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HYD 307

HYD 307

Denver, Colorado

September 20, 1950

Hyd. 307

Memorandum

To: E. W. Lane

From: A. C. Carter

Subject: Distribution of tractive forces around channel perimeters

Summary

In order to design a channel through erodible material which will be free from scour, a shape of channel must be selected which will result in maximum tractive forces smaller than the critical tractive force of the material composing the banks and bed.

The purpose of this investigation is to determine the magnitudes and distribution of tractive forces around the perimeter of earth canals having different shapes.

The method used to obtain these forces was as outlined in Hydraulic Laboratory Report No. 293 ³/_{*} by E. W. Lane. It consists of drawing isovels, or lines of equal velocity, on cross sections showing velocity distribution. Orthogonal lines or lines of zero net shear are then drawn perpendicular to the isovels. It was assumed that the area bounded by adjacent orthogonal lines and the included section of perimeter was proportional to the tractive or shearing force on that portion of the perimeter. Tractive force distribution curves could then be determined from area measurements.

Results of this investigation are summarized in two sets of curves, Figures 22 and 23, showing maximum tractive forces which exist on the sides and bottoms of channels having different side slopes and bottom width to depth ratios.

*Numbers refer to bibliography in appendix

Introduction

The problem of designing stable channels in erodible material is being approached in steps outlined by E. W. Lane in Hydraulic Laboratory Report No. 294 4/. Tractive force distribution around the perimeter of channel cross sections was given as an important phase of the problem which should be studied. To be free from scour, a channel must be so designed that tractive forces combined with gravity forces are everywhere just less than the critical tractive force of the bed material.

To evaluate the magnitude and location of tractive forces, all available data on flow distribution in straight rectangular, trapezoidal, and triangular shaped channels were used. Isovels, or lines of equal velocity, were sketched in and orthogonal lines, or lines of zero net shear, were drawn perpendicular to the isovels. It was assumed that the ratio of area bounded by two adjacent orthogonal lines to the total cross-sectional area was proportional to the ratio of tractive force on that section of the perimeter between the orthogonal lines to the total tractive force on the perimeter. By plotting percent area between orthogonal lines against percent distance along perimeter and converting to terms of tractive force, the distribution of tractive forces was obtained.

Rectangular, trapezoidal, and triangular shaped sections were studied. The ranges of bottom width to depth ratios and side slopes were somewhat limited since few cross sections were available for study.

Canal Sections Showing Velocity Distribution

All available channel cross sections showing velocity distribution, with the maximum velocity at the center, were taken from published experiments for study. This study was limited to straight channels, and unless the maximum velocity occurred at the center there was no assurance that the channel was straight.

From the experiments by Bazin, 1/ 33 rectangular cross sections were chosen which had B/D or bottom width to depth ratios varying from 0.60 to 10.50; 6 trapezoidal sections with B/D ratios of 1.81 to 4.95 and side slopes of 0.06:1, 0.1:1, and 1:1, and 2 triangular channels having side slopes of 1:1.

One cross section of the Festi-Rasini 2/ Canal was used which had a B/D ratio of 1.29 and side slopes of approximately 1:1. This canal was originally constructed of masonry with a rounded bottom and sloping sides. However, sediment filled the bottom until the cross section was trapezoidal in shape as shown in Figure 1.

Three trapezoidal sections having B/D ratios of 2.72 and side slopes of 1:1 were taken from experiments by J. Varwick 3/.

One trapezoidal cross section of the Main South Canal, Orland Project, California, with velocity measurements by Fred C. Scobey 6/ was used. The B/D ratio was 8.30 and side slopes 1.6:1.

Seventeen triangular sections with 1:1 side slopes were selected from "Experimental Studies of Velocity Distribution at a Section in a 90° Triangular Channel" by Harry D. Purdy, Jr. 5/.

Five cross sections of the Interstate Canal, North Platte Project, were included in the study. These had side slopes of 1.5:1 and 2:1 and B/D ratios of 2.55 to 18.66.

Three cross sections having side slopes of 2:1 and B/D ratios of 2.82, 3.08, and 3.52 were taken from notes on velocity distribution by V. M. Hegly 8/. These three channels had compound cross sections, and only the right half of each section corresponding to half of a simple trapezoidal section was used. The point of maximum velocity was considered to be the center of trapezoidal and rectangular sections.

Isovels and Orthogonal Lines

Most of the cross sections showed points of known velocity with the isovels or lines of equal velocity drawn in. On others it was necessary to draw in the isovels by interpolating between points of measured velocity.

Orthogonal lines, or lines of zero shear transfer, were then sketched in perpendicular to the isovels. All orthogonal lines met at the point of maximum velocity. A typical section is shown in Figure 1. Photostats of cross sections used are included in the appendix attached to the file copy of this report and are available for reference in hydraulic laboratory files.

Tractive Forces

Scouring forces due to velocities in a channel are difficult to measure because of turbulence and steep velocity gradients near the bed. For that reason, an analysis of the forces causing scour has been made using the concept of tractive force advanced by Du Boys in 1879. Stated briefly, tractive force is the force exerted by flowing water on the bottom of a channel and for a unit area is equal to the component of weight of a column of water above that unit area in the direction of flow. It is the force causing the volume of water to move and which tends to move the material composing the bed.

In a channel of infinite width and uniform depth, the traction on a unit area may be stated:

$$\tau = wDS$$

τ = tractive force per unit area

w = unit weight of water

D = depth

S = slope of energy gradient

For simplicity, all calculated values of tractive force will be expressed in terms of wDS.

Area Computations

On the premise that the area bounded by two adjacent orthogonal lines is proportional to the tractive force on the included section of perimeter, the areas were planimetered. The cross sections were divided into halves at the middle of the section. The area of each section was then expressed as a percent of one-half the total area of the cross section lying below the orthogonal line which originates at the edges of the water surface. Computations were made separately for the two halves of each cross section, and the results were averaged arithmetically as shown below. All results are given in detail in appendix to the file copy in the hydraulic laboratory files.

Since the point of maximum velocity usually occurs some distance below the water surface, an orthogonal line drawn to the edges of the water surface from this point and the water surface itself bound an area of the cross section which does not contact the wetted perimeter. It is not known how the tractive force due to this area is distributed around the perimeter, so it was assumed that the tractive force on each section of perimeter should be increased by an amount proportional to the area above the point of maximum velocity divided by the area lying below the point of maximum velocity. The same results are obtained by expressing individual areas as percentages of the area lying below the point of maximum velocity rather than the total area of the section.

The distance along the side was measured from the water surface to the intersection with each orthogonal line and expressed as a percent of the total side distance. The distance along the bottom was measured from the intersection of the side and bottom to the intersection with each orthogonal line and expressed as a percent of one-half the bottom width. Computations were made in this manner for the two halves of each cross section and the results were averaged arithmetically as shown below.

Cumulative Percent Area Curves

The percent of total tractive force acting on each section of perimeter between adjacent orthogonal lines could now be found by proportional

areas, but these would be average values for each interval of distance. In order to obtain tractive force distribution, the following method was used: the percent of the cross sectional area, as shown by the orthogonal lines, which exerted tractive force on the sides and on the bottom of different shapes of canals was plotted against B/D ratios, and smooth curves were drawn through the points, as shown in Figure 2. These curves were used later to determine the percent tractive force acting on the sides and bottoms of rectangles and trapezoids.

By plotting cumulative percent area against cumulative percent distance and joining points with smooth curves, mass diagrams were obtained for sides and bottoms of rectangles and trapezoids and sides of triangles having different side slopes and B/D ratios. Since for each section there were pairs of curves for the two sides and two halves of the bottom, these pairs were averaged arithmetically. Results were erratic, and it was necessary to systematize them; this was done as follows:

From the averaged pairs of curves cumulative percent area values were read for each 10 percent of perimeter distance along the sides and along the bottom of each cross section having a different B/D ratio. The values for each 10 percent of perimeter distance were then plotted against B/D ratio. Smooth curves with a similarity of shape and spacing were fitted to the plotted points as closely as possible, and cumulative percent areas read from the curves.

The resulting cumulative percent areas were replotted against percent distance for even and half numbered B/D ratios. These final curves are shown in Figures 3 to 7 and 14 to 17, and the calculations are included in the appendix attached to the file copy of this report in the hydraulic laboratory files.

Tangents of Cumulative Percent Area Curves

Cumulative percent area curves are actually integral curves of the area. The differential, or slope, at any point would therefore be the instantaneous area at that point. Slopes of these curves were then measured at 10 percent intervals of distance and the values for each interval were plotted against B/D ratio. Smooth curves for each 10 percent of distance were drawn to conform fairly closely to the plotted points with an attempt to eliminate irregularities in the shape and spacing. The resulting tangent values were plotted against percent distance for even and half-numbered B/D ratios.

Tractive Force Distribution Curves

The tangent curves were then evaluated in terms of tractive force as follows: an average tangent was computed for the sides and for the bottoms of channels having different side slopes and B/D ratios. These average tangents when multiplied by some constant, K, would equal the average tractive force on the side or on the bottom. The average tractive force multiplied

by the side or bottom distance then equaled that part of the total tractive force of the section which acts on the side or on the bottom as shown in Figure 2. Solution of this equation evaluates K, which converts the tangent value at any point on the side or on the bottom to tractive force at that point for that particular side slope and B/D ratio. Knowing K, it was possible to convert the tangent curves to tractive force distribution curves, Figures 8 to 12 and 18 to 21, and the maximum tractive forces for each side slope and B/D ratio were found from these curves. Tractive force data are included in the appendix attached to the file copy of this report which is available for reference in the hydraulic laboratory files.

The general solution of K for sides and bottoms of channels is as follows: referring to Figure 13, a trapezoidal channel of unit length has a volume,

$$V = BD + \overline{ss} D^2$$

V = volume of prism
 B = bottom width
 D = depth at center
 \overline{ss} = side slope (horizontal distance for a 1 ft. rise).

Multiplying by the unit weight and the slope of the energy gradient, the total tractive force of the prism is found

$$T = (B + \overline{ss} D) wDS \dots\dots\dots (1)$$

T = total tractive force
 w = unit weight of water
 S = slope of energy gradient.

The average tangent multiplied by K and the side distance equals the total tractive force on the sides

$$T_s = \text{avg. tan} (K) (2D \sqrt{1 + \overline{ss}^2}) \dots\dots\dots (2)$$

Solving equations (1) and (2) for K of the sides

$$K_s = \frac{\% T \text{ on side}}{\text{avg. tan}} \left(\frac{B/D + \overline{ss}}{2 \sqrt{1 + \overline{ss}^2}} \right) (wDS) \dots\dots\dots (3)$$

of the bottom

$$K_b = \frac{\% T \text{ on bottom}}{\text{avg. tan}} \left(\frac{B/D + \overline{ss}}{B/D} \right) (wDS) \dots\dots\dots (4)$$

where K = conversion factor from tangent to tractive force.

Maximum Tractive Force Curves

Maximum tractive forces on sides and bottoms were then plotted against B/D ratio for side slopes of 0:1, 1:1, 1.5:1, 1.6:1, and 2:1, and

points having the same side slopes were connected by straight broken lines. Smooth unbroken curves were then drawn to agree as nearly as possible with these points, with the following other considerations which indicated their position. One of these considerations was that the point representing the maximum shear for a 1:1 side slope and B/D ratio of 0, that is for a 90° V-shaped channel, was quite accurately established since it was based on velocity distribution observations in 19 channel sections, the results of which agreed quite closely with each other. Moreover, for this section the maximum velocity occurred at the surface, so there was no correction necessary for the area of flow above the line of maximum velocity, as was the case in most other channels. It seemed reasonable to expect that for V-shaped channels of flatter side slopes than 1:1, the maximum tractive force would be greater than for the 1:1 side slopes, approaching 100 percent as the side slopes became very flat. Another consideration was that, as the channel became wider, it seemed certain that a point would be reached at which further widening did not appreciably change the action of the water along the sides of the channel. The velocity distribution data of the various channels indicated that this was the case. This would mean that the curves of maximum shear values should show a nearly constant value for high B/D ratios. Although the values for rectangles indicate that the maximum tractive force on the sides reaches its greatest value for B/D ratios of 3 to 4, the shape of the orthogonal lines shown by the data on the velocity distribution for flatter side slopes indicates that this is not likely to occur for flatter side slopes. The data for 1.6:1 and 1.5:1 side slopes were based on so few sections and such irregular canals that it was largely ignored in drawing the lines representing values of maximum tractive force.

