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APPLICATION OF AN ANALOG COMPUTER TO THE HYDRAULIC PROBLEMS OF THE SACRAMENTO-SAN JOAQUIN DELTA IN CALIFORNIA

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The Delta area in California is a roughly triangular tract of land lying just to the east of Suisun Bay. This area, which extends for a distance of about 50 miles north and south and has a maximum width of about 25 miles, was originally a marsh with a network of channels threading through it. At the present time, this area is agricultural land which has been reclaimed by constructing dikes along the old channels to inclose areas which can be pumped out and farmed.

The Delta is traversed by the Sacramento River which enters it from the north, by the San Joaquin which comes into it from the south, and by the North and South Forks of the Mokelumne River which come in from the east. The old network of channels, which has been effectively preserved and stabilized by the process of reclamation, still carries the flow of these streams through the Delta.

Tides coming into San Francisco Bay from the Pacific Ocean propagate themselves through Suisun Bay and into the Delta channels. Since the tidal currents generally exceed the currents due to stream flow, the direction of flow in the channels are periodically reversed and a mechanism is provided for propagation of ocean salinity into them. The salinity encroachment is held in check by stream flow which tends to flush the salinity out of the channels. In times of flood the salinity is driven back but in times of low stream flow the tidal ebb and flow succeeds in carrying some salt into the channels. The presence of salinity is a matter of concern to the farmers of the Delta lands because the ground surface of these reclaimed lands or "islands" is commonly below sea level so that the gradients are such as to carry water from the channels into the islands. Construction of the Shasta Reservoir on the upper Sacramento River has made a water supply available which is desired for use on some of the lands in the San Joaquin Valley across the Delta. To supply this demand the Tracy Pumping Plant will lift water out of the Delta channels at the south end of the Delta and the water to supply these pumps must be brought across the Delta through it channels.

The problem to be solved is then how to bring the Sacramento water across the Delta to the San Joaquin side without upsetting the balance of forces which now holds the salinity in check.

Reasons for Use of an Analog

One of the first methods of attacking the problem was by means of an hydraulic model. The channels of this model were reproduced to a scale of 1:4800 horizontal and 1:100 vertical. A tide generating apparatus and means for introducing stream flow were provided. Provision was also made for extractions to represent diversions for use on the Delta lands. Dyes were introduced to represent salinity. Flow patterns in the Delta were extensively studied with this model both for historic conditions and for the anticipated conditions as altered by pumping. It also provided a means for testing analytical procedures for estimating salinity propagation. After these results were obtained, the model had served its purposes and was dismantled.

With the Tracy pumps in operation it will be necessary to increase the transfer of water from the Sacramento to the San Joaquin channels in order to replenish the water supply of the southern part of the Delta and thereby maintain a proper balance of flow. Studies of the possibilities of artificial channels connecting the Sacramento and Mokelumne channels to increase the transfer were carried out analytically using the Hardy Cross procedure for determining the division of flow and the methods previously established with the aid of the model for estimating salinity intrusion. A tidal phase difference existed at one of the sites which could be utilized to increase the transfer. Since gates would be necessary in any case for protection during floods, it would be possible to open the gates when the tidal currents were favorable and to close them when they were adverse. An analytical approach to this problem based on wave propagation formulas proved to be very difficult, and while some of these computations were actually made, the process proved to be so laborious as to make it desirable to search for some other method of solution.

The electronic analog computer built to expedite these computations not only was successful for this purpose but gave also a more rapid means of studying flow distribution in the Delta and a means for evaluating the effect of tidal currents on the effective flow resistance of the Delta channels. The appearance of the completed analog is shown in Figure 1.

Analog Requirements

In order to solve the Delta problem, it is required that the analog be able to reproduce the square law relation between friction and velocity which is characteristic of fluid flow. In addition, it is required to represent the wave motion associated with the tides. To do this, the factors of inertia and of storage due to water level changes must be accounted for. The factors employed in this analog to represent the hydraulic factors are shown in Table 1.

Table 1

CORRESPONDING HYDRAULIC AND ANALOG QUANTITIES

Hydraulic	Electrical
Quantity of flow	Current
Water surface elevations	Voltage
Inertia	Inductance
Storage	Capacity
Frictional drag	Resistance
Time	Time

Description of the Analog

The analog is designed on the basis of circuits of the type shown in Figure 2.

