 8 THU 14
300 Ω IN
1
8
10
11
12
13
14
31


VOLTAGE RECORDS
ZERO LINE
MILE 1
15
16
17
18
19
20
OUT
TIME


VOLTAGE RECORDS
MILES 1, 15 THU 20
300 Ω OUT
33
15
16
17
18
19
20
OUT


VOLTAGE RECORDS
MILES 1, 15 THU 20
300 Ω IN
1
15
16
17
18
19
20
OUT