It must be admitted that the values of maximum tractive force indicated by the lines drawn, particularly for the higher values of B/D, are not well established. They do, however, represent the best estimate that can be made from the data available. It is hoped that further studies will fix these values more definitely.

Results

The tractive force distribution curves on the sides and bottoms of trapezoids with 1:1 and 2:1 side slopes and rectangles all showed irregularities in shape and spacing. This was true despite the fact that families of curves having similar characteristics were drawn from data representing a number of channels having the same shape. The method of measuring slopes of mass area diagrams to obtain instantaneous tractive forces appears to be oversensitive judging from the inconsistent results.

In addition, the tractive forces on the sides and bottoms at the intersection of the side and bottom did not agree. It seems likely that the two tractive forces should be equal in the corner, but no attempt was made to so modify the results.

In some instances the tractive force at the water surface was zero, while in others it had a finite value. As a result, no conclusion could be reached regarding the validity of either condition.

Tractive forces of 19 triangular cross sections with 1:1 side slopes were averaged to obtain a single tractive force distribution curve as shown in Figure 12. Its reliability is probably much greater than that of the other channel shapes studied.

Tractive force distribution curves of trapezoids having miscellaneous side slopes and B/D ratios, as shown in Figures 20 and 21, showed considerable variation. Each curve was based on a single cross section, since no other cross sections with the same side slope were available for comparative study.

By plotting maximum tractive forces on sides and bottoms of channels against B/D ratios and connecting points having the same side slopes, Figures 22 and 23, sets of curves were obtained which showed trends but which were lacking in agreement.

Certain channel shapes resulted in tractive forces greater than wDS. In such cases an error very probably was involved since wDS is the maximum tractive force possible in a channel having the same depth as the cross section being considered and infinite width.

As a first approximation of the type of curves to be used in the design of stable channels, smooth curves having similar shapes were drawn for several side slopes without departing from the plotted points excessively. These results will probably depend on additional data for verification.

APPENDIX I

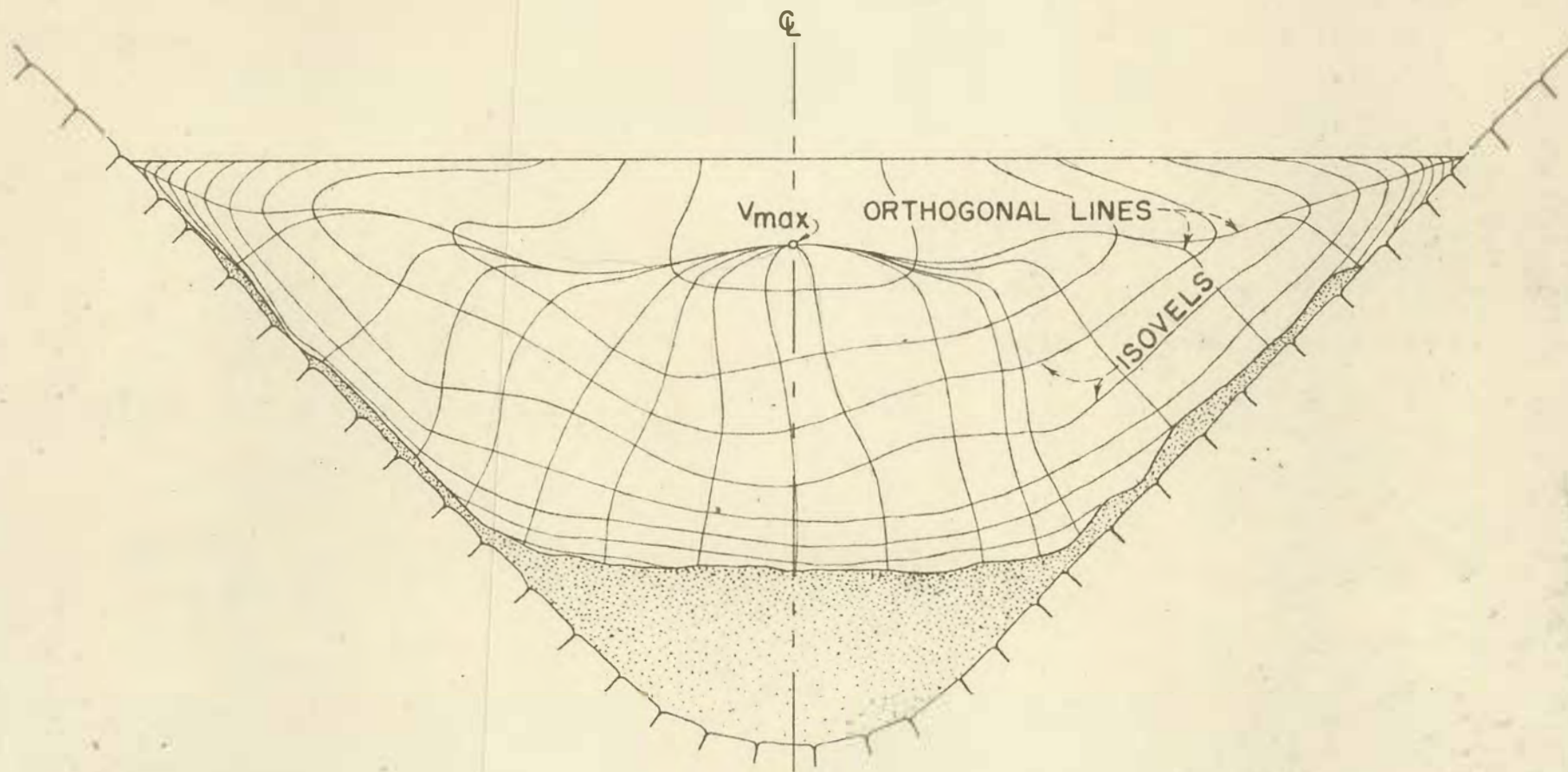
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APPENDIX II

LIST OF FIGURES

1. Typical cross section showing isovels and orthogonal lines.
2. Percent tractive force on sides and bottoms of rectangles and trapezoids.
3. Cumulative percent area--sides rectangles
4. Cumulative percent area--bottoms rectangles
5. Cumulative percent area--sides trapezoids, $ss = 1:1$
6. Cumulative percent area--bottoms trapezoids, $ss = 1:1$
7. Cumulative percent area--triangles, $ss = 1:1$
14. Cumulative percent area--sides trapezoids, $ss = 2:1$
15. Cumulative percent area--bottoms trapezoids, $ss = 2:1$
16. Cumulative percent area--miscellaneous trapezoids, sides
17. Cumulative percent area--miscellaneous trapezoids, bottoms
8. Tractive Force Distribution--sides rectangles
9. Tractive Force Distribution--bottoms rectangles
10. Tractive Force Distribution--sides trapezoids, $ss = 1:1$
11. Tractive Force Distribution--bottoms trapezoids, $ss = 1:1$
12. Tractive Force Distribution--triangles, $ss = 1:1$
18. Tractive Force Distribution--sides trapezoids, $ss = 2:1$
19. Tractive Force Distribution--bottoms trapezoids, $ss = 2:1$
20. Tractive Force Distribution--miscellaneous trapezoids, sides
21. Tractive Force Distribution--miscellaneous trapezoids, bottoms
13. Dimensions of trapezoidal cross section.
22. Maximum tractive forces, sides of channels.
23. Maximum tractive forces, bottoms of channels.