The inductances are air-cored coils which are either of commercial types or were wound as required. The condensers are commercial units of the paper or mica type. In the large channels, having very low frictional resistances, linear resistors, having appropriate average values for the currents flowing, were used. In some of the smaller channels, however, it was necessary to use some type of square-law resistor. This was obtained by taking advantage of certain vacuum tube characteristics which have approximately the the required form of variation. These were used with resistors in parallel and in series to obtain the desired characteristic. A biasing voltage was also required in this adjustment. The circuit used in such cases is shown in Figure 3.

A tube with two elements is used to permit current to flow in either direction. This type of resistor is not wholly satisfactory since the tubes show differences which make it necessary to adjust each one separately. The current carrying capacity is restricted within narrow limits, and it is necessary, therefore, to design the analog around these elements. Net current flows were read on d-c milliammeters. Tidal amplitudes and phase differences are read on a cathode-ray oscilloscope. The gate keeper was represented by a rectifier circuit using a 6N7 type tube. This also had some short comings near the zero point which introduced an effect analogous to gate leakage. In spite of these minor difficulties, the analog operates in a very satisfactory manner. Some idea of the speed with which the analog works may be obtained from the fact that the analog runs through about 500 days of actual tidal changes in each second of operating time.

Basic Equations

In setting up the correlation equations, the electrical circuits were assumed to have their inductance and capacity uniformly distributed along their length. In practice, these elements and the square law resistance were lumped. The inertia and storage factors were considered together, and the resistances were considered separately. For purposes of explanation, the following notation will be used:

In the hydraulic channel let:

g represent the acceleration of gravity
H the depth of the stream
L the length of a channel
M a constant applying to a channel specifying its flow resistance
Q the flow
t time
W the width of the stream at the surface
x distance along a stream
y the surface elevation above sea level
ρ weight of water per unit of volume

A longitudinal section of a stream channel is shown in Figure 4. The shaded element represents a lamina of width W, depth H, and length dx. For analytical purposes the actual channel is assimilated to a uniform rectangular channel which has the same top width and cross sectional area as the actual channel. As stated previously, frictional forces are not introduced into the dynamical equations, but are treated separately. Since x represents a distance measured along the stream from some fixed point on the bank, the planes defined by x and $x + dx$ do not change position with time. It is assumed that y is small compared to H.

The continuity condition requires that if the quantities of water flowing through the planes x and $x + dx$ differ, then the surface elevation must rise or fall as required to accommodate the changes of volume. If small quantities are neglected, this requirement is expressed by

$$W dx \frac{\partial y}{\partial t} \doteq +Q - \left(Q + \frac{\partial Q}{\partial x} dx\right)$$

If a surface gradient $\frac{\partial y}{\partial x}$ is present, the water depth on one side of the lamina will be greater than on the other by the amount $\frac{\partial y}{\partial x} dx$ and the additional pressure due to this head differential will cause the water within the lamina to be accelerated. Thus the requirements of Newton's law are expressed to a first order of approximation by:

$$\frac{\rho WH}{g} dx \frac{\partial}{\partial t} \left(\frac{Q}{WH}\right) = -\rho WH \frac{\partial y}{\partial x} dx$$

These two equations can be simplified by cancelling common terms and collecting. Then the equation of continuity is

$$\frac{\partial Q}{\partial x} + \frac{W \partial Y}{\partial t} = 0 \qquad (1)$$

and Newton's law takes the form

$$\frac{\partial y}{\partial x} + \frac{1}{gHW} \frac{\partial Q}{\partial t} = 0 \qquad (2)$$

It is of interest to note that if Q is eliminated from the two equations above, one obtains the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{gH} \frac{\partial^2 y}{\partial t^2} \qquad (3)$$

The relation between flow and gradient for the hydraulic channel can be expressed in the form

$$Q = M\sqrt{\frac{\partial y}{\partial x}} \qquad (4)$$

which may be recognized as a form of the Chezy formula.