FESTI - RASINI CANAL

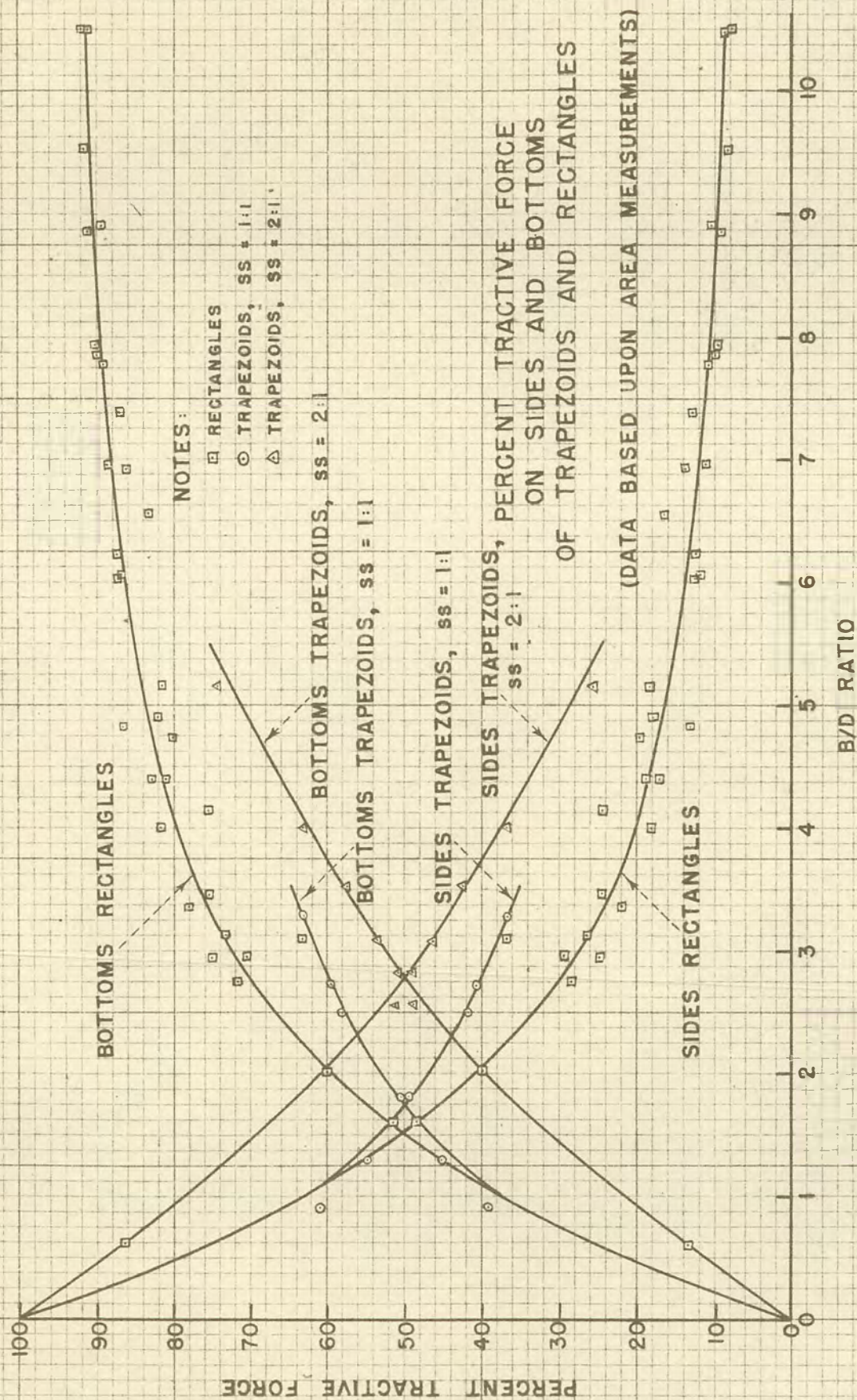
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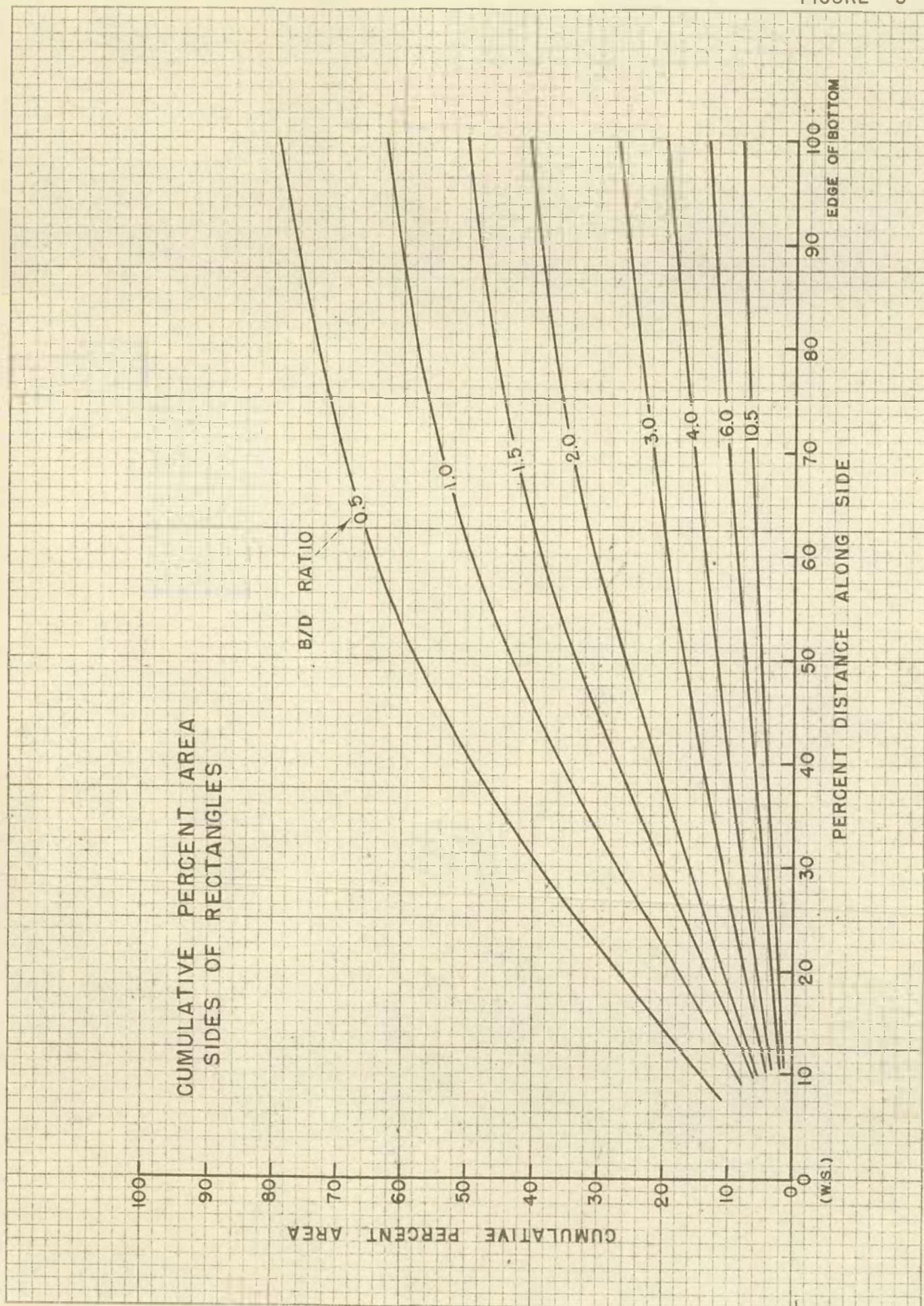
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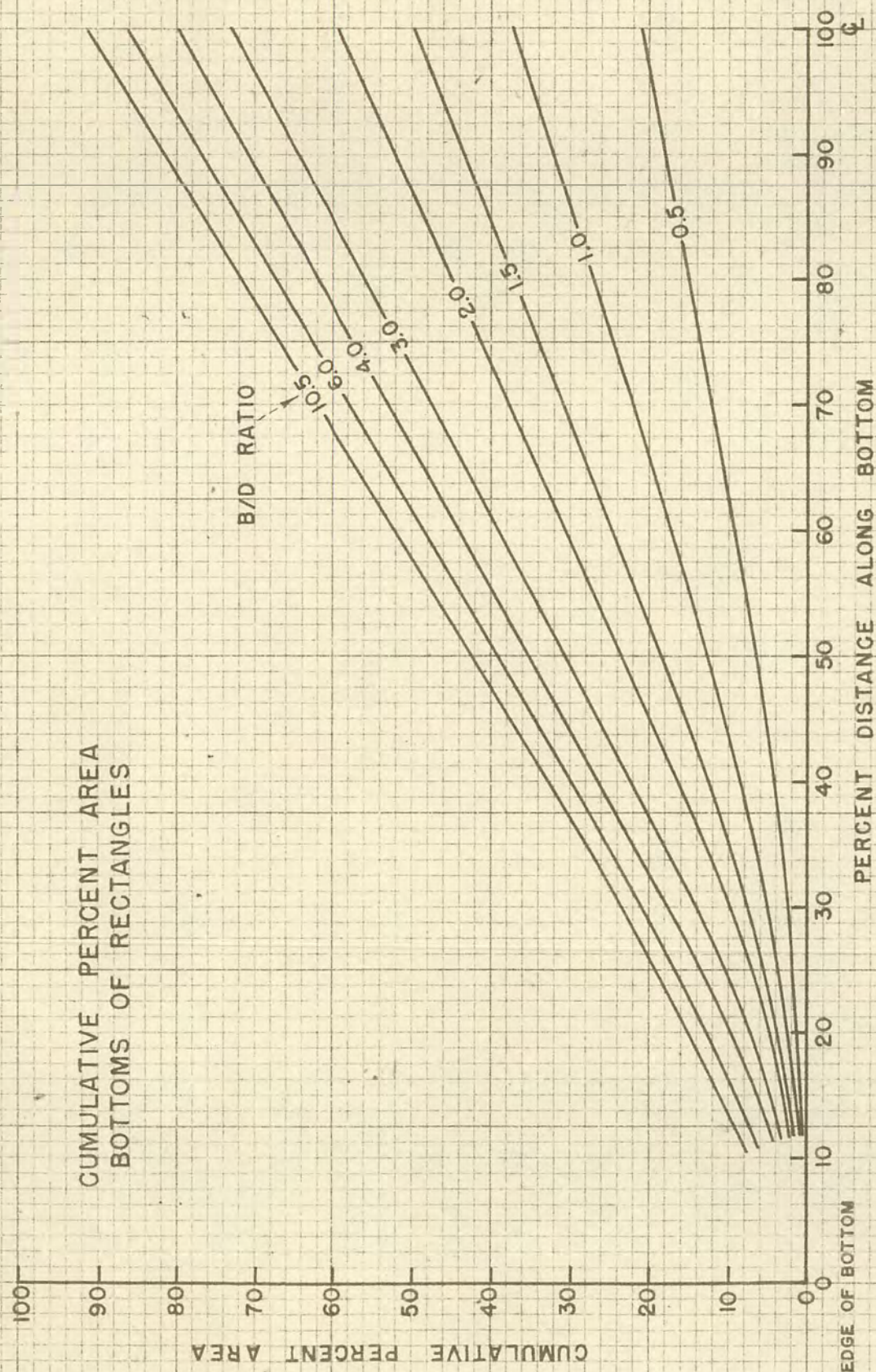
TYPICAL CROSS - SECTION
SHOWING ISOVELS AND
ORTHOOGONAL LINES

DRAWN A.C.C. CHECKED

DATE 9-1-50







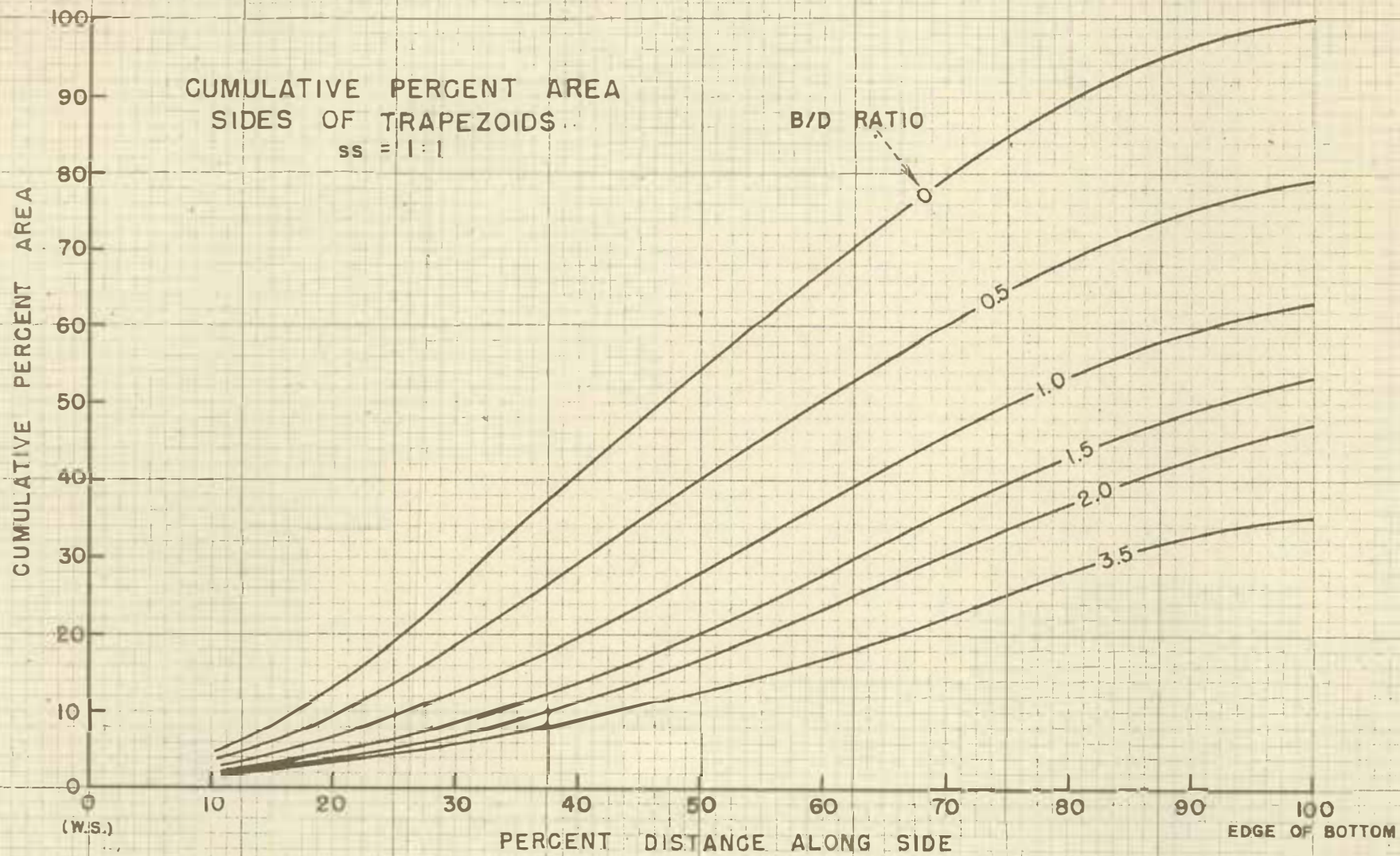


FIGURE 5

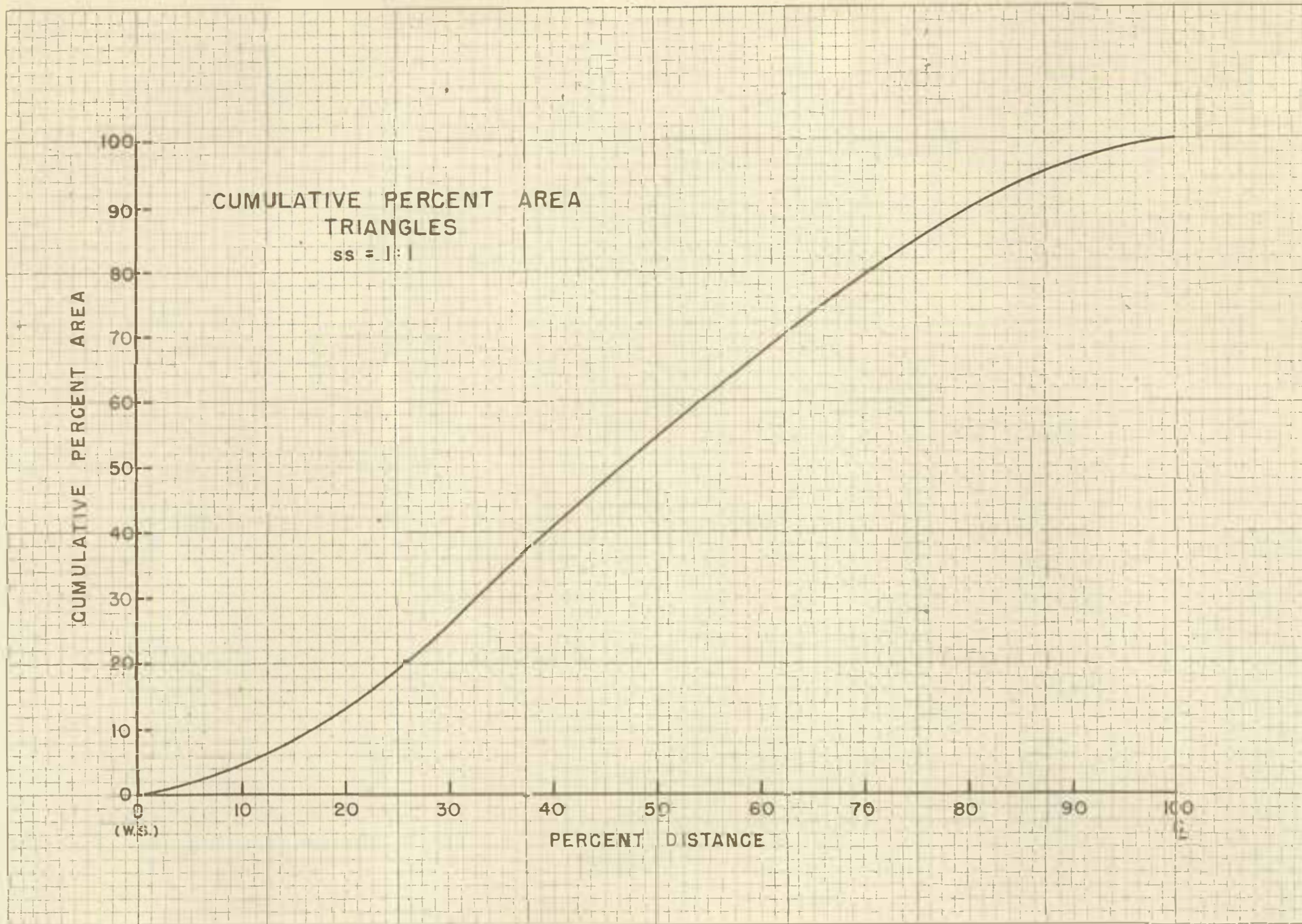


FIGURE 7

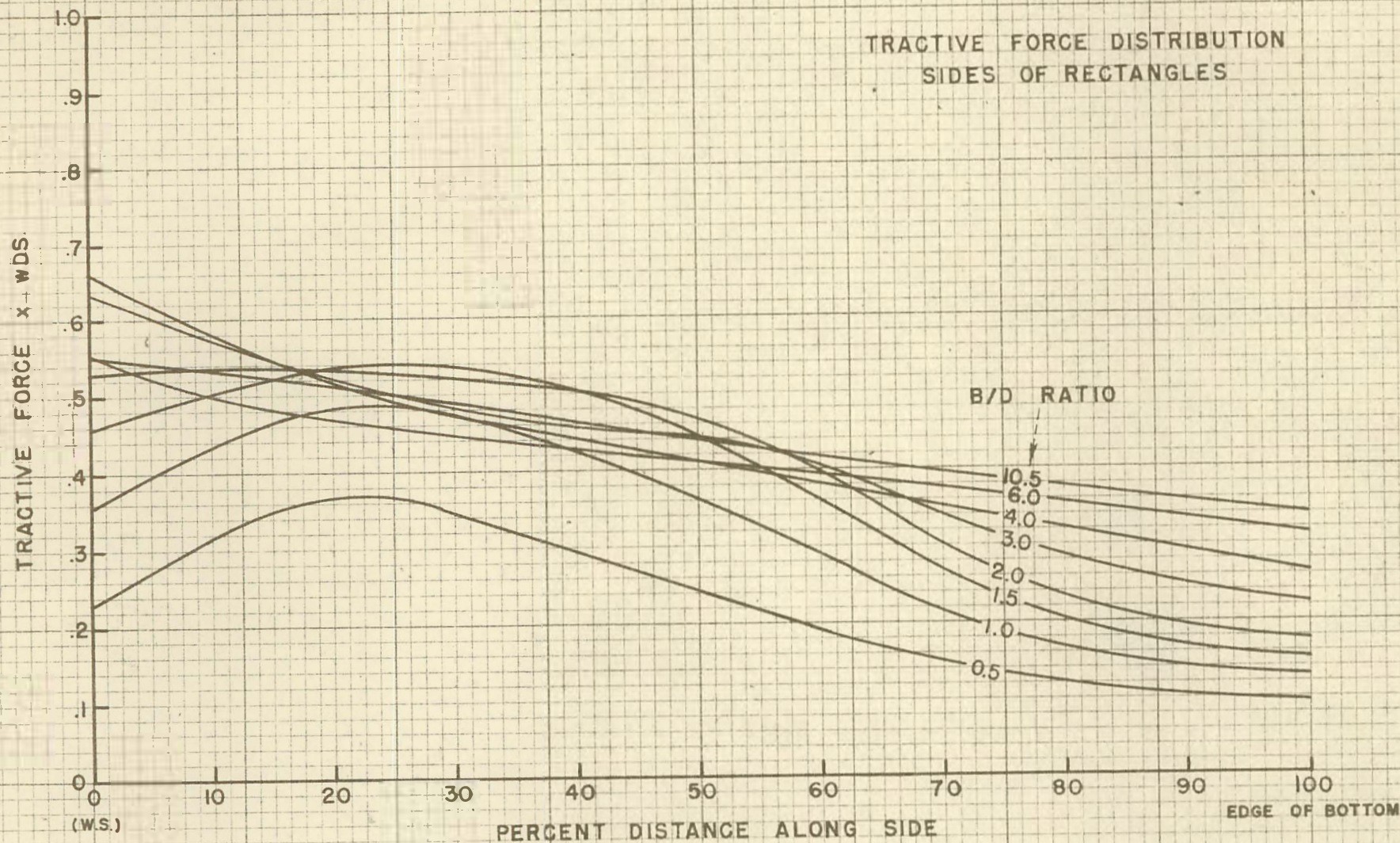
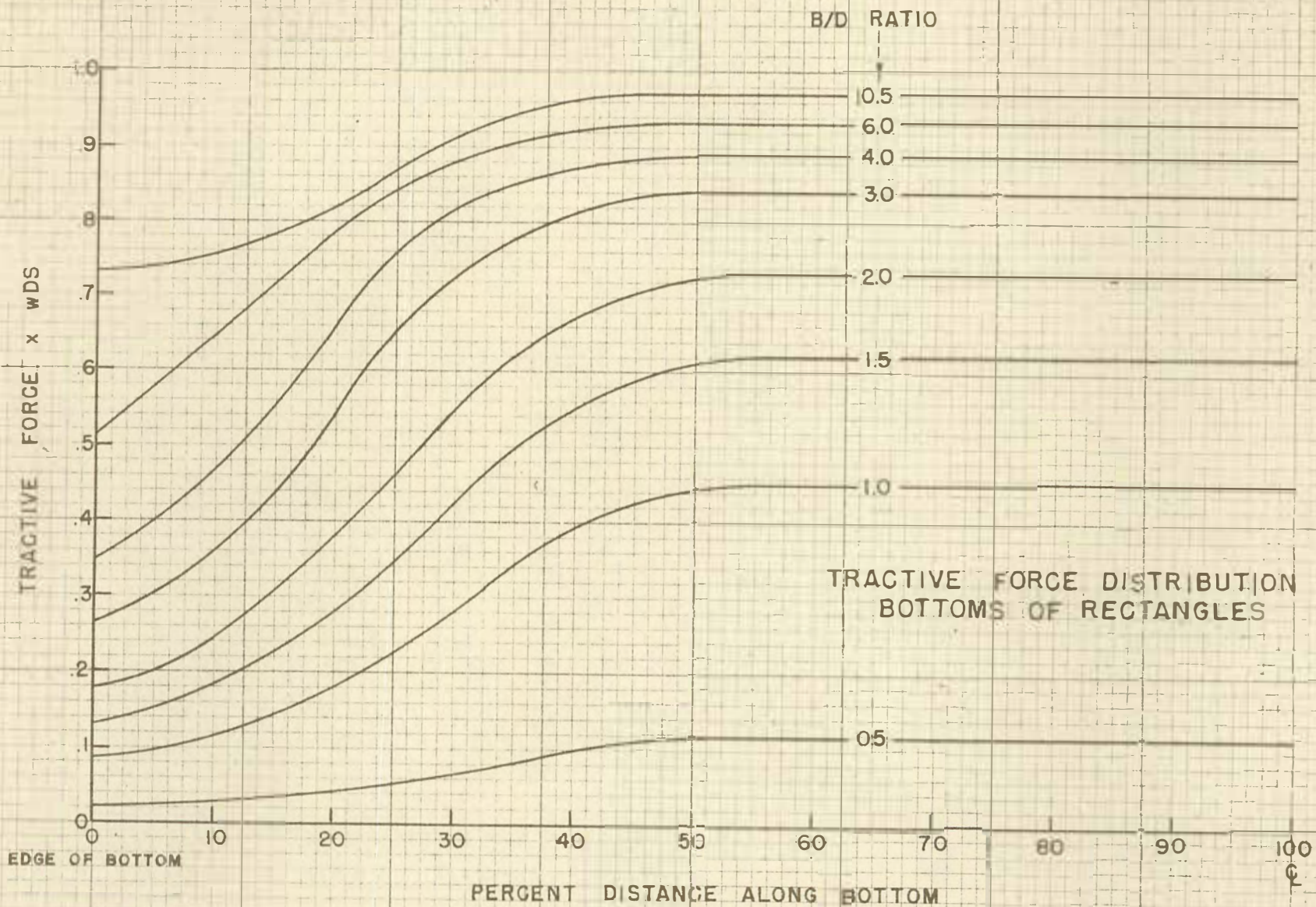
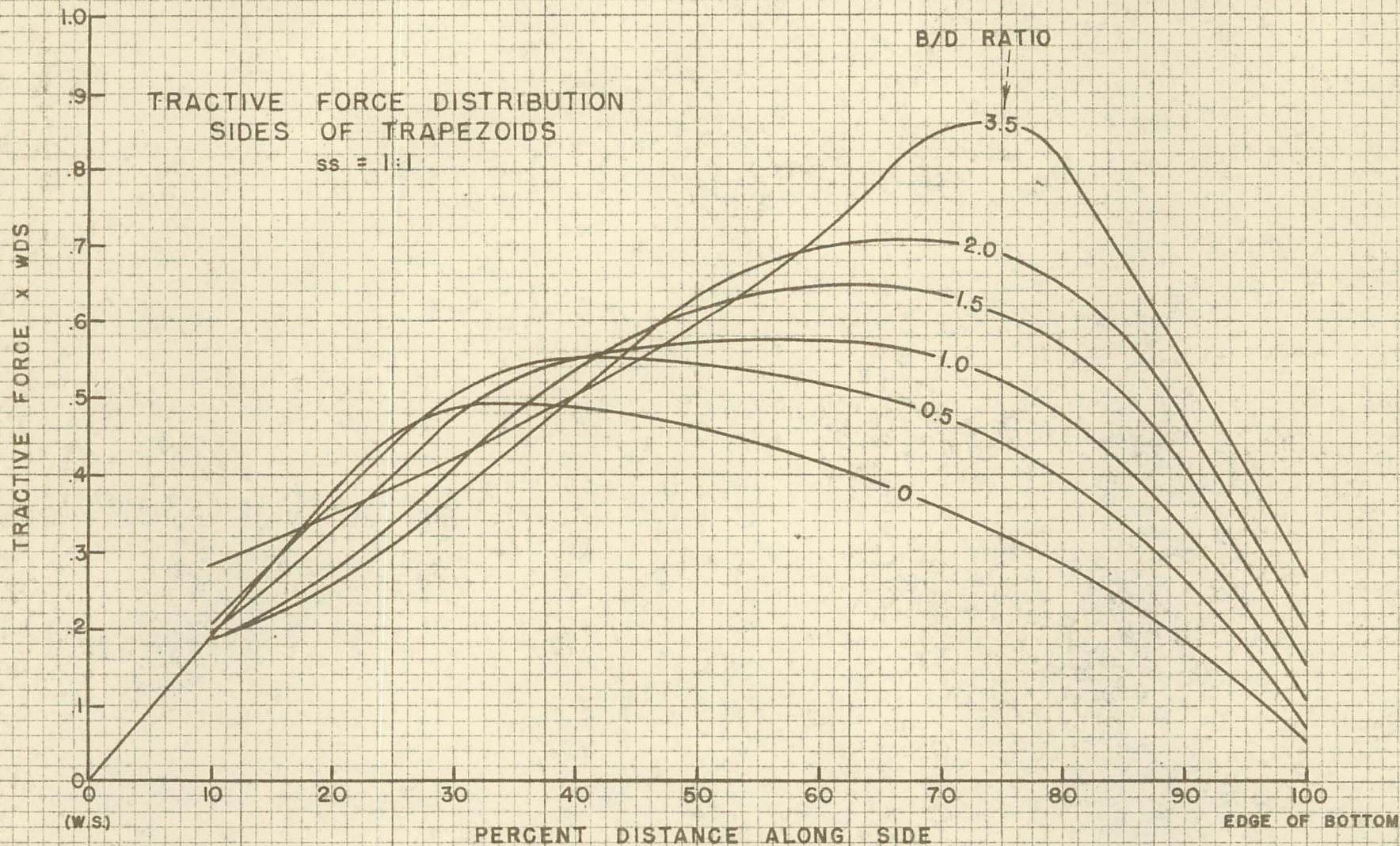
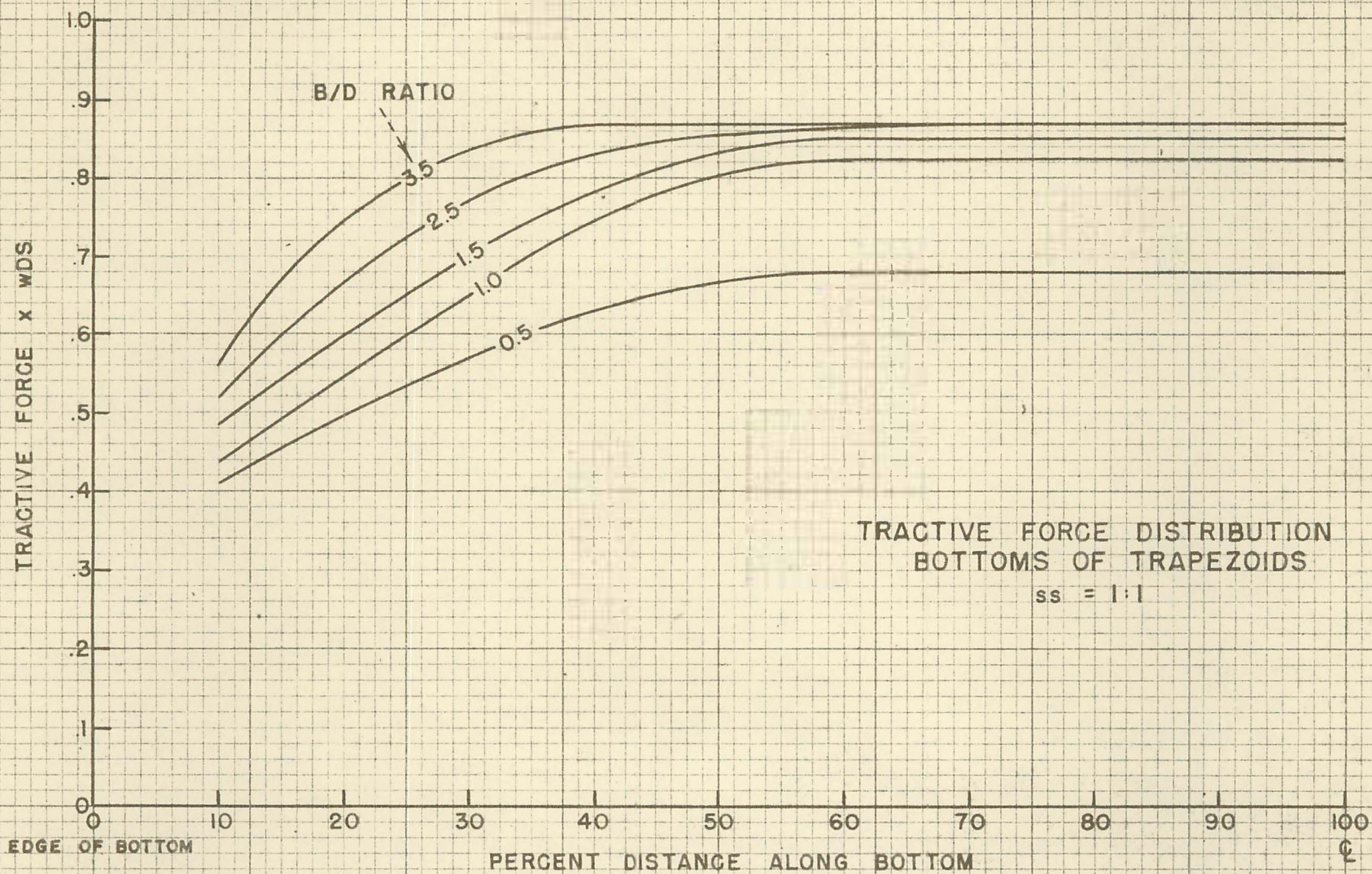