In the electrical circuits let

- C represent the capacity per unit length of circuit
- E the potential with respect to ground
- I current
- K a constant applying to a circuit
- r resistance per unit length of circuit
- η time in the analog
- λ inductance per unit length of circuit
- ξ distance along a circuit

Then the equations for the idealized electrical circuits* which correspond to equations (1) and (2) for the hydraulic channels are:

$$\frac{\partial I}{\partial \xi} + C \frac{\partial E}{\partial \eta} = 0 \qquad (5)$$

$$\frac{\partial E}{\partial \xi} + \lambda \frac{\partial I}{\partial \eta} = 0 \qquad (6)$$

from which there is obtained on elimination of I

$$\frac{\partial^2 E}{\partial \xi^2} = \lambda C \frac{\partial^2 E}{\partial \eta^2} \qquad (7)$$

For the circuits provided with an electronic resistor for representation of hydraulic resistances of the type expressed by equation (4)

$$I = K \sqrt{\frac{\partial E}{\partial \xi}} \qquad (8)$$

or if the circuit has a linear resistance

$$I = \frac{1}{r} \frac{\partial E}{\partial \xi} \qquad (9)$$

Correlation Equations

The electronic analog operates at a frequency of 1,000 cycles per second. The sinusoidal variations imposed on the analog approximately represent tidal oscillations having a frequency of

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*See, for example, "The Theory of Sound" by Lord Rayleigh, Volume 1, Paragraph 235x, page 467. The equations (5) and (6) can be obtained from Rayleigh's equation 1 by letting R = 0,K = 0. In this form they are a simplified version of Heaviside's equations for a long line.

about two cycles per day. The correlation equations which were found suitable for use with the available electrical components are:

$$
\begin{aligned}
y &= 0.1\ E \\
Q &= 10{,}000{,}000\ I \\
x &= 10{,}000\,\xi \\
t &= 45{,}000{,}000\,\eta
\end{aligned}
$$

Other applications would, of course, require other constants. An analogous electrical quantity is obtained by substituting the above relations into the hydraulic equations: for example, equation (4) is

$$Q = M\sqrt{\frac{\partial y}{\partial x}}$$

which on substitution becomes

$$10{,}000{,}000\ I = M\sqrt{\frac{0.1\ \partial E}{10{,}000\,\partial \xi}}$$

or

$$I = \frac{M}{3.2 \times 10^9}\sqrt{\frac{\partial E}{\partial \xi}}$$

Then the quantity $\frac{M}{3.2\times10^9}$ is the K value to use in the equation (8)

$$I = K\sqrt{\frac{\partial E}{\partial \xi}}$$

By this choice of constants the electrical circuit is given resistance characteristics which are analogous to the hydraulic friction in the corresponding actual channel. The other relations are treated in a similar way.

Boundary Conditions

To account for the stream flow it was necessary to introduce direct currents of specified amounts at certain points in the analog and to take them out at certain other points. In general the currents fed into the network represent river flows entering the Delta area, while currents leaving the network represent the draft of the Tracy pumps and the flow from the Delta area into Suisun Bay. To introduce these currents, voltages of controllable magnitude were introduced between the network and the ground wire (see Figure 2). Control of the currents was obtained by variable resistors located at the points where the currents enter and leave the network.

The tides were represented by alternating voltages of specified magnitude applied between the network and the ground wire at the point on the analog representing the entrance to Suisun Bay. A blocking condenser was used here to prevent the flow of direct current. The actual tides occurring at this point vary somewhat from day to day due to varying phase relations between the lunar and solar components. In the analog these tidal variations were replaced by a single sinusoidal variation of average amplitude. The connections arranged for introducing the direct currents representing stream flow would permit the alternating currents representing the tides to pass into the ground wire at other points than that representing the entrance to Suisun Bay. Since this would introduce errors inductive blocking impedences were introduced into the direct current circuit wherever necessary to confine the alternating currents to the proper network circuits. Where stream channels continued beyond the area represented by the analog, lumped impedences were introduced to represent those portions beyond the analog area. In most cases these were determined by trial so that known tidal behavior would be properly represented.

In order to protect the direct current meters where necessary from loss of field due to the alternating-current components, they were shunted by condensers having impedences which were low compared to the resistance of the meter.

Results

The analog has assisted materially in the solution of the problem of flow transfer through the Delta. The results obtained check well with those obtained by other means.