FIGURE 8







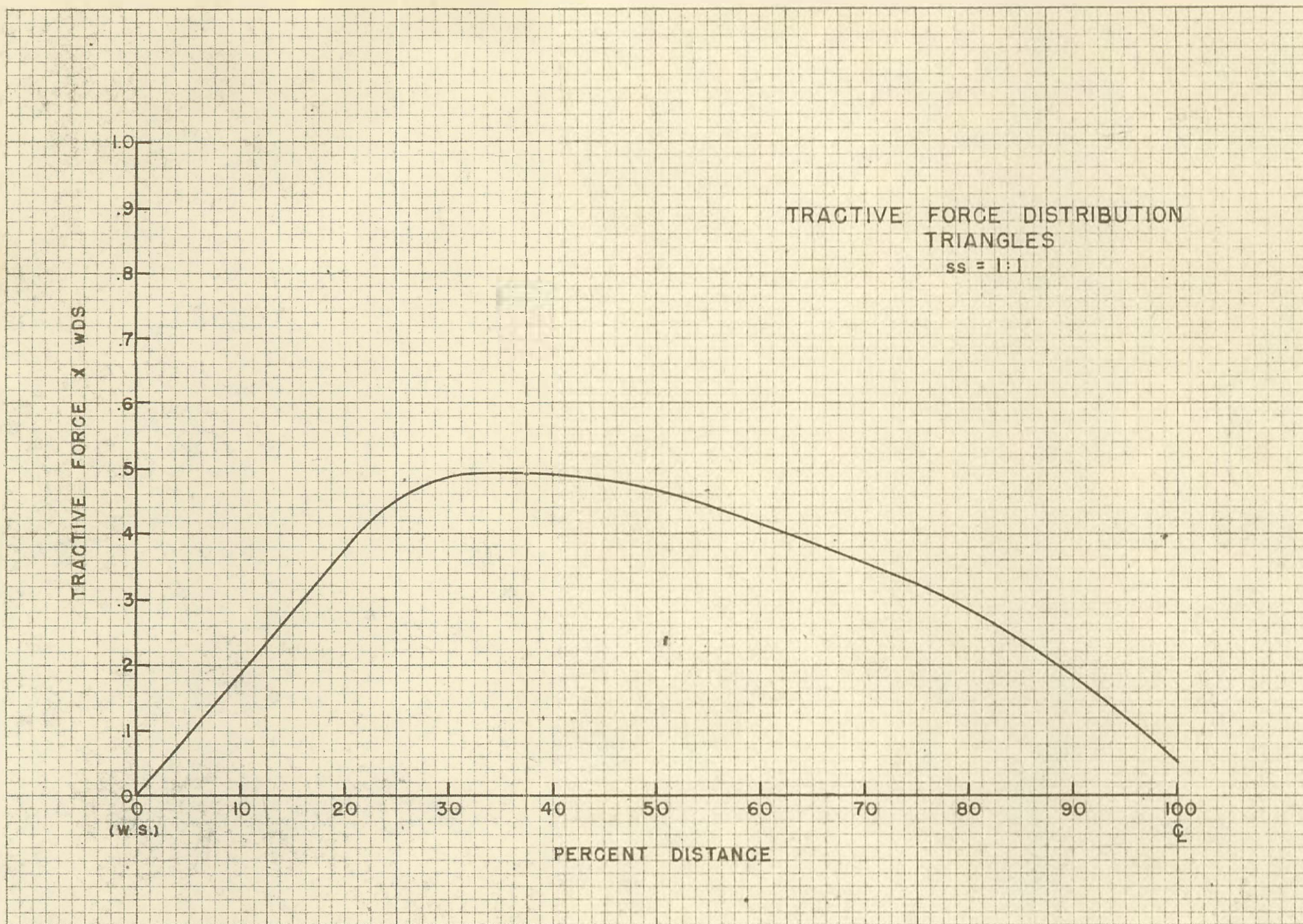
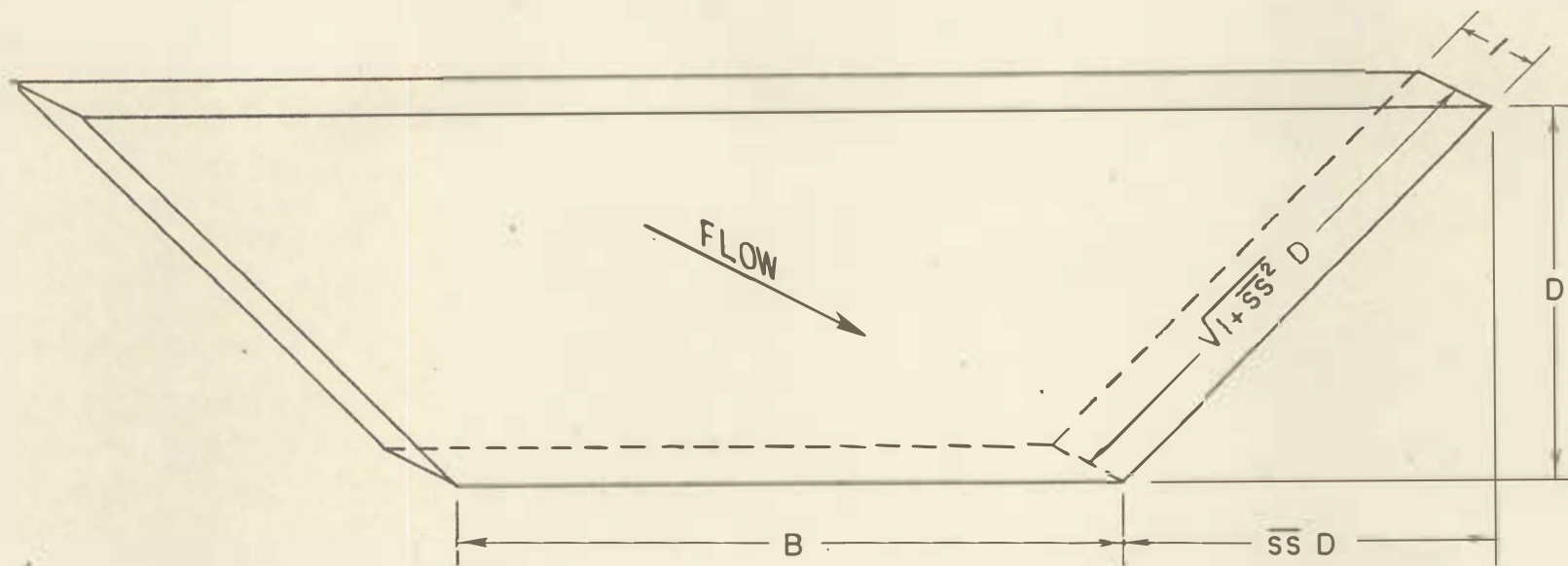


FIGURE 12

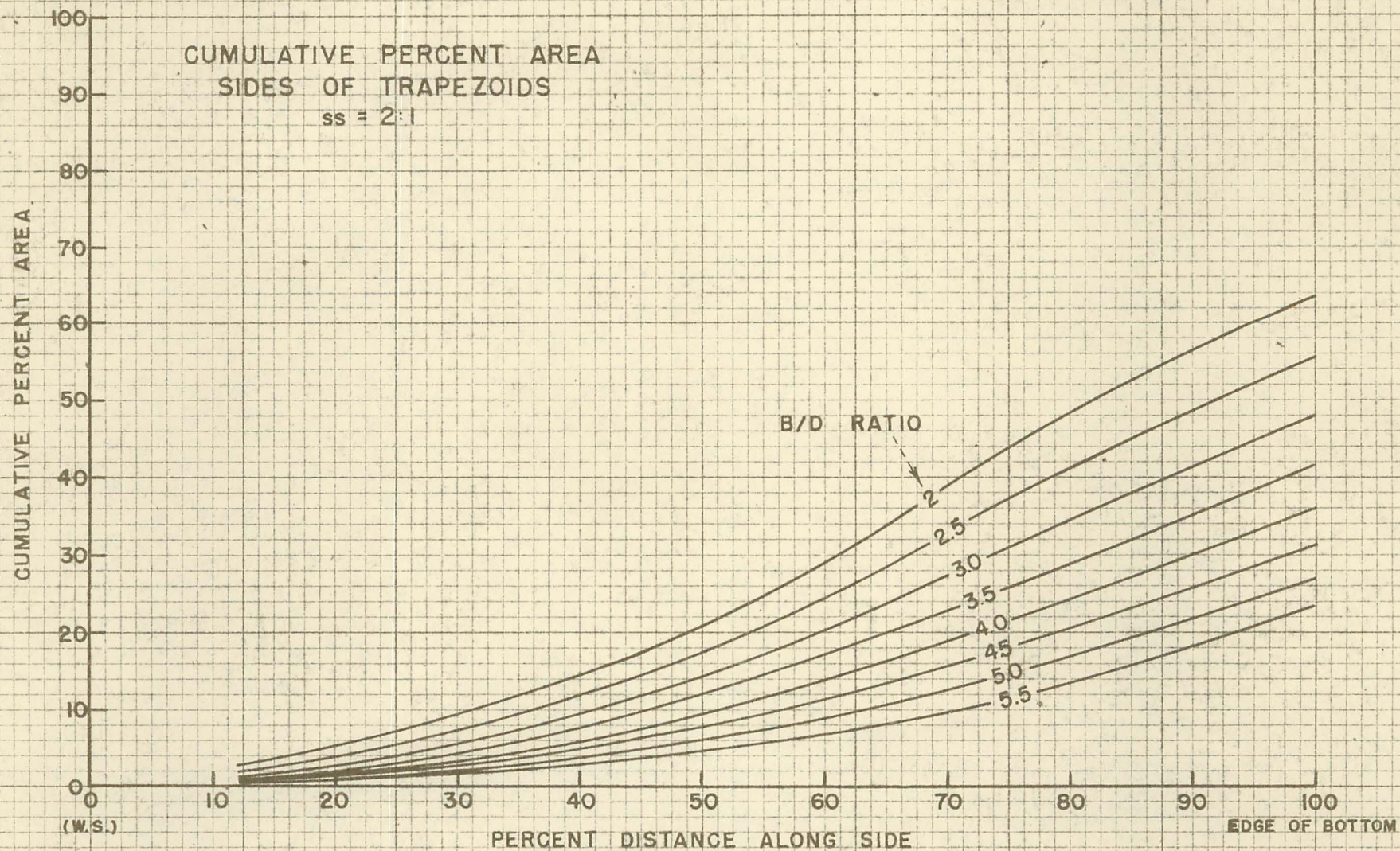


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DIMENSIONS OF TRAPEZOIDAL CROSS-SECTION