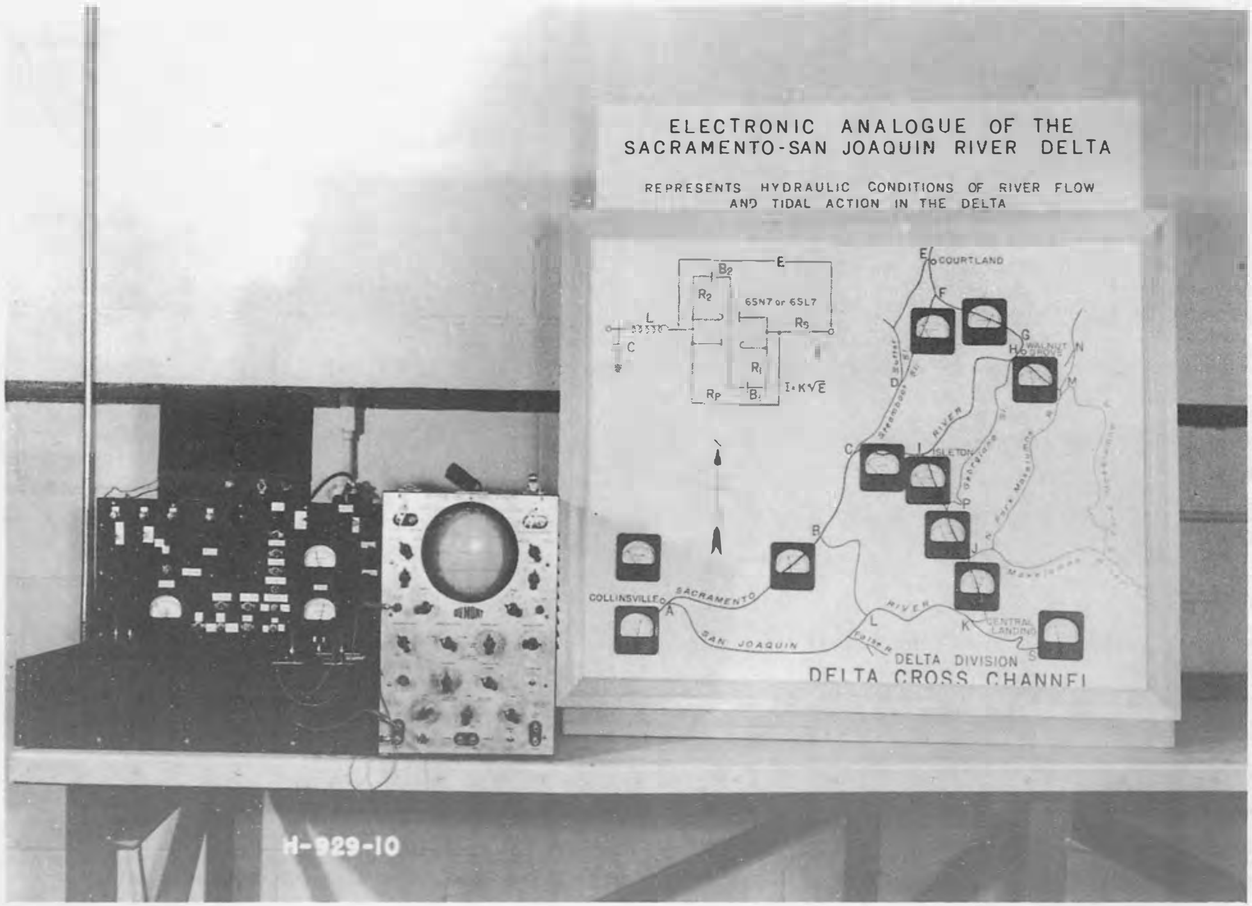

Figure 1. External appearance of the Electronic Analog.

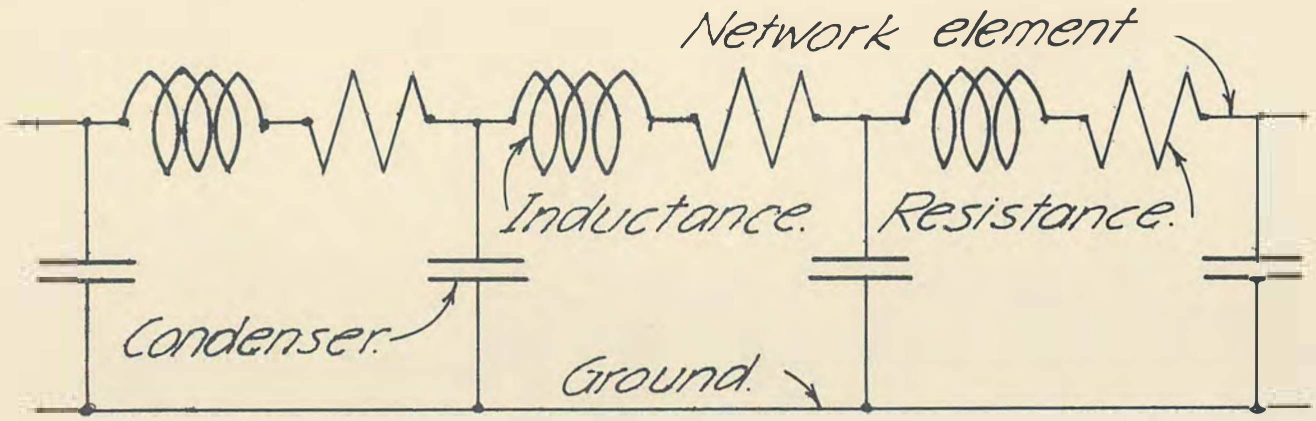

Fig 2. Basic Analog Circuit.