DRAWN **A.C.C.** CHECKED

DATE **9-6-50**



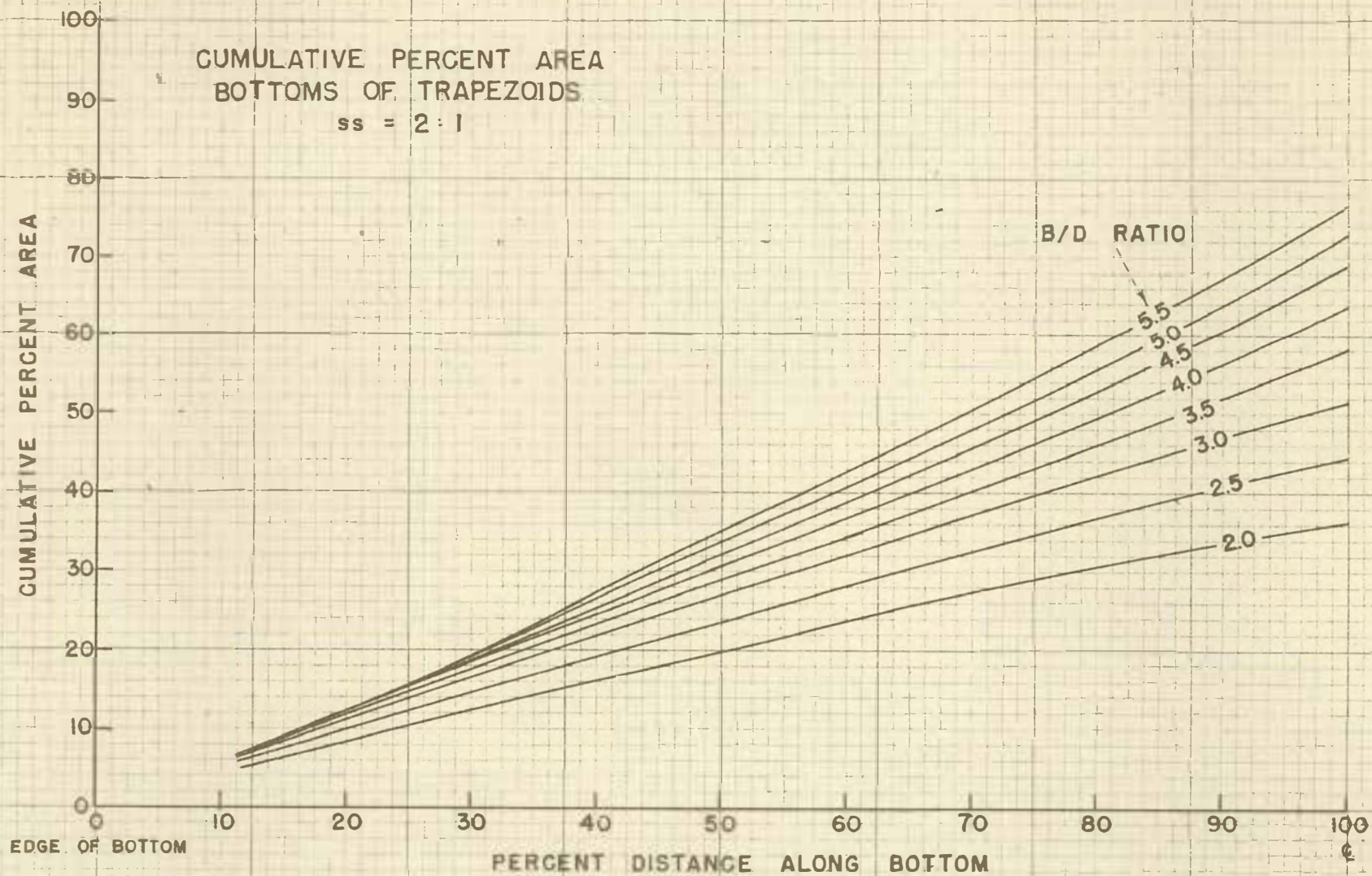


FIGURE 15

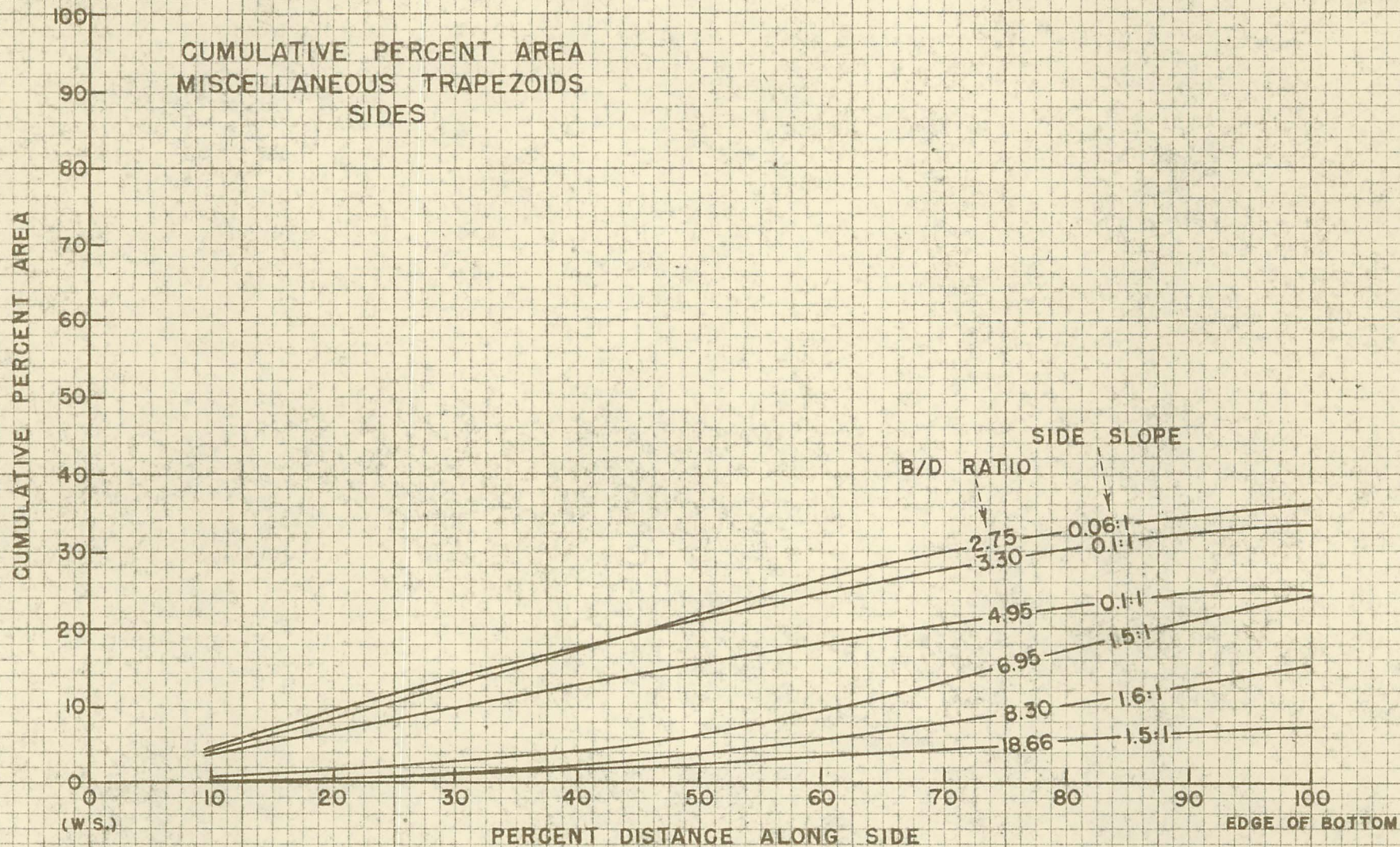


FIGURE 16

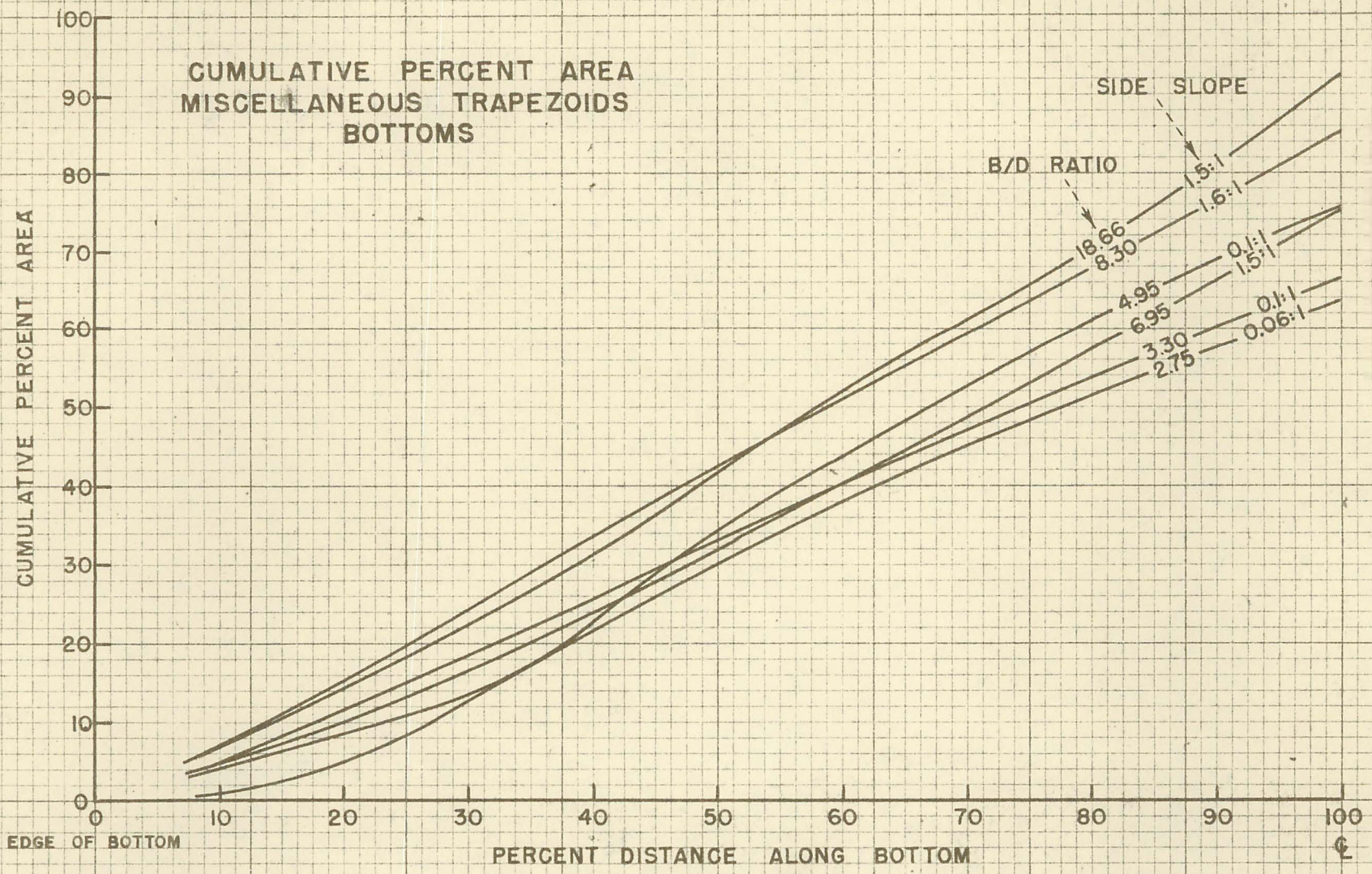


FIGURE 17

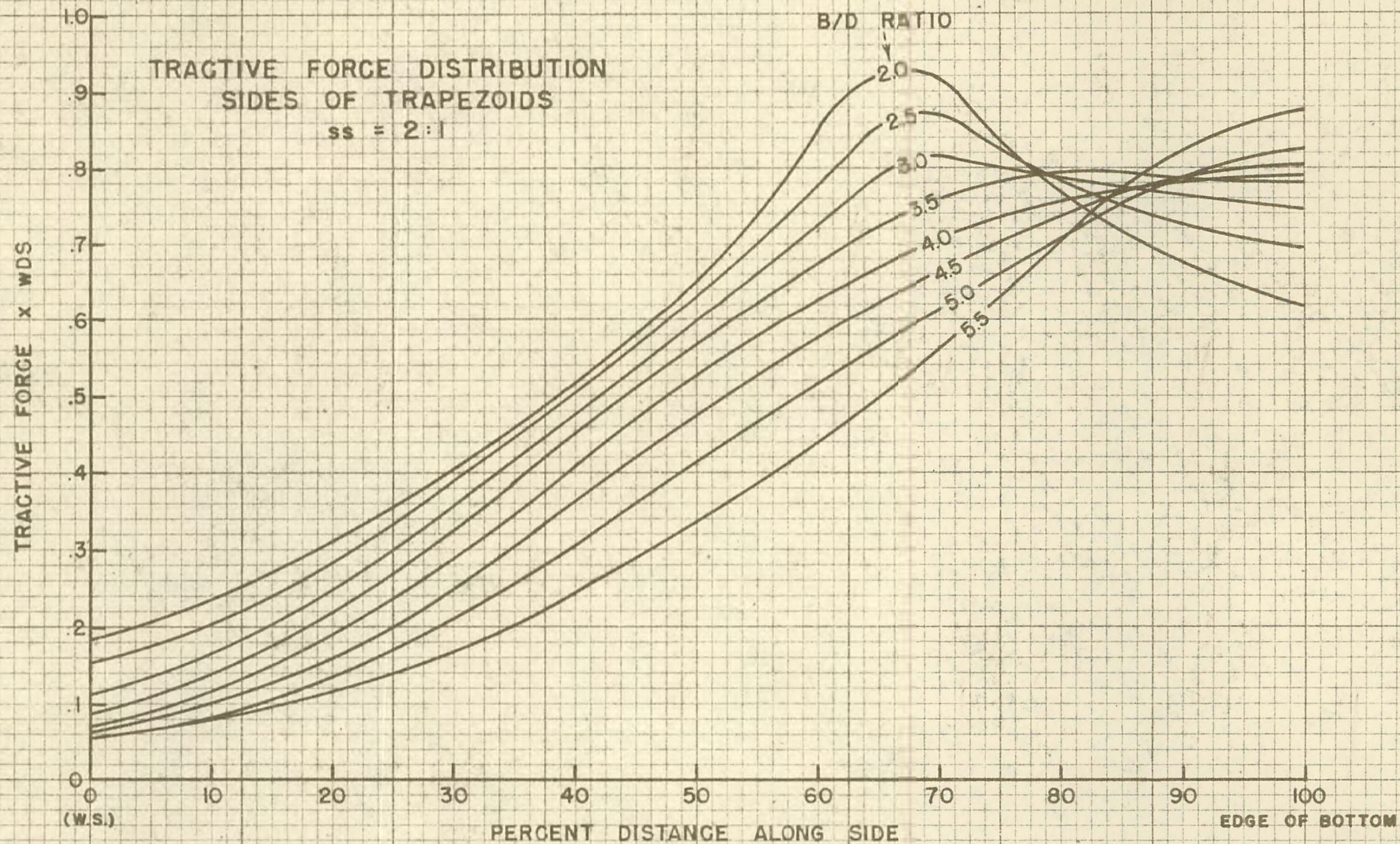


FIGURE 18

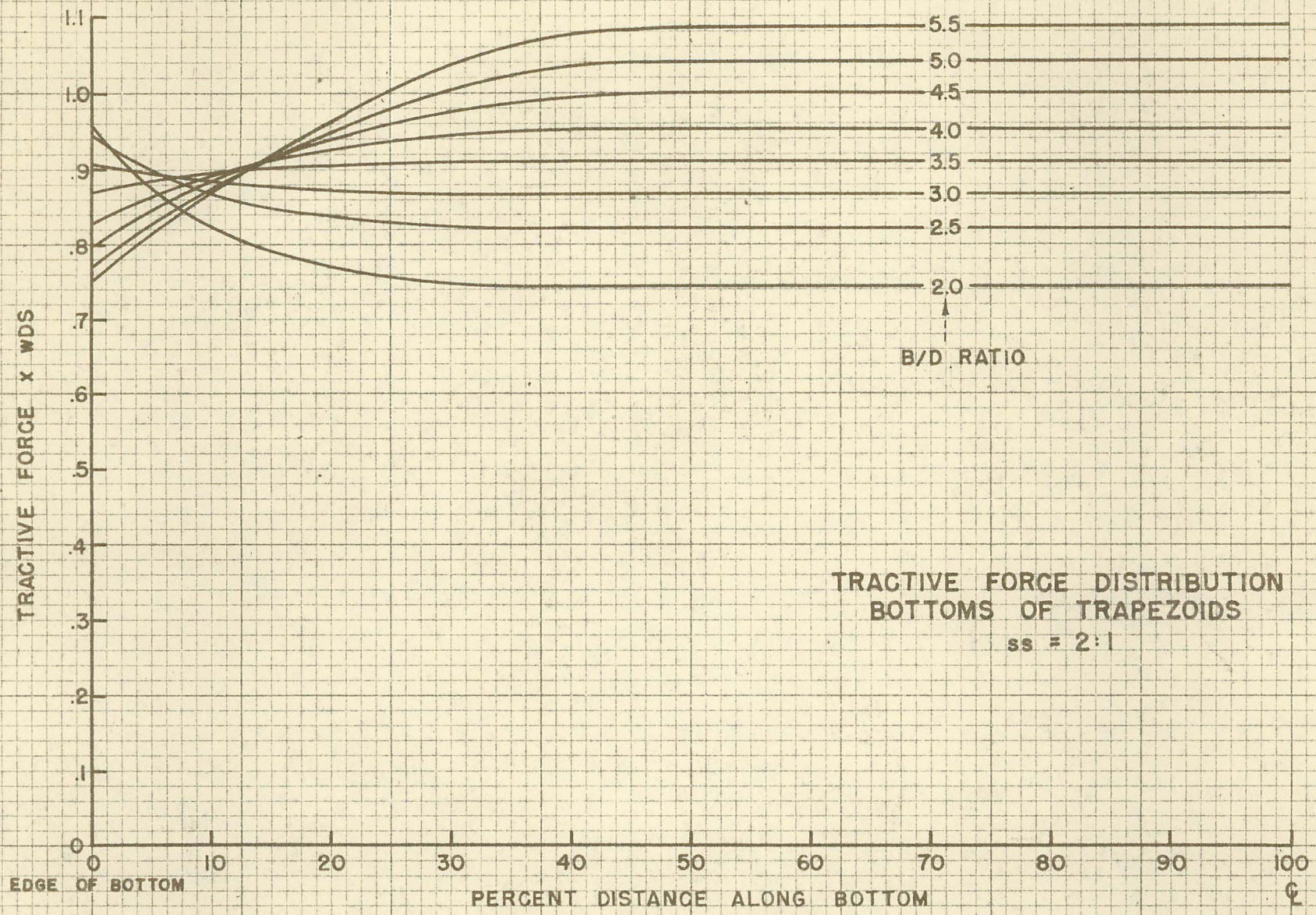
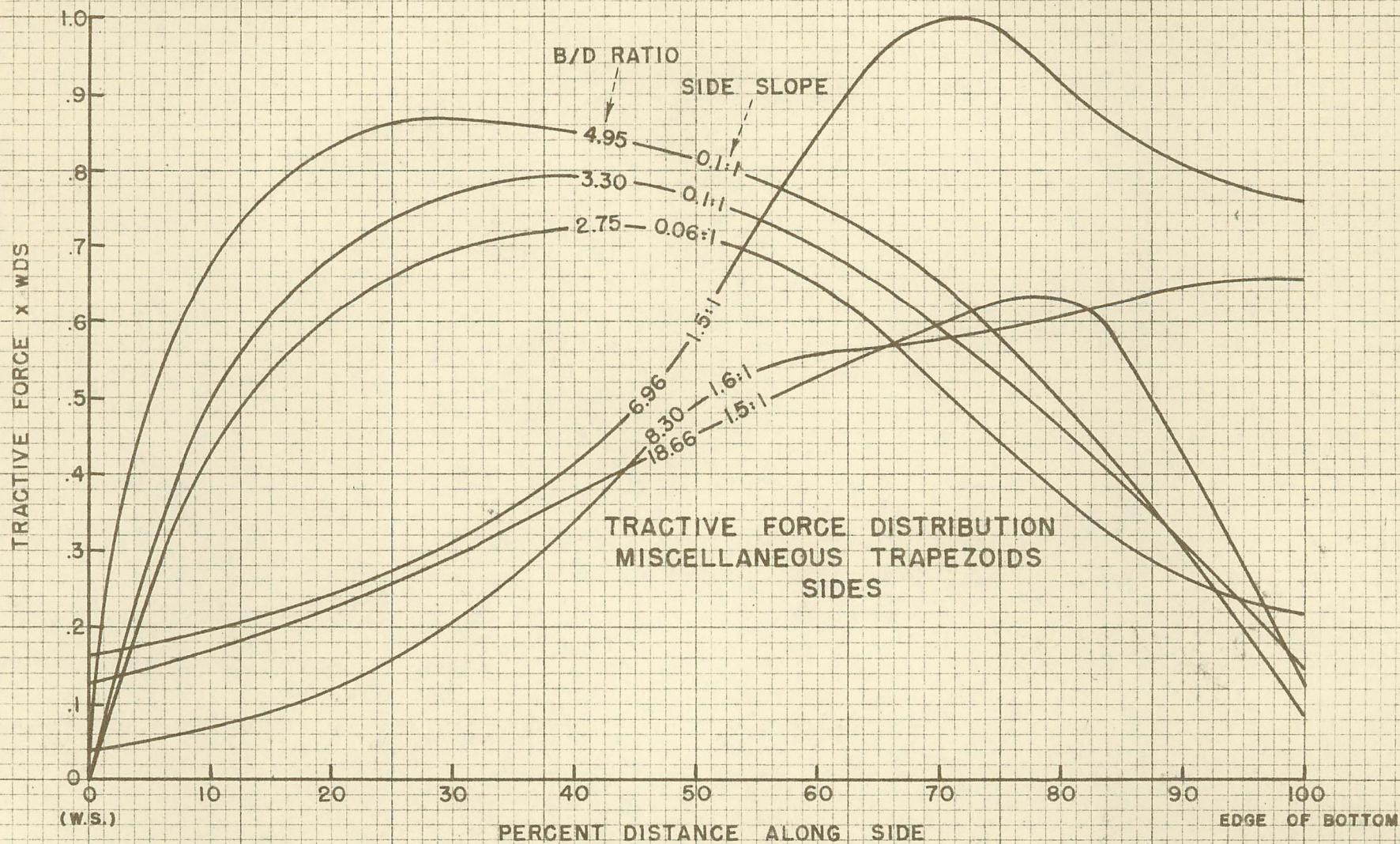
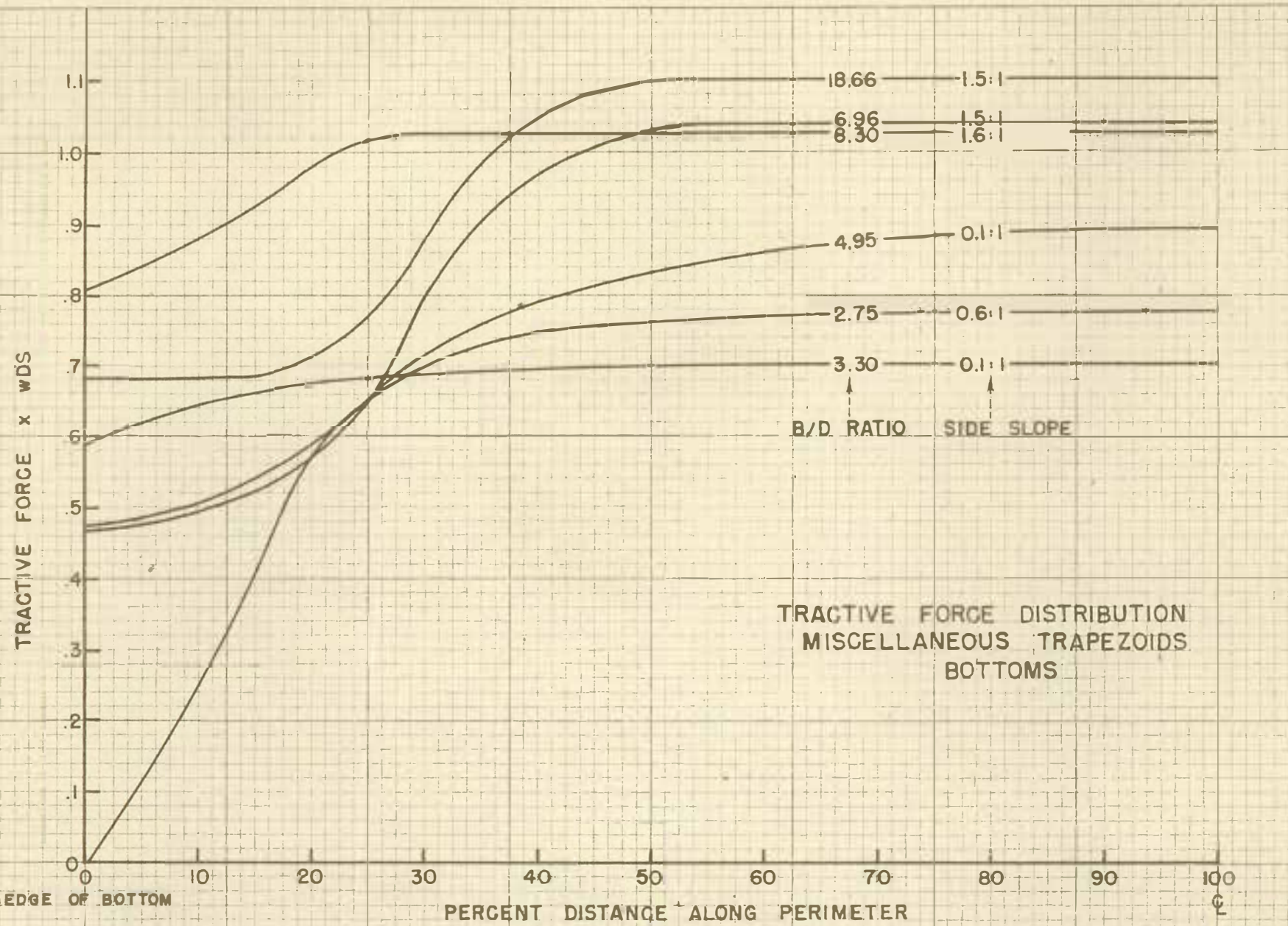


FIGURE 19





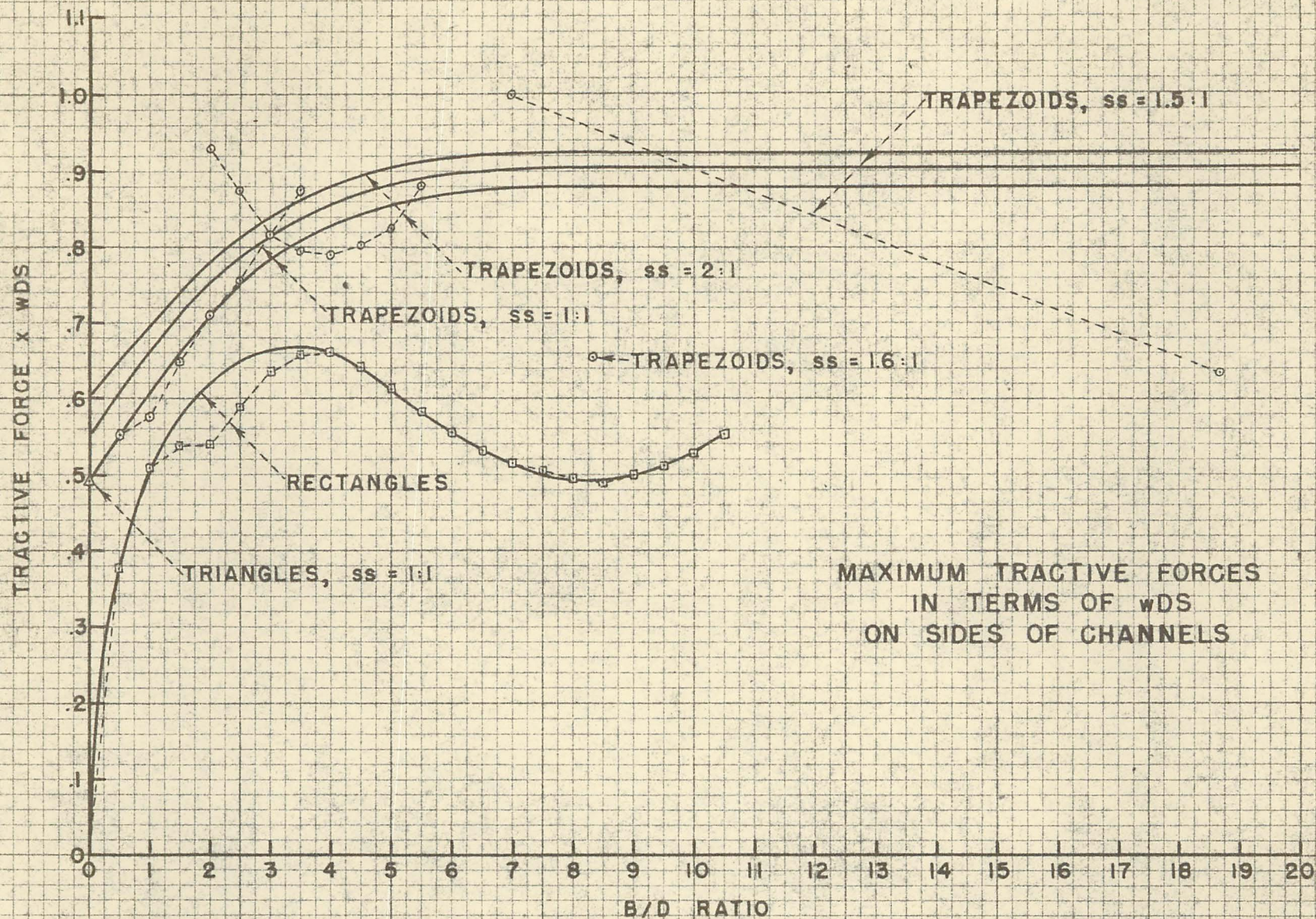
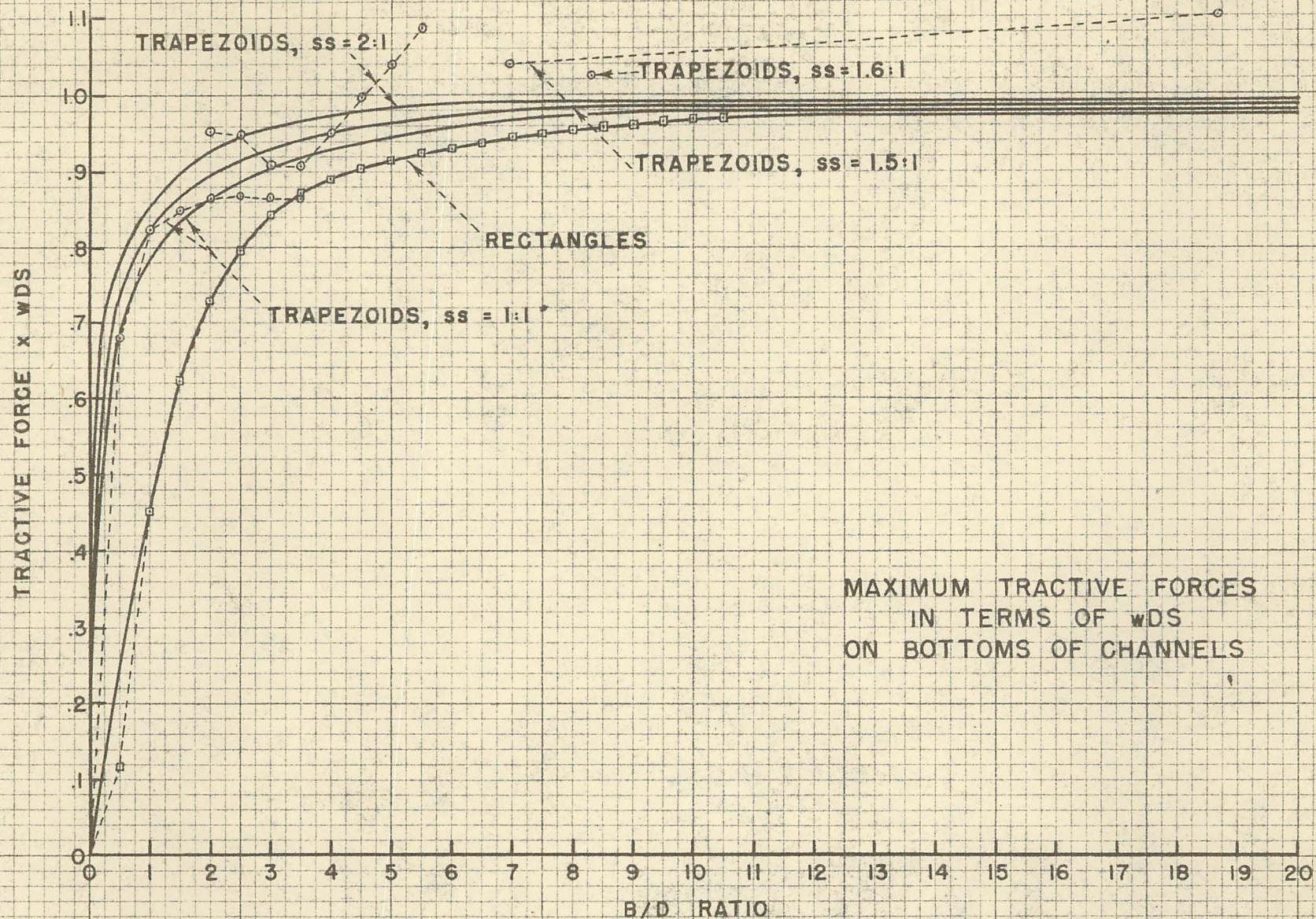


FIGURE 22



MAXIMUM TRACTIVE FORCES
IN TERMS OF wDS
ON BOTTOMS OF CHANNELS

FIGURE 23