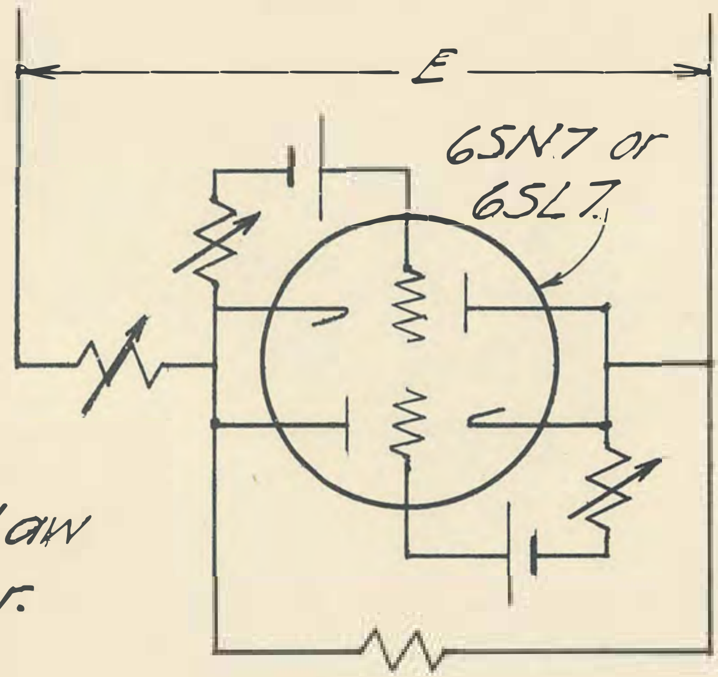

Fig 3.
Square Law
Resistor.

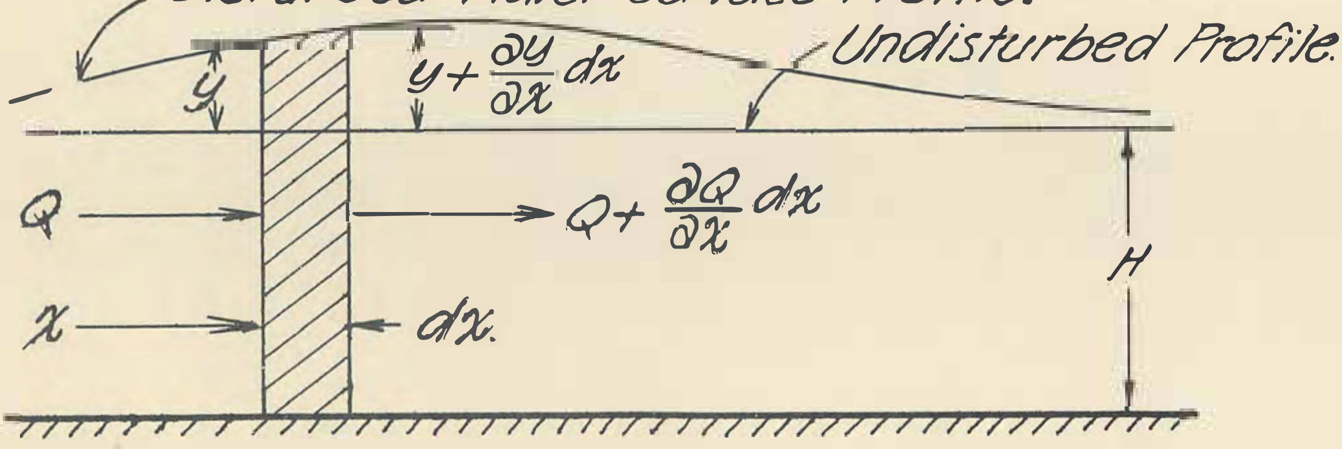

Fig 4. Longitudinal Section of a Channel.